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Inequality and Growth: Does Time Change Anything?

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Abstract

The econometric analysis of economic growth has always been subject to major flaws and shortcomings. Data scarcity and reliability, parameter heterogeneity, omitted variable bias, endogeneity problems, ... have seriously tainted estimation results. In this paper we propose an alternative framework that explicitly deals with these issues. We investigate the relation between income inequality and economic growth in a number of OECD countries in a cointegrated VAR-setting. Our results suggest that different models hold for different countries. However, for most countries the imperfect markets model better describes reality than the complete markets model.

Keywords: income inequality, economic growth, cointegrated VAR

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Introduction

Until the mid-1970s the Kuznets curve (Kuznets (1955)) was accepted as an empirical stylised fact. The Kuznets curve describes the relationship between income inequality and per capita income. According to the Kuznets hypothesis, economic progress is initially accompanied by rising inequality but as soon as per capita income reaches a certain threshold, inequality starts to decrease again. Note that the relationship between growth and inequality under the Kuznets hypothesis depends on the level of per capita income: once a certain threshold has been crossed (once societies are rich enough), further growth will reduce inequality. However, in the late 1970s the Kuznets picture was disturbed by a sudden increase (the UK, Germany) or a stagnation (France, Canada) in inequality in some rich countries. These events questioned the universal validity of the Kuznets curve and gave way to a new development in the empirical literature on inequality and economic growth. Although some authors remained faithful to the Kuznets idea and suggested modifications to the basic framework (o.a. Milanovic (1994)), others have questioned the causal linkages between economic growth and inequality. Ever since a closer look at data in the mid-1980s had shown that more inequality was always associated with lower long-run growth, the believe that inequality, rather than growth, is the determining factor in the relationship made way.

The question whether and how inequality is related to economic growth inspired a lot of empirical research over the past decade. In the early 1990s, several authors showed that higher inequality at the beginning of a longer-term period was linked to poorer growth performances (Alesina and Rodrik (1994), Perotti (1994, 1996), Persson and Tabellini (1994)). This resulted in a consensus that inequality worsens growth performances. Gradually the consensus weakened. First, it was argued that the relationship differs between poor and rich countries (Deininger and Squire (1998), Barro (1999)). A negative relationship was found in developing countries, but for richer countries there was no relation at all. Recently a new consensus with a very different content seems to take shape: inequality stimulates economic growth (Forbes (2000), Arjona et al. (2001)). Given this evolution, one might conclude that the world has changed drastically over the past decade thereby disturbing well-established economic relations.

However, a closer look shows that not the world but rather the econometric techniques to analyse it have been the subject of major changes. The earliest empirical contribution presented OLS and 2SLS estimates. Next, 3SLS and random and fixed effects panel estimators were used. These were in turn replaced by panel GMM estimators. Still, Durlauf (2001) notes that 'while we have seen remarkable advances in the econometric analysis of many areas of microeconomics and macroeconomics, growth economics has not experienced anything close to such progress' (p. 65).

We explore the possibilities of yet another econometric approach. A short overview of past contributions in the next section clarifies Durlauf's provocative statement and provides a motivation for our alternative. In this paper, we analyse the relationship between income inequality and economic growth using a cointegration approach within a VAR model.

The remainder of the paper is structured as follows. We motivate our econometric approach in section 1. Section 2 looks into two theoretical models for income inequality and economic growth: the complete markets model and the imperfect markets model. A description of the dataset follows in section 3. In section 4 the results of the cointegration analysis are reported. We try to identify the long run relations and check whether these relations allow us to discriminate between the structural models presented in section 2. The conclusion summarizes the most important insights of the paper.

The econometrics of inequality and growth

There has been a substantial evolution in growth econometrics since the beginning of the 1990s. In the bulk of early empirical work on growth a linear cross-country regression is estimated by means of OLS. Growth is regressed on several (lagged) explanatory variables as GDP, schooling, ... and income inequality (Barro and Sala-I-Martin (1995)). Several shortcomings of this approach have been noted, varying from an omitted variable bias and data heterogeneity to endogeneity.

If an important determinant is omitted in the estimation, the estimated coefficients of all included variables will be biased. This omitted variable bias is especially problematic in growth empirics, as the list of factors that can plausibly affect growth seems without limit. Durlauf (2003) discriminates between two kinds of regressors in traditional cross-country regressions: the ones offered by the Solow growth model (population growth, technological change, physical and human capital and savings rates) and those added by the new growth theories. While the former list is fixed, no consensus about the latter exists. As the number of data points available for growth estimates is not that large, a lot of potentially relevant regressors need to be excluded. The omitted variable problem seems hard to overcome as modern growth theories are fundamentally open-ended: one growth theory typically has no bearing on the empirical relevance of another.

Durlauf and Johnson (1995), Canova (1999), Krueger and Lindahl (2000), Kourtellos (2002) and Sonedda (2003) show that the assumption of parameter homogeneity in standard growth analyses is neither supported by the data, nor by theory. Durlauf (2001) argues that 'there is nothing in growth theory which would lead one to think that the marginal effect of a change in high school enrolment percentages on the per capita growth of the US should be the same as the effect on a country in sub-Saharan Africa' (p.67). He agrees that this argument is generally applicable in econometrics, but as any parsimonious growth regression will necessarily leave out many factors that from the perspective of economic theory affect the parameters of the included variables, it is particularly salient in the case of cross-country growth. The different 'income inequality – growth' relationship between richer and poorer countries (Deininger and Squire (1998), Barro (1999)) also illustrates that enforcing one overall relation will necessarily induce poor estimation results.

An important remark in the heterogeneity debate, is the lower data quality in developing countries (Schultz (1999)). One can imagine that lower income countries systematically underreport inequality due to a failure in data collection. If this is true, low inequality will be linked to a poor growth performance on the basis of a measurement error. Weede (1997) illustrates the importance of the data selection: he shows that the results of Persson and Tabellini (1994) are neither robust to different data sources, nor to the exclusion of a few original data points. Quah (2000) notes that 'researchers have long known about the biases and omissions in developing-country national income accounts. Comparison of those data with the data of developed countries can be unreliable even when within-country analysis over time for a given economy is perfectly sensible' (p. 5).

Given these heterogeneity problems, we choose to focus on individual OECD countries. Measurement error is probably less severe in those countries (although data collection is still far from perfect). Moreover it will enable us to evaluate the believed homogeneity within this group of richer countries. Our 'individual country approach' is also compatible with another comment by Durlauf (2001): 'Empirical growth studies virtually always assume that one theory is equally valid for all countries, whereas it is far more natural to think a given theory will explain the growth experience of each country more or less well depending on the country's individual characteristics' (p.69). Evans (1998) shows that different growth models may characterise the growth experiences of well-educated and poorly-educated countries. Park (1994) notes that although the suggestive empirical results established in the cross-country analyses can provide a useful guide for country studies, the challenge of empirical work is testing the theoretical insights against the economic evolution of individual countries

using time series data. Relevant country specific information gets lost amidst the large number of factors affecting growth performance in cross-country studies. Brock and Durlauf (2000) argue that theory and parameter heterogeneity (uncertainty) is of major importance in a policy-relevant empirical analysis of growth.

Endogeneity is a major problem in growth regressions. One can quite easily argue that education stimulates growth, but one can as easily explain why growth influences education decisions. The same holds for social security, investment, ... A lot of variables have an impact on the growth performance, but growth in turn influences almost all economic decisions. Using an instrumental variable approach to deal with endogeneity is not straightforward as it is problematic to identify instruments that simultaneously are correlated with the included growth determinants and uncorrelated with the residuals. Durlauf (2001) notes that 'those studies which attempt to use instrumental variables to address regressor endogeneity have not been persuasive in that the choices of instruments have not met the necessary exogeneity requirements for instrument validity' (p.66). To argue that an instrument is valid, means one has to show that it is uncorrelated with all growth theories not embodied by the regression. But because so many factors can plausibly influence growth, this condition is virtually impossible to satisfy.

A frequently used alternative solution, explaining subsequent growth by including explanatory variables at the beginning of the period, does not fully solve the endogeneity problem either, as expectations about economic growth will also matter in the decision-making process with respect to schooling, investment, ...

