BETWEEN LIBERALISM AND DEMOCRACY

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Abstract. We study a class of voting rules that bridge between majoritarianism and liberalism. An outcome of the vote specifies who among the voters are eligible to a certain right or qualification. Each outcome serves also as a permissible ballot. We characterize axiomatically a family of rules parameterized by the weight each individual has in determining his or her qualification. In one extreme case, the Liberal Rule, each individual's qualification is determined by her. In the other, an individual's qualification is determined by a majority. We also propose a formalization of self-determination, and apply it in a characterization of the Liberal Rule.

1. Introduction

1.1. **Liberalism and democracy.** The liberal and the democratic principles dominate modern political thought. The first requires that decisions on certain matters rest with the individual and not with society. The second assigns the power of decision making to majorities. The question of the right balance between these two principles is an ongoing subject of debate in the public at large and among students of political thought. An effort is continually made to draw the line between the domains in which each principle applies: when the majority is justified in becoming involved in an individual's affairs, and when the person is allowed to make the decision alone.

Thus, for example, it is acceptable nowadays in liberal democracies that questions regarding the reading of certain books should not be decided by a majority of any form, and should be left to each individual's discretion. It is also obvious, that although all citizens have the same political rights, the question who holds the highest political position (say the president) is decided, roughly speaking, by a simple majority. But in between these two extreme cases—the majoritarian rule, and the liberal rule—there are decisions, concerning an individual, that on the one hand do not require a majority, and on the other are not determined by the individual either. Consider, for example, the

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right to be a *candidate* for the top position in society, or some other political position. Exercising such a right by an individual is usually not up to him or her. But one does not need a majority decision to be included on the list of candidates; in most liberal democratic regimes one needs only a relatively small group of supporters to be considered as a candidate.

It is our purpose here to study and characterize, in the framework of a social choice model, a family of social procedures, called here Liberal-Democratic rules, that lie between the majoritarian and liberal rules. The dichotomy between liberalism and democracy has been much discussed and debated in liberal thought since its conception. Liberalism and majoritarianism, separately, have also been extensively studied in the social choice literature. But as far is we know, presenting them as the extremes of a whole spectrum of rules is new.

1.2. **The model.** In order to study the relation between the liberal and the majoritarian rules in the same model, we follow a recent work by Kasher and Rubinstein (1998) who studied group identification. We present the model against the background of the first and most familiar social choice model of liberalism introduced in Sen's pioneering work, Sen (1970). Sen studied liberalism in terms of Arrow's social choice functions, Arrow (1951). Such a function maps any profile of preference orders of individuals to a social preference order. An individual is said to be *decisive* on two alternatives, for a given social choice function, if the function orders these alternatives in the same way as the individual does. Sen's minimal liberalism axiom requires, that there are at least two individuals each of which is decisive on two alternatives. He shows, then, that this axiom contradicts Pareto optimality, referring to this contradiction as the Liberal Paradox.

The following three features of Sen's framework should be emphasized.

- Social alternatives, like in Arrow's model, are primitives of the theory. The decisiveness of an individual over alternatives is given exogenously, and nothing in the structure of the alternatives expresses its relation to the individual.
- Social ordering is a function of the real preference order of individuals. The mechanism that is used to select an alternative is disregarded.
- Liberalism is defined as the ability of an individual to guarantee that his *preference* over certain outcomes prevails, as opposed to the ability to guarantee a certain subset of the outcomes themselves.

Many social choice models of liberalism differ from that of Sen on these points. The model used here differs in each of them. First of all, social alternatives here are structured. More specifically, each alternative is a subset of individuals. Although, as we said, no such structure exists in Sen's framework, the example he used to illustrate the Liberal Paradox can be interpreted as having this structure. In this example society has to decide who of its members is allowed to read Lady Chatterley's Lover. Obviously, an answer to this question, that is, the social alternative chosen, would be a subset of society. Like in Sen's example we study one right at a time. Social alternatives are subsets of individuals, where each subset is interpreted as the group of individuals eligible to the said right.

Second, unlike Sen, we do not study social choice functions. We study, instead, a voting rule, in which each individual proposes a subset of individuals, and given the profile of the proposed subsets, the rule determines the eligible individuals.

Finally, liberalism is defined here as the ability of an individual to guarantee certain aspects of the outcomes. If we think of a voting rule as a game form in which the various ballots are the strategies of each individual, then liberalism means here that each individual has a strategy that enables her to determine certain aspects of the social outcome. This variant of liberalism says, in the example above, that the question of whether an individual reads a certain book or does not, can be determined by her, by choosing appropriate ballots, irrespective of others' ballots. Preferences over outcomes play no role in this definition of liberalism.

The model studied here, can be used in problems other than the endowments of rights on issues that are considered private and personal matter. For example, the right to drive a car in the public domain. It can also be applied to procedures of elections. Thus, when it is required to compose an agreed upon list of candidates for the election of a dean, it is natural to ask faculty members to propose each a list and then apply some aggregating rule to form a final list. In Kasher and Rubinstein (1998),the model is applied to the problem of socially defining the extension of a given nationality. Similarly, we can use such models in order to socially define who are the poor individuals in society, or even who are the tall people.

The feature common to all these problems is that a certain qualification of persons is considered. Any group of individuals is a candidate, or an alternative, for a specification of the qualified individuals. The qualifications, as demonstrated by all these examples, may have a significant subjective component; individuals may have different views concerning who is qualified. Therefore a rule needs to be found, that transforms the various views concerning qualification into a socially defined group of qualified individuals.

