Altruism, Spite and Competition in Bargaining Games

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Abstract

This paper shows that altruism may be beneficial in bargaining when there is competition for bargaining partners. In a game with random proposers, the most altruistic player has the highest material payoff if players are sufficiently patient. However, this advantage is eroded as the discount factor increases, and if players are perfectly patient altruism and spite become irrelevant for material payoffs.

Keywords: altruism, spite, bargaining, competition, coalition formation.

1 Introduction

Game theory usually assumes that players care only about their own material payoff. However, experiments suggest that many people care about others' material payoffs (see e.g. Fehr and Schmidt, 2003). The simplest possibilities are altruism - utility increases with other people's material payoffs - and spite - utility decreases with other people's material payoffs. This paper studies the consequences of allowing for limited altruism or spite in multilateral bargaining games with complete information.

It is well known that players can benefit from being spiteful in bilateral bargaining games: a spiteful player is committed to rejecting offers that would be acceptable to a selfish player, and, if this is anticipated by the other player, gets a higher payoff as a result. However, the situation is more complicated when players can choose bargaining partners: a spiteful player may get a better deal out of bargaining, but may not get any bargaining partners in the first place. On the other hand, even a spiteful player may lower his demands in view of competition.[1] It is then unclear whether spiteful players do better than selfish players in terms of material payoffs.

In this paper I address the question of what preferences are the most successful in terms of material payoffs when there are three players but only two of them need to cooperate. In a model with irrevocable choice of partner (players first chose a partner to bargain with and then negotiate on payoff division) neither altruism nor spite are unambiguously advantageous: the player with intermediate preferences has an advantage. On the other hand, if players can keep their options open until a payoff division is agreed upon, the most altruistic player has an advantage in terms of material payoffs when players are sufficiently patient. However, that advantage is eroded as the discount factor increases, and if players are perfectly patient altruism and spite become irrelevant for material payoffs.

2 General assumptions

There are three players, $N = \{1, 2, 3\}$. If two players cooperate, they can obtain one unit of money. All players are risk neutral and share a discount factor $\delta \leq 1$, but they may differ in their attitudes towards other players' material payoffs. We allow for all utility functions of the type $u_i = x_i + \alpha_i \frac{\sum_{j \neq i} x_j}{n-1}$, where $-1 < \alpha_i < 1$ for all i. Selfish players have $\alpha_i = 0$, altruistic players have $\alpha_i > 0$ and spiteful players have $\alpha_i < 0$. Notice the following consequences of this assumptions:

- The functional form follows Bester and Güth (1998) and Possajennikov (2000) and allows for altruism and spite, but not for inequality aversion. Unlike in the models of Fehr and Schmidt (1999) and Bolton and

[1] Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) point out that competition may make players behave as if they were selfish even if they are not. They give as an example the ultimatum game with proposer or responder competition.

Ockenfels (2000), players are not concerned about inequality as such: the marginal rate of substitution between own and others' payoffs is a constant.[2]

- As in Bester and Güth (1998), players care more about their own material payoffs than about other players'. If the total material payoff is X, player i's utility function can be rewritten as $u_i(.) = (1-\beta_i)x_i + \beta_i X$, where $\beta_i = \frac{\alpha_i}{n-1}$, $-1 < \beta_i < 1$. Thus, utility is a weighted average of own payoff and total payoff. If X is a constant, utility is maximized when i gets the whole cake, and thus players' altruism or spite is limited.

- Each player is equally altruistic or spiteful towards all other players.

- Preferences only depend on outcomes, and not on things such as past offers and counteroffers.

- Players do not care directly about other players' preferences (unlike in Levine (1998)).

- The assumption $-1 < \alpha_i$ guarantees that it is never a Pareto improvement to throw money away: if an additional ϵ of money becomes available and players divide it equally they will all be better-off.

Assuming that preferences are complete information, what preferences will be more successful in terms of material payoffs?

3 A benchmark: the two-player case

In two-player bargaining, a player's payoff is higher the more spiteful he is and the more altruistic the other player is. Assuming that no player can receive a negative share of the money, there are corner solutions in which the most altruistic player receives 0. These results are supported by the Nash (1950) bargaining solution, the Rubinstein (1982) bargaining model with alternating offers and Binmore's (1987b) variant with random proposers.

[2] For a model of multiplayer bargaining with inequity aversion see Montero (2005).

3.1 The Nash bargaining solution

The following lemma shows that a player's material payoff (weakly) increases as he becomes more spiteful or the other player becomes more altruistic.

Lemma 1 *Suppose no player receives any money in case of disagreement, and let x be the money received by player 1, $0 \leq x \leq 1$. Then*

a) If none of the players is much more altruistic than the other ($\alpha_i \leq \frac{1}{2-\alpha_j}$, $i = 1, 2$, $i \neq j$) the Nash bargaining solution is

$$x^* = \frac{1}{2} - \frac{\alpha_1 - \alpha_2}{2(1-\alpha_1)(1-\alpha_2)}.$$

This is decreasing in α_1 and increasing in α_2.

b) If one of the players is much more altruistic than the other, the Nash bargaining solution is $x^* = 0$ ($\alpha_1 > \frac{1}{2-\alpha_2}$) or $x^* = 1$ ($\alpha_2 > \frac{1}{2-\alpha_1}$).