A panel data estimation by fixed effects reduces the omitted variable problem as the country specific factors that are fixed over time are eliminated (Arjano et al. (2001), Forbes (2000)). Moreover, it becomes possible to evaluate the effects of changes in inequality, which is more relevant from a policy point of view. But a new problem arises as estimation by fixed or random effects is not consistent if the specification that needs to be estimated contains a lagged endogeneous variable (Nickell (1981)). Growth is known to be characterised by a catch-up effect (conditional convergence), which renders the fixed and random effects estimators useless for this kind of analysis. Besides this additional problem, estimation by random or fixed effects does not deal with the endogeneity issue.

An alternative is the use of a first-differenced GMM estimator (the Arellano-Bond estimator) (Forbes, 2000). This kind of estimator eliminates the country specific effects, but the need to identify the appropriate instruments remains. If the first stage relationship between differenced independent variables and lagged level variables is weak, the GMM estimates will be biased towards their fixed-effects counterparts (Stock *et al.* (2002)). In addition, Blundell, Bond and Windmeijer (2000) show that the instruments used in the first-differenced GMM become less informative with series that are highly autoregressive. As inequality series are characterised by a high degree of persistence, there is a substantial risk that the GMM-results have a large finite sample bias. The most fundamental criticism on the use of the Arellano-Bond estimator is that it is designed for micro data sets, i.e. for a cross section dimension that tends to infinity (Bond (2002)). This condition is gravely violated in the context of growth econometrics.

Given the above problems, how should one proceed? We choose not to resort to the 'classic' cross-country approach but focus on the time dimension in the data. This eliminates problems related to parameter and theory heterogeneity. Given the major endogeneity problems in growth econometrics, a VAR model seems to be a suitable framework. A VAR model also steers clear of a priori restrictions (with respect to stationarity, causality, ...) on the estimates. One exception is the assumption of a linear relationship between the different variables. This assumption is not undisputed (Banerjee and Duflo (2003), Krueger and Lindahl (2000)). As we only include OECD countries in our study, the dispersion of most variables is quite limited. Given this limited range, the linearity assumption seems less controversial.

Hauk and Wacziarg (2004) choose a different approach to the econometrical difficulties in growth regressions. Rather by reducing the biases in estimates by improving on the methodology, they evaluate the bias properties (e.g. size and direction) of common estimators in growth regressions.

The models

We want to discriminate between two models of growth and inequality: the complete markets model (CMM) and the imperfect markets model (IMM) (Perotti (1996)). Next to these models, Perotti (1996) also considers the socio-political instability model. We do not integrate this third model in our analysis, as we a priori consider it to be less relevant for a sample of relatively stable OECD countries.

In the CMM each economic agent can fully borrow against the present discounted value of future earnings. High inequality affects investment decisions, as a higher government intervention (more redistributive measures) will be demanded by the population. Redistribution reduces growth through tax distortions and reduced capital accumulation. In the IMM not all planned investment (especially in human capital) can be executed as the poor are credit constrained. Redistribution to the poor relaxes the credit constraint thereby stimulating growth and investment. At the same time redistribution has similar negative effects as in the complete markets model.

Both models predict a negative relation between growth and inequality. However the underlying mechanisms that result in this reduced form are different. This shows that it can be informative to look for structural relationships rather than to use the reduced form relation in empirical work.

We elaborate some more on the testable implications of both theories by means of a basic theoretical model inspired by Bénabou (1996) and Aghion and Howitt (1998). The models we present are deliberately kept simple which implies that some very stringent assumptions have to be made. The main aim of the theoretical elaboration is to motivate the empirical section of the paper. It seems needless to pursue a more 'sophisticated' approach. That is also the rationale for presenting two models that differ with respect to the redistribution system: integrating both systems in one model is possible, but severely complicates the discussion and has little added value. A different theoretical elaboration on both models is provided by Sonedda (2003).

The full mathematical derivations of some expressions can be found in appendix A.

Set up

We consider an overlapping generations model in which n individuals live for two periods. The intertemporal utility of an individual i born at time t is given by

$$U_t^i = \ln c_t^i + \rho \ln d_t^i \qquad \text{with } 0 < \rho < 1 \tag{1}$$

where c and d denote current and future consumption respectively (for reasons of notational convenience we omit the time subscript in the discussion of the model). The parameter ρ is a measure of time preference. There is only one good in the economy that serves both as capital and consumption good. Production of the future consumption good takes place at time t according to an AK technology

$$y_t^i = \eta k_t^i \tag{2}$$

The parameter η is an efficiency measure. In line with the work by Lindbeck (1985, 1988, 1993) we assume that efficiency is a decreasing function of the tax rate β

$$\eta = f(\beta) \qquad \text{with } f'(\beta) < 0 \tag{3}$$

For simplicity and without loss of generality we impose an efficiency loss that is proportional to the tax rate.

$$\eta = (1 - \kappa \beta) \eta^*$$
 with η^* the maximum efficiency (with zero tax rate) (3') $0 < \kappa < 1$

Using (3') we can rewrite the production function (2) as

$$y_t^i = (1 - \kappa \beta) \eta * k_t^i \tag{2'}$$

The second factor in (2') represents the 'society adjusted individual education level'

$$k_t^i = (e_t^i)^\delta (A_t)^{1-\alpha} \qquad \text{with} \quad 0 < \alpha < 1$$

where eⁱ denotes the education level attained by an individual i and A is the basic level of knowledge and skills in the society

To increase his education level, an individual can invest in human capital (hⁱ). Human capital investment is characterised by decreasing returns. For highly educated people it will take more time, money and effort to further increase their education level (eg. higher information costs, less qualified teachers, etc.).

$$e_t^i = (h_t^i)^{\gamma}$$
 with $0 < \gamma = \frac{\alpha}{\delta} < 1$ (5)

An individual's production level is thus determined by his own investment in human capital and by general knowledge and skills in society (i.e. the level of development). The accumulation of knowledge and skills follows from past production activities in the *private* sector (i.e. the private sector is characterized by a learning-by-doing process).

$$A_{t} = \frac{1}{n} \sum_{i} y_{t-1}^{i}$$
 (6)

We implicitly assume that the link between knowledge and former private production is one-to-one. If we allow for depreciation, i.e. only part of average past production results in current knowledge, the main results of the model will not change.

Individuals differ in their initial endowments. An individual's endowment upon birth at time t, w^i , is given by

$$w_t^i = \varepsilon_t^i A_t \tag{7}$$

with \mathcal{E}^i (≥ 0) an identically and independently distributed random shock with mean 1, that measures individual i's access to general knowledge at time of birth.

Individual i can either directly 'consume' his initial endowment¹, or invest it into the production of future consumption goods (according to (2'), (4) and (5)). *Complete markets model (CMM)*

Current consumption will be equal to the amount of initial endowments augmented with the amount of borrowing (bⁱ), less the amount of investment in human capital

$$c_t^i = w_t^i + b_t^i - h_t^i \tag{8}$$

We introduce a government that redistributes income (intra-generational transfers). It first takes away a fraction β of individual income and next adds a fraction β of the average income in the society (\overline{y}) to it. We implicitly assume that redistribution occurs at no direct cost, but we could easily introduce a deadweight cost by adding less than the fraction β to the average income. Note that redistribution will have an indirect cost due to its negative impact on efficiency.

Future consumption equals future production after redistribution, less the debt repayment

$$d_t^i = (1 - \beta)y_t^i + \beta \overline{y}_t - r_t b_t^i \tag{9}$$

with r (>1) the (gross) market interest rate endogenously determined by the loan market clearing condition: the sum of net-borrowings must equal 0.

$$\sum_{i} b_t^i = 0 \tag{10}$$

Each individual will spread his endowment over consumption in period 1 and production in period 1 (consumption in period 2) as to maximize intertemporal utility (expression (1)). In the case of a perfect capital market, no credit constraints exist. Everybody can borrow freely as long as the capital market is in equilibrium.

