ACCESSION GAMES: A DYNAMIC PER-MEMBER PARTITION FUNCTION APPROACH

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Abstract. In this paper we define and solve the accession game, a dynamic game containing a union and a set of applicants with a per-member partition function satisfying the conditions of Yi [17] to include negative externalities. The solution gives an equilibrium partition of the players as well as, after Morelli and Penelle [12], the optimal path, a subgame-perfect sequence of partitions, where each player maximises the present value of its payoffs subject to others' moves.

While this game can be applied in general our motivation was to model the ongoing extensions of the European Union.

1. Introduction

Economic and political integration has gained much attention recently. Mergers and acquisitions exceed all previous volumes, bourses coordinate their trading hours, stronger and stronger trading blocks develop just to mention a few examples. In this paper our focus is on the European Union (EU), on its extensions in particular. While our paper contains no empirical part the assumptions in our model are made with the aim to fit such an application very well.

WW2 redrew the political map of Europe. From 1 January 1958, Belgium, France, Western Germany, Italy, Luxembourg and the Netherlands formed the European Economic Community (EEC) with the ultimate aim of a total economic and political union. The enlargement of the EEC (then EU) has been on the agenda virtually from that date, with a number of countries joining: United Kingdom, Denmark and Ireland in 1970, Greece in 1981, Portugal and Spain in 1986, then Austria, Finland and Sweden in 1995. The applicants' queue did not decrease: since the fall of the communism a number of Central and East European Countries (CEECs) are seeking their place in the European Union.

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With its rapidly developing literature coalitional game theory contributes a lot to the better understanding of integration. Authors mostly use the characteristic function form, that lacks externalities: ignores the side effects of forming coalitions as Greenberg [9] points out. Thrall [14] and Thrall and Lucas [15] define the *partition function* [11, pp509-511], an extension of the characteristic function that assigns a value to a coalition given the whole partition and hence accounts for externalities too. The per-member partition function is its less general form, but, on the other hand, a lot more feasible computationally. We follow Yi [17] in assuming a set of conditions to express the negative externalities of mergers and some other features.

Although our findings will be largely theoretical and hence generally applicable, our main motivation is to understand the extension of the European Union better. After the introduction of the notation, terminology and some examples we will discuss the *accession game*, where a number of applicants try to get membership in a coalition larger than the others (cf. the apex game [3]), called the Union following the optimal path [12], or sequence of coalition structures that maximises the present value of future payoffs not independent of the behaviour of other players. We give the solution algorithm of the accession game with an arbitrary number of applicants.

2. Game Theoretic Foundations

2.1. **The extension of Yi's game.** [1] Yi [17] has introduced a two-stage non-cooperative per-member partition-function game with three conditions to contain negative externalities. We use a cooperative dynamic extension of this game:

- **Stage 1 :** The initial partition is given exogenously.
- **Stage 2a:** Players form their cooperative, so binding strategies.
- **Stage 2b:** Players execute the move they have "agreed" upon in stage 2a.

The game is repeated with Stage 2 of the kth game determining the initial partition for Stage 1 of the $k+1$st game.

Definition 1. A *coalition structure* $\mathcal{P} = \{P_1, ..., P_k\}$ is a partition of the player set $N = \{1, ..., n\}$; Π is the set of coalition structures, $|\Pi| < \infty$. We define *payoff* for individual players and coalitions too.

Assumption 1. Players are *ex-ante* identical [Yi's Assumption 2.1], and share the coalition payoff equally [p.205].

Yi [p.209] shows that ordered pair $(m, \mathcal{P})$ is fully describes player $i \in P \in \mathcal{P}, |P| = m$, hence its immediate strategies as well, and so with pure strategies a partition will always be followed by the same partition at any stage of the game giving rise to *loops*.

[1] In this subsection all references will be to Yi [17].

Now we can give further details as regards to how the game is played. By the above property all players belonging to coalitions of the same size have the same preferences and hence are identical, but we allow identical players to make a decision in which they have different fates. The players are assigned to these different roles by fair draws, and hence they choose this strategy if –not disregarding others' strategies– the expected worth of this lottery dominates other strategies. At each move players who enforce the move are called *perpetrators*, while the rest are the *residuals* (after Ray and Vohra [13]). As, in the definition of Ray and Vohra, the definition of the residual coalition is often arbitrary, we use the term less formally. We assume that the perpetrators' set is always minimal. As a corollary if the set of coalitions of size k is not smaller in partition $\mathcal{P}$ than in $\mathcal{P}'$ then the two are separated by a move, all players in a coalition k in $\mathcal{P}'$ are in k in $\mathcal{P}$, too. The significance of this rule will be understood later.

Definition 2. The *per-member partition function* is the function

$$v_i : \Pi \longrightarrow \mathbb{R}^N,$$
$$v = (v_1, \ldots, v_n)$$

The pair (N, v) is a *per-member partition game.*

Yi's conditions [17, Section 4.1.] on the per-member partition function form a very important part of our model:

Condition 1. $v(n_i, \mathcal{P}) > v(n_i, \mathcal{P}')$, where $n_i \in \mathcal{P} \cap \mathcal{P}'$ and $\mathcal{P}$ is a refinement of $\mathcal{P}'$. This expresses that mergers hurt residual players.

Condition 2. $v(n_j, \mathcal{P}) < v(k, \mathcal{P}')$, where $k = \sum_{i=1}^{j} n_i$

(1) $\mathcal{P} = \mathcal{P}' \cup \{n_1, n_2, \ldots, n_j\} \setminus \{k\}$ for some partition $\mathcal{P}_0$ of $n - k$,
(2) $n_i \geq n_j \quad \forall i,$

that is, a merger with coalitions that are not smaller is beneficial to the members of the coalition.

Condition 3. $v(n_j, \mathcal{P}) < v(n_i + 1, \mathcal{P}')$, where $\mathcal{P}' = \mathcal{P} \setminus \{n_i, n_j\} \cup \{n_i + 1, n_j - 1\}$, $n_i \geq n_j$, that is, a member of a coalition is strictly better off by leaving the coalition and joining another that is not smaller.

2.2. **Path dependence.** The concept of examining paths is due to Morelli and Penelle [12]. We give the basic definitions, introduce a more general notation and proceed to the set-up of our own model.

