Education and Job Market Signalling: A Comment.

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Abstract

In this paper the Signalling approach to the explanation of wage differentials is analyzed both under a microeconomic and a macroeconomic viewpoint. Departuring from the classical Spence's model, the introduction of inequalities in accessing to education leads to redistributive effects among workers and firms. Moreover the existence of factors related both to local and to parental externalities greately reduce the informative power of education about individual ability.

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Comments Welcome

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1 Introduction

The relationship between individual investment in education and earning has been extensively analyzed both from an empirical and a theoretical point of view and it plays a central role in the debate around human capital accumulation, but not only, since, in general, better educated people exibits qualitative features particularly attracting for firms. As an example *they have lower propensities to quit or to be absent, are less likely to smoke, drinks or use illecit drugs and are generally healthier*[1]. These characteristics are not directly observed by the firm but, according to the signalling approach, they can be inferred from the chosen lenght of schooling. But what is important to us, is that individuals attending succesfully a higher lenght of schooling are characterized by a particular ability which makes them valuable to the firms; the above quoted features are consequence of such a higher ability or "quality". In other words, education provides a "signal" about the degree of skill characterizing an individual randomly drawn from the population. The latter is the main focus of the pioneeristic Spence's work on Market Signalling and our departure point. The main idea is quite intuitive: at the birth, each individual is endowed with a given level of "intrinsic ability" according to some exogenous (natural?) distribution; high-skill individuals are more productive, when employed as workers, than low-skill ones and such heterogeneity has to be accomplished by a different salary. There exist hence two possible explanation to the nexus education - wages; according to the first one - the human capital theory - individuals acquire their productivity staying at school longer. In the second one - the signalling approach - education provides a signal about the individual ability, according to the idea that a longer stay at school is less costly, in effort term, for a more able individual. As in a sort of Darwinian mechanism, only more able individuals survive to a longer staying at school with success.

In the signalling approach, the nexus education - individual ability is exactly reversed w.r.t. the human capital theory; in the latter, the individual productivity is induced by a longer permanence at school while in the former a high education level is a consequence of a higher individual ability. The two sides of the coin are not necessarily rivals, and likely they are not, but

[1] Andrew Weiss, 1995, page 133.

nonetheless it is extremely hard to identify which of them is predominating in explaning the earnings distribution. The empirical analysis is not very useful in such a context, as the cited work of Andrew Weiss underlines, because of the non-observability of the individual ability differences. A positive and statistically significative relationship between wage and education can be interpreted as the sign that the signalling device, rather than the human capital one, is at work; in such a case the estimated coefficient would measure the private, rather than the social, return to education.

The signalling approach hence takes into consideration the "microeconomic" or individual point of view of the question, stressing on the "strategic interaction" between individual and firm on the job market and for such a reason the more conceivable methodology in order to analyze the matter is the Signalling Games approach. In fact when an individual meets a firm on the job market, the latter does not know, a priori, what type of worker it is facing, if a high or a low skill individual, nor it can expect the individual "reveals" voluntarily her type, since less-skilled individuals strictly prefer "to mime" skillful types in order to get higher wages. Hence an "asymmetric information game" is at the work here; there is a player (the worker) who has a greater information set than her opponent (the firm), i.e. the individual knows her type but not so for the firm.

In this paper we shall try to analyze the robustness of the signalling approach and if it could effectively provide a well established explanation to wage differentials. Our analysis departs from the idea that in the signalling approach, hereafter identified with the Spence's model and following refinements, it is implicitly assumed that individuals have free access to any amount of education they wish and only a subjective reason (the personal aversion towards education) "constraints" the individual choice. But if education is costly, and if individual has to face a budget constraint, is it still possible to acquire the necessary education level signalling properly own productivity? The answer, in general, depends on the magnitude of education necessary to such a goal. In the Spence's model, education has a subjective cost - the individual effort - but not a pecuniary one; we need to assume that education is provided in a free way by the scholastic system. Viceversa if education is costly, because we can not assume a completely free-fees scholastic context, then the above mentioned problem can arise several questions. Moreover when we "switch" from the microeconomic to the macroeconomic point of view of the question we have to take into consideration others factors which greately reduce the informative power of education in signalling individual ability.

The article is composed as follows: in paragraph 2 a review of the Spence's model is provided, with a particular emphasis on the signalling games language. It will be underlined the "separating property" of the solution, obtained by suitable refinements of the Nash equilibrium, due to Cho-Kreps, 1987, and Banks-Sobel, 1987.

In paragraph 3, we will use the same framework but with a "rationed agent" who can not properly signal her quality to the firm. We shall see that a separating equilibrium is still possible, although pooling equilibria are also possible, but with a particular result about the way according to which more able individuals are remunerated. Finally in paragraph 4, we look for the macroeconomic implications of the signalling approach, considering external factors affecting the signal quality. Conclusions follows.

2 The Spence's model: a review.

Since it is my intention to keep the discussion as intuitive as possible I am going to analyze the simplest case of signalling game. There exist two "types" of worker with different ability indexed by the integer t, viz low-skill ($t = 1$) and high-skill ($t = 2$). The opponent is a firm[2] asking for the services of the worker. Workers know their type but not so the firm; the latter observes only the education level chosen by the worker. Both players are optimiser: workers want to maximize their utility function choosing an education level $e \in (0, \infty)$[3], given their type, t, and the expected wage w paid by the firm. The latter is a profit-maximizer, hence pays a wage equal to the worker productivity which in turn depends on individual ability. But the firm does not know such a characteristic; it can only set-up a beliefs system about it on the basis of the observed education level. For such a reason workers choose the education level performing conjectures on what will be the firm response (wage) to the chosen education level. So both players, firm and workers, have to perform their choices basing on some beliefs about the opponent strategy.

