Endogenous TFP and Cross-Country Income Differences*

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Abstract

This paper explores the quantitative implications of a class of endogenous growth models for cross-country income differences. These models exhibit international spillovers, no scale effects and conditional convergence, and thus they overcome some difficulties faced by the early generation of endogenous growth models. Cross-country income differences arise in the model as the result of different distortions in the accumulation of rival factors of production, the objects, and in the accumulation of nonrival factor of production, the ideas. We show that object gaps play a much larger role to explain income gaps in models with endogenous TFP than in models with exogenous TFP. We also show, using a carefully calibrated version of the model, that most of the cross-country differences in output per worker are explained by barriers to the accumulation of rival factors (physical and human capital) rather than by barriers to the accumulation of knowledge.

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1 Introduction

What does growth theory tell us about the causes of cross-country income differences? Seminal papers by Parente and Prescott (1994), Klenow and Rodríguez-Clare (1997), and Hall and Jones (1999) have shown that differences in total factor productivity, or TFP, are key for understanding income differences, and Parente (1998) has called for a theory of TFP. Growth theories seek to understand the endogenous evolution of TFP over time, and therefore it is natural that they offer some explanation for the observed dispersion of TFPs across countries. However, endogenous growth models, as developed by Romer (1986, 1990), Lucas (1988), and Rebelo (1991) among others, have been criticized for their seemingly implausible predictions regarding scale effects and divergence, as well as other issues (see Easterly *et al.*, 1993; Jones, 1995; and Howitt, 2000).

To overcome some of the problems, a second generation of growth models has recently been proposed (Parente and Prescott, 1994; Eaton and Kortum, 1996; Howitt, 2000; and Klenow & Rodríguez-Clare, 2004, among others). These models allow for international diffusion of knowledge, and eliminate scale effects both in growth rates and in levels. Countries in these models share a common long-term growth rate, at least for the subset of growing countries, and their dynamics display conditional convergence. Long-term growth rates are determined worldwide, while country policies and specific conditions affect relative income levels. This second generation of growth models provides the needed theory of relative total factor productivities.

This paper explores the theoretical and quantitative implications of this second generation of growth models for cross-country income differences. A key insight of endogenous growth models is that 'income gaps' are caused by 'object gaps' and 'idea gaps' (Romer, 1993). More precisely, these models regard long-term income differences as the result of two types of distortions or frictions: distortions associated with the accumulation of rival factors of production, the objects, such as distortional taxes on capital and labor, as well as risks of expropriation, confiscation, thievery, squatting, extortion, kidnapping, etc.; and distortions associated with the accumulation of nonrival factors of production, the ideas, such as taxes on innovation and adoption activities, costly patent application and patent protection, limited intellectual property rights, and overall, risk of imitation and copying.

The main issue we address in this paper is the relative importance of each type of friction in explaining cross-country income differences. Our assessment resembles the one carried out by

Acemoglu and Johnson (2005) in “Unbundling Institutions”, but our decomposition is based on a specific theory of endogenous TFP and, therefore, it is fully microfounded. The fundamental frictions in our model are two rates of expropriation, one affecting physical capital, and the other affecting ideas via limited patent protection.

The main finding of the paper is that, according to growth theory, income differences are primarily explained by distortions to the accumulation of rival factors rather than distortions in the accumulation of ideas. This finding is surprising in light of the results of Klenow and Rodríguez-Clare (1997) –KR (1997) hereafter,– and Hall and Jones (1999) –HJ henceforth– who, using models of exogenous TFP, suggest instead that income differences are primarily explained by barriers to the accumulation of knowledge. We show analytically that frictions to the accumulation of rival factors are magnified when TFP is endogenous, and show numerically, using a carefully calibrated version of the model, that this amplification is large. In fact, our quantitative findings are closer to those of Mankiw, Romer and Weil (1992) -MRW hereafter- than to KR (1997) or HJ, but the mechanism and policy implications are completely different.

Our model builds on Howitt (2000), and particularly on Klenow and Rodríguez-Clare (2004) –KR (2004) henceforth. They extend a quality-ladder model of growth to include international diffusion of knowledge, and eliminate scale effects at the country level. A key component of these models is the “catch-up” externality, one that captures the idea that lagging behind the world technology frontier facilitates technological progress via adoption. This externality determines the speed of technological diffusion.

We differ from the two papers above in that we use Romer’s (1990) and Barro and Sala-i-Martin’s (1997) variety approach instead of the Shumpetarian approach. This alternative formulation is analytically more tractable and allows to obtain closed-form results that are easy to compare to related findings in the literature. For example, we show that the standard model of exogenous TFP is a particular case of our model when the speed of diffusion is infinite. We also show analytically that distortions to the accumulation of rival factors are amplified as the speed of diffusion decreases. Furthermore, our closed-form solutions allow us to perform *exact* variance decompositions analogous to those of KR (1997), and therefore we can compare our results directly to theirs.[1] Finally, we

[1] The steady-state solution for output in existing quality-ladder models has an additive structure, while it has a multiplicative structure in models with expanding varieties. This facilitates variance decomposition as well as other results.

document that this added tractability of the varieties model comes at no major cost because the quantitative results are similar to those of quality-ladder models.

For the quantitative assessment, we calibrate a version of our model following Howitt's (2000) suggestion of using the speed of convergence as a matching target. For this purpose, we estimate the growth regression equation implied by our model and find a slow speed of convergence, similar to other estimates in the literature. We further show that the model can only produce a slow speed of convergence if the ratio of increasing returns to speed of diffusion is large. This ratio is also the magnitude of amplification mentioned above. Therefore, the slow speed of convergence is what ultimately explains the main finding of the paper that distortions to the accumulation of rival factors of production are the prime determinant of cross-country income differences.

Our quantitative findings differ from those of Parente and Prescott (1994) and KR (2004) who stress the key role of barriers to the adoption of technology and the accumulation of knowledge. Although the models are different in important respects, for example increasing returns and externalities are central in our model while Parente and Prescott model only adoption decisions in a constant returns to scale framework, we think that the key difference is in the calibration procedure. As mentioned, we calibrate our model to match a slow speed of convergence similar to what has been documented in the growth regression literature. Parente and Prescott instead calibrate their model to Japanese data, a country that experienced a high speed of convergence. KR (2004) calibrate their model to match social returns to R&D. We show that their calibration also implies a high speed of convergence. We consider our results robust as they are based on well documented cross-country evidence.

Lucas (1988) asks: "Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's?" Recent growth theories provide an answer to this question. The long-term growth rate of India is likely tied to worldwide growth rate, but India's income level is tied to India's distortions. According to our calculations, India's barriers to the accumulation of factors explain up to 73% of the income gap relative to the US, and the remaining 27% is explained by distortions to the accumulation of knowledge. These figures suggest that an Indian government with the will and power to reduce the income gap must focus on eliminating barriers to the accumulation of rival factors rather than nonrival ones.

The paper is organized as follows. Section 2 reproduces benchmark results obtained in a framework of exogenous TFP using our database for 1996. Section 3 presents the main results of the paper using an extended Solow model with endogenous TFP, but exogenous saving rates and R&D investment rates. The section summarizes analytical results, estimation of growth regressions, calibration of the model, and it also reports the main quantitative findings. Section 4 endogenizes savings rates and R&D investment rates using a version of Romer's (1990) and Barro and Sala-i-Martin's (1997) variety model. We show that the steady of this model maps exactly into the Solow model of Section 3 but provides additional equations to determine savings rates and R&D investment rates. The two fundamental frictions in the microfounded model are an expropriation rate of physical assets, and an expropriation rate of ideas determined by the random duration of patents. We show that the results of Section 3 are reinforced when saving and R&D rates are endogenous. Section 5 concludes.

2 Exogenous TFP Models

The neoclassical growth model has been the workhorse of most existing attempts to quantify the sources of cross-country levels of output per worker. Prominent examples of these attempts have arrived at opposite conclusions. On the one hand MRW found that 78% of the world income variance could be explained by differences in human capital and saving rates. On the other hand, KR (1997), and HJ found that productivity differences are the dominant source of dispersion of output per worker, accounting for around 60% of the variance. The key reason why conclusions differ in these studies can be traced back to the measurement of human capital. While MRW uses only secondary schooling, KR (1997) in addition uses primary and terciary schooling, as well as experience and schooling quality.

In spite of their differences, these studies use a common framework; namely, the Solow model augmented with human capital in which total factor productivity evolves exogenously. More specifically, consider the following aggregate Cobb-Douglas production function

$$Y_t = K_t^{\alpha} \left(A_t H_t\right)^{1-\alpha} \equiv Z_t K_t^{\alpha} H_t^{1-\alpha},$$

where K is aggregate physical capital, H is aggregate human capital, A represents labor augmenting technological progress, Y aggregate output, and $Z \equiv A^{1-\alpha}$ is total factor productivity (TFP). In what follows, we loosely use the term TFP to refer both to A and Z. Aggregate human capital is defined as $H_t \equiv hL_t$, where L is the labor force, and h human capital per worker. Output per worker $y = Y/L$ is then given by

$$y_t = Z_t k_t^{\alpha} h^{1-\alpha}.$$

If TFP is exogenous and k endogenous, differences in k across countries reflect differences in Z_t. To account for this dependence MRW, KR (1997), and HJ rewrite output per worker, or 'income' for short, as

$$y_t = A_t \cdot X_t, \qquad (1)$$

where $X_t \equiv \kappa^{\frac{\alpha}{1-\alpha}} h$ and $\kappa \equiv K/Y$. The term $A_t = Z_t^{1+\frac{\alpha}{1-\alpha}}$ captures both the direct effect of TFP in output per worker, Z_t, and the indirect effect through capital, $Z_t^{\alpha/(1-\alpha)}$, while term X captures "factor intensities". If TFP is assumed exogenous, then equation (1) is appropriate to study the sources of cross-country variations in y because X is determined by parameters different from the productivity level A.[2] Moreover, studies typically assume $\alpha = 1/3$ so that the exponent $1 + \alpha/(1 - \alpha) = 1.5$ significantly enhances the role of TFP, and reduces the role of factors, in explaining income differences.

KR (1997) uses equation (1) to perform a variance decomposition exercise to assess the contributions of X and A to world income dispersion. An issue there is how to handle the covariance between A and X, a term that accounts for 35% of income dispersion, and for which exogenous theories of TFP have no predictions. As KR (1997) acknowledges, this large covariance suggests that the productivity level A is actually endogenous. In order to account for this possibility, they assign half of the covariance term as part of the contribution to X and the other half to A. They define the contributions of factors Φ_X and productivity Φ_A as

$$\Phi_X = \frac{var\left(\ln X\right) + cov\left(\ln A, \ln X\right)}{var\left(\ln y\right)}, \Phi_A = \frac{var\left(\ln A\right) + cov\left(\ln A, \ln X\right)}{var\left(\ln y\right)}. \qquad (2)$$

Using a database of 91 countries for 1996[3] and assuming $\alpha = 1/3$, we obtain $\Phi_X = 40\%$ and $\Phi_A = 60\%$, which is similar to the result reported by KR (1997) for 1985.

