

Output, Capital, and Labor in the Short, and Long-Run*

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Using a new series of capital stock and frequency domain analysis, this paper provides new empirical evidence on the relative importance of capital and labor in the determination of output in the short and long-run. Contrary to the common practice in the traditional growth accounting literature of assigning weights of 0.3 and 0.7 to capital and labor inputs respectively, the evidence presented here suggests that capital is a far more important factor than labor for determination of output at and near the zero frequency band. Furthermore, I show that the zero-frequency labor elasticity of output may well be close to zero, or even zero. Additional findings reported here support the traditional accelerator model of investment as a good description of the long-run investment process.

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Abstract

Using a new series of capital stock, and the frequency domain framework, the paper provides a new empirical evidence on the relative importance of capital and labor in the determination of output in the short, and long-run. Contrary to the common practice of assigning respectively 0.3 and 0.7 weights to capital and labor inputs in the traditional growth accounting equations, the evidence presented here suggests that capital is far more important factor than labor for determination of output at and near zero frequency band. Furthermore, I show that the zero-frequency labor elasticity of the output may well be close to zero, or even zero. Additional findings reported here support the traditional accelerator model of investment as a good description of the long-run investment process.

I. Introduction

In traditional growth accounting calculations that originated from Solow's [32; 33] seminal work, macroeconomists usually assign output elasticities of 0.30 and 0.70 to capital and labor inputs respectively. These values are based on the assumption that producers are operating in a competitive, profit maximizing, constant returns to scale environment in which factors of production are paid their marginal product. Brown [6], Douglas [12], and Intriligator [20] have provided empirical support for these assumptions for the pre-war and inter-war periods. Their estimated capital and labor elasticities of the output were around 0.25 and 0.75 respectively.

However, a recent study by Paul Romer [30] concludes that the contribution of capital accumulation to long-run growth is substantially underestimated in the conventional growth accounting analysis and that the true capital elasticity of output may actually be greater than its share in total income because of positive externalities associated with investment. On the other hand, Romer suggests that the contribution of labor is considerable overestimated and that the true labor elasticity may actually be smaller than its share in income because of negative externalities associated with labor. In particular, according to Romer's estimation, the long-run capital and labor elasticities of output probably lie in the range of 0.7–1.0 and 0.1–0.3 respectively. But his estimates come from historical data of output, capital, and labor averaged over 10- and 20-year intervals for the periods 1890–1980 and 1839–1979, respectively. Therefore, as Romer himself suggests, the signal-to-noise ratio may be too small for a sensible interpretation of these figures, since the above time series contain only 7–9 observations. In addition, a long-run averaging of the data may not have completely eliminated the effect of business cycle fluctuations.

Bernanke [3, 204] expresses doubts about the correctness of Romer's estimates since "it cannot literally be true that output is independent of labor input, [and therefore] this result must be caused by an estimation bias."

The importance of capital accumulation in the growth of the U.S. economy is

emphasized in other studies that analyze the sources of long-term growth in the U.S. economy [4; 5; 8; 10; 22; 23]. For example, Jorgenson [23, 25] argues that “comparing the contribution of capital input with other sources of output growth for the period 1948–1979 as a whole makes clear that capital input is the most significant source of growth.” Denison [11, 220] makes a similar argument: “I do not share the other extreme view, sometimes encountered, that capital can be ignored because its significance is hard to establish if one fits a production function by correlation analysis. I stress again: capital is an important growth source. It has sometimes contributed importantly to differences in growth rates between periods and places. More capital formation would raise the growth rate.”¹

From a theoretical point of view, this argument is not really new. The usual assumption used in standard microeconomic models that the stock of capital is fixed in the short-run but variable in the long-run implies that variations in the stock of capital will affect the output in the long-run.² More importantly, dynamic models that involve some kind of transaction costs usually make similar predictions.³

The purpose of this paper is to provide new empirical evidence on the relative importance of capital and labor in the determination of output in the short- and long-run. Unlike the studies cited above, the methodology applied here uses frequency domain analysis. The advantage of using the frequency domain framework is that it allows us to conduct the analysis on a frequency-by-frequency basis for describing empirical cyclical regularities in the data and examining the dynamic relationship between time series without the intervention of an econometric model. The frequency domain methodology used here is nonparametric and therefore requires no behavioral or distributional assumption about the time series of output, capital, and labor. The only requirement is that the series analysed be stationary. The quarterly time series of capital stock used here was constructed recently and thus differs from the data used by Romer [30] and others. Despite these differences, the findings reported in this paper indicate that capital indeed is a far more important factor than labor in the determination of output at the zero frequency band. Furthermore, I show that the zero-frequency labor elasticity of output may well be close to zero, or even zero. An

additional finding of this paper is related to the accelerator model of investment: it turns out that output leads capital at the zero frequency band which suggests that the traditional accelerator model may be a good description of the long-run investment process.

Statistical evidence supporting these ideas are derived below by examining the capital-output and the labor-output relationships using cross-spectral analysis. Spectral analysis provides a useful framework for studying the issues raised here because in the frequency domain short and long-run relationships between time series can be characterized and analysed by looking at the behavior of the series in the high and low-frequencies, respectively. The main disadvantage of ordinary time domain regression analysis in the context discussed here is the fact that it implicitly treats all frequencies equally. In addition, as Chow [9], Harvey [17], and many others argue, although the information gained from frequency domain analysis is theoretically a transformation of its time domain analog, some dynamic and cyclical features of the data are easier to identify and interpret in the frequency domain.

The paper is organized as follows. In the next section, I briefly review the statistical methodology used in this study. In section III, I describe the data set. Next, in section IV, I present and discuss the empirical results of the study. In section V, I discuss the implications of the findings for growth accounting in the context of U.S. business cycles. The paper ends with a brief summary of the main results and some concluding remarks.