Before to describe in detail the property of the beliefs-system, we return

[2] We assume for simplicity a single representative firm, but the result does not change if we assume two firms competing *a la* Bertrand.

[3] In reality we do not need to assume the real non-negative line as support for education; we can use a borelian-subset with a suitable finite measure. Nevertheless the assumption $e \in (0, \infty)$ underlines the lack of constraints in the maximizing program.

to the workers preferences. For each type $t = 1, 2$, arguments of the utility function are wage, w, and education, e, so both types can select a bundle (w, e) in the $\Re^+ \nwarrow \Re^+$ space. Individuals are characterized by a complete, reflexive and transitive preferences-system on the bundles (w, e) and so ordering is possible. This property allows us to work with a continuous, concave utility function which is strictly increasing in w and strictly decreasing in e. Moreover, since type 1 is more "education-averse" the curvature of her utility function is in any point greater than type 2.

This set of assumptions about preferences can be summarized by two families of indifference curves in the (w, e) space, one for each type. Such curves are continuous and increasing, but the curves of type 1 are in any point steeper than type 2, so, when they cross, they cross once. This single-crossing property is crucial for the result, but it seems to me quite reasonable, since it describes the subjective diversity characterizing the two types[4]. Finally, utility is increasing from South-East to North-West in the (w, e) space. We shall find very useful analyze the equilibrium by these indifference curves.

Now we return to describe the game at a less general level. With $t = 1, 2$, we identify types; $p(t) < 1$ means the fraction of type t in the workers population and it represents the (prior) probability to engage a type t in absence of additional information; this is common knowledge. Type-1 workers have a e productive value for the firms and, likewise, type- 2 individuals value is $2e$ so that, in equilibrium, firm pays $w(e) = e$ to less-skilled individuals and $w(e) = 2e$ to more able ones. The problem is to resume such a result in the asymmetric information context, i.e. when firm does not know a prior what type is facing. However, it is clear that the firm is never willing to pay a wage above $2e$ and, likewise, workers do not accept a wage below e. In other words, it is common knowledge that $e \leq w(e) \leq 2e$.

Each type is characterized by a conditional probability distribution $\rho\,(e|t)$; the latter represents the probability that a type t chooses e as education level. The firm responds to the education level e following some beliefs-system $\mu\,(t|e)$ which represent the probability, assigned by the firm, that the observed education level e comes from a t type. On this belief, firm chooses its best-reply, viz the wage w. It remains to define how the firm sets up the

[4]This property has several others implications, not last the fact that *"the supports of the signals* (education in our case) *sent by various type are increasing in the type. This does not preclude pooling, but a pool must be an interval of types pooling on a single signal, with any type in the interior of the interval sending only the pooling signal."* (Kreps-Sobel, 1994, page 855).

beliefs and the proposed solution is the following: for each education level e chosen by the worker with positive probability, $\mu(t|e)$ is generated by the Bayes rule:

$$\mu(t|e) = \frac{p(t)\rho(e|t)}{p(1)\rho(e|1) + p(2)\rho(e|2)} \in [0,1] \tag{1}$$

The (1) shows the conditional probability $\mu(\cdot|e)$ as a ratio between a joint probability (numerator) and a marginal probability (denominator). It is worth spending some word on it. The numerator is a joint probability since it is the product of a marginal and a conditional probability; it represents the probability that an individual t chooses the education level e, since $p(t)$ is the (marginal) probability to face a t-type and $\rho(e|t)$ is the probability that type t chooses the education level e. The denominator represents the (marginal) probability to choose an education level e whatever the type. According to such an assumption about the way beliefs are generated, the expected wage has to necessarily follow:

$$w(e) = \mu(1|e)e + \mu(2|e)2e \tag{2}$$

As it is easy to check, $e \leq w(e) \leq 2e$. For example, if the firm puts weight zero to the fact that the observed education level e is coming from a type 2, then $\mu(2|e) = 0$, $\mu(1|e) = 1$ and the firm best reply is $w = e$[5]. Nevertheless the (1) can be used only if $\rho(t|e) \neq 0$, $t = 1, 2$, i.e. only for education levels which are chosen, in equilibrium, by types.

For testing an equilibrium outcome robustness, we need to analyze "off-equilibrium" education levels for which the (1) does not hold, since denominator is zero. We need hence a beliefs-system also for the off-equilibrium choice and in order to restrict out of equilibrium beliefs we shall use the Cho-Kreps' *Intuitive Criterion* which, for a two types game, corresponds to Banks-Sobel' *Divinity*.

The reader does not take fright at this esoteric terminology; it involves only a certain dose of common-sense, as it will be clear in a while.

Now we are ready to show the game solution through the diagram in Figure 1

[5]The reader familiar with games theory, should recognize that this is a sequential equilibrium.

