

# Child Labor, Fertility, and Economic Growth\*

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## Abstract

This paper explores the evolution of child labor, fertility, and human capital in the process of development. In early stages of development the economy is in a development trap where child labor is abundant, fertility is high and output per capita is low. Technological progress, however, increases gradually the wage differential between parental and child labor, thereby inducing parents to substitute child education for child labor and reduce fertility. The economy takes-off to a sustained growth steady-state equilibrium where child labor is abolished and fertility is low. Prohibition of child labor expedites the transition process and generates Pareto dominating outcome.

Keywords: Child labor, Fertility, Growth.

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## 1. Introduction

Child labor is a mass phenomenon in today's world. According to the ILO Bureau of Statistics, 250 million children aged 5–14 were economically active in 1995, almost a quarter of the children in this age group world-wide.<sup>1</sup> The phenomenon is most widespread in the poorest continent, Africa, but was not always the sole province of the less developed countries: child labor was once common in Europe and in the United States, too. In 1851 England and Wales, 36.6 percent of all boys aged 10–14 and 19.9 percent of girls in the same age group worked. The historical evidence suggests that child labor has been part of the labor scene since time immemorial.<sup>2</sup>

Although the empirical literature on modern-day child labor is abundant, a theoretical examination of the phenomenon is rather scarce. Recent theoretical studies are Basu and Van (1998) and Baland and Robinson (2000). Basu and Van demonstrate the feasibility of multiple equilibria in the labor market; one equilibrium where children work and another where the adult wage is high and children do not work. Baland and Robinson (2000) study the implications of child labor for welfare. However, despite evidence about the positive relationship between child labor and fertility and a negative effect of income on child labor, existing theories have abstracted from the important dynamic interrelationship between child labor and the process of development.<sup>3</sup>

This paper explores the dynamic evolution of child labor, fertility, and human capital in the process of development.<sup>4</sup> In early stages of development the economy is in a development trap where child labor is abundant, fertility is high and output per

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<sup>1</sup> See Ashagrie (1998). Many definitions of child labor are available. See Ashagrie (1993) for a discussion of the definitions, classifications and the data available today on child labor. See also Basu (1999) and Morand (1999b) for data on contemporary child labor.

<sup>2</sup> These figures are from Cunningham (1990) who provides data on child employment in England and Wales from the 1851 census and evidences on earlier eras. An historical discussion of child labor in Europe, the United States and Japan appears in Wiener (1991).

<sup>3</sup> See Rosenzweig and Evenson (1977), Cain and Mozumder (1981), Levy (1985) and Grootaert and Kanbur (1995) for the positive relation between child labor and fertility. See Goldin (1979) and Levy (1985) for the negative relation between child labor and income, and Barro (1991) for the negative relation between fertility and income.

<sup>4</sup> Although the literature presents several theoretical studies of the joint dynamics of income and fertility, so far no theoretical analysis of child labor dynamics exists. Most of the literature which presents theoretical studies of the joint dynamics of income and fertility tends to explore the negative relation between income and fertility that has prevailed in developed countries since the mid-19th century, e.g., Becker, Murphy and Tamura (1990) and Galor and Weil (1996). Exceptions are Galor and Weil (1999, 2000) and Morand (1999a) who model non-monotonic relation between income and fertility. Namely that at first fertility increased with income and only at some stage this relation reversed.

capita is low. Technological progress, however, increases gradually the wage differential between parental and child labor, decreasing the benefit from child labor and permitting ultimately a take-off from the development trap. Parents find it optimal to substitute child education for child labor and reduce fertility. The economy converges to a sustained growth steady-state equilibrium where child labor is abolished and fertility is low.<sup>5</sup> Prohibition of child labor expedites the transition process and generates Pareto dominating outcome.

The analysis is based on four fundamental elements. First, we assume that parents control their children's time and allocate them between child labor and human capital formation. Secondly we assume that parents care about their descendants' future earnings. Thirdly we assume that the income generated by children is accrued to parents, and fourthly that child rearing is time intensive. As a result, an increase in the wage differential between parental and child labor increases the cost of child rearing and decreases the cost of schooling, which is the child's forgone earning in the labor market. Thus, the increase in the wage differential (between parental and child labor) decreases fertility and child labor and increases children's education, and therefore the wage differential between adults and children increases further in the next period.<sup>6</sup> Consequently, along the dynamic path to steady state, families become smaller and better educated. We thus show that, consistent with the empirical evidence, child labor tends to decrease as the household's dependency on child labor's income diminishes.

Since child labor abounds in today's world, the question whether policy should be applied to combat this phenomenon is of particular interest. Baland and Robinson (2000) show that child labor may be inefficiently high when bequests are zero or when capital markets are imperfect. In our model child labor is inefficiently high as well. To see why, consider the following contract: Parents allow their children to study their entire childhood and in exchange children promise to compensate their parents in the next period, when adults. As long as the potential income of an individual who invests in human capital her entire childhood is greater than the sum

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<sup>5</sup> This take-off out of the "pseudo steady state" resembles the endogenous demographic transition in Galor and Weil (1996) and Morand (1999a).

<sup>6</sup> In section 2 it is shown that the wage differential between adults and children may not increase when technology is constant, but when technological progress is introduced, this wage differential *must* increase.

of incomes of a child and an uneducated adult, this contract Pareto dominates the competitive equilibrium. However, Baland and Robinson (2000) claim that it is impossible to enforce such intergenerational contracts. Here we show that a government can solve this market failure by introducing compulsory schooling in the current period and a redistributive taxation from educated adults to the elders in the next period, a policy that needs to prevail for one generation only.<sup>7</sup> We show that this policy not only Pareto dominates the competitive outcome, but also that it can immediately launch the economy out of the poverty trap towards the high output steady state equilibrium. This take-off out of the poverty trap to a growth path toward the high output steady state is similar to the implication of policy in growth models that study income inequality in the face of capital market imperfection such as Galor and Zeira (1993) and Maoz and Moav (1999).

The paper is organized as follows: Section 2 presents the basic model of child labor and fertility with constant technology and derives the dynamic system implied by the model. Section 3 introduces technological progress and analyzes the resultant dynamics. Section 4 discusses policy implications of the model and section 5 concludes.

## **2. The Basic Structure of the Model**

Consider a small, open, overlapping-generations economy that operates in a perfectly competitive world and faces a given world rate of interest. Time is infinite and discrete. In every period, the economy produces a single good that can be used for either consumption or investment. Three factors of production exist in the economy: physical capital, raw units of labor, and efficiency units of labor.

### **2.1. Production**

In each period, there are two potential sectors. Production can take place in either one of them or in both. It is important to emphasize that the existence of one sector is independent of the existence of the other, and, as will become clearer later, the existence

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<sup>7</sup> This policy scheme formalizes the idea suggested by Becker and Murphy (1988) though their discussion ignores the dynamic applications.

of each sector is determined by individuals' optimal choices. In both sectors technology has constant returns to scale, but employs different factors: one technology employs only raw labor, while the other employs physical capital and efficiency units of labor. We refer to the former as "traditional" and to the later as "modern."<sup>8</sup>

The production function of the traditional sector is:<sup>9</sup>

$$Y_{1,t} = w^c L_t; \quad (1)$$

the modern production function satisfies all the neoclassical assumptions and is given by:

$$Y_{2,t} = F(K_t, \lambda_t H_t), \quad (2)$$

where  $L_t$ ,  $H_t$ ,  $K_t$ , and  $\lambda_t$  are the quantities of raw labor, efficiency labor, physical capital, and the level of technology (which we set in this section to equal 1) respectively, employed at time  $t$ , and  $w^c > 0$  is the marginal productivity in the traditional sector. Given the production technology, the competitiveness of markets and the world interest rate,  $\bar{r}$ , firms' inverse demand function for capital is:

$$\bar{r} = f'(k_t), \quad (3)$$

where  $k_t \equiv (K_t / \lambda_t H_t)$ , and therefore,

$$k_t = f'^{-1}(\bar{r}) \equiv \bar{k}. \quad (4)$$

The return to one unit of raw labor is  $w^c$  and the return to one unit of efficiency labor,  $w_t$ , is:

$$w_t = f(\bar{k}) - f'(\bar{k})\bar{k} \equiv \bar{w}. \quad (5)$$

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<sup>8</sup> The existence of these two sectors can represent the process of urbanization. Thus, the traditional sector can represent rural production and the modern sector can represent industrial production. The set up of two sectors that produce the same output but employ different factors of production is in the spirit of Galor and Zeira (1993).