II. The Methodology

Spectral analysis makes it possible to conduct time series analysis in the frequency domain, where we think of a stationary series as being made up of sine and cosine waves of different frequencies and amplitudes. In a univariate case, we are interested in determining how much of the total variance (‘power’) of the series is determined by each frequency component. In a bivariate setup, spectral analysis provides a description of a linear relationship between time series at different frequencies.⁴

For a covariance stationary univariate process y_t , the autocovariance function is given by the expression

$$\gamma(s) = E[(y_{t+s} - \mu)(y_t - \mu)] \quad (1)$$

where μ is the mean of the process. It is usually assumed that both $\gamma(s)$ and μ are time independent which is essential for past observations to be useful in describing the present or the future. It follows that $\gamma(s) = \gamma(-s)$. The spectrum of the series y_t is defined as the Fourier transform of its autocovariance function, and is given by

$$f_y(\omega) = (1/2\pi) \int_{-\infty}^{\infty} \gamma(s) e^{-is} ds \quad (2)$$

where ω is the frequency and is measured in cycles per period (in radians).

For a bivariate covariance stationary process (y_t, x_t) , the cross covariance function given by

$$\gamma_{yx}(s) = E[(y_{t+s} - \mu_y)(x_t - \mu_x)], \quad (3)$$

measures the degree of linear association between the two stochastic processes for different time lags and is independent of time. The cross spectrum is the Fourier transform of the cross covariance function and is given by

$$f_{yx}(\omega) = (1/2\pi) \int_{-\infty}^{\infty} \gamma_{yx}(s) e^{-is} ds \quad (4)$$

which is a complex-valued function of ω . Since the cross spectrum as given above cannot be examined directly, the usual practice is to compute and plot 'squared coherence', 'phase', and 'gain'. The squared coherence which is given by

$$C_{yx}(\omega) = |f_{yx}(\omega)|^2 / [f_y(\omega) f_x(\omega)] \quad 0 \leq C_{yx}(\omega) \leq 1 \quad (5)$$

is analogous to the square of the correlation coefficient between the series at each frequency. That is, it represents the degree to which one time series can be represented as a linear function of the other. The higher the $C_{yx}(\omega)$, the more closely related are the two series at frequency ω . By its construction the coherence says nothing about the sign of the relation between the two series, nor anything about the timing of any lead/lag in the relation.⁵

The phase, $P_{yx}(\omega)$, is a measure of the phase difference or the timing between the frequency components of the two series. It is measured in the fraction of a cycle that x leads y and is given by

$$P_{yx}(\omega) = (1/2\pi) \arctan \left\{ -\text{Im} [f_{yx}(\omega)] / \text{Re} [f_{yx}(\omega)] \right\}, \quad (6)$$

where Im and Re are the imaginary and real parts of the cross spectrum. It is worth noting that determining lead-lag relationship using the phase differs from the method used by the National Bureau of Economic Research (henceforth NBER). NBER determines the lead-lag relationship using only peaks and troughs without considering non-turning point periods. The phase, however, takes into account all the time points for which the data are available. The phase is known only up to adding or subtracting an integer since adding or subtracting one whole cycle to an angle will not change its tangent.⁶

The gain indicates how much the spectrum of y_t has been amplified or attenuated to approximate the corresponding frequency component of x_t . It is essentially the regression coefficient of the process y_t on the process x_t at frequency ω and is given by

$$G_{yx}(\omega) = |f_{yx}(\omega)| / f_y(\omega) \quad 0. \quad (7)$$

A small gain at frequency ω indicates that x has little effect on y at that frequency.⁷

In sum, we may interpret coherence, phase, and gain in terms of the ordinary regression analysis terminology if we imagine running a regression equation of y on x at each frequency. The squared coherence is the frequency domain analogue of the time domain correlation coefficient, R^2 , and is calculated at each frequency. The regression coefficient is just the gain if there is no time lag between y and x . If there is a time lag, the gain can be interpreted as the regression coefficient if the series were lagged just the right amount to eliminate any phase shift, and the phase is the angle by which they would have to be shifted.

An additional set of statistics crucial for a reliable interpretation of spectral analysis results include the significance test statistics associated with coherence, phase and gain. Unfortunately, they are rarely reported in published articles. I have computed these statistics and they are reported along with the estimates of coherence, phase, and gain. As a significance test for squared coherence, I test whether $H_0: C_{yx}(\omega) = 0$. For phase and gain, I provide a 95% confidence interval.⁸

III. The Data

The quarterly time series used in this study consists of real output, real capital stock, and employment data for the period 1948–1983 (144 observations).⁹ While there is nothing unique about the output, and employment data, the quarterly stock of capital data was recently constructed by Balke and Gordon [1] by using the corresponding annual capital stock data published in the *Survey of Current Business*. This was done by treating the annual series' values as the beginning and the ending values of the quarterly series using the fact that under a fixed exponential rate of depreciation, the quarterly series satisfy $K_t = I_t + (1 - \delta)K_{t-1}$. The depreciation rates (δ) of each capital stock component were iterated until the fourth quarter's value converged to the end of the year value from the annual series. The resulting estimated annual depreciation rates for the stock of nonresidential structures and producers' durable equipment are 6.036% and 14.3%, respectively.¹⁰ The quarterly employment figures are taken from *Business Conditions Digest*, Bureau of

Economic Analysis, February 1984, p. 101.

IV. Results of Spectral Analysis

Since spectral analysis methodology outlined here applies only to stationary processes, most economic times series require some kind of filtering prior to spectral analysis as they usually tend to be nonstationary. In order to determine whether the data I use is stationary or not, I formally examine the unit root properties of the time series of output, capital, and labor.

Recall that, if a time series x_t has to be differenced d times to make it stationary, then we say that x_t is integrated of order d , $x_t \sim I(d)$. The number of differencing needed to make a time series stationary corresponds to the number of unit roots the series contains. Therefore in order to determine whether the time series of output, capital, and labor are stationary, I examine each series for presence of unit roots using the Augmented Dickey-Fuller (henceforth ADF) univariate tests of the form

$$x_t = \alpha + \beta t + \gamma x_{t-1} + \sum_{i=1}^4 \delta_i x_{t-i} + \epsilon_t \quad (8)$$

where x_t is the series we are examining and t is a linear time trend. Engle and Yoo [14] and MacKinnon [26] recommend including a linear trend component in the test equation to avoid the dependence of the test statistic's distribution on the true value of α . The null hypothesis is that $\alpha = 0$, which means that the series contains an unit root and is thus nonstationary, i.e., $x_t \sim I(1)$. The alternative hypothesis is that the series are stationary, that is, $x_t \sim I(0)$.