[2] In the neoclassical growth model the steady state capital-output ratio is determined by preferences, α and the exogenous *growth* rate of TFP (see Barro and Sala-I-Martin, 2003).

[3] We use information from the Penn World Tables Mark 6.1 and the Barro and Lee data set for schooling. We only include countries with information on investment rates between 1960-1996, and construct capital stocks using the perpetual inventory method. The construction of human capital series is described below in Section 3.4.1.

HJ also uses equation (1) for levels accounting purposes. Rather than using the variance as KR (1997) do, they decompose output per worker in each country into the three multiplicative terms in equation (1): the contribution of physical capital intensity $\kappa^{\alpha/(1-\alpha)}$, the contribution of human capital per worker h, and the contribution of TFP. Table 1 updates HJ's results using our 1996 data set and bundling together the contribution of factors under X. All numbers in Table 1 are levels relative to the US in 1996. Similar to HJ, Table 1 shows that as one moves from richer to poorer countries down the table, X becomes gradually lower than in the US (for instance, in Kenya it is 29% of US' level), but A becomes proportionally much lower (it is around 16% of US' level in Kenya). On average, the five poorest countries in our sample (Mali, Malawi, Rwanda, Niger and Zaire) have 23% of US' factor intensity as measured by X, and only 12% of US' productivity level. Notice that the five poorest countries have just 2.6% of US' output per worker; i.e., output in the US is around 39 times higher. Table 1 implies that this 39-fold difference can be decomposed as the multiplication of a 4.55-fold difference in factors X, and a 8.55-fold difference in A. This result is consistent with the variance-decomposition exercise of KR (1997) reported above, and with the advantage that it does not have to deal with the distribution of the covariance term.

In any case, both levels accounting exercises reported above are based on equation (1), which assumes TFP to be exogenous. In the remainder of the paper we relax this assumption. Part of the motivation to relax this assumption is to better understand the covariance between X and A.

3 An Extended Solow Model with Endogenous TFP

3.1 The Model

Consider the following Solow model extended to incorporate endogenous accumulation of TFP along the lines of Jones (1995), and KR (2004), among others. Output in the economy is given by

$$Y_t = K_t^{\alpha} \left(A_t^{\beta} H_t \right)^{1-\alpha}, \tag{3}$$

where $H_t = hL_t$, h is exogenous, and A_t represents productivity. Parameter β determines the degree of increasing returns in the production of final goods. KR (2004), for example, assumes $\beta = 1$. Output can be consumed, invested in physical capital stock, or allocated to research and development (R&D)

$$Y_t = C_t + I_t + R_t. \tag{4}$$

In this section we treat both the saving rate s and the fraction of R&D expenditures on total output s_R as exogenous and constant, but potentially different across countries. We relax these assumptions in the next section by providing microfoundations in a decentralized model. The aggregates stocks of capital and human capital evolve according to:

$$\dot{K}_t = sY_t - \delta K_t, \tag{5}$$

$$\dot{H}_t = g_L H_t. \tag{6}$$

where g_L is the growth rate of population. Moreover, TFP evolves according to

$$\dot{A}_t = B_t R_t / L_t = B_t s_R y_t. \tag{7}$$

where B_t is the productivity of per-capita R&D expenditures, and $y_t \equiv Y_t/L_t$. This formulation states that technological progress depends on per-capita R&D expenditures rather than the total amount of R&D. This restriction is required to eliminate scale effects in levels, as discussed by KR (2004). This type of scale effects occur when the level of per-capita variables depend on the size of the population. They are hard to justify because large countries like China or India do not have particularly large levels of income per-capita. Moreover, we postulate the following functional form for the productivity parameter B_t:

$$B_t = d\left(\frac{A_t^*}{A_t}\right)^{\eta} A_t^{\nu}, \text{ with } \eta > 0, \tag{8}$$

where d is a parameter and A_t^* is the technology frontier, which could be country specific but it is assumed to be exogenous to the country. Equation (8) allows two types of externalities to TFP accumulation: a "catch-up" externality, $(A_t^*/A_t)^{\eta}$, and the standard research externality A_t^{ν}. The term $(A_t^*/A_t)^{\eta}$ captures the idea that lagging behind the technology frontier facilitates technological progress via adoption of existing ideas. This effect is often called "benefits to backwardness" because a more backward country would have a higher catch-up term. Parameter $\eta > 0$ captures the strength of this catching-up externality, and determines the speed of technological diffusion. The term A_t^{ν} allows for positive or negative externalities in domestic R&D activities. The endogenous growth literature, e.g. Jones (1995), typically assumes positive externalities so that $\nu > 0$.

Equations (7) and (8) imply that technological progress is always costly. It occurs only if some resources are diverted to technological advancement. This view is supported by Keller (2004) and Lederman and Maloney (2003), who argue that there is no indication that technology diffusion is inevitable or automatic, but rather, domestic investments are needed. It is instructive to compare our formulation to that of KR (2004). In their model, the law of motion for TFP satisfies

$$\dot{A}_t = \left(BR_t/L_t + \epsilon\right)\left(1 - A_t/A_t^*\right).$$

This formulation allows free technological diffusion governed by the parameter ϵ. It turns out that $\epsilon > 0$ is required in their model to guarantee the existence of a steady state with positive growth for countries with very small s_R. This is due to the fact that the productivity of per-capita R&D investment is bounded by the constant B, a bound obtained when $A_t/A_t^* = 0$. In contrast, in our formulation the productivity of R&D investment B_t goes to infinity as A_t/A_t^* goes to zero. This guarantees that countries with small s_R do not fall permanently behind and their steady state is well defined without the need of free diffusion. Moreover, parameter η allows us to replicate results obtained under free diffusion. For example, we show below that the standard Solow model with exogenous TFP is obtained in our framework by making $\eta = \infty$. The multiplicative formulation (8) is more convenient for analytical purposes, particularly regarding variance decomposition exercises as will be explained below. Defining $\phi \equiv \nu - \eta$, equation (8) can be written as

$$\dot{A}_t = dA_t^{*\eta} A_t^{\phi} s_R y_t. \tag{9}$$

We close the model by assuming an exogenous evolution of the technological frontier,

$$\dot{A}_t^* = gA_t^*. \tag{10}$$

3.2 Balanced Growth Characterization

Using (3), output per worker y_t can be written as

$$y_t = A_t^{\beta} X_t. \tag{11}$$

where $X_t \equiv \kappa^{\frac{\alpha}{1-\alpha}} h$ and $\kappa \equiv K/Y$. Equation (11) is similar to the one employed by KR (1997) and HJ, as given by (1), with the important difference that A_t is now endogenous. For this reason, this expression is not adequate anymore for variance decomposition because output is not written in terms of fundamentals. To obtain a proper expression, define the growth rate of a variable V as $g_{Vt} \equiv \dot{V}_t / V_t$. Then, use equation (3) and the definition of X to rewrite equation (9) as

$$g_{At} = dA_t^{*\eta} A_t^{\phi+\beta-1} s_R \left(\kappa_t^{\frac{\alpha}{1-\alpha}} h \right). \tag{12}$$

Furthermore, equation (5) implies that the capital-output ratio is given by

$$\kappa_t = \frac{s}{g_{Kt} + \delta}. \tag{13}$$

We can now determine the balanced growth rate of the economy. Along a balanced growth path (BGP) all variables grow at constant rates. Therefore, from equation (13) and the definition of X, κ_t and X_t are constant along such path. Moreover, from equation (12) it follows that $dg_{A_t}/dt = g_A \left[\eta g + (\phi + \beta - 1) g_A \right] = 0$. For this equality to hold, either $g_A = 0$ or $\eta g + (\phi + \beta - 1) g_A = 0$. Focusing on the relevant case $g_A > 0$, one finds that

$$g_A = \frac{1 - \phi - \beta}{\eta} g. \tag{14}$$

Thus, technological progress and economic growth in this economy is tied to the evolution of the technological frontier, and all countries grow at the same rate in the long run. It is natural to restrict parameter values so that countries do not fall behind or move ahead of the technological frontier. This requires $g_A = g$, which only occurs if the following restriction is satisfied.

Assumption 1. $1 - \phi - \beta = \eta$.

Notice that under Assumption 1, $\nu = 1 - \beta$ in equation (8), which means that there are positive externalities in the accumulation of TFP if $\beta < 1$. It is easy to derive the following results from equations (10)-(14) and Assumption 1:

$$g_A = g, \quad g_y = \beta g, \quad g_K = g_Y = \beta g + g_L, \tag{15}$$

$$\kappa = \frac{s}{\beta g + g_L + \delta}. \tag{16}$$

The last equation implies that the factor intensity of an economy, $X = \kappa^{\frac{\alpha}{1-\alpha}} h$, only depends on its saving rate and human capital.

3.3 Levels Accounting

We now derive the implications of the model for cross-country income and productivity differences. These implications are summarized in the following proposition.

Proposition 1 Let Assumption 1 hold. Then, along a balanced growth path

$$A_t = X^{\frac{1}{\eta}} \cdot \left(\frac{ds_R}{g}\right)^{\frac{1}{\eta}} A_t^* \tag{17}$$

$$y_t = \widehat{A}_t \cdot \widehat{X} \tag{18}$$

where

$$\widehat{A}_t \equiv \left(\frac{ds_R}{g}\right)^{\frac{\beta}{\eta}} A_t^{*\beta} \text{ and } \widehat{X} \equiv X^{1+\frac{\beta}{\eta}}.$$

Proof: Equation (17) is obtained from equation (12) using the balanced growth result that $g_{At} = g$ and Assumption 1. Equation (18) results from substituting (17) into (11).

There are four important results in Proposition 1. First, equation (17) states that the technological level of the economy depends positively on the factor intensity of the economy X as well as other R&D related parameters. The model predicts that economies with lower factor intensity will have, other things equal, lower TFP levels. Moreover, economies with lower R&D investment rates or with access to a lower technological frontier, say because of physical or cultural distances, will also display lower TFP levels.

Second, equation (18) describes the determination of steady state per-capita output in terms of components related to factors X and other components. This formulation is analogous to the one used by KR (1997) in equation (1), but the role of factors is adjusted to take into account their effect on TFP as described by equation (17). The adjustment increases the role of X by a factor of $\beta/\eta > 0$, a sort of 'amplification effect'. Thus, endogenizing TFP unambiguously increases the role of factors in explaining world income dispersion. The positive dependence of the amplification parameter on β reflects the indirect effect that additional factors of production have on output by increasing the TFP level. The negative dependence on η reflects the negative effect on long term output of closing the productivity gap relative to the frontier. A smaller gap reduces the productivity of the R&D sector and therefore total output.