Definition 3. In one *move* we allow one "action" per player, that is: an agreement is settled, the necessary draws are made, the proposed coalitions are formed and payoffs are paid out. Less relevant approaches would allow interaction between two players at a time or allow only self-enforcing strategies (too slow/too fast communication). Let also $\Pi^f(\mathcal{P})$ be the set of feasible partitions after a move starting from $\mathcal{P}$. (We will allow $\Pi^f(\mathcal{P}) \neq \Pi$ for some $\mathcal{P} \in \Pi$.)

Then a *path* π is a sequence $\{\mathcal{P}_i\}_{i>0}$ of partitions such that $\mathcal{P}_{i+1} \in \Pi^f(\mathcal{P}_i)$, that we write in the general form:

$$\pi = \left\{\mathcal{P}_1, \mathcal{P}_2, \ldots, \overline{\mathcal{P}_{\lambda-\kappa+1}, \ldots, \mathcal{P}_\lambda}\right\}$$

the *length* of the path $\lambda \in \mathbb{N}$ is maximal, such that for all $i \leq \lambda$ and $j \leq \lambda$,$\mathcal{P}_i = \mathcal{P}_j$ implies $i = j$. The last κ partitions form the equilibrium outcome that is repeated forever, forming a *loop* that is non-trivial unless $\kappa = 1$. Let $\mathcal{P}_t(\pi)$ denote the partition after playing the game t times along path π.

Definition 4. The present value for the player $(m, \mathcal{P})$ along path π from $\mathcal{P}$ is the discounted average of the payoffs along path π that players of type $(m, \mathcal{P})$ obtain, and is denoted by $w^m(\pi)$. Let Φ be the set of paths that can be enforced by the same player, assuming rational behaviour from the others. The *expected present value* or shortly *value* is defined by

$$w^m = \max_{\pi \in \Phi} w^m(\pi), \tag{1}$$

and the *optimal path*

$$\pi^m \in \arg\max_{\pi \in \Phi} w^m(\pi), \tag{2}$$

A path is a *solution* if the outcome cannot be improved and the corresponding game is coalition-proof and subgame perfect [4, 12].

3. The Accession Game

3.1. **Introduction.** In this section we define the accession game: a game of the extension of a special coalition S_t, with $|S_t| = s_t$ at time t that we refer to as the union. For simplicity we write S with s referring to the actual size of S. This will lead to no confusion.

Assumption 2 (Monotonicity)**.** We will restrict our attention to paths where the union does not secede, that is, if $S \in \mathcal{P}_t(\pi)$ then there exists $S' \in \mathcal{P}_{t+1}(\pi)$, such that $S \subseteq S'$, that is the size of the Union is monotone increasing.

As a result for all nontrivial games there exists $\mathcal{P} \in \Pi$ such that $\Pi^f(\mathcal{P}) \neq \Pi$.

Note that, although the above assumption is strong from a theoretical point of view the past of the European Union justifies it. The monotonicity is unlikely to change in the near future.

We can have two definitions of the applicants' sets, A: the set $N \setminus S_0$ for all t is the natural definition in the sense that it does not change in the course of the game. The alternative we use, $N \setminus S_t$, on the other hand does not preserve history; as soon as some of the applicants join S, a new game is considered with fewer applicants, giving rise to an inductive solution. We define value for A and S, as the mean of the members' values. As

$$w^{S_0}(\mathcal{P}) = \frac{1}{s_0}\sum_{i\in S_0} w^i(\mathcal{P}) = \frac{1}{s_t}\sum_{i\in S_t} w^i(\mathcal{P}) = w^{S_t}(\mathcal{P}),$$

and $S_0 \subseteq S_t$ the interests of the original and current members of the union coincide.

The focus of the solution is on the conflict between the interests of A and S. Although the game is not aimed to model the formation of the cooperative agreement, each step of the game can be pictured as a bargaining procedure: If no offers are made by the union or the offers are not accepted the applicants play the *disagreement strategy*, repartition themselves to obtain the highest value without acceptance. The union makes its most preferred offer. This is accepted if a subset of applicants is willing to take it, and is able to enforce it. If it is not, then the union makes further offers as long as these give improvement over the disagreement strategy.

The first of the two approaches we consider is the pure non-transferable utility game. In the other approach we allow transfers among applicants so the applicants' aggregated preferences are expressed by their total value in the proposed partition. This approach is preferred by the applicants as it maximizes their value along the optimal path. The difference in the approaches is small if the payoffs for the members of the Union S are considerably larger than for A: the benefit of one applicant being accepted outweighs the others' losses.

3.2. **An example.** The table below shows the payoffs of game G, as an example. The headings refer to the size of the coalition, so $v(s+1,\{s+1,1,1\})$ must be read from the column with $\geq s$ at the top, and along line $\{s+1,1,1\}$.

v_G		1	2,3	$\geq s$
$\mathcal{P}_6=$	$\{s+3\}$			4
$\mathcal{P}_5=$	$\{s+2,1\}$	0		5
$\mathcal{P}_4=$	$\{s+1,2\}$		2	6
$\mathcal{P}_3=$	$\{s+1,1,1\}$	1		7
$\mathcal{P}_2=$	$\{s,3\}$		3	3
$\mathcal{P}_1=$	$\{s,2,1\}$	1	4	4
$\mathcal{P}_0=$	$\{s,1,1,1\}$	2		5

As the union cannot secede, increasing Π by looking further back into the past of S to partitions, where more of its currents members were applicants does not increase the set of feasible outcomes Π^f. Thus we can have an inductive argument: first solve for the case when we have 0 applicants (partition $\mathcal{P}_6$), and then given the solution for i-applicants, we can solve for $i+1$ applicants. The number of applicants is finite, so in a finite steps we arrive to the case we aim to solve.

For 0 applicants the solution is trivial.

For 1 applicant we argue as follows: By monotonicity, set of possible strategies is $\{\mathcal{P}_5, \mathcal{P}_6\}$. If no acceptance offer is made by the union or the offers are not accepted (we call this the *disagreement strategy*), the applicants' maximal payoff is 0 by moving to $\mathcal{P}_5$. In this case the union S gets $\frac{5}{1-\delta}$, while at partition $\mathcal{P}_6$ it would get $\frac{4}{1-\delta}$ as calculated in the previous step. Hence it makes no offers for the applicant.

