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Mortality Reductions, Educational Attainment, and Fertility Choice*

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Abstract

This paper explores the role of life expectancy as a determinant of educational attainment and fertility, both during the demographic transition and after its completion. Two points distinguish our analysis from the previous ones. First, together with the investments of parents in the human capital of children, we introduce investments of adult individuals in their own education, which determines productivity in the goods and household sectors. Second, we let child mortality and adult longevity affect the way parents value each individual child. Increases in adult longevity eventually raise the investments in adult education. Together with the higher utility derived from each child, this tilts the quantity-quality trade off towards less and better educated children, and increases the growth rate of the economy. Reductions in child mortality may have similar effects. The setup is consistent with the demographic transition and the recent behavior of fertility and educational attainment in "post-demographic transition" countries.

1 Introduction

Major demographic changes swept the world in the course of the last century. Life expectancy at birth rose from 40 years to more than 70 years. Total fertility rates plummeted from around 6 points to close to 2 points or below. Today, over 60 countries, comprising almost 50% of the world population, have fertility rates below replacement level (2.1), and the vast majority of people lives in countries where population is expected to stabilize within the next fifty years (Robinson and Srinivasan, 1997). Furthermore, several developed countries have experienced increasingly low fertility levels. These include Austria, Canada, Greece, Japan, and Spain, all of which have fertility rates below 1.5. In short, recent reductions in fertility do not seem to be restricted to experiences of demographic transition. Time and again, developed countries, believed to have finished their transition long ago, experienced increasingly low fertility levels.

This phenomenon, largely overlooked both empirically and theoretically, points to the necessity of understanding the recent behavior of fertility from a more general perspective, not restricted to the demographic transition. The goal of this paper is to analyze the role of life expectancy gains, determined from technical developments in health technologies, as the driving force behind the changes in fertility, educational attainment, and growth observed during the process of demographic transition and thereafter[1] . The major role attributed to mortality in the empirical literature on the demographic transition suggests that life expectancy changes are an independent driving force.[2] This idea is further supported by the striking stability of the cross-sectional relationship between life expectancy, fertility and schooling, as opposed to the changing relationship between income and these same demographic variables (this evidence is discussed in detail in Section 2). In this paper, we look at how changes in child mortality and adult longevity affect the incentives of individuals to have children and to invest in education, and what the consequences of these changes are to the process of economic development. Changes in life expectancy can help explain the reductions in fertility that characterize the demographic transition, and the changes in demographic variables that accompany economic growth.

Extensive work has been done in the last decade on the determinants of fertility, and the relation between fertility and investments in human capital. A large part of this literature tries to explain the demographic transition as a consequence of increased investments in human capital due to technological change (Azariadis and Drazen,1990; Galor and Weil, 1996, 1999, and 2000; Hansen and Prescott, 1998; and Tamura, 1996).[3] A second strand of literature analyzes how changes in child mortality affect fertility decisions, occasionally incorporating investments of parents in the human capital of children (Blackburn and Cipriani, 1998; Boldrin and Jones, 2002; Ehrlich and Lui, 1991; Kalemli-Ozcan, 1999; Kalemli-Ozcan , Ryder, and Weil, 2000; Meltzer, 1992; Momota and Fugatami, 2000; and Tamura, 2001).

This paper improves upon this literature by stressing the importance of distinguishing between child and adult mortalities, and by explicitly incorporating adult investments in human capital. This allows the model to address the recent phenomenon of small and decreasing fertility in developed countries, ignored by the literature cited here and incompatible with most of its results. In addition, it extends the analysis to explain the observed behavior of educational attainment, and reveals the potential importance of adult longevity in determining the evolution of the economy after the demographic transition.

Two assumptions distinguish our model from the previous ones. First, we let child mortality

[1] The direct welfare implications of the gains in life expectancy, and their impact on the evolution of cross-country inequality, are discussed in Becker, Philipson and Soares (2003), and Philipson and Soares (2002).

[2] See, for example, Heer and Smith (1968), Cassen (1978), Kirk (1996), Mason (1997), and Macunovich (2000). In short, the view is that "if there is a single or principal cause of fertility decline, it is reasonable to ascribe it to falls in mortality, which was the major cause of destabilization" (Kirk, 1996, p.379).

[3] Galor and Weil (1999) mention that reductions in mortality could increase investments in human capital and reduce fertility via the quantity-quality trade-off, but they do not develop the idea formally.

and adult longevity affect the way in which parents value each individual child, in much the same way that number of children does in the traditional literature. This assumption incorporates the idea that parents care about number of children surviving into adulthood, and extends it to later ages. Once one considers that individuals are not only concerned with the survival of their children, but also with the continuing survival of their whole lineage, this is a natural assumption. Specifically, we draw on the evolutionary biology literature and assume that the utility that parents derive from each child depends on the number of children, the child mortality rate, and the lifetime that each child will enjoy as an adult. Acknowledging the importance of mortality to the way in which parents value each child has important consequences in terms of fertility choices.

Second, we incorporate explicitly the distinction between investment of parents' in the human capital of children and investment of adult individuals in their own human capital. This generates direct predictions about educational attainment and helps distinguish between the economic impacts of changes in adult and child mortality. This approach is also more realistic and brings the theory closer to the empirical accounts that justify the impacts of life expectancy on educational investments (see, for example, the discussion on rates of return in Meltzer, 1992).

These two assumptions play central roles in the mechanics of the model. Briefly, increases in adult longevity eventually raise the investments in education, which increase the productivity of individuals both in the labor market and in the household sector. Also, higher life expectancy tilts the quantity-quality trade-off towards less and better educated children and tends to move the economy out of a "Malthusian" equilibrium.[4] Once the economy abandons the "Malthusian" regime, increases in adult longevity reduce fertility, increase educational attainment, and increase the growth rate of the economy (or the steady state level of consumption). Reductions in child mortality may have similar effects. This setup is consistent with the demographic transition and with the recent behavior of fertility in "post-demographic transition" countries. In addition, it reconciles theory with the evidence on the changing relationship between income and several demographic variables.

Recent work suggests that individuals' predictions of their own life expectancies are considerably accurate, and react to exogenous events in consistent ways (see Hamermesh, 1985; Hurd

[4] The issue of fertility choice in underdeveloped economies is controversial. Nevertheless, evidence shows that there is always some margin of choice. Several kinds of actions taken in 'pre-modern' societies affect fertility outcomes, including marriage patterns, breast feeding habits, abortion, and sexual practices (see Demeney, 1979; Caldwell, 1981; Kirk, 1996; and Mason, 1997).

In relation to health, although some actions taken at the individual level affect mortality, our interest here is focused on the gains in life expectancy observed in the last two centuries, which were largely due to scientific and technical developments. At the individual level, these were partly exogenous. These gains were also exogenous to less developed countries, which experienced mortality reductions independent of improvements in economic conditions. The gains in life expectancy in less developed countries are thought to be consequence of the absorption of knowledge generated elsewhere and of the help provided by international aid programs (see Preston, 1975 and 1980; Kirk, 1996; and Becker, Philipson, and Soares, 2003).

and McGarry, 1997; and Smith et al, 2001). Therefore, the role of life expectancy in explaining changes in behavior may indeed be empirically relevant. Section 4 discusses evidence supporting the idea that recent changes in life expectancy were largely independent of improvements in economic conditions. In addition, it argues that the historical experiences of demographic transition display patterns consistent with the predictions of the model. Life expectancy gains seem to be a driving force behind the changes observed in the other variables.

The structure of the remainder of the paper is outlined as follows. Section 2 motivates the analysis by presenting a very simple but striking fact: while the cross-sectional relationship between income and some key demographic variables (life expectancy, fertility, and schooling) has been consistently shifting in the recent past, the relationship between life expectancy and fertility and schooling has remained considerably stable. This observation suggests that there is a dimension of changes in life expectancy that is not explained by material development (income), but that seems to explain changes in fertility and educational attainment. Section 3 presents the model. It starts by describing the structure of the model, and draws on the evolutionary biology literature to motivate the assumptions related to preferences for children. Following, it discusses the effects of changes in adult longevity and child mortality. Finally, it introduces the assumption of decreasing returns to human capital, and shows that the only change it introduces in the model is the elimination of long run growth. Under some normality conditions, results that previously held for growth rates, in this case hold for income levels, and the behavior of demographic variables remains unchanged. Section 4 discusses the causes of recent changes in mortality, the demographic transition, and the behavior of fertility and educational attainment after the transition. The historical evidence is consistent with the predictions and hypotheses of the model. The final section summarizes the main findings of the paper.

2 Recent Changes in Life Expectancy, Educational Attainment, and Fertility

The growth literature usually looks at income as a variable either driving or summarizing all the changes in relevant dimensions of development. In this perspective, gains in income per capita improve nutrition and demand for health, which reduces mortality rates; income gains also change the quantity-quality trade off in terms of number and education of children, which reduces fertility and increases human capital investment. Statements like these are common, and it is fair to say that they give an accurate description of the consensus regarding the main changes taking place during the process of development.

This view, although right, does not give a complete picture of reality. Recently, the relationship between income and some key demographic variables – such as life expectancy, educational attainment, and fertility – has been clearly unstable. Figures 1 to 3 plot the 1960 and 1995 cross sectional relation between income per capita (GNP per capita in constant 1995 US$) and, respectively, life expectancy at birth, total fertility rate, and average schooling in the population aged 25 and above.[5] To concentrate on economies that share the same demographic regime, the figures refer only to countries that had already started the demographic transition in 1960.[6]

Figure 1 shows that, for constant levels of income, life expectancy has been rising. This phenomenon was first noticed by Preston (1975), who analyzed data between 1930 and 1960. For lower levels of income, life expectancy at birth has increased by more than five years in the period between 1960 and 1995. This means, for example, that a country with GNP per capita of US$5,000 in 1995 had life expectancy roughly 10% higher than a country with the same income in 1960.

Figures 2 and 3 tell analogous stories for fertility and educational attainment. For constant levels of income, fertility has been falling and educational attainment has been rising. Reductions in fertility have been of up to 2 points for countries with per capita income around US$3,000. Gains in average schooling have been usually over 1 year.

The figures illustrate that there is a dimension of change in these three demographic variables that is not related to changes in income. It seems natural to ask whether the changes in life expectancy, education, and fertility are related to each other. An insight in this direction is gained by looking at the relation between life expectancy and the other two variables.

Figures 4 and 5 plot the 1960 and 1995 cross sectional relation between life expectancy and, respectively, fertility and educational attainment. Figure 4 shows that the curves for life expectancy and fertility are close to two segments of a single nonlinear function that remains stable throughout the period (portrayed as the darker line). This is even more obvious for the relation between life expectancy and educational attainment. Figure 5 shows that the curves for these two variables merge into each other for the region where observations for both periods exist. In both cases, countries seem to be sliding through time on a stable curve.

[5] The general results illustrated in Figures 1 to 5 do not depend on the specific statistics used, nor on the presence of any particular country in the sample. Income, fertility and life expectancy are from the World Development Indicators data set, and average schooling is from the Barro and Lee data set. Data are five-year averages centered on the reference year. The concave curves fitted to the data assume a logarithmic relation of the form $y = \alpha + \beta \ln(x)$, and the convex curves assume a power relation of the form $y = \alpha x^{\beta}$. The curves in Figure 5 are third order polynomials.