Third, (18) provides a decomposition in terms of the fundamentals of the model. On the one hand, changes in saving rates s and human capital only affect $\widehat{X}$ but not $\widehat{A}$. On the other hand, changes in s_R and A^* affect $\widehat{A}$ but not $\widehat{X}$. Finally, although changes in g affect both $\widehat{X}$ and $\widehat{A}$, g is identical across countries. Thus, equation (18) provides a clear separation between R&D investment rate s_R and the (country specific) technological frontier A^* on the one hand, and investment rate s and human capital h on the other hand. Recall that in this section we treat s and s_R as exogenous, so there is no feedback from s to s_R or viceversa. In Section 4 we provide microfoundations for these investment decisions and show that such feedback exists, and that the results from the levels accounting decomposition proposed in (18) are reinforced.

Finally, our model includes the standard Solow model with exogenous TFP as a special case. TFP becomes exogenous to the economy when the catching up parameter η goes to infinity. In this case, the productivity level is determined by the technological frontier, $A_t = A_t^*$, regardless of factor endowments. Moreover, in this particular case, equation (1) provides a meaningful factorization of income in terms of different parameters.

3.4 Calibration

We now explore the quantitative implications of the model for cross-country dispersion. The key equation for this purpose is (18), which provides a solution in terms of parameters of the model. We recover $\widehat{A}$ as a residual from (18) given values of the observable variables y, κ, and h, and of parameters α, β, and η. Once $\widehat{A}$ and $\widehat{X}$ are obtained, we can assess their relative contribution in explaining income dispersion.

In order to compare our results to KR (2004), we use their same parameter values for α, δ,

g_y, and g_L. In particular, we assume $\alpha = 1/3$, $\delta = 0.08$, $g_y = 0.015$ and $g_L = 0.011$ respectively. The key parameters for our purposes are β and η. We consider three alternative calibrations of these parameters. The first calibration exploits existing estimates of social returns to schooling provided by Bils and Klenow (2000); the second calibration uses existing estimates about the speed of convergence; the third uses estimates based on cross-country growth regressions derived from our model.

3.4.1 Calibration based on social returns to schooling

The cross-section of human capital h employed in this paper is constructed using the standard practice of transforming years of schooling and experience into stocks of human capital using estimates of private returns to education and experience. The specification used in this paper is the following one employed by Bils and Klenow (2000):

$$h = \exp\left(f(s) + 0.0512\,(a - s - 6) - 0.00071\,(a - s - 6)^2\right), \tag{19}$$

where s is average years of schooling, a is the average age of workers, and $f(s)$ is the private returns to schooling (a decreasing function of the years of schooling).

Equations (18) and (19) imply that the long term social returns to schooling exceed the private returns by a factor of $1 + \beta/\eta$. Bils and Klenow (2000) provide estimates of private returns to schooling and plausible bounds for the corresponding social returns. All human capital series used in this paper use their intermediate specification $f(s) = (0.18/(1 - 0.28))\, s^{1-0.28}$ that implies moderate decreasing private returns to schooling. This specification produces similar results to the one used by HJ. For this specification of private returns, Bils and Klenow provide an upper bound for $1 + \beta/\eta$ of 1.85, and use an intermediate value of 1.5 for their computations.[4] This motivates us to use a value of 0.5 for β/η as our first calibration.

3.4.2 Calibration based on speed of convergence

Extensive literature has documented that countries seem to converge very slowly to well-defined steady states. Barro (1997) summarizes existing literature on the topic and finds a speed of conver-

[4] They denote $1/(1 - \phi)$ the ratio of social to private returns to schooling. The intermediate and upper bound of ϕ in their Table 2 (p. 1169) are 1/3 and 0.46 respectively.

gence of 2.5%, a finding that has recently received further support from Hauk and Wacziarg (2004). Models with exogenous TFP cannot replicate this slow speed of convergence for plausible values of the capital share. On the other hand, models with endogenous TFP typically exhibit lower rates of convergence depending on the degree of long term increasing returns. Our second calibration exploits this connection to back up a value of β/η consistent with a speed of convergence of 2.5%.

The Appendix shows that the dynamic system of our model can be approximated around the steady state by the following system of log-linear differential equations:

$$\frac{d\ln(\overline{y}_t)}{dt} = -(1-\alpha)(g_L+\delta+g(1-\beta))\ln(\overline{y}/\overline{y}^*)+(1-\alpha)\beta\left[g_L+\delta+g(1-\beta-\eta)\right]\ln(\overline{a}/\overline{a}^*) \quad (20)$$

$$\frac{d\ln(\overline{a}_t)}{dt} = g\ln(\overline{y}/\overline{y}^*) - g(\eta+\beta)\ln(\overline{a}/\overline{a}^*), \quad (21)$$

where $\overline{y}_t = Y_t/\left(hL_tA_t^{*\beta}\right)$, and $\overline{a}_t = A_t/A_t^*$. The characteristic equation associated to this system is given by

$$\lambda^2 + \left(\psi + \frac{\eta}{\beta}g_y + \alpha g_y\right)\lambda + \psi\frac{\eta}{\beta}g_y = 0, \quad (22)$$

where $\psi \equiv (g_L + g_y/\beta + \delta)(1-\alpha)$. Since $\alpha g_y = 0.005$, this equation can be approximated by $(\lambda+\psi)(\lambda+(\eta/\beta)g_y) \approx 0$. Therefore the eigenvalues are approximated by $\lambda_1 \approx -(\eta/\beta)g_y$ and $\lambda_2 \approx -(g_L+g_y/\beta+\delta)(1-\alpha)$. For sufficiently large t, the speed of convergence is determined by the smaller root (in absolute value). For the parameter values assumed, and given that $\beta \leq 1$, this approximation yields $\lambda_2 \lesssim -0.07$, which would imply a large speed of convergence unless $|\lambda_1| < |\lambda_2|$. Therefore, a plausible speed of convergence of 2.5% requires $\lambda_1 \approx -0.025$, or $\beta/\eta \approx g_y/0.025 = 0.6$, which is our second calibration for β/η.

3.4.3 Calibration based on growth regressions

Our third calibration uses the full structure of the model and regression analysis. The Appendix shows that the solution to the system (20) and (21) can be expressed in the following form:

$$\ln(\overline{y}_t) = \ln(\overline{y}^*) + a_1\ln(\overline{y}_0/\overline{y}^*) + a_2\ln(\overline{y}_T/\overline{y}^*), \quad (23)$$

where $a_1 \equiv \left(e^{\lambda_2 t}e^{\lambda_1 T} - e^{\lambda_1 t}e^{\lambda_2 T}\right)/\left(e^{\lambda_1 T} - e^{\lambda_2 T}\right)$, $a_2 \equiv \left(e^{\lambda_2 t} - e^{\lambda_1 t}\right)/\left(e^{\lambda_1 T} - e^{\lambda_2 T}\right)$, $0 < T < t$, and $\lambda_1 < 0$, $\lambda_2 < 0$ are the solutions to the characteristic equation (22). A stochastic version of (23) can be estimated using cross-country data. Following Barro and Sala-i-Martin (2003), we estimate the equation above using OLS on a cross-section of 71 countries between 1960 and 2000.[5] We approximate the steady state value $\overline{y}^*$ by controlling for average saving rates. We choose $T = 10$, but the results are not sensitive to this choice. We use the estimated values of a_1 and a_2 to solve for λ_1 and λ_2, and then use these values to solve for η and β. We find $\beta = 0.35$ and $\eta = 0.41$. Thus, our third calibration for β/η is 0.85. Coincidentally, this value corresponds to the upper bound of social returns to schooling provided by Bils and Klenow (2000).

3.5 Results

3.5.1 Levels Accounting

We now show how the levels accounting results of KR (1997) and HJ are affected once the assumption of exogenous TFP is relaxed. The results clearly show that factors of production (i.e., saving rates and human capital) are significantly more important than other factors in explaining cross-country labor productivity and income differences. In particular, differences in R&D investment rates, in country-specific technological frontiers, or in the productivity of R&D investment play an important but secondary role in accounting for the cross-country variation of productivities.

Tables 2 and 3 report levels accounting results obtained from (18) for different values of β/η. Table 2 follows the same methodology of Table 1. The first row of the table reports the results for the exogenous TFP model. According to this model, the main cause of extreme poverty is low TFP productivity. The poorest countries in the world have only around 1/32 of the US income. While factor intensities in these countries are around 1/4 of US values, their TFP levels are only around 1/8 of US. This picture is completely reversed if TFP is endogenous. For $\beta/\eta = 0.5$, for example, factor differences now account for an income ratio of around 1/8, while differences in other TFP components, included under the label $\widehat{A}$, account for a ratio of only around 1/4. For $\beta/\eta = 0.85$, differences in factors can account for a ratio of incomes of around 1/16 while the remaining components can account for a ratio of only around 1/2.

For the average of the 86 countries, the first row of Table 2 suggests that both factors and TFP differences account similarly for the income ratio of around 1/3. However, the role of factors

[5] The sample of countries includes those with data available for 1960, 1970 and 2000 for the following variables: output per worker from Penn World Tables v. 6.1 (available electronically at http://pwt.econ.upenn.edu/), and average total years of schooling in population over 25 years of age from Barro and Lee's (2000) update.

increases dramatically once the endogeneity of TFP is taken into account. For $\beta/\eta = 0.6$, factor differences can account for an income ratio of 2/5 while other components can only account for a ratio of 4/5. Finally, for the case of $\beta/\eta = 0.85$, almost all the difference in income is accounted by factors.

Table 3 reports variance decomposition results using the methodology of KR (1997) described by equation (2). The first row reports results for the exogenous TFP models. According to these models, differences in factor intensities account for around 40% of income differences while TFP differences account for around 60%. These contributions split the covariance term, which accounts for 35% of the variance, equally between factors and TFP. Allowing TFP to be endogenous significantly affects these results. For $\beta/\eta = 0.5$, factors now account for around 60% of the variance and the covariance term is reduced to only 20% of the total variance. For $\beta/\eta = 0.6$ factors explain around 2/3 of the income variance and the covariance is reduced to 15%. Finally, for $\beta/\eta = 0.85$ factors explain almost 3/4 and the covariance is close to zero. This last result is similar to the one found by MRW although for different reasons. While MRW stress differences in human capital stocks across countries as well as a large share of capital (both physical and human) in production of 0.7, we stress the endogeneity of TFP and its effect on increasing the role of saving rates and human capital.

