# Information Acquisition: A (enhanced) Two-Stage Approach

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Abstract

This paper presents an alternative or enhanced approach to information acquisition in Cournot markets with stochastic demand in which the cost of information acquisition in endogenously determined by firms' information purchasing strategy. I propose a two stage model in which in the first stage each firm decides whether it will join a coalition to purchase information and therefore share the cost of information acquisition, to individually purchase information, or to remain uninformed. In the second-stage firms engage in Cournot competition to choose output. The model I propose encompasses the main assumptions of the current view on information acquisition, mainly those related to the role of information and how it affects firms' profits. However, I will argue that by adding natural assumptions on oligopolists' behavior, I can offer a model that provides a better description of firms' actions and trade-offs than the standard view.

Keywords: Information Acquisition, Cournot Markets, Subgame Perfect Nash Equilibrium, Strong Nash Equilibrium.

# 1 Introduction

This paper studies Cournot markets with stochastic demand where …rms have the possibility of perfectly knowledge the true realization of the demand parameter at a …xed and exogenously given cost $c > 0$: In this context, …rms have to decide whether or not to acquire this costly information before engaging in a Cournot competition to choose output.

The extensive literature[1] on information acquisition in Cournot markets with stochastic demand has mainly been concerned with understanding and modelling the protocol followed by …rms in deciding whether or not to purchase information. In what follows, information acquisition has almost exclusively been modelled as a two-stage game where in the …rst stage …rms decide whether or not to purchase information (and eventually the degree of precision of the information to be acquired) and in the second stage choose output.

In this paper I challenge this view on information acquisition claiming that the current literature has failed to identify an important step in the …rms' decision process. More speci…cally, I will claim that the set of actions available to …rms in the …rst stage of the game is larger than what has been assumed. Consequently, I shall argue that the current two-stage approach to information acquisition is incomplete and does not fully describe the …rms' behavior.

The main objective of this paper is to provide what is, in my opinion, a more plausible way to model information acquisition in Cournot markets with stochastic demand. I will propose an alternative or "enhanced" two stage model of information acquisition where in the …rst stage …rms decide whether or not to purchase information and their information purchasing strategy. That is, each …rm not only chooses whether to acquire information but also whether it will individually purchase infor-

[1] See, for instance, Chang and Lee, 1992; Gal-Or, 1985-1986; Hwang, 1993; Li, 1995; Li et al., 1987; Noveshek et al., 1982; Ponssard, 1979; ; Raith, 1996; Vives, 1988.

mation or join a coalition[2] to purchase information and therefore share the cost of information acquisition.

The model I propose encompasses the main assumptions of the "standard" or current view on information acquisition, mainly those related to the role of information and how it affects firms' profits. However, I will argue that by adding natural assumptions on oligopolists' behavior, I can offer a model that provides a better description of firms' actions and trade-offs than the standard view.

As we shall see, the change in firms' set of actions in the first stage of the game leads to a considerably different outcome compared to the standard two-stage approach. More importantly, the results obtained in regarding firms' incentive for cost sharing are not trivial.

The decision to join a coalition to share the cost of information acquisition affects firms' individual cost of information and consequently the incentives for acquiring information. Although common-sense would suggest that sharing the cost of information acquisition is the optimal strategy for all firms purchasing information, I will show that this is not always the case. The crucial result I obtain is that "not sharing" the cost of information may be the optimal strategy for a firm (or set of firms) if it prevents competitors from becoming informed, as long as the informational asymmetry results in higher profits to the informed firm(s) even if they are to pay the full cost of information acquisition.

The results in this paper build upon firms' incentive for purchasing information and how information asymmetry and private information affects informed firms' profits. Below I briefly discuss these points.

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[2] Throughout the paper the term coalition will be used in a neutral sense without structural or institutional implication to denote the subset of all firms in the market. The term coalition refers to a cost-reduction alliance with the sole objective to share the cost of information acquisition.

## 1.1 Incentives for information acquisition

It is well-recognized that when demand is stochastic and …rms simultaneously choose quantities, informed …rms have no incentive for information sharing because they may bene…t from the informational asymmetry in the market (See, for instance, Gal-Or 1985-1986; Raith, 1996; or Vives, 1994.) Here, the incentive for information acquisition is twofold: to be informed and, preferably, better informed than the competitors.

The literature on information acquisition in Cournot competition has extensively shown that, depending on the cost of information acquisition, just "to be informed" is not enough of incentive for a …rm to deviate from being uninformed and purchase information. A …rm may need "to be better informed" than some competitors in order for information acquisition to be optimal.

Generally, …rms acquire information when there is a bene…cial trade o¤ between the cost of information acquisition and its bene…ts (i.e., the increment in …rms' revenue caused by the acquisition of information), that is, when …rm's pro…t net of the cost of information acquisition is higher than what it would be if the …rm had remained uninformed.

A way to assess these bene…ts is to consider that information facilitates better decision making so that there is a direct gain[3] which emerges from information regardless of competitors' informedness. Hence, if the cost of information acquisition is a¤ordable to a …rm, it would seem that …rms always prefer to be informed than uninformed. However, in an oligopoly, the analysis of the role of information and its e¤ects on …rms' pro…ts would be incomplete if the e¤ect of private information on the behavior of competitors were not taken into account.

To illustrate this point, consider the following scenario: You and I producing a homogeneous good in a duopolistic market with stochastic demand. The demand

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[3] I borrowed this de…nition from Hauks and Hurkens (2001).

can be high or low with equal probability and we both have that knowledge. Upon observing the cost of information acquisition I decide to purchase information and you do not. I learn that the demand is high. You remain uninformed. You choose quantity conditional on the expected demand and consequently produce less than what you would if knew the true realization of the demand parameter. When I choose quantity, I take into account the true realization of the demand parameter and the fact that I am the only one with this information. As result, I respond more "aggressively" to the information I received (i.e., I produce more) than I would in the absence of private information. Consequently, I will increment my pro…ts because I will produce enough quantity to match also the part of demand you will not be able to supply. Thus, I will increase my pro…t not only because I am informed but also because I am better informed than you are.[4]

What I just described, is the indirect gain from information, which emerges from the strategic interaction between an informed and an uninformed agent. If we were both informed, the two of us would have an increase in pro…ts (direct gain) compared to the situation in which we are both uninformed. However, neither would have the extra gain (indirect gain) from being better informed than the competitor. Following Hauk and Hurkens (forthcoming), throughout the paper I will refer to the direct gain as the informational value of information and to the indirect gain as the strategic value of information. It is important to stress that the existence of the strategic value of information explains …rms' lack of incentive for sharing (exogenously or endogenously given) information in Cournot markets with stochastic demand as aforementioned in this introduction.

[4] Likewise, if I learn that the demand is low I will choose my quantity output taking into account that you will overproduce. Obviously, when …rms decide whether or not to purchase information they do not know if the signal to be received will be of "high" demand or "low" demand. The eventual losses that …rms have when the demand is low and the uninformed …rms overproduce is outweigh by the gains informed …rms have when the demand is high. Gal-Or (1985, footnote 3) shows an interesting numerical example.

The two-stage approach to information acquisition allows the strategic value of information to be included in the trade-off between the cost and benefit of information because decisions taken in the first-stage become common knowledge before firms choose output.[5]

Another important feature of the two-stage approach to information acquisition is that it relies on the so-called truth-telling assumption. As the name suggests, it is assumed that firms' informedness becomes common knowledge because, at the beginning of the second-stage, firms truthfully reveal their information acquisition decisions to the competitors.[6]

The model presented in this paper is also based on the truthtelling assumption and recognizes the strategic component of information. However, I show that the implications of this assumption go beyond those predicted by the standard two-stage approach to information acquisition.

In the enhanced model I propose, in the first stage upon observing the exogenously given cost of information acquisition c each firm i has to decide its information purchasing strategy. A firm would be willing to purchase information and share the cost of information acquisition if either, (i) cost sharing does not increase the number of informed firms compared to an equilibrium in which firms individually purchase information, and thus does not decrease the strategic value of information or; (ii)

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[5] This structure is crucial. If firms do not know the competitors' informedness, the effect of private information (strategic value) cannot be considered.