[6] A more precise reason for restricting the sample is given in the theoretical section. Empirically, some objective criterion defining whether a country already started the demographic transition has to be chosen. Our choice is the cutoff point "countries that had life expectancy at birth above 50 years in 1960," also to be justified later on. The results do not depend on the specific criterion chosen, and the countries included are the ones commonly regarded as having started the transition in the 1960's or before.

The evidence suggests that, for constant levels of income, life expectancy is rising, fertility is declining, and educational attainment is increasing. At the same time, changes in fertility and schooling are following very closely the changes in life expectancy. This has been happening in such a way that, for given life expectancy, fertility and schooling have remained roughly constant.

In short, there is a dimension of change in life expectancy that is not associated with income, but that seems to be associated with fertility and educational attainment. In addition, while fertility and education are direct objects of individual choice, life expectancy has a large exogenous component, related to scientific knowledge and technological development. This reasoning suggests that exogenous reductions in mortality, together with a stable behavioral relationship between life expectancy, educational attainment, and fertility, may be behind the observed changes. In what follows, we develop a theory along these lines. Our goal is to explain the evidence discussed here, together with the triggering of the demographic transition, as being determined by exogenous gains in life expectancy.

3 The Model

3.1 Structure of the Model

Consider an economy inhabited by adult individuals who live for a deterministic amount of time. Individuals invest in their own education, work, consume, have children, and invest in the education of each child. The model is a "one-sex" model. We abstract from uncertainty considerations to concentrate on the impact of adult longevity and child mortality on the direct economic incentives at the individual level. To make the model treatable, we also abstract from physical capital. Individuals, or households, have an endowed level of 'basic' human capital – determined from previous generations' decisions – based on which they decide on how much to invest in their own 'adult' education. Adult education determines productivity in the labor market and in the household sector. Households possess backyard technologies for producing adult human capital, goods, and basic human capital, and they decide on how to allocate their time across these activities in order to maximize utility.

A fraction β of the children born dies before reaching adulthood. Childhood is an instantaneous phase: as soon as individuals are born they face the child mortality rate and, if survivors, become adults. There is no decision to be made as a child. Adults live for T periods. They derive utility from their own consumption in each period of life ($\frac{c(t)^{\sigma}}{\sigma}$), and from the children they have.

We adopt a paternalistic approach and assume that adults are concerned directly with the level of basic human capital of their children, via a constant elasticity function $\frac{h_c^{\alpha}}{\alpha}$. The literature on fertility usually assumes that the value that parents place on the human capital of each child is an increasing and concave function of the number of children. Since we incorporate longevity and child mortality into the analysis, we also take into account the effect of these variables. We assume that, together with the number of children, parents also care about how long each child will live, in such a way that the discount factor applied to children's human capital is a function $\rho(n,T,\beta)$ of number of children (n), child mortality (β), and adult longevity (T). The function $\rho(n,T,\beta)$ is assumed be increasing and concave on n and T, and decreasing and concave on β. Additionally, we assume that there is a tendency towards satiation in terms of number of children, such that[7] $\rho_n(n,T,\beta)$ decreases rapidly as n increases, and eventually approaches zero. The specific assumption is that $\varepsilon(n,T,\beta) = \frac{\rho_n(n,T,\beta)n}{\rho(n,T,\beta)}$ – the elasticity of the altruism function ρ in relation to n – is decreasing, such that $\rho_{nn} - (\rho_n/\rho)(\rho_n - \rho/n) < 0$, and $\rho_n(n,T,\beta) = 0$ for n large enough. This captures the idea that there are natural constraints to the bearing of children, and it is biologically impossible for a woman to have an arbitrarily large number of children during her reproductive life. Specific assumptions regarding the interaction of n, T, and β inside $\rho(n,T,\beta)$ are very important in determining the behavior of fertility. This discussion is saved until the next section.

The utility function can be written as

$$\int_0^T \exp(-\theta t)\frac{c(t)^\sigma}{\sigma}dt + \rho(n,T,\beta)\frac{h_c^\alpha}{\alpha},$$

where θ is the subjective discount rate, and $0 < \sigma, \alpha < 1$. The first term denotes the utility that parents derive from their own consumption, and the second term denotes the utility that they derive from their children.[8]

Individuals face goods and time constraints: they have to allocate their total lifetime (T) between working (l), raising children (b), and investing in their own education (e); and they have to allocate their lifetime income (y) between their own consumption ($c(t)$ per period) and fixed costs of having children (f). Borrowing from future generations and bequests are not allowed. The time and goods constraints are given, respectively, by

[7] Throughout the paper, $f_z(z)$ denotes the partial derivative of $f(z)$ in relation to z. When the context is clear, we save on notation by writing f_z instead of $f_z(z)$.

[8] Alternatively, if we assume that parents enjoy having children only to the extent that they share part of their lifetime, and that adults have children at age τ, the second term in the expression has to be integrated over time from τ to T, and discounted at the rate θ. Another possible variation of the model would distinguish between parent's adult longevity and children's adult longevity. In this case, we could write T_p and T_c and analyze only the impacts of changes in T_c. Since our focus is on the effect of permanent, technologically induced, changes in β and T, we do not make this distinction. In any case, both variations of the model deliver qualitative predictions similar to the ones obtained here.

$$T \geqslant l + bn + e, \quad \text{and} \tag{1}$$

$$y \geqslant \int_0^T \exp(-rt)c(t)dt + \exp(-r\tau)nf \ , \tag{2}$$

where r is the interest rate, and it is assumed that adults have children at age τ.[9]

Adult human capital (H_p) is produced with the basic human capital that parents had once they entered adulthood (h_p), and time invested in adult education (e). Once adult human capital is accumulated, parents produce goods (or income) by using their stock of adult human capital (H_p) and time (l). Adult human capital, together with time invested in children (b), also determines the basic human capital that each child will inherit (h_c). We assume that human capital and time are complements in all production functions, such that adult human capital increases the productivity of time in the labor market and in the household sector, and basic human capital increases the productivity of education in generating adult human capital. Production functions take on simple multiplicative forms, with constant returns to human capital:

$$\begin{aligned} H_p &= Aeh_p + H_o, \\ h_c &= DbH_p + h_o, \text{ and} \\ y &= lH_p, \end{aligned} \tag{3}$$

where D, A, H_o, h_o are non-negative constants, and h_p is given.[10] In section 3.5, we consider the implications of having decreasing returns to human capital in this model.

This setup distinguishes between basic and adult human capital: h denotes human capital formed during childhood, in which parents can invest, related to basic education and skills, and emotional development; H denotes human capital obtained during young adulthood, related, for example, to college, graduate education, and professional training. The specification of the production functions takes into account the cumulative nature of the process of investment in these different forms of human capital. We assume that individuals enter adulthood with a given level of basic human capital (h_p), and then, by deciding on how much to invest in their own

[9] In reality, part of the costs of having children should apply to children born, and part should apply to children who survive into later ages. But the main economic effect of child mortality comes from the wedge between resources spent on children, and resources wasted due to mortality. Since distinguishing between expenditures on children born and children surviving would greatly complicate the model, we maintain this formulation. In general, specifying costs in terms of surviving children reduces the effects of changes in child mortality, because in this case it also affects the cost of acquiring survivors.

[10] There is no technological parameter in the production function of y because such parameter can be reabsorbed via relabeling of A, D, and H_p.

education, choose a level of adult human capital (H_p). h_c is the level of basic human capital that parents give to each of their children.[11] H_o and h_o denote the levels of adult and basic human capital that individuals have, even in the absence of explicit investments in education. These can be thought of as determined from innate skills and natural learning throughout life, as related to communication, hunting and gathering, and primitive household production techniques.

To concentrate on the issues of interest, we depart from the general formulation and introduce some simplifying assumptions. Since our main concern is the long run behavior of the economy, particularly the inter-generational fertility and human capital decisions, we abstract from life cycle considerations by assuming that subjective discount rates and interest rates equal zero. Given the time-separability of the utility function, this implies constant consumption throughout life.

Incorporating these hypotheses, the objective function and the goods constraint can be rewritten as

$$
\begin{aligned}
& T\frac{c^{\sigma}}{\sigma} + \rho(n, T, \beta)\frac{h_c^{\alpha}}{\alpha}, \quad \text{and} && (4)\\
lH_p \quad \geqslant \quad & Tc + fn. && (5)
\end{aligned}
$$

The full-income constraint is obtained by substituting for l in the budget constraint:

$$TH_p = Tc + fn + (bn + e)H_p. \tag{6}$$

Full lifetime income can be allocated between goods ($Tc + fn$), time spent raising children, and time spent investing in education, where the opportunity cost of time is given by its productive value (H_p). The problem of the individual is to maximize 4 subject to 6 and to the production functions in 3. This is the benchmark model that guides our discussion.

3.2 Preferences Over Fertility and Lifetime of Children

Two assumptions concerning the altruism function ρ are essential in the analysis. First, we assume that parents care for the life expectancy of children, possibly including ages beyond reproduction. Second, we assume that parents see number of children and lifetime of each child as substitutes, so that increases in life expectancy reduce the marginal utility of number of children. It is common to see these assumptions applied informally to discussions on child mortality, but their explicit consideration and their extension to later ages are new to our approach.

[11] To keep notation to a minimum, we are not indexing by generations, and are distinguishing parent's and children's basic human capital by the subscripts p and c. These are obviously related across generations. If we let i index different generations, $h_p^{i+1} \equiv h_c^i$.

We justify both assumptions with arguments from the evolutionary biology literature. In this tradition, preferences today are inherited from the effect of natural selection on two million years of human history in hunter-gatherer societies (see discussion in Bergstrom, 1996; Robson, 2001 and 2002; and Robson and Kaplan, 2003). Within this context, preferences that maximized evolutionary fitness were the ones that eventually dominated and were inherited by current populations.

An obvious critique to the incorporation of adult longevity into preferences for children might be that, from an evolutionary perspective, preferences over survival beyond reproductive ages cannot be justified, since they do not affect the reproductive success of individuals. But contrary to this simplest biological view, fitness does not depend exclusively on number of offspring (fertility). Beauchamp (1994), Robson (2001 and 2003), and Kaplan and Lancaster (2003), argue that fitness depends both on quantity and quality of offspring. As fitness refers to the continuing survival of the lineage, fitness maximization implies maximization of the long-term production of descendents. Natural selection on offspring quantity and quality should maximize the number of offspring that survive to reproductive ages in conditions to reproduce, or, indirectly, the reproduction and survival of later generations (Kaplan and Lancaster, 2003).

Particularly in human population, fitness was also affected by individuals' survival into post-reproductive ages (Robson and Kaplan, 2003; and Kaplan and Lancaster, 2003). This was the case because hunter-gatherer life-style involved a dramatic intergenerational transfer of resources. Given the slow process of human maturing – due to the biological formation of the brain in early stages of life and long periods of learning-by-doing thereafter – children and adolescents constituted a significative drain on society, and their survival depended upon the production of energy surplus by other members (mature adults). The evidence presented in Robson and Kaplan (2003) suggests that, in hunter-gatherer societies, individuals only "repay" the entirety of their energy debt when they are almost 50 years old. Although from a traditional perspective "it is even mysterious why individuals should live beyond [*reproductive*] age, (...) the biological purpose of this is clear: it is to provide resources to offspring" (Robson and Kaplan, 2003, p.157). Without food storage, preferences acknowledging the importance of offspring's survival to ages beyond reproduction would guarantee this intergenerational transfer of resources in future generations, and maximize fitness in the long run.