When no offer is made in the 2-applicant case A plays $\mathcal{P}_4$ giving $\frac{6}{1-\delta}$ to the union S. The union makes only offers with a higher payoff: $\mathcal{P}_3$. The applicants have homogeneous interests and hence a self-evident preference-ordering according to the expected payoffs along the various possible paths: $\mathcal{P}_6, \mathcal{P}_5, \mathcal{P}_4, \mathcal{P}_3$. As the state $\mathcal{P}_4$ can be achieved anyway, the applicants will only accept offers that are better than that; in this case we have no such moves, so $\mathcal{P}_4$ is played.

In he 3-applicant case we deal with $\mathcal{P}_0$ and $\mathcal{P}_2$ first. Although for the applicants $\mathcal{P}_2$ is no better than playing $\mathcal{P}_1$, it gives the union S a lower payoff and is still credible. The disagreement payoffs are therefore $w^A = 3\frac{1}{1-\delta}$ and $w^S = 3\frac{1}{1-\delta}$ which is the lowest of all strategies for S. The applicants prefer $\mathcal{P}_6$ most and hence this is played.

$\mathcal{P}_1$	Stage I	Stage II
1	$\mathcal{P}_4 > \mathcal{P}_6 > \mathcal{P}_3 > \mathcal{P}_1 > \mathcal{P}_5$	$\mathcal{P}_4 > \mathcal{P}_3 > \mathcal{P}_1 > \mathcal{P}_5$
2	$\mathcal{P}_5 > \mathcal{P}_6 = \mathcal{P}_3 = \mathcal{P}_1 > \mathcal{P}_4$	$\mathcal{P}_5 > \mathcal{P}_3 > \mathcal{P}_1 = \mathcal{P}_4$
s	$\mathcal{P}_3 > \mathcal{P}_4 > \mathcal{P}_5 > \mathcal{P}_1 = \mathcal{P}_6$	$\mathcal{P}_3 > \mathcal{P}_4 > \mathcal{P}_5 > \mathcal{P}_1$

$\mathcal{P}_1$	Stage III	Stage IV	Stage V
1	$\mathcal{P}_4 > \mathcal{P}_3 > \mathcal{P}_5$	$\mathcal{P}_4 > \mathcal{P}_3$	$\mathcal{P}_3$
2	$\mathcal{P}_5 > \mathcal{P}_3 > \mathcal{P}_4$	$\mathcal{P}_3 > \mathcal{P}_4$	$\mathcal{P}_3$
s	$\mathcal{P}_3 > \mathcal{P}_4 > \mathcal{P}_5$	$\mathcal{P}_3 > \mathcal{P}_4$	$\mathcal{P}_3$

In partition $\mathcal{P}_1$ the applicants have inhomogeneous preferences. Since the 2-coalition does not benefit from a merger with the singleton, the non-cooperative outcome is $\mathcal{P}_1$, giving $w^1 = \frac{1}{1-\delta}, w^2 = \frac{4}{1-\delta}, w^s = \frac{4}{1-\delta}$. The table above summarises the steps as the different preference orderings are evaluated with the relation signs expressing preferences. In stage I we can remove the strategies that are dominated by the disagreement strategy for the union ($\mathcal{P}_6$). From here the strategies are eliminated from backwards. The last chance to improve payoffs before disagreement is $\mathcal{P}_5$. In stage II the doubleton is willing to accept this and can enforce it after acceptance, so the "offer" $\mathcal{P}_1$ is never made. Foreseeing these actions, the singleton will accept the previous offer in stage III, $\mathcal{P}_4$, as it improves its payoff, and it can enforce it. This is the worst possible outcome for the doubleton, it is willing to accept the previous offer $\mathcal{P}_3$, and can enforce it, and hence this is the outcome for the game. The union exploited the tension among applicants very well: its first offer is accepted, for the applicants $\mathcal{P}_6$ Pareto-dominates this outcome. This table summarises the results for the various i-s for game G.

Game G	$\mathcal{P}$	m	π^*	w
0-applicants	$\mathcal{P}_6$	$s+3$	$\{\overline{\mathcal{P}_6}\}$	$4\frac{1}{1-\delta}$
1-applicant	$\mathcal{P}_5$	1	$\{\overline{\mathcal{P}_5}\}$	0
		$s+2$		$5\frac{1}{1-\delta}$
2-applicants	$\mathcal{P}_4$	2	$\{\overline{\mathcal{P}_4}\}$	$2\frac{1}{1-\delta}$
		$s+1$		$6\frac{1}{1-\delta}$
	$\mathcal{P}_3$	1	$\{\mathcal{P}_3, \overline{\mathcal{P}_4}\}$	$1+2\frac{\delta}{1-\delta}$
		$s+1$		$7+6\frac{\delta}{1-\delta}$
3-applicants	$\mathcal{P}_2$	3	$\{\mathcal{P}_2, \overline{\mathcal{P}_6}\}$	$3+4\frac{\delta}{1-\delta}$
		s		$3+4\frac{\delta}{1-\delta}$
	$\mathcal{P}_1$	1	$\{\mathcal{P}_1, \mathcal{P}_3, \overline{\mathcal{P}_4}\}$	$1+\delta+2\frac{\delta^2}{1-\delta}$
		2		$4\frac{1}{1-\delta}$
		s		$4+7\delta+6\frac{\delta^2}{1-\delta}$
	$\mathcal{P}_0$	1	$\{\mathcal{P}_0, \overline{\mathcal{P}_6}\}$	$2+4\frac{\delta}{1-\delta}$
		s		$5+4\frac{\delta}{1-\delta}$

If we allow transfers among applicants the singleton can compensate the 2-coalition when moving to $\mathcal{P}_2$, and thus the strategy offering the lowest value to the union S becomes a credible threat, and $\mathcal{P}_6$ is played. Remarkably, transfers never take place, as the threat is never executed; as soon as the union believes that transfers *could* take place, a better outcome is achieved.