[6] Even though there is some skepticism with respect to the plausibility of the truth-telling assumption, it is largely accepted in the literature on information acquisition. Meanwhile efforts have been made to strengthen this assumption (See, for instance, Milgrom (1981), Milgron and Roberts (1986)). If the truth-telling assumption is rejected so is the possibility to include the strategic value of information in the trade off between the cost of information and its benefits. If a firm believes that competitors are lying with respect to their informedness, firms will not take into account the informedness of the competitors when choosing output. In turn, this also affects the way information acquisition is modelled. If firms take into account only the direct gain of information, information acquisition ought to be modelled as an one-stage rather a two-stage game (see Hauk and Hurkens, 2001.)

the cost sharing scheme attracts otherwise uninformed …rms but there is a favorable trade o¤ between the endogenous cost of information acquisition and the strategic value of information, that decreases as the number of informed …rms increases.

On the other hand, a …rm would remain uninformed if there is negative trade-o¤ between the cost of information acquisition and its bene…ts, regardless of the information purchasing strategy chosen.

It is important to stress that the coalition to share the cost of information acquisition (if formed), is dissolved in the beginning of the second-stage and …rms choose output competitively.[7]

We shall see that, although the outcome of the enhanced two stage game substantially di¤ers from the standard two stage game, it is consistent with the standard assumption that in Cournot markets with stochastic demand …rms have no incentive for information sharing[8]. In addition, the model's prediction is also consistent with the common wisdom that oligopolists' incentive for cooperative e¤ort to obtain higher pro…ts is very strong. However, the enhanced two stage approach reveals that the standard two-stage modelling of information acquisition does not account for an important step in …rms' information acquisition process in which …rms reconcile the lack of incentive for information sharing with the incentive for sharing the cost of information acquisition.

Firms that hold information are aware that information sharing per se does not generate bene…ts to them. In turn, it is not plausible to assume that any game of information acquisition leading to a Cournot competition would start with any sort

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[7] It is well recognized that cost-reduction arrangements as I am proposing here often facilitates the formation of coalition in market, when …rms choose quantities. However, here I am discussing …rms incentive to form cost-sharing alliances to share the cost of information acquisition assuming that they will behave competitively when choosing quantities.

[8] See, for instance, Clarke (1983), Fried (1987), Ponsard (1979), Raith (1996).

of cooperative e¤ort by …rms. Cooperation would develop when …rms realize that, given the strategic value it would obtain, …rm could increase pro…ts by sharing the cost of information acquisition[9].

The fact that economists neglected to recognize that information acquisition decisions include the possibility of sharing the cost of information may bring adverse consequences in terms of policy measures. The outcome of the standard two-stage game points to a socially undesirable duplication of cost of information acquisition (See, Hauks and Hukens, 2001). The alternative approach I propose, on the other hand, shows that the duplication of costs (when it exists) is to a much lesser degree because there are con…gurations of the parameter value such that it is optimal for …rms to share the cost of information acquisition.

This paper is organized as follows. The next section presents the set-up of the model. The alternative two stage model of information acquisition is described in section 3. Section 4 presents the solution to the model. Section 5 concludes and discusses possible direction of further research. In the Appendix A I present a numerical example which illustrates the striking di¤erences between the two approaches to information acquisition.

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[9] Based upon this reasoning, in an earlier version of this paper I modeled information acquisition as a three stage game where in the …rst stage …rms decide whether or not to individually purchase information and then, in the second stage upon observing the competitors' decisions, decide whether it would be optimal to share the cost of information acquisition. However, in order for this approach to make sense, I had to assume that decisions in the …rst-stage are not binding. It would result in a "cheap talk" sort of equilibrium, as in the …rst stage any action would be optimal. The two-stage approach I propose eliminates this inconvenience. However, this approach by no means implies an assumption that in Cournot markets with stochastic demand informed …rms have the incentive for information sharing.

# 2 The Setup

Consider a Cournot oligopoly with $n$ identical firms that produce a homogeneous good for which the linear inverse demand function is given by $P = D - \sum_{i=1} y_i$; where $y_i \geq 0$ is firm $i$'s output choice. $D = d$, for $d = H; L$; is stochastic and with equal probability can be high ($D = H$) or low ($D = L$) and it is assumed to be known to all firms. Production is costless. Nevertheless, firms can learn the true realization of demand parameter $D$ at an exogenously given and fixed cost $c > 0$. Similar to the model proposed by Ponsard (1979), here the demand parameter is either perfectly learned or not learned at all. The cost of information acquisition is common knowledge among firms.

## 2.1 Baseline Model 1: The non-information game

The baseline model is the classical Cournot oligopoly model with stochastic demand without the possibility of information acquisition. The non-information game is solved as a static game in which firms simultaneously and individually choose quantity output, $y_{iU}$, to maximize their expected profit $E(\pi_i / D)$. Define $\pi_{iU}$

$$\pi_{iU} = E(P) y_{iU} \tag{1}$$

where $E$ is the expectation operator.

The maximization problem for each firm $i$ is to choose $y^*_{iU}$ such that

$$y^*_{iU} = \arg\max \pi_{iU}$$

Substituting $P$ into (??) we have

$$\pi_{iU}(y_{iU}) = \left( E(D) - \sum_{j=1} y_{jU} \right) y_{iU}, \; j = 1; \cdots; n$$

Solving this maximization problem we have $(y^*_{iU})$ $i = 1,\dots,n$ that satisfies the following set of linear equations

$$\sum_{j=1} y_{jU} + y_{iU} = E(D);\ i = 1,\cdots,n;\ j = 1,\cdots,n$$

This set of equations is easily solved by adding up all equations to obtain $\sum_{j=1} y_{jU}$ and then substituting to find each value of $y_{iU}$. Therefore, there is a unique pure strategy Nash equilibrium to this game in which,

$$y^*_{iU} = \frac{E(D)}{n+1} \tag{2}$$

and the expected profit of each uninformed firm at the equilibrium is

$$\pi_{iU} = \left(\frac{E(D)}{n+1}\right)^2. \tag{3}$$

## 2.2 Baseline Model 2: Information Acquisition Games (without cost sharing)

Here, I allow firms to individually acquire costly information. Let

$$\pi_{iI} = E(\pi_i / D = d)$$

be firm i's expected profit when the firm knows the true realization of the demand parameter D before the output decision is taken.

Regardless of the specification of the game and considering that information is costly, it is optimal for firm i to acquire information if and only if

$$\pi_{iI} \geq \pi_{iU}$$

that is, if the expected profit after acquiring costly information is greater than or equal to the expected profit when information is not available[10].

# 3 The Alternative Approach to Information Acquisition

As discussed earlier, this paper models information acquisition as a two-stage game. The game proceeds as follows:

**First stage:** Firms observe the exogenously given cost of information acquisition $c > 0$ and sequentially[11] decide whether to purchase information and their information purchasing strategy.

The actions available to each firm $i$ at this stage are:

(i) to share the cost of information. If a firm takes this action, it will be willing to purchase information and share the cost with any other firm which wants to do the same.

(ii) not to share the cost of information. If a firm chooses this action it will individually purchase information, that is, it will not participate in any cost sharing scheme.

(iii) do not purchase information (UN). If this action is chosen, the firm will not purchase information and remain uninformed.

Hence, actions at this stage are $a_{1i} \in \{S; NS; UN\}$:

After firms have decided their purchasing strategy, the uncertainty of the state of demand is resolved for those that have decided to become informed.

The underlying assumption is that firms have no veto power meaning that, once

[10] Here I assume that when firms are indifferent between acquiring information or to remain uninformed they will always prefer to become informed.

[11] It will be assumed that in the first stage firms move sequentially. The reason for this is to avoid multiple equilibria in the final solution of the game (See remark 2).

a …rm proposes to share information (action S) no other …rm can be banned from joining such an "information sharing" scheme (see remark 1 below.)

Decisions taken in this stage are binding. De…ne $I$ as the set of informed …rms at the end of the second-stage and let $0 \leq K \leq n$ be the number of …rms in this set. The set $I$ is partitioned into two subsets. The subset $S$ consists of …rms sharing the cost of information acquisition such that $S = \{i \mid a_{1i} = S\}$ and the subset $N$ of …rms individually purchasing information such that $N = \{i \mid a_{1i} = NS\}$. De…ne $0 \leq K^* \leq K$ as the number of …rms in $S$ and $(K - K^*)$ as the number of …rms in $N$. Notice that $S \cup N = I$ and $S \cap N = \emptyset$. For a …rm $i \in S$ its expected pro…t is denoted by $\pi_{iS}$, that is, …rm's pro…t is a function of the number of …rms sharing the cost of information. For a …rm $i \in N$, its expected pro…t is denoted by $\pi_{iN}$. In this case …rms are individually purchasing information and hence will pay the exogenously given cost of information acquisition c.