<sup>9</sup> For the sake of simplicity we assume that the marginal productivity in the traditional sector is constant. As long as income in the traditional sector would grow at a lower rate than income in the modern sector the qualitative results of the paper would not change.

## 2.2. Individuals

In each period,  $t$ , a generation of  $L_t$  individuals joins the labor force. Each individual has a single parent. Individuals within a generation are identical in their preferences and levels of human capital. Members of generation  $t$  live for three periods. In the first period (childhood),  $t-1$ , individuals are endowed with single unit of time that is allocated by their parent between schooling and labor-force participation. Children can offer only  $\theta \in (0, 1)$  units of raw labor due to their (purportedly) inferior physical ability and can work only in the traditional sector.<sup>10,11</sup> Their earnings accrue to the parent.<sup>12</sup> In the second period of life (parenthood),  $t$ , individuals save their income and allocate their single unit of time between childrearing and labor-force participation. They choose the number of children and the children's time allocation between schooling and labor; they then direct their own remaining time to the labor market. They decide whether to supply raw labor (and to work in the traditional sector), or to supply efficiency units of labor (and to work in the modern sector).<sup>13</sup> The decision is made according to the number of efficiency units of labor,  $h_t$ , they have. Specifically, they will choose the sector that maximizes their income, that is:

$$I_t = \max \{ \bar{w} h_t, w^c \}, \quad (6)$$

where  $I_t$  is potential income.<sup>14</sup> In the third period, this generation consumes its savings with the accrued interest.

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<sup>10</sup> "Children rarely receive an income even approaching the minimum wage, and their earnings are consistently lower than those of adults, even where the two groups are engaged in the same tasks" (Bequele and Boyden, 1988, pp. 4–5). Thus,  $\theta$  can also be interpreted as discrimination against children in the labor market in the sense that they get less than their marginal productivity. For evidence see, for example, Bequele and Boyden (1988, Chapter 5).

<sup>11</sup> The literature on child labor suggests that children are usually employed in industries where technologies are simple and production labor-intensive. Many studies show that the development of capital-intensive production has the effect of displacing child labor. See, e.g., Bequele and Boyden (1988). Galbi (1994) shows the same impact for the industrial revolution. Hence, we assume that children can be employed in the traditional sector only.

<sup>12</sup> The role of children as assets is important in developing economies. See Dasgupta (1993) and Razin and Sadka (1995). Morand (1999a) introduces the old-age support motive when modeling the demographic transition.

<sup>13</sup> Canagarajah and Coulombe (1997), Jensen and Nielsen (1997) and Psacharopoulos (1997) support the assumption of a trade off between child labor and their human capital formation.

<sup>14</sup>  $I_t$  is potential income because it is the income per one unit of time. However, parents devote some of their time to childrearing and hence earn an income equals to  $(1 - zn_t)I_t$ .

### 2.2.1. Preferences

We assume that individuals derive utility from consumption and from the potential income of their offspring in period  $t+1$ . For the sake of simplicity, we assume that individuals consume only in the third period. Thus, the utility function of an individual who is a member of generation  $t$  is:<sup>15</sup>

$$u^t = \alpha \ln(c_{t+1}) + (1 - \alpha) \ln(n_t I_{t+1}), \quad (7)$$

where  $c_{t+1}$  is consumption in period  $t+1$ ,  $n_t$  is the number of children of individual  $t$  and  $I_{t+1}$  is the potential income of each child in period  $t+1$ , determined by the rule given in equation (6).

### 2.2.2. The budget constraint

As in Galor and Weil (1996, 2000), we follow the standard demand model of household fertility behavior. We assume that a parent faces a time constraint when choosing how many children to have. More specifically, we assume that time is the *only* input required in raising children. We denote by  $z \in (0, 1)$  the amount of time needed to raise one child, implying that  $(1/z)$  is the maximum number of children that can be raised.

As mentioned earlier, it is the parent who allocates the time endowment of children between schooling and labor-force participation.<sup>16</sup> Let  $\tau_t \in [0, 1]$  be the fraction of time allocated to schooling and  $(1 - \tau_t)$  the fraction of time allocated to labor-force participation of each child in period  $t$ . Thus, given the assumption on the physical ability of a child, each child supplies  $\theta(1 - \tau_t)$  units of raw labor to labor-force participation.

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<sup>15</sup> This form of preferences and utility function follows Galor and Weil (2000).

<sup>16</sup> Parents do not discriminate between children: each child gets the same schooling as its siblings.

Schooling is free and hence the only cost of schooling is the opportunity cost, i.e., the forgone earnings of the child.<sup>17</sup> Therefore the budget constraint of the household is:

$$(1 - zn_t)I_t + \theta(1 - \tau_t)n_t w^c = s_t \quad (8)$$

In the third period individuals consume their savings with accrued interest. Hence,

$$c_{t+1} = s_t(1 + \bar{r}) . \quad (9)$$

### 2.2.3. *The production of human capital*

The level of human capital of member of generation  $t+1$ ,  $h_{t+1}$ , is predetermined in period  $t$  through schooling. We assume that an individual is born with some basic human capital and can achieve more by attending school. As in Galor and Weil (2000) we assume that the level of human capital is an increasing, strictly concave function of the time devoted to schooling. In order to simplify, we assume that the production function of human capital is:

$$h_{t+1} = h(\tau_t) = a(b + \tau_t)^\beta, \quad (10)$$

where  $a, b > 0$  are constants and  $\beta \in (0, 1)$  is the “adjusted” elasticity of human capital with respect to schooling.<sup>18</sup> Note that since  $\tau_t \in [0, 1]$ , the level of human capital is bounded from below by  $ab^\beta$ , the level of human capital that the child is born with, and from above by  $a(b + 1)^\beta$ , the maximum level of human capital that can be achieved if the child’s time is allocated entirely to schooling.<sup>19</sup>

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<sup>17</sup> Introducing direct schooling costs does not change the qualitative result of the model, as long as they are constant. Kanbargi (1988) shows that in some Indian states, where education (and even books and meals) are provided free of charge, enrolment is low due to the indirect costs of schooling, namely, the child’s forgone earnings.

<sup>18</sup> By “adjusted” we mean that  $\beta$  includes not only schooling but also innate ability,  $b$ . Note that if  $b=0$ ,  $\beta$  would be exactly the elasticity of human capital with respect to schooling.

<sup>19</sup> Salazar and Glasinovitch (1996) and Schiefelbein (1997) point that child labor adversely affects children’s schooling performance. If we take this finding into account we should specify the human capital production function as follows:  $h_{t+1} = h(\tau_t) = a(b + \eta\tau_t)^\beta$  where  $\eta=1$  if the child does not



#### 2.2.4. Optimization

A member of generation  $t$  chooses the number of her children, the time allocation of her children between schooling and labor-force participation, and consumption, so as to maximize her utility function (7) subject to her budget constraint (8) and the constraints on  $\tau_t$  and  $n_t$ , that is,  $\tau_t \in [0, 1]$  and  $n_t \in [0, 1/z]$ .<sup>20</sup> Substituting (8) and (9) into (7), the optimization problem facing the individual of generation  $t$  is:

$$(n_t, \tau_t) = \arg \max \left\{ \alpha \ln \left[ (1 + \bar{r}) \left[ (1 - zn_t) I_t + \theta (1 - \tau_t) n_t w^c \right] \right] + (1 - \alpha) \ln(n_t I_{t+1}) \right\} \quad (11)$$

s.t.:  $0 \leq \tau_t \leq 1$   
 $0 \leq n_t \leq 1/z.$

The following assumption is needed to ensure the existence of child labor and that parents devote some of their time to labor force participation.