3.3. **The general form.** In the general accession game we allow transfers among the applicants. Such a game even with an arbitrary number of applicants simplifies to a two-player game between the union S and the applicants A. At each partition $\mathcal{P}$, given the corresponding $\Pi^f(\mathcal{P})$ the next move is determined as follows.

Given $\mathcal{P}$, both S and A can assign a value to any outcome in $\Pi^f(\mathcal{P})$. Given these, the union proposes its favoured partition. The applicants can either accept this, or reject it, in which case the Union makes further proposals as long as these are better than the disagreement strategy:

$$\mathcal{P}_D \in \arg\max_{\substack{\mathcal{P}' \in \Pi^f \\ |S| \in \mathcal{P}'}} \left\{ w^A(\mathcal{P}') \right\}$$

By perfect knowledge, applicants may choose their most preferred offer. This is an equilibrium by construction and formally we have the solution:

$$\mathcal{P}^* = \arg \max_{\mathcal{P}\in\Pi^f(\mathcal{P}_0)} \left\{ w^A(\mathcal{P}) \,\middle|\, w^S(\mathcal{P}) > w^S(\mathcal{P}_D) \right\} \tag{3}$$

Alternatively, if offers are made by the applicants, then

$$\mathcal{P}^* = \arg \max_{\mathcal{P}\in\Pi^f(\mathcal{P}_0)} \left\{ w^S(\mathcal{P}) \,\middle|\, w^S(\mathcal{P}) \geq w^A(\mathcal{P}_D) \right\}$$

If the decision is made in a symmetrical way, the solution is more complex, we have something similar to the classical problem of the Battle-of-Sexes except that we do not allow randomised strategies. In many real life situations one of the strategies becomes a *focal point* , but in a theoretical problem gives little help.

3.4. **Solving the general accession game.** Our solution will be inductive. Let $\Pi^a = \{\mathcal{P} \in \Pi |\, n - s = a\}$ the set of partitions with exactly a applicants. For $a = 0$, $\Pi = \{\{n\}\}$ and the solution is trivial. When solving for $a + 1$ we assume that for all $\mathcal{P} \in \Pi^k$ with $0 \leq k < a$ the solution is known. Now let

$$\Pi^a_D = \arg \max_{\mathcal{P}\in\Pi^a} v^A(\mathcal{P}) \text{ the disagreement set,}$$

$$\Pi_+ = \left\{ \mathcal{P} \in \bigcup_{a=0}^{n-s_0} \Pi^a \,\middle|\, w^S(\mathcal{P}) > \min_{\mathcal{P}_D\in\Pi^a_D} w^S(\mathcal{P}_D) \right\} \text{ the set of offers,}$$

$$\mathcal{P}^a \in \arg \max_{\mathcal{P}\in\Pi_+} w^A(\mathcal{P}) \text{ the accepted offer in the case of } a \text{ applicants.}$$

Let $b \in \mathbb{N}$ such that $\mathcal{P}^a \in \Pi^b$. Then starting from the initial partition $\mathcal{P} \in \Pi^a$ we have the following results:

$$\pi^*(\mathcal{P}) = \begin{cases} \{\mathcal{P}, \mathcal{P}^a\} & \text{if } b = a \\ \{\mathcal{P}\} \cup \pi^*(\mathcal{P}^a) & \text{otherwise,} \end{cases} \tag{4a}$$

$$w^A(\mathcal{P}) = \begin{cases} v^A(\mathcal{P}) + \delta \frac{v^A(\mathcal{P}^a)}{1-\delta} & \text{if } b = a \\ v^A(\mathcal{P}) + \delta w^A(\mathcal{P}^a) & \text{otherwise,} \end{cases} \tag{4b}$$

$$w^S(\mathcal{P}) = \begin{cases} v^S(\mathcal{P}) + \delta \frac{v^S(\mathcal{P}^a)}{1-\delta} & \text{if } b = a \\ v^S(\mathcal{P}) + \delta w^S(\mathcal{P}^a) & \text{otherwise,} \end{cases} \tag{4c}$$

4. Solving more general games

4.1. **Motivation.** Like Morelli and Penelle [12], having discussed the extensions of the EU, we must raise the question: how is the Union formed? So far we have very strongly used the assumed dominance of the Union, but now we want to know what happens if it is smaller, not dominant, or even non-existent. Hence, in this part we consider games without a dominant coalition. By removing it, we also remove the preordering of the strategies that was so far determined by the union's preferences. In the lack of a dominant player it would be very arbitrary to assign this role to any other player, and so a similar solution cannot be established.

4.2. **Solving games without a dominant coalition.** The monotonicity of the union provided a natural termination state, and even for most of the others we could say that one is more likely to be a termination state, and hence an outcome than the other simply by looking at the size of the Union. This enabled us to define an inductive solution algorithm. It is clear that in games where moves between any two partitions are possible, such algorithm cannot work. Here we argue as follows: Given the the solution and assuming that it contains no non-trivial loops the final partition is a coalition-proof, subgame-perfect equilibrium outcome. While such outcomes cannot, in general, be pointed out at once, as there might be several of them depending the initial partition. By condition 2 coalitions of the same size benefit from merging so partitions with coalitions of the same size can be excluded. For a large set of players this leaves relatively few partitions to test. (If $n = 15$ we have 27 such partitions out of a total of 176)[2] These partitions are interesting even if they fail as equilibrium outcomes, as a further state can only be achieved by cooperation between coalitions of different sizes in which case –by rationality– all benefit, or by secession, where, by condition 1 all outsiders get to a state with a higher payoff that often means a higher expected present value, too.

Given these outcomes, we construct subgame perfect (and coalition-proof) paths by extending these "1-paths" (as they have length 1). In order to generate the k-paths first we take all partitions $\mathcal{P}_i$ of n and all $k-1$-paths and seek the equilibrium path of length k. The strictly preferred one of this path and π_i^{k-1} ($k \geq 2$, otherwise π_i^{k-1} does not exist) becomes π_i^k. If $\mathcal{P}_i$ appears twice in π_i^k then either the path had already contained a loop or we have just identified one. This argument is repeated until no improvement is possible, no new paths are generated. By construction the paths are the ones we have been looking for.

It is not our aim to define a concept to aggregate the players' interests. We can discuss some simple examples, where the solution is evident, but the general problem is open for further research.