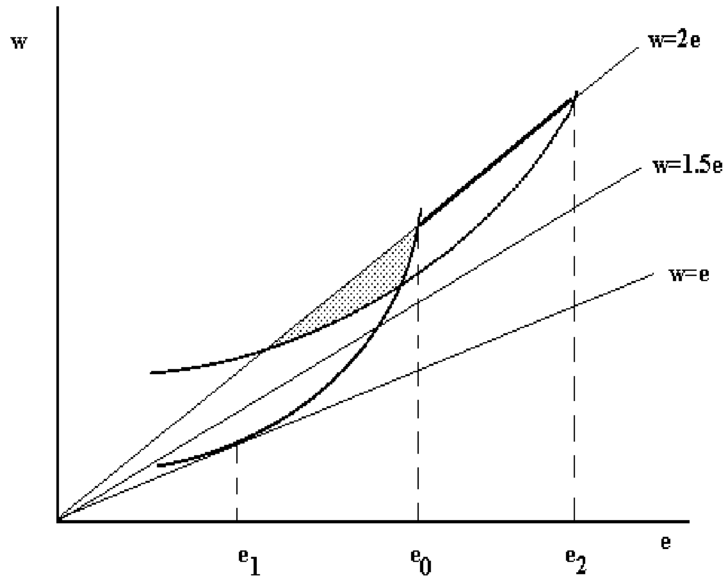

Figure 1 - A separating equilibrium

Such a figure shows a "separating equilibrium" where type 1 chooses the education level e_1 and type 2 chooses e_2. Such a result is supported by beliefs putting zero weight on the fact that education levels $e < e_0$ come, in equilibrium, by type 1, hence $\mu(2|e) = 0$, $\mu(1|e) = 1$ and the firm "best reply" is $w = e$ $\forall e < e_0$, according to (2). Given such a firm reply, type 1 selects e_1 which provides the first best to her; note in fact that in e_1 the indifference curve displays a tangency point with the ray $w = e$. Likely, the reader is wondering: *Why, in such an equilibrium, are we sure that any education level above* e_0 *reveals exactly type 2*? The answer is quite simple: because $\forall e > e_0$ type 1 is worse off than in equilibrium, i.e. in e_1. As the diagram shows indeed, such education levels lie under the indifference curve of type 1 characterizing the equilibrium, hence $\forall e > e_0$ type 1 is worse off than the equilibrium outcome. According to such reasoning, if firm observes an education level lying in this region it can put $\mu(2|e) = 1, \mu(1|e) = 0$ and $w = 2e$, according to (2).

For the sake of simplicity in the diagram I put the type 2 equilibrium point in e_2 but this is not a very efficient solution, since type 2 is signalling too much. It is easy to see that all education levels lying on the plotted bold line along the $2e$ ray provide a better situation for type 2 since they involve higher indifference curves; a rational agent characterized by this kind of preferences should choose the highest possible curve compatible with the separating condition, i.e. $e = e_0$. In such a point in fact type 1 has a weak incentive to deviate while type 2 has a strong one, and, according to the *Divinity* criterion, we can prune the signal e_0 from the type-1 message set. Nevertheless it is clear that in the asymmetric information context we obtain an over-signalling w.r.t. the full information context; in the latter type 2 would choose the education level which provides her a first best, viz at the tangency point with the $2e$ ray. Given the preferences in Figure 1, such education level would be cast under e_0. In other words, the presence of asymmetric information produces an over-signalling whose cost is paid by more able individuals in term of a lower utility level.

Now we have to demonstre that only separating equilibria are possible in such a game. Let us consider a point in the shaded region in Figure 1[6]; this region is above both indifference curves, hence it represents strictly preferred bundles w.r.t. the equilibrium situation, yet it is not take into consideration by the workers. In fact if the firm had observed an education level lying in this region, then it would not have could exclude any types since both workers could gain by sending this signal. If we assume that types are drawn randomly from a uniform distribution, i.e. $p(1) = p(2) = 1/2$, the more reasonable beliefs-system is $\mu(1|e) = \mu(2|e) = 1/2$[7]; then the firm best reply would be to offer $w = 1.5e$ (by the (2)). Workers are able to reply this firm strategy and choose a common education point $e \in (e_1, e_0)$ on the mix-line $w = 1.5e$, since this is the better choice in such a situation; in other words seems that our original *separating equilibrium* collapses in a *pooling equilibrium;* yet, this is a false statement. In fact in a pooling equilibrium

[6] It is worth stressing that these points are off-equilibrium, hence we can not set up a beliefs-system by the (1), viz we can not assume $p(1)\rho(e|1) + p(2)\rho(e|2) > 0$. We are going to use the above mentioned Nash refinements in order to restrict beliefs.

[7] Such beliefs are coerent with the Bayes rule because if the observed level e were the equilibrium result, then we necessarily should have $\rho(e|1) = \rho(e|2) = 1$ which, togheter with the prior probability $p(1) = p(2) = 1/2$, provides exactly the above beliefs. Moreover it is not by chance that the posterior probabilities (beliefs) matches the prior probabilities, since, in absence of a new information on types, the Bayes rule provides no beliefs updating.

type 2 is worse off than in the separating one, since the pooling involves lower indifference curves for type 2. Hence she has an incentive to departure from the pooling equilibrium, by sending a higher signal in order to signal properly herself, breaking down the pooling equilibrium; in such situation the best strategy for type 1 is e_1 since it can not mime a type 2.

Concluding, in the Spence's model only separating equilibria survives, solving the informative problem thanks to a workers self-selection by the mean of education. Despite education does not increase individual ability at all, it provides the right signal for identifying the worker productivity. The result provides a simple but powerful microeconomic theory justifying a positive relationship between wage and education, which in turn hides a positive relationship between wage and individual productivity.

3 A slightly different game.

So far the story sounds quite standard. However, as pointed out in the introduction, the Spence's model does not pose any pecuniary cost to education; each type can choose an education level $e \in [0, \infty)$ without additional constraints other than we have seen in the previous paragraph. In other words, we have to assume either education is entirely free or individuals are rich enough for achieving the necessary education level.