**Assumption 1:**

- $\alpha z > \theta$
- $w^c < a\bar{w}(b+1)^\beta < \frac{z}{z-\theta} w^c.$

$\alpha z > \theta$  is needed to ensure that  $(1 - \alpha)/(z - \theta) < 1/z$ , i.e., that the parent devotes positive amount of time to labor-force participation.<sup>21</sup> The second part of assumption 1 is needed to ensure that if parental income is at its lowest possible level, the parent would choose a positive level of child labor. Note that the middle term is the maximum level of potential income in the modern sector and can be thought of as the gain from child schooling in terms of future potential income. The term on the right is the ratio between the cost (in terms of output) of child rearing when each child just goes to school and the cost of child rearing when each child just works. Thus, this term can be thought of as the relative cost of child schooling. The second part of

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work at all and  $0 < \eta_0 < 1$  if the child spends some time working. For simplicity we ignore this finding since adding this element will only strengthen our results.

<sup>20</sup> We ignore integer problems and allow the number of children per household to be in the segment  $[0, 1/z]$ .

assumption 1 implies, therefore that, if household's income is the lowest possible one, the relative cost of child schooling is greater than the gain from child schooling.

Let us now describe the solution to the optimization problem (11). Note that  $h_t$  is determined in period  $t-1$  and hence the parent chooses the sector to which she supplies her labor independently of the optimal choice of the number of children and their time allocation to schooling, which we denote by  $(n_t^*, \tau_t^*)$ . The optimization is done in two stages. In the first stage, the parent considers the possibility that her children will work in the modern sector in the next period, that is, she assumes  $I_{t+1} = \bar{w}h_{t+1}$ . She maximizes (11) with respect to  $(n_t, \tau_t)$ , derives a solution denoted by  $(\hat{n}_t, \hat{\tau}_t)$ , substitutes  $\hat{\tau}_t$  into the production function of human capital [equation (10)] and obtains a solution to  $h_{t+1}$ , denoted by  $\hat{h}_{t+1}$ . In the second stage, she compares her children's potential income in the next period if they work in the modern sector,  $\bar{w}\hat{h}_{t+1}$ , to their potential income in the next period if they work in the traditional sector,  $w^c$ . If  $\max\{\bar{w}\hat{h}_{t+1}, w^c\} = \bar{w}\hat{h}_{t+1}$ , then  $(\hat{n}_t, \hat{\tau}_t) = (n_t^*, \tau_t^*)$  is the solution to the problem. Otherwise, If  $\max\{\bar{w}\hat{h}_{t+1}, w^c\} = w^c$ , then the parent chooses  $\tau_t^* = 0$  and differentiates (11) with respect to  $n_t$ .<sup>22</sup>

Depending on the parameters of the model two cases can arise regarding the solution to the maximization problem. The first case occurs when  $\theta q < z$  where

$$q \equiv \left[ \left( \frac{w^c}{a\bar{w}} \right)^{\frac{1}{\beta}} \frac{1-\beta}{\beta} + 1 + b \right].$$

In this case,  $\tau_t^*$  is positive, regardless of  $h_t$ . In the second

case, when  $\theta q \geq z$ ,  $\tau_t^*$  is equal to zero for sufficiently low levels of  $h_t$  and positive only for higher levels of  $h_t$ . Note that  $\hat{\tau}_t$  and therefore  $\hat{h}_{t+1}$  are monotonically increasing functions of parental income,  $I_t(h_t)$ . If the parameters of the model are such that some schooling is optimal even when the level of parental income is the lowest possible one,  $w^c$ , then it would be optimal to choose schooling when parental income

<sup>21</sup>A less restrictive assumption,  $z > \theta$  is needed to rule out an uninteresting case. We thank an anonymous referee for pointing this out.

<sup>22</sup>Note that if  $\max\{\bar{w}h_{t+1}, w^c\} = w^c$  then  $\tau_t = 0$  is optimal, because any fraction of time devoted to schooling is chosen only to maximize children's future potential income. If future potential income is  $w^c$ , then education is a waste of time. Unpalatable as it may seem, this is probably true. See Bequele and Boyden (1988), especially p. 6; Bonnet (1993); and Grootaert and Kanbur (1995), p. 193.

is higher than  $w^c$ , i.e., for every  $h_t$ . Alternatively, if the parameters are such that when the level of parental income is the lowest possible one,  $w^c$ , no schooling is optimal, then there exists a threshold level of parental human capital (and a corresponding parental income's threshold), denoted by  $\tilde{h}$ , such that whenever parental human capital is below it, zero schooling is optimal and vice versa.

Equation (12a) gives the optimal schooling for the case where  $\tau_t^*$  is equal to zero for sufficiently low levels of  $h_t$ .

$$\tau_t^* = \begin{cases} 0 & \text{if } h_t < \tilde{h} \\ \frac{\beta z \bar{w} h_t - \beta w^c \theta - b w^c \theta}{w^c \theta (1 - \beta)} & \text{if } \tilde{h} \leq h_t \leq \frac{w^c \theta (1 + b)}{\beta z \bar{w}} \\ 1 & \text{if } \frac{w^c \theta (1 + b)}{\beta z \bar{w}} \leq h_t \end{cases} \quad (12a)$$

where  $\tilde{h} \equiv \frac{q \theta w^c}{z \bar{w}}$ .

The first line of (12a) shows that children do not receive any schooling when parental human capital below the threshold  $\tilde{h}$ . Only when parental human capital is above this threshold, children receive positive level of schooling as can be seen from the second and the third lines of (12a).

Equation (12b) gives the optimal number of children for this case.

$$n_t^* = \begin{cases} \frac{1 - \alpha}{z - \theta} & \text{if } h_t \leq \frac{w^c}{\bar{w}} \\ \frac{(1 - \alpha) \bar{w} h_t}{z \bar{w} h_t - w^c \theta} & \text{if } \frac{w^c}{\bar{w}} \leq h_t < \tilde{h} \\ \frac{(1 - \alpha)(1 - \beta) \bar{w} h_t}{z \bar{w} h_t - w^c \theta (1 + b)} & \text{if } \tilde{h} \leq h_t \leq \frac{w^c \theta (1 + b)}{\beta z \bar{w}} \\ \frac{1 - \alpha}{z} & \text{if } \frac{w^c \theta (1 + b)}{\beta z \bar{w}} \leq h_t \end{cases} \quad (12b)$$

The first two lines of (12b) are relevant for the case where  $\tau_t^*$  is equal to zero. The third and the fourth lines of (12b) are relevant for the case where some schooling is optimal.<sup>23</sup>

Equation (13a) gives the optimal schooling for case where  $\tau_t^*$  is positive, regardless of  $h_t$ .

$$\tau_t^* = \begin{cases} \frac{\beta z - \beta \theta - b \theta}{\theta(1 - \beta)} & \text{if } h_t \leq \frac{w^c}{\bar{w}} \\ \frac{\beta z \bar{w} h_t - \beta w^c \theta - b w^c \theta}{w^c \theta(1 - \beta)} & \text{if } \frac{w^c}{\bar{w}} \leq h_t \leq \frac{w^c \theta(1 + b)}{\beta z \bar{w}} \\ 1 & \text{if } \frac{w^c \theta(1 + b)}{\beta z \bar{w}} \leq h_t \end{cases} \quad (13a)$$

And (13b) gives the optimal number of children for that case.

$$n_t^* = \begin{cases} \frac{(1 - \alpha)(1 - \beta)}{z - \theta(1 + b)} & \text{if } h_t \leq \frac{w^c}{\bar{w}} \\ \frac{(1 - \alpha)(1 - \beta) \bar{w} h_t}{z \bar{w} h_t - w^c \theta(1 + b)} & \text{if } \frac{w^c}{\bar{w}} \leq h_t \leq \frac{w^c \theta(1 + b)}{\beta z \bar{w}} \\ \frac{1 - \alpha}{z} & \text{if } \frac{w^c \theta(1 + b)}{\beta z \bar{w}} \leq h_t \end{cases} \quad (13b)$$

In contrast to (12), the distinction between the first line and the second line of equation (13) merely represents the sector to which the parent supplies her labor and the solution for schooling as well as for fertility is continuous in parental income.