1.3. **The main result.** We consider four axioms, which in one form or another are very standard in theories of social choice. The special twist of the rules that are characterized by these axiom stems from the particular feature of the model, where alternatives are subsets of the individuals.

The first axiom requires *monotonicity*. Consider the qualification of individuals as either tall and handsome (qualification a) or just tall (qualification b). Individuals may disagree on who is qualified as a or as b. But obviously, it is unanimously agreed that being qualified as a, implies being qualifies as b. The axiom requires that this unanimous agreement is reflected in the social definition of qualification. That is, all persons who are socially qualified as a, must also be socially qualified as b.

Next we require *independence*. The social qualification of an individual should depend only on the views held by all individuals concerning *this* individual and not others.

The data required to distinguish between qualified individuals and unqualified ones, come in two forms. Individuals can specify their opinions about either who is qualified, or who is not. The *Duality* axiom requires that applying the rule to either data results in the same distinction.

Finally, the *Symmetry* axiom requires that a rule does not depend on the names of individuals.

It is not surprising that these axioms are related to Majority Rules. Similar axioms characterize Majority Rules in various models. It is somewhat less obvious that these axioms characterize a family of rules which are a mixture of a Majority Rule and a Liberal Rule. We call the rules that satisfy the four axioms *Liberal-Democratic Rules*. This family of rules is parameterized by the *support* s, which is an integer that does not exceed half of the population size by more than 1. According to the rule with parameter s, one's characterization of oneself, whether qualified or not, is adopted by society if there are at least $s - 1$ other individuals who agree with it. Otherwise, the majority of individuals who do not agree with one's qualification of oneself have the say.

The two extreme values of s give rise to rules which express the two principles in their purest form. When $s = 1$, one's qualification of oneself is accepted by society. This is the Liberal Rule. When s takes the other extreme value, then the social qualification of an individual is determined by a Majority Rule which depends on the parity of the population size n. When n is odd, then the social qualification of an individual is determined by a simple majority of all opinions concerning her qualification. For even n, the largest value of s is $n/2+1$, and in this case an individual's qualification is determined by a simple majority of all individuals *other* than her.

The Liberal Rule was characterized axiomatically by Kasher and Rubinstein (1998). One of their axioms, which they call the Liberal axiom, requires that individuals can force certain outcomes. It says that if there is an individual who considers herself qualified, then there must be someone who is socially qualified, and if there is an individual who considers herself *un*qualified, then there must be someone who is socially *un*qualified. Thus, the special status of one's qualification of oneself is stated explicitly in the Liberal axiom. In our model, none of the axioms requires explicitly, neither that individuals has any power to determine certain outcomes, nor that one's qualification of oneself is more significant than others' qualification of one. The emergence of the weight given to one's qualification of oneself, in our model, depends on the combination of all four axioms. But we would like to highlight the role of the Symmetry axiom with this respect.

Symmetry of any object refers to the transformations of that object that leave it the same. Indeed, symmetry defines, sameness. It defines what is essential to the object and what is not. Social symmetries define the meaningful features of society. In our model, the Symmetry axiom says that the naming of individuals is insignificant. And as rules are expressed in terms of names, this means that individuals are indistinguishable, which reflects the idea of equality that underlies the democratic principle. However as social alternatives in our model are subsets of individuals, the Symmetry axiom says more. It allows name swapping only as long the same permutation is carried out both for the individuals as voters, and as elements of the social alternatives. Thus, breaking the linkage between an individual and a certain issue is not allowed by the symmetry axiom, or in other words, the axiom allows such a linkage to be socially meaningful. In some cases this linkage seems to be necessary. Suppose, for example that the issue is the reading of book X. If Adam changes his name to Barry, then Adam's reading book X, now becomes Barry's reading it. The linkage between one and one's reading X cannot be broken. This sounds almost tautological: it is impossible to think of *my* reading of a certain book as an issue which in some transformation of society is not linked to me, let alone linked to someone else. But the linkage between me and *my* apple—the one that I consider my property—is less obvious. We can easily think about transformation of the societal environment in which my apple is no longer mine. Indeed, some of the arguments made to justify property rights try to establish a logical link between individuals and the objects over which they have property rights. Thus, for example, Locke's argument is based on the work and efforts invested by the individuals in their property.

Our result seems to indicate, then, that rights are not necessarily primitive notions. Rights can be based, among other things, on the more primitive assumption that certain linkages between individuals and issues are considered socially meaningful and relevant. This assumption is more primitive, since it does not say how these meaningful linkages should be reckoned with: they can be liabilities, for example, rather than rights.

1.4. **Related works.** Structuring social alternatives, in order to study the liberal paradox, was proposed by Gibbard (1973a). Each individual, in his model, is associated with certain *issues*. An issue can be the color of John's shirt, or whether Marry reads book X. A social alternative specifies how all issues are resolved. Gibbard's liberal axiom requires that an individual should be decisive on *any* two alternatives that differ on only one issue which is associated with that individual. The alternatives in our model can viewed as a special case of Gibbard's. For each individual there is one issue, which can be resolved in one of two ways: it is the question whether or not the individual is qualified. Gibbard shows that there can be no social choice function that satisfies the liberal axiom. In contrast, in our model, where liberalism is outcome based, liberal rules obviously exist; these are the simple rules which allow individuals to resolve the issues related to them as they wish.