3.5.2 Further implications

Social rates of return to R&D We now discuss some implications of the model for the social rates of return to R&D. KR (2004) calibrate their model to match a target value of the social rate of return to R&D. Jones and Williams (1998) show that if $\dot{A} = G(A, R)$, the social rate of return to R&D r_s can be expressed as $r_s = (\partial Y/\partial A)/P_A + \partial G/\partial A + g_{PA}$, where $P_A = (\partial G/\partial R)^{-1}$ is the price of ideas. In our model, G is defined from equation (9) as $G(A, R) = \textit{constant} \cdot A_t^{\phi} R_t$. This functional form implies that along the balanced growth path

$$r_s = \left(\frac{1-\alpha}{s_R} - \frac{\eta}{\beta}\right) g_y + g_L,$$

and solving for β/η yields

$$\frac{\beta}{\eta} = \left[\frac{g_y s_R}{(1-\alpha) g_y - (r_s - g_L) s_R}\right]. \tag{24}$$

One can compute β/η from this equation given some target values of r_s and s_R. These values must satisfy the restriction $(1-\alpha)\, g_y - (r_s - g_L)\, s_R > 0$ so that the β/η remains positive.[6] This restriction means that a high social rate of return must correspond to a relative low R&D investment rate and vice versa. Finally, notice that β/η computed in this way is an increasing function of both s_R and r_s (given $r_s > g_L$).

There are three shortcomings in using (24) to compute β/η. First, feasible target values for the social rate of return vary widely and are typically very large. For example, Coe and Helpman (1995) estimate rates of return to R&D of 123% for the G7 and 85% for the remaining OECD countries, while van Pottelsberghe de la Potterie and Lichtenberg (2001) find returns of 68% in the G7 and 15% for the remaining OECD countries. For the U.S, Griliches and Lichtenber (1984) estimate returns of 71%, and Terlecky (1980) and Scherer (1982) find returns above 100%. Jones and Williams (1998) suggest even larger values. KR (2004) calibrate their model using a target value for the social returns to R&D of 26%. This is a conservative value given the existing estimates.

Second, available estimates of R&D investment rates provide only lower bounds for the relevant R&D investment rates. For example, the OECD reports series of R&D expenditures as percentage of GDP for different countries. These expenditures only include formal research, i.e., research performed in an R&D departments of corporations and government institutions. These figures do not include expenditures in technology adoption by all types of entities, nor any informal type of research. As we suggest below, a key message from this paper, as well as KR's (2004) paper, is that informal research efforts are probably the most important source of technological progress, both for developed and developing countries. KR (2004) calibrate their model using a value of $s_R = 2.5\%$, the R&D investment rate reported by OECD for the U.S in 1995.

Third, the estimated values of β/η are extremely sensitive to the precise target values of s_R and r_s. This is because the denominator in the right hand side of (24) can easily approach zero for reasonable values of r_s and s_R. The values used by KR (2004) for r_s and s_R imply a modest $\beta/\eta = 0.12$. This low amplification effect explains why they conclude that differences in factor intensities still play a secondary role in accounting for international income differences in models with endogenous TFP. However, this low ratio of β/η implies a speed of convergence of 12.5% rather than 2.5%. Moreover, their conclusion is not robust to the choice of social returns nor

[6] Negative values for either β or η would be hard to interpret.

R&D investment rates. Assuming $s_R = 2.5\%$ but social returns of 38.5%, which are still on low spectrum of existing estimates, one obtains $\beta/\eta = 0.61$. Alternatively, assuming $r_s = 26\%$, but $s_R = 3.65\%$ also yields $\beta/\eta = 0.6$. This is also the value of β/η if $r_s = 32\%$ and $s_R = 3\%$, which seem reasonable targets as well. We conclude that existing evidence about social returns to R&D and R&D investment rates is consistent with relatively large values for β/η, in the order of the ones we used in our calibration.

R&D investment rates We now compare implied R&D investment rates from our model, KR's (2004) model, and some existing evidence. According to Proposition 1, the ratio of R&D investment rates in countries i and j satisfies

$$\frac{s_{iR}}{s_{jR}} = \frac{d_j}{d_i}\frac{X_{jt}}{X_{it}}\left(\frac{A_i}{A_j}\frac{A_j^*}{A_i^*}\right)^{\eta}.$$

Assuming identical technological frontier and technological parameter d for all countries, this expression simplifies to

$$s_{iR} = s_{jR}\frac{X_{jt}}{X_{it}}\left(\frac{A_i^{\beta}}{A_j^{\beta}}\right)^{\frac{\eta}{\beta}}. \tag{25}$$

This expression can be easily computed using s_R of the U.S as country j and computing A_i^{β} using equation (11). For purposes of comparison, we follow KR (2004) and assume $s_{j,R} = 2.5\%$. As expected, we find that the correlation between the s_R series from our model and ones from KR (2004) model (reported in Table A1 of their paper) increases as β/η decreases. This correlation is 83% if $\beta/\eta = 0.5$ but only 64% if $\beta/\eta = 0.85$. To keep close to KR (2004), the remaining of this section assumes $\beta/\eta = 0.5$. Figure 1 plots both R&D investment rates series. The graph reveals that both models produce similar R&D investment rates although our series are slightly higher.

KR (2004) assesses their model using evidence compiled by Lederman and Saenz (2003) on R&D investment rates for a cross section of countries. This data set has the same shortcomings mentioned in the previous section. Moreover, it may be particularly biased for poor countries in which most R&D efforts are likely informal and take the form of technology adoption. Lederman and Saenz also warn that the data "does not include investments in mining exploration or soil analysis, and thus might imply a bias against natural resource activities and agriculture" (p. 3).

We now compare the Lederman and Saenz data with the series of R&D investment implied by the model. The correlations between the models' series and the data is very similar, 13.27% for our model and 14.34% for KR's (2004) model. This correlation, however, is rather low for both models and it its hard to increase by changing the degree of increasing returns (for our model, the maximum correlation is 16.64% and it is obtained for $\beta/\eta = 1/3$). The problem becomes more apparent if one considers only OECD countries, for which the data is likely better. For this subset of countries, both models predict very similar R&D investment rates (their correlation is 90%). However, the correlation between the models' predicted values and the data is actually negative (-33% for our model and -21% for KR's (2004) model). The data is plotted in Figure 2.

These correlations are puzzling for both models and suggest that there are important R&D efforts not captured in the data. We think that the understanding of this puzzle is a major pressing issue for the development literature in general, but more specifically for endogenous growth models with innovation. Part of the issue is that some countries, particularly poor countries, appear to have levels of TFP productivity that seem "too high" given their factors of production and low measured s_R. One possibility is that the Cobb-Douglas production function is not appropriate for poor countries because it does not capture differences in sectoral composition across countries. Córdoba and Ripoll (2005) explore this issue and find a more "proportionate" relationship between the amount of factors and TFP productivity.

4 A Full-Fledged Model

The extended Solow model of the previous section assumes that the saving rate and R&D investment rates are exogenous. Under these assumptions, equation (18) solves steady state output per worker in terms of its ultimate determinants. However, if s and s_R are endogenous, equation (18) is not a solution in terms of fundamentals, and it may provide misleading results. For example, it could be that a R&D parameters, say R&D taxes, affect both s and s_R in the same way. This would explain the positive correlation between A and X, and therefore increase the role of R&D variables in accounting for income differences. In this section we study this issue by endogenizing s and s_R using a model of endogenous growth with expanding variety of products and erosion of monopoly power, in the spirit of Barro and Sala-i-Matin (1997 and 2003, chapter 6). We introduce capital accumulation to Barro and Sala-i-Martin's model, as well as distortions to capital accumulation. In addition, we eliminate scale effects from the model and calibrate it. We show that the results from the extended Solow model of the previous section are reinforced by endogenizing s and s_R.

In the microfounded model of this section, there are two key exogenous forcing variables, a 'patent protection' parameter $1/p$, which captures the degree of rent protection for innovators, and an 'expropriation' parameter q, which captures the rate of confiscation of physical assets. Both these parameters proxy for policy or institutional characteristics. It turns out that in equilibrium, both s and s_R depend on p, but only s depends q. Moreover, s turns out to depend positively on p while s_R depends negatively on p. Thus, R&D distortions cannot explain the positive correlation between X and A. The reason is that better patent protection, lower p, strengthens the monopolistic power of innovators, reduces the fraction of competitive firms, and reduces the steady state returns on capital investments. This is because monopolies demand less capital than competitive firms. As a result, better patent protection reduces incentives to accumulate capital.

In the calibrated model, the effect of p on s is second order, and as a result, changes in p primarily affect s_R while changes in q only affect s. We use the calibrated model to assess the effects on cross-country income dispersion of eliminating differences in p and q respectively. Consistent with the message of Section 3, differences in $p's$ have a secondary role in explaining cross-country income differences.

4.1 The Model

There are two type of goods in this economy, a final good and N varieties of intermediate goods. There is production of final goods, intermediate goods, and blueprints for new varieties or "ideas". Technological progress takes the form of an increase in the number of varieties.

4.1.1 Production of Final Good

Final goods are produced by competitive firms using the following constant return to scale production function at the firm level:

$$Y_t = \left[\sum_{j=1}^{N_t} x_{jt}^{\gamma} \right]^{1/\gamma}, \tag{26}$$

where $0 < \gamma < 1$, Y is output, x_j is the input of the j^{th} intermediate good, N is the number of varieties of inputs (or more exactly, the set of domestic usable blueprints), and $1/(1-\gamma)$ is the elasticity of substitution between inputs. To simplify, consider only one single price-taker final-good producer.

Denote P_j the price of input j and let the price of final good be the numeraire. Profit maximization by the final good producer yields the following demand functions for intermediate inputs

$$x_{jt}^{d} = Y_t P_{jt}^{-\frac{1}{1-\gamma}} \text{ for } j = \{1, .., N\}. \tag{27}$$

4.1.2 Production of Intermediate Goods

Intermediate goods are produced using capital and human capital according to $x = k^{\alpha} h^{1-\alpha}$ (we omit the subscript j). Factor markets are competitive. This implies that the cost of producing one unit of intermediate good is given by $e_t \equiv r_{kt}^{\alpha} w_t^{1-\alpha}$, where r_k is the rental rate of capital, and w is the wage rate per unit of human capital.