Finally, de…ne $U$ as the set of …rms that remain uninformed at the end of the second stage such that $U = \{i \mid a_{1i} = UN\}$. Then $(n - K)$ is the number of …rms in this set. For …rm $i \in U$ the expected pro…t is given by equation (??):

**Second stage** The numbers $K$ and $K^*$ of …rms become common knowledge and …rms simultaneous and individually choose output. Let $y_{iI}$ and $y_{iU}$, be the quantity of output chosen by the informed and uninformed …rms, respectively. Actions at this stage are in the set

$a_{2;i \in I} \in \{y_{iI} : y_{iI} \in R_+\}$

$a_{2;i \in U} \in \{y_{iU} : y_{iU} \in R_+\}.$

Notice that the output choice of informed …rms is the same regardless of the purchasing strategy chosen[12].

**Remark 1 (Cost sharing scheme)** Although a general transaction cost paid by …rms for cooperative e¤orts to share the cost of information is intuitively appealing, I do not model such a charge. I instead assume that there is a trade association

[12] This is because the marginal cost of information acquisition is zero.

(TA, hereafter) that shares the information it receives from members with all firms in the industry (members or non members). That is, in my extensive form, it is as if at the end of the first-stage, firms (members and non-members) reveal their information purchasing strategy to the trade association (TA) that will be responsible for purchasing the information on behalf of each coalition. Once a firm reports its decision, it is bound by it. I will assume that if a firm decides to purchase information (individually or collusively) it will be in its best interest to communicate its decision to the other firms through the TA. Thus, if a firm does not communicate its decision to the TA, it will be assumed that this firm decided not to purchase information and remained uninformed. Notice, though, that the information that the TA releases to all firms in the industry is the firms' informedness and information purchasing strategy (that is, which firm is or is not informed how it will purchase information.. The TA does not reveal industry-wise the signal (high or low demand) informed firms received.

**Remark 2 (Sequential protocol)** I will mainly be interested in the total number of firms that, in equilibrium, acquire information (recall that firms are identical). Suppose, for instance, that there were $n = 3$ firms and, in equilibrium, only two firms become informed. If we had assumed that firms move simultaneously, three possible equilibria with two informed firms would arise. If firms move sequentially, for any sequential protocol, there will be a unique subgame perfect equilibrium in which only the first and second firms to move become informed. Thus, I rely on the sequential protocol to avoid this type of multiple equilibria. The nature and quality of my results would remain unchanged in a model without sequential protocol.

# 4 The Solution to The Enhanced Two Stage Game

The equilibrium notion used to solve the game is the one of Subgame Perfect Nash Equilibrium (SPNE) and Strong Nash Equilibrium (SNE). The game is solved by backwards induction starting with the continuation game, the output choice game.

Following the baseline model described in Section 2.1, if a firm $i$ remains uninformed its equilibrium output $y^*_{iU}$, is given by equation (??) and each uninformed firm's expected profit, $\pi^*_{iU}$; is given by equation (??).

However, if a firm decides to become informed by the time it chooses output it perfectly knows the realization of the demand parameter. The uncertainty is resolved and firms engage in a Cournot competition to determine output. Thus, to solve the continuation game, we have to …nd the pure strategy Nash equilibrium to the Cournot subgame. At this stage informed …rms play a game of complete information (with respect to the opponents' pro…t function and informedness, and the demand function) but imperfect information (…rms choose output simultaneously). The equilibrium output determined in the continuation game will be function of the total number of informed …rms $K$; and the realization of the demand parameter $D$.

Once we …nd the equilibrium output choice in the continuation game, we substitute these values into the expected pro…t function of each of the $K$ informed …rms in the set $I$: The expected pro…t of …rms in $I$ depends on the cost of information and, consequently, …rms' information purchasing strategy. On these expected pro…ts …rms base their equilibrium information purchasing strategy (share, not share, do not purchase) in the …rst stage and the Subgame Perfect Nash Equilibrium to the entire game is determined.

We can now proceed to formally solve the enhanced two stage game of information acquisition.

## 4.1 Solving the continuation game when …rms take informed decisions

I will now …nd the Nash equilibrium of the continuation game when …rms make informed decisions.

The next lemma displays the equilibrium to the continuation game, specifying the equilibrium output to the informed …rms, $y^{¤}_{iI}$: Recall that the equilibrium output to the uninformed …rms, $y^{¤}_{iU}$; is the same as in the …rst baseline model (refer to equation ??).

**Lemma 1** In the pure strategy (symmetric) Nash equilibrium of the continuation game

$$y^*_{iI} = \frac{D - (n - K)y^*_{iU}}{K + 1} \text{ when } i \in I \text{ and } K \leq n, \text{ for any } K^* \leq K.$$

Proof. See Appendix.

When a firm decides to become informed, its decision depends on the output choice of the $K$ informed and the $(n - K)$ uninformed competitors. The value of $K$ is common knowledge at the beginning of the continuation game (truth-telling assumption):

On the other hand, uninformed firms take into account only the expected value of demand and the total number of competitors (see equation (??)). Uninformed firms, obviously, do not know the true realization of the demand parameter. Thus, when choosing output uninformed firms rely on the expected value of the demand function to estimate competitors' output choice. Thus, to the uninformed firms, the informedness of the competitors does not really matter.

## 4.2 Solving the first stage of the game

Solving the model backwards, in the first stage I calculate firms' incentive to become informed and share the cost of information acquisition given the equilibrium quantities calculated in the second stage. The intuition is as follows. For each realization of $D$, in the first stage firms anticipate the optimal choice of output in the second-stage, which depends on $K$, the total number of informed firms. In the first stage each firm has to verify whether it is optimal to pay the full cost of information acquisition to obtain information on $D$, to join a cost sharing scheme to purchase information, or to remain uniformed. This decision will be based on the trade-off between the cost of information (shared or not) and its benefits.

A way to relate the benefits from information to the optimal output of informed firms is by observing that the true realization of $D$ determines the informational value of information, while $K$, the total number of informed firms will determine the strategic value of information. If a firm decides to purchase information, the decision of whether or not to share the cost of information surely does not affect $D$, however it may affect $K$, the total number of informed firms.

Substituting $y^*_{iI}$ into the expected profit function $\pi_{iS}$ and $\pi_{iN}$ we obtain the informed firm's expected profit.

In the first stage the cost of information for each firm is endogenously determined and depends on the firms' decision on whether to individually purchase information or to join an information sharing scheme. Notice that the cost of information acquisition is evenly shared among the participants of the scheme. Thus, if $K^* \geq 2$, that is, if at least two firms agree to participate in a cost sharing scheme, the cost of information acquisition for each firm $i \in S$, is[13]

$$c_i(K^*) = \frac{c}{K^*}.$$

A firm $i \in N$ that individually purchases information pays the full fixed cost of information[14], $c > 0$.

The expected profit is a function of the true realization of the demand parameter $D$, the total number of informed firms $K$ and the cost of information acquisition (shared or not shared).

If firm $i$ in the first stage decides to purchase information and share the cost of

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13 If the set $S$ is a singleton, that is, if $K^* = 1$, the cost of information acquisition for this firm, will be equal to $c$. Nevertheless, even in this case, the information acquisition cost is said to be endogenous because it was determined by the firm's decision of not sharing the cost of information.

14 It is interesting to notice that if only one firm decides to share the cost of information acquisition, although this firm belongs to $S$, it will pay the full cost of information acquisition $c > 0$. In other words, a firm $i$ would be indifferent between sharing or not sharing the cost of information acquisition if $(n - 1)$ firms decide to not share the cost of information acquisition. Thus, firms benefit from a cost sharing scheme if and only if $K^* \geq 2$. In what follows, this paper will be interested in joint deviations instead of single deviations from "not share the cost" to "share the cost" of information.

information acquisition its expected profit function will be given by

$$\pi_{iS} = E\left[\frac{D^2 - (n-K)ED^2 - (n-K)^2(K+1)}{(K+1)^2} - c_i(K^*)\right] \qquad (4)$$

which depends on $K$, the total number of firms in $I$; and $K^*$ the total number of firms in the set $S$ that share the exogenous cost of information acquisition $c$. (See Appendix for algebra).

Alternatively, if firm $i$ decides not to share the cost of information acquisition and individually purchase information its expected profit is given by

$$\pi_{iN} = E\left[\frac{D^2 - (n-K)ED^2 - (n-K)^2(K+1)}{(K+1)^2} - c\right] \qquad (5)$$

and depends on the total number of firms actually purchasing information and the exogenously given cost of information acquisition, $c$.