In this paragraph we shall assume that education is costly (not only as individual effort) and in order to analyze the effect of such an assumption we introduce a third worker. The latter is a type 2 but with a different characteristic: she is poorer than the other workers, and in particular than the typical type-2; we shall call, conventionally, this "rationed" agent as 2^1 and the non-rationed one as 2^2. The only difference between these types is in the feasible education set, viz, while 2^2 selects $e \in [0, \infty)$, type 2^1 can choose only in the $e \in [0, \bar{e}]$ domain - with $\bar{e} < e_0$[8]- because of her income constraint.

By so doing we restrict the feasible message set for player 2^1 to a compact set with the upper bound less than the signalling threshold e_0. We have now to investigate how such an assumption can affect the previous section results; we can distinguish two cases: in the first one $\bar{e} < e_0$ but $\bar{e} > e^*$ and $\bar{e} < e^*$ in the second one.

[8]The case $\bar{e} > e_0$ has not any reveleance to us, since it does not affect the standard separating property.

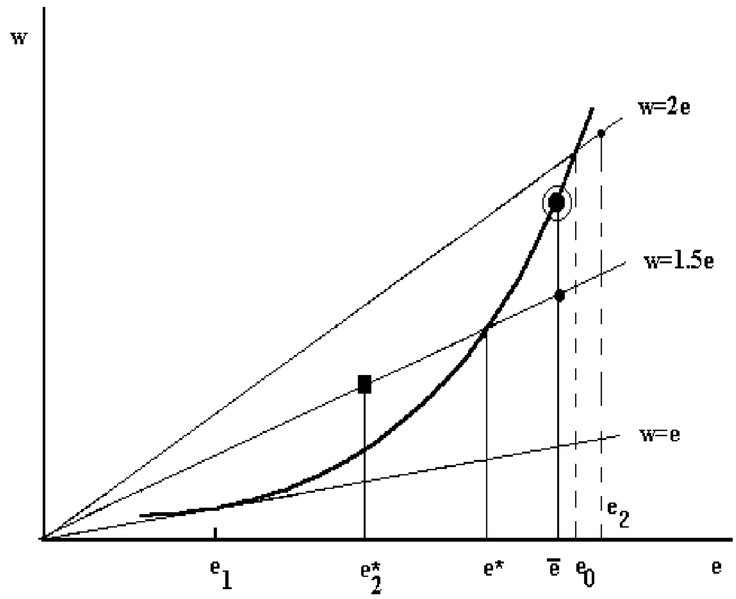

Figure 2

In the Spence's model, because of the kind of assumptions on the individual preferences we adopt, "*high Sender* (workers) *types are stronger than low ones; The higher t is, the more R* (the firm) *is willing to pay. Consequently, ... any Sender type t would like R to believe that t=T* (the highest type). *... higher types are more willing to send higher signals than lower types"*[9]. In other words, in such a signalling game, the highest type - type 2 in our case - is always better off in a separating rather than in a pooling equilibrium, since she can achieve a higher indifference curve. For such a reason, this kind of sender prefers reaveling properly himself by the mean of education and lower types try to mime, until possible, her behaviour. I hope the reader has not difficult to recognize that this is exactly the way in which we breaked down the pooling equilibrium in the previous section: the highest type is always willing to choose any necessary amount of education separating herself by

[9]Cho-Sobel, 1990, page 392.

the lower ones.

When the highest individual is constrained to a compact message set, a pooling equilibrium can arise only at the upper bound of the message set ; this is one of the contributes of the Cho and Sobel article[10]. For a rigorous demonstration of such a result the reader is urged upon reading the cited article; nevertheless an intuitive justification can be provided on the basis of the previous reasoning. Since the highest type always has an advantage to separate, she will choose the maximum education level signalling herself but such a value could be outside the feasible choice set. In such a case the message set is not sufficient to separate workers, but the highest type will use the maximum feasible education level in the tempative to signal her productivity and in this way will be mimed by the lowest type. The final result is a pooling equilibrium at the maximum feasible education level which "survives" both the *Intuitive Criterion* and the *Divinity* test. So we have achieved an interesting result: when individuals, and in particular higher types, can not access to a suitable education level, then education does not provide any explanation to wage differentials, at least in this class of models. Moreover in a pooling equilibrium a part of the more able individuals "ability rents"[11] is redistributed to the less able ones, since the latter get a higher wage than in a separating equilibrium. In other words we assist to a typical market failure operating as a tax on high-skill individuals accruing to low-skill ones under the form of a subside.

But this is not the unique theoretical embarrassment; let us come back to our example beginning from the $\bar{e} > e^*$ case. In such a case a pooling equilibrium could be respresented by the bold dot in Figure 2 on the 1.5 ray; yet such an equilibrium can be easily breaked down. If we show as $u^*(t)$ the utility achieved by type t in equilibrium, it is easy to note that $u^*(1) < u(1, e_1, e_1)$, i.e. type 1 has an incentive to deviate from the pooling equilibrium. She prefers reveal herself as a low-skill worker and receive the "right" firm response to such a signal. In other words, we obtain a separating equilibrium once more. Such a result is still possible to the extent that $\bar{e}$ makes type 1 indifferent between e_1 and $\bar{e}$, i.e. until to $\bar{e} = e^*$. Let us consider Figure 3:

[10] See Proposition 4.1 page 395.

[11] I mute this terminology from Stiglitz, 1975.