N_{mt} intermediate goods are produced by monopolistic firms and $N_{ct} = N_t - N_{mt}$ intermediate goods are produced by competitive firms. Monopolistic power vanishes randomly according to a Poisson process with parameter $p \geq 0$: a variety j presently monopolized becomes competitive in the next interval dT with probability pdT. Thus, $p = 0$ describes everlasting monopolies while $p = \infty$ means no monopolistic power at all. Monopolists set prices P_{mt} to maximize the expected present value of profits

$$V(t) = \int_t^{\infty} \pi_{mt} e^{-(\overline{r}(v,t)+p)(v-t)} dv,$$

where $\pi_{mt} = (P_{mt} - e_t)\, x_t^d$, $\overline{r}(v,t) \equiv (1/(v-t)) \int_t^v r(s)ds$ is the average interest between t and v, and r is the risk-free interest rate. This maximization yields the familiar pricing of mark-up over marginal cost, while competitive prices equal marginal cost:

$$P_{mt} = e_t/\gamma;\ P_{ct} = e_t. \tag{28}$$

Plugging prices into (27), the quantities produced by monopolists, x_{mt}, and competitive producers, x_{ct}, are given by

$$x_{mt} = Y_t \left(\frac{e_t}{\gamma}\right)^{-\frac{1}{1-\gamma}} \text{ and } x_{ct} = Y_t e_t^{-\frac{1}{1-\gamma}}. \tag{29}$$

4.1.3 Innovators

The cost (in terms of final good) of creating a new variety in the form of a blueprint is given by

$$\lambda_t = N_t^{-\phi} \left(N_t^*\right)^{-\eta} L_t. \tag{30}$$

where the dependence on N_t reflects domestic research externalities (both positive and negative), and N_t^* is the technological frontier (a set of internationally available blueprints). In this formulation, international blueprints cannot be used domestically unless some costly adjustment is made. Parameter $\eta > 0$ captures the idea that the farther the country is from the frontier, the less costly it is to create (imitate or adopt) a new variety. The dependence on L_t is required to eliminate scale effects in levels, and can be motivated by a duplication of efforts in the process of discovering new ideas.

The inventor of a variety is granted monopolistic power in the production of that variety during a period of random duration, as described above. We think of p as capturing the degree of patent protection. Free entry into the production of ideas guarantees that $\lambda_t = V(t)$, or[7]

$$\lambda_t = \int_t^\infty \pi_{mv} e^{-(\overline{r}(v,t)+p)(v-t)} dv. \tag{31}$$

4.1.4 Households and Government

A representative household maximizes its lifetime utility described by

$$\int_0^\infty e^{-\rho t} L_t u(c_t) dt,$$

where c_t is consumption per-capita, $u(c) = c^{1-1/\sigma} / \left(1 - 1/\sigma\right)$, $\sigma > 0$ is the intertemporal elasticity of substitution, and $\rho > 0$ is the rate of time preference. The household budget constraint is

$$\dot{K}_t = w_t L_t + T_t + \left[\left(r_{kt} - \delta\right)(1-q) - q\right] K_t + N_m \pi_{mt} - L_t c_t + T_t,$$

where q is a rate of confiscation of physical capital and capital income. This parameter captures distortions associated with the accumulation of capital. Notice that if $q = 0$ this budget constraint

[7] It is further required that inventors are risk neutral or that they own a large set of innovations.

reduces to a standard one, while if $q = 1$ all physical capital and capital income are confiscated. In the latter case household savings, $w_t L_t + T_t + N_m \pi_{mt} - L_t c_t$, are used to replace the confiscated capital K_t and to increase the capital stock $\dot{K}_t$. T_t are lump sum transfers. The government runs a balanced budget so that $T_t = [(r_{kt} - \delta) q + q] K_t$. Moreover, we assume that monopoly profits cannot be confiscated, although both monopolistic power and profits are lost if $p = \infty$.

The optimal consumption path and capital accumulation path must satisfy

$$g_{ct} = \frac{\dot{c}_t}{c_t} = \sigma [r_t - \rho], \tag{32}$$

and $1 + r = (1 + r_k - \delta)(1 - q)$. This last condition can be written as

$$r_{kt} = \frac{r_t + q + \delta (1 - q)}{1 - q}. \tag{33}$$

4.1.5 Resource constraints

Final goods are used to consume, increase the stock of capital, and for R&D spending, as described by (4). We continue to assume that the average level of human capital h is exogenous. Therefore, the law of motion of aggregate human capital is given by (6). The following are additional aggregate restrictions:

$$\dot{K}_t = I_t - \delta K_t, \tag{34}$$

$$\dot{N}_c = p N_{mt}, \tag{35}$$

$$K_t = k_{mt} N_{mt} + k_{ct} N_{ct}, \ H_t = h_{mt} N_{mt} + h_{ct} N_{ct}, \tag{36}$$

$$R_t = \lambda_t \dot{N}_t, \tag{37}$$

$$\dot{N}_t^* = g N_t^*. \tag{38}$$

These restrictions are easily interpreted. The final equation describes the exogenous evolution of N_t^*.

4.2 Equilibrium

The equilibrium of this economy is defined in a standard way. The following proposition provides the equilibrium solutions for output, profits, and the rental rate of capital.

Proposition 2 The equilibrium levels of final goods production Y_t, monopolistic profits π_{mt}, and rental rate of capital r_{kt}, are given by

$$Y_t = \frac{\left[N_{mt} + a_1^{\gamma} N_{ct}\right]^{1/\gamma}}{N_{mt} + a_1 N_{ct}} K_t^{\alpha} H_t^{1-\alpha}, \tag{39}$$

$$\pi_{mt} = \frac{(1-\gamma)\, Y_t}{N_{mt} + a_1^{\gamma} N_{ct}}, \tag{40}$$

$$r_{kt} = \gamma\alpha \frac{1 + a_1 N_{ct}/N_{mt}}{1 + a_1^{\gamma} N_{ct}/N_{mt}} \frac{Y_t}{K_t}. \tag{41}$$

where

$$a_1 \equiv \frac{x_{ct}}{x_{mt}} = (1/\gamma)^{\frac{1}{1-\gamma}} > 1. \tag{42}$$

Proof: Equation (42) follows from (29). Furthermore, all producers face the same factor prices and therefore employ the same capital-labor ratios, $k_c/h_c = k_m/h_m = K/H$. One can thus write $x_i = h_i\,(k_i/h_i)^{\alpha} = h_i\,(K/H)^{\alpha}$ so that $x_c/x_m = h_c/h_m = a_1$. Using this result into (36) gives $h_m = H/\left(N_m + a_1 N_c\right)$. Similarly, $k_m = K/\left(N_m + a_1 N_c\right)$. Thus,

$$x_{mt} = h_{it}\left(\frac{K_t}{H_t}\right)^{\alpha} = \frac{K_t^{\alpha} H_t^{1-\alpha}}{N_{mt} + a_1 N_{ct}}. \tag{43}$$

Substituting this result and (42) into (26) produces (39). Moreover, substituting (39) and (43) into (29) and solving for e gives

$$e_t = \gamma\left[N_{mt} + a_1^{\gamma} N_{ct}\right]^{\frac{1-\gamma}{\gamma}}. \tag{44}$$

Furthermore, monopolistic profits are given by $\pi_t = (P_{mt} - e_t)\, x_t^d$. Equation (40) results from substituting (28), (43) and (44) into this expression, and using (39). Finally, monopolistic prices implies that $r_k = \alpha\gamma P_m x_m / k_m = \alpha\gamma P_m x_m \left(N_m + a_1 N_c\right)/K$ so that the marginal product of capital for monopolies is larger than the rental price of capital. Equation (41) is obtained by substituting (28), (44), and (43) into the previous expression.

Notice that the returns to physical capital investment r_k decrease with N_m/N_c. It equals $\gamma\alpha\left(Y_t/K_t\right)$ if $N_m/N_c = \infty$, and $\alpha\left(Y_t/K_t\right)$ if $N_m/N_c = 0$. This negative relationship is what explains the main result of this section described below, that better patent protection, which increases N_m/N_c, also reduces savings.

4.3 Balanced Growth

Along a balanced growth path (BGP) all variables grow at constant rates. A BGP with positive growth requires $g_{N_c} = g_{N_m} = g_N$, which together with equation (35) imply

$$\frac{N_c}{N_m} = \frac{p}{g_N} \text{ and } \frac{N_c}{N} = \frac{p}{g_N + p}.$$

Substituting this result into (39) and simplifying produces

$$Y_t = K_t^{\alpha}\left(A_t^{\beta} H_t\right)^{1-\alpha} \tag{45}$$

where

$$A_t \equiv \chi(p)N_t, \quad \beta \equiv \frac{1-\gamma}{\gamma}\frac{1}{1-\alpha}, \quad \chi(p) \equiv \left[\left(\frac{g_N + a_1^{\gamma} p}{g_N + p}\right)^{1/\gamma}\frac{g_N + p}{g_N + a_1 p}\right]^{\frac{\gamma}{1-\gamma}},$$

so that $\chi(0) = \chi(\infty) = 1$ and $\chi(p) \simeq 1$ for g_N close to zero.

We can now show that along a BGP this economy collapses into the extended Solow model of Section 3, with the important difference that the saving rate $s \equiv I/Y$, and the R&D investment rate $s_R \equiv R/Y$ are endogenous in this model. To see the exact mapping between the two models, note that equation (3) is identical to equation (45), equation (4) follows from (34) and the definition of s, and (6) is also a resource constraint in this economy. It remains to show that this economy also satisfies equations (9) and (10). For this purpose, define $A_t^* \equiv N_t^*$. This definition and (38) produces (10). Moreover, it follows from (37) and (30) and the definitions of A_t and A_t^* that

$$\dot{A}_t = dA_t^{\phi}\left(A_t^*\right)^{\eta} s_R y_t, \text{ where } d \equiv \chi(p)^{1-\phi},$$

which is exactly as equation (9), with the important difference that d is a function of p. This last result implies that not only s and s_R are endogenous in the microfounded model, but also the R&D productivity parameter d which was assumed constant and equal across countries in Section 3.

Given the equivalence between the two models along a balanced growth path for particular but constant values of d, s, and s_R, the results from Section 3.1 apply for the microfounded model. In particular, Assumption 1 is still required to prevent a country from falling behind or moving ahead of the technological frontier. Therefore, we assume in the remainder of the paper that Assumption 1 holds. The following Proposition summarizes the balanced growth properties of the economy, and the determination of s and s_R.