Thus, in the first stage of the game firms' decisions are as follows:

a firm $i$ shares the cost of information acquisition if,[15],[16]

$$\pi^*_{iS} \geq \max\{\pi^*_{iN}; \pi^*_{iU}\}; \qquad (6)$$

that is, if a firm's profit when it purchases information and shares the cost of information, $\pi^*_{iS}$; is higher compared to profits when the firm either individually purchases information, $\pi^*_{iN}$; or remains uninformed, $\pi^*_{iU}$.

Firm $i$ becomes informed but does not share the cost of information if

$$\pi^*_{iN} \geq \max\{\pi^*_{iS}; \pi^*_{iU}\}; \qquad (7)$$

[15] Just recall that the number of informed firms changes depending on firms' purchasing strategy.

[16] To ease the notation, from now on and whenever it is clear, I will omit the argument of the profit functions.

that is, if individually purchasing information gives higher profit to firm $i$ compared to purchasing information and sharing the cost of information acquisition, $\pi^*_{iS}$; or remaining uninformed, $\pi^*_{iU}$.

Finally, a firm $i$ does not purchase information and remains uninformed if

$$\pi^*_{iI} < \pi^*_{iU} \tag{8}$$

where the expected profit of the uninformed firms is given by equation (??) of the baseline model in section 2.1. In words, a firm $i$ does not purchase information if it has higher profits when uninformed, regardless of the purchasing strategy.

In the first stage firms' decisions ultimately determine the values of $K$ and $K^*$, that is the total number of informed firms and the number of firms sharing the cost of information acquisition. Thus, before proceeding to the formal analysis to determine the subgame perfect Nash equilibrium, it is important to discuss how firms' profits are affected by these two variables.

Keeping everything else constant, the expected profit for each informed firm $i \in S$ increases as $K^*$ increases. That is,

$$\left.\frac{\partial \pi_{iS}}{\partial K^*}\right|_{y^*_{iI}} = \left(\left.\frac{\partial \pi_{iS}}{\partial c_i(K^*)}\right|_{y^*_{iI}}\right)\left(\frac{\partial c_i(K^*)}{\partial K^*}\right) > 0 \text{ for all } K^* \leq K$$

where the inequality follows from $\frac{\partial \pi_{iS}}{\partial c(K^*)}, \frac{\partial c(K^*)}{\partial K^*} < 0$ for all $K^* \leq K \leq n$.

This result is straightforward as an increase in $K^*$ leads to a decrease in the information acquisition cost $c_i(K^*) = \frac{c}{K^*}$.

On the other hand, the expected profit of each firm $i \in I = S \cup N$ decreases as $K$, the number of firms taken informed decisions, increases. That is, as the informational asymmetry in the market decreases, the strategic value of informed firms (the gain from being better informed than the competitors) also decreases:

$$\left.\frac{\partial \pi_{iI}}{\partial K}\right|_{y^*_{iI}} < 0 \text{ for all } K \leq n.$$

The alternative two stage game poses an interesting problem to firms, that is, to reconcile the incentive to share the cost of information acquisition and the lack of incentive for information sharing.

Following the discussion above and proceeding to find the equilibrium to the information acquisition game, let us now calculate firms' benefit from being informed as opposed to being uninformed. This can be done by calculating the difference between the expected profit of informed and uninformed firms[17]. Define

$$C_i = \begin{cases} c \text{ if } i \in N \\ c(K^*) = \frac{c}{K^*} \text{ if } i \in S \end{cases}$$

then,

$$\underbrace{E[\pi_i / D = d]}_{\text{expected profit if informed}} - \underbrace{E[\pi_i / D]}_{\text{expected profit if uniformed}} = \frac{Var D}{(K+1)^2} - C_i \text{ for any } 1 \leq K \leq n. \tag{9}$$

(see Appendix for algebra).

If we denote $\frac{Var D}{(K+1)^2} = TV(K)$ the total value of information we can rewrite equation (??) as

$$E[\pi_i / D = d] - E[\pi_i / D] = TV(K) - C_i.$$

Hence, firms become informed if and only if,

$$TV(K) \geq C_i \tag{10}$$

17 This result is based on the second baseline model and holds regardless of the specification of firms' actions in the first stage of the game.

Let us now observe how the total value of information $TV(K)$ changes as the number of informed firms $K$ changes. This is given by the expression below.

$$\frac{\partial TV(K)}{\partial K} = -\frac{2}{(K+1)^3} Var D < 0 \text{ for any } 1 \leq K \leq n. \tag{11}$$

As discussed earlier, equation (??) shows that $TV$ is a decreasing function of the total number of informed firms, $K$.

For firms that share the cost of information, the marginal cost of information acquisition is given by

$$\frac{\partial c(\cdot)}{\partial K^*} = -\frac{c}{(K^*)^2} < 0 \text{ for any } 2 \leq K^* \leq K. \tag{12}$$

It is easy to verify that for any fixed value of $c$ and $K$, the negative value of expression (??) is maximized when $K = K^*$. Not surprisingly, then, for any given $K$, the information acquisition cost is minimized when all firms are included in the coalition. Based on this result I state the next Lemma.

**Lemma 2** In the equilibrium of the enhanced two-stage game of information acquisition in which $K$ is the number of informed firms and $c(K^*)$ is the information acquisition cost function, when a coalition is formed $K^* = K$, that is all informed firms share the cost of information acquisition and consequently only one coalition is formed.

**Proof.** See Appendix.

Lemma 2 rules out the possibility of more than one coalition to be formed in equilibrium. Thus, either all firms purchasing information share the cost amongst themselves; or firms that purchase information do so individually. Notice that lemma 2 does not say anything about the total number of firms that become informed or how the decision to share or not to share information is taken. This will be determined by the next lemmas and proposition.

Following Lemma 2 we can substitute $K^* = K$ into equation (??) and compare the rate of changes in the total value of information ($TV$) and in the marginal cost for any given value of $K$ (equations ?? and ??, respectively):

For any value of $K$ and c, a decrease in the marginal cost due to a cost-sharing scheme will be offset by a reduction in the strategic value, and consequently is not optimal for firms to share the cost of information acquisition, if

$$\left|\frac{\partial TV}{\partial K}\right| > \left|\frac{\partial c(\cdot)}{\partial K}\right|$$

that is, if

$$VarD > \frac{c(K+1)^3}{2K^2}. \tag{13}$$

We can now proceed to find the subgame Nash equilibrium of the game; we now know that when a coalition is formed it encompasses all firms that purchase information (Lemma 2), that is, $K^* = K$. This implies that, in equilibrium either $S = \emptyset$ or $N = \emptyset$; that is, the set of informed firms $I$ may have either only firms sharing information ($I = S$) or firms individually purchasing information ($I = N$). Let $\#S$ denote the cardinality of the set $S$. If $I = S$ we have that $\#S = K_S$. That is, $K = K_S$ is the total number of informed firms when in equilibrium, a coalition is formed. Similarly, if $I = N$ we have that $\#N = K_N$, where $K = K_N$ is the number of informed firms that in equilibrium individually purchase information.

Bear in mind, however, that for the same initial cost c; the total number of informed firms ($K_S$ or $K_N$) may differ depending on whether, in equilibrium, a coalition is formed or not.

In order to determine the equilibrium of the game we must relate the cost of information acquisition ($C_i$) with the total value of information ($TV$). In what follows it is useful to introduce some parameters.

Let

$$\underline{c} = \frac{VarD}{(n+1)^2} = TV(n):$$

be the total value of information when the total number of informed …rms is $K = n$, and let

$$\overline{c} = \frac{VarD}{(2)^2}$$

be the total value of information acquisition when there is only one informed …rm, that is, $K = 1$. Recall that total value of information represents the increment in …rms' revenue caused by the acquisition of information and does not directly depend on the information purchasing strategy used by …rms but on the total number of informed …rms. Nevertheless, …rms' pro…ts depend on the information purchasing strategy.

### 4.2.1 Cooperative re…nements of Nash equilibrium and the subgame Nash equilibrium

Continuing to solve the game backwards, in order to …nd the subgame perfect Nash equilibrium for the whole game, we have to …nd the equilibrium of the reduced game where …rms decide whether to become informed and the purchasing strategy after identifying (pure strategy) Nash equilibria in the output choice subgame. If we use the concept of Nash equilibrium to solve the …rst stage, multiple equilibria might arise for di¤erent values of c and some of them would clearly be ine¢ cient (see remark 3 below). We need, therefore, to obtain a sharper prediction about …rms incentive to form a coalition to share the cost of information acquisition. To this end, I will use an equilibrium concept which allows deviations by a group of …rms not just individual deviations. The concept I will use to solve the …rst stage of the game and to determine the subgame perfect Nash equilibrium to the whole game is the one of strong Nash Equilibrium (SNE)[18]. This equilibrium concept is appropriate to analyze the model

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in this paper as a coalition is formed if and only if at least two firms deviate from purchasing alone and decide to share the cost of information acquisition.