Proof. See Appendix.

3.2 Rubinstein bargaining

In the Rubinstein bargaining model, the two players alternate making offers until an agreement is reached. Every time an offer is rejected, a period elapses. For $\delta < 1$, Rubinstein (1982) shows that the subgame perfect equilibrium of this game is unique and stationary (strategies do not depend on past play). Binmore (1987a) shows that this subgame perfect equilibrium converges to the Nash bargaining solution as δ tends to 1. One of Rubinstein's assumptions on preferences is that for any given share of the pie x, all players prefer to have x now rather than later. This assumption allows for altruistic players but excludes spiteful ones. However, Rubinstein's and Binmore's results hold in this case as well.

Lemma 2 *There is a unique subgame perfect equilibrium of the Rubinstein game. The equilibrium payoffs converge to the Nash bargaining solution as δ tends to 1.*

Proof. See Appendix.

The same result holds for the random proposers variant of the Rubinstein's game introduced by Binmore (1987b).

4 A model with irrevocable choice of partner

Suppose there are three players potentially differing in α_i, and any two of them can divide one dollar. The players play the following two-stage game:

- Stage 1: Irrevocable choice of partner. A pair of players emerges from this stage, and the third player no longer plays a role.
- Stage 2: Once a pair is formed, the players play the Rubinstein game (or the game with random proposers) described above. If no agreement is reached, all players receive zero.

Clearly, each player would prefer to form a pair with the most altruistic of the other two, and the coalition of the two most altruistic players is likely to emerge. In terms of material payoffs, the most spiteful player gets nothing and the intermediate player does best.

Example 1 *Consider* $\alpha_1 = -\frac{1}{5}$, $\alpha_2 = 0$, $\alpha_3 = \frac{1}{4}$ *and* δ *close to* 1.

Coalition\Payoffs	1	2	3
$\{1,2\}$	0.58	0.42	–
$\{1,3\}$	0.75*	–	0.25
$\{2,3\}$	–	0.67*	0.33*

The numbers in the matrix are material payoffs, not utilities. However, given our assumptions every player wants as large a material payoff as possible. Both players 2 and 3 prefer coalition $\{2,3\}$ to form. Player 2 gets the best deal while player 1, who is the toughest, gets nothing.[3]

Conclusion 1 *In the model with irrevocable choice of partner, neither altruism nor spite is unambiguously beneficial. It is the player with intermediate preferences who does best in material terms.*

[3]Since the third player is no longer in the game, the calculations assume $u_i = x_i + \alpha_i x_j$ rather than $u_i = x_i + \alpha_i \frac{x_j + x_k}{2}$. This makes no difference to the players' preferences over bargaining partners.

This result echoes Binmore's (1985) result on bargaining with different discount factors: the player who can get a better deal out of bilateral bargaining is excluded when there is a choice of bargaining partners.[4]

5 A competitive model with choice of partner

In the model with irrevocable choice of partner, the most spiteful player suffers from a sort of hold-up problem. Since he gets nothing out of the situation he would be willing to moderate his demands in order to get into a coalition. However, once he is alone with the other player he has no incentive to do so, thus he is always excluded from the coalition that forms.

Suppose instead that this problem is not present. Players can agree on a payoff division at the same time they form a coalition, and there is effective competition between the players. A way of modelling the competition between players is to consider bargaining games with random proposers (introduced by Binmore, 1987b and extended by Baron and Ferejohn (1989) and Okada (1996)).

5.1 Preliminaries

In a game with random proposers, each of the three players is selected to be proposer with probability $\frac{1}{3}$. The proposer i chooses a responder j and offers him a division of the monetary payoff. Player j then accepts or rejects. If j rejects, a period elapses and a new proposer is selected - again each of the players with probability $\frac{1}{3}$ -. We focus of stationary subgame perfect equilibria (SSPE). These are subgame perfect equilibria in which players' strategies do not condition on elements of history other than the current proposal.

Given a strategy combination, we will use the following notation:

$\overline{y}_i$ for the expected material payoff for player i.

[4]In Binmore's "telephone bargaining" model, later generalized by Chatterjee et al. (1993), players are not irrevocably committed to a bargaining partner. However, competitive pressures in the model are very weak: for example, in a market with one seller and $n \geq 1$ identical buyers, the equilibrium price is independent of n.

r_i for the probability that i receives an offer.

λ_{ij} for the probability that i makes a proposal to j, conditional on i being selected to be proposer.

x_i for the offer that makes player i indifferent between accepting it or rejecting it (given that the proposer gets $1 - x_i$).

Because each player would like to keep as much of the dollar as possible, x_i will play an important role in the analysis. In equilibrium the proposer will choose the player with the lowest x_i and offer him exactly x_i. A particular class of equilibria that will be important has the property that each player is indifferent between proposing to any of the other two players. Then $x_i = x$ for all i. We will refer to this type of equilibria as *mixed-strategy SSPE* because players typically randomize between partners.