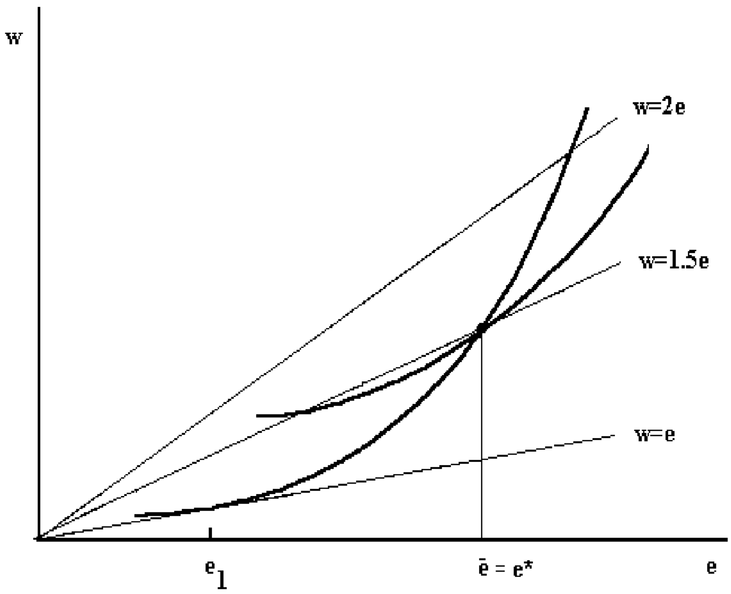

Figure 3

In such a situation, type 1 is indifferent between separating herself at e_1 or pooling at $\bar{e}$; type 2 is worse off in any other situation than $\bar{e}$[12]. Hence signalling $\bar{e}$ is more important for type 2 than type 1; in other words, $\bar{e}$ is strictly preferred by type 2 so as we can assign weight one to the belief that $\bar{e}$ is coming from a type 2[13]. According to such a reasoning, we can conclude that Figure 3 represents a separating equilibrium but it represents the frontier for the separating equilibria set. At the left of $\bar{e}$ only pooling equilibria are possible; let us consider indeed the bold box in figure 2, viz for $\bar{e} = e_2^*$. In such a situation only a pooling equilibrium is possible at the maximum feasible signal $\bar{e}$[14].

Summing up, depending on the $\bar{e}$ value, it is possible identify either a separating or a pooling equilibrium; these equilibria sets are parametrized to

[12] It is worth recalling that the game does not allow pre-commitment among types.
[13] The argument follows from the Divinity criterion, as previously remarked.
[14] This is a case where the Cho-Sobel proposition 4.1 applies.

$\bar{e}$ and, varying the latter, we obtain a continuum of equilibria spanning from pooling to separating[15].

So far, we have intentionally skipped any reference to the firm best reply to the type 2^1; now we are in for filling the gap. In a separating equilibrium - as, for example, the one depicted in figure 2 where type 1 chooses e_1 and type 2^1 chooses the maximum feasible signal $\bar{e}$ - type 2^1 is rightly identified as a high-skill worker but the firm best reply can not be $w = 2\bar{e}$, since in this case type 1 has an incentive to deviate. In such a stituation the firm best reply is fix a wage level which is compatible with the type 2^1 preferences, as for example a wage $\bar{w}$ corresponding to the circled bold dot in Figure 2. Note that such a wage level is lower than the one type 2^1 should receive: $\bar{w} < 2\bar{e}$; nonetheless this is the best that both players can do. The presence of a constraint on the type 2^1 signals set, jointly to the presence of a type 1, produces an inefficency in the job market in sense that type 2^1 workers are paid less than their productivity; firms hiring this kind of worker attain positive profits in equilibrium thanks to the minor ability rents accruing to the 2^1 workers. Moreover if there exists, as previously assumed, a second high-skill worker who can freely chooses the properly separating education level, then another theoretical embarrassment arises as a result of the presence of two equally productive individuals but receiving a different earning. If we assume that the economy consists of two sectors, one using low-skill and the other one high-skill workers, then in the "high-tech" sector firms strictly prefers type 2^1 to 2^2 since the former owns the same skill level but she is willing to accept a lower wage than her productivity level, since she can not do better.

It is clear that in such a situation no firm demands for type 2^2. The only way for 2^2 not to be expelled from the labour market is to mime type 2^1 behaviour, but then the only separating equilibrium surviving is the one characterized by the bundles (e_1, e_1) for type 1 and $(2\bar{e} - \delta, \bar{e})$ for both types 2, where δ is a part of the more able workers ability rents accruing to the firms because of the presence of rationed individuals.

The results obtained underlines that, when inequalities in accessing to education are introduced in the Signalling approach, then this theoretical scheme arises a series of market failures characterized by redistributive effects among players. When pooling equilibria are met, then there is a partial transfer of ability rents from more able to less able individuals. Even worse,

[15] In other words, the Nash equilibria set for such a game is connected.

in such a case the signalling approach fails to provide an explanation to earning distribution. Viceversa, when separating equilibria operates, under the assumptions above mentioned, the recipients of the redistributive effect are the firms. In both cases, in the economy some workers receive an earning which is less than their productivity.

4 The Macroeconomic point of view

The previous section underlined that the result of a perfect separating equilibrium in the theoretical microeconomic model lies in the idea that education is a type of commodity that individuals can purchase in any desired amount; more able individuals gain from separating in any situation, since a pooling equilibrium operates a private redistribution from them to the less able agents. In order to avoid this market failure more able individuals are willing to send the highest possible signal[16]. But if the latter is not sufficient to prevent miming by the less able individuals, then pooling equilibria can arise, as we have seen; moreover, also if separating operates, in presence of a constrained high-skill individual it involves some unpleasent characteristics which greatly reduce the real application of such a theoretical scheme. Slightly changes in the basic assumptions lead to private redistributions among players. In a pooling equilibrium surviving the mentioned Nash refinement, more able individuals get a lower wage than their productivity and viceversa for the low-skill ones; the former pay a tax on their "ability rents" which is translated to the latter. In a separating equilibrium with rationed agents instead a part of the more able individuals ability rents accrues to the firms.