Initially, I test the hypothesis of an unit root in the series measured in levels. The results are reported in the first column of Table I. As the values of the ADF t -statistics indicate, the hypothesis of nonstationarity cannot be rejected for either of the series. Next, I test whether first differences of the same series are nonstationary. Based on the figures presented in the last column of Table I, the hypothesis of an unit root in the differenced series can be rejected with 1% significance. Therefore, I conclude that all three series are $I(1)$, and thus can be represented as

difference stationary processes. Consequently, the time series of output, capital, and labor are all log-differenced prior to the application of spectral analysis.¹¹

Next, using the methodology described in Section 2, I estimate the squared coherence, phase, and gain between the time series of real GNP and real stock of capital on one hand, and between the real GNP and employment level on the other.¹² The results are reported in Figures 1–6. On these figures the frequency along the horizontal axis, ω , is measured in radians. i.e., $0 \leq \omega < \pi$. Each frequency corresponds to a particular periodicity (or a cycle length) according to the mapping, $\omega = 2\pi / T$, where T denotes the length of a cycle.¹³ In the analysis that follows, the frequency range $0 \leq \omega < \pi$ is divided into long-run, business cycle, and short-run frequency bands. The cut-off points of these frequency bands are identical to those used in the modern business cycle literature. For example, according to Englund et al. [15] and Hassler et al. [18], most students of business cycles and growth define business cycles as 12–32 quarter cycles. Their estimate of the average length of a business cycle is about 20 quarters, which corresponds to the frequency of $\omega = 0.31$. The frequencies below business cycle frequency band correspond to the long-run, while the frequencies above business cycle frequency band correspond to the short-run. The shortest identifiable cycle is a two-period cycle and it corresponds to frequency $\omega = \pi$, also known as the Nyquist frequency.

The estimated squared coherence between the real GNP and the real stock of capital is plotted in Figure 1. The horizontal line at 0.31 is the 95% critical value derived from testing the hypothesis, $H_0: C_{yx}(\omega) = 0$. Thus, any coherence value above 0.31 is statistically bigger than zero.¹⁴ As the figure shows, the squared coherence between real output and the stock of capital is statistically significant at most frequency bands including business cycle frequencies, 0.19–0.51, which correspond to cycles of 12–32 quarters length. In particular, the coherence is statistically significant at the zero frequency band, suggesting a long-run relationship between capital and output. The coherence is not statistically significant at the frequency bands 1.98–2.23, and 3.06–3.14, which corresponds to a 2–3 quarter cycle. The coherence is highest at the frequency corresponding to 12 quarter cycle.

The phase of the real GNP with the stock of capital is provided in Figure 2 along with the 95% confidence interval. Note that the confidence interval is smaller, the higher the squared coherence.¹⁵ The phase diagram indicates an upward trend over the frequency band $0 < \omega < 0.29$, which corresponds to cycles of about five years and longer. Then it remains relatively stable up to about $\omega = 0.76$, followed by another downward trend up to about $\omega = 1.57$, and then an upward trend up to $\omega = 1.79$. In the range of frequencies $2.25 < \omega < 2.5$, the phase indicates another downward trend.¹⁶

A visually observable trend in the phase diagram usually indicates a fixed time lead-lag structure which implies that there is a fixed phase differential between the series at the corresponding frequency band. When there is a time delay, the phase is a linear function of frequency, the slope representing the magnitude of the delay. Thus, the fixed time delay can be estimated by approximating the slope of the phase trend at the frequency band. For instance, a perfect one-period lag relationship would result in a straight trend line with a slope of one (radian per radian). In case of a horizontal phase at some constant value (e.g., at the frequencies $0.29 < \omega < 0.76$), the lead-lag relationship is variable (fixed angle lag). That is, the smaller the frequency, the larger the time lag between the corresponding components.

Following these guidelines, the estimated phase diagram of Figure 2 suggests the following: at the zero-frequency band (specifically for $\omega < 0.29$), the real GNP seems to lead the capital stock by about 6 quarters ($1.71/0.29$). Afterwards, the lead-lag mechanism seems to be varying with frequency. Along business cycle frequencies (12–32 quarters), the smaller the frequency, the larger is the time-lead of the real GNP over the capital stock. In the range of frequencies $2.25 < \omega < 2.5$, the phase indicates a downward trend, which implies that capital stock leads real GNP. The diagram indicates several phase shifts (including at the seasonal frequency, 4 quarters). Overall, the lead-lag relationship between real GNP and capital stock is complicated and varies with frequencies. The finding that output leads capital in the long-run, that is, at the zero frequency band, suggests that the traditional accelerator model of investment may be a good description of the long-run investment process.

The gain of real GNP with capital stock is plotted in Figure 3 along with 95%

confidence interval. As the diagram indicates, the gain starts at about 0.36 at the zero frequency band and keeps declining continuously all the way to about 0.03–0.04 at the short-run frequency band. This means that the increase in output associated with an increase in the stock of capital is much larger in the long-run than in the short run.

In sum, cross spectral analysis of real output and capital stock suggests that there is a significant correlation between output and capital across almost the entire frequency band. However, in magnitude, this relationship is much more important in the long-run than in the short-run. The implied lead-lag relationship supports the accelerator model of investment as a good description of the U.S. long-run investment process.

Figures 4–6 provide the plottings of coherence, phase, and gain of the real output with employment. The coherence diagram reveals that at frequencies close to zero (specifically, for $\omega = 0.12$, which corresponds to 13 year or longer cycles), there is no statistically significant correlation between output and employment. The correlation is not significant also at $\omega = 2.08$ – 2.35 , which corresponds to about a three-quarter cycle. But there is a sharp increase in the coherence immediately after $\omega = 0.12$, with the peak occurring at $\omega = 0.39$, which corresponds exactly to a 16 quarter cycle. This obviously identifies the output-labor comovement at this frequency as a typical business cycle phenomenon.