Lemma 3 *Let $\delta \leq 1$. All SSPE of the game are such that a coalition is formed immediately and the proposer offers a responder i exactly x_i.*

Proof. Given any SSPE, we can calculate the expected utility player i would get from rejecting a proposal. Because equilibrium strategies are stationary, this utility does not depend on what has happened in the game so far. Because the utility function is linear, the utility of rejecting a proposal can be written as $\delta \left[\overline{y}_i + \alpha_i \frac{\overline{y}_j + \overline{y}_k}{2}\right]$; only the average material payoffs matter.

Let i be a player with $\overline{y}_i < 1$. Consider the situation of player i as proposer. If he makes a proposal that is rejected, his utility is $\delta \left[\overline{y}_i + \alpha_i \frac{\overline{y}_j + \overline{y}_k}{2}\right]$. But he can do better by proposing to the player j with the lowest $\overline{y}_j$ and offering him $\delta \overline{y}_j + \frac{1 - \delta \overline{y}_i - \delta \overline{y}_j}{2}$, keeping $\delta \overline{y}_i + \frac{1 - \delta \overline{y}_i - \delta \overline{y}_j}{2}$ for himself. This proposal will be accepted by j and is strictly preferred by i to a proposal that would be rejected (equal division of the extra payoff ensures this even if i and/or j are spiteful).

Finally, no player can have $\overline{y}_i = 1$ because he would receive no proposals, contradicting $\overline{y}_i = 1$. Thus, all proposals made in equilibrium are accepted.

Also, if i is a responder he must be offered exactly x_i, where $x_i + \alpha_i \frac{1 - x_i}{2} = \delta \left[\overline{y}_i + \alpha_i \frac{\overline{y}_j + \overline{y}_k}{2}\right]$; otherwise the proposer would be better-off by reducing i's share. ■

5.2 Patient players ($\delta = 1$)

With patient players, altruism and spite play no role.

Proposition 1 *If $\delta = 1$, the unique SSPE is such that the first proposer forms a coalition with one of the other players and keeps $\frac{2}{3}$ of the total payoff, regardless of $(\alpha_i)_{i \in N}$.*

Proof. Because of lemma 3, the players will always agree to divide the whole payoff. Thus, material payoffs add up to 1 and the utility function can be written as $u_i(.) = (1 - \frac{\alpha_i}{2})z_i + \frac{\alpha_i}{2}$, where z_i is player i's material payoff. But then we can take $\frac{\alpha_i}{2}$ from both sides and divide by $1 - \frac{\alpha_i}{2}$ in the relevant equations, and altruism and spite become irrelevant.

As an illustration, suppose we have an equilibrium and let i be a player who receives proposals with positive probability and j a player who receives proposals from player i. In equilibrium x_i must be such that i is indifferent between accepting and rejecting, thus

$$(1 - \frac{\alpha_i}{2})x_i + \frac{\alpha_i}{2} =$$
$$= \frac{1}{3}\left[(1 - \frac{\alpha_i}{2})(1 - x_j) + \frac{\alpha_i}{2}\right] + \frac{\lambda_{ji} + \lambda_{ki}}{3}\left[(1 - \frac{\alpha_i}{2})x_i + \frac{\alpha_i}{2}\right] + \frac{2 - \lambda_{ji} - \lambda_{ki}}{3}\frac{\alpha_i}{2}$$

Taking $\frac{\alpha_i}{2}$ from both sides and dividing by $1 - \frac{\alpha_i}{2}$, we obtain

$$x_i = \frac{1}{3}(1 - x_j) + \frac{1}{3}(\lambda_{ji} + \lambda_{ki})\, x_i.$$

This is precisely the same equilibrium condition that we would have with selfish players.

The equilibrium of this game with selfish players is described by Baron and Ferejohn (1989). They show that in any equilibrium $x_i = \frac{1}{3}$ for all i, thus the proposer offers $\frac{1}{3}$ to one of the other two players and keeps $\frac{2}{3}$ for himself. Equilibrium strategies are not unique, but they must be such that each player is equally likely to be in the final coalition; for example, each player can propose to each of the others with probability $\frac{1}{2}$. ■

5.3 Impatient players ($\delta < 1$)

Altruism or spite will play a role because if a proposal is rejected there is a period in which all players get 0, and different players may feel differently about this. Altruism in this context is similar to impatience, and would seem to reduce bargaining power. However, it turns out that if players are sufficiently patient the most altruistic player does best.

Proposition 2 *If δ is sufficiently close to 1, there is an SSPE with the property that the most altruistic player has the highest expected material payoff.*

To prove this result, we show that in a mixed-strategy SSPE the more altruistic players must do better in material terms. We then go on to construct such an equilibrium.