This arguments asserts that preferences towards offspring life expectancy should naturally arise within a hunter-gatherer society, and that these preferences should eventually dominate the population. The simple fact that post-reproductive longevity did represent a dimension of fitness implies that there is an evolutionary basis for preferences over adult life expectancy to arise. This justifies including both β and T inside the function ρ, and deals with the first of our assumptions.

The second assumption refers to the substitutability between number of children and lifetime of each child.

The central idea is that there is a trade-off between quantity and quality of offspring in determining the fitness of any evolutionary strategy. The simple fact that fitness is determined by both quantity and quality of offspring, together with full use of resources, should imply such trade-off. Smith and Fretwell (1974) discuss the analytical aspects of this trade-off in terms of the maximization of fitness.

Virtually all the papers cited above mention the importance of this quantity-quality trade-off in determining the evolution of human preferences towards reproduction. Various examples from other species corroborate the presence of this trade-off in nature. These include variation in number of offspring and post-natal care to each offspring across different species (Kaplan and Lancaster, 2003), as well as variation in number of offspring and different dimensions of quality within species. In a study on bird reproduction, Lack (1968) shows that there is a within species trade-off between clutch size and egg size (or weight and survival of the newborn), and that this trade-off is more intense for species with longer breeding periods. Smith and Fretwell (1974) discuss evidence on a similar type of trade-off among mammalian species – including beavers, chimpanzees, and humans – due to the reduction in the energy available to each offspring as parental care has to be spread out among different litters.

Our assumption requires only the existence of this trade-off in nature, and the recognition that adult longevity can be seen as an additional dimension of offspring quality. The last part of the argument follows immediately from the discussion in the previous paragraph, where we argued that survival into adulthood affected fitness in earlier hunter-gatherer populations. In this case, natural selection would impose a trade-off between life expectancy and number of offspring (fertility) that, if recognized by preferences, would imply a dominant evolutionary strategy. This is the logic underlying a recent model developed by Robson (2003, section 3), in which preferences regarding life expectancy and fertility as substitutes arise as fitness maximizing and dominant in the long run. In his model, life expectancy and fertility arise as substitutes for purely biological reasons. But the sense in which they are substitutes in the induced preferences is exactly the same that we will have here: exogenous increases in life expectancy reduce the marginal utility of fertility.

In the remainder of the paper, we assume that preferences determined by evolutionary forces – throughout two million years of human life in a hunter-gatherer environment – were carried on to the ten thousand years of modern history. These preferences are defined over number of children and lifetime of each child, and regard these variables as substitutes. Increases in adult longevity and reductions in child mortality reduce the marginal utility of fertility ($\rho_{nT}(n,T,\beta) < 0$ and $\rho_{n\beta}(n,T,\beta) > 0$).

3.3 The Role of Adult Longevity

3.3.1 Static Implications of Longevity Gains

This section looks at the individual decision taking the initial level of basic human capital as given (h_p). Following, we discuss the implications of this decision process to the growth rate and dynamic behavior of the economy, and look at the properties of an equilibrium with zero growth and no investments in human capital.

Consider an equilibrium with growth. In this case, the parameters f, h_o, and H_o become asymptotically irrelevant. Substituting for h_c in the utility function and for H_p in the full-income constraint, the first order conditions (foc's) for, respectively, c, n, b, and e, are:

$$\begin{aligned} c^{\sigma-1} &= \frac{1}{Ah_pe}\lambda, && (7)\\ \rho_n(n,T,\beta)\frac{h_c^{\alpha}}{\alpha} &= b\lambda, && (8)\\ \rho(n,T,\beta)h_c^{\alpha-1}ADeh_p &= n\lambda, && (9)\\ \rho(n,T,\beta)h_c^{\alpha-1}ADbh_p &= \left(1-\frac{Tc}{Ah_pe^2}\right)\lambda; && (10)\end{aligned}$$

where λ is the multiplier on the full-income constraint.

Using equations 8 and 9 from the foc's, we get:

$$\frac{\rho_n n}{\rho} = \alpha. \tag{11}$$

Define $\varepsilon(n,T,\beta) = \frac{\rho_n(n,T,\beta)n}{\rho(n,T,\beta)}$, the elasticity of the altruism function ρ in relation to fertility (n). The expression above states that the agent will equate the elasticity of the altruism function to the constant elasticity of the h_c sub-utility: $\varepsilon(n,T,\beta) = \alpha$.

This expression determines the response of n to exogenous changes in T and β. The implicit function theorem yields

$$\frac{dn}{dT} = -\frac{\varepsilon_T(n,T,\beta)}{\varepsilon_n(n,T,\beta)} = -\frac{\rho_{nT}\rho n - \rho_T\rho_n n}{\rho n[\rho_{nn} - (\rho_n/\rho)(\rho_n - \rho/n)]} < 0, \tag{12}$$

where the sign comes from the assumptions of decreasing elasticity of ρ in relation to n ($\varepsilon_n < 0$),[12] and "substitutability" between n and T ($\rho_{nT} < 0$).

[12] Decreasing elasticity of ρ in relation to n can be restated as "strong" concavity of the function ρ, in the sense that $\rho_{nn} - (\rho_n/\rho)(\rho_n - \rho/n) < 0$.

The equalization of elasticities expressed in equation 11 comes from the fact that n and b enter multiplicatively in the objective function (via the sub-utility functions) and the constraint. But the simple expression obtained above hinges on the additional assumption of constant elasticity for the h_c sub-utility function. What this buys us is the independency of n in relation to all other exogenous variables apart from T and β. With a more general specification, h_c would show up in the right hand side of 11, and it would allow other exogenous variables to affect the optimal choice of n. But even in this case, the force working towards a negative relationship between n and T would still be present, though possibly weakened by adjustments on h_c. The important point here is the presence of T in the altruism function ρ, and the way in which T and n interact inside this function. This is the role played by the assumption that parents see number of children and lifetime of each child as substitutes.[13]

Using equations 7, 9, and 10:

$$
\begin{aligned}
Ah_p e^2 &= Tc + Ah_p ebn, && (13)\\
\rho {h_c}^{\alpha-1} D &= nc^{\sigma-1}. && (14)
\end{aligned}
$$

The constraint gives $Tc + Ah_p ebn = TAh_p e - Ah_p e^2$, which, with equation 13, yields

$$
e = \frac{T}{2}, \text{ and } \frac{de}{dT} = \frac{1}{2}. \qquad (15)
$$

Educational attainment increases with longevity. This should be expected, since increases in longevity increase the period over which the returns from investments in education can be enjoyed. Technological parameters, such as A and D, do not appear in expression 15 because they affect costs and benefits of investments in education in symmetric ways.[14] Although we see e here as a measure of educational attainment, it can also be understood more generally as the specialization of individuals in the social division of labor. In this sense, this result is analogous to the one obtained by Becker (1985) and Becker and Murphy (1992), where increases in the total time available for labor market activities increase specialization.

With expressions 12 and 15, we can use equations 13 and 14 to determine the effect of exogenous changes in T on c and b. Appendix A.1 shows that $\frac{dc}{dT}$ and $\frac{db}{dT}$ can be either positive or negative, but they cannot be both negative at the same time. Either c or b must increase as T increases,

[13] More generally, as long as we have a specification where n and T have similar effects on ε, in the sense that $sign\{\varepsilon_n\} = sign\{\varepsilon_T\}$, the negative effect of T on n (or positive effect of β on n) will be obtained. This includes the case assumed here ($\varepsilon_n < 0$ and $\rho_{nT} < 0$), but is not restricted to it.

[14] This result is analogous to the one obtained by Ben-Porath (1967), regarding the effect of the price of services of human capital.

and both may increase at the same time. This is obvious once we realize that an increase in T also means an expansion in the constraint set. Since n goes down as T increases, and e increases only proportionally to T, the additional resources have to be 'consumed' either *via* increased b or increased c, and possibly both.

The specific signs of $\frac{dc}{dT}$ and $\frac{db}{dT}$ depend on the values of the parameters, but the forces at work can be understood by looking at the problem of the individual. We know that, as T increases, the shadow price of the time b invested in h_c (n) goes down, and the productivity of this investment goes up (e), so that h_c must increase in the new optimum, even though b itself may decrease. Depending on the magnitude of the reduction in this shadow price, and on the concavity of the sub-utility functions (σ and α), it will be worthwhile for the individual also to increase c together with h_c, or to let c decrease as h_c increases.

It is easy to show that h_c unequivocally increases as T rises. Since $h_c = ADh_p be$, we have that $\frac{dh_c}{dT} = ADh_p(b\frac{de}{dT} + e\frac{db}{dT})$, which gives:

$$\frac{dh_c}{dT} = \frac{D[T\Psi\frac{dn}{dT} - TDh_c^{\alpha-1}\rho_T + \frac{Ah_p}{2}(\sigma-1)nc^{\sigma-2}(\frac{T}{2}+nb)]}{(\sigma-1)n^2c^{\sigma-2} + D^2(\alpha-1)\rho h_c^{\alpha-2}T} > 0,$$

where $\Psi = c^{\sigma-1} - Dh_c^{\alpha-1}\rho_n - (\sigma-1)nc^{\sigma-2}Ah_p\frac{b}{2} > 0$ (see Appendix for proof).

It may seem counter-intuitive that c may actually go down as T increases, but it is important to keep in mind exactly what this theoretical experiment corresponds to. Here, we analyze an increase in T holding constant the level of basic human capital of parents (h_p). The result means that individuals entering adulthood that face a permanent increase in longevity will increase their own education and the basic education that they give to their children. And it may be the case that they reduce their own consumption in each period in order to be able to invest more in the children's human capital. This is different from analyzing what the effect of T on the consumption pattern across generations will be. As we will see now, the model predicts that increases in T increase the growth rate of consumption across generations.

3.3.2 Dynamic Implications of Longevity Gains

In order for a steady-state to exist in this economy, preferences have to be homothetic over c and h_c. This guarantees that, as the economy grows, individuals from different generations will make optimal decisions such that c and h_p will grow at the same constant rate, and b, n, e, and l will be constant. In our set up, this is equivalent to imposing the condition $\sigma = \alpha$ (see Appendix A.2 for discussion).[15]

[15] The existence of a steady-state is not essential. Nevertheless, it greatly simplifies the discussion. A formal analysis of the condition $\sigma = \alpha$ and of the consequences of deviating from this assumption is contained in Appendix A.2.

Assuming that this condition holds, the production function of h_c implies that the growth rate of basic human capital is given by[16] $(1+\gamma) = \frac{h_c}{h_p} = DAbe$. From the goods constraint, we have that $Ah_p le = Tc$, which implies that, in steady-state, c will grow at the same rate of h_p. The same will be true for the level of adult human capital (H_p), as can be seen from $H_p = Aeh_p$.