An individual's decision then becomes

$$\operatorname{Max}\left\{\operatorname{ln}c_{t}^{i} + \rho \operatorname{ln}d_{t}^{i}\right\} \quad \text{w.r.t. } b_{t}^{i}, \ h_{t}^{i}$$
s.t. expression (10)

Or after substitution of (8) and (9) into (11)

$$\operatorname{Max}\left\{\ln\left(w_{t}^{i}+b_{t}^{i}-h_{t}^{i}\right)+\rho\ln\left((1-\beta)y_{t}^{i}+\beta\overline{y}_{t}-r_{t}b_{t}^{i}\right)\right\} \qquad \text{w.r.t. } b_{t}^{i},\ h_{t}^{i}$$
s.t. expression (10)

Some straightforward manipulation of the first order conditions, leads to the following expression for an individual's investment

$$h_{t} = h_{t}^{i} = \frac{\rho \alpha (1 - \beta)}{1 + \rho \alpha (1 - \beta)} A_{t}$$

$$\tag{12}$$

¹ Each individual can employ the efficiency units of labour he is endowed with to produce current consumption according to a linear 'one-for-one' technology. Thus $c^i = w^i + b^i - k^i$ (see Aghion and Howitt (1998), p.283)

Every agent will invest the same amount of capital in the production process (irrespective of his initial endowment). The first derivative with respect to β is negative, so redistribution reduces investment levels

$$\frac{\partial h_{t}}{\partial \beta} = \frac{-\rho \alpha}{\left(1 + \rho \alpha (1 - \beta)\right)^{2}} A_{t} + \frac{\rho \alpha (1 - \beta)}{1 + \rho \alpha (1 - \beta)} \left(-\frac{\kappa}{n} \sum_{i} k_{t-1}^{i}\right) < 0$$
(13)

Result 1a (CMM): "If agents can borrow freely, redistribution reduces individuals' optimal investment in human capital".

Next we derive an expression for the steady state growth:

$$g = \ln \left(\frac{\sum_{i} y_{t}^{i}}{\sum_{i} y_{t-1}^{i}} \right) = \ln \eta * + \ln(1 - \kappa \beta) + \alpha \ln \rho \alpha + \alpha \ln(1 - \beta) - \alpha \ln[1 + \rho \alpha (1 - \beta)]$$

$$(14)$$

The partial derivative of g with respect to β is

$$\frac{\partial g}{\partial \beta} = -\frac{\alpha}{1-\beta} - \frac{\kappa}{1-\kappa\beta} + \frac{\rho\alpha^2}{1+\rho\alpha(1-\beta)} < 0 \tag{15}$$

If there is more redistribution (larger β), growth will slow down. Bénabou (1996) shows that the preferred tax rate by the median voter will depend on the relative position of his income to the mean of the income distribution. The larger the gap between the median and the mean income is (i.e. the more skewed to the left the income distribution is), the higher the preferred tax rate will be. This brings us to result 2:

Result 2a (CMM): "If capital markets are perfect and more inequality leads to more redistribution, then more inequality will hamper growth".

Imperfect market model (IMM)

Now, assume that capital markets are absent (bⁱ is equal to 0 for all individuals) and the government chooses to redistribute income across generations (intergenerational transfers). Current consumption is now represented by

$$c_t^i = w_t^i - h_t^i + \beta A_t \tag{16}$$

Each individual receives an equal share of the tax revenues collected from the *previous* generation:

$$\frac{1}{n}\sum_{i}\beta y_{t-1}^{i}=\beta A_{t} \tag{17}$$

Future consumption equalizes production minus taxes

$$d_t^i = y_t^i (1 - \beta)$$
 (18)

The individual's maximization problem becomes (substitution of (17) and (18) into (1)):

$$\operatorname{Max}\left\{\ln\left(w_{t}^{i}-h_{t}^{i}+\beta A_{t}\right)+\rho \ln y_{t}^{i}(1-\beta)\right\} \quad \text{w.r.t. } h_{t}^{i}$$
(19)

which leads to

$$h_t^i = \frac{\rho \alpha}{1 + \rho \alpha} (\varepsilon_t^i A_t + \beta A_t) = \frac{\rho \alpha}{1 + \rho \alpha} \left(\frac{\eta^*}{n} \sum_i k_{t-1}^i \right) (1 - \kappa \beta) \left(\varepsilon_t^i + \beta \right)$$
(20)

In contrast to the perfect market case, investment will differ across individuals. The first derivative of this expression with respect to redistribution is equal to

$$\frac{\partial h_t^i}{\partial \beta} = \frac{\rho \alpha}{1 + \rho \alpha} \left(\frac{\eta^*}{n} \sum_i k_{t-1}^i \right) \left(-\kappa \varepsilon_t^i + 1 - 2\kappa \beta \right) \tag{21}$$

The sign of expression (21) will be determined by the sign of the last factor:

$$(-\kappa \varepsilon_{t}^{i} + 1 - 2\kappa \beta) > 0 \quad \text{if} \quad \varepsilon_{t}^{i} < \frac{1 - 2\kappa \beta}{\kappa}$$

$$< 0 \quad \text{if} \quad \varepsilon_{t}^{i} > \frac{1 - 2\kappa \beta}{\kappa}$$

$$(22)$$

As redistribution relaxes credit constraints, the poorly endowed (ϵ^i sufficiently low) will invest more. The 'rich' will invest less. The higher κ , the higher the 'cost' of the tax system in terms of 'lost efficiency' and the lower the number of people that will benefit from redistribution. The effect of redistribution on aggregate investment is positive for 'normal' values of κ (see appendix (A7)),

Substituting (20) into (5) gives in terms of the individual education level

$$e_t^i = (h_t^i)^{\gamma} = \left(\frac{\rho\alpha}{1 + \rho\alpha}\right)^{\gamma} (A_t)^{\gamma} \left(\varepsilon_t^i + \beta\right)^{\gamma}$$

Total education can then be expressed as:

$$e_t^{tot} = \left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\gamma} (A_t)^{\gamma} \sum_{i} \left(\varepsilon_t^i + \beta\right)^{\gamma}$$
 (23)

The effect of redistribution is double: a higher β negatively influences efficiency (and thereby the accumulated knowledge and skills, A) but at the same time leads to in increase of the third factor of expression (23). For normal parameter values the total effect of redistribution will be positive (parallel to the effect on aggregate investment).

The effect of more equality (in the sense of a lower variance of incomes) on total education is univocally positive: x^{γ} is a concave function of x (0< γ <1), so by Jensen's inequality we know that a more unequal distribution of endowments (larger variance of ε_t^i), for a given amount of redistribution (fixed β), tends to lower total education.

Result 1b (IMM):	"If agents cannot borrow and returns to investment in human capital					
	are decreasing, more inequality reduces total education.					
	If the efficiency loss is not excessive, redistribution has a positive					
	impact on investment in human capital and education levels".					

We can again derive an expression for growth

$$g = \ln \eta * + \ln(1 - \kappa \beta) - \ln n + \alpha \ln \left(\frac{\rho \alpha}{1 + \rho \alpha}\right) + \ln \left[\sum_{i} \left(\varepsilon_{i}^{i} + \beta\right)^{\alpha}\right]$$
 (24)

In contrast to the CMM, the effect of redistribution on growth is ambiguous. On the one hand, a negative effect is still present through the second term of expression (24). But now there is also a positive impact through the fourth term.

Result 2b (IMM)::	"If capital	markets	are	absent,	redistribution	will	stimula	te grow	rth
	through th	ne relaxa	tion	of credit	constraints	Th	ne 'full'	effect	of
	redistribution is less clear".								

Before we can further explore the potential of the VAR-framework to evaluate the empirical validity of the above theoretical models, we need to take a closer look into the data. Given the above results we need to collect time series for income inequality, human capital, economic growth and redistribution.

The data

We include 9 OECD countries in the empirical investigation. The choice of the countries was somewhat forced upon us due to the limited data availability. Fortunately the sample seems representative for the entire OECD. Firstly, with Canada, France, Italy, the UK and the USA, 5 members of the G-7 are present, next to 4 smaller countries: Belgium, Finland, the Netherlands and Sweden. Secondly, the sample contains countries with an extensive social security system (Belgium, Finland and Sweden) as well as countries with a limited one (the UK and the USA). Thirdly, also non-EU countries are present. It would be informative to include Germany in the estimates, but we lack sufficient data to do so.

Although we have selected countries based on data availability and data consistency, we admit that measurement error remains a problem, especially with respect to the enrolment and inequality series. Measurement error increases the noise to signal ratio of the series which blurs the estimation results, especially if one uses first differences (Krueger and Lindahl, 2000). Therefore, the empirical results should be approached with a healthy amount of scepticism.