In a mixed-strategy equilibrium, each player must be indifferent between proposing to any of the other two. All players make the same proposal: they offer x to one of the other players, and keep $1-x$ for themselves. Because the responder must be indifferent between accepting and rejecting, a mixed-strategy SSPE must be a solution to the following system (where $i = 1, 2, 3$ and $\{j, k\} = N \backslash \{i\}$)

$$x + \alpha_i \frac{1-x}{2} = \frac{\delta}{3}\left[1 - x + \alpha_i \frac{x}{2}\right] + \frac{\delta}{3}(\lambda_{ji} + \lambda_{ki})\left[x + \alpha_i \frac{1-x}{2}\right] + \frac{\delta}{3}(2 - \lambda_{ji} - \lambda_{ki})\frac{\alpha_i}{2}$$

There are three additional equations of the form $\lambda_{ij} + \lambda_{ik} = 1$. Thus, we have six equations and seven unknowns (x and six λ_{ij}'s). We will show that for δ sufficiently close to 1 we can find values for the λ_{ij}'s such that $0 \le \lambda_{ij} \le 1$ for all $i \ne j$, and a unique solution for the equilibrium proposal x, the expected material payoffs for each player $(\overline{y}_i)_{i \in N}$ and the probabilities of receiving proposals for each player $(r_i)_{i \in N}$.

Lemma 4 *In a mixed-strategy SSPE, the equilibrium values of x, $(\overline{y}_i)_{i \in N}$ and $(r_i)_{i \in N}$ are uniquely determined given $(\alpha_i)_{i \in N}$ and δ.*

Proof. Because players are risk neutral, we can write the indifference conditions of the players in terms of $(\overline{y}_i)_{i\in N}$ and x. The equilibrium values of $(\overline{y}_i)_{i\in N}$ and x must solve the following system

$$\begin{aligned} x+\alpha_i\frac{1-x}{2} &= \delta\left[\overline{y}_i+\alpha_i\frac{1-\overline{y}_i}{2}\right] \quad i=1,2,3 \\ \sum_{i\in N}\overline{y}_i &= 1 \end{aligned}$$

The solution to this system is

$$x = 1-\frac{2\delta}{3}-\sum_{i\in N}\frac{2(1-\delta)}{3(2-\alpha_i)}. \tag{1}$$

$$\overline{y}_i = \frac{1}{3}+\frac{4(1-\delta)}{3\delta(2-\alpha_i)}-\frac{2}{3}\sum_{j\in N\backslash\{i\}}\frac{1-\delta}{\delta(2-\alpha_j)} \tag{2}$$

Given the equilibrium value for x, the value for r_i can be found from the indifference condition of player i, now rewritten in terms of r_i

$$x+\alpha_i\frac{1-x}{2}=\delta\left[\frac{1}{3}\left(1-x+\alpha_i\frac{x}{2}\right)+r_i\left(x+\alpha_i\frac{1-x}{2}\right)+\left(\frac{2}{3}-r_i\right)\frac{\alpha_i}{2}\right]. \tag{3}$$

■

Lemma 5 *In a mixed-strategy SSPE, the value of* $\overline{y}_i$ *is increasing in* α_i *and decreasing in* α_j*.*

Proof. Differentiating expression (2), we can see that $\frac{\partial\overline{y}_i}{\partial\alpha_i}=\frac{4(1-\delta)}{3\delta(2-\alpha_i)^2}>0$ and $\frac{\partial\overline{y}_i}{\partial\alpha_j}=-\frac{2(1-\delta)}{3\delta(2-\alpha_j)^2}<0$. ■

The intuition for this result is that, in order for a mixed-strategy SSPE to exist, all players must have the same value of x_i, even though they may have different values for α_i. Equilibrium strategies should then balance two sources of bargaining power: the way players react to the possibility of delay, and how often people receive proposals. Because more altruistic players suffer more from delay they must receive proposals more often.

Thus, altruism is beneficial if players are playing a mixed strategy equilibrium. However, this effect becomes smaller as players become more patient, as the following corollary shows.

Lemma 6 *Let $\alpha_i > \alpha_j$, and suppose we have a mixed-strategy SSPE. Then $\overline{y}_i - \overline{y}_j$ is decreasing in δ.*

Proof. $\overline{y}_i - \overline{y}_j = \frac{2(1-\delta)(\alpha_i-\alpha_j)}{\delta(2-\alpha_i)(2-\alpha_j)}$. The derivative of this expression with respect to δ is $-\frac{2(\alpha_i-\alpha_j)}{\delta^2(2-\alpha_i)(2-\alpha_j)} < 0$. ■

In order for a mixed strategy equilibrium to exist, players must be sufficiently patient as the following lemmas show.

Lemma 7 *The value of x given by (1) is decreasing in each α_i and increasing in δ, and is positive if δ is close enough to 1.*

Proof. $\frac{\partial x}{\partial \alpha_i} = -\frac{2(1-\delta)}{3(2-\alpha_i)^2} < 0$. $\frac{\partial x}{\partial \delta} = \frac{2}{3}\left(\sum_i \frac{1}{2-\alpha_i} - 1\right)$, which is positive because $\alpha_i > -1$ for all i.