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<sup>23</sup> Note that in (12b)  $n_t^*$  is continuous at  $h_t = w^c/\bar{w}$  and therefore the distinction between the first line and the second, merely reflects the fact that the parent switches from the traditional sector to the modern one. However,  $n_t^*$  is discontinuous at  $h_t = \tilde{h}$ . This happens because  $I_t$  is continuous at  $h_t = \tilde{h}$  and the household's income is divided proportionally between consumption and the potential income of the children. Since schooling changes from zero to a positive level, child labor declines discontinuously and therefore the income generated by the children decreases discontinuously. To prevent a discrete fall in consumption the parent has to supply more of her time to labor force participation. To do so, she has to rear fewer children and thus fertility decreases discontinuously.

Note that from (12a) and (13a) it follows that the optimal schooling time,  $\tau_t^*$ , increases in time required to rear children,  $z$ ; in the elasticity of human capital with respect to schooling time,  $\beta$ ; and in the wage in the modern sector,  $\bar{w}$ . Also,  $\tau_t^*$  decreases in the wage in the traditional sector,  $w^c$ ; in the children's units of raw labor,  $\theta$ ; and in  $b$ , which represents part of their innate human capital. Similarly, from (12b) and (13b) it follows that the optimal number of children,  $n_t^*$ , decreases in  $z$ ,  $\beta$  and  $\bar{w}$  and increases in  $w^c$ ,  $\theta$  and  $b$ . Finally, it can be verified from (12) and (13), that optimal schooling is a non-decreasing function of parental income,  $I_t$ , and that fertility is a non-increasing function of  $I_t$ .<sup>24</sup>

The following assumption is needed to assure that at the highest rate of fertility the population does not contract.

**Assumption 2:**  $1 - \alpha > z - \theta$ .

It follows from the solution to the household's maximization problem that as long as child labor exists, the optimum number of children is greater than  $(1 - \alpha)/z$ , which is the optimum number of children without child labor.<sup>25</sup> Hence, child labor increases fertility. Moreover, as the wage differential between parental and child labor increases, the optimum number of children declines. Along with the decline in the number of children, the time allocated to children's schooling increases because the relative importance of children's earnings declines. This result might explain a familiar feature of demographic transition: a rapid decline in fertility accompanied by higher rates of growth in output per capita. It also implies a trade-off between quantity and quality of children and that as the economy develops individuals prefer quality to quantity.

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<sup>24</sup> Child labor increases the family income for any level of parental income and weakens the income effect of the parent's wage relative to the substitution effect. Thus, the result that fertility decreases in parental income in a model with child labor, holds in any case where in the absence of child labor, the substitution effect dominates the income effect, equals it or is dominated by the income effect by less than the magnitude of weakening the income effect due to child labor.

<sup>25</sup> Note that if the maximization problem was formulated without child labor, i.e., with the same utility function, but a different budget constraint,  $(1 - zn_t)I_t = s_t$ , the optimum number of children would be  $(1 - \alpha)/z$ , regardless of  $I_t$ .

### 2.3. The Dynamical System

The level of human capital in period  $t+1$ ,  $h_{t+1}$ , is uniquely determined by the time allocated to schooling in period  $t$ . Since  $\tau_t^*$  is uniquely determined by the level of human capital in period  $t$ ,  $h_t$ , the level of human capital in period  $t+1$ ,  $h_{t+1}$ , is a real valued function of  $h_t$ . Thus, the solution of the maximization problem in each period generates a first-order, nonlinear dynamical system in  $h_t$ , denoted here by  $\Psi(h_t)$ , which is given by substituting  $\tau_t^*$  from (12a) or (13a) into (10).

**PROPOSITION 1:** If  $\theta q < z$ , i.e., if  $\tau_t^*$  is given by (13a), then the dynamical system,  $\Psi(h_t)$ , has a unique steady state equilibrium.

**Proof:** First, note that for all  $h_t \in \left[0, \frac{w^c}{\bar{w}}\right]$ ,  $\Psi(h_t) = a \left( b + \frac{\beta z - \beta \theta - b \theta}{\theta(1-\beta)} \right)^\beta > \frac{w^c}{\bar{w}}$  since  $\bar{w} a \left( b + \frac{\beta z - \beta \theta - b \theta}{\theta(1-\beta)} \right)^\beta > w^c$  must hold. Otherwise choosing a positive level of schooling is not optimal. Thus  $\Psi(h_t) > h_t$  for all  $h_t \in \left[0, \frac{w^c}{\bar{w}}\right]$ .

Secondly, note that  $\Psi(h_t)$  is continuous at  $h_t = \frac{w^c}{\bar{w}}$  since

$$\lim_{h_t \rightarrow \left(\frac{w^c}{\bar{w}}\right)} a \left( b + \frac{\beta z \bar{w} h_t - \beta w^c \theta - b w^c \theta}{w^c \theta (1-\beta)} \right)^\beta = a \left( b + \frac{\beta z - \beta \theta - b \theta}{\theta(1-\beta)} \right)^\beta.$$

Thirdly, note that for all  $h_t \in \left(\frac{w^c}{\bar{w}}, \frac{w^c \theta (1+b)}{\beta z \bar{w}}\right)$ ,  $\Psi'(h_t) > 0$  and  $\Psi''(h_t) < 0$  which

implies that  $\Psi(h_t)$  is strictly concave and strictly monotonically increasing in that

range. Fourthly, note that  $\Psi(h_t)$  is continuous at  $h_t = \frac{w^c \theta (1+b)}{\beta z \bar{w}}$  since

$$\lim_{h_t \rightarrow \left[\frac{w^c \theta (1+b)}{\beta z \bar{w}}\right]} a \left( b + \frac{\beta z \bar{w} h_t - \beta w^c \theta - b w^c \theta}{w^c \theta (1-\beta)} \right)^\beta = a(b+1)^\beta. \text{ Finally, note that for all}$$

$h_t \in \left[ \frac{w^c \theta (1+b)}{\beta z \bar{w}}, \infty \right)$ ,  $\Psi(h_t) = a(b+1)^\beta$ . Thus, there exists a unique  $\bar{h}$  such that  $\Psi(\bar{h}) = \bar{h}$

**PROPOSITION 2:** If  $\theta q \geq z$ , i.e., if  $\tau_t^*$  is given by (12a), then there can be either multiple equilibria or a unique equilibrium.

**Proof:** First, note that for  $h_t \in [0, \tilde{h})$ ,  $\Psi(h_t)$  is constant and equals  $ab^\beta$ . Secondly, note that  $\tilde{h} > ab^\beta$  because  $\theta q \geq z$  implies  $\tilde{h} \geq \frac{w^c}{\bar{w}}$  and  $\tau_t^* = 0$  implies  $\frac{w^c}{\bar{w}} > ab^\beta$ . Thus, the low stable steady state equilibrium,  $\bar{h}_t = ab^\beta$ , exists. Thirdly, note that  $\Psi(h_t)$  is discontinuous at  $\tilde{h}$  because  $\tau_t^*$  changes from 0 to a positive value and  $\lim_{h_t \rightarrow \tilde{h}^+} \Psi(h_t) > ab^\beta$ . If  $\Psi(\tilde{h}) > \tilde{h}$ , the high stable steady state equilibrium must exist because  $\Psi(h_t)$  is bounded from above and thus  $\Psi(h_t)$  has two stable steady state equilibria (see fig. 1.d). If not, either  $\Psi(h_t) < h_t$  for all  $h_t > \tilde{h}$  and therefore only the low steady state equilibrium exists (see fig 1.b), or,  $\Psi(h_t) > h_t$  for some  $h_t > \tilde{h}$  and then an unstable steady state and the high stable steady state equilibria exist (see fig 1.c).

