Stakeholders in Bilateral Conflict

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Abstract

The resolution of a conflict often has an impact which extends beyond the remits of the parties directly involved in the confrontation (e.g. labour negotiations in sectors of public interest, where a strike would impact on the public at large). Once this is recognised, models addressing negotiations in such situations ought to account for the role and interests of the stakeholder - a third party whose stake is linked to the original negotiations. In this paper we address the strategic role of stakeholders in bilateral confrontations that take the form of a war of attrition; we assume that the bilateral confrontation runs concurrently with the parties interaction with the stakeholder, that chooses strategically her timing to intervene and take action to promote agreement. We show that under complete information the interplay of different interests in this tripartite timing game results in delayed outcomes. We also explore the role of incomplete information and show that asymmetries of information do not necessarily translate in increased inefficiency.

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1 Introduction

Stalemate and conflict in negotiations often produce considerable and direct effects on agents other than the prime sides to the dispute. These externalities are often internalised by involving the concerned stakeholders directly, as for instance in matters of public concern, such as transport, health, the supply of utilities. Whenever industrial action looms on negotiations in any of these domains, governments have a vested interest in a speedy resolution of the dispute to minimise the adverse consequences on the public at large. Thus the remits of the original negotiations over wages, layoffs, working conditions and so on extend well beyond the concerns of the parties directly involved; the consequence is that the third party which is unwillingly pulled into negotiations has a concrete interest to put up resources and protect his own stake in the dispute.[1] Our objective in this paper is to study the equilibrium outcomes that obtain in this type of negotiations with interested third party with a model that emphasises tripartite interactions in the timing of mutual concessions.

We model the bilateral conflict as a timing game where confronted parties choose a time to concede to each other's demands; and we assume that the bilateral conflict runs concurrently with the stakeholder's timing decisions on actions that promote agreement. We begin by considering a setup where agents are perfectly informed about each other's stakes. A situation where parties are all perfectly informed might be suitable to model those circumstances where the issues under negotiation are straightforward and negotiating stances well publicised (e.g. the renegotiation of a national employment contract, where all data and accounts are publicly available). Even so, we show that protracted stalemate can occur.

Obviously there are many instances where negotiators lack an accurate assessment of the means at the stakeholder's disposal. We turn to this issue in the second part of the paper, where we show that introducing incomplete information does not necessarily make things worse. While delay cannot be ruled out altogether, the situation might improve in the sense that a 'large' degree of uncertainty might result in immediate agreement with substantial probability.

To get a flavour of our results, consider the complete information setup first. Our starting

[1] Recent examples include the Fire Service dispute in the UK in 2003-4 and the Fiat plant closures in southern Italy in 2003.

point is that the presence of a stakeholder (e.g. the government) implies that the original set of negotiations in effect runs in parallel with another set of negotiations involving the stakeholder. The crucial issue is that the resources bargained upon on the two negotiating tables are *interconnected*, in the sense that now the terms of each agreement can be conditioned on the outcome of the other one. The resolution of the 'meta'-dispute which encompasses all parties hinges upon the resolution of each stalemate: this creates the potential for a strategic interplay between the two sets of negotiations (the original one and the one with the stakeholder), which may be the cause of severe inefficiency, even in the absence of informational asymmetries.

The reason for this is simple, and easier to illustrate with an example. Consider the case of a firm threatening to close a plant during an economic downturn. The impact that job losses may have on the economy at large and on voter behaviour may be enough to warrant the involvement of the government, that could intervene with a handout to the firm. So in effect two deals need to be stricken, one between the firm and its workforce (e.g. on reduced pay versus layoffs), and one between the firm and the government (over financial support). The central feature is that in considering a concession over the terms of employment to the workforce, the firm will bear in mind the development of its negotiations with the stakeholder. Similarly, no financial support will be forthcoming from the government unless agreement is reached on employment issues. In this setting the firm is pivotal, in that its agreement is needed in both sets of negotiations for a successful resolution of the dispute. Neither the workforce nor the government can on their own guarantee a quick resolution, as neither of them can influence directly the set of negotiations in which they are not a bargaining party. To the contrary, the firm can single-handedly draw the dispute to a speedy conclusion, by conceding on *both* negotiating tables; or impose a drawn out process to *both* of the other sides. Provided the gains from obtaining its most favoured alternative are sufficiently high, the latter may be an attractive prospect for the firm.

One could contend that this sort of inefficiency could be removed if all parties were involved in negotiations over all issues, so as to have a trilateral negotiating table at the outset. Besides the fact that this may be sometimes either unfeasible or undesirable for some of the parties, multilateral negotiations generally do not solve inefficiencies[2].

[2] It is well known that strategic models of multilateral bargaining generate delayed agreement, unless restrictions are imposed on the strategies or the bargaining protocols (see e.g. Osborne and Rubinstein [16], Muthoo [15]

Alternatively, one could argue that inefficiencies may be removed if governments strengthened their reliance on legislation; in the example above, the introduction of tighter rules on consultation of the workforce and the imposition of higher levels of redundancy pay could arguably eliminate the need for the government to take an active role in specific negotiations, thus removing the source of inefficiency outlined above.[3] In fact, though, the tendency of modern governments it is towards less and less *mandatory* intervention, with the consequence that governments become progressively more active players in negotiations, rather than 'referees'[4].

On the other hand, we show that introducing incomplete information might improve the situation in the sense that a 'large' degree of uncertainty might result in immediate agreement. We introduce uncertainty on the part of bargainers who are unsure about the size of the stakeholder's stake. Note that besides being more realistic, this type of framework can account for situations where the stakeholder is genuinely *super partes*, or even antagonistic[5] to one or both of the other negotiators. Although, as one would expect, uncertainty does not remove inefficient (i.e. delayed) outcomes, we find however that if the uncertainty is sufficiently large, the two litigants reach an efficient agreement with substantial probability. Such behavior is driven by a low

and references therein). This is even more so when a stakeholder that can give out resources is present; see e.g. Manzini and Ponsatí [13] and Ponsatí [18].

[3]Reality appears to contradict this argument, as in industrial relations the rules for consultation with the trade unions in effect do not prevent massive layoffs when companies feel this is needed. A case in point is the one involving the Anglo-Dutch steel group Corus, that in February 2001 announced its intention to cut 6050 jobs in the UK. After months of negotiations, trade unions eventually failed to persuade Corus to fund retraining of employees for employment in other sectors, and in May Corus confirmed it was to go ahead with the announced layoffs. In the event the government tried and failed to induce a rethink of the massive job cuts, and finally intervened with substantial financial support, in a package comprising lump sum payments to laid off workers and their retraining .

[4]For example, in 2002 in the UK the then Secretary of State for the Department of Trade and Industry Byers declared "Government needs to be active but must not be interventionist. In doing so I reject the approaches of the past. That of the new right who took the line that the market should be the 'be all and end all' and that government should simply keep out. And that of the old left who believed in large scale intervention, coupled with massive state subsidies which either sought to back winners or to rescue failing companies." See Insitute of Public Policy Research [8].

[5]This can be the case for instance when the government (the *indirect* employer) oversees negotiations between a union and a local government (the *direct* employer) which is controlled by the opposition party.

expectation on the stakeholder's concern; expecting that intervention in case of stalemate is an unlikely event, it is worthwhile for the bargainers to get to an agreement quickly. Consequently, it is optimal for the stakeholder to dither, thereby pressurising the two bargainers into reaching a speedy conclusion.

At the formal level, our modelling strategy starts from the observation that negotiations typically do not take place in a vacuum: they may come at the end of a previous contract, in which case there may be some natural default option (e.g. the previous agreement), and the negotiating parties may avail themselves of external independent advice[6]. In short, there will be preferred settlements for each side which, if incompatible, trigger negotiations. A war of attrition framework seems best suited to model this class of situations.