Because x is decreasing in each α_i, it suffices to find a value of δ that guarantees $x \geq 0$ when all α_i's are close to 1. If we replace each α_i by 1, $x = \frac{4\delta-3}{3}$, which is positive for $\delta \geq \frac{3}{4}$. ■

It remains to show that we can find a collection $(\lambda_{ij})_{i \neq j}$ (with $\lambda_{ij} \in [0, 1]$ and $\lambda_{ij} + \lambda_{ik} = 1$) so that each player offering x to one of the others is an equilibrium. This will be possible provided that δ is sufficiently close to 1. Lemma 8 shows that, for δ sufficiently close to 1, the equilibrium value of r_i is between 0 and $\frac{2}{3}$ for any preference profile $(\alpha_i)_{i \in N}$. This is clearly a necessary condition for the existence of a suitable collection $(\lambda_{ij})_{i \neq j}$. Lemma 9 shows that it is also sufficient.

Lemma 8 *The value of r_i that solves (3) is between 0 and $\frac{2}{3}$ if δ is sufficiently close to 1.*

Proof. The expression for r_i as a function of $(\alpha_i)_{i \in N}$ can be found by solving for r_i in (3) and then replacing x by its equilibrium value found in (1). Because exactly one player is the responder, the solution satisfies $\sum r_i = 1$.

It can be shown[5] that $\frac{dr_i}{d\alpha_i} > 0$ and $\frac{dr_i}{d\alpha_j} = -\frac{2(1-\delta)(\alpha_i(2\delta-3)+2\delta)}{9\delta x^2(2-\alpha_i)(2-\alpha_j)^2} < 0$ for $\delta \geq \frac{3}{4}$. This implies that the condition $r_i \geq 0$ will be most difficult to

[5] A proof is available from the author.

satisfy when α_i is close to -1 and α_j and α_k are close to 1. Analogously, the condition $r_i \leq \frac{2}{3}$ will be most difficult to satisfy when α_i is close to 1 and α_j and α_k are close to -1.

The value of r_i associated to $\alpha_i = -1$ and $\alpha_j = \alpha_k = 1$ is $\frac{8\delta^2+19\delta-24}{3\delta(8\delta-5)}$, which is positive for $\delta \geq \frac{\sqrt{1129}}{16} - \frac{19}{16} \approx 0.913$. The value of r_i associated to $\alpha_i = 1$ and $\alpha_j = \alpha_k = -1$ is $\frac{4\delta^2-25\delta+24}{3\delta(4\delta-1)}$, which is smaller than $\frac{2}{3}$ for $\delta \geq 0.902$. ■

We have shown that the candidate equilibrium values for r_i are between 0 and $\frac{2}{3}$ provided that δ is large enough. It is also the case that $\sum r_i = 1$.

Lemma 9 *Suppose we have a vector $(r_i)_{i\in N}$ such that $0 \leq r_i \leq \frac{2}{3}$ for all i and $\sum_{i\in N} r_i = 1$. Then there are mixed strategies that implement $(r_i)_{i\in N}$.*

Proof. Without loss of generality, suppose $r_3 \geq r_2 \geq r_1$.

Equilibrium strategies must satisfy the following system of equations

$$\frac{1}{3}(\lambda_{ji} + \lambda_{ki}) = r_i, \;\; i = 1, 2, 3$$

Since $\lambda_{23} = 1 - \lambda_{21}$, $\lambda_{32} = 1 - \lambda_{31}$ and $r_3 = 1 - r_1 - r_2$, we can write the system as

$$\begin{aligned} \frac{1}{3}(\lambda_{21} + \lambda_{31}) &= r_1 \\ \frac{1}{3}(\lambda_{12} + 1 - \lambda_{31}) &= r_2 \\ \frac{1}{3}(2 - \lambda_{12} - \lambda_{21}) &= 1 - r_1 - r_2 \end{aligned}$$

The three equations are not linearly independent: the first two imply the third. Taking λ_{31} as a parameter we find:

$$\begin{aligned} \lambda_{12} &= 3r_2 - 1 + \lambda_{31} \\ \lambda_{21} &= 3r_1 - \lambda_{31} \end{aligned}$$

We want to set a value for λ_{31} such that λ_{12} and λ_{21} are between 0 and 1. Notice that, because $r_3 \geq \frac{1}{3}$, $\frac{1}{3} \leq r_1 + r_2 \leq \frac{2}{3}$. There are two cases:

a) $r_2 \geq \frac{1}{3}$: then $\lambda_{31} = 0$ is suitable.

b) $r_2 \leq \frac{1}{3}$: then $\lambda_{31} = 1 - 3r_2$ is suitable. ■

Even thought equilibrium strategies are not unique, equilibrium payoffs are. Uniqueness of equilibrium payoffs can be shown by considering each possible type of equilibrium in turn. We have seen that an equilibrium exists with $x_i = x_j = x_k$; the other three possibilities are $x_i > x_j = x_k$, $x_i = x_j > x_k$ and $x_i > x_j > x_k$. These other possibilities can be eliminated by calculating the equilibrium values of $(x_i)_{i \in N}$ implied by player's preferences (for example, if $x_i > x_j > x_k$, players i and j always propose to k and player k always proposes to j) and reaching a contradiction (in the case $x_i > x_j > x_k$, one would actually find $x_k > x_i$).

The following example illustrates how the most altruistic player is the most successful in purely material terms and how this effect is less pronounced when players are more patient.