Note that the existence of the development trap, i.e., the low steady state equilibrium, depends positively on  $\tilde{h}$  and thus from the properties of  $\tilde{h}$  the existence of the development trap depends negatively on the time required to rear children,  $z$ ; on the elasticity of human capital with respect to schooling time,  $\beta$ ; and on the wage in the modern sector,  $\bar{w}$ . In contrast, it depends positively on the wage in the traditional sector,  $w^c$ ; on the children's units of raw labor,  $\theta$ ; and on  $b$ , which represents part of their innate human capital.

The four possible shapes of  $\Psi(h_t)$  are drawn in Figure 1 and can be divided into three groups: (i) The dynamical system drawn in Figure 1a. This system has unique and stable equilibrium characterized by high income, a small number of children in

each household, and almost no child labor.<sup>26</sup> (ii) Figure 1b, where equilibrium is also unique and stable, but is characterized by low income, a large number of children in each household, and extensive child labor, which we refer to as a development trap.<sup>27</sup> Note that for these two groups, the initial condition of the economy, i.e., the level of human capital at time 0,  $h_0$ , has no effect on the long-run equilibrium. (iii) The third group consists of the dynamical systems drawn in Figures 1c and 1d. In Figure 1c, there are three steady state equilibria: the low and the high ones are stable, and the “middle” one is unstable; in Figure 1d only the low and high steady-state equilibria exist. For these two dynamical systems, the initial level of human capital is crucial because it determines the characteristics of the long-run equilibrium.

### 3. Technological Progress

In this section we extend the basic model to allow for technological progress. We show that under this process the poverty trap is only “pseudo steady state equilibrium”, that is, child labor, fertility and output per capita are constant at their development trap levels for long periods. However, at some period it becomes optimal to launch the modern sector and begin the process of investing in more advanced technology. At this stage, the wage differential between parental and child labor starts to increase and the process of development as described in the introduction “kicks in”.

Our modeling of technological progress is rather abstract. Since the focus of this section is the effect of technological progress on child labor and fertility dynamics (and not technological progress per se) we do not model an R&D sector explicitly. Nonetheless, firms choose the level of technology to be employed in each period optimally. We rely on the non-rivalry property of technology as emphasized by Romer (1990), resulting in increasing returns to technological progress. The dynamic implication implied by this assumption is similar to the Goodfriend and McDermott’s (1995) model where, in the first stage, development is driven by population growth and in later stages development is driven by human capital accumulation.<sup>28</sup>

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<sup>26</sup> The parameters can be adjusted so that child labor is abolished in the high equilibrium case.

<sup>27</sup> Actually in this steady state children work all the time and get no education (see Grootaert and Kanbur, 1995, p.191).

<sup>28</sup> The process of development in the Goodfriend and McDermott’s (1995) model is driven in its first stage by increasing in population size, which allows for specialization and in the second stage by human capital accumulation. In our model development is driven by the stock of human capital. However, in the first stage this stock is increasing over time due to population growth while in the



Consider the production function described in (2) and define  $x_t$  as the increment to the level of the technology employed in the modern sector from period  $t-1$  to period  $t$ , i.e.,  $x_t \equiv \lambda_t - \lambda_{t-1}$ . We assume that the process of upgrading the technology level incurred costs to the firms, which are represented by the following cost function:

$$P = x_t^\phi, \quad (14)$$

where  $\phi > 1$ . In each period, firms choose  $(K_t, H_t, x_t)$  as to maximize their profits. The solution is characterized by the following equations:

$$\begin{aligned} k_t &= f^{r-1}(\bar{r}) \equiv \bar{k} \\ w_t &= (\lambda_{t-1} + x_t) \bar{w} \end{aligned} \quad (15)$$

$$x_t = \left( \frac{1}{\phi} \bar{w} H_t \right)^{\frac{1}{\phi-1}},$$

where  $\bar{k}$  and  $\bar{w}$  are defined by (4) and (5). Note that the increment of the technology level,  $x_t$ , is positively related to the aggregate level of human capital. This is due to our assumption that the cost of technology change is independent of the size of the economy.<sup>29</sup> Note also that the existence of the modern sector still depends on the optimal choice made by the individuals as given by (6) where the modification needed in (6) is that the potential income in the modern sector now becomes  $\lambda_t \bar{w} h_t$ . Thus firms invest in more advanced technology (i.e.,  $x_t > 0$ ) only if the modern sector is launch, i.e., only if  $\max\{(\lambda_{t-1} + x_t) \bar{w} h_t, w^c\} = (\lambda_{t-1} + x_t) \bar{w} h_t$ .

Let us now describe the evolution of the economy under this specification of technological progress. Suppose that at date  $t=0$ ,  $\lambda_0$ ,  $x_0$  and  $H_0$  are such that  $I_0 = w^c$ .<sup>30</sup> It follows that as long as  $I_t = w^c$  i.e., as long as  $\left\{ \lambda_0 + \left[ \frac{1}{\phi} \bar{w} H_t \right]^{1/(\phi-1)} \right\} \bar{w} h_t < w^c$ , the

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second stage this stock is increasing (mainly) due to direct investment in human capital through schooling.

<sup>29</sup> This qualitative result would not change as long as we assume that the average cost of technology progress is decreasing in the population size. Formally, if we assume that  $P = P(L_t, x_t)$  where  $P$  is increasing and strictly concave in  $L$  the economy would follow the same qualitative dynamic path.

economy behaves as if it is trapped in poverty: fertility is at its highest level, child labor is extensive and consumption is at its lowest level. Note however, that the potential income in the modern sector is increasing due to the growth of the population over time.<sup>31</sup> Hence there exists a period  $t$  such that upgrading the technology level to  $\lambda_0 + x_t$  and launching the modern sector is optimal. Denote this  $t$  by  $\tilde{t}$  and suppose that the economy is at period  $\tilde{t} - 1$ . An individual who supplies raw labor and works in the traditional sector solves the maximization problem [the first line of (13) gives the solution]. She finds it optimal to provide her children with some schooling. From this period on, the level of technology employed in the economy is increasing in every period and thus potential income is increasing over time too. Consequently, fertility declines as well as child labor while consumption increases over time. At a certain level of potential income the economy reaches its new steady state: Child labor is abolished while fertility reaches its lowest level. However, consumption (and output) continues to grow forever since the investment in more advanced technology continues in every period. The evolution of the economy is described in figure 2.

#### 4. Pareto Improving and Policy Implications

As we explained in the introduction, the equilibrium of the model presented in section 2 is not Pareto efficient.<sup>32</sup> This suggests that a government policy may enhance the welfare in the economy. Moreover, in the context of this model we are more interested in the long run consequences of such a policy, namely, we are interested in the question whether such a policy can immediately launch the economy out of the low steady state towards the high steady state equilibrium.

Consider then an economy, which is characterized by two stable steady state equilibria and is trapped in poverty (see figures 1c and 1d) and for simplicity ignore

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<sup>30</sup> In period  $t = 0$   $x_0$  is normalized to zero and hence the optimal level of feasible technology to be employed in the modern sector is  $\lambda_0$ .

<sup>31</sup> Galor and Weil (2000) assume that the rate of technological progress is positively related to the population size. Kremer (1993) argues that regions that started with larger initial populations experienced faster technological progress.