The next Lemma introduces the strong Nash equilibrium to the first stage of the two stage information acquisition game.

**Lemma 3** In the unique (strong) Nash Equilibrium of the two-stage game of information acquisition, in which firms decide whether to purchase information and the purchasing strategy

1. If $c \leq \underline{c}$, $U = \emptyset$; $I = S \Rightarrow \#S = n$,
2. If $c > \bar{c}$, $I = \emptyset$; $U \neq \emptyset \Rightarrow \#N = n$;
3. If $\underline{c} < c \leq \bar{c}$, $U \neq \emptyset$, $I \neq \emptyset$ such that.
   (a) If $VarD \leq \frac{c(K+1)^3}{2K^3}$; $I = S \neq \emptyset \Rightarrow \#S = K_N$;
   $U \neq \emptyset \Rightarrow \#U = n - K_S$:
   (b) If $VarD > \frac{c(K+1)^3}{2K^3}$; $I = N \neq \emptyset \Rightarrow \#N = K_N$;
   $U \neq \emptyset \Rightarrow \#U = n - K_N$:

**Proof.**

1. The exogenous cost of information acquisition is

$$c \leq \underline{c} = \frac{VarD}{(n+1)^2} = TV(n):$$

In this case, the exogenous cost of information acquisition c is less than or equal to the increment in firms' revenue (total value of information) when n

The concept of Strong Nash Equilibrium was introduced by Aumann (1959). Briefly, a strategy profile is a strong Nash equilibrium if and only if it is Pareto-efficient and immune to any coalitional or joint deviation. For a definition of SNE see also Myerson (1991).

firms purchase information, $TV(n)$. Thus, individually purchasing information strictly dominates (or weakly dominates if $c = \underline{c}$) the strategy "to remain uninformed" ($UN$) because,

$$\pi^*_{iN}(K = n) - \pi^*_{iU} = \frac{Var D}{(n+1)^2} - c$$
$$= TV(n) - c \geq 0$$

Now we have to check whether the strategy "to individually purchase information" ($NS$) is dominated by the strategy "to share the cost of information" ($S$)[19].

This case is trivial, as an equilibrium in which $n$ firms would individually purchase information is vulnerable to the joint deviation in which $K^* \leq n$ firms share the cost of information acquisition. The total number of informed firms would not change and consequently the total value of information would not change. On the other hand, the individual cost of information decreases as

$$C_i = c(K^*) = \frac{c}{n} < c.$$

Thus, if $c < \underline{c}$, each firm $i \in I$ shares the cost of information acquisition because,

$$\pi^*_{iS}(K_S = n) - \pi^*_{iN}(K_N = n) > 0$$
$$TV(K_S) - TV(K_N) > -\left(c - \frac{c}{n}\right).$$

That is, the strategy "to share the cost of information" strictly dominates the

[19] It is important to recall that, according to Lemma 2, when firms purchase information there are only two possible equilibria, that is, an equilibrium in which firms individually purchase information or an equilibrium in which firms form a coalition. The possibility of an equilibrium in which the number of firms sharing information is different from the total number of firms purchasing information is ruled out by Lemma 2.

strategies "to individually purchase information" and "to remain uninformed".

If $c < \underline{c}$, firm i shares the cost of information acquisition because

$$\pi^*_{iS}(K_S = n) > \max\{\pi^*_{iN}(K_N = n); \pi^*_{iU}\}$$

and in equilibrium,

$$U = \emptyset, \; I = S \Rightarrow \#S = n.$$

2. The cost of information acquisition is

$$c > \bar{c} = \frac{VarD}{2^2} = TV(1).$$

If $c = \bar{c}$ it is optimal for a firm to purchase information if and only if it is the only informed firm. If $c > \bar{c}$, however, no uninformed firm in this equilibrium deviates and individually purchase information because for any[20]

**Remark 3 (multiple Nash equilibria)** In order to stress the benefit of using the concept of strong Nash equilibrium, it is interesting to observe inefficiencies that arise

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[20] Recall that for Lemma 2, if a coalition is formed $K^* = K$. Recall also that $K$, the total number of informed firms differ depending on firms purchasing strategy. That is, $K = K_N$ or $K_S$ depending on whether, in equilibrium firms decide to individually purchase information or to join a coalition. As a shortcut, I will always use $K_N$ or $K_S$ to indicate the total number of informed firms instead of $K = K_N$ or $K = K_S$.

$K_N \geq 1$;

$$\pi^*_{iN}(K_N) - \pi^*_{iU} = TV(K_N) - c < 0.$$

Thus, if $c > \bar{c}$, the strategy "to remain uninformed" strictly dominates the strategy "to individually purchase information" for any $K_N \geq 1$.

Now we have to check whether if it is optimal for at least $K_S \geq 2$ firms to deviate and purchase information sharing the cost of information acquisition. That is, if for $c > \bar{c}$ there is a $K_S \geq 2$ such that,

$$\pi^*_{iS}(K_S) \geq \pi^*_{iU}. \tag{14}$$

We start by evaluating the inequality (??):

$$\begin{aligned} \pi^*_{iS}(K_S) - \pi^*_{iU} &\geq 0 \\ \frac{VarD}{(K_S+1)^2} - \frac{c}{K_S} &\geq 0 \end{aligned}$$

if instead we look for the Nash equilibrium of the reduced game (first-stage). In lemma 3 (1), for instance, for $c \leq \underline{c}$ there would be two Nash equilibria: Notice that, keeping everything else constant, if $(n - 1)$ firms decide not to share the cost

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Rearranging the above inequality we have

$$\frac{K_S}{(K_S+1)^2} \geq \frac{c}{VarD} \tag{15}$$

Recall, however, that $c > \overline{c} = \frac{VarD}{4}$. Thus, we can rewrite $c$ as

$$c = \frac{VarD}{4} + \varepsilon \text{ for } \varepsilon \in (0, 1). \tag{16}$$

Substituting the value of $c$ into the inequality (??) we have

$$\frac{K_S}{(K_S+1)^2} \geq \frac{VarD + 4\varepsilon}{4VarD}.$$

Solving the above expression for $\varepsilon$ we have that the inequality (??) holds, if and only if

$$\varepsilon \leq VarD\left(\frac{K_S}{(K_S+1)^2} - 0.25\right).$$

However, a coalition is formed if and only if at least 2 firms ($K_S \geq 2$) deviate from being uninformed and purchase information sharing the cost.

Notice that for $K_S \geq 2$,

$$\frac{K_S}{(K_S+1)^2} \leq 0.22 \Rightarrow -1 < \varepsilon < -0.03VarD.$$

That is, inequality (??) holds only for strictly negative values of $\varepsilon$. However, strictly negative values of $\varepsilon$ imply $c \leq \overline{c}$ (refer to equation ??). That is, it is optimal for $K_S \geq 2$ to become informed and share the cost of information if and only if $c \leq \overline{c}$. Thus, for any $c > \overline{c}$,

$$\pi^*_{iU} > \pi^*_{iS}(K_S) \text{ for any } K_S \geq 2.$$

In other words, "to remain uninformed" strictly dominates the strategy "to purchase information" regardless of the purchasing strategy used by firms.

Thus, in equilibrium, $c > \overline{c}$ implies that $I = \emptyset, U \neq \emptyset \Rightarrow \#U = n$.

3. The cost of information acquisition is $\underline{c} < c \leq \overline{c}$.

If $\underline{c} < c \leq \overline{c}$, there is a number $1 \leq K_N < n$ of firms individually purchasing information such that,

$$\pi^*_{iN}(K_N) - \pi^*_{iU} = \frac{VarD}{(K_N+1)^2} - c \geq 0.$$

Thus, for $1 \leq K_N < n$ the strategy "to individually purchase information" dominates the strategy "to remain uniformed".

Now we have to check it is optimal for a firm to deviate and purchase information sharing the cost.