The procedural, game theoretic, aspects of social choice were first studied by Farquharson (1969). He characterized families of voting procedures axiomatically and analyzed their game theoretic aspects. Gibbard's manipulability result, Gibbard (1973b), further highlighted these game theoretic aspects.

Barbera et al. (1991) studied voting procedures in which the subsets of some fixed finite set serve both as the alternatives voted for, and as the ballots. The model here is a special case where this finite set is the set of individuals. They characterize the rules of voting by committees as those rules which are strategy-proof and satisfy voter sovereignty over separable preferences. In our terminology, voting by committees are rules that satisfy Monotonicity and Independence.

The distinction between the two variants of liberalism, preference based and outcome based, has been discussed by many authors. Nozick (1974) criticized Sen's preference based liberalism. Gärdenfors (1981) formalized outcome based liberalism by a game theoretic model. Liberalism as it is understood here corresponds to the notion of dichotomous veto power in Deb et al. (1997). A definition of liberalism, applied to a game forms, is discussed in Riley (1989). There, a voting rule is said to be liberal if for every profile of preferences there exists a strategic equilibrium in pure strategies. Following this reasoning, a Liberal Paradox occurs if for some profile every strategic equilibrium is not efficient. Riley uses the concept of strong equilibrium instead of efficient strategic equilibrium.

2. The main characterization

2.1. **Preliminaries and notations.**

Let $N = \{1, \ldots, n\}$ be a set of individuals. These individuals are facing the problem of collectively choosing a certain subset of N, of those individuals who have a certain qualification. The input for this collective choice is the personal views that individuals have concerning who is qualified. These views are summarized by a *profile* which is an $n \times n$ matrix $P = (P_{ij})$, the elements of which are 0's and 1's. When $P_{ij} = 1$, we say individual i qualifies individual j. Thus, row i in the matrix P describes i's personal view of the group of qualified individuals; it is the set $\{j \mid P_{ij} = 1\}$. Column j tells us who are the individuals i who qualify j.

A *Rule* is a function f which associates with each profile P, a vector $f(P) = \big(f_1(P), \ldots, f_n(P)\big)$ of 0's and 1's, which is the indicator function of the group of socially qualified individuals, $\{j \mid f_j(P) = 1\}$.

For $x \in \{0, 1\}$ we use the standard notation $\overline{x} = 1 - x$. Accordingly, $\overline{P} = (\overline{P}_{ij})$, and $\overline{f}(P) = \big(\overline{f_j(P)}\big)$. For arrays A and B (matrices or vectors) of the same dimension we write $A \geq B$ if this inequality holds coordinatewise.

2.2. **The axioms.**

Suppose we are interested in socially qualifying persons who have the right to read book X and persons who have the right to read all books. Obviously, from each individual's point of view, any person who has the right to read all books, has the right to read X. In terms of the profiles P' and P which correspond to these two qualifications, this is equivalent to saying that $P \geq P'$. We should expect, then, that every one who is *socially* qualified as having the right to read all books is also *socially* qualified as having the right to read X.

Monotonicity. If $P \geq P'$, then, $f(P) \geq f(P')$.

We require, next, that the social qualification of individual j is independent on what individuals think about the qualification of individuals other than i.

Independence. If P and P' are profiles such that for some $j \in N$, $P_{ij} = P'_{ij}$ for all $i \in N$, then $f_j(P) = f_j(P')$.

Qualifications come in pairs; an individual is either qualified as one who is allowed to read Lay Chatterley's Lover, or he is qualified as one who is not allowed to read it. He is either qualified as a Jew, or as a Non-Jew. We can socially qualify persons as Jews by asking individuals whom they qualify as Jews, or we can socially qualify Non-Jews by asking individuals to qualify Non-Jews. We require that either way we arrive at the same distinction between Jews and Non-Jews. We note that if the profile P describes individuals' views concerning a certain qualification Q, then the profile $\overline{P}$ describes the views of the same individuals concerning the qualification non-Q. Our requirement is expressed then as follows.

Duality. $f(\overline{P}) = \overline{f}(P)$.

We require that social qualification should not change if individuals switch their names. Name switching is described by a permutation π of N. We think of $\pi(i)$ as the *old* name of the person whose *new* name is i. For a profile P, given in terms of the old names, we denote by πP the profile after the name switching. To say that i qualifies j, using the new names, means that the individual whose old name is $\pi(i)$ qualifies the person whose old name is $\pi(j)$. Thus, $(\pi P)_{ij} = P_{\pi(i)\pi(j)}$. The axiom requires that given the profile πP, in terms of the new names, individual i is socially qualified, if and only if individual $\pi(i)$ was qualified when the profile was P. Denoting by $\pi f(P)$ the vector $\big(f_{\pi(1)}(P), \ldots, f_{\pi(n)}(P)\big)$, the axiom can be succinctly stated as follows.

Symmetry. For any permutation π of N, $f(\pi P) = \pi f(P)$.

2.3. **Liberal-Democratic Rules.**

Definition 1. Let s be an integer, such that $1 \leq s \leq n/2 + 1$. A *Liberal-Democratic Rule* with *support* s is a social rule f^s that satisfies for each individual j and profile P,

$$f^s_j(P) = P_{jj} \quad \text{if and only if} \quad |\{i \mid P_{ij} = P_{jj}\}| \geq s.$$

According to the rule f^s, j's view of herself is accepted by society, if and only if j's view is shared by at least s individuals (including her).

The support s expresses the importance of an individual's view of herself to her social qualification; the lower s is, the more influential is the way individuals perceive themselves. The two extreme values of s deserve special attention.