4.3. **Examples.** We solve a game to illustrate the introduced concepts and to draw attention to some difficulties encountered when solving games. The table below shows the payoff values for game A. Games with 5 players have 6 possible partitions. We aim to find the optimal path from $(1, N)$.

[2] Generated using http://sue.csc.uvic.ca/~cos/gen/nump.html.

v_A		1	2	≥ 3
$\mathcal{P}_6 =$	$\{5\}$			6
$\mathcal{P}_5 =$	$\{4,1\}$	-3		7
$\mathcal{P}_4 =$	$\{3,2\}$		2	5
$\mathcal{P}_3 =$	$\{3,1,1\}$	1		6
$\mathcal{P}_2 =$	$\{2,2,1\}$	1	4	
$\mathcal{P}_1 =$	$\{2,1,1,1\}$	2	5	
$\mathcal{P}_0 =$	$\{1,1,1,1,1\}$	3		

Beyond the validity of the general concepts, looking at game A we notice that in partition $\mathcal{P}_4$ all players benefit from cooperating and playing $\mathcal{P}_6$. Thus the set of outcomes is reduced to $\mathcal{P}_5$ and $\mathcal{P}_6$. For each player we compute the value along all 1-path alternatives. The table below summarises our calculations: The sign # marks enforceable outcomes, that is, outcomes where cooperation with other players is not required. This means that the player can achieve at least this value.

		1-paths				
$\mathcal{P}_i$	m	$\{\mathcal{P}_4\}$	$\{\mathcal{P}_5\}$	$\{\mathcal{P}_6\}$	π^*	$w(\pi^*)$
$\mathcal{P}_6$	5	$\frac{19}{5}\frac{1}{1-\delta}\#$	$5\frac{1}{1-\delta}\#$	$6\frac{1}{1-\delta}\#$	$\{\overline{\mathcal{P}_6}\}$	$6\frac{\delta}{1-\delta}$
$\mathcal{P}_5$	1	$2\frac{1}{1-\delta}$	$-3\frac{1}{1-\delta}\#$	$6\frac{1}{1-\delta}$	$\{\overline{\mathcal{P}_5}\}$	$-3\frac{1}{1-\delta}$
	4	$\frac{17}{4}\frac{1}{1-\delta}$	$7\frac{1}{1-\delta}\#$	$6\frac{1}{1-\delta}$		$7\frac{\delta}{1-\delta}$
$\mathcal{P}_4$	2	$2\frac{1}{1-\delta}\#$	$2\frac{1}{1-\delta}$	$6\frac{1}{1-\delta}$	$\{\mathcal{P}_4,\mathcal{P}_6\}$	$2+6\frac{\delta}{1-\delta}$
	3	$5\frac{1}{1-\delta}\#$	$7\frac{1}{1-\delta}$	$6\frac{1}{1-\delta}$		$5+6\frac{\delta}{1-\delta}$
$\mathcal{P}_3$	1	$2\frac{1}{1-\delta}\#$	$2\frac{1}{1-\delta}$	$6\frac{1}{1-\delta}$	$\{\mathcal{P}_3,\mathcal{P}_6\}$	$1+6\frac{\delta}{1-\delta}$
	3	$5\frac{1}{1-\delta}$	$7\frac{1}{1-\delta}$	$6\frac{1}{1-\delta}$		$6\frac{1}{1-\delta}$
$\mathcal{P}_2$	1	$5\frac{1}{1-\delta}$	$-3\frac{1}{1-\delta}$	$6\frac{1}{1-\delta}$	$\{\mathcal{P}_2,\mathcal{P}_5\}$	$1-3\frac{\delta}{1-\delta}$
	2	$\frac{7}{2}\frac{1}{1-\delta}$	$7\frac{1}{1-\delta}\#$	$6\frac{1}{1-\delta}$		$4+7\frac{\delta}{1-\delta}$
$\mathcal{P}_1$	1	$5\frac{1}{1-\delta}\#$	$\frac{11}{3}\frac{1}{1-\delta}$	$6\frac{1}{1-\delta}$	$\{\mathcal{P}_1,\mathcal{P}_6\}$	$2+6\frac{\delta}{1-\delta}$
	2	$2\frac{1}{1-\delta}$	$7\frac{1}{1-\delta}$	$6\frac{1}{1-\delta}$		$5+6\frac{\delta}{1-\delta}$
$\mathcal{P}_0$	1	$\frac{19}{5}\frac{1}{1-\delta}\#$	$5\frac{1}{1-\delta}\#$	$6\frac{1}{1-\delta}\#$	$\{\mathcal{P}_0,\mathcal{P}_6\}$	$3+6\frac{\delta}{1-\delta}$

Within $\mathcal{P}_6$ and $\mathcal{P}_0$ players are of the same type, the best path is the one offering the highest value.

In $\mathcal{P}_1$ the singletons can get to $\mathcal{P}_4$ without cooperating with the 2-coalition to get $5\frac{1}{1-\delta}$. This dominates strategy $\mathcal{P}_5$, so the 2-coalition will never obtain the value $7\frac{1}{1-\delta}$. But now both the singletons and the 2-coalition can improve their payoffs by choosing the path $\{\mathcal{P}_6\}$.

In $\mathcal{P}_2$ the 2-coalition disregards the singleton and plays strategy $\{\mathcal{P}_5\}$.

In $\mathcal{P}_3$, similarly to $\mathcal{P}_1$ the singletons can get to $\mathcal{P}_4$, and so, although this is not strictly better than playing $\mathcal{P}_5$ it is a credible threat forcing the 3-coalition to play $\mathcal{P}_6$.

The same argument goes for $\mathcal{P}_4$. Here we must add that this was the end one of the possible paths considered, hence in theory wherever it was selected as an outcome of optimal paths (in our example it was not) the paths can be further improved upon.