For each country in our sample we collected annual data for income inequality, secondary and tertiary enrolment, economic growth and social security expenditure. More detailed information on the inequality series is provided in Gobbin and Rayp (2003). The enrolment series are described in appendix B. Graphs of all series are presented in appendix C. We first motivate the variable choice.

Income inequality

Although Cowell (1995) shows that the generalized entropy indices are theoretically superior to the gini coefficient for the measurement of inequality, we mainly use the latter in this paper. The reason is twofold. On the one hand the availability of the gini coefficient is far greater than that of any other inequality index. On the other hand the use of the gini coefficient is commonplace in empirical applications. Using the gini coefficient allows a comparison of our results with other studies. However, data availability was the main determinant of our choice. As we need an annual inequality measure over the longest possible time period, the gini coefficient turns out to be the only viable alternative. The one exception is France for which we lack data on the gini coefficient and instead use the 5% top income share as a proxy for income inequality.

A lot of recent empirical work in the field of inequality and economic growth (Banerjee and Duflo (2003), Forbes (2000), Barro (1999)) is based on the Deininger and Squire (DS) data set (Deininger and Squire (1996)). Because the DS data set has substantially increased data comparability, both over countries and time, it has somewhat become the standard for data sources on inequality. Nevertheless, Atkinson and Brandolini (1999) show that there remain important problems, even with the so-called 'high quality' data in the DS data (see also Gobbin and Rayp (2003)).

In its current form, the DS data set is only applicable to cross section and panel data estimates. As we intent to explore a time series approach, we need to compile a suited data set.

In our sample, the gini coefficients for the different countries are not all based on the same income concept, nor were they all collected in the same manner. Some gini coefficients were derived from census data, others were calculated out of income tax data. Both types have some drawbacks. Income inequality measures based on income tax data might underestimate inequality as only those incomes that are high enough to be taxable are included in the calculation. The quality of census data will depend on the representativity of the sample. Atkinson (2003) and Gobbin and Rayp (2003) describe the problems related to the measurement of income inequality in more detail.

The testable implications derived in section 3 are based on inequality of incomes before redistribution. Unfortunately income data are scarce and do not always allow for a perfect test of the theoretical models. Only for Belgium and for France we have data on income before taxes. Even then the available time series are only imperfect proxies for the 'theoretically optimal choice of income', as a. o. the government support for education is not taken into account. Hence our data series serve as rough proxies for the theoretically optimal inequality concepts. For most countries data availability forces us to use net income or household disposable income. This has implications for the empirical exercise. Firstly, we will less likely detect the link between inequality and redistribution predicted by the CMM. Secondly, as in the IMM redistribution matters for enrolment because it reduces 'post-redistribution-inequality', we should not expect to detect a positive effect of redistribution on enrolment if we do not use it in combination with pre-redistribution inequality in the estimates.

Based on the theoretical models we can also make some reservation with respect to the choice of inequality measure. To test for the CMM, the middle incomes (median voter) should be highlighted. In the IMM the bottom incomes (credit constraints) and the dispersion of incomes matter more. As the gini coefficient is especially sensitive to changes in the middle incomes, it is an a priori acceptable choice in case of the CMM. To test for the IMM one might prefer a poverty line index. In this light the use of the top 5% income share for France is questionable.

All of the above remarks also matter if one uses the DS data. Still most panel data studies using the DS data ignore the potential problems and use different income concepts for different countries in a single panel estimation (Knowles (2001)). Rehme (2002) illustrates how mixing measures of gross and net income inequality can blur the estimation results if redistribution negatively affects growth. Some authors correct the data for the differences in income concept, but Atkinson and Brandolini (1999) doubt that these adjustments really solve the problem.

A related issue is that our income definition might be too narrow. Deininger and Squire (1998) argue that it is the asset distribution that really matters for the systematic effect of inequality on growth. They believe land distribution is to be preferred to income distribution as a proxy for asset distribution.

Enrolment rates

The data set includes enrolment rates in secondary and higher education as a proxy for human capital. In the estimations enrolment appears as a proxy for the investment in as well as for the stock of human capital, which is in line with earlier work (eg. Perotti (1996), Barro and Sala-I-Martin (1995)).

Although existing theoretical literature does not force us to interpret investment in this narrow way, human capital formation is likely to be most gravely affected due to credit constraints as the collateral is highly insecure. The choice of this narrow investment variable has a drawback: in the absence of credit constraints the significance of the effect of redistribution on investment might be underestimated. This is only a minor problem if investment in human and physical capital are complements rather than substitutes.

We include both secondary and tertiary education in the estimates to capture the full effects of enrolment. It seems plausible that the evolution of enrolment in secondary education was a driving force at the beginning of our sample (the 1960s), but that enrolment in tertiary education has gradually taken over this leading role.

There is some arbitrariness in the exact definition of the enrolment rates, which also makes it hard to compare them across countries. Firstly, the education system differs across countries. The study length can differ, as can the age at the time of first enrolment. This is of importance when computing the number of potential students, which is the denominator of the enrolment rate. It is even problematic in countries that have delegated the design of the education system to the constituent regions (the UK and Canada). Secondly, the education system can change over time. Especially the division between primary and secondary education has been subject to major changes in Finland and Sweden. For the number of potential students for tertiary education we look at the first 5 years after the normal end date of secondary education, which is of course a rough approximation. Although the result might not be the best possible measure for the 'average' enrolment rate, we are confident that it is a good proxy for it (and our results robust to different choices).

For higher education we use total tertiary education as well as university education (again a choice based solely on data availability). Although the levels are clearly different, the correlation between both series is very high (above 95% for all countries). Therefore, we can safely use the term 'higher education' without further distinction and concentrate on the longest and most consistent series.

In appendix B we describe the enrolment data in more detail.

Economic growth

We use the 'GDP at market prices, in volume and at local currency' and 'total population' from the OECD Economic Outlook database (1960-2000) (OECD 2002b). In the estimation we use the first difference of the logarithm of the ratio of GDP to total population. As there is no cross-section dimension in the estimates, it is not necessary to adjust for purchasing power parity.

Social security

As a proxy for redistribution we use social security expenditure. Sinn (1994) and Wigger (2001) document two ways by which an increase in social security expenditure redistributes resources in a society. We use the public and mandatory private social security expenditure as a percentage of GDP. The data are collected from two sources. Our basic series is taken from the OECD Social Expenditure Database (OECD (2002b)). As for most countries this series starts in 1980 (except for Italy (1982), the Netherlands (1995) and Sweden (1993)), we need to extend it backwards (the final observation is for 1997). To this end we use the growth rate of the comparable ILO data series. Although the levels of both series are sometimes quite different, the growth rates of the OECD series and the ILO series in the overlapping period (1980-1993) are very similar. The start date for the ILO series is 1960, but the most recent year available is at best 1993. Missing values were obtained by linear interpolation.

The use of social security expenditure in our set-up can be questioned. Given the theoretical models we need a variable that captures both the redistributive efforts in a society (IMM) and the distortionary effects of fiscal policy (CMM). Our variable choice partially meets both requirements. The ideal variable again depends on the theoretical model we have in mind. For the IMM we want to measure the redistributive efforts that reduce capital constraints with respect to human capital formation. On the one hand, social security expenditure seems too broad a measure as also expenditure related to health, housing, ... is taken into account. Note, however, that relaxing capital constraints in one field, opens up resources for other purposes. On the other hand not all relevant redistributive efforts are included (for instance study loans might be overlooked). The CMM deals with all types of government expenditure that are redistributive in nature. From this perspective our broad variable choice seems acceptable. However, in the CMM not the magnitude of the expenditure, but the distortionary effects of the related taxation matter. If taxation were fully lump sum, a redistributive fiscal policy would not distort growth. Easterly and Rebelo (1993) and Perotti (1996) propose the average marginal tax rate as an alternative. Sonedda (2003) uses marginal and average tax rates. But tax-based indicators also have their drawbacks as tax revenues are used for multiple purposes besides redistribution (e.g. defence). Moreover, it is hard to obtain a time series that goes back as far as the 1960s. As it is possible to construct a long enough and consistent annual time series for social security expenditure, and as no alternative variable seems a priori superior, we believe it is an acceptable choice.