We call the rule f^1, the *Liberal Rule*. As the set $\{i \mid P_{ij} = P_{jj}\}$ always contains j, it follows that for all profiles P, $f^1_j(P) = P_{jj}$. In other words, j's social qualification is solely determined by him.

The other extreme case is, when s is the largest integer that does not exceed $n/2+1$. We will show that when n is odd, j's view of himself does not have more weight than other's views, and when n is even his view does not count at all.

Consider first an odd n. The highest value that s can take in this case is $(n+1)/2$, and the rule f^s is a Simple Majority Rule in which j does not have any special role. Indeed, suppose that a majority of individuals qualify j as x, where x is either 0 or 1. That is, $|\{i \mid P_{ij} = x\}| \geq s$. Now, if $P_{jj} = x$ then $f_j(P) = x$ by definition. If $x \neq P_{jj}$, then $|\{i \mid P_{ij} = P_{jj}\}| \leq n - s < s$, and $f_j(P) = x$.

Suppose now that n is even, then $s = n/2+1$. In this case a simple majority of all individuals *other* than j (i.e., $n/2$, or more, out of the $n-1$ individuals) determines his qualification. Indeed, suppose that $|\{i \mid P_{ij} = x,\ i \neq j\}| \geq n/2$. If $P_{jj} = x$ then $|\{i \mid P_{ij} = P_{jj},\}| \geq n/2+1 = s$ and $f^s_j(P) = x$. If $x \neq P_{jj}$, then $|\{i \mid P_{ij} = P_{jj}\}| \leq n/2 < s$, and again, $f^s_j(P) = x$.

Theorem 1. *A social rule satisfies the axioms of Monotonicity, Independence, Duality, and Symmetry, if and only if it is a Liberal-Democratic Rule. Moreover, all four axioms are independent.*

3. Self-determination

The political principle of Self-determination says that a group of people recognized as a nation has the right to form its own state and choose its own government. One of the main difficulties in applying Self-determination is that it grants the right to exercise sovereignty to well defined national identities; it assumes that the self is well defined. In many cases the very distinct national character of the group is under dispute. Such disputes can be resolved, at least theoretically, by a voting rule. Here we want to examine rules which grant the self the right to determine itself. For brevity we refer to this property of voting rules as Self-Determination and not Determination of the self, despite the new meaning we give it.

Suppose we want to define the nationality of Hobbits. On first examination the rquirement that Hobbits determine who Hobbits are seems to be circular. But this circularity can be avoided in two ways. In the first, we require that after defining Hobbits using the rule f, changing non-Hobbits' opinion about Hobbits and applying again the rule will result in the same definition of Hobbits. We call this axiom *Exclusive Self-determination,* because it is expressed in terms of excluding non-Hobbit from those who have the power to define Hobbits. We call the second axiom *Affirmative Self-determination,* because it states directly the right of Hobbits to define Hobbits. In order to formulate it we use the rule f to qualify not only Hobbits, but also *definers* of Hobbits. Inclusive Self-determination says that the two groups, of Hobbits and of definers of Hobbits, should coincide. We show that each of these axioms combined with three of the previously defined axioms characterize the Liberal Rule. That is, the right of a collective to define itself is reduced to the right of each individual to define herself as part of this collective.

The formulation of the first version of Self-determination is straightforward.

Exclusive Self-Determination. For a profile P, let

$$H = \{j \mid f_j(P) = 1\}.$$

If Q is a profile such that $P_{i,j} \neq Q_{i,j}$ only if $i \notin H$ and $j \in H$, then $f(Q) = f(P)$.

Theorem 2. *The Liberal Rule is the only one that satisfies the axioms of Monotonicity, Duality, Independence, and by Exclusive Self-determination.*

For the formulation of Affirmative Self-determination we need to be able to qualify individuals as definers of Hobbits. For this purpose we look closer at the working of profiles and rules The social profile P, which we use to define Hobbits, can be thought of as a binary relation on the set of individual, where a pair of individuals (i, j) belongs to this relation if i defines j as a Hobbit. Individuals play two roles in this relation: an individual in the first place of the relation plays the role of a *definer* of Hobbits, and in the second place—the role of one who is *defined* as a Hobbit. Being defined as a Hobbit in the profile requires a definer. The rule f, which constructs from P the group of Hobbits, generates, in terms of the binary relation, a set of individuals who play the role of the second place of the relation.

Thus, f can be though of as a rule that generates for any given binary relation on individuals a subset of them which can be described as "individuals of the second-place-type." In particular f can be applied to the *inverse* relation: "j is defined by i as a Hobbit". The role individuals play in the second place of the inverse relation is that of *definer* of Hobbits. Thus, f generates from this binary relation a subset of individuals who are the definers of Hobbit.

We can now express, in terms of the rule f, the idea that Hobbits have the right to define themselves as such. It says that the group of individuals qualified as Hobbits and the group of individuals qualified as definers of Hobbits are the same. An underlying assumption here is that the rule f is universally used for all binary relationships on the set of individuals.