<sup>32</sup> Even if individuals were two sided altruistic it might be that the compensation from children when adults to parents would be too small and thus parents would find it optimal to send their children to work. Baland and Robinson (2000) provide a comprehensive discussion on the issue of two-sided altruism.

technological progress. We demonstrate the existence of a government policy that induces an allocation, which Pareto dominates the competitive one and pulls the economy out of its trap by formalizing the idea of Becker and Murphy (1988) into a dynamic setting.<sup>33</sup> We assume that there exists a government in the economy, one that can execute a policy if it is Pareto improving.<sup>34</sup> Suppose that at some period  $t$  the government declares the following two period's policy (for period  $t$  and  $t+1$ ) before individuals allocate their resources. At the current period, compulsory schooling is introduced at a certain amount. In the following period, the government collects a lump-sum tax from workers in the modern sector and transfers the revenues to compensate the elders for the foregone earnings of their children in the previous period. Denote the policy by  $(\tau_t^{cs}, \rho_{t+1}, \sigma_{t+1})$  where  $\tau_t^{cs}$  is the minimum time that must be allocated to schooling of each child in period  $t$ , and  $\rho_{t+1}$  and  $\sigma_{t+1}$  are the lump-sum tax levied on each worker in the modern sector and the compensation each elder gets in period  $t+1$ , respectively.<sup>35</sup> Note that the individuals observe the government policy and then choose the optimal number of descendants.<sup>36</sup> It is important to emphasize that when the government picks the policy  $(\tau_t^{cs}, \rho_{t+1}, \sigma_{t+1})$  it takes into consideration the optimal number of descendants for each feasible policy it chooses. Given the compulsory schooling  $\tau_t^{cs}$  and the compensation  $\sigma_{t+1}$ , consumption (in period  $t+1$ ) and the optimal number of children are uniquely determined. Let  $n_t(\tau_t^{cs}, \sigma_{t+1})$  be the optimal number of descendants for each  $(\tau_t^{cs}, \sigma_{t+1})$ .

The government scheme  $(\tau_t^{cs}, \rho_{t+1}, \sigma_{t+1})$  is Pareto improving and pulls the economy from its development trap if it meets the following sufficient conditions:

1.  $n_t(\tau_t^{cs}, \sigma_{t+1})\rho_{t+1} = \sigma_{t+1}$
2.  $\bar{w}h_{t+1} - \rho_{t+1} > \bar{w}\bar{h}$

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<sup>33</sup> Becker and Murphy (1988) suggest a policy of subsidies to education and redistributive taxation from educated children, when adults, to their parents through a social security system.

<sup>34</sup> A more realistic assumption would be that the government could execute policies that take into consideration the welfare of the current generation alone. We take this restrictive assumption because our objective is to examine whether government intervention can affect the long run equilibrium.

<sup>35</sup> It is possible that the policy takes place more than two periods. If it does it can be denoted by  $\{(\tau_t^{cs}, \rho_{t+1}, \sigma_{t+1})\}_{t=0}^{\infty}$ . However we show that a two periods policy is sufficient.

<sup>36</sup> Note that under this policy the number of descendants and consumption are the only choice variables since compulsory schooling is binding.

3. (i)  $\alpha w^c (1 + \bar{r}) \leq [(1 - z n_t(\tau_t^{cs}, \sigma_{t+1})) w^c + n_t(\tau_t^{cs}, \sigma_{t+1})(1 - \tau_t^{cs}) \theta w^c] (1 + \bar{r}) + \sigma_{t+1}$
- (ii)  $\frac{1 - \alpha}{z - \theta} w^c \leq n_t(\tau_t^{cs}, \sigma_{t+1}) [\bar{w} h_{t+1} - \rho_{t+1}]$ ,

where  $\bar{h}$  is the level of the unstable equilibrium level of human capital (see figures 1c and 1d).

Condition 1 implies a balanced government budget in period  $t+1$ . Condition 2 ensures that each child, when adult in period  $t+1$ , can earn in the modern sector a net income, which is larger than the minimum income, which ensures that her dynasty will converge to the high steady state. Condition 3 is a sufficient condition for (weak) improving of parent's welfare. The LHS of (i) is parent's consumption in the low steady state (without government intervention) while the RHS is parent's consumption when the government policy is introduced. The LHS of (ii) is potential income of children at  $t+1$  in the low steady state, whereas the RHS is the potential income of children at  $t+1$  net of taxes when government scheme is executed.<sup>37</sup>

**PROPOSITION 3:** Suppose that the government decides on compulsory schooling  $\tau_t^{cs}=1$ . Then for a certain domain of the parameters  $(\theta, \bar{w}, w^c, \alpha)$  there exists  $(\rho_{t+1}, \sigma_{t+1})$  such that the generated allocation Pareto dominates the competitive one, and takes off the economy out of its poverty trap. (See the proof in the appendix)

The rationale behind this proposition is straightforward. The introduction of compulsory schooling solves the under investment of parents in their children's human capital. On the other hand, the redistributive taxation solves the compensation problem from children, when adults, to their parents. In order to achieve an allocation that Pareto dominates the competitive one, we only need to assume that the potential income of an individual who study her entire childhood is greater than the sum of incomes of a child and an uneducated adult.<sup>38</sup> The more restrictive assumptions on the parameters are imposed to assure that the induced allocation would not only Pareto

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<sup>37</sup> For the sake of simplicity we choose to (weakly) increase both components of the parent's utility function.

<sup>38</sup> Formally the condition is:  $\bar{w}a(b+1)^\beta > \theta w^c(1+r) + w^c$

dominates the competitive one, but would enable the immediate take-off of the economy out of its poverty trap towards the high steady state equilibrium.

## 5. Concluding Remarks

In this paper we have explored the dynamic evolution of child labor, fertility, and human capital in the process of development. In early stages of development the economy is in a development trap where child labor is abundant, fertility is high and output per capita is low. Technological progress, however, increases the wage differential between parental and child labor, decreases the benefit from child labor and ultimately permits a take-off out of the development trap. Parents find it optimal to substitute child education for child labor and reduce fertility. The economy converges to a sustained growth steady-state equilibrium where child labor is abolished and fertility is low. We have also argued that the competitive equilibrium is not Pareto efficient due to the fact that children do not have access to capital market and lack of enforcement of intergenerational contracts. We have suggested a policy that not only Pareto dominates the competitive outcome but also expedites the take-off out of the poverty trap towards the high steady state.

Our result regarding the negative relation between fertility and income is well established in the literature, e.g., Becker, Murphy and Tamura (1990) and Galor and Weil (1996). However we have shown that it still holds when child labor is introduced into the household's decision.

As for policy, we suggested the introduction of compulsory schooling in a given period and a redistributive taxation from the adults to the elders in the following period. The need for such a policy arises since as Baland and Robinson (2000) claim, the intergenerational contract where the parent allows her children to study their entire childhood and in exchange children promise to compensate their parent in the next period, when adults, cannot be enforced. Basu and Van (1998) find that a ban on child labor is not Pareto improving since in the equilibrium without child labor, firms' profits are lower. In contrast, Baland and Robinson (2000) show that a ban on child labor can be Pareto improving if it induces certain changes in children's wages in current and next period and in the supply of efficiency units of labor in the next period. In their model a ban on child labor is equivalent to compulsory schooling

since schooling is given for free (in terms of output) as in our model. However, unlike in their model, we have assumed an open economy and thus a change in the supply of labor has no effect on wages. We therefore suggested a redistributive taxation to compensate parents for the foregone earnings of their children. Nonetheless, our policy suggestion captures the essence of the intergenerational contract discussed in Baland and Robinson (2000) and in this paper.