Finally, note that at the conceptual level our approach differs markedly from that followed for instance in a series of papers by Jehiel and Moldovanu (e.g. [9] and [10]) for the analysis of externalities in bargaining: In our approach the pre-existing externality produced on the stakeholder by the original negotiations is internalised by the direct involvement of the stakeholder, who is drawn into negotiations together with his stake.

2 Complete information

The government is engaged in two sets of negotiations over financial support, one with the employees (e.g. voluntary redundancies and/or re-training), and one with the firms (e.g. debt refinancing). All agents discount utility at the istantaneous rate r_i (with $i = f, u, g$ for firm, worker and government respectively), so that an agreement reached at time t over a payment x to agent i yields to this agent a utility of $xe^{-r_i t}$ in present discounted value. To capture the feature that the two bargaining problems belong to the larger 'meta' dispute, agents obtain their payoff only when both sets of negotiations are over[7].

[6] For instance in the UK Pay Review Bodies make recommendations to the government about the 'appropriate' level of pay for various public sector employees. In Canada conciliators and mediators can be appointed by the government to help resolve management-union disputes. See Gunderson, Hyatt and Ponak [6].

[7] This model is distinct from the standard war of attrition in that there are two sets of *interrelated* negotiations running alongside. This also sets my setup apart from generalised war of attrition models (see Bulow and Klemperer [2] or more recently La Casse, Ponsatí and Barham [12]), where $m \geq 2$ agents compete for $n < m$

The overall government stake in negotiations is S, part of which can be used to settle the union's side and part of which the firm's side of the dispute.

In negotiations between the firm and the union, let w_u be the worker's preferred settlement, and $w_g < w_u$ the wage preferred by the government. If agent $i = u, g$ concedes at time t_i, then this implies agreement on wage $w_{j \neq i}$. If both agents concede at the same time, then each of the two agreements is implemented with equal probability. The time when an agreement is enjoyed depends on whether or not the parallel set of negotiations (between the firm and the government) is over or not. If it is, then the wage settlement is implemented immediately upon agreement. If not, then the wage settlement is implemented as soon as agreement is reached in the firm-government negotiations. If we denote by $\tau = \min\{\tau_g, \tau_f\}$ the time at which an agreement is reached in firm-government bargaining, the union's payoff in wage negotiations, denoted by $v_u(t_g, t_u; \tau)$, can be written as:

$$v_u(t_g, t_u; \tau) = \begin{cases} w_i e^{-r_u t_j} & \text{if} \quad \tau \leq t_j < t_i \text{ for all } i, j = g, u \text{ and } i \neq j \\ \frac{w_u + w_g}{2} e^{-r_u \max\{t, \tau\}} & \text{if} \quad t_g = t_u = t \\ w_i e^{-r_u \tau} & \text{if} \quad t_j \leq \min\{\tau, t_i\} \text{ for all } i, j = g, u \text{ and } i \neq j \end{cases}$$

where the top (bottom) row refers to payoffs when wage negotiations terminate with a single concession *after* (resp., before) the time τ of agreement in firm-government negotiations, and the middle row considers those cases where the union and the government concede at the same time (either before or after the other set of negotiations has terminated, depending on whether $t \gtreqless \tau$).

The concurrent set of negotiations between the firm and the government develops in similar fashion. Let h_g be the handout to the firm favoured by the government, and $h_f > h_g$ the outcome preferred by the firm. If agent $i = g, f$ concedes at time τ_i, this means that an agreement is struck on government payment $h_{j \neq i}$. If both agents concede at the same time, then each of the two agreements is implemented with probability of $\frac{1}{2}$. As above, if there is already agreement in the government-union wage negotiations, then the settlement over the government's payout to the firm is implemented immediately. Alternatively, it is implemented as soon as an agreement

prizes. In that case multiple agents compete in the *same* war of attrition, and the game terminates once $n - m$ agents have conceded. To the contrary, this paper is concerned with multiple wars of attrition.

is reached over wages. Let $t = \min\{t_u, t_g\}$ denote the time at which an agreement is reached in the parallel set of negotiations between the government and the union. The payoff for the firm, denoted by $v_f(\tau_f, \tau_g; t)$, is:

$$v_f(\tau_f, \tau_g; t) = \begin{cases} h_p e^{-r_u \tau_q} & \text{if} \quad t \leq \tau_q < \tau_p \text{ for all } p, q = g, f \text{ and } p \neq q \\ \frac{h_g + h_f}{2} e^{-r_f \max\{t, \tau\}} & \text{if} \quad \tau_g = \tau_f = \tau \\ h_q e^{-r_f t} & \text{if} \quad \tau_p \leq \min\{t, \tau_q\} \text{ for all } p, q = g, f \text{ and } p \neq q \end{cases}$$

The government's payoff $v_g(t_g, t_u, \tau_f, \tau_g)$ is

$$v_g(t_g, t_u, \tau_f, \tau_g) = \begin{cases} (S - h_p - w_j) e^{-r_g \max\{t_i, \tau_q\}} & \text{if} \quad \tau_q < \tau_p \text{ and } t_i < t_j \\ \left(S - h_p - \frac{w_g + w_u}{2}\right) e^{-r_g \max\{t, \tau_q\}} & \text{if} \quad \tau_q < \tau_p \text{ and } t_g = t_u = t \\ \left(S - \frac{h_g + h_f}{2} - w_j\right) e^{-r_g \max\{t_i, \tau\}} & \text{if} \quad \tau_g = \tau_f = \tau \text{ and } t_i < t_j \\ \left(S - \frac{h_g + h_f + w_g + w_u}{2}\right) e^{-r_g \max\{t, \tau\}} & \text{if} \quad \tau_q = \tau_p = \tau \text{ and } t_g = t_u = t \end{cases}$$

$$\text{for all} \quad \begin{matrix} i, j = g, u \text{ with } i \neq j \\ p, q = g, f \text{ with } p \neq q \end{matrix}$$

As a preliminary it is useful to define τ_f^* and t_u^* as the times such that in negotiations with the government the firm and the union, respectively, are indifferent between *(i)* holding out and obtaining the preferred proposal with delay, and *(ii)* conceding immediately to the government's proposal. Then τ_f^* solves $h_f e^{-r_f \tau} = h_g$ and t_u^* solves $w_u e^{-r_u t} = w_g$, yielding

$$\tau_f^* = \frac{1}{r_f} \ln\left(\frac{h_f}{h_g}\right) > 0$$
$$t_u^* = \frac{1}{r_u} \ln\left(\frac{w_u}{w_g}\right) > 0$$

Similarly, define for the government the threshold stopping time $t_g^* = \frac{1}{r_g} \ln\left(\frac{S - w_g - h_g}{S - w_u - h_f}\right)$ which makes the government indifferent between conceding immediately and holding out in *both* sets of negotiations, that is $(S - w_g - h_g) e^{-r_g t_g^*} = (S - w_u - h_f)$.

Note that at any time *before* these threshold times τ_f^*, t_g^* and t_u^*, the agent prefers to hold on rather than concede, since the payoff if conceding is less than the payoff if the opponent concedes at the threshold, that is

$$h_f e^{-r_f \tau} \geq h_g \Leftrightarrow \tau \leq \tau_f^*; \; w_u e^{-r_w t} \geq w_g \Leftrightarrow t \leq t_u^*; \text{ and } (S - w_g - h_g) e^{-r_g t} \geq (S - w_u - h_f) \Leftrightarrow t \leq t_g^*$$

A pure strategy for either the union or the firm is simply a concession time, i.e. $t_u, \tau_f \in \mathcal{R}_+$; a pure strategy for the government is instead a pair of concession times, one for each war of attrition in which it is involved, i.e. $(t_g, \tau_g) \in \mathcal{R}_+^2$. Consequently a pure strategy profile is a vector $s = (t_u, \tau_f, (t_g, \tau_g))$ of concession times.