Example 2 *Consider* $\alpha_1 = -\frac{1}{5}$, $\alpha_2 = 0$, $\alpha_3 = \frac{1}{4}$.

$\delta = 0.65$ *(*$x = 0.21$*)*	*Spiteful*	*Selfish*	*Altruistic*
Probability of being in coalition	0.39	0.62	0.99
Expected material payoffs	0.28	0.32	0.40
$\delta = 0.85$ *(*$x = 0.28$*)*	*Spiteful*	*Selfish*	*Altruistic*
Probability of being in coalition	0.60	0.66	0.75
Expected material payoffs	0.31	0.33	0.36

If δ is not sufficiently close to 1, the most altruistic player is not necessarily the one that does best in terms of material payoffs, as the next example illustrates.

Example 3 *Consider* $\alpha_1 = -\frac{2}{3}$, $\alpha_2 = -\frac{1}{2}$, $\alpha_3 = \frac{3}{4}$ *and* $\delta = 0.8$. *The unique SSPE equilibrium is such that players* 1 *and* 2 *propose* $(0.87, 0.13)$ *to player* 3, *and player* 3 *proposes* $(0.68, 0.32)$ *to either player* 1 *- with probability* 0.44 *- or player* 2 *- with probability* 0.56 *-. Expected material payoffs are* $(0.34, 0.35, 0.32)$, *thus player* 2 *is doing best.*

6 Conclusion

In purely material terms, spite is beneficial in two-player situations (or in general in unanimous bargaining). Intermediate preferences (relative to the preferences of the other two players) are best in three-player games with irrevocable choice of partner. In more competitive environments the most altruistic player does best in material terms provided that players are sufficiently patient. However, when δ equals 1 altruism and spite become irrelevant.

The results imply that the same characteristics of preferences that are beneficial in two-player bargaining can be detrimental when there is competition for bargaining partners. Related results have been found by Harrington (1990) for risk aversion and Kawamori (2005) for impatience. The driving force behind this type of results is that, if every player were to receive offers with the same probability, the 'weakest' player would have the lowest continuation value and be the most desirable partner; thus in equilibrium the weakest types receive proposals more often.

Altruism matters because it affects the players' attitudes towards disagreement. With discounting, disagreement occurs temporarily and altruism interacts with discounting so that more altruistic players behave like more impatient players. In the infinite horizon game without discounting, agreement is guaranteed and altruism and spite become irrelevant.[6]

An important assumption behind the results is that players are indiscriminately altruistic or spiteful. If instead we allow for utility functions of the type $u_i(x) = x_i + \sum_{j \neq i} \alpha_{ij} x_j$, altruism may be detrimental. As an illustration, suppose player 1 is altruistic towards player 2 (that is, $u_1(x) = x_1 + \alpha_1 x_2$ with $\alpha_1 > 0$) and players 2 and 3 are selfish. Because player 1 is altruistic towards player 2, he will prefer to propose to player 2 unless 2 has a significantly higher continuation value than 3. Moreover, player 1 requires a lower payoff in order to accept a proposal if the proposer

[6] In a finite-horizon bargaining game without discounting, spite would still play some role because the most spiteful player never receives a proposal in the last period; altruism and selfishness however would be indistinguishable because both altruistic and selfish players would accept 0 in the last period. See Norman (2002) for an analysis of the bargaining procedure with a finite horizon and selfish players.

is player 2. In the mixed-strategy SSPE, player 2 has the highest material payoff, and 1 and 3 are equally hurt in material terms by 1's altruism towards 2.[7]

In Bester and Güth (1998), if an altruistic player is paired with an egoistic one, the egoistic player does better than the altruistic one in material terms. The possibility of evolutionary stability of altruistic players arises because the presence of altruistic players increases efficiency. Because of this, an altruistic player may do better against another altruistic player than a selfish player would do against an altruistic player. In this paper there is no possibility of efficiency gains. Total material payoffs always add up to 1 provided that players reach an agreement immediately, and even the most spiteful players manage to do that. There is no possibility for altruistic preferences to be stable in bilateral situations. Altruistic preferences however have an advantage when there is competition for bargaining partners. Altruistic players are more popular in equilibrium, despite the fact that other types of players have moderated their demands due to the competitive pressures. If we made the (very arbitrary!) assumption that players always interact in triads, altruistic preferences would not only be evolutionarily stable, but could successfully invade a population made by any other preferences provided that players are sufficiently patient.

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7 Appendix

Proof of lemma 1. Because no player receives any money in case of disagreement,[8] the *utility* associated with disagreement is 0 regardless of

[7]Calvert and Dietz (1998) allow for identity-dependent altruism, but require $\alpha_{ij} = \alpha_{ji}$.

[8]If one or both players receive something in case of disagreement, they will come to an agreement provided that the sum of material payoffs in case of disagreement is less than 1. This is because the extra money can be divided equally.

preferences. The *Nash bargaining solution* would be the solution to

$$\begin{aligned} & \max && (x+\alpha_1(1-x))\,(1-x+\alpha_2 x) \\ \text{s. t. } x & \in && [0,1] \\ x+\alpha_1(1-x) & \geq && 0 \\ 1-x+\alpha_2 x & \geq && 0 \end{aligned}$$

The maximization problem takes into account that players will never agree to throw money away, so player 2's share equals $1-x$. The requirement $x \in [0,1]$ is a feasibility requirement. The other two constraints are individual rationality constraints: the agreement must guarantee a utility of 0 to both players.