The phase diagram plotted in Figure 5 shows that the phase is relatively stable at the frequencies, $\omega = 2.09$, fluctuating around zero. This is an indicator of a stable contemporaneous relationship between output and labor at frequencies that correspond to 3 quarter and longer cycles. That is, output and labor are in phase in the long-run as well as across business cycles (12–32 quarters). The diagram also has a negative trend for $\omega = 2.52$, which suggests that in the very short-run, employment leads output by about 6 quarters $[2.52 - (-1)] / (3.14 - 2.52)$.¹⁷

The gain plot of output with employment suggests a relatively stable relationship across almost the entire frequency band. At frequencies close to zero (specifically, for $\omega = 0.12$, which corresponds to 13 year or longer cycles), its value is not statistically significant, which

again suggests that in the zero-frequency band, labor does not matter for the determination of output. That is, the zero-frequency labor elasticity of output seems to be zero. At the frequencies where the coherence is not statistically significant, the confidence interval of the gain contains zero, because the lower the coherence, the larger the sample variance of the estimated gain. Although the gain attains a maximum at $\omega = 2.20$, which corresponds to a three quarter cycle, its confidence interval is very wide at that frequency as the squared coherence is low, and thus the gain's estimate at that frequency is very imprecise.

In sum, the cross-spectral statistics indicate that the zero-frequency labor elasticity of the output is very small and may even be zero. Across other frequency bands (except at very short-run frequencies) output and labor are in phase, moving contemporaneously.¹⁸ At the short-run frequencies, labor leads output by about 6 quarters. Overall, the relationship between labor and output is far more stable than between capital stock and output.

V. Implications for Growth Accounting

As shown in the previous section, at business cycle frequencies, that is at frequencies that correspond to 3–8 year cycles, the coherences between output and labor input and between output and capital stock are both high, suggesting that labor as well as capital are procyclical. This finding is in line with the general findings documented by Burns and Mitchell [7] about durations of various business cycles in the U.S. As Table II indicates, the American business cycles from 1854 to the present have varied in length between 2–10 years. The average length of cycle has been about 4–5 years, with the most common length being 3–4 years.

In Table III, I have computed the ratio of the estimated output-labor to output-capital gain functions for $\omega = 0.51$. Comparison of the estimated gain functions at the business cycle frequencies indicates that the ratio of output-labor to output-capital gains is in the range 1.7–2.6. The commonly used ratio of labor and capital shares in income, $0.7/0.3 = 2.3$, falls in this interval. As the figures presented in Table III indicate, the ratio of the estimated gains exactly equals 2.3 at the frequency $\omega = 0.27$, which corresponds to about a 6 year cycle,

which is slightly higher than the average length of a business cycle. On the other hand, at and immediately near the zero frequency band (say, for <0.20) the ratio is very low, which indicates that at these frequencies the contribution of capital to output far exceeds the contribution of labor. This puzzling result obviously leads to the following question: which frequencies should be used for the determination of the relative weights of capital and labor inputs in growth accounting equations?

From the theoretical point of view, the classical model considers a state of the economy at a point in time, but under the assumption that prices have adjusted to clear markets. Solow's [32] growth model, on the other hand, explains the growth pattern of an economy over many decades. Capital stock is a key factor in that model. But, capital stock is a slow moving variable as changes in it require the building of new factories and production lines, new machines, new structures, etc. Therefore, in the context of Solow's model, which is what I believe most students of growth and business cycles have in mind when they think of long-run growth, a period of just a few years might be considered short-run, since it may take many, many years for an economy to adjust to its steady-state equilibrium.

This obviously leads to the well-known discussions about the difficulties practitioners face with defining and measuring long-run trends. According to Kydland and Prescott [24, 8] "the trend component for real GNP should be approximately the curve that students of business cycles and growth would draw through a time plot of this time series." This definition of long-run trend identifies growth time trend as a very low frequency phenomenon. The long-run then might be measured in decades rather than years. Therefore, in order to determine proper weights of capital and labor in growth accounting equations, we need to look at *zero and near-zero frequencies*. Consequently, based on the findings reported here, it follows that the conventional weights of capital and labor used in traditional growth accounting equations indeed overestimate labor's share and underestimate capital's share in national income.

VI. Summary and Conclusions

The purpose of this paper was to compare and contrast relative importance of labor and capital inputs for the determination of output in the short, and long-run. The findings indicate that at the frequency band, 0.12 , which corresponds to cycles of 13 years or longer, capital is statistically important for the determination of output, while the level of employment does not seem to matter. That is, the zero-, and near zero-frequency elasticity of output with respect to labor seem to be close to zero. In the short run, the opposite is true: the variations in output due to a given change in labor is much bigger than due to a change in capital. Thus, the statistical evidence provided here supports the view that in conventional growth accounting analysis the contribution of capital accumulation to long-run growth is indeed underestimated, while the contribution of labor is substantially overestimated.

The analysis of the lead-lag structure of capital-output and labor-output relationships suggest that across most frequency bands, output and labor are in phase, moving together contemporaneously. At the short-run frequencies, labor leads output by about 6 quarters. The lead-lag structure of capital and output varies with the frequency band. In the long-run, output leads the stock of capital as predicted by the accelerator model of investment.

Thus, this paper contributes to the literature on the sources of growth of the U.S. economy by showing that stock of capital plays a far more important role in the determination of output in the long-run than in the short-run, while employment level has a significant effect on output in the short, and medium-run, but not in the long-run.

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Footnotes

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1. Statistically, the increasing importance of the capital stock in the growth of the U.S. economy has been documented in other studies as well, but the figures are not as high as Romer's estimates. For example, the figures cited in Maddison [27] for the weight of the capital stock are in the range 0.21–0.40. The estimates of the post-war capital elasticity of the output reported by Levy [25] are in the range 0.44–0.55. However, these figures were estimated in studies that cover much shorter periods than Romer [30].