Since joint deviations are allowed, firm $i \in S$ shares the cost of information acquisition if and only if,

$$\pi^*_{iS}(K^* = K_S) \geq \pi^*_{iN}(K = K_N)$$

of information acquisition, firm i would be indifferent between sharing or not sharing

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that is if,

$$\pi^*_{iS}(K^* = K_S) - \pi^*_{iN}(K = K_N) \geq 0$$

$$\frac{VarD}{(K_S+1)^2} - \frac{c}{K_S} - \left[\frac{VarD}{(K_N+1)^2} - c\right] \geq 0$$

or

$$\frac{VarD}{(K_S+1)^2} - \frac{VarD}{(K_N+1)^2} \geq -\left[c - \frac{c}{K_S}\right] : \qquad (17)$$

Denote $\Delta TV = \left[\frac{VarD}{(K_S+1)^2} - \frac{VarD}{(K_N+1)^2}\right]$ as the change in the total value of information when firms change their purchasing strategy from individually purchasing information to sharing the cost of information acquisition.

Similarly, denote $\Delta C = -\left[c - \frac{c}{K_S}\right]$ as the change in the individual cost of information acquisition when firms change their information purchasing strategy from individually purchasing information to sharing the cost of information acquisition.

Thus, we can rewrite inequality (??) as

$$\Delta TV \geq -\Delta C$$

and thus, it is optimal for a firm i to share the cost of information if

$$\frac{\Delta TV}{\Delta C} \geq -1;$$

and firm i does not share the cost of information acquisition if

$$\frac{\Delta TV}{\Delta C} < -1$$

Notice that for any $K_S \geq 2$, $c - \frac{c}{K_S} > 0 \Rightarrow \Delta C < 0$:

(a) According to equations (??, ?? and, ??), if $VarD \leq \frac{c(K+1)^3}{2K^3}$ the reduction in the cost of information acquisition offsets the eventual reduction in the strategic value of information. That is, $\frac{\Delta TV}{\Delta C} \geq -1$ and it is easy to verify that this inequality holds for $K_S \geq K_N$: Thus, in this case it is optimal for firms to share the cost of information acquisition and in equilibrium $I = S \neq \emptyset \Rightarrow \#S = K_S; U \neq \emptyset \Rightarrow \#U = n - K_S$:

(b) If, $VarD > \frac{c(K+1)^3}{2K^3}$, the reduction in the cost of information acquisition is offset by the losses in strategic value, that is, $\frac{\Delta TV}{\Delta C} < -1$: It is easy to verify that this inequality holds for $K_S > K_N$. Thus if, it is optimal for firms not to share the cost of information acquisition and in equilibrium, $I = N \neq \emptyset \Rightarrow \#N = K_N$;
$U \neq \emptyset \Rightarrow \#U = n - K_N$:
Notice that there is no equilibrium in which firms share cost and $K_S < K_N$: This is trivial, if there exists an equilibrium in which $K_N$ individually purchase information, that is, if

$$\pi^*_{iN}(K_N) \geq \pi^*_{iU};$$

there will be an equilibrium in which $K_S = K_N$ such that

$$\pi^*_{iS}(K_S) > \max\{\pi^*_{iU}; \pi^*_{iN}(K_N)\}: \blacksquare$$

the cost of information acquisition. In this case, …rms' pro…t when sharing or not sharing would be the same. On the other hand , if at least one …rm has decided to share the cost of information acquisition, …rm i$^0$s best response is to share the cost of information acquisition. Thus, if $c \cdot \underline{c}$, there would be two Nash equilibria: one equilibrium in which all …rms share the cost of information and a second one in which no …rms shares the cost of information acquisition. Although, both pro…le would be Nash equilibria, only the strategy pro…le in which all …rms share information is a strong Nash equilibrium (see footnote 13 for a de…nition of Nash equilibrium). In other word, only the equilibrium pro…le in which all …rms share information is not vulnerable to joint deviation, that is, more than one …rms deviating from this equilibrium. Why is the equilibrium pro…le in which no …rms share the cost, is not a SNE? Because while one …rm alone would have no incentive to deviate and share the cost of information since it would not a¤ect its information acquisition cost, a "group" of at least two …rms would have the incentive to deviate because doing so would a¤ect their individual cost of information acquisition. Hence, "no …rm shares the cost" is not "stable" and consequently cannot be a strong Nash equilibrium.

The next proposition states the subgame perfect Nash equilibrium for the information acquisition game in which strong Nash equilibrium for the …rst-stage has been found

**Proposition 4** The n-tuples

(a) $(\#S = n;\ y^{¤}_{iS}$ for $i\ 2\ S)$ if $c \cdot \underline{c}$;

(b) $(\#N = n;\ y^{¤}_{iN}$ for $i\ 2\ N)$ if $c > \bar{c}$;

(c) $(\#S = K_S;\ \#U = n\ ¡\ K_S;\ y^{¤}_{iS}$ for $i\ 2\ S$, $y^{¤}_{iU}$ for $i\ 2\ U)$ if $\underline{c} < c \cdot \bar{c}$ and $VarD \cdot \frac{c(K+1)^3}{2K^3}$;

(d) $(\#N = K_N;\ \#U = n\ ¡\ K_N;\ y^{¤}_{iN}$ for $i\ 2\ N$, $y^{¤}_{iU}$ for $i\ 2\ U)$ if $\underline{c} < c \cdot \bar{c}$ and $VarD > \frac{c(K+1)^3}{2K^3}$:

constitute a subgame perfect Nash Equilibrium of the information acquisition game.

**Remark 4 (Subgame Perfect Nash Equilibrium)** A pro…le of strategy is a subgame perfect Nash equilibrium if it induces a Nash equilibrium in every subgame. Here, in the …rst subgame, which solves the whole game, I opted for using a re…nement of the Nash equilibrium to eliminiate ine¢cient equilibria. However, the fact that I used this re…nement of Nash equilibrium does not a¤ect the essence of or is inconsistent with the formal de…nition of a subgame perfect Nash equilibrium.

According to proposition 4, if the cost of information acquisition is sufficient low ($c \leq \underline{c}$), all firms in the market purchase information and share the cost of information acquisition. In this case, the total number of informed firms in the enhanced model is the same as in the standard two stage model to information acquisition. However, in the former, the total cost of information acquisition is considerable smaller. If the cost of information acquisition is sufficiently large ($c > \overline{c}$) all firms remain uninformed. This outcome is also similar to the outcome of the standard two stage approach to information acquisition. The intuition for this result is as follows. If the exogenous cost of information acquisition ($c$) is too high, there would be necessary a coalition with a large number of firms in order to make the cost of information acquisition affordable. However, in this case, the strategic gain for each informed firm would be very low. Thus, the eventual strategic and informational gain informed firms would have would not be enough to compensate the shared cost of information acquisition.

For intermediate levels of information acquisition costs ($\underline{c} < c \leq \overline{c}$), the outcome of the enhanced and the standard approach to information acquisition are substantially different. Within this range, according to the enhanced two stage approach more firms would become informed. That is, depending on the exogenous cost of information acquisition, firms may decide to form a coalition to share the cost of information. The coalition allows firms, that would otherwise be uninformed, to purchase information. The decision on whether or not to form a coalition depends on how the cost sharing affects the strategic value and the individual cost of information acquisition.

# 5 Concluding Remarks

This paper presented an alternative or enhanced approach to information acquisition in Cournot markets with stochastic demand. It was shown that the current literature has failed to observe that despite the lack of incentive to information sharing, firms may find beneficial to share the cost of information acquisition even if it implies an increase in the total number of informed …rms. The decision on whether or not to form a coalition in the …rst-stage of the game depends on the trade-o¤ between the endogenously determined cost of information acquisition and the bene…ts from being informed as opposed to uninformed when choosing output in the second stage.

In the enhanced approach to information acquisition, …rms acquire more information, which is socially desirable and at a lower cost, compared to the standard two stage-game. In addition, …rms' pro…ts are generally higher compared to the standard two-stage approach to information acquisition. When the individual cost of information is exogenously given (standard approach), …rms trade-o¤ the cost of information acquisition against its bene…ts (strategic and informational value). However, when the possibility of cost sharing is considered, …rms face a more complex trade-o¤ namely, the trade-o¤ between the endogenously determined cost of information acquisition and its e¤ect on the strategic value. Firms that, in the equilibrium of the standard two-stage game, would remain uninformed may deviate and purchase information if there is a trade o¤ between the endogenized cost of information and its bene…ts (strategic and informational value). The new trade o¤ introduced by the alternative approach, con…rms the lack of incentive for information sharing though proves that …rms may bene…t from cost sharing if the reduction in cost is not o¤ set by the eventual increase in the number of informed …rms.

It has already been shown that when the strategic value of information is included in the trade-o¤ between cost and bene…t of information, …rms acquire more information than when only the informational value of information is included (and thus information acquisition is modelled as a one-stage game). However, the two-stage approach reveals an unfortunate consequence namely, the duplication of costly research, which is socially undesirable (see Hauk and Hurkens, 2001). What the enhanced two stage approach of information acquisition presented in this paper shows, however, is that although the duplication of costly research may occur, it is on a much smaller scale than what has been believed so far. Moreover, industries are more informed and total cost of information acquisition proven to be lower.