Ignoring the constraints for the moment, the FOC of this maximization problem is

$$1-\alpha_1(2-\alpha_2)-2x(1-\alpha_1)(1-\alpha_2)=0$$

or

$$x=\frac{1}{2}-\frac{\alpha_1-\alpha_2}{2(1-\alpha_1)(1-\alpha_2)}.$$

Since $-(1-\alpha_1)(1-\alpha_2)<0$, the second order condition for a maximum is always satisfied. The solution also satisfies individual rationality for both players, but it is not always between 0 and 1. In some cases (when α_1 and α_2 are sufficiently dissimilar) we will have a corner solution. If $\alpha_1 \geq \frac{1}{2-\alpha_2}$ we have $x=0$; if $\alpha_2 \geq \frac{1}{2-\alpha_1}$ we have $x=1$. If both players are selfish or spiteful, the solution is always interior. Thus, corner solutions are always individually rational because the player getting 0 must be altruistic.

Proof of lemma 2. Let us look for a stationary subgame perfect equilibrium. Denote player 1's proposal by $(x,1-x)$ and player 2's proposal by $(y,1-y)$. Because each player (however altruistic) will offer the other as little as possible, in an interior solution each responder is indifferent between accepting a proposal and rejecting it, thus:

$$\begin{aligned} y+\alpha_1(1-y) &= \delta\,(x+\alpha_1(1-x)) \\ 1-x+\alpha_2 x &= \delta\,(1-y+\alpha_2 y) \end{aligned}$$

The solution to this system is $x = \frac{1-\alpha_1(1+\delta(1-\alpha_2))}{(1-\alpha_1)(1-\alpha_2)(1+\delta)}$ and $y = \frac{\delta-\alpha_1(1+\delta-\alpha_2)}{(1-\alpha_1)(1-\alpha_2)(1+\delta)}$. In order for these values to be between 0 and 1, we need $\alpha_i \leq \frac{\delta}{1+\delta-\alpha_j}$ for $i, j = 1, 2$, $i \neq j$. Both x and y converge to $\frac{1}{2} - \frac{\alpha_1-\alpha_2}{2(1-\alpha_1)(1-\alpha_2)}$ as δ tends to 1.

If $\alpha_1 \geq \frac{\delta}{1+\delta-\alpha_2}$, there is a corner solution. Player 2 claims the whole payoff when he proposes; player 1 makes the proposal that makes 2 indifferent. Player 1 prefers to accept nothing in the current period rather than endure delay. The corresponding equations would be:

$$\begin{aligned} 1 - x + \alpha_2 x &= \delta \text{ (indifference condition for player 2)} \\ \alpha_1 &\geq \delta\left[x + \alpha_1(1-x)\right] \text{ (player 1 prefers to accept } (0,1)) \end{aligned}$$

The solution to the first equation is $x = \frac{1-\delta}{1-\alpha_2}$, always positive. In order for $x \leq 1$ we need $\alpha_2 \leq \delta$. In order for player 1 to be willing to accept $(0, 1)$ we need $\alpha_1 \geq \frac{\delta}{1+\delta-\alpha_2}$. This bound becomes more demanding with δ and at the limit becomes $\alpha_1 \geq \frac{1}{2-\alpha_2}$.

If $\alpha_i > \delta$ for $i = 1, 2$, we have a corner solution in which 1 proposes $(1, 0)$ and 2 proposes $(0, 1)$. Because $\alpha_i < 1$ for $i = 1, 2$, this type of corner solution is not relevant for sufficiently large values of δ.

Uniqueness of equilibrium can be shown adapting the arguments in Sutton (1986), taking into account that a player's utility depends on the other's share.

Denote by U_i^* (u_i^*) the supremum (infimum) of the utility player i can get in any subgame perfect equilibrium as the proposer. Because equilibria could in principle be inefficient and player 1's utility depends on both players' share, there isn't a unique payoff division associated to these utility levels. However, we can find the share $M_i \in [0, 1]$ such that player i's utility when he gets M_i and j gets $1 - M_i$ equals U_i^* (we can define m_i analogously).[9]

Since in order to keep player 1 indifferent we need to give player 2 more than half of the (possible) extra payoff, player 2 weakly prefers $(M_1, 1 - M_1)$

[9] The existence of $M_i \geq 0$ is guaranteed even if i is altruistic: if the equilibrium is not efficient we need to reduce player i's share to make him indifferent, but i cannot prefer to let j get the whole payoff rather than play the equilibrium, since i can always offer the whole payoff to j and j would accept.