		2-paths					1-paths		
$\mathcal{P}_i$	m	$\{\mathcal{P}_0, \mathcal{P}_6\}$	$\{\mathcal{P}_1, \mathcal{P}_6\}$	$\{\mathcal{P}_2, \mathcal{P}_5\}$	$\{\mathcal{P}_3, \mathcal{P}_6\}$	$\{\mathcal{P}_4, \mathcal{P}_6\}$		π^*	$w(\pi^*)$
$\mathcal{P}_6$	5	$3+6\gamma\#$	$\frac{16}{5}+6\gamma\#$	$\frac{17}{5}+6\gamma\#$	$4+6\gamma\#$	$\frac{19}{5}+6\gamma\#$	$6\frac{1}{1-\delta}\#$	$\{\overline{\mathcal{P}_6}\}$	6γ
$\mathcal{P}_5$	1	$3+6\gamma$	$2+6\gamma$	$1-3\gamma$	$1+6\gamma$	$2+6\gamma$	$-3\frac{1}{1-\delta}\#$	$\{\overline{\mathcal{P}_5}\}$	$-3\frac{1}{1-\delta}$
	4	$3+6\gamma\#$	$\frac{7}{2}+6\gamma\#$	$4+7\gamma\#$	$\frac{19}{4}+6\gamma$	$\frac{17}{4}+6\gamma$	$7\frac{1}{1-\delta}$		7γ
$\mathcal{P}_4$	2	$3+6\gamma$	$5+6\gamma$	$4+7\gamma$	$1+6\gamma$	-	$6\frac{1}{1-\delta}$	$\{\mathcal{P}_4, \mathcal{P}_6\}$	$2+6\gamma$
	3	$3+6\gamma$	$2+6\gamma\#$	$3+\frac{11}{3}\gamma\#$	$6\frac{1}{1-\delta}$		$6\frac{1}{1-\delta}$		$5+6\gamma$
$\mathcal{P}_3$	1	$3+6\gamma$	$2+6\gamma$	$\frac{5}{2}+2\gamma$	-	$2+6\gamma\#$	$6\frac{1}{1-\delta}$	$\{\mathcal{P}_3, \mathcal{P}_6\}$	$1+6\gamma$
	3	$3+6\gamma$	$4+6\gamma$	$4+7\gamma$		$5+6\gamma$	$6\frac{1}{1-\delta}$		$6\frac{1}{1-\delta}$
$\mathcal{P}_2$	1	$3+6\gamma$	$2+6\gamma$	-	$1+6\gamma$	$5+6\gamma$	$-3\frac{1}{1-\delta}$	$\{\mathcal{P}_2, \mathcal{P}_5\}$	$1-3\gamma$
	2	$3+6\gamma\#$	$\frac{7}{2}+6\gamma\#$		$\frac{19}{4}+6\gamma$	$\frac{7}{2}+6\gamma$	$7\frac{1}{1-\delta}\#$		$4+7\gamma$
$\mathcal{P}_1$	1	$3+6\gamma$	-	$3+\frac{11}{3}\gamma\#$	$\frac{8}{3}+6\gamma$	$5+6\gamma\#$	$6\frac{1}{1-\delta}$	$\{\mathcal{P}_1, \mathcal{P}_6\}$	$2+6\gamma$
	2	$3+6\gamma$		$4+7\gamma$	$6\frac{1}{1-\delta}$	$2+6\gamma$	$6\frac{1}{1-\delta}$		$5+6\gamma$
$\mathcal{P}_0$	1	-	$\frac{16}{5}+6\gamma\#$	$\frac{17}{5}+5\gamma\#$	$4+6\gamma\#$	$\frac{19}{5}+6\gamma\#$	$6\frac{1}{1-\delta}\#$	$\{\mathcal{P}_0, \mathcal{P}_6\}$	$3+6\gamma$

This way we have chosen a unique path of length at most 2 starting from each of the partitions. These are the 2-paths. In the table we compare the values along these paths with each other and with the values of the 1-paths ($\gamma = \frac{\delta}{1-\delta}$). Arguing like before we find that all possible 3-paths are dominated by previously calculated 2-paths, no paths are extended and column π^* contains the optimal paths. In particular the solution of the game, and the corresponding value are:

$$\pi = \left\{\mathcal{P}_0, \overline{\mathcal{P}_6}\right\} \tag{5a}$$

$$w_0^1 = 3 + 6\frac{\delta}{1-\delta} \tag{5b}$$

4.4. **Loops.** To illustrate the significance of loops, before moving to more complex issues, first we will solve the simplest game L containing a loop with its payoff function below. This game has four players[3] Notice the relatively high value of v_1^2. We can guess that the players want to exploit this.

v_L		1	≥ 2
$\mathcal{P}_4 =$	$\{4\}$		9
$\mathcal{P}_3 =$	$\{3, 1\}$	0	13
$\mathcal{P}_2 =$	$\{2, 2\}$		8
$\mathcal{P}_1 =$	$\{2, 1, 1\}$	4	15
$\mathcal{P}_0 =$	$\{1, 1, 1, 1\}$	7	

In our solution we exploit the game's simplicity, the fact that at each stage there are coalitions of at most 2 different sizes and so the game simplifies to a 2-player game, there is always a unique disagreement strategy where the two sets of players form two coalitions, alternatively

[3] Using the conditions it is easy to prove that less players will not engage in a loop.

they cooperate in some way. The game lacks partitions with more than one cooperative strategy dominating disagreement.

Finding paths that include loops is similar, except that calculations are done recursively. Instead of a proof we give some evidence: By construction the method supports coalition-proofness, and subgame perfectness, the question is only whether it terminates or not. When looking for paths with loops we cannot assume stable outcomes to begin with. However, the number of paths with loops is finite, too, and at each stage where a new path is created we get rid of another one, hence the algorithm must terminate. As opposed to paths without loops due to the recursive nature of the algorithm , we need separate calculations to get the exact expected present values.

In stage I we look at the 1-paths. In stage II players improve their payoffs by selecting another path[4], and repeat this in stage III as well, but using the values calculated in stage II. Further steps do not change the result, all paths end in stable loops, hence we have the optimal paths.