In this paper it was considered that firms can either perfectly learn the realization of the demand parameter or not learn it at all. A natural extension to this model is to assume that information can be learned at different degrees of precision and firms have to decide on the precision of the information they are willing to acquire. This treatment has already been applied in the two-stage approach to information acquisition but it would interesting to verify how (and if) an information sharing scheme is formed under these assumptions.

Further, although not directly related to the topic of information acquisition, it would be interesting to investigate how firms that supply information would react to collusive consumers, in this case, firms that join a cost sharing scheme to purchase information.

Last but not least, there is still room for research with the objective to strengthen the truth-telling assumption. This assumption is the backbone of most of the information acquisition models and efforts have to be made in order to mitigate or eliminate questions regarding the plausibility of these assumptions.

# A Appendix

## A.1 Standard vs Enhanced Approach to Information Acquisition: An Example

The objective of this example is to show the striking difference between the outcome of the two approaches to information acquisition.

Consider a Cournot Oligopoly as previously described, with $n = 3$ firms for which the inverse demand function is given by:

$$P = d - Q$$

where $d = 50$ or $100$ with equal probability and $Q$ is the aggregate demand. Let

$$EP = a - Q$$

where $a = \frac{50+100}{2} = 75.$

If a firm remains uninformed its expected profit is $\pi^*_{iU} = 351.56$

If all $K_N = 3$ firms decide to individually purchase information, the expected profit of each informed firm is

$$\pi^*_{iN}(K_N = 3) = 390.625 - c$$

Thus, $K_N = 3$ firms individually purchases information if

$$\begin{aligned} \pi^*_{iN}(K_N &= 3) \geq \pi^*_{iU} \\ c &\leq 39.065. \end{aligned}$$

If $c \leq 39.065$ and all firms are informed, the total value of information ($TV$) is given by the informational value of information ($IV$) only. Though the $K_N = 3$ firms benefit from taking better informed decision, none of them have the extra strategic gain which emerges when a firm(s) is (are) better informed than its competitor(s). Thus, in this case we have that the total value of information is

$$\begin{aligned} TV &= IV = \pi^*_{iN}(K_N = 3) - \pi^*_{iU} \\ &= 39.065 \end{aligned}$$

Consider now the possibility of cost sharing among firms. If $c \leq 39.065$ and $K_N = 3$, firms share the cost of information acquisition because for any $c > 0$

$$\pi^*_{iS}(K_S = 3) > \pi^*_{iN}(K_N = 3).$$

Hence, if $c \leq 39.065$ all firms purchase information and the individual cost of information acquisition to each informed firm is $\frac{c}{3} \leq \frac{39.065}{3} = 13.02.$

If only $K_N = 2$, firms decide to individually purchase information and the expected profit of each informed firm $i$ is

$$\pi^*_{iN}(K_N = 2) = 421.09 - c$$

Thus, $K_N = 2$ firms would individually purchase information if and only if,

$$\begin{aligned} \pi^*_{iN}(K_N &= 2) \geq \pi^*_{iU} \\ 39.065 &< c \leq 69.53 \end{aligned}$$

In this case the two informed firms benefit from the information asymmetry in the market. The strategic value of information ($SV$) is given by

$$SV = \pi^*_{iN}(K_N = 2) - \pi^*_{iN}(K_N = 3) = 30.465$$

and the informational value ($IV$) of information

$$IV = [\pi^*_{iN}(K_N = 2) - \pi^*_{iU}] - SV = 39.065.$$

The total value of information ($TV$) is given by

$$\begin{aligned} TV &= IV + SV \\ &= \pi^*_{iN}(K_N = 2) - \pi^*_{iU} \\ &= 69.53 \end{aligned}$$

Note that the informational value of information does not change as the number of informed firms decreases from 3 to 2. On the other hand, the strategic value of information increases when instead of three there are only two informed firms.

Let us allow for the possibility of cost sharing. Firms do not share the cost of information acquisition if

$$\pi^*_{iN}(K_N = 2) \geq \pi^*_{iS}(K_S = 3)$$

$$421.09 - c \geq 390.625 - \frac{c}{3}$$
$$c \leq 45.70.$$

Thus, if

$$39.065 < c \leq 45.70$$

we have that $\# N = 2$; $S = \emptyset$ and $\# U = 1$. That is, there will be only two informed firms and they will not share the cost of information acquisition.

On the other hand, if

$$45.70 < c < 69.70$$

we have that $\# S = 3$; $N = \emptyset$ and $U = \emptyset$, that is, three firms will acquire information, sharing the cost of information acquisition, because in this case

$$\pi^*_{iN}(K_N = 2) < \pi^*_{iS}(K^*_S = 3)$$

If only $K_N = 1$ firm acquires information, its expected profit is

$$\pi^*_{iN}(K_N = 1) = 507.81 - c.$$

Thus, $K_N = 1$ firm individually purchases information if and only if

$$\pi^*_{iN}(K_N = 1) \geq \pi^*_{iU}$$
$$69.53 < c \leq 156.25.$$

The strategic value $(SV)$ of information of the informed firm is

$$SV = \pi^*_{iN}(K_N = 1) - \pi^*_{iN}(K_N = 3) = 117.185$$

and the informational value $(IV)$ of information

$$IV = [\pi^*_{iN}(K_N = 1) - \pi^*_{iU}] - SV = 39.065.$$

The total value of information $(TV)$ is given by

$$\begin{aligned} TV &= IV + SV \\ &= \pi^*_{iN}(K_N = 1) - \pi^*_{iU} \\ &= 156.25. \end{aligned}$$

If

$$69.53 < c \leq 156.25$$

only one firm $(K_N = 1)$ firm purchases information.

Let us now allow firms to share the cost of information acquisition. If $69.53 < c \leq 156.25$ three firms would purchase information sharing the cost of information acquisition if and only if

$$\begin{aligned} \pi^*_{iS}(K_S &= 3) \geq \pi^*_{iU} \\ 390.625 - \frac{c}{3} &\geq 351.56 \\ 69.53 &< c \leq 117.95. \end{aligned}$$

Likewise, two firms would become informed sharing the cost of information acquisition if and only if

$$\pi^*_{iS}(K_S = 2) \geq \pi^*_{iU}$$

$$390.625 - \frac{c}{3} \geq 351.56$$
$$117.95 < c \leq 139.06$$

For the cost range

$$139.06 < c \leq 156.25$$

only one firm acquires information and $K_N$ because at this cost range

$$\pi^*_{iS}(K_S \geq 2) < \pi^*_{iU}.$$

A similar result occurs when

$$c > 156.25.$$

In this case

$$\pi^*_{iS}(K_S \geq 1) < \pi^*_{iU}.$$

| c | $K_N$ | IV | SV | TV | c(k) |
|---|---|---|---|---|---|
| c · 39:625 | 3 | 39:065 | 0 | 39:065 | c(k) · 39:625 |
| 39:53 < c · 69:53 | 2 | 39:065 | 30:465 | 69:53 | 39:625 < c(k) · 69:625 |
| 69:53 < c · 156:25 | 1 | 39:065 | 117:185 | 156:25 | 69:625 < c(k) · 156:25 |
| c > 156:25 | 0 | 0 | 0 | 0 | c(k) = 0 |

Table 1: Standard Two-Stage Game of Information Acquisition

| c | $K_N$ | $K_S$ | IV | SV | TV | $c(K^¤) = \frac{c}{K^¤}$ |
|---|---|---|---|---|---|---|
| c · 39:625 | | 3 | 39:065 | 0 | 39:065 | $c(K^¤)$ · 13:208 |
| 39:625 < c · 45:70 | 2 | | 39:065 | 30:465 | 69:53 | 39:625 < $c(K^¤)$ · 45:70 |
| 45:70 < c · 69:53 | | 3 | 39:065 | 0 | 39:065 | 15:23 < $c(K^¤)$ · 23:10 |
| 69:53 < c · 117:95 | | 3 | 39:065 | 0 | 39:065 | 23:17 < $c(K^¤)$ · 39:32 |
| 117:95 < c · 139:06 | | 2 | 39:065 | 30:465 | 69:53 | 58:975 < $c(K^¤)$ · 69:53 |
| 139:06 < c · 156:25 | 1 | | 39:065 | 117:185 | 156:25 | 139:06 < $c(K^¤)$ · 156:25 |
| c > 156:25 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 2: Two-Stage Game of Information Acquisition with Cost Sharing

The table 1 shows the outcome of the two stage game and table 2 the outcome of the two stage game of information acquisition with cost sharing.