Proposition 3 Let Assumption 1 hold. Along a balanced growth path

$$g_N = g, \ g_c = g_y = g_k = \beta g, \ g_\lambda = -(\phi + \eta) g + g_L, \tag{46}$$

$$r = \frac{\beta g}{\sigma} + \rho, \tag{47}$$

$$\kappa = \frac{s}{\beta g + g_L + \delta} = \alpha\gamma \frac{g + a_1 p}{g + a_1^{\gamma} p} \frac{1-q}{r + q + \delta(1-q)}, \tag{48}$$

$$s_R = \frac{(1-\gamma) g}{r + p + (\phi + \eta) g - g_L} \frac{g + p}{g + a_1^{\gamma} p}. \tag{49}$$

Proof: Equation (46) follows from (15), the definition of A in (45), and the cost function in (30). Equation (47) results from (32) and (46). To obtain (48), equate (41) to (33), solve for K/Y, and substitute the result into (34). Finally, to obtain s_R, notice that along a BGP equation (31) becomes $\lambda_t = \int_t^\infty \pi_{mv} e^{(r+p)(t-v)} dv$. Taking derivatives of this expression with respect to time and rearranging yields $\pi_{mt}/\lambda_t = r + p - g_\lambda$. This result together with (40) produces $Y_t = (r + p - g_\lambda) \ \lambda_t \ (N_{mt} + a_1^{\gamma} N_{ct}) \ / \ (1-\gamma)$. Dividing (37) by this expression, and using (46) provides the expression for s_R.

Equation (46) states that the long run growth rate of the economy is determined by the exogenous rate of worldwide technological progress. Equations (48) and (49) are the key equations. They provide the solution for s and s_R in terms of the fundamental parameters of the economy, in particular p and q. Equation (49) states that the R&D investment rate depends on the degree of patent protection p, but not on the degree of expropriation q. In contrast, the saving rate depends both on the degree of patent protection and the degree of expropriation. Moreover, it is easy to check that s_R depends negatively on p while κ and s depend positively on p. This implies that better patent protection, lower p, increases the R&D investment rate s_R but reduces the savings rate s. As explained above, this last result arises because better patent protection reduces the returns on capital investments. Thus, according to the microfounded model of this Section, the observed positive correlation between X and A cannot be explained by R&D distortions.

We now proceed to derive a solution for y in terms of long-term fundamentals. As noticed, the common dependence of s and s_R on p invalidates the decomposition of Section 3 which is based on the premise that $\widehat{A}$ and $\widehat{X}$ are determined by different sets of parameters. Instead, by substituting (48) and (49) into (18), one finds:

$$\widehat{X}_t = \widehat{X}(q,p,h) \equiv \left(\left(\alpha\gamma \frac{g+a_1 p}{g+a_1^{\gamma} p} \frac{1-q}{r+q+\delta(1-q)} \right)^{\frac{\alpha}{1-\alpha}} h \right)^{1+\frac{\beta}{\eta}}; \tag{50}$$

$$\widehat{A}_t = \widehat{A}_t(p) \equiv \left(\frac{(1-\gamma)\,\chi(p)^{1-\phi}}{r+p+(\phi+\eta)\,g-g_L} \frac{g+p}{g+a_1^{\gamma} p} \right)^{\frac{\beta}{\eta}} N_t^{*\beta}. \tag{51}$$

Finally, incorporating these two results into (18) provides the following result:

Proposition 4 Let Assumption 1 hold. Along a balanced growth path

$$y_t = \widetilde{A}(p) \cdot \widetilde{X}(q,h) \tag{52}$$

where

$$\widetilde{A}_t(p) = \Omega(p)\widehat{A}_t, \quad \widetilde{X}(q,h) = \Omega(p)^{-1}\widehat{X}_t, \quad \Omega(p) \equiv \left(\frac{g+a_1 p}{g+a_1^{\gamma} p} \right)^{\frac{\alpha}{1-\alpha}\left(1+\frac{\beta}{\eta}\right)}.$$

Equation (52) factorizes output in two fundamental components, one that depends only on the frictions in the R&D sector p, and one that depends only on the frictions to capital accumulation q. According to (52), $\widehat{A}_t$ and $\widehat{X}_t$ need to be adjusted by the factor $\Omega(p)$, a factor that is nearly independent of p if g is close to zero. Since g is actually close to zero in the data, one expects that a decomposition based on (18) must provide similar results as one based on (52). We confirm this intuition next.

4.4 Revisiting Levels Accounting

In order to use (52) for variance decomposition, we need to construct series $\widetilde{A}(p)$ and $\widetilde{X}(q)$, which according to Proposition 4 requires to estimate values of p for each country, compute $\Omega(p)$, and adjust $\widehat{A}_t$ and $\widehat{X}_t$ from Section 3. We use the same parameter values for $[\phi,\ \beta/\eta,\ \beta, \alpha,\ \delta,\ g,\ g_L]$. Given β/η, the exact value of η has only marginal effects on the calculations. We thus pick $\eta = 0.41$, consistent with Section 3. Parameter γ is computed using the definition of β given in equation (45), and we assume a risk-free rate r of 2% for all countries.[8] This last assumption implicitly pins down values for σ and ρ according to equation (47).

To compute p for each country we use equation (51). Given a value for the technological frontier N_t^* the only unknown in that equation is p. To find N_t^* notice that $\widehat{A}_t(p)$ is a decreasing function of p and that $1/p$ is the life expectancy of a monopoly. Assuming a life expectancy of 100 years for monopolies in the country with the highest value of $\widehat{A}_t(p)$, one can use (51) to solve for $N_t^{*\beta}$ as[9]

$$N_t^{*\beta} = \max\left(\widehat{A}_t\right)\left(\frac{r + 1/100 + (\phi+\eta)\,g - g_L}{(1-\gamma)\,\chi(1/100)^{1-\phi}}\,\frac{g + a_1^{\gamma}/100}{g + 1/100}\right)^{\frac{\beta}{\eta}}.$$

Finally, we assume that this technological frontier is identical for all countries. Figure 3 portrays the expected life of a monopoly as a function of the country's output per worker relative to the US. Given p, we use equation (50) to find q for each country. The implied values of q are portrayed in Figure 4.

Tables 4 and 5 show the results of levels accounting for different values of β/η using equation (52). Table 4 follows the same methodology of Tables 1 and 2. It shows that the role of factors $\widetilde{X}$ increases even more when s and s_R are endogenous. The differences between Table 2, corresponding to the extended Solow model, and Table 4, corresponding to the microfounded model, are quantitatively small for the lower values of β/η but they are significant when β/η is large. This is particularly true for last column of the tables which captures the role of the ideas gap. This gap is further narrowed which makes apparent that the large income gaps must be explained mainly by objects gap.

Finally, Table 5 reports the decomposition of variance based on equation (52). Results are almost identical to those reported in Table 3. The explanatory power of the factor intensity term $\widetilde{X}$ increases marginally by 1 percentage point for all values of β/η. The only significant difference is that all covariance terms are positive in Table 5, while one covariance term is negative in Table 3. Overall, the microfounded model confirms that differences in frictions associated with the

[8] The value of r determines the level of q, but it is unconsequential for the results.

[9] Results are not sensitive to the assumed life expectancy of monopolies in the country at the frontier.

accumulation of knowledge p likely account for only 26 to 36% of the cross-country income variance.

5 Concluding Comments

Countries with higher factor intensities X tend to exhibit higher total factor productivity A. In our database, the covariance term accounts for around 35% of the cross-country dispersion of output per worker. What explains this large covariance? Models with exogenous TFP, such as the Solow model, provide no explanation for this relationship. However, the covariance term is crucial for understanding the ultimate causes of cross-country income differences. In fact, two landmark studies by Mankiw, Romer and Weil (1992) and KR (1997) arrived to opposite conclusions about the relative importance of X versus A partly because they impute the covariance term differently. Their way to assign the covariance is arbitrary because their underlying model, a neoclassical growth model, has no predictions about this covariance. In contrast, growth theory provides a natural explanation for the covariance between X and A. If technological progress is costly, then economies with more factors abundance can undertake more R&D activities, accumulate a larger stock of knowledge, and become more efficient. Thus, growth theory suggests that the covariance term, or part of it, must be assigned to X.

The focus of the paper is to assess the quantitative predictions of a recent second generation of growth models characterized by international spillovers and no scale effects. In these models, the extent to which factors abundance limit efficiency levels depend on the speed of diffusion of international knowledge. The slower the speed of diffusion, the smaller the role of international knowledge, and the larger the role of local factors in determining local efficiency levels. A way to quantitatively assess the speed of diffusion is through the speed of convergence. We show that these two speeds are closely and positively associated. We argue that the slow speed of convergence, extensively documented in the growth regression literature and confirmed by growth regressions consistent with our model, provides evidence that the speed of diffusion is slow. This is the basis for our main quantitative finding: that factor intensities play a major role in determining efficiency levels and income differences, a role significantly larger than what has been traditionally recognized. Alternatively, we find that most of the covariance between X and A is explained by X affecting A rather than the other way around. In fact, we find that if the A to X channel were strong, then the covariance term would be negative rather than positive. This is because, in the model, policies directed toward enhancing the accumulation of TFP, such as better patent protection, increase the fraction of monopolistic firms in the economy. The inefficiency associated with monopolies turns out to reduce the saving rate and therefore X.

Our quantitative finding is supportive of earlier findings by Mankiw, Romer and Weil (1992) who argue that differences in factor intensities are the main sources of income differences. However, the data, the model, and potential policy implications are completely different. Regarding the data, our human capital series are obtained using Mincerian equations, as suggested by KR (1997). Moreover, our model is an endogenous growth model with increasing returns and externalities rather than a neoclassical model. Therefore, there is a potentially large scope for policy intervention, both within countries and across-countries, a scope that is unclear in the Mankiw, Romer and Weil framework.

Naturally, X and A are not deep parameters in a well microfounded model. However, we show that a fully microfounded model in which the deep parameters are two distortions, one affecting the accumulation of capital and one affecting the accumulation of knowledge, reinforce the basic accounting result. Most differences in cross-country incomes are explained by distortions in the accumulation of rival factors of production rather than nonrival factors or knowledge.

Our analysis opens two important future research avenues. First, the R&D investment rates predicted by the model are at odds with the existing data, a problem already uncovered by KR (2004). Whether the problem is measurement of R&D investment, or the production technology used to construct Solow residuals, further research is warranted. Second, in our model accumulation of physical capital and TFP are endogenous, while the level of human capital is exogenous. This is a simplification also made by KR (2004) in order to focus on a single engine of growth. Future work should endogenize human capital and TFP simultaneously, and address issues that arise when there are two growth engines.