2. Although it seems that Marshall [28] was the first to explicitly argue that the stock of capital is fixed in the short-run, the idea itself can be traced back to Smith and Ricardo. Marshall assumed that the industry in the short period can be treated as if it were in static equilibrium, which together with the assumption of fixed equipment stock, resembles the agricultural sector model of Ricardo. Marshall's fixed capital plays the same role as Ricardo's land, 'original' and 'indestructible' within the period. Marshall treated his short period as a single period in the manner of Smith or Ricardo and invoked the constancy (or approximate constancy) of the capital stock of the industry as a justification for treating the single period as self-contained [19]. According to Robinson [29], Keynes treated the existing stock of capital in the short-run simply as a "part of the environment in which labour works."

3. It is interesting to note that historically Marshall [28] was the first to formally distinguish between different time periods in the context of the intertemporal production process. He actually considered four different time periods: (i) the period where all inputs are fixed ('very short-run'); (ii) the period where inputs are variable, and thus supply can be increased up to the highest level possible for given capital stock ('short-run'); (iii) the period where all the inputs are variable given the available technology ('long-run'); and (iv) the period where even the technology is variable ('very long-run').

4. Historically, spectral analysis seems to have originated in the work of Schuster [31], who has developed the method of periodogram analysis for finding hidden periodicities in sunspot data. The development of the modern formal theory of spectral analysis was initiated by Wiener [34]. It was initially applied to engineering and physical science data where large data sets are generated by experiments, and was imported to economic time series data much later. The description of spectral methodology presented here follows Engle [13], Fishman [16], and Jenkins and Watts [21].

5. A useful property of coherence is its *invariance under linear filtering*. This means that the degree of linear association between time series as measured by coherence is preserved under linear filtering of the time series.

6. A similar ambiguity is present in time domain cross-correlation analysis if both negative as well as positive cross-correlation coefficients are found.

7. Note that the squared coherence may be constant over the entire frequency range. However, as Fishman [16] notes, the corresponding gain function will not necessarily be so because even if the same linear association exists at all frequencies, some frequency components may still be amplified or attenuated more than others.

8. The statistics used in these tests are provided by Jenkins and Watts [21].

9. The sample period stops in 1983 since the capital stock data series used here is not available thereafter.

10. See Balke and Gordon [1] for more details. Note that these figures are much higher than the depreciation rates used by the Bureau of Economic Analysis (BEA) in constructing its National Income and Product Accounts' (NIPA) estimates. According to John Musgrave of the BEA (personal communication), the BEA's estimated annual depreciation rates of the stock of nonresidential structures and producers' durable equipment are 1.5% and 6.0%, respectively. It is not clear to me why there are such large differences between the two estimates. It may be that because of the iterative nature of the estimation method Balke and Gordon [1] use, their estimate is nonlinear, while the BEA's estimates are constructed using a linear life time depreciation path.

11. The difference filter has been commonly used in frequency domain literature even before the recent development of unit root literature since it turns out that spectral representations of the original and the differenced series are related. An additional advantage of difference filter is the fact that it belongs to an important class of symmetric digital filters called *nonnegative definite filters*. These filters have zero phase shift for all frequencies and therefore passing time series through them does not alter the lead-lag relationship between the time series.

12. Various statistics (coherence, phase, and gain) reported in this study were all computed by first constructing the cross periodograms, which then were smoothed in order to get consistent estimates of the series' cross spectral densities. The smoothing was done by averaging the neighboring periodogram ordinates using a flat window with a width of 9. That is,

$$\hat{f}_{yx}(\omega_k) = 1/9 \sum_{i=-4}^4 \hat{I}_{yx}(\omega_{k-i}),$$

where I_{yx} is the estimated cross-periodogram, and $k = 128$ is the

number of ordinates.

13. Thus the frequency $\omega = 0.524$, e.g., corresponds to a three-year cycle if quarterly data is used.

14. The 99% critical value of coherence equals 0.43.

15. This is because the higher the coherence, the lower is the variance of the estimated phase.

16. I ignore frequencies at which the squared coherence is not statistically significant, as at these frequencies the gain is not statistically different from zero. Some portions of the confidence interval are not shown on Figures 2 and 4 since at those frequency bands the phase's confidence interval is not defined. To see this, note that phase's confidence interval is given by $\hat{P}_{yx}(\omega) \pm \arcsin \left[\frac{2\sqrt{1 - C_{yx}(\omega)}}{2 + C_{yx}(\omega)} \right]^{1/2}$, where $C_{yx}(\omega)$ and $P_{yx}(\omega)$ are the estimated squared coherence and phase, respectively. Under the null hypothesis $H_0: C_{yx}(\omega) = 0$, the quantity $\frac{2 + C_{yx}(\omega)}{2 - C_{yx}(\omega)}$ follows a Snedecor's $F_{2, \infty}$ distribution. Therefore, the expression under the square root will be greater than 1 for any $C_{yx}(\omega) > C_{yx}^*$, where C_{yx}^* is the 95% critical value of the coherence. However, arcsin is not defined if its argument is bigger than 1. Therefore, at frequency bands where the squared coherence is not statistically significant, the confidence interval of phase will be undefined.

17. I again ignore the frequencies at which the squared coherence is not statistically significant.

18. Baxter and King [2, 5] find that "major upward and downward movements in output and labor input occur together."

Table I. ADF Unit Root Test t -Statistics^a

Variable	ADF t -statistic	Variable	ADF t -statistic
Output	-2.39	(Output)	-5.32*
Capital	-0.30	(Capital)	-4.05*
Labor	-1.62	(Labor)	-5.18*

a. The * indicates a significance at 1%. Critical values as tabulated in MacKinnon [26], for $\alpha = 1, 5,$ and 10% are: $-4.02, -3.44,$ and $-3.14,$ respectively. The null hypothesis is $H_0: x \sim I(1)$.

Table II. Length of American Business Cycles, 1854–83

Length in Years	Number of Cycles
2	2
3	10
4	9
5	4
6	2
7	1
8	1
9	0
10	1

Source: Burns and Mitchell [7].