2.1 Delayed equilibria

It is immediate to verify that, as in the standard war of attrition, here as well there are subgame perfect equilibria (henceforth s.p.e.) with immediate agreement. In all these equilibria one agent backs down immediately, giving in to the (credible) threat that his opponent will hold on for long enough (i.e. beyond the threshold t_i^* of the conceding agent i). It is however more interesting to consider whether in this simple setup delayed equilibria can arise in pure strategies. This can occur only if both sets of negotiations are delayed; furthermore, as the next lemma establishes, in any delayed equilibrium both negotiations must end at the *same* time:

Lemma 1 *In any delayed s.p.e. it must be* $\tau = t$.

Proof. Suppose not, and assume $\tau < t$. Then the conceding player in wage negotiations has a profitable deviation, since by anticipating his concession to a time $t' = \tau$ he would enjoy the same agreement earlier. Similarly for the case $t < \tau$. ■

Lemma 1 highlights the distinctive feature of any delayed equilibrium, which is that the government uses strategically its participates in both negotiations, exploiting the possibility to impose single-handedly delays to both the other negotiations. As long as this threat is credible, the other side of the negotiations will concede. Consequently, *the firm never concedes in both negotiations*. This is stated more formally in the following proposition, which establishes that in equilibrium either the government holds out in handout negotiations with the firm while conceding in wage negotiations, or in wage negotiations while conceding to the firm, or in both:

Proposition 1 *Let* $w_u > w_g \geq 0$ *and* $0 \leq h_g < h_f$. *Define* $\bar{t}_L = \frac{1}{r_g} \ln\left(\frac{S-w_u-h_g}{S-w_u-h_f}\right)$, $\bar{t}_R = \frac{1}{r_g} \ln\left(\frac{S-w_g-h_f}{S-w_u-h_f}\right)$ *and* $\bar{t}_I = \frac{1}{r_g} \ln\left(\frac{S-w_g-h_g}{S-w_u-h_f}\right)$. *Then any delayed pure strategy s.p.e. will be of one of the following types:*

1. Concession over Wage (**CW**): *for any time* $t \in (0, \bar{t}_L)$ *there exists a pure strategy s.p.e. where the government concedes in wage negotiations but holds out in handout negotiations, and where agreement is reached at time* t*.*

2. Concession over Handout (**CH**): *for any time* $t \in (0, \bar{t}_R)$ *there exists a pure strategy s.p.e. where the government concedes in wage negotiations but holds out in handout negotiations, and where agreement is reached at time* t*.*

3. No Concessions (**NC**): *for any time* $t \in (0, \bar{t}_I)$ *there exists a pure strategy s.p.e. where the government holds out in both wage and handout negotiations, and where agreement is reached at time* t*.*

Proof. See Appendix.

The supporting strategies are specified in the appendix: the common feature for all equilibria is that the firm and the union are at a disadvantage in negotiations with the government, which always prevails in at least one of the two negotiations. No unilateral deviation by either the firm or the union can impact on the overall outcome of negotiations: an *earlier* concession than as specified by the strategies supporting proposition 1 would not anticipate the overall end of negotiations. On the other hand, a deviation to holding on even further would not be profitable: although this may mean obtaining the most preferred settlement, this would come at a time which is late enough to make this proposition unappealing. To the contrary, unilateral deviations by the government can indeed change the implementation date for all agreements, and it is this which gives the firm a stronger bargaining position.

Note that the maximum delay can be quite lengthy, as it depends on the level of conflict, that is on the difference between w_g and w_u, and between h_g and h_f; furthermore, the more the union and the firm's claims eat into the government's stake (so that $S - w_u - h_f$, the denominator for $\bar{t}_L$, $\bar{t}_R$ and $\bar{t}_I$), and the more patient the government is (i.e. the lower r_g), the longer the potential delay for agreement. Intuitively, the more the government has to lose by conceding, the more it pays to hold out and force the preferred settlement.

2.2 Incompatible demands?

So far we have assumed that negotiators start at the outset with incompatible demands. Although this is not an unreasonable assumption, it is worth investigating whether these demands can be endogenised in some 'pre-war of attrition' stage, and what the impact is on the behaviour in the war of attrition, which we do here.

Each of the two negotiations (the one over wages and the one over the government handout) is thus composed of two phases. In the *bargaining phase* each of the two sides in the negotiations tables its proposal. If demands are compatible, then an agreement is reached. Alternatively, negotiations enter the *war of attrition phase* (as described in section 2). As in the base model, agents obtain their payoff only when both sets of negotiations are over. The bargaining phase is structured as a simple Nash demand game, where both sides involved table their claims/offers independently. Proposed wage settlements w and government contributions h are allowed to vary within some bounded interval, i.e. $w \in [\underline{w}, \overline{w}]$ and $h \in \left[\underline{h}, \overline{h}\right]$, with $\underline{w} \leq \overline{w}$ and $\underline{h} \leq \overline{h}$. For instance, there may be either budgetary or legal constraints to how much the stakeholder can contribute to support the firm, or there may be a minimum wage in operation.

In wage negotiations, w_g and w_u now denote the wage settlements put forward by the government and the worker, respectively. If $w_g \geq w_u$, then these wages are compatible, and this set of negotiations ends in agreement, with the worker accepting w_g. If instead $w_g < w_u$, then negotiations enter the war of attrition stage described in section 2.

Negotiations between the firm and the government develop in similar fashion. Now h_g is the handout to the firm proposed by the government, and h_f that claimed by the firm. If $h_g \geq h_f$, negotiations end with the firm obtaining h_g. If instead $h_g < h_f$, negotiations enter the war of attrition stage.

In this setup, the bargaining phase is in effect the first stage of a negotiation game, with the war of attrition as second and final stage. Obviously there are subgame perfect equilibria in which initial demands are compatible, so that agreement is reached immediately. In these equilibria any four-tuple of compatible demands can be supported in equilibrium with immediate agreement. A more interesting issue is to examine whether the inefficient (delayed) equilibria of the base model are wiped out by the introduction of the first stage, in which demands are chosen strategically. As we show below, the strategic incentives to delay agreement are still paramount, and delayed equilibria still occur in pure strategies. As a preliminary note that a strategy for a player now has to specify an initial claim and a concession time, so that a strategy profile is now a vector $\widetilde{s} = ((w_u, t_u), (h_f, \tau_f), (w_g, h_g, t_g, \tau_g))$. As the next proposition establishes, even when demands are endogenised the same pattern of delayed agreements can be supported in an equilibrium. There are three classes of delayed equilibria, one where the government is sympathetic to the union, and eventually concedes to its claim while resisting requests from the firm ('leftwing government equilibrium'); one where the governments holds firm with the union but concedes the maximum payoff to the employer ('rightwing government equilibrium'); one in which the government does not concede to any party ('intransigent government equilibrium'):

Proposition 2 *Let* $w \in [\underline{w}, \overline{w}]$, $h \in [\underline{h}, \overline{h}]$, *and define* $\overline{\tau}_R = \frac{1}{r_g} \ln\left(\frac{S-\underline{w}-\overline{h}}{S-w-\overline{h}}\right)$, $\overline{\tau}_L = \frac{1}{r_g} \ln\left(\frac{S-\overline{w}-\underline{h}}{S-\overline{w}-h}\right)$, $\overline{\tau}_I = \frac{1}{r_g} \ln\left(\frac{S-\underline{w}-\underline{h}}{S-w-h}\right)$. *Then:*

1. *'**Leftwing government' equilibrium:** For any time* $\tau \in (0, \overline{\tau}_L)$ *there exists a pure strategy s.p.e. where the government concedes in wage negotiations but holds out in handout negotiations, and where agreement is reached at time* τ.