Cointegration analysis

A number of explorative univariate unit root tests (results not shown) indicated that more than half of the time series in the data set potentially display non-stationary behaviour (the tests for the income inequality series are discussed at length in Gobbin and Rayp (2003)). Given these results and the endogeneity problems in growth econometrics, we choose to test for cointegration in a VAR-framework (eg. Johansen and Juselius (1994)).

As the time dimension of our series is rather limited, we can not use asymptotic theory and need to perform small sample corrections. Given the small time dimension, we only allow for 2 lags in the VAR. The results are only marginally altered by including a third lag. The choice for 2 lags in the VAR results in a 1st order VECM of the following form:

$$\Delta X_{t} = \mu_{0} + \Pi X_{t-1} + \Gamma \Delta X_{t-1} + \varepsilon_{t} \tag{1}$$

where X_t is a 5x1 vector containing enrolment in secondary and tertiary education, growth, income inequality and social security expenditures in year t and μ_0 a vector of constants. Based on our univariate analysis, we do not restrict the constant to the cointegration space,

as some individual series display trending patterns (Franses (2001)). This might not be in line with economic intuition in the longer term, but it is acceptable over the short period we consider.

If not all variables in X_t are (trend) stationary, the matrix π will not be of full rank. If the system is cointegrated, i.e. there exist linear combinations of the non-stationary variables which are stationary, we can rewrite π as the product of two full column rank matrices, $\Pi = \alpha \beta^T$. Both matrices are of dimension 5xr, with r being the number of cointegrating relations. Expression (1) can be rewritten as:

$$\Delta X_{t} = \mu_{0} + \alpha \beta^{T} X_{t-1} + \Gamma \Delta X_{t-1} + \varepsilon_{t}$$
(2)

The matrix β contains the long run (cointegrating) relationships, the matrix α the short run adjustments towards these long run equilibria.

The cointegrating rank

For the determination of the cointegrating rank we use the trace statistic evaluated against its 95% critical value. However, this is an asymptotic critical value. Johansen (2002) illustrates that the actual probability of rejecting a correct null hypothesis in a finite sample is much larger than the 5% nominal value. In other words: we should use higher critical test values than the asymptotic ones.

Johansen (2002) introduces a correction factor for the trace statistic which should lead to a good approximation of these corrected critical values. If we reject the null hypothesis with the asymptotic critical values, we also look at the corrected ones. In the table the corrected values are in italic. We report the asymptotic values between brackets if the conclusion about the rank changes due to the correction. It should be noted that the correction factor is only an approximation of the actual small sample distortion (Johansen *et al.* (2002)). Its reliability also depends on the parameter values. However, throughout the correction appears to be a useful supplement to the 'classic' analysis (Johansen (2002)).

<insert table 1 around here>

We do not reject two cointegrating relations for all countries, except for Italy, Sweden and the UK. If we do not apply the small sample correction, we do not reject 3 cointegrating relations for Canada, Finland and the Netherlands.

In the presence of multiple cointegrating relations (r>1), the estimates are not unique and directly interpretable. We can identify the long term relationships between the 5 variables by imposing coefficient restrictions and the long term relations (β) and the short run adjustments (α). Based on the identified relationships we try to distinguish between the IMM and the CMM. Note that the number of model results (cf. section 2) we can test *simultaneously* is equal to the number of cointegrating relationships.

Identification of the long term relationships

We identify the long run relations by imposing coefficient restrictions on the cointegrating vectors (beta-vector) and the short run adjustments (alpha-vector). The choice of restrictions is based on the testable results of the IMM and CMM. Again, Johansen (2002b) notes that the asymptotic results of the estimates are not accurate enough for small samples. His results indicates that the actual size can be quite distorted (much larger than the nominal size) in small samples. We do not provide a robust correction for this small sample bias, but, based on Johansen's results, a correction factor for the likelihood ratio of between 1.3 and

1.7 seems probable. Therefore, to convincingly reject the restrictions, the p-value should be sufficiently below 5%.

We use the following 2-step testing procedure:

- 1. Can we identify the long run relations in line with the results of the IMM? growth = f[enrolment (+); social security(-); ...] (result 2b) enrolment = f[gini (-); social security(+); ...] (result 1b)
- 2. Can we identify the long run relations in line with the results of the CMM? $growth = f[social\ security(-); ...]$ (result 2a) $social\ security = f[gini\ (+), ...]$ (result 2a)

Our objective is to check whether it is possible to detect an identification of the long term relations in line with the theoretical models (given the limitations of the available data). The identification of multiple cointegrating vectors is not unique and changing the order in which restrictions are imposed can sometimes drastically change the results. Therefore, we systematically explore the different sequences of restrictions. If we find both the correct growth and/or enrolment relation and/or inequality relation in the first step, we can argue that the data support the IMM. But we can not reject the CMM. Next we check whether the implications of the CMM are compatible with the long run relations. We check whether growth is negatively related to social security, and whether social security is positively related to inequality. The latter requirement might be somewhat too strict given the limitations of the dataset. We should not be surprised if we fail to find a significant effect of inequality on social security expenditure if we can not use an inequality measure for income before taxes (i.e. for all countries except Belgium and France). A similar argument holds for the effect of credit constraints on enrolment rates in the IMM. But now the effect should not vanish: higher post redistribution inequality should still result in less enrolment irrespective of the amount of redistribution taking place. Combined with our earlier remark that the investment variable might suit the IMM better than the CMM (cf. supra) the above comments indicate that the tests give a slight preferential treatment to the IMM, although the gini coefficient better fits the CMM.

Note that the ambition of this empirical exercise is limited: we want to check if the data support the models. Even if the data fit a model perfectly, we can not yet call this model the 'true model'. We only give an indication which (if any) model suits reality better. Also recall that the variables in the estimates are only approximations of the 'theoretical' variables in the models (cf. supra).

<insert table 2 around here>

In table 2 we present the beta matrix (after imposing restrictions), i.e. the long term relations. The standard errors are between brackets. We also impose zero restrictions on the alpha matrix, the short run adjustments, if the coefficients have a wrong sign or if they are highly insignificant. These 'corrections' are obviously somewhat ad hoc but they can be thought of as a kind of robustness check. We do not present the alpha matrix as it is not essential in our analysis.

For Belgium we find an enrolment relationship compatible with the IMM. We can however remove income inequality after taxes from the enrolment relation. If we use inequality before taxes, the elimination of the gini coefficient can not be accepted. Social security is negatively related to growth, which is compatible with both models. Enrolment in secondary education is positively related to growth. The impact of enrolment in higher education on growth is not significant. We can not identify a social security relation in line with the CMM.

Inequality is detrimental for enrolment in Canada. As the data do not confirm the positive link between enrolment and growth, there is only partial support for the IMM. The data neither indicate that inequality induces social security expenditure.

We again find a robust enrolment relation for Finland. It is hard to identify the second cointegration relationship in terms of the IMM or the CMM. The data do not fit a long term social security nor a long term growth relationship. The only meaningful interpretation seems to be an inequality relationship: income inequality is reduced by an increase in social security expenditure.

For France we find a long term enrolment relationship in line with the IMM: more inequality reduces enrolment. Different from other countries, economic growth is not significant in the relationship. However, as enrolment rates do not significantly affect growth, the data only partially support the IMM. We also find a negative impact of social security expenditure on economic growth. Next we abandon the enrolment relationship and try to identify a social security relationship instead. We detect a long run relationship in which social security expenditure are positively influenced by inequality and enrolment, and negatively by growth. This is a specification in line with the CMM. Moreover the negative effect of social security on economic growth is still present. Now we also find a (robust) positive link between enrolment in secondary education and growth. So the CMM might be more appropriate for France

For Italy there was only 1 cointegration relationship which does neither seem to fit the CMM nor the IMM. Allowing for 2 cointegration relationships does not reduce these identification difficulties.

For the Netherlands we can not identify an enrolment relation. We do find a growth relation and social security relation consistent with the CMM. More inequality leads to more social security expenditure and social security leads to less growth. However, the results for the Netherlands might be less trustworthy, as our measure of redistribution does not capture its extensive system of students loans (Guille (2000)). The fact that we can not identify an enrolment relation in line with the IMM might stem from that shortcoming. Credit constraints will be less severe for investment in education.