Table III. The Ratio of the Estimated Output-Labor to Output-Capital Gains, U.S., 1948–83^a

	$2 / \text{in}$	$G_{YL}(\)$	$G_{YK}(\)$	$G_{YL}(\)/G_{YK}(\)$
0.000		0.088	0.262	0.338
0.025	256.456	0.091	0.294	0.309
0.049	127.967	0.098	0.367	0.268
0.074	85.369	0.143	0.367	0.389
0.098	63.984	0.203	0.316	0.643
0.123	51.208	0.280	0.342	0.820
0.147	42.656	0.392	0.287	1.365
0.172	36.573	0.387	0.281	1.375
0.196	32.008	0.471	0.280	1.680
0.221	28.444	0.504	0.261	1.931
0.245	25.604	0.538	0.256	2.106
0.270	23.271	0.543	0.237	2.293
0.294	21.335	0.575	0.224	2.570
0.319	19.690	0.579	0.211	2.749
0.344	18.286	0.586	0.219	2.672
0.368	17.065	0.581	0.220	2.637
0.393	16.000	0.600	0.218	2.749
0.417	15.060	0.583	0.205	2.839
0.442	14.222	0.566	0.200	2.827
0.466	13.475	0.561	0.220	2.542
0.491	12.799	0.542	0.218	2.486
0.515	12.191	0.557	0.210	2.655

a. in denotes the frequency, $2 / \text{in}$ denotes the periodicity in quarters, and $G_{YL}(\)$ and $G_{YK}(\)$ denote output-labor and output-capital gain functions, respectively.