Formally, if the profile P describes a binary relation, then the inverse relation is described by the transposed matrix P^t, where $P^t_{ij} = P_{ji}$. Thus, if in the example above, the groups of Hobbits and of definers of Hobbits are constructed in the same way, then these two groups are given by $f(P)$ and $f(P^t)$, respectively. Affirmative Self-determination is now stated by,

Affirmative Self-determination. $f(P) = f(P^t)$

Theorem 3. *The Liberal Rule is the only one that satisfies, Monotonicity, Duality, Independence and Affirmative Self-determination.*

4. Discussion

4.1. **The Symmetry axiom.** In our model a society N is required to qualify its own individuals. A more general model is one in which society is required to qualify individuals of some set M, disjoint of N. A social profile, in this case, would be a matrix P, not necessarily a square one, where the rows are labeled by individuals in N, and columns are labeled by individuals in M. All three axioms other than Symmetry can be stated in the same way, and equally motivated for the more general case.

The Symmetry axiom is special, though, to our model. In the general model there are two axioms that are related to Symmetry. First, we can require *Anonymity* of the individuals of society. That is, if the names of the members of N—the qualifying individuals—are permuted, then the socially qualified group remains the same. Formally, for any permutation τ of N, we require that $f(\tau P) = f(P)$, where $(\tau P)_{ij} = P_{\tau(i)j}$.

Second, we can require *neutrality* of the qualified persons, as follows. Let σ be a permutation of M, and P a profile. Suppose, now, that we change the profile such that each individual i in the society qualifies j iff he qualified $\sigma(j)$ in P. We require that socially qualified persons in the new profile are those obtained by permuting the qualified persons in the original profile. Formally, it is required that $f(\sigma P) = \sigma f(P)$, where $(\sigma P)_{ij} = P_{i\sigma(j)}$.

Requiring Anonymity in our model implies the disregarding of the natural identification of society with the set of qualified individuals. In particular, the linkage between the individual and the issue to which her qualification is considered becomes socially irrelevant. Among the Liberal-Democratic Rules only the pure Majority Rule, for odd n, satisfies this.

Requiring neutrality, on top of Independence, implies that the same rule is used by society to determine the qualification of each individual. Again, only the pure Majority Rule, for odd n, satisfies this, among all the Liberal-Democratic Rules.

We conclude, then, that by adding either of these two axioms, or both, to the four axioms in Theorem 1 results in a unique mechanism, the Simple Majority Rule, when n is odd, and no possible rule for even n. Observe also, that these two axioms imply the Symmetry axiom, and therefore omitting Symmetry and adding Anonymity and neutrality gives the same characterization as adding both on top of Symmetry.

4.2. **The Independence axiom.** In certain social situations the Independence axiom is untenable. Consider for example election of a committee, of a certain fixed size, from a list of candidates. The list may consist of all voters, like the model we discuss in this paper. Suppose, moreover, that each individual votes by specifying a subset of individuals (which are also the candidates). A social rule, in this case, will be defined exactly as it is defined here, with one difference. The range of such a rule should be restricted to subsets of the size of the committee. In this case the Independence axiom is not reasonable. The question whether a certain individual should qualify as a member of the committee should depend not only on what voters think about her, but also on what they think about others, and how the others compare to her. A rule for such a problem is approval voting (see Brams and Fishburn (1978)), in which the elected committee consists of the individuals with the highest score, where the score of an individual is the number of voters who includes her in their votes. Obviously, approval voting does not satisfy Independence.

The Independence axiom is not only unreasonable when such restrictions on the range of rules are imposed, it also implies that such restrictions are impossible. Indeed, the range of a rule that satisfies Independence includes any subset of individuals whose qualification depends on the social profile. Formally,

Proposition 1. *Let f be a rule that satisfies the axiom of Independence. Then, there is a subsets, T of N such that the projection of* *the range of f on $\{0,1\}^T$ is all of it, and for each individual $j \notin T$, $f_j(P)$ is independent of the profile P. If, moreover, the rule f satisfies Symmetry, then its range is either $\{0,1\}^N$, or $(1,\ldots,1)$, or $(0,\ldots,0)$.* [1]

The Independence axiom is inappropriate not only when individuals should be compared. It is also inconsistent with rules that relegate the social power to a subset of individuals. Suppose, for example, that the qualification of an individual is socially determined by the way others view only *him*, but not *all* others—only views of individuals who are unanimously qualified are counted (see, for example, the rule in the proof of Theorem 1, that does not satisfy Independence). Here, an individual's qualification depends on views concerning other individuals; such views determine who are the "judges" of qualification, and through them who is qualified.

4.3. **An alternative presentation of Liberal-Democratic Rules.** Consider for each individual j, and each even number e which satisfies $0 \le e \le n$, a weighted majority game v_j^e. The weight of all individuals in this game is 1, except for j whose weight is $n - e$. The quota in the game is half of the total weight. Obviously, for each coalition S, at least one of S and its complement is a winning coalition. Since e is even, the total weight is odd, and therefore only one of the two coalitions is winning. This enables us to define a rule g^e as follows. For any profile P and individual j, consider the two complementary subsets $\{i \mid P_{ij} = 0\}$ and $\{i \mid P_{ij} = 1\}$. One, and only one of these subsets is a winning coalition in v_j^e. Now, $g_j^e(P)$ is 0 or 1, according to which one of the two coalitions is winning.

Proposition 2. *For each Liberal-Democratic Rule f^s, $f^s = g^{e(s)}$, where $e(s)$ is the one to one map of the domain $\{s \mid 1 \le s \le n/2+1\}$ onto $\{e \mid e \text{ is even}, 0 \le e \le n\}$ defined by $e(s) = 2s - 2$.*

4.4. **The Liberal Rule and the Liberal Axiom.** We assume, now, that each individual i has a strict preference relation $\succ_i$ over subsets of individuals (or equivalently over $\{0,1\}^N$). For given preferences of individuals we can select a subset of individuals by applying the Liberal

[1] Rubinstein and Fishburn (1986) also record a connection between Independence and restrictions on the range of a rule. They studied models in which the rows of P and the range of f can be neither $(1,\ldots,1)$ nor $(0,\ldots,0)$. They show that the only rules that satisfy the axioms of Independence and Consensus are the dictatorial Rules. The requirement that the range of a rule is full plays an important role in Barbera et al. (1991) and is called voter sovereignty.