The effect of longevity gains on the growth rate of this economy is

$$\frac{d(1+\gamma)}{dT} = DA\left(b\frac{de}{dT} + e\frac{db}{dT}\right) > 0,$$

where the sign comes from the fact that $\frac{dh_c}{dT} > 0$. Longevity gains increase the steady-state growth rates of consumption and all forms of human capital across generations.

The intuition for this result is as follows. As longevity increases, incentives to invest in adult human capital increase, so that e – the amount of time devoted to parent's own education, or the educational attainment – increases. Once educational attainment and adult human capital (H_p) are higher, the individual becomes more productive in investing in children's human capital. The higher life span of each child also tilts the quantity-quality trade off towards less and better educated children, which reduces fertility. Together with the higher adult productivity in the household sector, this increases the level of basic human capital given to each child. Higher basic human capital, and more investments in adult education (higher educational attainment), end up increasing the growth rate of the economy.

The goal of this section is to stress the role played by adult longevity, through changes in the return to education and the way parents value each child, in the fertility and educational choices. Our approach shows that, under reasonable conditions, longevity gains can reduce fertility, increase educational attainment, and increase the growth rate of the economy.

3.3.3 The Malthusian Equilibrium

The model can also accommodate a so called Malthusian equilibrium, where investment in all forms of human capital are at corner solutions and fertility varies positively with consumption and production. This allows the characterization of the fertility transition as a natural consequence of the escape from this steady-state, caused by successive increases in adult longevity.

We reincorporate the goods fixed cost of children (f) and the lower bound levels of basic and adult human capital (h_o and H_o). In an equilibrium with consumption and all forms of human capital growing, these constant terms become irrelevant, and all conclusions discussed in the previous section hold. But in an equilibrium with zero growth and no investment in human

[16] If $DAbe < 1$, there is no growth in steady-state. In this case, H_o and h_o will be important in determining the human capital and consumption levels in equilibrium.

capital, these elements play a key role.

A Malthusian equilibrium is a situation where $h_p = h_o$, and the optimal choice of the individual implies $b = e = 0$. This equilibrium is characterized by the following foc's on $\{c, n, b, e\}$:

$$
\begin{aligned}
c^{\sigma-1} &= \frac{\lambda}{H_o}, && (16)\\
\rho_n \frac{h_o{}^{\alpha}}{\alpha} &= \frac{f}{H_o}\lambda, && (17)\\
\rho h_o^{\alpha-1} D H_o &< n\lambda, && (18)\\
0 &< \left[1 - \frac{A h_o (Tc + fn)}{H_o^2}\right]\lambda. && (19)
\end{aligned}
$$

We call this corner solution a Malthusian equilibrium because, in a situation like this, changes in productivity are positively related to changes in both consumption and fertility. Changes in productivity can be brought about, for example, by exogenous changes in H_o. Using the first two foc's and the constraint:

$$
\begin{aligned}
\frac{dn}{dH_o} &= \frac{f(\sigma-1)c^{\sigma-2}}{\rho_{nn}\frac{h_o^{\alpha}}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} > 0, \text{ and}\\
\frac{dc}{dH_o} &= \frac{\rho_{nn}\frac{h_o^{\alpha}}{\alpha}}{\rho_{nn}\frac{h_o^{\alpha}}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} > 0.
\end{aligned}
$$

Fertility and consumption respond positively to exogenous increases in productivity. With a minor modification, this setup can display all the features of a Malthusian regime, including its "positive checks" mechanism and the constant long run consumption level. In order to achieve that, we must incorporate the assumption of decreasing returns to population (scarcity of land) into the model. Defining P as aggregate population, this can be done by substituting H_o by some function $F(H_o, P)$, with $F_{H_o}(H_o, P) > 0$ and $F_P(H_o, P) < 0$. As before, changes in H_o capture exogenous shocks to productivity, while $F_P(H_o, P) < 0$ captures decreasing marginal product in the agriculture sector. For given H_o, average productivity decreases with total population.

This formulation implies that the benefits of exogenous gains in H_o are 'exhausted' in the long run by the increased population generated by higher fertility. With time, n returns to its long run equilibrium – constant population, such that $(1-\beta)n = 1$ – which pins down the long run value of consumption. There are no long run improvements in living standards, and population grows only to the extent allowed by exogenous technological or natural shocks (changes in H_o). This modification of the model captures all the properties of what is known as a Malthusian regime, but to keep things simple we analyze the case where $F(H_o, P) = H_o$.

While the corner solution holds, changes in T will be associated with changes in c and n only. Working with the first two foc's and the constraint:

$$
\begin{aligned}
\frac{dn}{dT} &= \frac{f(\sigma-1)c^{\sigma-2}\frac{(H_o-c)}{T} - \rho_{nT}\frac{h_o^{\alpha}}{\alpha}}{\rho_{nn}\frac{h_o^{\alpha}}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} \lesseqgtr 0, \text{ and} \\
\frac{dc}{dT} &= \frac{\rho_{nn}\frac{h_o^{\alpha}}{\alpha}(H_o-c) + f\rho_{nT}\frac{h_o^{\alpha}}{\alpha}}{T\rho_{nn}\frac{h_o^{\alpha}}{\alpha} + f^2(\sigma-1)c^{\sigma-2}} > 0.
\end{aligned}
$$

Increases in longevity increase consumption and have an ambiguous effect on fertility. The ambiguous effect on fertility is due to the substitutability between T and n, and the absence of investments in human capital in this equilibrium. The "income" effect from the increased T tends to increase fertility, but the "substitution" effect ($\rho_{nT} < 0$) tends to reduce it. At low levels of income (or consumption), the income effect – first term in the numerator of dn/dT – tends to be larger in absolute value, so that $dn/dT > 0$. We assume this to be the case in the Malthusian equilibrium.

While stuck in this equilibrium, increases in longevity are associated with increases in fertility and consumption. But as T keeps growing, incentives to invest in both adult and basic human capital increase, so that the inequalities characterizing the Malthusian equilibrium are eventually broken. This is clear from the foc's. The two inequalities characterizing corner solutions on e and b can be written, respectively, as: $Th_oA/H_o < 1$ and $\varepsilon(n,T,\beta) > \alpha Df/h_o$. As T rises, the first inequality is eventually broken, so that individuals start investing in adult human capital. Also, as T rises, and n rises in response to it, ρ increases and ε is reduced, so that the second inequality also tends to be broken, and individuals start investing in their children's basic human capital.

The intuition for the escape from the Malthusian regime is the following. As adult longevity increases, returns from investment in adult education also rise, because of the longer period over which education is productive. If gains in adult longevity are large enough, parents start investing in their own education ($e > 0$). In addition, as adult longevity gains take place, fertility rises. Generally, depending on the properties of ρ, it could be the case that fertility would grow indefinitely and the corner solution on b would never be broken. The role played by the assumption that the elasticity of ρ is reduced as n increases is exactly to guarantee that, for n and T sufficiently large, fertility will stop increasing and investments in children's human capital will be undertaken ($b > 0$). If this assumption holds, and the minimum value of ε is not bounded above $\alpha Df/h_o$, sufficiently large adult longevity can always guarantee positive investments in adult and basic human capital (b and $e > 0$). After this threshold is reached, further increases in longevity trigger the demographic transition, and the economy moves to a sustained growth path. Appendix A.3.1 proves these claims.

When this transition happens, the economy enters in the dynamic process described in the previous sections, where consumption and human capital grow from one generation to the next, and fertility declines with further increases in longevity. In this case, the only engine behind the demographic transition and the escape from the Malthusian steady-state is the exogenous change in longevity. In the next section, we show that reductions in child mortality can play a similar role, though some differences exist.

3.4 The Role of Child Mortality

3.4.1 Child Mortality in the Equilibrium with Growth

We start by analyzing the static implications of child mortality reductions, and then discuss its effects on the growth rate of the economy and the possibility of escape from the Malthusian steady-state. First order conditions are given by equations 7 to 10, plus the constraint. The derivation follows the same steps outlined in section 3.3.1. As before, $\varepsilon(n,T,\beta)=\alpha$, so that we have

$$\frac{dn}{d\beta}=-\frac{\varepsilon_\beta(n,T,\beta)}{\varepsilon_n(n,T,\beta)}=-\frac{\rho_{n\beta}\rho n-\rho_\beta\rho_n n}{\rho n[\rho_{nn}-(\rho_n/\rho)(\rho_n-\rho/n)]}>0. \tag{20}$$

Remember that β refers to the mortality rate, so that $\rho_\beta<0$, $\rho_{n\beta}>0$, and reductions in child mortality are represented by reductions in β.

Investments in adult human capital depend only on adult longevity ($e=T/2$), which implies $\frac{de}{d\beta}=0$. With equation 7 and the constraint, this yields:

$$\begin{aligned}\frac{db}{d\beta} &= \frac{\Psi\frac{dn}{d\beta}-Dh_c^{\alpha-1}\rho_\beta}{D(\alpha-1)\rho\frac{h_c^{\alpha-1}}{b}+Ah_p(\sigma-1)n^2\frac{c^{\sigma-2}}{2}}<0, \text{ and}\\ \frac{dc}{d\beta} &= \frac{ADh_pn\rho_\beta h_c^{\alpha-1}}{2D(\alpha-1)\rho\frac{h_c^{\alpha-1}}{b}+Ah_p(\sigma-1)n^2c^{\sigma-2}}>0.\end{aligned}$$

In addition, since $h_c=DAh_peb$, we have $\frac{dh_c}{d\beta}=DAh_pe\frac{db}{d\beta}<0$.

In an equilibrium with growth, reductions in child mortality reduce fertility, increase investments in basic human capital, and leave adult educational attainment unchanged (so that h_c will increase). Parents' consumption is reduced in order to enhance investments in basic human capital.

The growth rate of the economy is given by $(1+\gamma)=DAeb$. So $\frac{d(1+\gamma)}{d\beta}=DAe\frac{db}{d\beta}<0$. Increases in child mortality reduce the steady-state growth rate of the economy, via reductions in investments in basic human capital.

In this case, all effects of child mortality work through fertility. As child mortality decreases and fertility is reduced, resources are freed up to be used either in producing c or h_c. But the reduction in n also reduces the shadow price of h_c in relation to c, and increases the marginal utility of h_c (via ρ), such that h_c is increased (via b), and consumption is reduced.

3.4.2 Child Mortality and the Malthusian Equilibrium

The Malthusian equilibrium is characterized by the foc's discussed in section 3.3.3. The corner solutions on e and b can be written, respectively, as $TAh_o/H_o < 1$ and $\varepsilon(n, T, \beta) > \frac{\alpha D f}{h_o}$. From the foc's and the constraint, changes in β have the following effects on n and c:

$$
\begin{aligned}
\frac{dn}{d\beta} &= \frac{-\frac{h_o^\alpha}{\alpha}\rho_{n\beta}}{\frac{h_o^\alpha}{\alpha}\rho_{nn} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} > 0, \text{ and} \\
\frac{dc}{d\beta} &= \frac{f\frac{h_o^\alpha}{\alpha}\rho_{n\beta}}{T\frac{h_o^\alpha}{\alpha}\rho_{nn} + f^2(\sigma-1)c^{\sigma-2}} < 0.
\end{aligned}
$$

Since child survival and fertility are substitutes, increases in child mortality lead to increases in the number of children. As β does not affect the resources constraint, increases in fertility take place at the expenses of reductions in consumption.