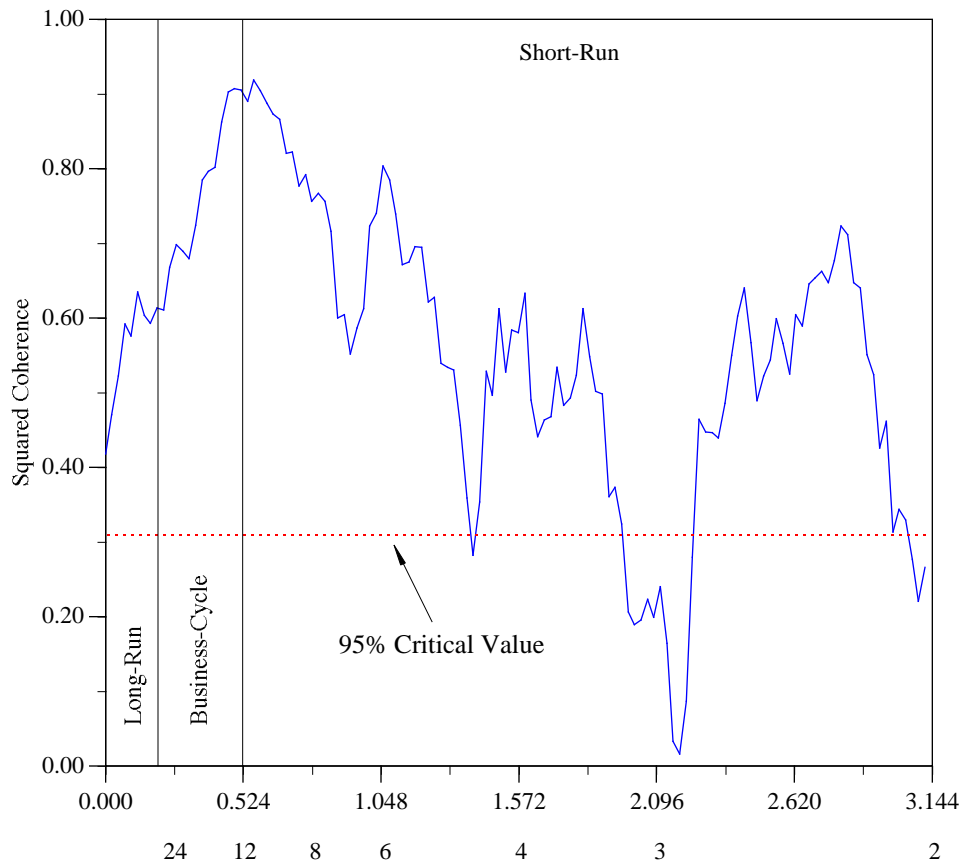


Figure 1. Coherence of Output and Capital, U.S., 1948–83

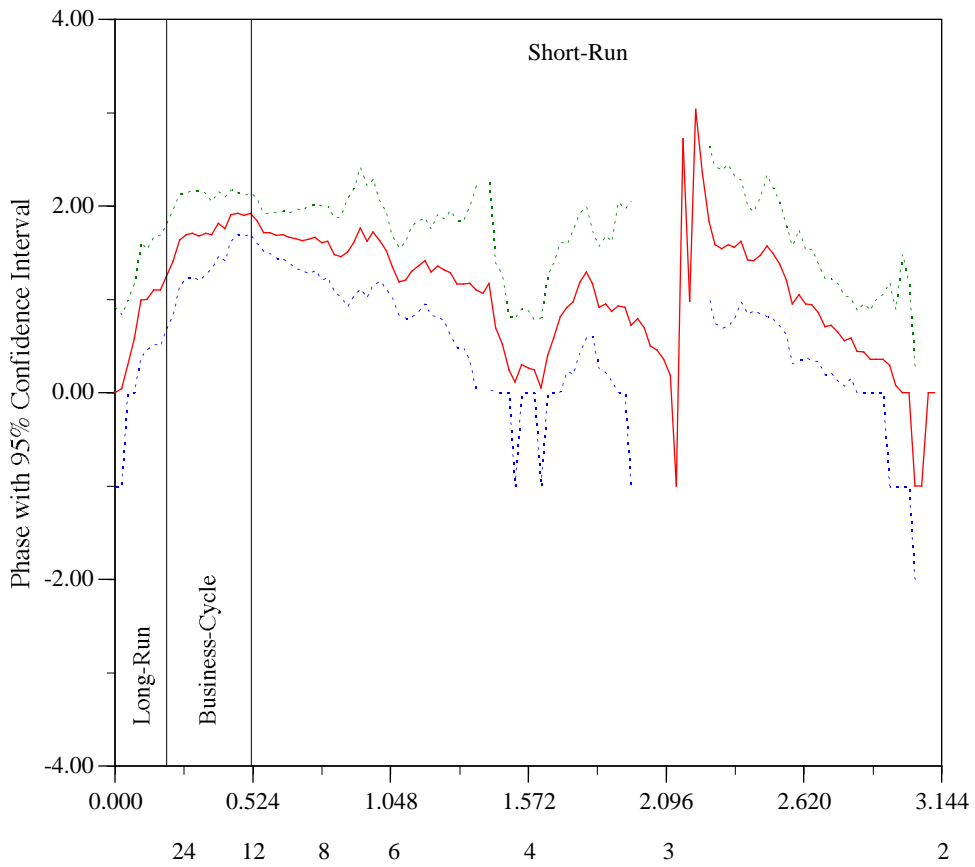


Figure 2. Phase of Output and Capital, U.S., 1948–83

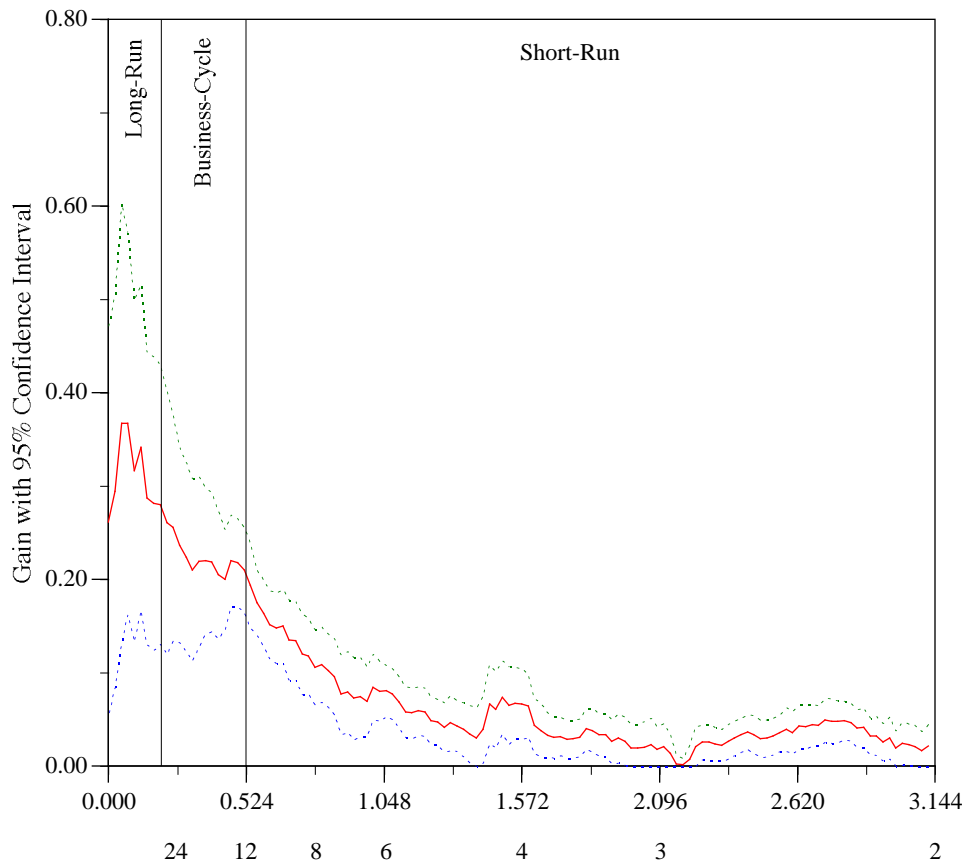


Figure 3. Gain of Output and Capital, U.S., 1948–83