2. ***'Rightwing government' equilibrium**: For any time* $\tau \in (0, \overline{\tau}_R)$ *there exists a pure strategy s.p.e. where the government concedes in wage negotiations but holds out in handout negotiations, and where agreement is reached at time* τ.

3. ***'Intransigent government' equilibrium:** For any time* $\tau \in (0, \overline{\tau}_I)$ *there exists a pure strategy s.p.e. where the government holds out in both wage and handout negotiations, and where agreement is reached at time* τ.

Proof. See appendix.

Thus delays can still occur even when agents can formulate their initial demands: it pays to risk delay by setting as high a claim as possible if the claim is eventually successful. To see this, recall from the discussion of proposition 1 that in both sets of negotiations the firm and the union are somewhat at a disadvantage, in that any unilateral deviation is not sufficient to *anticipate* agreement. A deviation may be sufficient to *postpone* agreement, but this is never profitable. To the contrary, because it is involved in both sets of negotiations, the government is the only player that could single-handedly impose an immediate agreement. This, however, would mean accepting the opponents' proposals, and as long as the demand put forward is sufficiently attractive, the government has an incentive to haggle. The crucial feature of the delayed equilibria of the game is that in each set of negotiations both agents start out with incompatible demands, but one of them will prevail. Consequently, it is optimal for such 'strong' negotiator to set his initial demand as high as possible.

3 Incomplete information

The results derived in the previous section apply to a situation in which all parties are perfectly informed. In this section we model the situation where the government is uncertain about the need for public intervention while the union and the firm ignore the value of the government's stake.

For simplicity, we assume that the firm-union pair face no uncertainty about their bilateral surplus, and we treat their joint decisions as those of a single player to which we refer as 'the bargainers'. Consequently the interaction among the three agents is modelled as a game of incomplete information between two players only, the bargainers and the government. Formally, the bilateral surplus between the bargainers is $1 - c_b$ and the government stake is $G - c_g$, where c_b and c_g are random variables uniformly distributed respectively in $[0, C_b]$, $C_b > 1$ and $[0, C_g]$, $C_g > G$.[8] The government privately observes the realization of c_g while the realization of c_b is privately observed by the bargainers.

The game between the bargainers and the government is again a timing game: each side can terminate the game at any $t \in [0, \infty)$. Bargainers terminate by reaching a bilateral agreement without the government intervention. Upon such termination at date t, the bargainers split $1 - c_b$ - each obtains $\frac{1-c_b}{2} e^{-t}$ - and the government receives $(G - c_g) e^{-t}$.[9] Alternatively the bargainers can delay, that is, they may remain in disagreement hoping that the government will intervene. On its side, the government may decide to intervene at any t, or simply delay and

[8] Uniform distributions are assumed for simplicity.

[9] Identical discount factors are assumed for simplicity.

wait for a bilateral agreement. The government decision to intervene at t is a terminal move as well because, if the trilateral negotiation ensues, all private information is revealed and the three parties reach an immediate agreement dividing the total surplus equally so that each agent receives $\frac{G+1-c_b-c_g}{3}e^{-t}$.

A *strategy* of player i, $i = b, g$ is a function t^i selecting, for each $c_i \in [0, C_i]$, the date $t^i(c_i)$ at which player i of type c_i terminates the game. Let $V^i(c_i, t, t^j)$ denote the expected gains from termination at t for type c_i, given the opponent's strategy t^j. The focus of our attention will be pairs of strategies that constitute a *(Bayesian) Equilibrium*; that is (t^g, t^b) such that

$$t^g(c_g) = \arg\max V^g(c_g, t, t^b)$$
$$t^b(c_b) = \arg\max V^b(c_b, t, t^g)$$

for all $c_i \in [0, C_i]$.[10] We limit our attention to equilibria in strategies that are differentiable in t a.e.[11]

So what happens along the equilibrium path? Three observations are in order.

1. Equilibrium strategies must be type-monotone[12], i.e. types of agent with larger surplus/stake available (i.e. 'lower' types) give in sooner than those with smaller surplus available (i.e. 'higher' types), that is: $t^i(c_i) \leq t^i(c_i')$ for all $c_i < c_i'$, where the inequality is strict unless $t^i(c_i) = 0$. The intuition for this result is pretty straightforward: the larger the surplus, the larger the opportunity cost in case of disagreement, so that a concession looks more attractive than haggling. Type-monotonicity implies our next observation.

2. An equilibrium strategy profile $\left(t^g, t^b\right)$ is fully characterized by strictly increasing functions $(g(.), b(.))$ - the inverses of $\left(t^s, t^b\right)$ - mapping dates into types such that $(g(t), b(t)) = (c_g, c_b)$ if and only if $t^g(c_g) = t$ and $t^b(c_b) = t$. An equilibrium is thus characterized via the ordinary differential equation system obtained from the first order conditions of the players' optimization problems with the appropriate boundary condition.

[10] Observe that, in the present informational environment, Perfect Bayesian Equilibrium profiles do not refine the set of equilibrium outcomes relative to Bayesian Equilibria.

[11] This is a mild assumption since it is not hard to show that Lipschitz continuity of the strategies (and thus differentiability almost everywhere) is a necessary condition for BE. See Ponsatí and Sákovics [19] for a proof that differentiability is necessary for BE in a very related model.

[12] See Appendix for a Proof.

3. Letting $P^i(t)$ denote the probability that player i concedes at a date $\tau \leq t$, then the boundary condition follows from the requirement that

$$P^g(0)\,P^b(0) = 0.$$

(If $P^g(0) > 0$ and $P^b(0) > 0$ any type conceding at $t = 0$ would benefit from a deviation in which she waits to see if the opponent does yield first).

Our next result assures the existence and uniqueness of an equilibrium consistent with observations 1 to 3.

Proposition 3 *The following strategy profile (t^s, t^b) constitutes the unique equilibrium:*

$$t^g(c_g) = \begin{cases} 0 \text{ iff } c_g < g(0), \\ t \text{ iff } c_g = g(t), \\ \infty \text{ iff } c_g \geq G; \end{cases} \quad t^b(c_b) = \begin{cases} 0 \text{ iff } c_b < b(0), \\ t \text{ iff } c_b = b(t), \\ \infty \text{ iff } c_b \geq 1; \end{cases}$$

where $g(.)$ and $b(.)$ are the unique strictly monotone solutions to the system of differential equations

$$g' = \tfrac{3(C_g - g(t))(1-b(t))}{2(G-g(t))-(1-b(t))}, \quad b' = \tfrac{(C_b - b(t))(1+G-g(t)-b(t))}{5(G-g(t))-(1+C_b-2b(t))}, \tag{1}$$

such that

$$b(0) \times g(0) = 0. \tag{2}$$

The detailed proof of Proposition 3 is in the Appendix. Let us now discuss its implications.

System (1) is obtained from the maximisation of the expected gains $V^i(c_i, t, t^i)$ of each agent i of type c_i if terminating (i.e. conceding) at time t. Since equilibrium strategies must satisfy type monotonicity, the expressions on the left hand side of the equations in the autonomous system (1) must be positive. System (1) only applies to $c_g \leq G$ and $c_b \leq 1$ because types that cannot generate surplus never concede, i.e.

$$t^g(c_g) = \infty \text{ for all } c_g > G \text{ and } t^b(c_b) = \infty \text{ for all } c_b > 1. \tag{3}$$

Hence the numerators in the expressions at the right hand side of system (1) are positive[13], and therefore so must be the denominators, that is $2(G - g(t)) - (1 - b(t)) \geq 0$ and $5(G - g(t)) -$

[13] Recall that $C_g > G$ and $C_b > 1$ by assumption.