	Stage I		Stage II		Stage III	
	w	π^*	w	π^*	w	π^*
v_0^1	$\frac{7}{1-\delta}$	$\{\mathcal{P}_0\}$	$7+\frac{19}{2}\frac{\delta}{1-\delta}$	$\{\mathcal{P}_0,\mathcal{P}_1\}$	$7+\frac{\frac{19}{2}\delta+9\delta^2}{1-\delta}$	$\{\mathcal{P}_0,\overline{\mathcal{P}_1,\mathcal{P}_4}\}$
v_1^1	$\frac{4}{1-\delta}$	$\{\mathcal{P}_1\}$	$4+9\frac{\delta}{1-\delta}$	$\{\mathcal{P}_1,\mathcal{P}_4\}$	$4+\frac{9\delta+\frac{19}{2}\delta^2}{1-\delta}$	$\{\overline{\mathcal{P}_1,\mathcal{P}_4}\}$
v_1^2	$\frac{13}{1-\delta}$		$13+9\frac{\delta}{1-\delta}$		$13+\frac{9\delta+\frac{19}{2}\delta^2}{1-\delta}$	
v_2^2	$\frac{8}{1-\delta}$	$\{\mathcal{P}_2\}$	$8+\frac{19}{2}\frac{\delta}{1-\delta}$	$\{\mathcal{P}_2,\mathcal{P}_1\}$	$8+\frac{\frac{19}{2}\delta+9\delta^2}{1-\delta}$	$\{\mathcal{P}_2,\overline{\mathcal{P}_1,\mathcal{P}_4}\}$
v_3^1	0	$\{\mathcal{P}_3\}$	0	$\{\mathcal{P}_3\}$	0	$\{\overline{\mathcal{P}_3}\}$
v_3^3	$\frac{13}{1-\delta}$		$\frac{13}{1-\delta}$		$\frac{13}{1-\delta}$	
v_4^4	$\frac{9}{1-\delta}$	$\{\mathcal{P}_4\}$	$7+\frac{19}{2}\frac{\delta}{1-\delta}$	$\{\mathcal{P}_4,\mathcal{P}_1\}$	$\frac{9\delta+\frac{19}{2}\delta^2}{1-\delta}$	$\{\overline{\mathcal{P}_4,\mathcal{P}_1}\}$

The game below, M has 5 players. The outcome of this game (a loop) does not include states where all players are of the same type, so differences never vanish. As a result values have to be calculated separately.

v_M		1	2	≥ 3
$\mathcal{P}_6=$	$\{5\}$			4
$\mathcal{P}_5=$	$\{4,1\}$	0		6
$\mathcal{P}_4=$	$\{3,2\}$		2	5
$\mathcal{P}_3=$	$\{3,1,1\}$	1		8
$\mathcal{P}_2=$	$\{2,2,1\}$	1	5	
$\mathcal{P}_1=$	$\{2,1,1,1\}$	2	11	
$\mathcal{P}_0=$	$\{1,1,1,1,1\}$	3		

[4]Of course the improvement need not apply to all players.

	Stage I		Stage II		Stage III	
	w	π^*	w	π^*	w	π^*
v_0^1	$\frac{3}{1-\delta}$	$\{\mathcal{P}_0\}$	$3+\frac{28}{5}\frac{\delta}{1-\delta}$	$\{\mathcal{P}_0,\mathcal{P}_1\}$	$3+\frac{28}{5}\delta+\frac{24}{5}\frac{\delta^2}{1-\delta}$	$\{\mathcal{P}_0,\mathcal{P}_1,\mathcal{P}_5\}$
v_1^1	$\frac{2}{1-\delta}$	$\{\mathcal{P}_1\}$	$2+4\frac{\delta}{1-\delta}$	$\{\mathcal{P}_1,\mathcal{P}_5\}$	$2+4\delta+5\frac{\delta^2}{1-\delta}$	$\{\overline{\mathcal{P}_1,\mathcal{P}_5}\}$
v_1^2	$\frac{11}{1-\delta}$		$11+6\frac{\delta}{1-\delta}$		$11+6\delta+\frac{13}{2}\frac{\delta^2}{1-\delta}$	
v_2^1	$\frac{1}{1-\delta}$	$\{\mathcal{P}_2\}$	$1+2\frac{\delta}{1-\delta}$	$\{\mathcal{P}_2,\mathcal{P}_1\}$	$1+2\delta+4\frac{\delta^2}{1-\delta}$	$\{\mathcal{P}_2,\mathcal{P}_1,\mathcal{P}_5\}$
v_2^2	$\frac{5}{1-\delta}$		$5+\frac{13}{2}\frac{\delta}{1-\delta}$		$5+\frac{13}{2}\delta+5\frac{\delta^2}{1-\delta}$	
v_3^1	$\frac{1}{1-\delta}$	$\{\mathcal{P}_3\}$	$1+3\frac{\delta}{1-\delta}$	$\{\mathcal{P}_3,\mathcal{P}_5\}$	$1+3\delta+\frac{17}{4}\frac{\delta^2}{1-\delta}$	$\{\mathcal{P}_3,\mathcal{P}_5,\mathcal{P}_1\}$
v_3^3	$\frac{8}{1-\delta}$		$8+6\frac{\delta}{1-\delta}$		$8+6\delta+\frac{13}{2}\frac{\delta^2}{1-\delta}$	
v_4^2	$\frac{2}{1-\delta}$	$\{\mathcal{P}_4\}$	$2+3\frac{\delta}{1-\delta}$	$\{\mathcal{P}_4,\mathcal{P}_5\}$	$2+3\delta+\frac{17}{4}\frac{\delta^2}{1-\delta}$	$\{\mathcal{P}_4,\mathcal{P}_5,\mathcal{P}_1\}$
v_4^3	$\frac{3}{1-\delta}$		$5+6\frac{\delta}{1-\delta}$		$5+6\delta+\frac{13}{2}\frac{\delta^2}{1-\delta}$	
v_5^1	0	$\{\mathcal{P}_5\}$	$2\frac{\delta}{1-\delta}$	$\{\mathcal{P}_5,\mathcal{P}_1\}$	$2\delta+4\frac{\delta^2}{1-\delta}$	$\{\overline{\mathcal{P}_5,\mathcal{P}_1}\}$
v_5^4	$\frac{6}{1-\delta}$		$6+\frac{13}{2}\frac{\delta}{1-\delta}$		$6+\frac{13}{2}\delta+5\frac{\delta^2}{1-\delta}$	
v_6^5	$\frac{4}{1-\delta}$	$\{\mathcal{P}_6\}$	$4+\frac{28}{5}\frac{\delta}{1-\delta}$	$\{\mathcal{P}_6,\mathcal{P}_1\}$	$4+\frac{28}{5}\delta+\frac{24}{5}\frac{\delta^2}{1-\delta}$	$\{\mathcal{P}_6,\mathcal{P}_1,\mathcal{P}_5\}$

TABLE 1. Solution of game M.