In this final paragraph, we shall underline some macroeconomic consequences of the signalling approach. In the microeconomic model used so far, a t-type individual who studies e has a value te for the firm, where t is the individual ability. Although education does not affect the latter at all, a sort of *learning by doing* is at work here, since individuals productivity depends both on t and on e. More precisely, the signalling device involves a "by-product" to the extent that more able individuals must invest in education in order to be properly separated; the final result is that they are more productive not only because they are more able but also because they stay

[16]For such a reason, the measure of the signals set is increasing in the type, as previously noted.

longer at school. From this point of view, the learning component produces a scale effect in the aggregate production as consequence of the signalling device. If argument of a social utility function were the level of GDP, then any policy increasing the amount of education necessary to provide a fully separating mechanism should be considerated. For example an increase in the minimal attendancy age for schooling would force the less able types to accumulate more education which in turn would induce more able types to increase their signal in order to be separated . Nevertheless the positive shift would produce an over-signalling (there is too much education) which would make worse off both types. In this case the private and social return to education will be different. But such a conclusion needs further considerations; in fact, as Stiglitz underlines, the wages accruing to the types "*are best thought to be lifetime incomes, i.e. present discounted values of wage streames*"[17]. Although individuals would prefer a lower wage, they are undoubtedly richer and if intergenerational transfers are at the work, they make the whole stream of future generations better endowded, operating a utility redistribution from the current generation to the future ones.

From a macroeconomic point of view, a perfect signalling - hereafter identified with the model of paragraph 2 - increases the aggregate output, since it establishes a monotonically increasing relationship between individual ability and amount of education, $e(t)$, $e'(t) > 0$, where t is the individual ability. In general, we can define the aggregate output, when the separating device works, as:

$$Q_s = \int_T te(t)\pi(t)dt$$

where $T \subset \Re^+$ is the types set and $\pi(t)$ the relative numerosity of type t in the workers population. It is clear that if the separating device does not work at all $e(t) = \bar{e} \in (e_1, e_T)$, then $Q = \bar{e} \int_T t\pi(t)dt < Q_s$ where $\bar{e} \in (e_1, e_T)$ is a common education level chosen by types as "best reply" to the mean wage rate $\int_T t\pi(t)dt$ offered by the firm. Moreover, in such a situation, the earnings distribution collapses to a Dirac-delta function, i.e. to a zero dispersion function around $\bar{e}$. On the other side, the separating device induces the highest dispersion among earnings and the direct consequence of such an unpleasent result is that more able individuals have a major weight in the aggregate output.

[17]Stiglitz, 1975, page 284.

There is hence a trade-off between efficency and equality; the presence of a fully separating mechanism involves the highest degree of efficency, since there is no redistribution of ability rents among types, but the highest variance among wages. On the other hand, the absence of such a device induce a loss in efficency, with an implicit tax burdening on more able types and accruing to less skilled individuals, but it is characterized by a perfectly egalitarian distribution.

In paragraph 3 we pointed out on the effects of inequalities in accessing to school for underlining some weaknesses of the theoretical scheme. Now we are going to point out the role played by others macroeconomic factors in conditionating the functioning of the signalling device. We depart from the consideration that the microeconomic model lies on a set of assumptions like: i) individuals have free access to schooling, or alternatively, they can purchase any amount of education which is required by their optimal program; ii) individuals are perfectly informed about their types; iii) players know the game rules, i.e. the opponent adopted inferential process; iv) there exists no uncertainty about the transmission of the information so that signals are correctly decoded by the receiver (noiseless communication); v) the quality of the signal is homogenous over the types, viz. there exist no types signalling better than others.

This kind of assumptions, and in particular points iv) and v), leave out many relevant questions concerning differences in individuals related to the socio-economic as well as parental context - where the individual was born and grew up - which is composed by a a series of local and parental externalities that do affect individual aptitudes. Sex, race, parental income, place of birth, quality of schooling, differences in access to school are only a few of the exogenous factors affecting the relationship between individual ability and education[18]; hereafter we appeal such external factors as "field effects". It is clear that such factors introduce a noise component in the transmission of the information at a macroeconomic level, and in general we can have different levels of signalling spanning from a low-efficency signalling mechanism, where some types are not able to be properly identified, to a perfect signalling device, as the one we have seen in paragraph 2. In other words the reduction, by the schooling, of the uncertainty related to the actual

[18]For a more detailed discussion about the role played by these exogenous factors in explaining the empirical findings about the nexus wage-education, see Andrew Weiss, 1995.

individual ability is depending, at different stages, on the above mentioned macroeconomic factors.

In general we can describe the signalling device as a system characterized by a sending and a receiving "equipment" communicating through a noisy channell; it is not possible to observe directly the state of the sending device and such information has to be inferred decoding the received signal. To fit such assumptions into our scheme we identify the receving device with the observed education level while the sending one represents the individual ability. Hereafter, we mean with $\mathcal{E}$ the amount of education that can be observed in a certain community; $\mathcal{E}$ assumes discrete values $\varepsilon_j > 0$, $j = 1, 2, ...n$ with probability r_j. Likewise the individual ability, T, can assume discrete values $t_i, i = 1, 2, ...n$, with probability p_i. We assume that, in a noiseless communication, the observed education level provides a complete information about the individual ability, viz. $\mathcal{E}$ is equivalent to T from an informative point of view (perfect signalling). The quantity $H(X) = -\sum_i \phi_i \log \phi_i = -E[\log \phi_i]$ defines the Shannon's entropy of the system X, i.e. the degree of uncertainty characterizing X. The latter depends both on the number of X states (n) and on the related probabilities ϕ_i. Likewise, $H(\mathcal{E}, T) = -E[\log P(\mathcal{E}, T)]$ is the joint entropy of the system $(\mathcal{E}, T)$ under investigation, where $P(\cdot, \cdot)$ is the joint distribution.