$(1 + C_b - 2b(t)) \geq 0$. These two inequalities can be rearranged as

$$g(t) \leq \min\left\{G - \frac{1 + C_b}{5} + \frac{2b(t)}{5}, G - \frac{1 - b(t)}{2}\right\} \tag{4}$$

The loci of points satisfying $2(G - g) - (1 - b) = 0$ and $5(G - g) - (1 + C_b - 2b) = 0$ can be represented in a (g, b) space where, if measuring g on the horizontal axis, the relevant solution to system (1) must therefore lie *above* both loci. Depending on the relative magnitudes of C_b (i.e. the government's uncertainty about the bargaining surplus) and G (i.e. the government's potential concern), one constraint or the other, or both, will be binding.

These constraints combined with the constraint on initial values $g(0) b(0) = 0$ yield either *i)* a solution with $b(0) > 0$, where the bargainers have a positive mass of types $c_b < b(0)$ that agree at the outset (i.e. concede to the government); or *ii)* a solution with $g(0) > 0$, where the government has a positive mass of types $c_g < g(0)$ that intervene at $t = 0$.

Let us now illustrate two simple scenarios where the solution is of type *i)*, so that there is a positive probability of bilateral agreement at the outset and, if the government intervenes, it does so only after some delay:

1. Let $G < \frac{1+C_b}{5}$. Then $g(t) \leq G - \frac{1+C_b}{5} + \frac{2b(t)}{5}$ implies that $g(t) < \frac{2b(t)}{5}$. In turn this implies that $b(t) > g(t)$ for all t, which together with the initial condition $g(0) b(0) = 0$ yields $b(0) > g(0) = 0$.

2. Let $G < \frac{1}{2}$. Now $g(t) \leq G - \frac{1-b(t)}{2}$ implies that $g(t) < \frac{b(t)}{2}$ and again we obtain that $b(t) > g(t)$ for all t.

We provide a graphical illustration of these two scenarios in the two figures below, which display the direction field corresponding to our autonomous system, where the arrows are tangent to the solution curves with slope $\frac{db}{dg}$. The flatter and the steeper, darker line, respectively, represent the loci of points $2(G - g) - (1 - b) = 0$ and $5(G - g) - (1 + C_b - 2b) = 0$ (corresponding to $g' \geq 0$ and $b' \geq 0$, respectively). Note that the smaller G is, the higher the two lines.

Figure 1 is an example illustrating case 1, where $g(0) = 0$ (so that no type of government ever intervenes at the outset of negotiations) and $b(0) > 0$. A set of parameter values which satisfies the conditions for case 1 is $C_b = C_g = 2$ and $G = \frac{1+C_b}{5} = \frac{3}{5}$. Since in this case $C_b > \frac{3}{2}$, it is easy to verify[14] that in condition 4 the relevant constraint is $g(t) \leq G - \frac{1+C_b}{5} + \frac{2b(t)}{5}$, so that the solution must lie above the highest of the two loci in figure 1, so that our solution must lie to the north-west of the highest curve.

Figure 2 depicts one example of case 2, where our choice of parameter values is $C_b = \frac{5}{4} < \frac{3}{2}$, $C_g = 2$ and $G = \frac{1}{2}$. Now since $C_b \leq \frac{3}{2}$ either constraint may hold, depending on how large $b(t)$ is relative to $g(t)$. Now the solution lies to the north west of the upper envelope of the two curves.

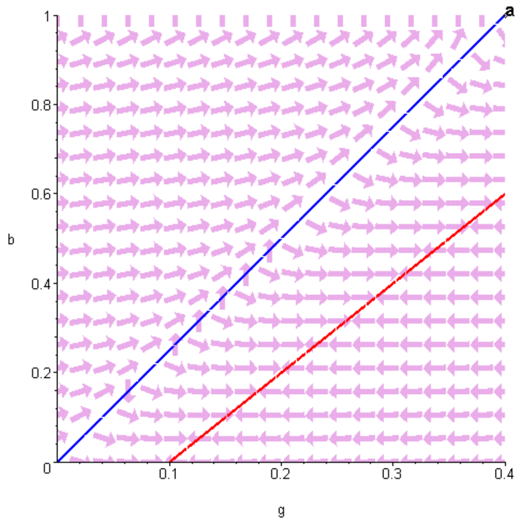

Figure 1: An example with $C_b = 2 > \frac{3}{2}$, $C_g = 2$ and $G = \frac{1+C_b}{5} = \frac{3}{5}$.

Having settled the equilibrium behavior at $t = 0$, we remark[15] that as $t \to \infty$, the unique strictly monotone solution to (1) compatible with $g(0)\, b(0) = 0$ satisfies

$$\lim_{t\to\infty} b(t) = 1, \ \lim_{t\to\infty} g(t) = a \equiv G - \frac{C_b - 1}{5}. \tag{5}$$

That is, over time, all bargainers' types that can concede, i.e. with $c_b \leq 1$, eventually do so. For the government, only types $c_g \leq G - \frac{C_b-1}{5} < G$ concede at some $t < \infty$. So a government might be totally committed not to intervene even if it has positive resources to contribute. This leads

[14] The value of the expression $\min\left\{G - \frac{1+C_b}{5} + \frac{2b(t)}{5}, G - \frac{1-b(t)}{2}\right\}$ is given by the first term in brackets if $C_b > \frac{3}{2} - \frac{1}{2}b$. When $b = 0$ this reduces to $C_b > \frac{3}{2}$.

[15] This claim is established in the proof of Proposition 3.

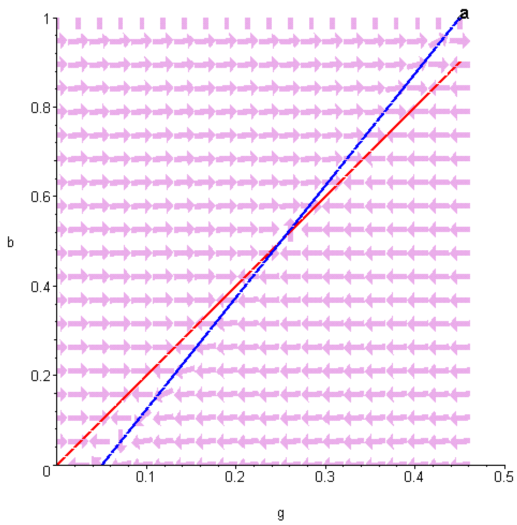

Figure 2: An example with $C_b = \frac{5}{4} < \frac{3}{2}$, $C_g = 2$ and $G = 1/2$.

naturally to ponder about the probability that the government will intervene in negotiations and contribute part of its stake at any one time t. This is easily measured as the government's type conditional on having positive resources to contribute, that is $\frac{g(t)}{G}$. In the limit as t increases this converges to $\frac{a}{G} = \frac{1}{5}\frac{5G+1-C_b}{G}$, which is bounded away from unity.

These considerations are summarised below:

Proposition 4 *The following hold in the unique equilibrium:*

1. *If the maximum stake of the government G is not too large, the bargainers reach a bilateral agreement at $t = 0$ with (strictly) positive probability.*

2. *In the preceding scenarios, if the government eventually enters a multilateral negotiation it does so with delay.*

3. *The probability that a government with positive stake, $G - c_g > 0$, enters a multilateral negotiation decreases with the range of its uncertainty C_b, and is never greater than $\frac{a}{G} = \frac{1}{5}\frac{5G+1-C_b}{G}$.*

Thus, in the presence of asymmetric information, the unique equilibrium may produce immediate agreement among the bargainers. It is precisely the bargainers uncertainty about the government's concern that supports immediate agreements - since an unconcerned government will surely let the bargainers to "fight it out" before intervening. In fact, the larger the (maximum) bargainers bilateral surplus is, the less likely intervention is. Provided the government stake is not too high (corresponding to the necessary conditions on G outlined above), there is a positive probability that bargainers will settle their dispute immediately, without government intervention; but the government is still pulled into negotiations with a positive probability that depends on how large the uncertainty on the bargaining surplus is (i.e. how large C_b is). At the other extreme, a very uncertain government with a great potential stake will intervene right away with substantial probability.