Appendix

A Empirical Implementation

Define $\overline{y}_t = Y_t / \left(hL_t \left(A_t^*\right)^{\beta} \right)$, $\overline{k}_t = K_t / \left(hL_t \left(A_t^*\right)^{\beta} \right)$ and $\overline{a}_t = A_t / A_t^*$. Using balanced growth restrictions, equation (9) in the text reads

$$\dot{A}_t = dA_t^{*\eta} A_t^{-(\eta+\beta-1)} s_R Y_t / L_t = dh\overline{a}_t^{-(\eta+\beta)} s_R \overline{y}_t A_t$$

and since by definition $\dot{A}_t/A_t = \dot{A}_t^*/A_t^* + \dot{\overline{a}}_t/\overline{a}_t = g + g_{\overline{a}_t}$, then

$$\frac{\dot{A}_t}{A_t} = dh\overline{a}_t^{-(\eta+\beta)} s_R \overline{y}_t = g + g_{\overline{a}_t}.$$

Using this result, our Solow model with endogenous TFP from Section 3 can be summarized by the following three equations

$$\overline{y}_t = \overline{k}_t^{\alpha} \overline{a}_t^{(1-\alpha)\beta}, \tag{53}$$

$$\dot{\overline{k}}_t = s\overline{y}_t - (g_L + \beta g + \delta)\,\overline{k}_t,$$

$$g_{\overline{a}_t} = dh\overline{a}_t^{-\eta-\beta} s_R \overline{y}_t - g.$$

This system can be reduced to the following two equations:

$$g_{\overline{k}_t} = s\left(\frac{\overline{a}_t^{\beta}}{\overline{k}_t}\right)^{1-\alpha} - (g_L + g + \delta)\,,$$

$$g_{\overline{a}_t} = dh\overline{a}_t^{-\eta} s_R \left(\frac{\overline{k}_t}{\overline{a}_t^{\beta}}\right)^{\alpha} - g.$$

Log-linearization of these two equations around the balanced growth path yields

$$g_{\overline{k}_t} \simeq (g_L + g + \delta)\,(1-\alpha)\left(\beta\frac{\triangle\overline{a}}{\overline{a}^*} - \frac{\triangle\overline{k}}{\overline{k}^*}\right)$$

$$g_{\overline{a}_t} \simeq g\left(-\,(\eta + \alpha\beta)\,\frac{\triangle\overline{a}}{\overline{a}^*} + \alpha\frac{\triangle\overline{k}}{\overline{k}^*}\right). \tag{54}$$

In addition the two preceding equations together with (53) imply

$$g_{\overline{y}_t} = -\,(1-\alpha)\,(g_L + \delta + g\,(1-\beta))\,\frac{\triangle\overline{y}}{\overline{y}^*} + (1-\alpha)\,\beta\,[g_L + \delta + g\,(1-\beta-\eta)]\,\frac{\triangle\overline{a}}{\overline{a}^*}. \tag{55}$$

Equations (54) and (55) form a system of log-linearized differential equations. In matrix form this system can be written as:

$$\begin{bmatrix} \frac{d\ln(\overline{y}_t)}{dt} \\ \frac{d\ln(\overline{a}_t)}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} -\,(g_L + \delta + g\,(1-\beta))\,(1-\alpha) & (1-\alpha)\,\beta\,[g_L + \delta + g\,(1-\beta-\eta)] \\ g & -g\,(\eta+\beta) \end{bmatrix}}_{A} \begin{bmatrix} \ln(\overline{y}/\overline{y}^*) \\ \ln(\overline{a}/\overline{a}^*) \end{bmatrix},$$

where the determinant of matrix A is $\det A = (g_L + \delta + g)\,(1-\alpha)\,\eta g > 0$. The eigenvalues of the system are solution to

$$\lambda^2 + \underbrace{[(g_L + g + \delta)\,(1-\alpha) + (\eta + \alpha\beta)\,g]}_{b>0}\lambda + \underbrace{(g_L + g + \delta)\,(1-\alpha)\,\eta g}_{c>0} = 0.$$

The eigenvalues satisfy:

$$\begin{aligned} 2\lambda_i \;=\; & -\left[(g_L + g + \delta)\,(1-\alpha) + (\eta + \alpha\beta)\,g\right] \pm \\ & \left[\left[(g_L + g + \delta)\,(1-\alpha) + (\eta + \alpha\beta)\,g\right]^2 - 4\,(g_L + g + \delta)\,(1-\alpha)\,\eta g\right]^{1/2}. \end{aligned}$$

It can easily be checked that both roots are negative (or at least will have negative real parts if η is too large and the roots are complex), so that the system is stable. Note that $\lambda_i \to 0$ as $\eta \to 0$. We find, numerically, that the eigenvalues are increasing (in absolute terms) in η. Thus, a slow speed of convergence requires a low value of η.

Given the two eigenvalues, the log-linearized solutions take the form

$$\ln(\overline{y}_t) = \ln(\overline{y}^*) + v_{11}c_1 e^{\lambda_1 t} + v_{12}c_2 e^{\lambda_2 t},$$

$$\ln(\overline{a}_t) = \ln(\overline{a}^*) + v_{21}c_1 e^{\lambda_1 t} + v_{22}c_2 e^{\lambda_2 t},$$

where v_{ij} are elements of the eigenvector matrix V as given by $V^{-1}AV = D$, where D is the diagonal matrix of eigenvalues. The i column of V is given by $Av_i = \lambda_i v_i$, or

$$\begin{bmatrix} -\left(g_L + \delta + g(1-\beta)\right)(1-\alpha) & (1-\alpha)\beta\left[g_L + \delta + g\left(1-\beta-\eta\right)\right] \\ g & -g\left(\eta+\beta\right) \end{bmatrix} \begin{bmatrix} v_{1i} \\ v_{2i} \end{bmatrix} = \lambda_i \begin{bmatrix} v_{1i} \\ v_{2i} \end{bmatrix}.$$

Normalizing $v_{1i} = 1$, and using only the second row of this system yields $v_{2i} = \frac{g}{\lambda_i + g(\eta+\beta)}$. Substituting this result into the system above gives:

$$\ln(\overline{y}_t) = \ln(\overline{y}^*) + c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}; \tag{56}$$

$$\ln(\overline{a}_t) = \ln(\overline{a}^*) + \frac{g}{\lambda_1 + g\left(\eta+\beta\right)} c_1 e^{\lambda_1 t} + \frac{g}{\lambda_2 + g\left(\eta+\beta\right)} c_2 e^{\lambda_2 t}.$$

Note that asymptotically, $\ln(\overline{y}_t) \simeq \ln(\overline{y}^*) + c_1 e^{\lambda_1 t}$ since $|\lambda_1| < |\lambda_2|$. Constants c_1 and c_2 can be solved for from the following two equations:

$$c_1 + c_2 = \ln(\overline{y}_0/\overline{y}^*),$$

$$c_1 e^{\lambda_1 T} + c_2 e^{\lambda_2 T} = \ln(\overline{y}_T/\overline{y}^*),$$

where T is an alternative year for which we have information. These two equations can be simplified to

$$c_1 = \frac{1}{e^{\lambda_1 T} - e^{\lambda_2 T}} \left[\ln(\overline{y}_T/\overline{y}^*) - e^{\lambda_2 T} \ln(\overline{y}_0/\overline{y}^*)\right]$$

$$c_2 = \frac{1}{e^{\lambda_1 T} - e^{\lambda_2 T}} \left[e^{\lambda_1 T} \ln(\overline{y}_0/\overline{y}^*) - \ln(\overline{y}_T/\overline{y}^*)\right].$$

Substituting these two results into (56) one obtains equation (23) in the text.

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Table 1. Productivity Calculations: Ratios to U.S Values
Exogenous TFP Models

Country	Y/L	Ratio	
		X	A
United States	1.000	1.000	1.000
Canada	0.791	1.009	0.784
Italy	0.892	0.716	1.246
France	0.789	0.858	0.919
United Kingdom	0.709	0.709	1.001
Hong Kong	0.903	0.789	1.144
Singapore	0.754	0.840	0.897
Japan	0.663	1.102	0.602
Mexico	0.374	0.547	0.685
Argentina	0.449	0.676	0.665
India	0.095	0.341	0.278
China	0.086	0.464	0.185
Kenya	0.045	0.288	0.157
Zaire	0.011	0.291	0.038
Average, 86 countries	0.336	0.531	0.546
Standard deviation	0.287	0.261	0.315
Average, 5 poorest countries	0.026	0.228	0.121
Correlation with Y/L (logs)	1.000	0.846	0.928
Correlation with A (logs)	0.928	0.588	1.000

Table 2. Productivity Calculations: Ratios to U.S. Values
Extended Solow Model

	Average 5 poorest:		Average 87 countries:	
	Ratio of		Ratio of	
Model	$\hat{X}$	$\hat{A}$	$\hat{X}$	$\hat{A}$
$\beta/\eta = 0.00$	0.23	0.12	0.54	0.56
$\beta/\eta = 0.50$	0.11	0.27	0.43	0.77
$\beta/\eta = 0.60$	0.10	0.31	0.41	0.83
$\beta/\eta = 0.85$	0.07	0.47	0.38	1.00

Table 3. Variance Decomposition
Extended Solow Model

Model	% contribution to variance of *log(Y/L)*		
	$\hat{X}$	$\hat{A}$	Covariance
$\beta/\eta = 0.00$	39	61	35
$\beta/\eta = 0.50$	59	41	20
$\beta/\eta = 0.60$	63	37	15
$\beta/\eta = 0.85$	73	27	-3

Table 4. Productivity Calculations: Ratios to U.S. Values
Varieties Model

	Average 5 poorest:		Average 87 countries:	
	Ratio of		Ratio of	
Model	$\tilde{X}$	$\tilde{A}$	$\tilde{X}$	$\tilde{A}$
$\beta/\eta = 0.00$	0.23	0.12	0.54	0.56
$\beta/\eta = 0.50$	0.10	0.29	0.40	0.83
$\beta/\eta = 0.60$	0.09	0.35	0.37	0.91
$\beta/\eta = 0.85$	0.06	0.56	0.32	1.16

Table 5. Variance Decomposition
Varieties Model

Model	% contribution to variance of *log(Y/L)*		
	$\tilde{X}$	$\tilde{A}$	Covariance
$\beta/\eta = 0.00$	39	41	35
$\beta/\eta = 0.50$	60	40	21
$\beta/\eta = 0.60$	64	36	16
$\beta/\eta = 0.85$	74	26	1