By the entropy it is possible to quantify the information necessary to reveal completely the state of a system:

$$I_X = H(X) - 0$$

that is, the quantity of information gained by the complete knowledge of the system X,$(H(X) = 0)$, is equal to the entropy of the system itself.

In our case, we do not observe directly the source of the signal, T, but the receiver $\mathcal{E}$, and we are interested to the following question: *How much information on T is it contained in $\mathcal{E}$?* Before to provide an answer we need a bit of terminology. We define the conditional entropy $H(Y \mid x_i)$ as the entropy of the system Y when the system X is in the x_i state, i.e. the uncertainty degree of Y after the revelation on X. Likewise we define $H(Y \mid X) = \sum_i p_i H(Y \mid x_i)$ the total conditional entropy. This kind of measure plays a crucial role in our problem since it represents the entropy of Y "destroyed" by X. With a bit of probabilistic calculus it is easy to see that the following relationship holds:

$$H(Y, X) = H(X) + H(Y \mid X)$$

If X and Y were independent, then $H(Y \mid X) = H(Y)$ and the entropy of the joint system would be just the sum of the single entropies. From here, let us conclude that, in general, the joint entropy can never be higher than the sum of the single entropies: $H(Y, X) \leq H(X) + H(y)$, since the uncertainty degree of the joint system can not increase with the revelation on a single component. The conditional entropy provides an interesting instrument to analyze the informative power of education relatively to individuals ability, allowing us to identify the amount of information on T contained in $\mathcal{E}$, $I_{\mathcal{E}\to T}$. The latter is defined as:

$$I_{\mathcal{E}\to T} = H(T) - H(T \mid \mathcal{E}) \tag{3}$$

The rationale of the above equation is quite simple: before of observing the $\mathcal{E}$ system, the entropy of T is $H(T)$; when the data on $\mathcal{E}$ are available the entropy about T reduces to $H(T \mid \mathcal{E})$; $I_{\mathcal{E}\to T}$ represents the entropy which is destroyed by the communication and $H(T \mid \mathcal{E})$ the residual one. By definition, $I_{\mathcal{E}\to T}$ is the total information about T contained in $\mathcal{E}$. Moreover it is possible show that $I_{\mathcal{E}\to T} = I_{T\to\mathcal{E}}$, i.e. $H(T) - H(T \mid \mathcal{E}) = H(\mathcal{E}) - H(\mathcal{E} \mid T)$.

When two systems are equivalent the information is at the maximum value: $I_{\mathcal{E}\to T} = H(T)$ and the residual entropy is zero. This case corresponds to a perfect signalling or communication. When the transmission is affected by noise, it is interesting to have an idea about the revelating power of the education in such a context and, if possible, to identify conditions which make $I_{\mathcal{E}\to T}$ a maximum. At first glance it seems that this kind of approach, although using a similar probabilistic framework, is rather different from the signalling model we have discussed in paragraph 2 and 3, since now we are "testing" the informative process as a whole, taking into consideration for every communication engaging the joint system, rather than analyze the strategic interaction between individual and firm. Nevertheless it is worth noting that we are assuming that, in absence of noise, the educational system provides a complete information about the individual ability, hence that the microfoundation provided by the Spence's model is perfectly compatible with such a macroeconomic approach to the problem. Our task is simply to investigate the role played by the noise component, induced by the field effects, in the signalling functioning.

In order to apply the (3) we need to calculate the conditional entropy; the latter is the average of the partial conditional entropies $H(T \mid \mathcal{E} = \varepsilon_j)$. Indicating with $P(T = t_i \mid \mathcal{E} = \varepsilon_j)$ the (conditional) probability that the received signal ε_j originates from a source t_i, we obtain a system:

$$H(T \mid \mathcal{E}=\varepsilon_j) = -\sum_i P(T_i \mid \mathcal{E}=\varepsilon_j)\log P(T_i \mid \mathcal{E}=\varepsilon_j) \qquad j=1,2,...n$$

It is clear that any of such entropies depends on the noise affecting the communication; moreover the total conditional entropy $H(T \mid \mathcal{E})$=$\sum_j p_j H(T \mid \mathcal{E}=\varepsilon_j)$ depends only on the distribution law - hence on the moments - of the noise and not on the entropy characterizing the single systems. In other words, whatever the uncertainty degree of the "transmission equipment" the quality of the information depends only on the noise magnitude. If the latter is remarkable then the observation of the education level provides a poor indication about individual ability, even if the "equipment" is properly working, i.e. the players strategies are correct.

To make clearer the latter point we focus on two types only, t_1, t_2, and two corresponding education levels, e_1, e_2. In absence of noise, $\mathcal{E}$ reveals completely T, i.e. $e_1 \to t_1, e_2 \to t_2$; the error probability in decoding the signal is μ. We have four conditional probabilities: $P(e_1 \mid t_1)=1-\mu; P(e_1 \mid t_2)=\mu; P(e_2 \mid t_1)=\mu; P(e_2 \mid t_2)=1-\mu$. The total conditional base-2 entropy is[19]:

$$H(\mathcal{E} \mid T) = -\left[\mu\log_2\mu + (1-\mu)\log_2(1-\mu)\right]$$

As noted, the lack of information depends only on the noise probability μ while it is indipendent on the a-priori entropy characterizing the system. The quantity of information that $\mathcal{E}$ provides on T, in presence of noise, is then:

$$I_{\mathcal{E}\to T} = -r\log_2 r - (1-r)\log_2(1-r) - \left[-\mu\log_2\mu - (1-\mu)\log_2(1-\mu)\right]$$

where r is the a-prior probability to receive e_1. The quantity of information provided by $\mathcal{E}$ describes a saddle in the space $[0,1]^2 \nwarrow \Re^+$. For a given μ, $I_{\mathcal{E}\to T}$ achieves an interior maximum for $r=1/2$, i.e. for the maximum entropy of $\mathcal{E}$. In this case we obtain:

$$I_{\mathcal{E}\to T} = 1 - \left[-\mu\log_2\mu - (1-\mu)\log_2(1-\mu)\right]$$

where the term into the square brackets represents the lack of information due to the noise. If for example $\mu=1/2$, i.e. the maximum noise possible, then the lack of information is total: $I_{\mathcal{E}\to T}=0$.

[19]Since the transmission is composed by two "digits"- t_1, t_2 - it is useful adopt the base 2 for the logaritmic operator.

This simple example illustrates how the signalling process depends both on the receiver entropy level and the noise probabilities and, for such a reason, a wide range of situations arise, spanning from a poor informative process to a good one. Even if $\mathcal{E}$ provides a perfect signalling in a noiseless situation, the presence of field effects reduce drammatically the informative power of education.

5 Conclusions

The goal of the paper was to analyze in depth the signalling explanation to wage differentials as opposed to the human capital point of view. In paragraph 2 we analyzed the game theory approach, underlining that only separating equilibria do exist in such a game. In paragraph 3 we altered a key assumption of the model by introducing some heterogeneity among more able individuals, showing as inequality in accessing to school leads to a serious problem of "market failure" in the model. Finally in paragraph 4 we have analyzed the signalling approach as macroeconomic extension to the microeconomic problem.

It seems to me that three main conclusions arise. The first one concerns inequalities in accessing to education; if a part of more able individuals can not send the right amount of signal, then they suffer of a utility loss which induces a redistributive effect to the firms. The consequence is that each more able individual, rationed and not, is underpaid with respect to her true productivity. This kind of market failure affects both the private and the social return to education, since a lower aggregate output follows. Moreover it is not possible to rule out such a distorsion by subsidizing individuals, since, because of the asymmetric information, we do not know, a-priori, the recipients. Neither borrowing is able to solve out completely the problem, since the borrower would be certainly able to attain the right amount of education necessary to her for signalling, but in equilibrium we would have two individuals characterized by an identical productivity but by a different satisfaction, since borrower gains the same wage of the non-rationed individual but with an additional cost due to refund; also in this case there is a redistributive effect of a share of the borrower ability rents to the lender, despite the presence of a fully separating equilibrium. The only way to avoid such a problem is to guarantee to everyone a free access to schooling. This does not necessarily imply that only public school should exist, provide that in the economy does not arise beliefs according to which individuals coming from private school are providing an implicit high-quality signal.

The second main point concerns the actual functioning of an educational informative channell. There exist several external factors which can strongly reduce this kind of communication at a macroeconomic level. When the field effects make noisy the communication of the individual ability through schooling, the separating device does not work properly and more able individuals can be grouped with the less able ones and viceversa. Also in this case there is not only a private effect but also a social one, since, as noted by Stiglitz, 1975, a productive line composed by individuals with different abilities is less productive than two distinct lines characterized by homogenous workers. The presence of field effects induce a wide range of possible missallocations of individuals into the right "slots", altering the sorting mechanism.

The third and last point takes into consideration the trade-off between efficency and equity. Even if we believe in the sorting device, we can not neglect this relevant by-product of the analysis. In an economy characterized by fully separated individuals, which in turn calls for a large dispersion among wages, the productive process is primarily driven by the high-skill individuals, confering to them a preminent role in the economy. Moreover, being them the principal recipients of the full operating of the sorting mechanism, they will be naturally driven to invest a relevant GDP share in improving the educational system. Such an effect produces an over-signalling effect reducing efficency and worsening the earning distribution. If intergenerational transfers are at the work, then more able individuals tends to increase their income over time, and the dynamics of the income distribution depends on the initial conditions[20].

The distributive question related to the earning distribution is particulary relevant in the real economy. As underlined by Rebecca M. Blank[21], during the 80's, "*wages for less-skilled workers declined steadily. Between 1983 and 1989, GDP growth of 1 percent was correlated with a $0.32 decline in weekly wages for the poorest decile of the population....Among men with less than a high school diploma who work full time all year, real weekly wages declined 22 percent between 1979 and 1993....In short, for the last 15 years, economic growth has not been as powerful in fighting poverty as it was in the past.* A possible explanation for such a result can be provided by the signalling

[20] Giannini, 1996.

[21] NBER Reporter, Fall 1996, page 11.

approach, where more able individuals play a leading role in the production process and consequently in the distributive one; although less-skilled workers choose voulantarily a low education level according to their preferences, if signalling works fully, they tend to be marginalized by the growth process which is primarily driven by more able workers, getting a negligibale share of GDP growth.

The more the signalling is noisy, the more the distributive process is biased since individuals are erroneously sorted; in such a case not only the market fails to allocate workers according to their real ability, but it creates additional inequalities.

In conclusion, the signalling approach to the explanation of wage differentials arises several questions related to the real functioning of such a thereotical scheme; inequality in accessing to school and the presence of field effects reduce remarkably the possibility that a signalling device works properly. Obviously this does not mean that it does not operate at all, nor that it contains only an abstract meaning, but simply that the utility of such an approach in understanding wage differentials can be greately reduced by several factors, some of which are not under control neither by the market nor by the economist.

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