With a general function ρ, it is possible that reductions in child mortality end up reducing the total utility derived from children (after the adjustments in n). But the realistic case is the one where reductions in child mortality lead unequivocally to increased utility from children (see discussion in Appendix A.3.2). In this case, reductions in child mortality – followed by reductions in fertility – lead to increases in ρ and reductions in ε, increasing the return to investments in children. At first, as child mortality is reduced, investments in basic human capital are undertaken, but nothing happens to investments in adult human capital (first inequality). Only after basic human capital is accumulated from one generation to the next, incentives to invest in adult education rise (as h_p grows in $TAh_p/H_o < 1$). If child mortality reductions are large enough, the economy leaves the Malthusian equilibrium, and moves into a steady-state with growth and positive investments in human capital. These claims are proved and discussed in detail in the Appendix A.3.2.

3.5 Decreasing Returns to Human Capital

Throughout the paper we assume constant returns to human capital. This hypothesis is essential in generating long run growth in the equilibrium with positive investments in human capital, but it does not affect the responses of fertility and educational attainment to changes in adult longevity and child mortality. With decreasing returns to human capital, most of our results remain unchanged, and, under some normality conditions, the results that previously applied to growth rates now apply to income levels.

In the Malthusian regime, the economy is characterized by the absence of investments in human capital. In addition, the escape from such equilibrium depends only on the marginal returns to human capital. So the behavior of the economy in the Malthusian equilibrium is not be affected by the specific shape of the human capital production function. Therefore, we concentrate the discussion on the equilibrium with positive investments in human capital. In this section, we introduce decreasing returns to human capital in the model and analyze its impact on the long run behavior of the economy.

Since there are two types of human capital in the model, decreasing returns can be introduced in different ways. We choose the simplest one, which is also the most basic, in the sense that the decreasing returns are transmitted throughout the different uses of human capital.

As before, assume that adult human capital is produced with basic human capital and time invested in education. But now consider the case where there are decreasing returns to basic human capital, such that $H_p = Aeh_p^{\phi}$, where $0 < \phi < 1$. These decreasing returns are transmitted to other household technologies in the sense that $h_c = DbH_p = ADbeh_p^{\phi}$ and $y = lH_p = Aleh_p^{\phi}$.

This modelling choice is the most convenient one because, from the perspective of an individual entering adulthood with given h_p, the problem remains unchanged. Following the same steps discussed in section 3.3, it is easy to show that two main results still hold: $\varepsilon(n, T, \beta) = \alpha$ and $e = T/2$. This means that increases in adult longevity increase educational attainment and reduce fertility, and reductions in child mortality reduce fertility. Results related to b, c, and h_c, conditional on h_p, are also the same. In particular, $d(eb)/dT > 0$ and $db/d\beta < 0$, such that $dh_c/dT > 0$ and $dh_c/d\beta < 0$.

The difference lies in the long run behavior of basic human capital. As $h_c = ADbeh_p^{\phi}$, and the steady state implies constant e and b, there is no possibility of long run growth. Asymptotically, the economy converges to a constant level of basic human capital ($h_c = h_p$), given by $h^* = (ADb^*e^*)^{1/(1-\phi)}$, where the asterisk denotes steady state. This immediately implies constant adult human capital and consumption in the long run.

The effect of changes in adult longevity on the equilibrium levels of human capital is given by the following expressions:

$$\begin{aligned} \frac{dh^*}{dT} &= \frac{1}{1-\phi}(ADbe)^{\phi/(1-\phi)}AD(b\frac{de}{dT} + e\frac{db}{dT}) > 0, \text{ and} \\ \frac{dH^*}{dT} &= \frac{Ah^{*\phi}}{2} + Ae\theta h^{*\phi-1}\frac{dh^*}{dT} > 0. \end{aligned} \tag{21}$$

Similarly, the effect of changes in child mortality on the equilibrium levels of human capital is given by:

$$\begin{aligned}\frac{dh^*}{d\beta} &= \frac{1}{1-\phi}(ADbe)^{\phi/(1-\phi)}ADe\frac{db}{d\beta} < 0, \text{ and} \\ \frac{dH^*}{d\beta} &= Ae\phi h^{*\phi-1}\frac{dh^*}{d\beta} < 0.\end{aligned} \tag{22}$$

Increases in adult longevity or reductions in child mortality lead to increases in the long run stock of all forms of human capital. In the case of adult longevity, this happens mainly because of higher educational attainment, though investments in children may also increase. In the case of child mortality, it happens exclusively because of higher investments in children.

In equilibrium, consumption is given by

$$c^* = H^*\frac{l}{T} = H^*\frac{(T-b^*n^*-e^*)}{T} = H^*\left(\frac{1}{2}-\frac{b^*n^*}{T}\right).$$

Therefore, changes in T and β lead to the following changes in the long run level of consumption:

$$\begin{aligned}\frac{dc^*}{dT} &= \left(\frac{1}{2}-\frac{b^*n^*}{T}\right)\frac{dH^*}{dT} + \frac{H^*}{T}\frac{b^*n^*}{T}\left[1-\frac{d(b^*n^*)}{dT}\frac{T}{b^*n^*}\right], \text{ and} \\ \frac{dc^*}{d\beta} &= \left(\frac{1}{2}-\frac{b^*n^*}{T}\right)\frac{dH^*}{d\beta} - \frac{H^*}{T}\frac{d(b^*n^*)}{d\beta}.\end{aligned} \tag{23}$$

Theoretically, both expression can be either positive or negative, depending on the specific shape of preferences. But the realistic case is the one where increases in adult longevity or reductions in child mortality lead to increases in the long run level of consumption. As long as the total amount of time spent raising children (nb) decreases as fertility is reduced, which is the empirically relevant case, we have $\frac{dc^*}{dT} > 0$ and $\frac{dc^*}{d\beta} < 0$. In fact, even less is needed for the case of changes in adult longevity: as long as the elasticity of the total amount of time spent raising children in relation to longevity is not above unit, gains in longevity lead to increases in long run consumption.

Increased female participation in the labor market is probably the most obvious evidence of reduced demand for total time spent on children as fertility is reduced, even though time spent per child may well increase. In this case, the model with decreasing returns to human capital reproduces virtually every feature of the model outlined in previous sections, with the exception that results that previously held for growth rates now hold for consumption levels.

4 The Nature and Timing of Mortality Changes

The model predicts that a Malthusian economy experiencing increases in life expectancy ($(1-\beta)$ or T) would go through an initial phase with consumption increasing, fertility possibly changing in unpredictable ways – depending on the particular value of the parameters and the changes in T and β– and population increasing rapidly.[17] Population growth would be driven mainly by gains in life expectancy. If these gains were large enough, individuals would start investing in human capital and the economy would move to a new equilibrium, with the possibility of long run growth (depending on the returns to human capital). From this point on, educational attainment would rise with gains in adult longevity, and fertility would be reduced by either reductions in child mortality or increases in adult longevity. Further increases in life expectancy would be associated with further reductions in fertility, increases in human capital accumulation, and increases in the growth rate (or consumption level).

For this theory to be empirically relevant, it must be the case that life expectancy gains actually preceded fertility reductions in the real experiences of demographic transition. In addition, it must also be the case that mortality reductions were somewhat exogenous to economic development, so that they can be seen as an independent driving force.

The Nature of Mortality Changes

Figure 1 depicts evidence that a large fraction of the recent changes in life expectancy was not determined by development. Preston (1975) presents similar evidence for the period between 1930 and 1960. Together, these data suggest that a large part of the mortality changes during the twentieth century was unrelated to changes in income.[18] Similar evidence is available for the relation between life expectancy and nutrition. Preston (1980, p.305) presents data on life expectancy at birth and nutrition for a cross-section of countries in 1940 and 1970. He shows that life expectancy gains took place at every nutrition level. For the lowest nutrition group (less than 2,100 calories daily), there was an increase of 10 years in life expectancy at birth. He also

[17] At any point in time, population is an intricate function of adult longevity, and of the cumulative effect of past fertility and child mortality rates on initial population levels. If we normalize our model such that parents have children in the end of their first period of life ($\tau = 1$), and we call P_s the population at period s, we have

$$P_s = \sum_{j=s-T}^{s-1} \left[\prod_{i=s-T}^{j} (1-\beta_i) n_i \right] P_{s-T-1} = \sum_{j=s-T}^{s-1} \left[\prod_{i=0}^{j} (1-\beta_i) n_i \right] P_0,$$

where $s > T$, and P_0 is the initial population.

[18] We do not claim that improvements in living conditions do not affect life expectancy. This is, indeed, what is behind the positive logarithmic relationship between life expectancy and income in Figure 1. Our claim is that changes in life expectancy at birth from 40 to more than 70 years, like the ones experienced during the demographic transition, are not entirely due to material improvements.

relates life expectancy changes to both income and calories consumption in a regression setting, and concludes that approximately 50% of the changes in life expectancy was due to 'structural factors,' unrelated to economic development.

Further support to this idea is provided by the diseases responsible for mortality reductions in different countries. Preston (1980, p.300-313) argues that the role of economic development in reducing mortality operated mostly through influenza/pneumonia/bronchitis, for which there was no effective deployment of preventive measures, and diarrheal diseases, for which the gains came mainly through improvements in water supply and sewerage. Apart from these diseases, preventive measures were the most effective ones. Simple changes in public practices and personal health behavior, brought about by knowledge previously inexistent or unavailable, allowed for significant reductions in mortality at very low costs (Preston, 1996, p.532-4).[19] This view generates numbers similar to the ones obtained in the income–nutrition–mortality analysis, with a little more than 50% of life expectancy gains being unrelated to economic development *per se*. The evidence discussed in Becker, Philipson and Soares (2003) also supports this view. They show that reductions in mortality due to infectious, respiratory and digestive diseases, congenital and perinatal conditions were the most important factors producing the convergence in life expectancy observed between 1965 and 1995. This suggests that the large changes in mortality observed in the developing world were due to the absorption of previously available knowledge and, in this sense, were exogenous to these countries.

Lichtenberg (2003) presents a different type of evidence that also supports the idea of exogenous, technologically induced, reductions in mortality. He uses a cross-country panel to show that launches of new drugs – associated with "new chemical entities" – explain 40% of the life expectancy gains observed in 52 countries between 1986 and 2000. His estimates control for a series of other determinants of life expectancy, such as education, income, nutrition, environment, and lifestyle.

The Timing of Mortality Changes

The consensus in the demographic literature depicts mortality reductions starting the transition, implying a period of intense population growth, which phases out as fertility declines. Initial

[19] Most dramatically, the acceptance of the germ theory – developed on the turn of the nineteenth to the twentieth century – allowed for inexpensive gains in life expectancy via simple preventive measures (Vacher, 1979; Ram and Schultz, 1979; Preston, 1980 and 1996; Ruzicka and Hansluwka, 1982). Also, throughout the twentieth century, health programs became increasingly dissociate of the countries' economic conditions, and more dependent on the concerns of the developed world. Even though the monetary value of the help was relatively small, the larger contributions came in the form of development of low cost health measures, training of personnel, initiation of programs, and more effective and specific interventions (see Preston, 1980, p.313-5; and Ruzicka and Hansluwka, 1982). To some extent, this helped to dissociate gains in life expectancy from improvements in economic conditions.

economic conditions are extremely diverse in the different experiences (see Heer and Smith, 1968; Cassen, 1978; Kirk, 1996; Mason, 1997; and Macunovich, 2000). This is true both for the classic histories of demographic transition – such as England or Sweden – and for the post-war experience of Asian and Latin American countries. If we look at developing countries, we see modest longevity gains without fertility reductions, but we do not see fertility reductions without life expectancy gains (see Soares, 2002a). The features of the data are consistent with the theory. Initial gains in life expectancy, while the economy is still in the Malthusian equilibrium, may have distinct effects on fertility. But further mortality reductions eventually move the economy out of this equilibrium. Once this threshold is reached, fertility decreases with gains in life expectancy.