### Appendix: Proof of proposition 3

**Proof:** Suppose that individuals observe  $(\tau_t^{cs}, \sigma_{t+1})$ . From the solution to the optimization problem (equation 11) it follows that:

$$n_t(\tau_t^{cs}, \sigma_{t+1}) = \frac{(1-\alpha)I_{t+1}(\tau_t^{cs})(1+\bar{r})w^c + (1-\alpha)I_{t+1}(\tau_t^{cs})\sigma_{t+1} + \alpha(1+\bar{r})w^c [z - (1-\tau_t^{cs})\theta]}{\alpha(1+\bar{r})w^c [z - (1-\tau_t^{cs})\theta]I_{t+1}(\tau_t^{cs})}, \quad (16)$$

substituting  $\tau_t^{cs} = 1$  into  $n_t(\tau_t^{cs}, \sigma_{t+1})$  we get:

$$n_t(\tau_t^{cs} = 1, \sigma_{t+1}) = \frac{1-\alpha}{z} + \frac{(1-\alpha)}{(1+\bar{r})w^c z} \sigma_{t+1} + \frac{\alpha}{\bar{w}a(b+1)^\beta} \sigma_{t+1}, \quad (17)$$

Condition 3(i) holds with strict inequality for all  $\sigma_{t+1} > 0$ . By substituting (17) into condition 3(ii) we get:

$$\sigma_{t+1} \geq \frac{\frac{1-\alpha}{z-\theta} w^c - \frac{1-\alpha}{z} \bar{w}a(b+1)^\beta}{\frac{\bar{w}a(b+1)^\beta}{w^c} \frac{1-\alpha}{(1+\bar{r})z} - (1-\alpha)}, \quad (18)$$

and from the time restriction of the parent  $n_t \leq 1/z$  we get:

$$\sigma_{t+1} \leq \frac{\frac{\alpha}{z}}{\frac{1-\alpha}{(1+\bar{r})w^c z} + \frac{\alpha}{\bar{w}a(b+1)^\beta}}. \quad (19)$$

Thus condition 3(ii) implies the following inequality:

$$\frac{\frac{1-\alpha}{z-\theta} w^c - \frac{1-\alpha}{z} \bar{w} a(b+1)^\beta}{\frac{\bar{w} a(b+1)^\beta}{w^c} \frac{1-\alpha}{(1+\bar{r})z} - (1-\alpha)} \leq \sigma_{t+1} \leq \frac{\frac{\alpha}{z}}{\frac{1-\alpha}{(1+\bar{r})w^c z} + \frac{\alpha}{\bar{w} a(b+1)^\beta}} \quad (20)$$

Note that the RHS of (20) is strictly positive. Note also that the LHS of (20) is positive and continuous in  $\theta$  (see assumption 1), and as  $\theta$  approaches to  $\frac{z[\bar{w} a(b+1)^\beta - w^c]}{\bar{w} a(b+1)^\beta}$  the LHS converges to zero. Thus from continuity there is a sufficiently small  $\theta$  for which there exists  $\sigma_{t+1}$  satisfying inequality (20).

Substituting (17) into condition 1 and condition 1 into condition 2 we get:

$$\bar{w} [a(b+1)^\beta - \check{h}] \frac{1-\alpha}{z} \geq \left\{ 1 - \bar{w} [a(b+1)^\beta - \check{h}] \left[ \frac{(1-\alpha)\bar{w} a(b+1)^\beta + \alpha(1+\bar{r})w^c z}{(1+\bar{r})w^c z \bar{w} a(b+1)^\beta} \right] \right\} \sigma_{t+1} \quad (21)$$

Note that the LHS of (21) is strictly positive while the sign of the RHS is negative if

$$(1-\alpha) \frac{\bar{w} [a(b+1)^\beta - \check{h}]}{(1+\bar{r})w^c z} + \alpha \frac{\bar{w} [a(b+1)^\beta - \check{h}]}{\bar{w} a(b+1)^\beta} > 1 \text{ for all } \sigma_{t+1} > 0. \text{ This is sufficient for}$$

conditions 1 and 2 to hold.

Thus for a certain domain of  $\theta, \bar{w}, w^c, \alpha$  the desired  $\sigma_{t+1}$  exists. <sup>39</sup>

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<sup>39</sup> Note that the choice of  $\theta$  does not affect inequality (21).

## References

- Ashagrie, Kebebew (1993). "Statistics on Child Labour," *Bulletin of Labour Statistics*. Geneva: International Labour Organization, 3: pp. 11–24.
- (1998). "Statistics on Working Children and Hazardous Child Labour in Brief" ILO Bureau of Statistics, <http://www.ilo.org/public/english/120stat/actrep/childhaz.htm>
- Baland, Jean-Marie and James A. Robinson (2000). "Is Child Labor Inefficient?," *Journal of Political Economy*, 108 (No. 4, August): pp. 663-79.
- Barro, J. Robert (1991). "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics*, 106 (No.2, May): pp. 407-43.
- Basu, Kaushik (1999). "Child labor: Cause, consequence, and cure, with remarks on international labor standards," *Journal of Economic Literature*, 37 (No. 3, September): pp. 1083-119.
- and Pham Hoang Van (1998). "The Economics of Child Labor," *American Economic Review*, 88 (No. 3, June): pp. 412–27.
- Becker, Gary S. and Kevin M. Murphy (1988). "The Family and the State." *Journal of Law and Economics*, 31 (April) pp. 1-18.
- and — and Robert Tamura (1990). "Human Capital, Fertility, and Economic Growth," *Journal of Political Economy*, 98 (No. 5, Part 2, October): pp. S12–S37.
- Bequele, Assefa and Jo Boyen (eds.) (1988). *Combating Child Labour*. Geneva: International Labour Organization.
- Bonnet, Michel (1993). "Child Labour in Africa," *International Labour Review*, 132 (No. 3): pp. 371–89.
- Cain, Mead and A. B. M. Khorshed Alam Mozumder (1981). "Labour Market Structure and Reproductive Behaviour in Rural South Asia." In Gerry Rodgers and Guy Standing (eds.), *Child Work, Poverty, and Underdevelopment*. Geneva: International Labour Organization, pp. 245–87.



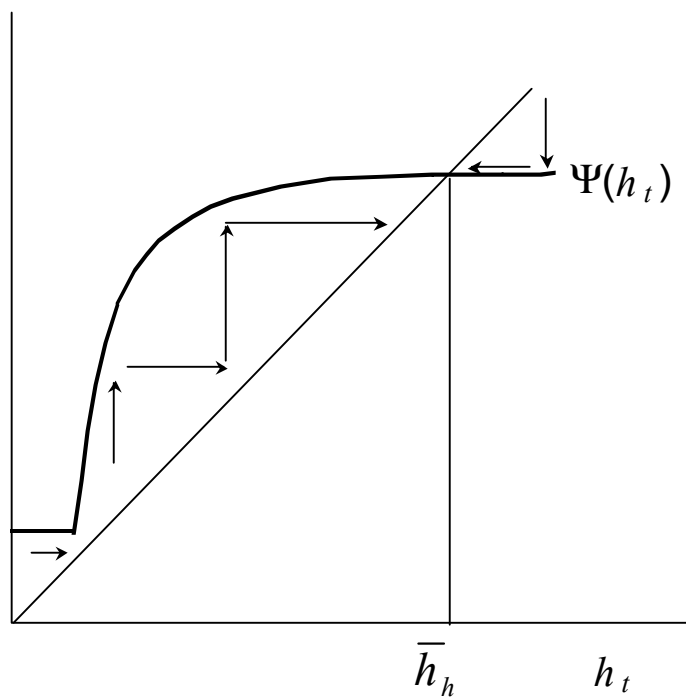
- Canagarajah, Roy and Harold Coulombe (1997). "Child labor and Schooling in Ghana," Human Development Tech. Report (Africa Region). Washington: World Bank.
- Cunningham, Hugh (1990). "The Employment and Unemployment of Children in England c. 1680–1851," *Past and Present*, 126 (February): pp. 115–50.
- Dasgupta, Partha (1993). *An Inquiry into the Well-Being and Destitution*. Oxford: Clarendon Press.
- Galbi, D. (1994). "Child Labour and the Division of Labour." Cambridge: Centre for History and Economics, King's College. Mimeograph.
- Galor, Oded and Joseph Zeira (1993). "Income distribution and Macroeconomics," *Review of Economic Studies*, 60: pp. 35–52.
- and David N. Weil (1996). "The Gender Gap, Fertility, and Growth," *American Economic Review*, 86 (No. 3, June): pp. 374–87.
- and — (1999) "From Malthusian Stagnation to Modern Growth," *American Economic Review*, 89 (No. 2, May): pp. 150-154.
- and — (2000). "Population, Technology, and Growth: From the Malthusian stagnation to the Demographic Transition and Beyond," *American Economic Review*, 90 (No. 4, September): pp. 806–28.
- Goldin, Claudia (1979). "Household and Market Production of Families in a Late Nineteenth Century American Town," *Explorations in Economic History*, 16 (No. 2, April): pp. 111–31.
- Goodfriend, Marvin and John McDermott (1995). "Early Development," *American Economic Review*, 85 (No. 1, March): pp. 116–33.
- Grootaert, Christiaan and Ravi Kanbur (1995). "Child Labor: An Economic Perspective," *International Labour Review*, 134 (No. 2): pp. 187–203.
- Jensen, Peter and Helena S. Nielsen (1997). "Child Labor or School Attendance? Evidence form Zambia," *Journal of Population Economics*, 10 (October): pp. 407-24.
- Kanbargi, Ramesh (1988). "Child Labour in India: The Carpet Industry of Varanasi." In Assefa Bequele and Jo Boyen (eds.), *Combating Child Labour*. Geneva: International Labour Organization, pp. 93–108.