Table 1 shows the process of solving game M. By stage III it is clear that all paths lead to the loop $\overline{\mathcal{P}_1,\mathcal{P}_5}$, but further stages give different values for the expected present values. The following simultaneous equations for the values at each stages in the loop and the subsequent calculations give the correct figures.

$$w_1^1 = v_1^1 + \delta\frac{2w_5^4 + w_5^1}{3} \tag{6a}$$

$$w_1^2 = v_1^2 + \delta w_5^4 \tag{6b}$$

$$w_5^1 = v_5^1 + \delta w_1^1 \tag{6c}$$

$$w_5^4 = v_5^4 + \delta\frac{w_1^1 + w_1^2}{2} \tag{6d}$$

This is a set of simultaneous equations, 4 of them, with 4 unknowns that can be rearranged as follows:

$$\begin{array}{lllll}
v_1^1 = & w_1^1 & & -\frac{2\delta}{3}w_5^1 & -\frac{2\delta}{3}w_5^4 \\
v_1^2 = & & w_1^2 & & -\delta w_5^4 \\
v_5^1 = & -\delta w_1^1 & & +w_5^1 & \\
v_5^4 = & -\frac{\delta}{2}w_1^1 & -\frac{\delta}{2}w_1^2 & & +w_5^4
\end{array}$$

In general there might be zero, one, or infinite solutions to these equations, but our assumption that we found the equilibrium path after observing convergence in the values implies that the solution is unique. Solving these equations we get the final results:

	w	π^*
$(5, \mathcal{P}_6)$	$\frac{4}{5}\frac{5+7\delta+\delta^2}{1-\delta^2}$	$\left\{\mathcal{P}_6, \overline{\mathcal{P}_1, \mathcal{P}_5}\right\}$
$(1, \mathcal{P}_5)$	$4\frac{\delta(3+6\delta+4\delta^2)}{6-7\delta^2+\delta^4}$	$\left\{\overline{\mathcal{P}_5, \mathcal{P}_1}\right\}$
$(4, \mathcal{P}_5)$	$\frac{36+39\delta-12\delta^2+11\delta^3}{6-7\delta^2+\delta^4}$	
$(2, \mathcal{P}_4)$	$\frac{1}{2}\frac{24+36\delta+23\delta^2+12\delta^3+9\delta^4}{6-7\delta^2+\delta^4}$	$\left\{\mathcal{P}_4, \overline{\mathcal{P}_5, \mathcal{P}_1}\right\}$
$(3, \mathcal{P}_4)$	$2\frac{9+18\delta+9\delta^2-6\delta^3+-4\delta^4}{6-7\delta^2+\delta^4}$	
$(1, \mathcal{P}_3)$	$\frac{1}{2}\frac{12+36\delta+37\delta^2+12\delta^3+\delta^4}{6-7\delta^2+\delta^4}$	$\left\{\mathcal{P}_3, \overline{\mathcal{P}_5, \mathcal{P}_1}\right\}$
$(3, \mathcal{P}_3)$	$\frac{48+36\delta-17\delta^2-12\delta^3-3\delta^4}{6-7\delta^2+\delta^4}$	
$(1, \mathcal{P}_2)$	$\frac{6+12\delta+17\delta^2+16\delta^3+\delta^4}{6-7\delta^2+\delta^4}$	$\left\{\mathcal{P}_2, \overline{\mathcal{P}_1, \mathcal{P}_5}\right\}$
$(2, \mathcal{P}_2)$	$\frac{30+39\delta-5\delta^2-11\delta^3-\delta^4}{6-7\delta^2+\delta^4}$	
$(1, \mathcal{P}_1)$	$4\frac{3+6\delta+4\delta^2}{6-7\delta^2+\delta^4}$	$\left\{\overline{\mathcal{P}_1, \mathcal{P}_5}\right\}$
$(2, \mathcal{P}_1)$	$2\frac{33+18\delta-19\delta^2-6\delta^3}{6-7\delta^2+\delta^4}$	
$(1, \mathcal{P}_0)$	$\frac{1}{5}\frac{15+28\delta+9\delta^2}{1-\delta^2}$	$\left\{\mathcal{P}_0, \overline{\mathcal{P}_1, \mathcal{P}_5}\right\}$

5. Conclusions

We have demonstrated a number of problems arising when trying to solve the general accession game. If we exclude loops and assume the monotonicity of the union's size the game can be solved. The solution of games without these restrictions is open for future research. However, despite its limitations the model matches the selected application rather well, and at the same time demonstrates a mechanism how small players may be able to enforce their interest against the much larger union by cooperation, while the lack of cooperation may allow the union to exploit the division of the applicants.

Baldwin [2, pp130-139] describes the "hub-and-spoke bilateralism" model, where the European Union has arrangements with each applicant separately. In our model this corresponds to the union and a set of singletons. This state is favourable for the Union, but not for the applicants. Candidates may improve their average position by forming a coalition, such as the Visegrád group or the Baltic countries were in the past. These lost their importance as the EU –probably not malevolently– favoured some of their members over others, who then in turn made every effort to exploit their advantage. Such divisions reduce the average chance for acceptance. Allowing transfers between the applicants, for instance in the form of tax exemptions, could help to overcome this difficulty and give a more efficient outcome.

The encountered difficulties give rise to some questions that we leave open. Loops are very uncommon in games and they might offer a new way to explain cyclic behaviour by firms or countries in forming alliances. Morelli and Penelle [12] discuss how different utility transfers affect the accession path; our example shows that even the uncertainty about the transfers can influence the outcome.

There are many ways to extend or generalise this model. Most of the restrictions introduced are to reduce the level of complexity and to match our limited tools. Multi-level integration in the sense of Baldwin [2] is an interesting problem possibly modelled by games, with gradual membership in coalitions such as games with fuzzy coalitions [5].

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