to the actual equilibrium at the subgame in which player 1 proposes and get his supremum payoff. Moreover, player 2's utility associated to the supremum must be at least δu_2^*, that is, $\delta u_2(1-m_2, m_2)$. Thus

$$u_2(M_1, 1-M_1) \geq \delta u_2(1-m_2, m_2). \quad (4)$$

On the other hand, a proposal of player 1 never needs to give player 2 a utility higher than $\delta u_2(1-M_2, M_2)$. Nevertheless, there is one case in which 1 prefers to give 2 a higher utility: if 2 prefers to accept $(1,0)$ rather than wait one period to obtain $u_2(1-M_2, M_2)$. Thus[10]

$$u_2(m_1, 1-m_1) \leq \max\left[u_2(1,0), \delta u_2(1-M_2, M_2)\right]. \quad (5)$$

There are two analogous equations for player 1.

$$\begin{aligned} u_1(1-M_2, M_2) &\geq \delta u_1(m_1, 1-m_1). & (6) \\ u_1(1-m_2, m_2) &\leq \max\left[u_1(0,1), \delta\left(u_1(M_1, 1-M_1)\right)\right]. & (7) \end{aligned}$$

There are four possible cases, depending on what the maximum is on the right-hand side of expressions (5) and (7). We examine each case in turn.

1. If $\max\left[u_2(1,0), \delta u_2(1-M_2, M_2)\right] = \delta u_2(1-M_2, M_2)$ and $\max\left[u_1(0,1), \delta\left(u_1(M_1, 1-M_1)\right)\right] = \delta u_1(M_1, 1-M_1)$, we can replace $u_i(.)$ by its value in the four equations above, and manipulate then to show that $M_1 = m_1$ and $M_2 = m_2$.

   Equation (4) can be written as $1 - M_1 + \alpha_2 M_1 \geq \delta\left(m_2 + \alpha_2(1-m_2)\right)$. Note that since $\alpha_2 < 1$, the right-hand side is increasing in m_2.

   Equation (7) becomes $1 - m_2 + \alpha_1 m_2 \leq \delta(M_1 + \alpha_1(1-M_1))$, or, since $\alpha_1 < 1$, $m_2 \geq \frac{1-\delta(M_1+\alpha_1(1-M_1))}{1-\alpha_1}$. Combining these two expressions leads to

$$M_1 \leq \frac{1-\alpha_1(1+\delta(1-\alpha_2))}{(1-\alpha_1)(1-\alpha_2)(1+\delta)}.$$

[10]If the actual equilibrium proposal is not efficient, it cannot be the case that $u_2(m_1, 1-m_1) > \delta u_2(1-M_2, M_2)$, because then player 1 could have made a Pareto improving proposal that player 2 would accept and we would not have an equilibrium.

Analogously, from (5) and (6) we obtain

$$m_1 \geq \frac{1-\alpha_1(1+\delta(1-\alpha_2))}{(1-\alpha_1)(1-\alpha_2)(1+\delta)}.$$

Thus, player 1 obtains the same payoff as a proposer in any SPE of this type.

Analogously, we can show that $M_2 = m_2 = \frac{1-\alpha_2(1+\delta(1-\alpha_1))}{(1-\alpha_1)(1-\alpha_2)(1+\delta)}$.

In order for the equilibrium payoffs we have calculated to be between 0 and 1, we need $\alpha_i \leq \frac{\delta}{1+\delta-\alpha_j}$ for i, $j = 1, 2$.

2. If $\max[u_2(1,0), \delta u_2(1-M_2, M_2)] = \delta u_2(1-M_2, M_2)$ and $\max[u_1(0,1), \delta(u_1(M_1, 1-M_1))] = u_1(0,1)$, player 2 proposes $(0,1)$ in any SPE. Thus, $M_2 = m_2 = 1$. Player 1's proposal is found from the equation $u_2(m_1, 1-m_1) = \delta u_2(0,1)$. The equilibrium value of m_1 is then

$$m_1 = \frac{1-\delta}{1-\alpha_2}.$$

In order for this value to be smaller than 1, we need $\alpha_2 \leq \delta$. In order for player 1 to prefer $(0,1)$ now rather than $(m_1, 1-m_1)$ in the next period, we need $\alpha_1 \geq \frac{\delta}{1+\delta-\alpha_2}$.

3. Analogously, if $\max[u_2(1,0), \delta u_2(1-M_2, M_2)] = u_2(1,0)$ and $\max[u_1(0,1), \delta(u_1(M_1, 1-M_1))] = \delta(u_1(M_1, 1-M_1))$, player 1 proposes $(1,0)$ and player 2 proposes $(1-m_2, m_2)$ where $m_2 = \frac{1-\delta}{1-\alpha_2}$ in any SPE. This case requires $\alpha_1 \leq \delta$ and $\alpha_2 \geq \frac{\delta}{1+\delta-\alpha_1}$.

4. Finally, if $\max[u_2(1,0), \delta u_2(1-M_2, M_2)] = u_2(1,0)$ and $\max[u_1(0,1), \delta(u_1(M_1, 1-M_1))] = u_1(0,1)$, player 1 proposes $(1,0)$ and player 2 proposes $(0,1)$ in any SPE of this type. This case requires $\alpha_1 \geq \delta$ and $\alpha_2 \geq \delta$.

Since each possible combination (α_1, α_2) corresponds to only one type of equilibrium, subgame perfect equilibrium payoffs are unique.

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