- Kremer, Michael (1993). "Population Growth and Technological Change: One Million B.C. to 1990," *Quarterly Journal of Economics*, 108 (August): pp. 681–716.
- Levy, Victor (1985). "Cropping Patterns, Mechanization, Child Labour, and Fertility Behavior in a Farming Economy: Rural Egypt," *Economic Development and Cultural Change*, 33 (No. 4, July), pp. 777–91.
- Maoz, Yishay D. and Omer Moav (1999). "Intergenerational Mobility and the Process of Development," *Economic Journal*, 109, pp. 677-97.
- Morand, Olivier F. (1999a). "Endogenous Fertility, Income Distribution, and Growth," *Journal of Economic Growth*, 4 (No. 2, September), pp. 331-49.
- Morand, Olivier F. (1999b). "Growth, Development, and Child Labor" Manuscript, University of Connecticut, Department of Economics.
- Psacharopoulos, George (1997). "Child Labor versus Educational Attainment: Some Evidence from Latin America," *Journal of Population Economics*, 10 (October): pp. 377-86.
- Razin, Assaf and Efraim Sadka (1995). *Population Economics*. Cambridge: The MIT press.
- Rodgers, Gerry and Guy Standing (eds.) (1981). *Child Work, Poverty, and Underdevelopment*. Geneva: International Labour Organization.
- Romer, Paul M. (1990). "Endogenous Technological Change," *Journal of Political Economy*, 98 (No. 5, Part 2, October): pp. S71–S102.
- Rosenzweig, R. Mark and Robert E. Evenson (1977). "Fertility, Schooling, and the Economic Contribution of Children in Rural India: An Econometric Analysis," *Econometrica*, 45 (July) 1065-79.
- Salazar, M.C. and W.A. Glasinovitch (1996). "Better Schools: Less Child Work. Child Work and Education in Brazil, Columbia, Ecuador, Guatemala and Peru" Innocenti Essays No. 7. Florence, Italy: International Child Development Centre.
- Schiefelbein, E. (1997). "School-Related Economic Incentives in Latin America: Reducing DropOut and Repetition and Combating Child Labour" Innocenti

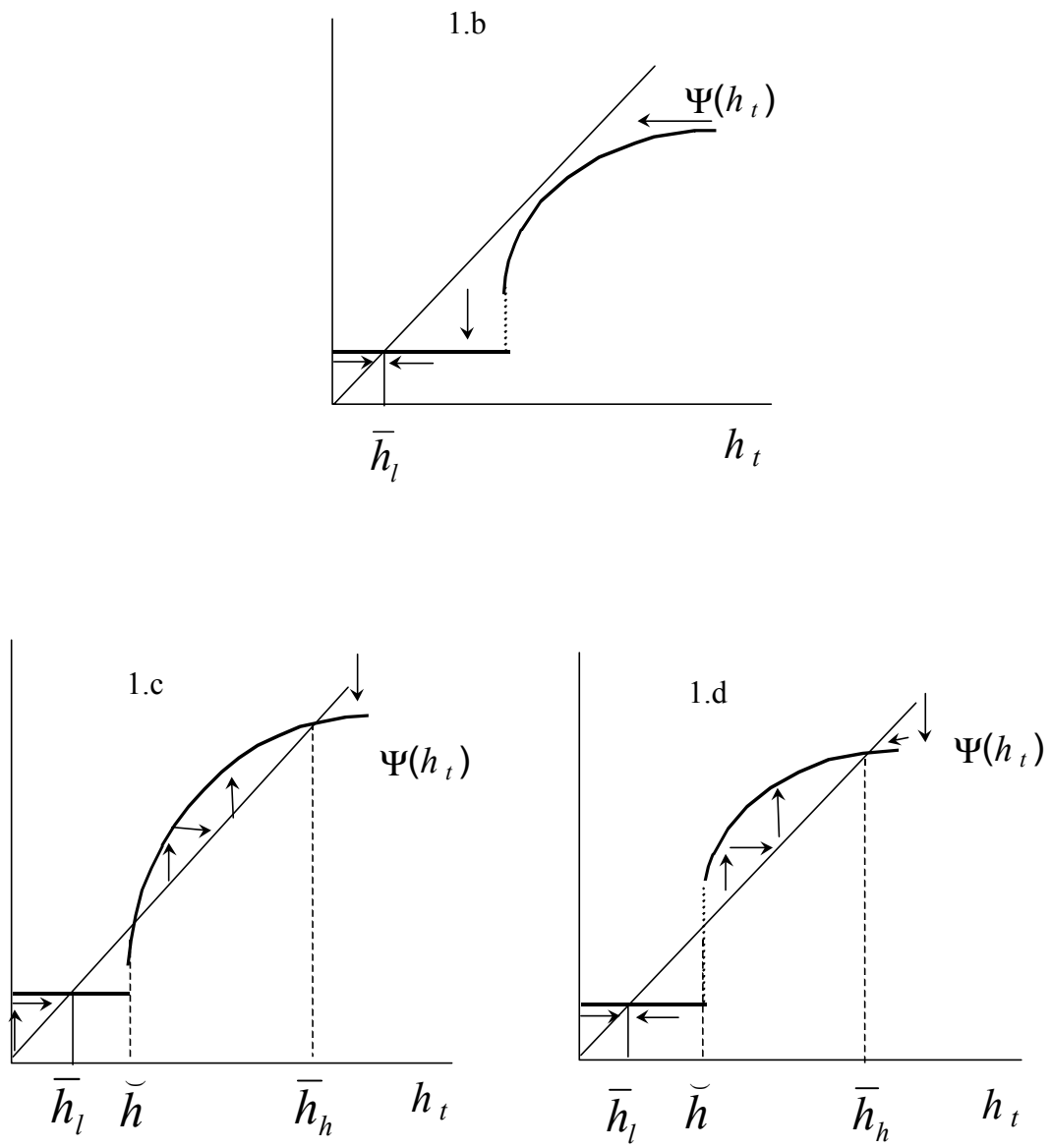
Occasional Paper CRS No. 12. Florence, Italy: International Child Development Centre.

Weiner, Myron (1991). *The Child and the State in India: Child Labor and Education Policy in a Comparative Perspective*. Princeton, NJ: Princeton University Press.

Figure 1.a The shape of  $\Psi(h_t)$  when  $\theta q < z$



Figures 1.b-d: The shape of  $\Psi(h_t)$  when  $\theta q > z$



**Figure 2. Consumption, Fertility and Schooling along the Path to Steady State under Technological Progress**