## A.2 Proofs and Solutions

### A.2.1 Proof of Lemma 1

The informed …rms maximize their pro…t conditional on D, thus

$$\text{if } i \ 2 \ S; ¼_{iS} = (D \ ¡ \sum_{j2I} y_{jI}(D) \ ¡ \sum_{j2U} y_{jU})y_{iI}(D) \ ¡ \ c(K^¤)$$

The …rst order condition is given by

$$\sum_{j2I} y_{jI}(D) + \sum_{j2U} y_{jU} + y_{iI}(D) = D$$

for all $i \in S$. For $K = n$ let $\sum_{j \in U} y_{jU} = 0$.

$$\text{If } i \in N;\ \pi_{iN} = \Big(D - \sum_{j \in I} y_{jI}(D) - \sum_{j \in U} y_{jU}\Big) y_{iI}(D) - c$$

the first order condition is given by

$$\sum_{j \in I} y_{jI}(D) + \sum_{j \in U} y_{jU} + y_{iI}(D) = D$$

for all $i \in N$. For $K = n$ and thus, $\sum_{j \in U} y_{jU} = 0$. ■

### A.2.2 Derivation of equations 4 and 5

$$E(\pi_{iS}/D = d) = \pi_{iS}(K; K^*) = E[y^*_{iI} P(Q)] - c(K^*) \tag{18}$$

where

$$P(Q) = D - [K(y^*_{iI}) + (n - K) y^*_{iU}].$$

Substituting $P(Q)$ into equation (??) we obtain equation 1.4.

Similarly if we substitute the value of $P(Q)$ into the equation

$$E(\pi_{iN}/D = d) = \pi_{iN}(K; c) = E[y^*_{iI} P(Q)] - c$$

we obtain equation (3.5).

### A.2.3 Derivation of equation 9

Following Ponssard (1979, p. 247)'s proof for the difference between the expected profit knowing the realization of the demand parameter $D = d$ and the expected profits not knowing d, observe that $E[\alpha Y + \beta]^2 - [E[\alpha Y + \beta]]^2 = \alpha^2 Var Y$ where

$\alpha, \beta > 0$. Applying this result to the following expression

$$\begin{aligned} E[\pi_i / D &= d, K, K^*]^2 - (E[\pi_i / D])^2 = E\left(\frac{D}{K+1}\right)^2 - \left(\frac{ED}{K+1}\right)^2 \\ &= \frac{VarD}{(K+1)^2} \end{aligned}$$

■

### A.2.4 Proof of Lemma 2

Observe that

$$c(K^*) = \frac{c}{K^*}$$

is a decreasing linear function of $2 \le K^* \le K$. For any fixed $c$ and $K$, $c(K^*) \to 0$ as $K^* \to K$.

■

## A.3 Solving the Examples of Appendix A.1

### A.3.1 Expected profit when all firms are uninformed:

If all firms remain uninformed each firm $i$ chooses $q_i$ to maximizes the expected profit

$$\begin{aligned} \pi_{iU} &= E[\pi_i(q_i, q_{-i})/D] = q_{iU} P^e(Q) \\ &= q_{iU}[75 - q_1 - q_2 - q_3] \end{aligned} \tag{19}$$

Finding the first order condition we have

$$\frac{\partial E[\pi_i(q_i, q_{-i})/D]}{\partial q_{iU}} = 75 - 2q^*_{1U} - q^*_{2U} - q^*_{3U} = 0 \tag{20}$$

Assuming that are symmetric we have that

$$q^*_{1U} = q^*_{2U} = q^*_{3U} = q^*_{iU} \tag{21}$$

Substituting eq.(??) into eq.(??) we have that

$$q^*_{iU} = \frac{75}{4} = 18.75 \tag{22}$$

Substituting eq(??) into eq(??) we have that

$$\begin{aligned} \pi^*_U &= 18.75(75 - (3)18.75) \\ &= 351.56. \end{aligned}$$

**Expected profit when all firms are informed.** If all firms are informed each firm i chooses $q_{iI}$ to maximize

$$\begin{aligned} \pi_{iI} &= E[\pi_i(q_i, q_{-i}) | D = d] = q_{iI} P(Q) - C_i \qquad (23) \\ &= \frac{1}{2}[\pi_{iI}(q_i, q_{-i}) | D = 50] + \frac{1}{2}[\pi_{iI}(q_i, q_{-i}) | D = 100] - c \\ &= \frac{1}{2}[q_i(50 - q_1 - q_2 - q_3) - C_i] + \frac{1}{2}[q_{iI}(100 - q_1 - q_2 - q_3) - C_i]. \end{aligned}$$

Finding the first order condition we have

$$\frac{\partial E[\pi_i(q_i, q_{-i}) | D = d]}{\partial q_{iI}} = D - 2q^*_{1I} - q^*_{2I} - q^*_{3I} = 0 \tag{24}$$

Assuming that firms are asymmetric we have that

$$q^*_{1I} = q^*_{2I} = q^*_{3I} = q^*_{iI} \tag{25}$$

Substituting eq(??) into eq.(??) we have

$$\begin{aligned} q^*_{1I} &= \frac{D}{4} \\ &= 12.5 \text{ if } D = 50 \\ &= 25 \text{ if } D = 100 \end{aligned}$$

Substituting the values of $q^*_{1i}$ into equation (??) we have that

$$\pi^*_{il} = 390.625 - C_i.$$

The algebra for the expected profits when there are 2 or 1 follows the same procedure and thus omitted.

# References


[1] Aumann, R. (1959) "Acceptable Points in General Cooperative n-Person Games", Ann. Math. Stud. 40, 287-324.

[2] Chang, C.L. and Lee, C.W.J,.(1992), "Information Acquisition as Business Strategy," Sourthen Economic Journal, 58, 750-61.

[3] Clarke, R.N. (1983), "Collusion and Incentive for Information Sharing," Bell Journal of Economics, 14, 383-94.

[4] Fried, D. (1987), "Incentive for Information Production and Disclosure in a Duopolistic Enviroment," Quarterly Journal of Economics 99, 367-381.

[5] Gal-Or, E. (1985), "Information Sharing in Oligopoly," Econometrica, 36, 329-343.

[6] Gal-Or, E. (1986), "Information Transmission-Cournot and Bertrand equilibria," Review of Economic Studies, (53), 85-92.

[7] Hauk, E. and Hurkens, S.(2001), "Secretey Information Acquisition in Cournot Markets," Economic Theory.

[8] Hwang, H-S. (1995), "Information Acquisition and Relative Efficiency of Competitive, Oligopoly and Monopoly Markets," International Economic Review, 36, 325-40.

[9] Li,L. (1985), “Cournot Oligopoly with Information Sharing,” The Rand Journal of Economics, 16, 521-536.

[10] Li, L., McKelvey, R., and Page, T. (1987), “Optimal reserach for Cournot Oligopolist,” Journal of Economic Theory, 1(42), 140-66.

[11] Mas-Colell, A. Whinston, M. and Green, J. (1995). Microeconomics Theory, Oxford University Press,

[12] Milgrom, P. (1981), “Good News and Bad News: Representation Theorems and Applications,” Bell Journal of Economics, 12, 380-391.

[13] Milgrom, P. and Roberts, J. (1986), “Relying on The Information of Interested Parties,” The Rand Journal of Economics, 17, 18-32.

[14] Myerson, Roger (1991), Game Theory: Analysis of Con‡ict, Havard University Press.

[15] Novshek, W. and Sonnenschein, H. (1982), “Full…lled Expectations Cournot Duopoly with Information Acquisition and Release,” Bell Journal of Economics, 13, 214-218.

[16] Ponssard, J.-P, (1979), “The Strategic Role of Information on the Demand Function in an Oligopolistic Market,” Management Science, 25, 243-50.

[17] Raith, M. (1996), “A General Model of Information Sharing in Oligopoly,” Journal of Economic Theory, 71, 260-88.

[18] Vives, X., (1984), “Duopoly Information Equilibrium: Cournot and Bertrand,” Journal of Economic Theory, 34, 71-94.

[19] Vives, X. (1988), “Aggregation Information in Large Cournot Markets,” Econometrica, 56, 851-76.

[20] Vives, X. (1990), "Trade Association Disclosure Rules, Incentives to Share Information, and Welfare," The Rand Journal of Economics, 21, 409-30.