4 Discussion

The upshot of this paper is that the presence of stakeholders in the shadow of negotiations changes the nature of the negotiations, and inefficiencies are rife: although the model does allow for efficient equilibria - where agreement is immediate - delayed equilibria are a pervasive phenomenon. One prediction of the model is thus that industrial relations in sectors of public interest should be characterised by a higher degree of conflict: as discussed in the introduction, in this case the possibility of strikes has an impact on the public at large, so that the government has a stake in the firm-workforce negotiations. Consequently, one should really look at the 'meta-negotiations' which involve the firm and its workforce on one side, and the government and one of the other agents on the other, depending on the exact nature of the dispute. Indeed, there is evidence in support of this. For instance, in Canada the share of total strike days lost in the public sector relative to the private sector has increased[16] since the mid 1970s.

As discussed above, the rationale for this result is the fact that two sets of negotiations are interdependent implies that an agreement can be implemented only once all issues have been settled; this may introduce an incentive for the pivotal agent (i.e. that involved in both negotiations) to delay agreement in order to win the maximum concessions from one's opponents. This feature also makes it possible for inefficiencies to persist even once the initial demands are endogenised: although there exist equilibria with immediate agreement, once more the linkage

[16] See Gunderson, Hyatt and Ponak [6].

between the two sets of negotiations creates an incentive for delay. Interestingly, this would not be possible if the first stage of negotiations was followed by a standard war of attrition game, where delays are generally obtained only under incomplete information[17].

Turning next to the case of incomplete information, we showed that in the presence of uncertainty the fact that the stakeholder may delay his involvement supports equilibria were bargainers agree immediately. Furthermore, our incomplete information model also provides a rationale for the introduction of advisory bodies which make salary recommendations to the government as an employer[18]. In situations were central government is not a *direct* employer, in practice the effect of such bodies is to remove a potentially sympathetic[19] management (the *direct* employer) from salary negotiations altogether. Moreover, because such bodies have a purely advisory role, as a result the government has total freedom in its decision whether or not to accept the recommendation. In terms of our framework, bargainers' failure to reach an immediate agreement is equivalent to a call for a pay review; the government's acceptance of a recommendation (which"fixes" the random variable c_g) corresponds to our stakeholder "entering" negotiations[20].

Thus it would appear that it is the existence of a wider interest which generates the possibility for one party to take a pivotal role and act strategically to its maximum advantage.

[17] See Abreu and Gul [1] and Kambe [11], who study the equilibria of bargaining games under incomplete information when players can be "stubborn", i.e. insist on receiving a particular share.

[18] For instance, in the UK Pay Review Bodies are committees of experts appointed by the Prime Minister which make mainly salary recommendations for various categories of workers in the public sector (see e.g Fredman and Morris[5] and Flynn [4]). In Italy a Commission of nine experts is appointed by the President of the Republic, following indications from the two legislative Chambers (see Sciarra [20]).

[19] A case in point is the UK Fire Service industrial dispute in 1980. A pay increase of 18.8% was reached when the Labour party gained control of the employer's side of the National Joint Council under a Conservative government (see Morris [14]).

[20] In actual fact British Governments have hardly rejected any recommendation from a Pay Review Body. However, it is pretty common to either implement only a subset of the recommendations (which may cover a number of other issues additional to salaries) or just delay taking any decision. This delaying tactics is in effect tantamount to rejecting the reccommendations, although it carries less political stigma than a straight rejection - a feature which our model allows for. See Elgar and Simpson [3].

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5 Appendix

Proof of proposition 1

*1. Concession over Wage (**CW**):* The supporting equilibrium strategy profile $s = (t_u, \tau_f, (t_g, \tau_g))$ is $\left(t_u > t_g^* + t, \tau_f = t, \left(t_g = t, \tau_g > \tau_f^* + \tau\right)\right)$, with corresponding equilibrium payoffs $v_u(s) = w_u e^{-r_u t}$, $v_f(s) = h_g e^{-r_f t}$ and $v_g(s) = (S - w_u - h_g)\, e^{-r_g t}$. To see that this is an equilibrium, consider the union first. Conceding at any later time is payoff irrelevant, as agreement will still be struck at t. It cannot be profitable to concede any earlier either, as this would mean receiving a lower wage at the same time t, since in order to enjoy its payoff the union will still have to wait until agreement is reached in the other set of negotiations at time t. Consider now the firm. By conceding earlier it would obtain the same handout at the same time (since the other set of negotiations is still terminating at t). On the other hand, delaying a concession can only be payoff relevant if the delay exceeds the concession time for the government. Since however $\tau_g > \tau_f^*$ by the definition of τ_f^* it is suboptimal for the firm to concede at such a later date. Finally consider the government. A deviation to conceding at a later time t' in wage negotiations can have an impact on payoffs only if $t' > t_u > t_g^* + t$, in which case the resulting deviation payoff is

$$(S - w_g - h_g) e^{-r_g t'} > (S - w_u - h_g) e^{-r_g t} \Leftrightarrow \frac{1}{r_g} \ln \frac{(S - w_g - h_g)}{(S - w_u - h_g)} > t' - t > t_g^* \tag{6}$$

This generates a contradiction, as

$$t_g^* = \frac{1}{r_g} \ln \left(\frac{S - w_g - h_g}{S - w_u - h_f} \right) > \frac{1}{r_g} \ln \left(\frac{S - w_g - h_g}{S - w_u - h_g} \right) \Leftrightarrow h_f > h_g$$

A deviation to some earlier time $t' < t$ in both negotiations[21] would also be non profitable, as the government's payoff after this deviation would be $(S - w_u - h_f) e^{-r_g t'}$, which is smaller than the equilibrium payoff $(S - w_u - h_g) e^{-r_g t}$ since

$$(S - w_u - h_f) e^{-r_g t'} < (S - w_u - h_g) e^{-r_g t} \Leftrightarrow t - t' < \frac{1}{r_g} \ln \left(\frac{S - w_u - h_g}{S - w_u - h_f} \right)$$

where the last inequality holds true since $t < \bar{t}_L = \frac{1}{r_g} \ln \left(\frac{S - w_u - h_g}{S - w_u - h_f} \right)$.

*2. Concession over Handout (**CH**):* The supporting equilibrium strategy profile $s = (t_u, \tau_f, (t_g, \tau_g))$ is $\left(t_u = t, \tau_f > t_g^* + t, (t_g > t_u^* + t, \tau_g = t)\right)$, with corresponding equilibrium payoffs $v_u(s) = w_g e^{-r_u t}$, $v_f(s) = h_f e^{-r_f t}$ and $v_g(s) = (S - w_g - h_f) e^{-r_g t}$. The union has no profitable deviation: by conceding earlier than t it would obtain the same wage at the same time (since the other set of negotiations is still terminating at t), while delaying a concession can only be payoff relevant if the delay exceeds the concession time for the government. Since however $t_g > t_u^*$, by the definition of t_u^* it is suboptimal for the firm to concede at such later date. Nor does the

[21] Recall that if the government were to give in earlier in wage negotiations over wages only, the overall time of agreement would be left unchanged, but a higher wage would be settled.