For Sweden we should again only allow for 1 cointegration relation. We can identify this relation as an enrolment or as a growth relationship (results not shown). If we identify it as an enrolment relation, enrolment is positively related to growth and social security expenditure. Income inequality is not significant. If we choose the second relationship social security expenditure reduce growth. Enrolment is not significant. If we impose homogeneity between countries with respect to the cointegrating rank, i.e. ignore the test results in table 7 and always allow for 2 cointegration relations, we are able to identify a growth relation and a social security relation consistent with the CMM (results not shown). Sweden also has an extensive system of students loans (Guille (2000)), which might again explain the lack of IMM-compatible relationship in the Swedish data.

Also for the UK we concluded that there was only 1 cointegration relationship. This relationship can be reduced to the stationary behaviour of economic growth. If we allow for 2 cointegration relations, the second relation can be identified as an enrolment relation in which income inequality reduces enrolment and social security stimulates it (results not shown).

For the USA we find an enrolment relation and a growth relation consistent with the IMM. We can not identify the social security relation implied by the CMM. Carneiro and Heckman (2002) find that currently only a very marginal fraction of the US population is credit constrained. Their results do not necessarily contradict ours as they themselves note that 'the limited role of short run credit constraints in explaining American educational gaps is, no doubt, in part due to the successful operation of policies that were designed to eliminate such constraints'.

We never detect a significant positive relation between social security and enrolment, which is not really surprising given the 'imperfect' inequality measures (cf. supra).

The available data support both the IMM (3 countries) and the CMM (2 countries, 3 if we impose a higher cointegrating rank on the Swedish data). Some relations can not be identified in terms of either the CMM or the IMM. For France the evidence is somewhat mixed. Thus different models seem to be appropriate for different countries.

With some reservation we can draw two more general conclusions: income inequality reduces enrolment in most countries and the direct impact of social security expenditures on growth is negative.

The findings for the Netherlands and Sweden show that it might be useful to select the redistribution variable on a country specific basis. Given their extensive system of students loans it might not be surprising that the results for these countries are more in line with the implications of the CMM. However, one needs an extensive knowledge of the particularities in the social security and education systems of the different countries to further explore this possibility.

Limitations of the time series approach

The time series approach deals with a number of problems that disturb the reliability of cross section estimates but it is unable to solve all of them. The most important drawback is the limited length of the time series. While there exist longer time series for some of the variables included in the VAR system, most data series (and by extension the entire VAR system) are limited to the 1960s (at best). As economic growth is a long term phenomenon, we would prefer to include much more lags in the VAR specification. However, we face a trade-off between theoretical considerations and practicability.

If we do not include sufficient lags, the correlation between the error terms and the lagged variables might differ from zero resulting in inconsistent estimates. We already argued that lagged values of the endogenous variables are not necessarily good instruments because of the importance of expectations about the future for current decision making. So we need to include enough lags to cover a normal 'planning horizon'. Although the appropriate horizon is still a subject of discussion, its length will surely exceed 2 or 3 years.

To evaluate the potential inconsistency of our results due to the presence of autocorrelation in the error terms, we looked at a vector error autocorrelation test. Based on the findings of Doornik (1996) we applied the F-approximation of the Lagrange-multiplier test because of its superior behaviour in small samples. The results indicate that autocorrelation is not much of a problem for the estimates involving Belgium, Canada, Sweden and the UK (we obtain a p-value above 0.7 for each of these countries). For France, Finland and the USA the test values do not reject 'the absence of autocorrelation' at the 10%-level, but they do reject it at the 20%-level for the former and at the 15%-level for the latter two. For Italy and the Netherlands the results clearly reject 'no autocorrelation'. On the one hand these test results further strengthen our earlier suspicions of the Dutch data (and results), but on the other hand they deepen our believe in some of the other results.

Due to the short time series, it is hard to check whether the estimated parameters are stable over time. We argued that parameters differ across countries, but they might also depend on the level of development in an individual country. Maddala and Wu (2000) find some evidence of instability over time in growth relationships. On the other hand, because of the short time period covered in the estimates, this potential 'time heterogeneity' will be much lower than the 'cross country heterogeneity'.

In summary, our results are not unquestionable, but nonetheless the new methodology can serve as a valuable supplement to the techniques currently used in growth econometrics. The time series approach will gradually lead to more robust results as longer time series become available.

Conclusion

Durlauf (2001) urged growth economists to advance in the field of growth econometrics. The empirical analysis of economic growth is indeed tainted by some very serious flaws and shortcomings. We accepted Durlauf's challenge and this paper is a concise report of an attempt to deal with the problems typical of growth empirics. Based on an overview of the most serious problems noted in the literature, we propose a methodology that deviates in two ways from existing work: firstly we propose a time series approach instead of a cross section or panel analysis, and secondly we resort to the Johansen cointegration framework, a methodology that, to our knowledge, has not been applied before in growth econometrics. These 'innovations' deal with heterogeneity and endogeneity problems. However, new problems arise as existing data sets are not suited for this new approach. Even for OECD countries it is hard to collect long and reliable time series for inequality and enrolment. On the one hand, these data problems limit the workability of our method, but on the other hand also open up new perspectives for the future. As longer and more reliable time series will become available for more countries, the trustworthiness of the results will strongly increase. We applied the methodology to the analysis of the relation between income inequality and economic growth. Keeping the above data reservation in mind, we dare present some cautious conclusions. Both the imperfect market model (Belgium, Canada, the USA) and the complete market model (the Netherlands, France and with some flexibility: Sweden) find support in the data. The fact that the negative effect of inequality on enrolment seems relevant for most countries, enhances the credibility of the IMM. But for most countries we find also a negative relation between social security expenditures and economic growth, a central mechanism in the CMM. However, the support for the CMM is weakened if we take the extensive system of students loans in the Netherlands and Sweden into account. An indirect conclusion is that different models seem to hold for different countries (even in our subset of rich countries). If this conclusion proves to be robust, it questions the appropriateness of panel estimates for this kind of research and thereby also the validity of previous studies.

Table 1: cointegrating rank (growth, social security, income inequality, secondary and tertiary education)

Country	Rank	Trace	Trace 95% - Corrected values	Conclusion
Belgium	H ₀ : r=0; H ₁ : r>0	95.22	74.37	2 cointegration relations is not
Income after taxes	H ₀ : r≤1; H ₁ : r>1	57.39	51.92	rejected
(2 lags)	H ₀ : r≤2; H ₁ : r>2	28.90	29.70	(asymptotic critical values: 2)
Belgium	H ₀ : r=0; H ₁ : r>0	91.67	74.40	2 cointegration relations is not
Income before taxes	H ₀ : r≤1; H ₁ : r>1	54.19	52.16	rejected
(2 lags)	H ₀ : r≤2; H ₁ : r>2	26.20	29.70	(asymptotic critical values: 2)
Canada	H_0 : r=0; H_1 : r>0	115.50	79.53	2 cointegration relations is not
(2 lags)	H ₀ : r≤1; H ₁ : r>1	72.09	58.39	rejected
(2 lays)	H ₀ : r≤2; H ₁ : r>2	43.15	45.57 (25.80)	(asymptotic critical values: 3)
France	H_0 : r=0; H_1 : r>0	90.58	77.27	2 cointegration relations is not
(2 lags)	H ₀ : r≤1; H ₁ : r>1	59.01	54.31	rejected
(2 lays)	H ₀ : r≤2; H ₁ : r>2	29.20	50.37 (29.70)	(asymptotic critical values: 2)
Finland	H_0 : r=0; H_1 : r>0	100.10	75.98	2 cointegration relations is not
(2 lags)	H ₀ : r≤1; H ₁ : r>1	55.21	52.35	rejected
(2 lags)	H ₀ : r≤2; H ₁ : r>2	33.06	34.15 (29.70)	(asymptotic critical values: 3)
Italy	H_0 : r=0; H_1 : r>0	83.10	78.47	1 cointegration relation is not
(2 lags)	H ₀ : r≤1; H ₁ : r>1	45.47	52.67 (47.20)	rejected
(2 lags)	H ₀ : r≤2; H ₁ : r>2			(asymptotic critical values: 1)
Netherlands	H_0 : r=0; H_1 : r>0	101.90	72.36	2 cointegration relations is not
(2 lags)	H ₀ : r≤1; H ₁ : r>1	59.83	59.57	rejected
(2 lags)	H ₀ : r≤2; H ₁ : r>2	30.13	33.14 (29.70)	(asymptotic critical values:3)
Sweden	H ₀ : r=0; H ₁ : r>0	51.24	50.13	1 cointegration relation is not
(2 lags)	H ₀ : r≤1; H ₁ : r>1	20.14	26.70	rejected
				(asymptotic critical values: 1)
UK (2 lags)	H_0 : r=0; H_1 : r>0	77.40	74.20	1 cointegration relation is not
	H_0 : $r \le 1$; H_1 : $r > 1$	44.06	47.20	rejected
				(asymptotic critical values: 1)
USA	H_0 : r=0; H_1 : r>0	85.75	75.58	2 cointegration relations is not
(2 lags)	H ₀ : r≤1; H ₁ : r>1	52.59	50.99	rejected
(2 1495)	H ₀ : r≤2; H ₁ : r>2	23.42	29.70	(asymptotic critical values: 2)

Note: The figures in *italic* in the fourth column are corrected values. If we do not reject the null hypothesis (H_0) on the basis of the asymptotic value for the trace statistic, we give the asymptotic value (not in italic) as the corrected one can only be higher. If the use of the corrected values changes the conclusion, we also report the asymptotic value (between brackets).