The model predicts that, conditional on the value of the parameters, different combinations of adult longevity and child mortality may move the economy out of the Malthusian equilibrium. Generally, this does not give a single life expectancy number that characterizes the transition. This is even truer once we realize that different countries may have different parameter values, due to differences in cultures, institutions, etc. Nevertheless, the data seem consistent with the idea that there may be a cut off level of life expectancy that determines the escape from the Malthusian equilibrium. The evidence discussed above is consistent with a common threshold around 50 years of life expectancy at birth. If this is the case, reaching this level of life expectancy would mark the transition of a country from a Malthusian regime to an equilibrium with investments in human capital and growth or, alternatively, higher level of long run consumption.

In Figures 6 and 7 we explore this point by analyzing the behavior of fertility and educational attainment before and after the year when life expectancy at birth reaches 50 years. Every country for which data is available that reaches this level of life expectancy within the interval 1960-95 is included in the figures. Countries are aligned in time according to the year when the threshold was reached, such that year T is the 'year when life expectancy at birth reached 50.' Other years are measured as deviations from this reference point. This specific moment in time is obviously not the precise point at which all the different countries start their demographic transition. But if it is a roughly reasonable approximation, and life expectancy is rising throughout the period, fertility and educational attainment should show clear trends after year T, while there should be no clear trend in either variable before year T.

Figure 6 shows the behavior of fertility before and after year T, measured as the deviation of fertility from its initial transitional level. The pattern arises clearly. While fertility behaves erratically before year T, it shows a clear downward trend for all countries after that point. Figure 7 does the same exercise for average schooling in the population aged 25 and above. The result shows an analogous pattern: while educational attainment does not have any clear trend before year T, it shows a clear upward trend for all countries after that point. In both cases, it may be argued that the transition point is actually slightly before year T, which would imply a cut off between 45 and 50 years of life expectancy at birth. We do not argue against this possibility. As mentioned before, the evidence should be seen just as suggestive that a level of life expectancy at birth around 50 years is, on average, associated with changes in the demographic regime. The particular point is likely to be country specific and to depend differentially on child mortality and adult longevity. Further research is needed to pinpoint the precise timing of regime switch in each experience of demographic transition.

The few African countries that have not yet started the transition – such as Ethiopia, Guinea-Bissau, Niger, Sierra Leone, and Congo – also support this interpretation. Even though most of them experienced significant life expectancy gains, the levels are still very low, usually below 50 years. In addition, there are no consistent reductions in fertility or increases in educational attainment (Soares, 2002a).

Finally, the behavior of population in the second half of the twentieth century also supports the theory. Heuveline (1999) uses counter factual projections of the behavior of mortality and fertility between 1950 and 2000 to disentangle their effects on the evolution of world population. He extends the methodology applied by White and Preston (1996), by dividing the world into regions and projecting four counter factual scenarios for each of them. The projections are obtained by applying age and sex specific survival rates to initial populations, and by applying age specific fertility rates to initial female populations. His analysis shows that mortality reductions of the second half of the twentieth century contributed to increase the world population by 33%, while fertility changes reduced it by 26%. Interestingly, had the fertility and mortality levels remained at their 1950 values, the world population today would be virtually the same as it is. Contrary to common belief in the economics profession, the population explosion of the twentieth century was caused almost entirely by gains in life expectancy, with fertility changes working to slow down the process.

Other Evidence

Specific predictions of the model are also in line with a vast array of evidence from studies that try to estimate the economic impact of life expectancy gains. Most notoriously, these include the positive effect of life expectancy on growth in the empirical growth literature, summarized and discussed in Barro and Sala-i-Martin (1995). Other examples are the case study for India of the effects of life expectancy gains on schooling and productivity (Ram and Schultz, 1979), and the simulation exercises performed by Bils and Klenow (2000), analyzing the role of life expectancy in explaining cross-country differences in schooling, productivity, and fertility.

Additionally, Soares (2002a) shows that, even though issues of causality are a concern, the correlations between adult longevity, child mortality, fertility, and educational attainment implied by the theory are present in cross-country panel data (after controlling for country and time fixed effects, and development level). In the same direction, Kalemli-Ozcan (2001) shows that the spread of AIDS in Africa, which was associated mostly with increases in young adult mortality, had a positive impact on fertility (after controlling for female schooling, urbanization, infant mortality, income per capita, and time and country fixed effects). Finally, Soares (2002b) argues that these same correlations can be detected at the micro level. He uses family specific mortality indicators and micro data from Brazil to show that adult longevity is positively related to educational attainment and negatively related to fertility, after child mortality and a large set of demographic variables is accounted for.

5 Concluding Remarks

This paper explores the link between life expectancy, educational attainment, and fertility choice. We show that, under reasonable conditions, mortality reductions can explain the movement of economies from a Malthusian equilibrium, with no investments in human capital, to a steady-state with the possibility of growth. Further reductions in mortality in this steady-state with growth reduce fertility, increase educational attainment, and, thus, increase the growth rate of the economy. These features of the model help explain the demographic transition throughout the world and the recent behavior of fertility in post-demographic transition countries.

Two aspects of the model drive these effects, and distinguish our theoretical work from the previous literature. The utility that parents derive from each child is assumed to depend on the number of children, on child mortality, and, additionally, on the lifetime that each child will enjoy as an adult. The way number of children and lifetime of each child interact in the parent's utility function is an important force behind the mechanics of the model.

In addition, human capital investments are broken down in two pieces: basic investments, that take place during childhood and are done by parents; and adult investments, that take place during adulthood and are done by the individuals themselves. We interpret educational attainment as the time that adult individuals spend on their own education. Apart from being more realistic, this approach allows the model to distinguish between the effects of adult longevity and child mortality on investments in education and growth, and stresses the sequential nature of human capital investments.

Through these two channels, gains in adult longevity can move an economy out of a steady state without growth and with no investments in human capital (Malthusian equilibrium) into an equilibrium with growth. Also, increases in adult longevity in the equilibrium with growth reduce fertility, increase educational attainment, and increase the growth rate of the economy. Child mortality reductions may have similar effects.

The possibility of long run growth rests on the assumption of constant returns to human capital, but the behavior of the demographic variables does not depend on this assumption. We show that, under decreasing returns to human capital, most of the implications of the model hold, and the results that previously held for growth rates in this case apply to long run consumption.

We justify the exogenous role played by life expectancy by arguing that a large share of the changes in this variable during the last century were unrelated to economic development. We also discuss other sets of evidence showing that the chronology of events during demographic transitions, and the behavior of fertility and educational attainment, seem to agree with the predictions of the model.

The theory supports the idea that gains in life expectancy are a major force determining the onset of the demographic transition. Also, it suggests that life expectancy changes may be relevant in determining the behavior of the economy after the transition. In particular, adult longevity – a variable largely overlooked in both demographic and economic literature – is a potentially important factor determining fertility and educational choices.

A Appendix

A.1 The Effect of T on c and b in an Equilibrium with Growth

Using equations 11 to 15:

$$\frac{db}{dT}=\frac{\Psi\frac{dn}{dT}-Dh_c^{\alpha-1}+\frac{Ah_p}{2}[\frac{(\sigma-1)nc^{\sigma-2}}{2}-D^2(\alpha-1)\rho h_c^{\alpha-2}b]}{\frac{Ah_p}{2}[(\sigma-1)n^2c^{\sigma-2}+D^2(\alpha-1)\rho h_c^{\alpha-2}T]}\gtrless 0,$$

and

$$\frac{dc}{dT}=\frac{\begin{array}{c}Ah_pD^2(\alpha-1)\rho h_c^{\alpha-2}(\frac{T}{2}+bn)+2nDh_c^{\alpha-1}\rho_T-\\ -\{2n\Psi+Ah_pb[(\sigma-1)n^2c^{\sigma-2}+D^2(\alpha-1)\rho h_c^{\alpha-2}T]\}\frac{dn}{dT}\end{array}}{(\sigma-1)n^2c^{\sigma-2}+D^2(\alpha-1)\rho h_c^{\alpha-2}T}\gtrless 0,$$

where $\Psi=c^{\sigma-1}-Dh_c^{\alpha-1}\rho_n-(\sigma-1)nc^{\sigma-2}Ah_p\frac{b}{2}>0$, because first order conditions give $c^{\sigma-1}=Dh_c^{\alpha-1}\frac{\rho}{n}>\alpha Dh_c^{\alpha-1}\frac{\rho}{n}=\frac{\rho_n n}{\rho}Dh_c^{\alpha-1}\frac{\rho}{n}=\rho_nDh_c^{\alpha-1}$.

If $\dfrac{db}{dT}<0$:

$$\Psi\frac{dn}{dT} - Dh_c^{\alpha-1}\rho_T + \frac{Ah_p}{2}[\frac{(\sigma-1)nc^{\sigma-2}}{2} - D^2(\alpha-1)\rho h_c^{\alpha-2}b] > 0$$

$$\Longrightarrow 2n\Psi\frac{dn}{dT} - 2nDh_c^{\alpha-1}\rho_T - Ah_pD^2(\alpha-1)\rho h_c^{\alpha-2}bn + \frac{Ah_p}{2}(\sigma-1)nc^{\sigma-2} > 0$$
$$\Longrightarrow Ah_pD^2(\alpha-1)\rho h_c^{\alpha-2}bn - 2n\Psi\frac{dn}{dT} + 2nDh_c^{\alpha-1}\rho_T < \frac{Ah_p}{2}(\sigma-1)nc^{\sigma-2} < 0$$

$\Longrightarrow \frac{dc}{dT} > 0$.

If $\dfrac{dc}{dT} < 0$:

$$Ah_pD^2(\alpha-1)\rho h_c^{\alpha-2}(\frac{T}{2} + bn) + 2nDh_c^{\alpha-1}\rho_T - \\ - [2n\Psi + Ah_pb(\sigma-1)n^2c^{\sigma-2} + Ah_pD^2b(\alpha-1)\rho h_c^{\alpha-2}T]\frac{dn}{dT} > 0$$

$$\Longrightarrow \Psi\frac{dn}{dT} - \frac{Ah_pD^2}{2}(\alpha-1)\rho h_c^{\alpha-2}b - Dh_c^{\alpha-1}\rho_T < \\ < \frac{Ah_pD^2}{2n}(\alpha-1)\rho h_c^{\alpha-2}\frac{T}{2} - [\frac{Ah_p}{2}b(\sigma-1)nc^{\sigma-2} + \frac{Ah_p}{2n}D^2b(\alpha-1)\rho h_c^{\alpha-2}T]\frac{dn}{dT} < 0$$

$\Longrightarrow \frac{db}{dT} > 0$.