Rule, f^1, to the profile of most preferred subsets. We may ask, now, whether the Liberal Rule satisfies the Liberal Axiom.

Since social alternatives in our model are a special case of those in Gibbard (1973a), it is most appropriate to consider Gibbard's version of the Liberal Axiom. In terms of our setup, a rule f satisfies this axiom when each individual is decisive on certain alternatives as follows. If x and y are two elements of $\{0,1\}^N$, such that $x_j = y_j$, for each $j \neq i$, and i prefers x to y, then $f(P) \neq y$. As is shown in Gibbard (1973a) there is no rule, defined over all preference orders, that satisfies this axiom. However, he shows that there are rules that satisfy a Restricted Liberal Axiom, in which decisiveness of individuals is required only when preferences are *unconditional*. Individual i's preference is said to be unconditional (for his issue), when for any x and y as above, if i prefers x to y, then he prefers also x' to y', whenever, $x'_i = x_i$, $y'_i = y_i$ and $x'_j = y'_j$ for all $j \neq i$. In terms of subsets this means that if for some S where $i \notin S$, $S \cup \{i\} \succ_i S$, then for all T such that $i \notin T$, $T \cup \{i\} \succ_i T$, and if for some S where $i \notin S$, $S \succ_i S \cup \{i\}$, then for all T such that $i \notin T$, $T \succ_i T \cup \{i\}$.

It is straightforward to see that f^1 satisfies the Restricted Liberal Axiom. Moreover, Gibbard's proof for the existence of a social choice function that satisfies the Restricted Liberal Axiom is carried out by constructing a function that extends the Liberal Rule to the more general case studied in Gibbard (1973a).

4.5. **Sincerity.** Individual preferences combined with the rule f^1 define a game. We can ask, then, under which conditions will it be an equilibrium for the individuals to be *sincere* (as defined by Farquharson (1969)), i.e., to propose their most preferred subset. It is easy to see that if an individual's preference are unconditional, then proposing his most preferred subset is a dominant strategy. Indeed, an individual's proposal determines only whether he belongs to the socially selected subset of qualified persons or not. If individual i belongs to her most preferred set, then by conditionality she prefers to join any subset, which she achieves by proposing her most preferred subset. The argument is similar when she does not belong to her most preferred subset.

The model here is a special case of the one in Barbera et al. (1991), where individuals vote for a subset of a given set K. A preference order $\succ$ on subsets of K is called *separable*, in that work, when for any subset T and $x \notin T$, $T \cup \{x\} \succ T$ if and only if x is an element of the most preferable subset. In our model, separability implies unconditionality, and therefore guarantees that proposing the most preferred subset of individual is a dominant strategy for f^1. It is easy to see also, that the stronger condition of separability guarantees that truth telling is dominant for every rule which satisfies Monotonicity and Independence (and hence in all Liberal-Democratic Rules). This is the "easy" part of the characterization of voting by committees in Barbera et al. (1991).

4.6. **The Liberal and the Pareto axioms.** As noticed by Gibbard, even the Restricted Liberal Axiom contradicts Pareto efficiency. Indeed, suppose there are two individuals who have to determine who reads X. Individual 1 cares foremost for 2's education: he prefers any alternative in which 2 reads X to any alternative in which 2 does not. In the second place, 1 cares for his own education: other things being equal (i.e., given 2's behavior) he would rather read X than not. Similarly, 2 cares foremost for 1's moral fiber, and therefore she prefers any alternative in which 1 does not read X to any alternative in which 1 does. Likewise, other things being equal she prefers refraining from reading salacious X. Proposing their most preferred alternatives (1 proposes that both read it, 2—that both do not), the Liberal Rule allows 1 to read X and 2 not to. Yet both prefer that 1 does not read it, and 2 does.

Each individual preference in this case is unconditional for his/her issue. The reason for the failure of Pareto efficiency is due to the excessive nosiness of the individuals: each individual minds foremost the other's business. If we require the opposite, that each individual cares in the first place for his/her issue and only in the second place for others', than the Liberal Rule satisfies Pareto efficiency. We say that i's preference is *moderately nosy* if when i prefers x to y he also prefers x' to y', whenever $x'_i = x_i$, and $y'_i = y_i$. When individuals are moderately nosy, there cannot be an alternative x which is unanimously preferred to $f^1(P)$. Indeed, if $x \neq f^1(P)$, then for some j, $x_j \neq f^1_j(P) = P_{jj}$. As $(P_{j1}, \ldots, P_{jn})$ is the most preferred alternative by j, it follows by moderate nosiness, that it is also preferred to x.

5. Proofs

Proof of Theorem 1. It is easy to see that any Liberal-Democratic rule satisfies Independence. To prove that it satisfies Monotonicity, assume that $P \geq P'$ and let $f = f^s$. Suppose that $f_j(P) = 1$. Then either $P_{jj} = 1$ and $|\{i \mid P_{ij} = 1\}| \geq s$, or $P_{jj} = 0$ and $|\{i \mid P_{ij} = 1\}| \geq n - s + 1$. If $P'_{jj} = P_{jj}$ then in either case $f_j(P') = 1$, since $|\{i \mid P'_{ij} = 1\}| \geq |\{i \mid P_{ij} = 1\}|$. If $P_{jj} = 0$ and $P'_{jj} = 1$, then $|\{i \mid P'_{ij} = 1\}| \geq n - s + 2 \geq s$ and therefore $f_j(P') = 1$.