Table 2: The long term relations

	Growth	Gini	Secondary education	Higher education	Social security
Belgium before taxes					
Enrolment relation	-14.222	5.822	1.000	-3.832	0.000
Lin onnent relation	(2.573)	(1.610)	1.000	(0.489)	0.000
Growth relation	1.000	0.000	-0.527	0.000	0.0083
(p-value: 0.942)	1.000	0.000	(0.117)	0.000	(0.0024)
Belgium after taxes			(0.111)		(0.002+)
Enrolment relation	-13.615	3.564	1.000	-2.481	0.000
Linonnent relation	(1.912)	(0.923)	1.000	(0.332)	0.000
Growth relation	1.000	0.000	-1.049	0.000	0.0137
(p-value: 0.872)	1.000	0.000	(0.274)	0.000	(0.0059)
Canada			(0.214)		(0.0039)
Enrolment relation	-2.101	6.316	1.000	-1.075	0.000
Linonnent relation	-2.101 (0.268)	(1.097)	1.000	(0.088)	0.000
Growth relation	1.000	22.765	0.000	0.000	0.0818
(p-value: 0.319)	1.000	(5.787)	0.000	0.000	(0.0118)
Finland		(3.767)			(0.0116)
	-2.104	6.300	1.000	-1.077	0.000
Enrolment relation	-		1.000	-	0.000
la consolito sucletica	(0.267)	(1.089)	0.000	(0.088)	0.0000
Inequality relation	0.000	1.000	0.000	0.000	0.0033
(p-value: 0.379)					(0.0004)
France	0.000	0.740	4.000	0.407	0.000
Enrolment relation	0.000	2.746	1.000	-0.487	0.000
0	4.000	(0.572)	0.000	(0.1067)	0.0007
Growth relation	1.000	0.000	0.000	0.000	0.0027
(p-value: 0.576)					(0.0003)
Growth relation	1.000	0.000	0.000	-0.031	0.0030
On state and the state	250.00	70.00	40.700	(0.012)	(0.0002)
Social security relation	352.80	-72.88	-18.792	0.000	1.000
(p-value: 0.729)	(12.42)	(23.48)	(4.589)		
The Netherlands	4.000	0.000	0.500	0.000	0.0407
Growth relation	1.000	0.000	-0.566	0.000	0.0137
	040.00	400.74	(0.073)	0.000	(0.0015)
Social security relation	219.82	-123.74	0.000	0.000	1.000
(p-value: 0.225)	(25.96)	(15.43)			
The USA	0.5.15	0.555	4.655	0.5	0.000
Enrolment relation	-0.316	0.585	1.000	-0.275	0.000
	(0.074)	(0.104)		(0.029)	
Growth relation	1.000	0.0000	-0.508	0.000	0.0045
(p-value: 0.718)			(0.156)		(0.0009)

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Appendix A: Mathematical derivations

Expression 12:

$$\operatorname{Max}\left\{\ln\left(w_{t}^{i}+b_{t}^{i}-h_{t}^{i}\right)+\rho\ln\left((1-\beta)y_{t}^{i}+\beta\overline{y}_{t}-r_{t}b_{t}^{i}\right)\right\} \qquad \text{w.r.t. } b_{t}^{i},\ h_{t}^{i}$$
s.t. expression (10)

Maximizing with respect to h:

$$\frac{1}{w_t^i + b_t^i - h_t^i} = \frac{\rho(1 - \beta)(1 - \kappa \beta)\eta * \alpha (h_t^i)^{\alpha - 1} A_t^{1 - \alpha}}{(1 - \beta)y_t^i + \beta \bar{y}_t - r_t b_t^i}$$
(A1)

Maximizing with respect to b:

$$\frac{1}{w_{t}^{i} + b_{t}^{i} - h_{t}^{i}} = \frac{\rho r_{t}}{(1 - \beta) y_{t}^{i} + \beta \overline{y}_{t} - r_{t} b_{t}^{i}}$$
(A2)

Equations (A1) and (A2) reveal the following expression for r:

$$r_{t} = \eta * \alpha \left[\frac{A_{t}}{h_{t}^{i}} \right]^{1-\alpha} (1 - \kappa \beta)(1 - \beta), \qquad (A3)$$

which is of course equal to the after tax marginal product of human capital.

From (A3) we can deduce that investment levels have to be equal across individuals $(h_i^i = h_i)$.

From (A2) we can derive that:

$$(1 - \beta)y_t^i + \beta \bar{y}_t - r_t b_t^i = \rho r_t (w_t^i + b_t^i - h_t)$$
(A4)

If we aggregate this expression over the entire population (n individuals) and use the restriction that the sum of net-borrowings has to equal 0 (market clearing condition), we get

$$\sum_{i} ((1 - \beta) y_t^i + \beta \overline{y}_t) = \sum_{i} \rho r_t (w_t^i - h_t)$$

$$\Rightarrow n[(1-\beta)\overline{y}_t + \beta\overline{y}_t] = \rho r_t \left[\sum_i w_t^i - nh_t\right]$$

$$\Rightarrow n\overline{y}_{t} = \rho r_{t}(nA_{t} - nh_{t}) \tag{A5}$$

$$\Rightarrow (1 - \kappa \beta) \eta * h_t^{\alpha} A_t^{1-\alpha} = \rho \eta * \alpha A_t^{1-\alpha} h_t^{\alpha-1} (1 - \kappa \beta) (1 - \beta) (A_t - h_t)$$

$$\Rightarrow h_t = \rho \alpha (1 - \beta) (A_t - h_t)$$

which gives expression (12).

Expression 14:

$$\frac{\sum_{i} y_{t}^{i}}{\sum_{i} y_{t-1}^{i}} = \frac{(1 - \kappa \beta) \eta * h_{t}^{\alpha} A_{t}^{1-\alpha}}{A_{t}} = \frac{(1 - \kappa \beta) \eta * \left[\frac{\rho \alpha (1 - \beta)}{1 + \rho \alpha (1 - \beta)}\right]^{\alpha} A_{t}^{\alpha} A_{t}^{1-\alpha}}{A_{t}} = \eta * \left[\frac{\rho \alpha (1 - \beta)}{1 + \rho \alpha (1 - \beta)}\right]^{\alpha} (1 - \kappa \beta)$$

In the first step we use the assumption that there is no population growth. Taking the natural logarithm of this expression results in expression (14).