In words, $\frac{dc}{dT}$ or $\frac{db}{dT}$ may be negative, but both cannot be negative at the same time. If one is negative, the other must be positive.

A.2 The Possibility of a Steady-State

The possibility of a steady-state in this economy rests on the values of the parameters α and σ. Technological factors summarized by the goods constraint imply that, in any steady-state, c and h_p must necessarily grow at the same constant rate from one generation to the next. But the individual maximization problem tells us, through equation 14, that c and h_p growing at the same rate will not be consistent with the optimal choices of the different generations, unless $\alpha = \sigma$. Therefore, for a steady-state to exist in this economy, it must be the case that $\alpha = \sigma$, so that individuals from different generations will make optimal choices such that c and h_p will grow at the same constant rate, and b, n, e, and l will be constant.

This can be formally seen once we realize that, in terms of the individual's problem, for a steady-state to exist it must be the case that agents will not change their decisions regarding n, b, l, and e as h_p increases. This means that the different generations, who differ only in terms of their endowed h_p and see it as a given parameter, will translate the higher levels of basic human capital in increased consumption, leaving b, n, e, and l unchanged.

From the results obtained before, we already know that $\frac{dn}{dh_p} = \frac{de}{dh_p} = 0$. We can use equations 13 and 14 to show how b and c respond to changes in h_p. This gives the following expressions:

$$\begin{aligned}
\frac{db}{dh_p} &= \frac{(\sigma-1)be}{h_p[(\sigma-\alpha)bn+(\alpha-1)e]} - \frac{b}{h_p} \gtreqless 0, \text{ and} \\
\frac{dc}{dh_p} &= \frac{Ae^2(1-\alpha)(bn-e)}{T[(\sigma-\alpha)bn+(\alpha-1)e]} > 0,
\end{aligned}$$

where the sign of $\frac{dc}{dh_p}$ comes from the fact that $\sigma < 1$.

As mentioned before, a steady-state requires a constant b with an increasing h_p. This will only happen here if $\sigma = \alpha$, in which case we have $\frac{db}{dh_p} = 0$ and $\frac{dc}{dh_p} = \frac{A}{T}e(e-bn)$. It is immediate to see that, in this case, c and h_p will grow at the same constant rate, given by $(1+\gamma) = \frac{h_c}{h_p} = DAbe$.

If $\sigma \neq \alpha$, there is no steady-state, and b will increase or decrease over time (with the increase in h_p) until a corner solution is reached. Rewrite $\frac{db}{dh_p}$ in the following way:

$$\frac{db}{dh_p} = \frac{b}{h_p}\frac{(\sigma-\alpha)(e-bn)}{[(\sigma-\alpha)bn+(\alpha-1)e]}.$$

So, if $\alpha > \sigma$, we have $\frac{db}{dh_p} > 0$; and if $\alpha < \sigma$, we have $\frac{db}{dh_p} < 0$, since $\sigma < 1$.

The intuition for this result is clear. If $\alpha > \sigma$, the sub-utility function related to h_c is less concave than the one related to c, such that when h_p grows from one generation to the next, younger generations tend to increase h_c more than proportionately to c, and this is achieved through increases in b. The same sort of argument works for the case where $\alpha < \sigma$, implying that h_c is increased less than proportionately to c, and that this is achieved through reductions in b. When $\alpha = \sigma$, every generation is just happy to increase c and h_p in the same proportion in relation to the previous generation, in which case b remains unchanged and we have a steady-state.

A.3 The Escape from the Malthusian Equilibrium

A.3.1 T and the Escape from the Malthusian Steady-State

This section of the Appendix discusses what happens to the two last first order conditions in the Malthusian equilibrium as T increases. We start by analyzing the steady-state where investment in both forms of human capital is zero, and show that, as T increases, an interior solution tends to be achieved in both b and e. We then show that, when an interior solution is actually achieved in one of these variables, further increases in T still tend to break the remaining inequality ($e > 0$ and $b = 0$, or $b > 0$ and $e = 0$).

i) $e = 0$ **and** $b = 0$

The last two foc's can be rewritten as $TAh_o/H_o < 1$ and $\varepsilon(n,T,\beta) = \rho_n n/\rho > \alpha Df/h_o$.

So, sufficiently large increases in T can always break the first inequality, making $e > 0$.

Since we assume that consumption is low (H_o not too big) in the Malthusian equilibrium, we have $dn/dT > 0$. This means that the value of ρ increases as T rises, so that the elasticity ε is reduced, and the second inequality may be broken. This will be the case only if ε is not bounded from below above $\alpha Df/h_o$. In this case, starting from $e = b = 0$, exogenous gains in adult longevity will tend to eliminate the corner solutions on both e and b.

ii) $e > 0$ **and** $b = 0$

This solution is characterized by the following foc's:

$$\begin{aligned} c^{\sigma-1} &= \frac{1}{Aeh_o + H_o}\lambda, \\ \rho_n \frac{{h_o}^{\alpha}}{\alpha} &= \frac{f}{Aeh_o + H_o}\lambda, \\ \rho {h_o}^{\alpha-1} D(Aeh_o + H_o) &< n\lambda, \\ (Tc + fn)Ah_o &= (Aeh_o + H_o)^2; \end{aligned}$$

with the constraint $T - e = \frac{Tc+fn}{Aeh_o+H_o}$.

The constraint together with the last foc gives $e = \frac{T}{2} - \frac{H_o}{2Ah_o}$. Differentiating the foc's:

$$\frac{dn}{dT} = \frac{f(\sigma-1)c^{\sigma-2}\frac{(Ah_oT+H_o-2c)}{2T} - \rho_{nT}\frac{h_o^{\alpha}}{\alpha}}{\rho_{nn}\frac{h_o^{\alpha}}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} \lesseqgtr 0.$$

For the same reasons discussed before, at low levels of consumption we have $dn/dT > 0$. The corner solution on n can be characterized by the same expression $\varepsilon(n,T,\beta) > \frac{\alpha Df}{h_o}$. Again as before, as the value of ρ increases as T rises, the elasticity ε is reduced, and the inequality may be broken.

iii) $b > 0$ **and** $e = 0$

The third and fourth foc's in this case are:

$$\begin{aligned} \rho {h_c}^{\alpha-1} DH_o &= n\lambda, \\ \rho {h_c}^{\alpha-1} DbAh_o &< \left[1 - \frac{(Tc + fn)Ah_o}{H_o^2}\right]\lambda; \end{aligned}$$

and the constraint is $T = bn + \frac{Tc+fn}{H_o}$. Using the foc's and the constraint have the same inequality of the first case: $TAh_o/H_o < 1$. As T increases, an internal solution on e tends to be achieved.

A.3.2 β and the Escape from the Malthusian Steady-State

Starting from a position where $e = b = 0$, changes in β do not affect the foc related to e. The effect on the foc for b depends on the behavior of ρ, as β changes. Given the expression for $\frac{dn}{d\beta}$:

$$\frac{d\rho}{d\beta} = \rho_n \frac{dn}{d\beta} + \rho_\beta = \frac{-\rho_n \rho_{n\beta} \frac{h_o^\alpha}{\alpha} + \rho_\beta \rho_{nn} \frac{h_o^\alpha}{\alpha} + \rho_\beta \frac{f^2}{T}(\sigma - 1)c^{\sigma-2}}{\rho_{nn} \frac{h_o^\alpha}{\alpha} + \frac{f^2}{T}(\sigma - 1)c^{\sigma-2}} \gtrless 0.$$

The realistic case is the one where reductions in child mortality lead unequivocally to increased utility from children. If this is the case, the expression above is positive. This is true if the substitutability between survival rates and number of children is not too strong ($\rho_{n\beta}$ quantitatively small when compared to ρ_{nn}). It is also likely to happen when consumption is low in the Malthusian equilibrium (H_o relatively small). In both scenarios, the positive terms in the numerator dominate, so that $d\rho/d\beta < 0$ (remember that $\rho_\beta < 0$). Reductions in child mortality increase the value of ρ, reducing ε, and increasing the return to investments in basic human capital, possibly breaking the corner solution $\varepsilon > \alpha Df/h_o$.

So, differently from increases in adult longevity, reductions in child mortality tend unequivocally to move the economy to a transitional situation where $b > 0$ and $e = 0$. In this case, the corner solution in e is still characterized by the same inequality as before, just substituting h_o by h_p: $TAh_p < H_o$. For the first generation of parents experiencing reductions in their children's mortality, $h_p = h_o$, and there is no tendency to break the corner solution on e. But as children who receive positive investments in basic human capital become adults, their h_p in period t is $h_{p,t} = h_{c,t-1} > h_o$. If child mortality is consistently reduced one generation after another – such that $h_{p,t+1} > h_{p,t} > h_{p,t-1}$ – the inequality $TAh_p < H_o$ is eventually be broken, and the economy reaches a steady-state with growth and positive investments in all forms of human capital.

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Figure1: Relationship Between Per Capita Income and Life Expectancy at Birth - Transitional Countries (1960-95)

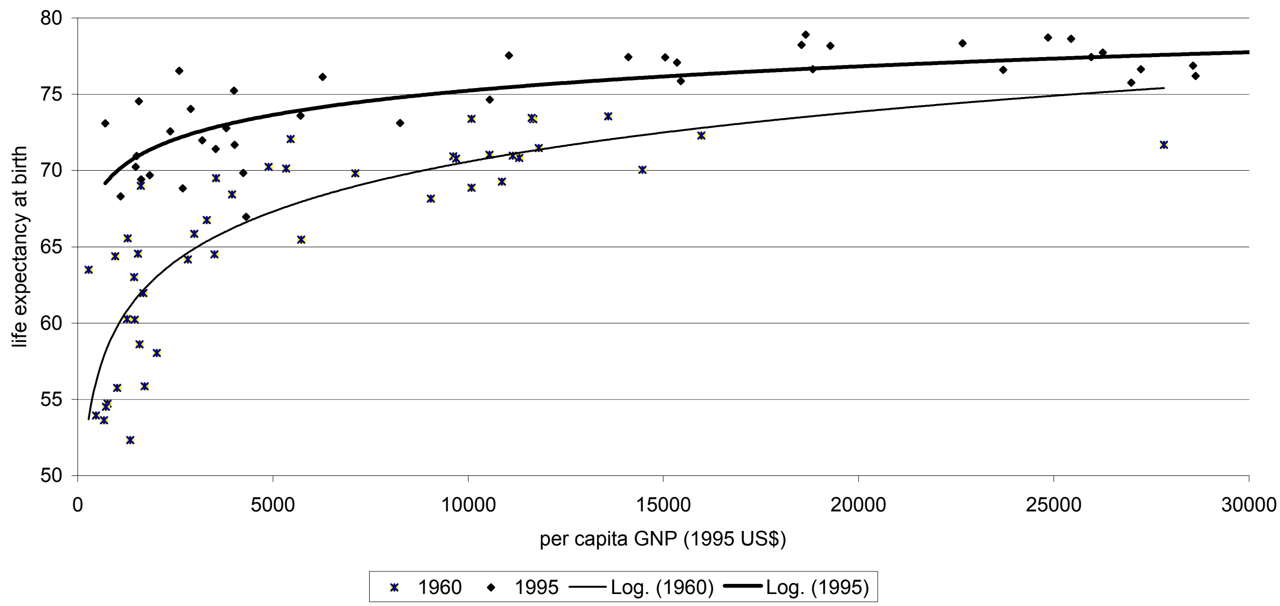

Figure 2: Relationship Between Per Capita Income and Fertility Rate - Transitional Countries (1960-95)