firm has a profitable deviation: conceding at any later time would lower its discounted payoff, while conceding any earlier would mean receiving a lower handout at the same time t. As in the previous equilibrium, the government has no profitable deviation either. Giving in at a later time τ' in negotiations with the firm is payoff relevant only if $\tau' > \tau_f > t_g^* + t$, in which case the resulting deviation payoff is

$$(S - w_g - h_g)\, e^{-r_g \tau'} > (S - w_u - h_g)\, e^{-r_g t} \Leftrightarrow \frac{1}{r_g} \ln \frac{(S - w_g - h_g)}{(S - w_u - h_g)} > \tau' - t > t_g^*$$

This is the same as (6) with τ' in place of t', thus generating a contradiction as before. Finally a deviation to some earlier time $\tau' < t$ in both negotiations[22] would also be non profitable, as the government's payoff after this deviation would be $(S - w_u - h_f)\, e^{-r_g \tau'}$, which is smaller than the equilibrium payoff $(S - w_g - h_f)\, e^{-r_g t}$ since

$$(S - w_u - h_f)\, e^{-r_g \tau'} < (S - w_g - h_f)\, e^{-r_g t} \Leftrightarrow t - \tau' < \frac{1}{r_g} \ln \left(\frac{S - w_g - h_f}{S - w_u - h_f} \right)$$

where the last inequality holds true since $t < \bar{t}_R = \frac{1}{r_g} \ln \left(\frac{S - w_g - h_f}{S - w_u - h_f} \right)$.

*3.**No Concessions (NC):*** The supporting equilibrium strategy profile $s = (t_u, \tau_f, (t_g, \tau_g))$ is $\left(t_u > t, \tau_f = t, \left(t_g > t_u^* + t, \tau_g > \tau_f^* + \tau \right) \right)$, with corresponding equilibrium payoffs $v_u(s) = w_g e^{-r_u t}$, $v_f(s) = h_g e^{-r_f t}$ and $v_g(s) = (S - w_g - h_g)\, e^{-r_g t}$. It is easy to verify that neither the union (see the proof for case 2 above) nor the firm (see the proof for case 2 above) have a profitable deviation. To see that this is the case for the government, too, note that the only potentially profitable deviation is to concede before t in both negotiations (which is the only way in which the government's payoff can be positively affected - if the government conceded in only one bargain, it would decrease its payoff in that negotiation without any impact on the implementation date). So, if the government is to deviate, it will set $t_g = \tau_g = t - \varepsilon \geq 0$. The corresponding payoff is thus $S - w_u - h_f$, so that this deviation is not profitable since

$$(S - w_u - h_f)\, e^{-r_f (t - \varepsilon)} < (S - w_g - h_g)\, e^{-r_g t} \Leftrightarrow \varepsilon < \frac{1}{r_f} \ln \left(\frac{S - w_g - h_g}{S - w_u - h_f} \right) = \bar{t}_I$$

which concludes the proof. ■

Proof of proposition 2.

[22] Recall that if the government were to give in earlier in wage negotiations over handout only, the overall time of agreement would be left unchanged, but a higher handout would be settled.

Supporting strategies are as follows:

1. '**Leftwing government' equilibrium (LGE):**

$$\widetilde{s} = \left(\left(w_u = \overline{w}, t_u > t_g^* + \tau\right), \left(h_f = h, \tau_f = \tau\right), \left(w_g = w, h_g = \underline{h}, t_g = \tau, \tau_g > \tau_f^* + \tau\right)\right)$$

2. **'Rightwing government' equilibrium (RGE)**:

$$\widetilde{s} = \left(\left(w_u = w, t_u = \tau\right), \left(h_f = \overline{h}, \tau_f > t_g^* + \tau\right), \left(w_g = \underline{w}, h_g = h, t_g > t_u^* + \tau, \tau_g = \tau\right)\right)$$

3. **'Intransigent government' equilibrium (IGE):**

$$\widetilde{s} = \left(\left(w_u = w, t_u = \tau\right), \left(h_f = h, \tau_f > t_g^* + \tau\right), \left(w_g = \underline{w}, h_g = \underline{h}, t_g > t_u^* + \tau, \tau_g > \tau_f^* + \tau\right)\right)$$

Consider the war of attrition phase first. Substitution of $w_u = \overline{w}$, $w_g = w$, $h_f = h$ and $h_g = \underline{h}$ into $\overline{t}_L$ from the proof of proposition 1 yields $\overline{\tau}_L = \overline{t}_L$, so that by proposition 1 there exist strategies supporting the $\boldsymbol{LGE}$ in the war of attrition stage. Similarly, for the $\boldsymbol{RGE}$ after letting $w_u = w$, $w_g = \underline{w}$, $h_f = \overline{h}$ and $h_g = h$ into $\overline{t}_R$, so that $\overline{\tau}_R = \overline{t}_R$; and for the $\boldsymbol{IGE}$ after substituting $w_u = w$, $w_g = \underline{w}$, $h_f = h$ and $h_g = \underline{h}$, so that $\overline{\tau}_I = \overline{t}_I$. Consider now deviations in the first stage, starting with the union. If it were to deviate to a compatible demand, i.e. to some $w' \leq w_g$, its payoff would not be affected, as because of incompatible demands in the parallel set of negotiations the game would still enter the second stage, with termination at time $t = \tau$. Similarly for the firm. Consider now the government. For equilibrium $\boldsymbol{LGE}$ in case 1, a deviation in the first stage to compatible payoffs would yield the government

$$S - \overline{w} - h < \left(S - \overline{w} - \underline{h}\right) e^{-r_g \tau} \Leftrightarrow \tau < \frac{1}{r_g} \ln \left(\frac{S - \overline{w} - \underline{h}}{S - \overline{w} - h}\right) = \overline{\tau}_L$$

Consider now the equilibrium $\boldsymbol{RGE}$ of case 2 above. A deviation in the first stage to compatible payoffs would yield the government

$$S - w - \overline{h} < \left(S - \underline{w} - \overline{h}\right) e^{-r_g \tau} \Leftrightarrow \tau < \frac{1}{r_g} \ln \left(\frac{S - \underline{w} - \overline{h}}{S - w - \overline{h}}\right) = \overline{\tau}_R$$

Finally consider equilibrium $\boldsymbol{IGE}$. The government's equilibrium continuation payoff in the second stage is $\left(S - \underline{w} - \underline{h}\right) e^{-r_g \tau}$. By deviating to compatible demands[23] the government would

[23] This deviation must affect both demand games, as otherwise the war of attrition stage would be triggered, and the government would still receive its payoff with delay, but after having contributed larger amounts.

get $S-w-h$ immediately. But

$$S-w-h<(S-\underline{w}-\underline{h})\,e^{-r_g\tau}\Leftrightarrow\tau<\frac{1}{r_g}\ln\left(\frac{S-\underline{w}-\underline{h}}{S-w-h}\right)=\overline{\tau}_I$$

To conclude, observe that in the bargaining phase for the player who anticipates that his proposal will prevail in the second stage it is suboptimal to set his claim/offer to a value which is different from either the highest (for either the union or the firm) or the lowest (for the government). Thus those characterised above are the only equilibrium configurations possible for delayed agreement in pure strategies. ■

Lemma 2 *Equilibrium strategies are type monotone, that is:* $t^i(c_i)\leq t^i(c_i')$ *for all* $c_i<c_i'$*, where the inequality is strict unless* $t^i(c_i)=0$.