Figure 1. R&D as a Percentage of GDP
KR's (2004) Model versus Ours

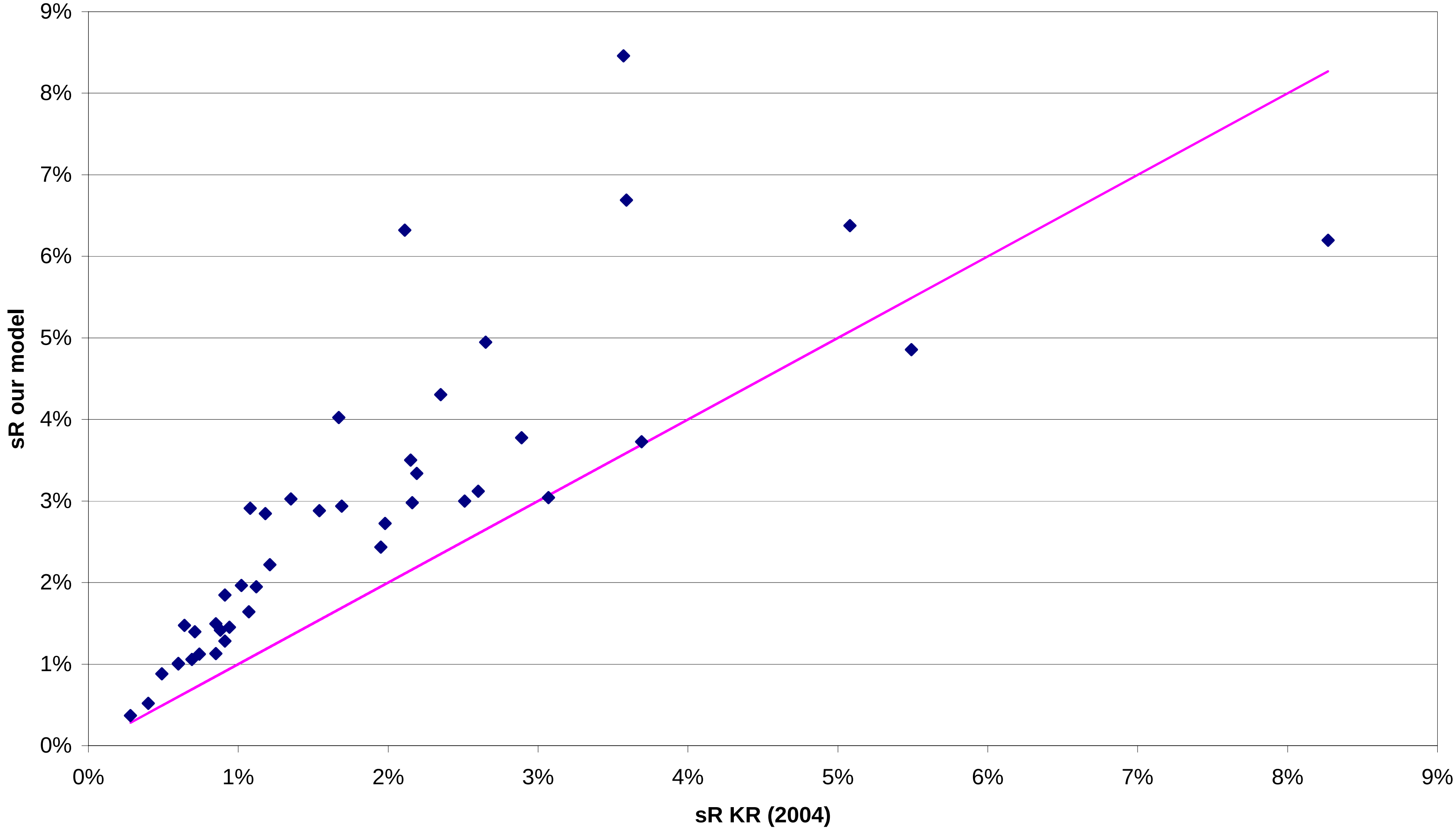

**Figure 2. R&D as Percentage of GDP: Models versus Data
OECD Countries**

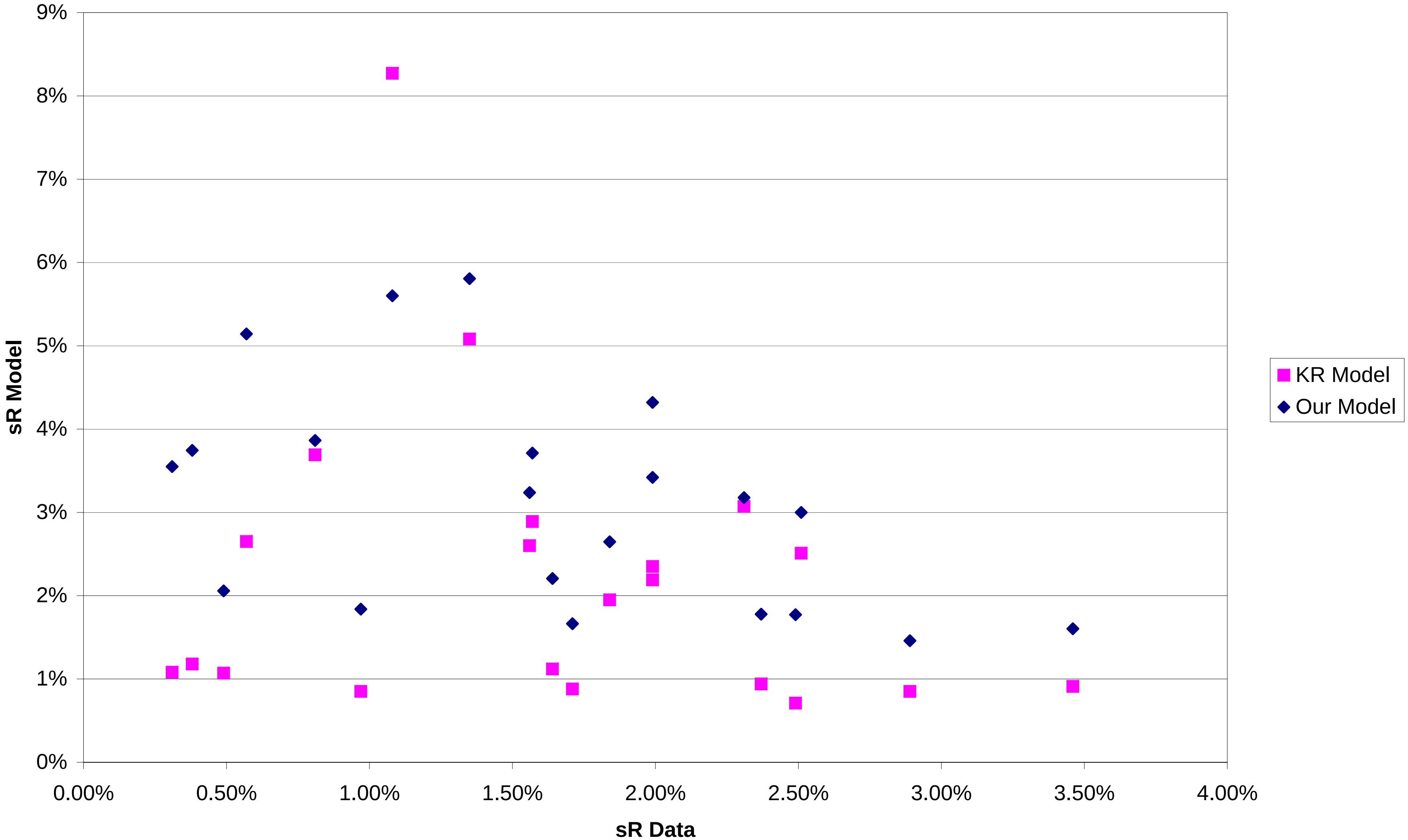

Figure 3. Implied Expected Life of a Monopoly
Microfounded Model

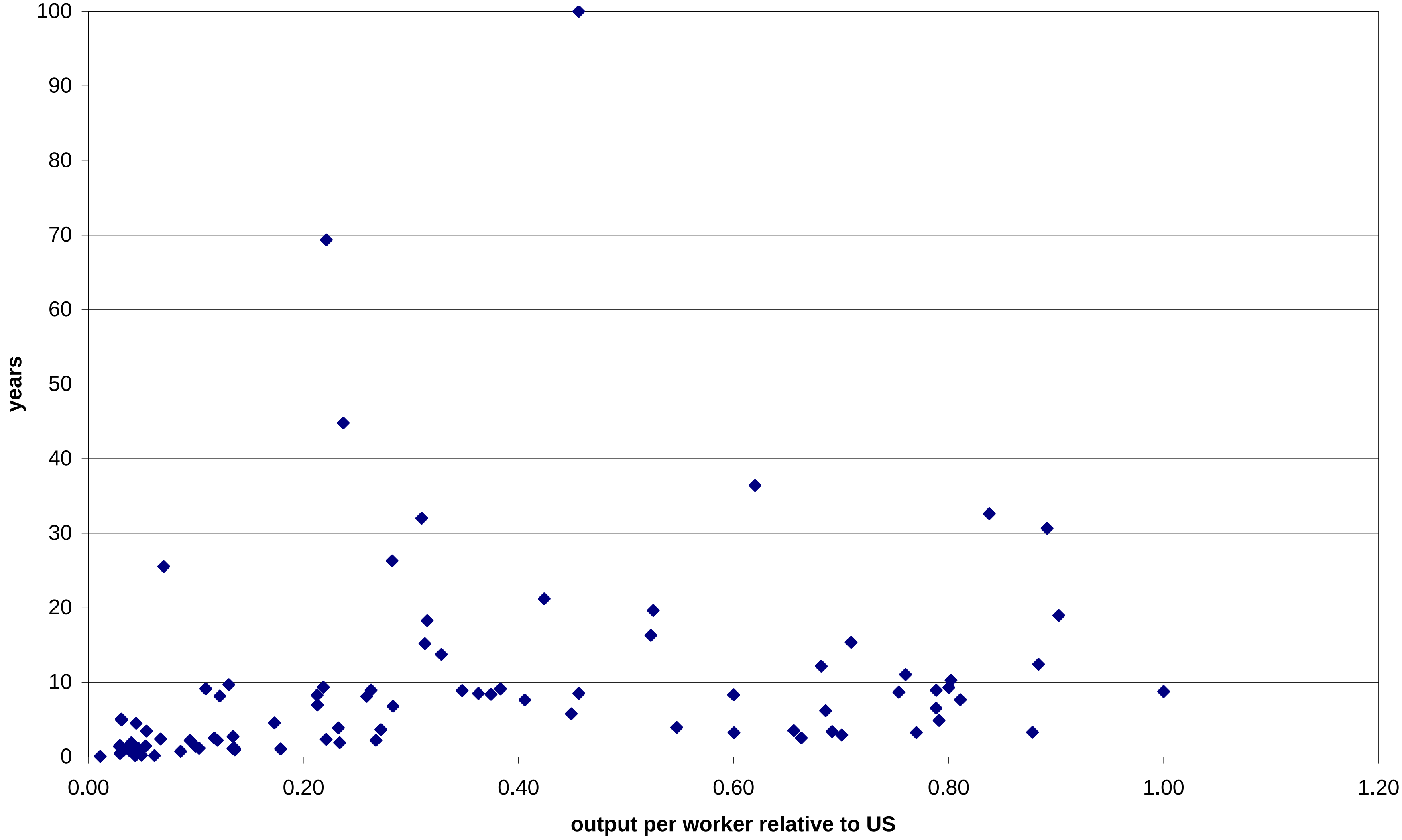

Figure 4. Implied Rate of Confiscation of Physical Capital
Microfounded Model