To show that Symmetry holds for $f = f^s$, let π be a permutation of N. Then, $f_j(\pi P) = (\pi P)_{jj} = P_{\pi(j)\pi(j)}$ iff $|\{i \mid (\pi P)_{ij} = (\pi P)_{jj}\}| \geq s$, i.e., $|\{i \mid P_{\pi(i)\pi(j)} = P_{\pi(j)\pi(j)}\}| \geq s$. As π is one-to-one, the latter condition is equivalent to $|\{\pi(i) \mid P_{\pi(i)\pi(j)} = P_{\pi(j)\pi(j)}\}| \geq s$. But this is exactly the necessary and sufficient condition that $f_{\pi(j)}(P) = P_{\pi(j)\pi(j)}$. Thus, $f_j(\pi P) = f_{\pi(j)}(P)$.

To show that Duality holds, observe that $f_j(\overline{P}) = \overline{P}_{jj}$ iff $|\{i \mid \overline{P}_{ij} = \overline{P}_{jj}\}| \geq s$. This inequality is equivalent to $|\{i \mid P_{ij} = P_{jj}\}| \geq s$, which holds iff $f_j(P) = P_{jj}$, or equivalently $\overline{f}_j(P) = \overline{P}_{jj}$.

We now prove that any social rule f that satisfies the four axioms is a Liberal-Democratic Rule. By the Independence axiom, $f_j(P)$ depends only on column j in P. Therefore, for each j there exists a function $h_j\colon \{0,1\}^N \to \{0,1\}$ such that $f_j(P) = h_j(P_{1j}, \ldots, P_{nj})$. Using the Duality axiom we conclude,

$$h_j(\overline{P}_{1j}, \ldots, \overline{P}_{nj}) = f_j(\overline{P}) = \overline{f}_j(P) = \overline{h}_j(P_{1j}, \ldots, P_{nj}).$$

Hence, for any $x \in \{0,1\}^N$,

$$h_j(\overline{x}) = \overline{h}_j(x). \tag{1}$$

From Monotonicity it follows that if $P \geq P'$, then

$$h_j(P_{1j}, \ldots, P_{nj}) = f_j(P) \geq f_j(P') = h_j(P'_{1j}, \ldots, P'_{nj}).$$

Thus, h_j is monotonic, that is, for $x, y \in \{0,1\}^N$, if $x \geq y$, then $h_j(x) \geq h_j(y)$.

Let π be a permutation of N. Then,

$$f_j(\pi P) = h_j\big((\pi P)_{1j}, \ldots, (\pi P)_{nj}\big) = h_j(P_{\pi(1)\pi(j)}, \ldots, P_{\pi(n)\pi(j)}).$$

and

$$f_{\pi(j)}(P) = h_{\pi(j)}(P_{1\pi(j)}, \ldots, P_{n\pi(j)}).$$

Since by the Symmetry axiom, $f_j(\pi P) = f_{\pi(j)}(P)$ it follows that,

$$h_j(P_{\pi(1)\pi(j)}, \ldots, P_{\pi(n)\pi(j)}) = h_{\pi(j)}(P_{1\pi(j)}, \ldots, P_{n\pi(j)}). \tag{2}$$

Assume, now, that $\pi(j) = j$. Then, for such a permutation, equation (2) yields $h_j(P_{\pi(1)j}, \ldots, P_{\pi(n)j}) = h_j(P_{1j}, \ldots, P_{nj})$. Therefore, for any such permutation, and $x \in \{0,1\}^N$, $h_j(x_{\pi(1)}, \ldots, x_{\pi(1)}) = h_j(x)$. It follows that if x and y are such that $x_j = y_j$, and $\sum_{i \neq j} x_i = \sum_{i \neq j} y_i$, then $h_j(x) = h_j(y)$. Thus, there exists a function $g_j(a, b)$, where $a \in \{0,1\}$ and $b \in \{0, \ldots, n-1\}$, such that $h_j(x) = g_j(x_j, \sum_{i \neq j} x_i)$. Moreover, by (2), for any permutation π, $h_j(x_{\pi(1)}, \ldots, x_{\pi(n)}) = h_{\pi(j)}(x)$. But then $g_j(x_{\pi(j)}, \sum_{i \neq j} x_{\pi(i)}) = g_{\pi(j)}(x_{\pi(j)}, \sum_{i \neq \pi(j)} x_i)$, and this means that $g_j = g_{\pi(j)}$. Therefore, we can write g for all g_j.

As h_j is monotonic, g is monotonic in both arguments. Also, by (1),

$$g(a, b) + g(1-a, n-1-b) = 1. \tag{3}$$

By (3), g can not be identically 0, and therefore, there exist a and b such that $g(a, b)$=1. By the monotonicity of g, $g(1, b) = 1$. Let c be the first integer for which $g(1, c) = 1$. If $c = 0$, then the monotonicity of g implies $g(1, \cdot) = 1$, and by (3) $g(0, \cdot) = 0$. If $c > 0$, then by the definition of c, and (3),

$$\begin{aligned} g(1, c-1) &= 0, & g(1, c) &= 1, \\ g(0, n-c) &= 1, & g(0, n-1-c) &= 0. \end{aligned}$$

Denote $s = c + 1$, then $g(1, b) = 1$ iff $1 + b \geq s$, and $g(0, b) = 0$ iff $b \leq n - s$. To see that $s \leq n/2 + 1$ observe that either $s = 1$, or else $g(0, n-c) = 1$ and by Monotonicity, $g(1, n-c) = 1$. Hence, $n - c \geq c$, or $c \leq n/2$.