Expression 20:

$$\operatorname{Max}\left\{\ln\left(w_{t}^{i}-h_{t}^{i}+\beta A_{t}\right)+\rho \ln y_{t}^{i}(1-\beta)\right\} \qquad \text{w.r.t. } h_{t}^{i}$$
(19)

Results in

$$\frac{1}{w_{t}^{i} - h_{t}^{i} + \beta A_{t}} = \frac{\rho(1 - \beta)\eta * (1 - \kappa\beta)A_{t}^{1 - \alpha}\alpha(h_{t}^{i})^{\alpha - 1}}{(1 - \beta)\eta * (1 - \kappa\beta)A_{t}^{1 - \alpha}(h_{t}^{i})^{\alpha}} \Leftrightarrow \frac{1}{w_{t}^{i} - h_{t}^{i} + \beta A_{t}} = \frac{\rho\alpha}{h_{t}^{i}}$$

$$\Leftrightarrow h_t^i = \frac{\rho \alpha}{1 + \rho \alpha} (\varepsilon_t^i A_t + \beta A_t) = \frac{\rho \alpha}{1 + \rho \alpha} \left(\frac{\eta *}{n} \sum_i k_{t-1}^i \right) (1 - \kappa \beta) \left(\varepsilon_t^i + \beta \right)$$
 (20)

Total investment is then equal to

$$\sum_{i} h_{t}^{i} = \sum_{i} \left[\frac{\rho \alpha}{1 + \rho \alpha} (\eta * \overline{k}_{t-1}) (1 - \kappa \beta) (\varepsilon_{t}^{i} + \beta) \right] = \frac{\rho \alpha}{1 + \rho \alpha} (\eta * \overline{k}_{t-1}) (1 - \kappa \beta) \sum_{i} (\varepsilon_{t}^{i} + \beta)$$

$$= \frac{\rho \alpha}{1 + \rho \alpha} \left(\eta * \overline{k}_{t-1} \right) (1 - \kappa \beta) n (1 + \beta) \tag{A6}$$

Redistribution will have a positive effect on total investment in human capital as long as

$$\beta < \frac{1}{2} \left(\frac{1}{\kappa} - 1 \right). \tag{A7}$$

This condition is satisfied for most plausible values of κ and β . For instance for values of κ below 1/3, the effect is positive independent of the value of β . For values of β comparable to the highest average effective tax rates in the OECD (Carey and Tchilinguirian (2000)), κ can be as high as $\frac{1}{2}$. So if the efficiency loss is not too high, redistribution will positively affect total investment.

Expression 24:

$$\sum_{i} y_{t}^{i} = \eta A_{t}^{1-\alpha} \sum_{i} \left(h_{t}^{i} \right)^{\alpha} = \eta A_{t}^{1-\alpha} \left(\frac{\rho \alpha}{1 + \rho \alpha} \right)^{\alpha} A_{t}^{\alpha} \left[\varepsilon_{t}^{i} + \frac{\beta}{1 - \kappa \beta} \right]^{\alpha}$$
(A8)

$$\Rightarrow \frac{\sum_{i} y_{t}^{i}}{\sum_{i} y_{t-1}^{i}} = \frac{\eta A_{t} \left(\frac{\rho \alpha}{1 + \rho \alpha}\right)^{\alpha} \sum_{i} \left[\varepsilon_{t}^{i} + \frac{\beta}{1 - \kappa \beta}\right]^{\alpha}}{n A_{t} \frac{1}{1 - \kappa \beta}} = \eta (1 - \kappa \beta) \frac{1}{n} \left(\frac{\rho \alpha}{1 + \rho \alpha}\right)^{\alpha} \sum_{i} \left[\varepsilon_{t}^{i} + (1 - \kappa)\beta\right]^{\alpha}$$
(A9)

Taking the natural logarithm of (A9) gives expression (24).

Appendix B: Enrolment rates (data description)

For Belgium we have a shorter series for enrolment at university (1956 – 1992, Mitchell (1998)). The correlation between this series and tertiary education is very high (98,6%).

For Canada no distinction between students in primary or secondary education is made, which is partly due to the existence of different education systems in the different Canadian regions. However, the sum of years in primary and secondary education is always identical. We suspect that the division is rather arbitrary (i.e. not based on some kind of minimum education programme) and have approximated secondary education by subtracting the population aged 5 to 11 from the original series. As education was obligatory over the period 1960-1998 for children at these ages, the result of the subtraction should be a reasonable proxy for the number of pupils in secondary schools (although officially they might belong to another category). UNESCO reports total enrolment in secondary education in Canada for the period 1980-1995. The ratio of our data and the UNESCO data is systematically between 1,05 and 1,08. So our series might slightly overestimate the number of students in secondary education. However, this error seems to be quite consistent over time. As our estimation result depends on changes in enrolment, we choose not to adjust the series to eliminate the 'error'. Enrolment at university is strongly correlated (.97) with enrolment in tertiary education reported by the OECD (2002) over the period 1985-1997.

The enrolment in secondary education in Finland is again an approximation. First, we added the students in primary and secondary education, and next we subtracted the population aged 7 to 12 from that sum. The rationale for this operation are some clear inconsistencies in the basic series that seem to stem from a changing division between primary and secondary education over the years. Although the levels in the new series are substantially lower than the levels reported by the OECD (2002) over the period 1985-1993, the correlation between the two series is quite high (.93).

For Italy we did not find suitable demographical data before 1970. We extended the population series backwards until 1967 by shifting back age cohorts from 1970. By doing so we ignore the impact of migration and death. The latter will not be very important as we deal with young people. The former might be more relevant. However, if we look at the first comparable period, 1970-1972, there are only very small changes in these age cohorts. Therefore, we can safely assume that the error will be minor. The correlation between university enrolment and tertiary education (OECD) over the period 1985-1995 is again very high (.97).

Secondary enrolment in the Netherlands does not include vocational training. If we only consider university enrolment (WO), the enrolment rate in higher education drops considerably but the correlation between both series is high (.98).

The population statistics for the UK were collected from 3 regional sources: the Office for National Statistics (England and Wales), the General Register Office (Scotland) and Northern Ireland Statistics.

We did not include secondary enrolment in Sweden, as the available series clearly lacked consistency. Different from the Finnish case, it seems hard to cure this problem in a reliable manner.

Table B.1: Enrolment rates

Country	Enrolment rate	Source	Period
Belgium	Secondary education /	NIS – Statistical Yearbook	1956 – 2000
J	Population aged 12-17	NIS – Population Statistics	1960 – 2000
	Tertiary education / Population	NIS – Statistical Yearbook	1956 – 2000
	aged 18-22	NIS – Population Statistics	1960 – 2000
Canada	Secondary education	Statistics Canada	1960 – 1998
	(approximation) /		
	Population aged 12-17	Statistics Canada	1971 – 2001
	University education /	Statistics Canada	1960 – 1997
	Population aged 18-22	Statistics Canada	1971 – 2001
Finland	Secondary education	Mitchell (1998)	1960 – 1993
	(approximation)/	, ,	
	Population aged 13-18	Statistics Finland	1960 – 2001
	Tertiary education /	SF – Statistical Yearbook	1960 – 1993
	Population aged 19-23	Statistics Finland	1960 – 2001
France	Secondary education /	Insée – Statistical Yearbook	1968 – 2000
	Population aged 12-18	Insée – Statistical Yearbook	1960 – 2001
	3		
	University education /	Insée – Statistical Yearbook	1968 – 2000
	Population aged 19-23	Insée – Statistical Yearbook	1960 – 2001
Italy	Lower secondary education	ISTAT – Statistical Yearbook	1963 – 1995
	(scuola media) / Population aged 12-14	Eurostat – New Cronos	1970 – 2000
	Higher secondary education (scuole secondari superiori) /	ISTAT – Statistical Yearbook	1966 – 1995
	Population aged 15-19	Eurostat – New Cronos	1970 – 2000
	Secondary education	Combination of lower and higher secondary education	1966 – 1995
	University education/	ISTAT – Statistical Yearbook	1966 – 1995
	Population aged 20 – 24	Eurostat – New Cronos	1970 – 2000
The Netherlands	Secondary education /	CBS (MAVO + HAVO + VWO)	1950 – 1996
The Netherlands	Population aged 12 – 17	CBS	1950 – 2000
	Tertiary education /	CBS (WO + HBO)	1950 – 1996
	Population aged 18 – 22	CBS	1950 – 2000
The UK	Secondary education /	ONS	1950 – 1999
me en	Population aged 12 – 17	ONS, GRO, NIStat	1961 – 1999
	University education /	Mitchell (1998)	1950 – 1993
	Population aged 18 - 22	ONS, GRO, NIStat	1691 – 1999
The USA	High School students /	USBC	1960 – 2000
	Population aged 12 – 17	USBC	1960 – 2000
	College students /	USBC	1960 – 2000
	Population aged 18 - 22	USBC	1960 – 2000
Sweden	Undergraduate students /	Statistics Sweden	1960 – 2000
	Population aged 19 – 23	Eurostat – New Cronos Statistical Yearbook Sweden	1970 – 2001 1960 – 1969

Appendix C: Inequality, social security expenditure, enrolment and growth in graphs



