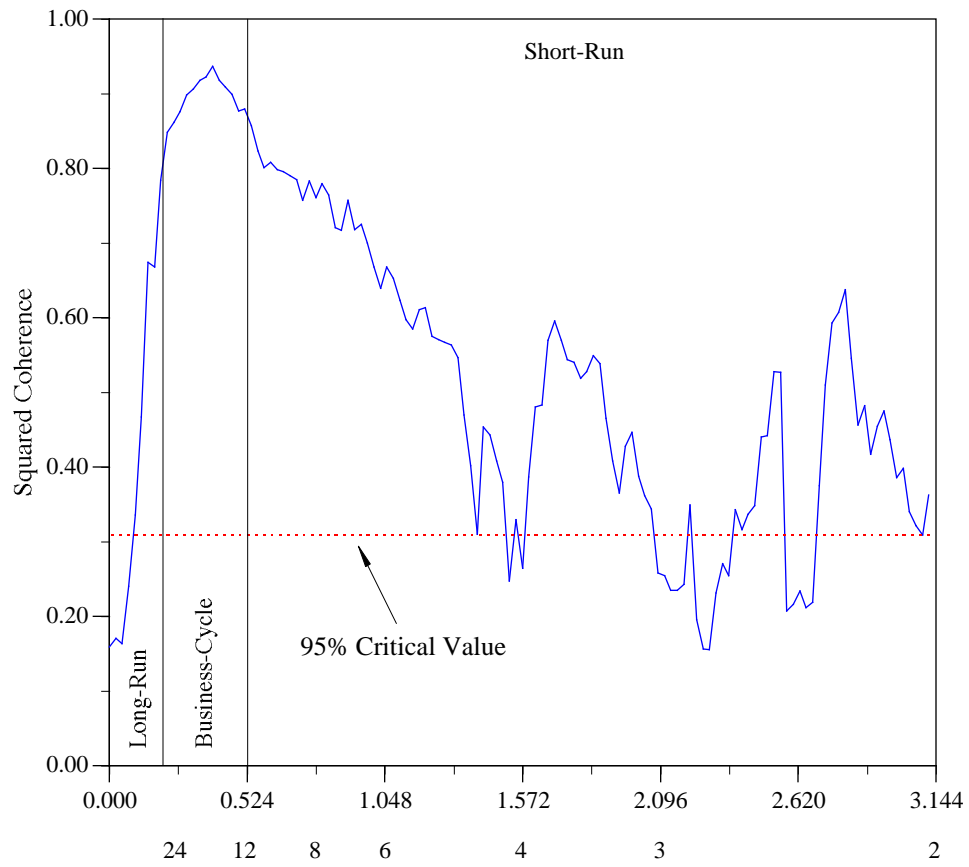


Figure 4. Coherence of Output and Labor, U.S., 1948–83

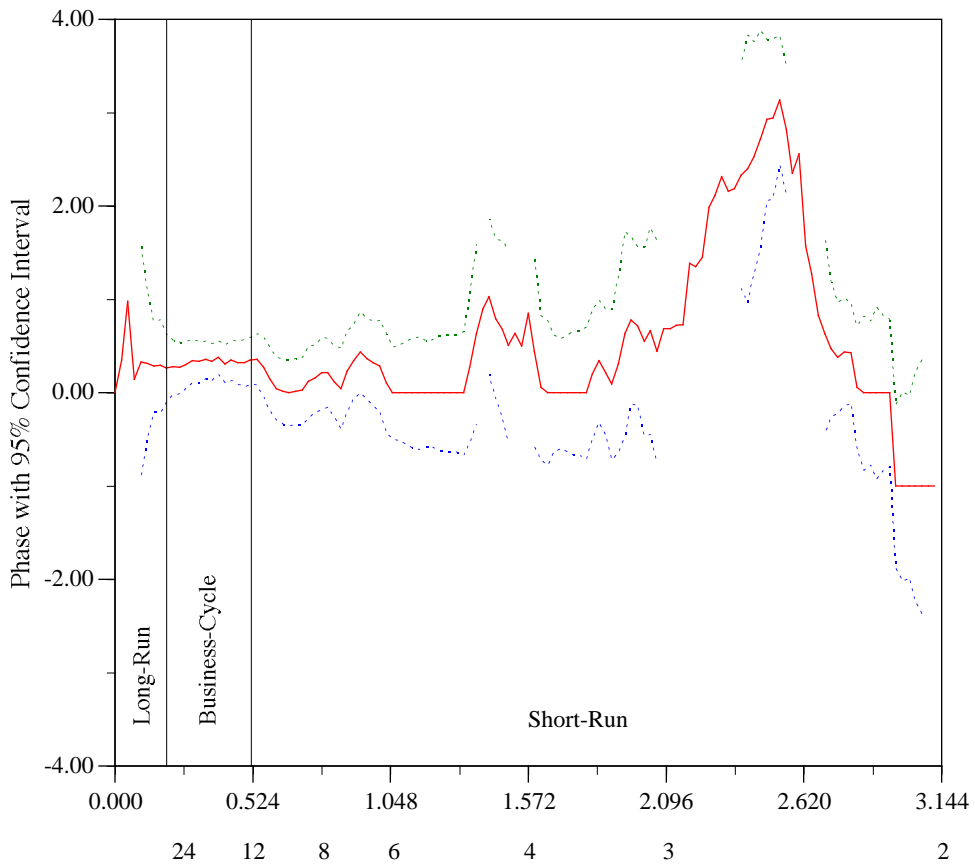


Figure 5. Phase of Output and Labor, U.S., 1948–83

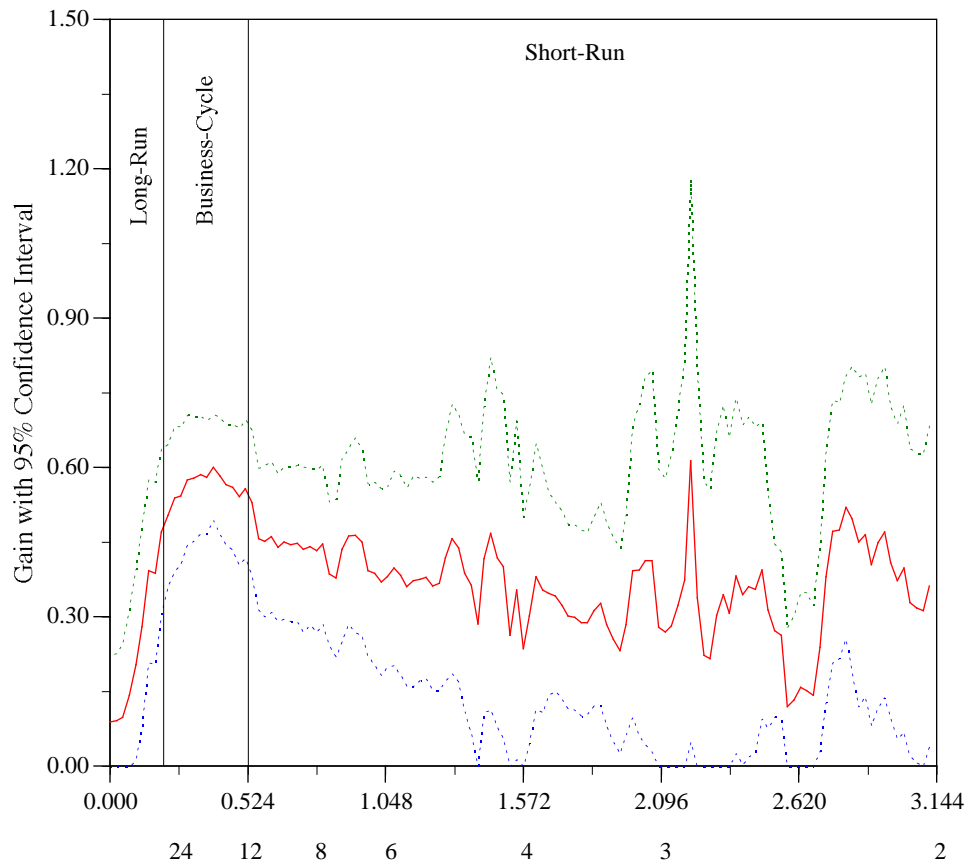


Figure 6. Gain of Output and Labor, U.S., 1948–83