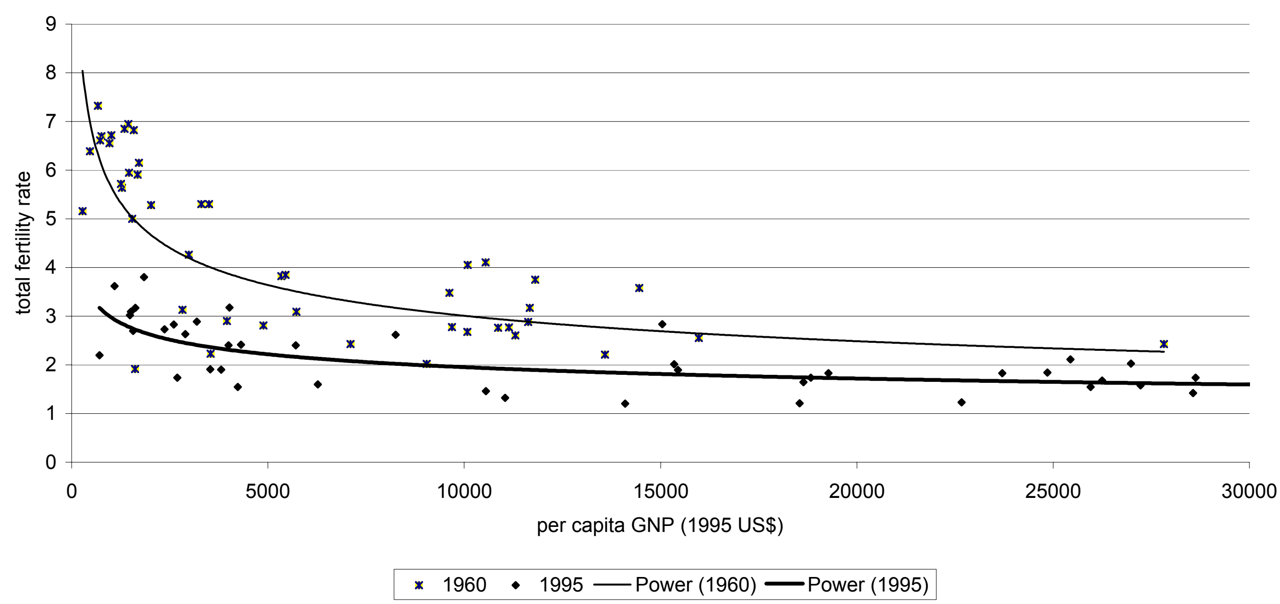

Figure 3: Relationship Between Per Capita Income and Educational Attainment - Transitional Countries (1960-95)

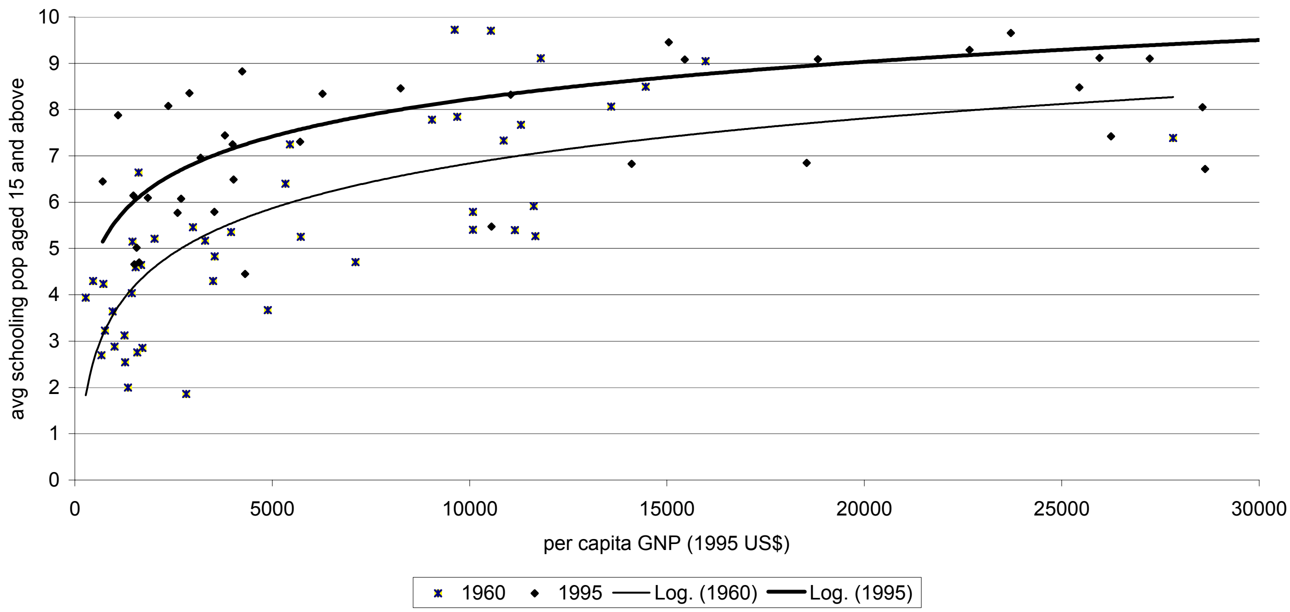

Figure 4: Relationship Between Life Expectancy at Birth and Fertility Rate - Transitional Countries (1960-95)

Figure 5: Relationship Between Life Expectancy at Birth and Educational Attainment - Transitional Countries (1960-95)

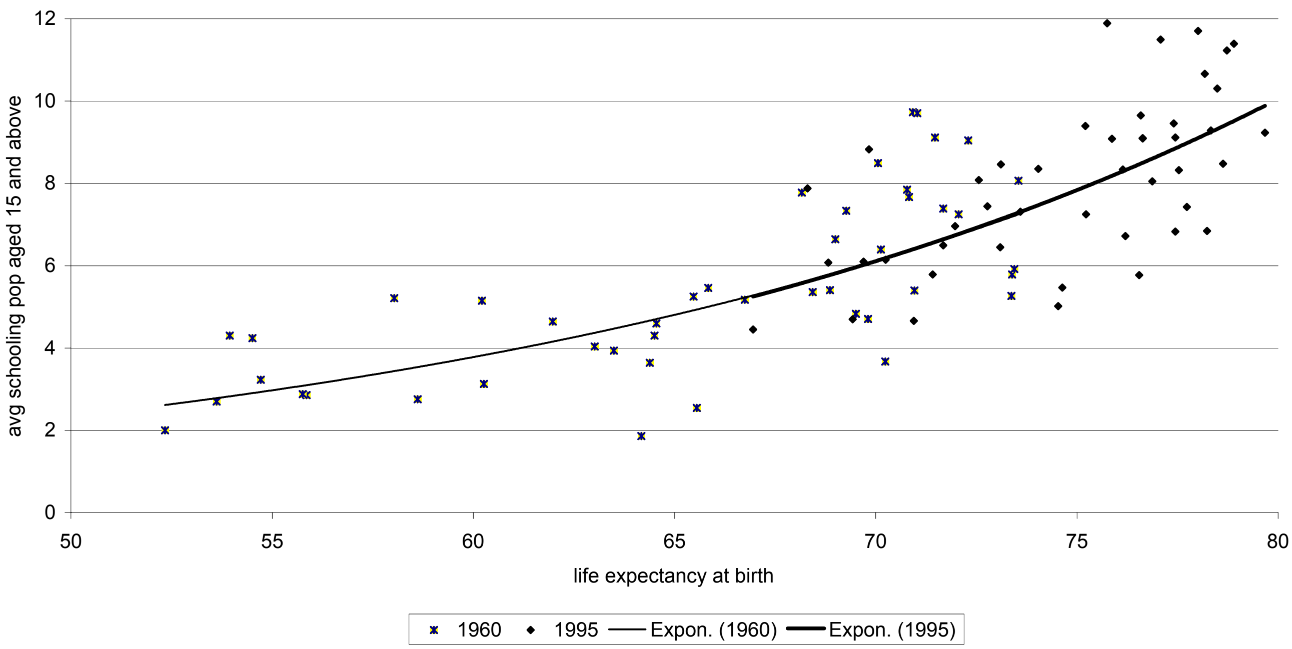

Figure 6: Fertility Deviation from Initial Transitional Level Before and After the Year when Life Expectancy at Birth Reached 50

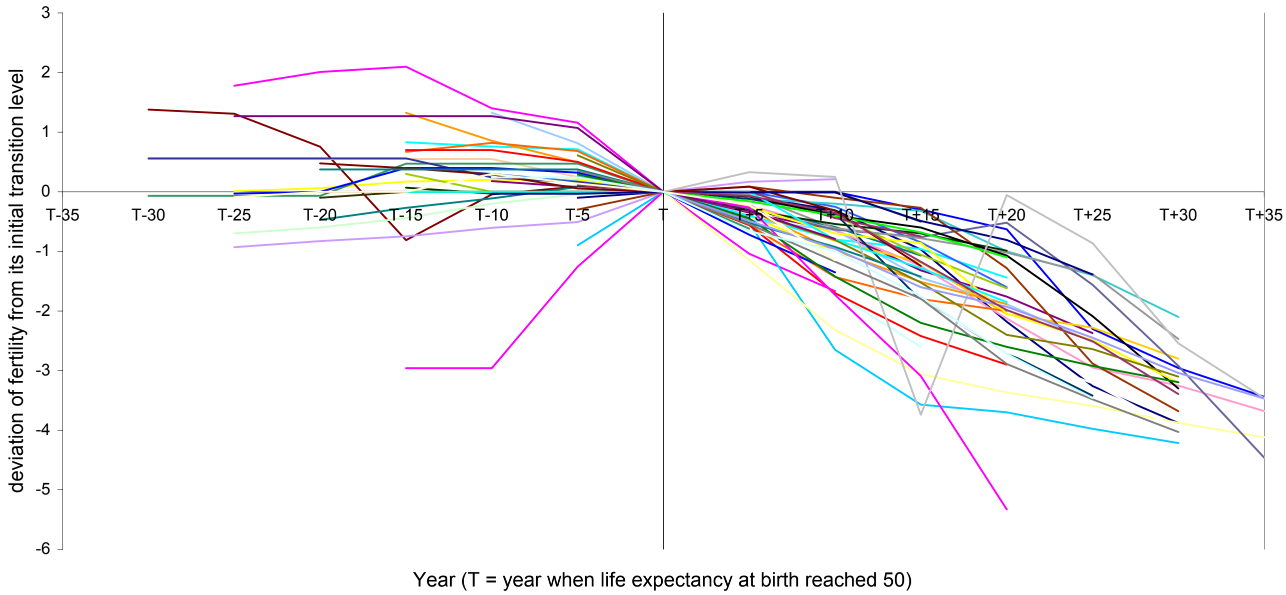

Figure 7: Schooling Deviation from Initial Transitional Level Before and After the Year when Life Expectancy at Birth Reached 50