Proof: Fix an equilibrium profile. Let $t^b(c_b)=t$ and $t^b(c_b')=t'$, if $c_b>c_b'$, $t>0$ and $t\neq t'$ then $t'<t$. Consider the payoff that the bargainers of type c_b obtain from termination at t:

$$V(c_b,t)=\int_0^t\tfrac{G+1-Ec_b(\tau)-c_b}{3}e^{-\tau}dP^g(\tau)\quad+(1-P^g(t))\tfrac{(1-c_b)}{2}e^{-t}$$

where $E(c_b|t\leq t^b(c_b)<\infty)$ is the government expectation of the bargainers' type given that they have not yet conceded at time t and let $\delta(t)=\left(\int_0^t e^{-\tau}dP^g(\tau)+(1-P^g(t))e^{-t}\right)$. In an equilibrium $V(c_b,t)\geq V(c_b,t')=V(c_b',t')+(c_b-c_b')\,\delta(t')$. On the other hand $V(c_b',t')\geq V(c_b',t)=V(c_b,t)-(c_b-c_b')\,\delta(t)$, hence $V(c_b,t)\geq V(c_b,t)+(c_b-c_b')\,(\delta(t')-\delta(t))$, and therefore $(\delta(t')-\delta(t))\leq 0$ which is equivalent to $t'<t$.

A similar argument shows the result for the stakeholder. ■

Proof of Proposition 3:

Fix an equilibrium profile (t^b,t^g). That is,

$$\begin{aligned}t^b(c_b)&=\arg\max\ \int_0^t\tfrac{G+1-g(\tau)-c_b}{3}e^{-\tau}dP^g(\tau)+\quad(1-P^g(t))\,\tfrac{(1-c_b)}{2}e^{-t},\\ t^g(c_g)&=\arg\max\ \int_0^t(G-c_g)\,e^{-\tau}dP^b(\tau)+\quad\left(1-P^b(t)\right)\tfrac{G+1-c_g-b_t(c_g)}{3}e^{-t},\end{aligned}$$

where $b_t\,(c_g)=E(c_b|b(t)<c_b\leq G+1-c_g)$.

The following two claims are obvious but important:

Claim 1 $t^g(c_g) = \infty$ *for all* $c_g > G$ *and* $t^b(c_b) = \infty$ *for all* $c_b > 1$.

Claim 2 $P^g(0)\, P^b(0) = 0$.

Claim 1 simply states that types that can not generate a surplus have a dominant strategy, namely never to yield. Claim 2 points out that if there is a strictly positive probability that the opponent will yield at the start of the game, then any type planning to yields at $t = 0$ must benefit from a deviation in which she waits to see if the opponent does yield first.

By Claims 1 and 2 there are types such that $t^i(c_i) = t \in (0, \infty)$, and those types must satisfy a first order condition. Differentiating $V^b(c_b, t, t^g)$ with respect to t the first order condition for type $b(t)$ is

$$\frac{dg(t)}{dt} = \frac{3(C_g - g(t))(1 - b(t))}{[2(G - g(t)) - (1 - b(t))]}. \tag{7a}$$

Similarly, differentiating $V^g(c_g, t, t^b)$ with respect to t, and taking into account that $b_t(c_g) = \frac{1+G-c_g+b(t)}{2}$, the first order condition for type $g(t)$ simplifies to:

$$\frac{db(t)}{dt} = \frac{(C_b - b(t))[(G - g(t)) + (1 - b(t))]}{5(G - g(t)) - (1 + C_b - 2b(t))}. \tag{7b}$$

Hence the inverse of (t^b, t^g) must be characterized by a solution to the autonomous dynamical system1.

Moreover, by Lemma 2, the relevant solution of 1 must be strictly increasing; and by Claim 2 its initial condition must be in the set $I = \left\{(x, y) \in \Re^2 \text{ such that } xy = 0\right\}$. Consider the open set D,

$$D = \{(x, y) \in (-\epsilon, 1) \times (-\epsilon, G),$$
$$\text{such that } y < \min\left\{G - \tfrac{1}{2} + \tfrac{x}{2}, G - \tfrac{1-C_b}{5} + \tfrac{2x}{5}\right\}\}$$

Note that a solution of 1 such that $(g(t), b(t)) \notin D$ for some t, $0 < t < \infty$, cannot describe an equilibrium strategy profile: either because it is decreasing, in contradiction with Lemma 2, or because it prescribes that types $c_g > G$ or $c_b > 1$ choose a finite termination date, contradicting Claim 1. On the other hand, by the Fundamental Theorem of ordinary differential equations, a unique solution to 1 goes though each $(x, y) \in I \cap D$. And observe, moreover, that each such solution is strictly increasing, and approaches the boundary of D. Consider the point $(1, a)$, $a = G - \frac{C_b - 1}{5}$ in the boundary of D, and note that there is a unique $(x, y) \in I \cap D$ such that the solution to 1 through (x,y) approaches $(1,a)$. Denote this unique point (γ^*,β^*) and let (g^*,b^*) denote the unique solution to 1 with initial condition (γ^*,β^*) . Observe that for all t, $0<t<\infty$, $b^*(t)<1$ and $g^*(t)<a$, and $\lim_{t\to\infty} b^*(t)=1, \lim_{t\to\infty} g^*(t)=a$. To see that condition (5) is indeed necessary assume that the equilibrium strategy profile is described by another solution to 1 $\left(\widetilde{g},\widetilde{b}\right)$, with initial condition $(\gamma,\beta)\in I\cap D$, $(\gamma,\beta)\neq(\gamma^*,\beta^*)$. Since $\left(\widetilde{g}\left(t\right),\widetilde{b}\left(t\right)\right)$ approaches the boundary of D at $(z,u)\neq(1,a)$, then either i) $z=1, u<G$; or ii) $z<1$, $u=G-\frac{1+C_b}{5}+\frac{2z}{5}$; or iii) $z<1, u=G-\frac{1}{2}+\frac{z}{2}$. Consider case i): then $\left(\widetilde{g}\left(T\right),\widetilde{b}\left(T\right)\right)=(z,u)$ for $T<\infty$,that is, no type of either player terminates the game after T; this cannot be equilibrium behavior since any type c_g , $g(T)<c_g<G$, such that $t^g(c_g)=\infty$ according to the alleged strategy, is strictly better off deviating to $t'^g(c_g)=T+\Delta$. In cases ii) and iii) $t^b(c_b)=\infty$ for $c_b<1$. Along this profile, for each $\pi>0$,there is a $t_\pi<\infty$, such that $P\left(t^g(c_g)<\infty | t^g(c_g)\geq t_\pi\right)\leq\pi$; and for each $c_b<1$ there is a $\pi_b>0$ such that if $P\left(t^g(c_g)<\infty | t^g(c_g)\geq t_\pi\right)\leq\pi_b$, then

$$\int_t^\infty \frac{G+1-g(\tau)-c_b}{3}e^{-\tau}dP^s(\tau)\leq \quad \pi_b\frac{G+1-g(\tau)-c_b}{3}e^{-t}< \quad \frac{(1-c_b)}{2}e^{-t},$$

contradicting that $t^b(c_b)=\infty$ is a best response for any $c_b<1$. Hence the equilibrium profile must be characterized by(g^*,b^*).

To complete the proof we check that the necessary conditions are indeed sufficient for an equilibrium, i.e. that each type indeed maximizes her expected payoff given that the opponent plays the alleged strategy. Since the agents' preferences satisfy the single crossing property[24] the first order conditions together with the monotonicity of the opponent's strategy imply the second order condition and are therefore sufficient for a maximum for all types such that $t^b(b)<\infty$ and $t^g(s)<\infty$. Observe moreover, that since for each $t<\infty$ there are types $c_g<a$ maximizing her payoff by $t^g(c_g)=t$, by monotonicity any $c_g\geq a$ must maximize her payoff with $t^g(c_g)\geq \sup\{t^g(c_g),c_g<a\}=\infty$. Therefore $\left(t^g,t^b\right)$ is indeed an equilibrium. ■

[24] For an agent of type c a surplus share x at date t yields a utility $(x-c)\,e^{-t}$. This preferences satisfy the *single crossing property* : utility is strictly monotone in x and the slope of the indiference curve in the (t,x) outcomes space is strictly monotone in c.