Now, $f_j(P) = g(P_{jj}, \sum_{i \neq j} P_{ij})$. When $P_{jj} = 1$, then $f_j(P) = P_{jj}$ iff $\sum_i P_{ij} \geq s$ iff $|\{i \mid P_{ij} = P_{jj}\}| \geq s$. If $P_{jj} = 0$, then $f_j(P) = P_{jj}$ iff $\sum_i P_{ij} \leq n - s$, which is equivalent to $|\{i \mid P_{ij} = P_{jj}\}| \geq s$. This is precisely the condition that defines the Liberal-Democratic Rule with support size s.

To show that the axioms are independent we describe for each axiom a rule that does not satisfy the axiom but does satisfy all other axioms. We omit the detail of the proof.

Monotonicity. Consider the rule defined by $f(P) = \overline{f^1}(P)$, where $f^1(P)$ is the Liberal Rule. Fix individual j. Then, $f_j(0) = 1$, where 0 here is the zero matrix. For P' with $P'_{jj} = 1$, $P' \geq 0$, and $f_j(P') = 0$.

Note also that if we define f^s exactly like Liberal-Democratic Rules, but with $s > n/2 + 1$, then by the proof of theorem 1, f^s satisfies all axioms but Monotonicity.

Duality. Consider the rule which defines the group of qualified as the subset of all individuals that are unanimously qualified. That is, $f(P) = 1$ iff $P_{ij} = 1$, for all i.

Symmetry. This axiom is obviously violated by dictatorial Rules, like $f(P) = 1$ iff $P_{1j} = 1$.

Independence. Define $f(P)$ as follows. Let $S = \{j \mid P_{ij} = 1, \text{ for all } i\}$ and $T = \{j \mid P_{ij} = 0, \text{ for all } i\}$. The sets S and T consist of the individuals who are unanimously considered qualified or unqualified, correspondingly. The rule accepts this unanimous verdict. That is, for each $j \in S$, $f_j(P) = 1$ and for each $j \in T$, $f_j(P) = 0$. Qualification of individuals that are not in $S \cup T$ is determined by the members of $S \cup T$ as follows. Define for each $j \in N \setminus (S \cup T)$, $s_j = |\{i \mid P_{ij} = 1\} \cap S|$ and $t_j = |\{i \mid P_{ij} = 0\} \cap T|$. If $s_j > t_j$ then $f_j(P) = 1$; if $t_j > s_j$, then $f_j(P) = 0$. Finally, if $s_j = t_j$ then j's qualification is determined by herself, i.e., $f_j(P) = P_{jj}$. ∎

Proof of Proposition 1. Let S be the subset of all individuals j for which $f_j(P)$ is independent of P, and let $T = N \setminus S$. For every $j \in T$, there must be profiles Q^{j0} and Q^{j1} such that $f_j(Q^{j0}) = 0$ and $f_j(Q^{j1}) = 1$. Given any $x \in \{0,1\}^T$, construct a profile Q as follows. For $j \in T$, if $x_j = 0$, than $Q_{ij} = Q^{j0}_{ij}$ for all i, and if $x_j = 1$, than $Q_{ij} = Q^{j1}_{ij}$ for all i. The rest of Q can be defined arbitrarily. By Independence, for all $j \in T$, $x_j = f_j(Q)$.

Suppose that f satisfies also the Symmetry axiom. If $S = \emptyset$ then $T = N$, and the range of f is $\{0,1\}^N$. Assume, now, that $j \in S$. Then, $f_j(P)$ is independent of P. We show, moreover, that $f_k(P) = f_j(P)$ for each k and P, and hence the range of f is either $(1,\ldots,1)$ or $(0,\ldots,0)$. Indeed, let π be a permutation of N such that $\pi(j) = k$. Then, $f_j(P) = f_j(\pi P) = f_{\pi(j)}(P) = f_k(P)$, where the first equality follows from the definition of S and the second from Symmetry. ∎

Proof of Theorem 2. As in the previous proof, it is easy to see that the Liberal Rule satisfies the axioms. Assume now that f satisfies the axioms. We show that for any profile P and each individual k, $f_k(P) = P_{kk}$. By duality it is enough to consider the case $P_{kk} = 1$. Assume, per absurdum, $f_k(P) = 0$.

For a subset T of N, denote by P^T the profile which is the same as P, except that all individuals in T, qualify k. (i.e., $P^T_{jk} = 1$ for $j \in T$, and $P^T_{jl} = P_{jl}$ when $j \notin T$ or $l \neq k$.) By Duality, Monotonicity, and Independence, $f_k(P^N) = 1$. Let $S = \{j \mid P_{jk} = 1\}$. Then, $P^S = P$, and hence $f_k(P^S) = 0$. Therefore, by Monotonicity, there are T and m, such that $S \subseteq T \subseteq N \setminus \{m\}$, and $f_k(P^T) = 0$, and $f_k(P^{T\cup\{m\}}) = 1$. Obviously, $m \neq k$ because otherwise, as $P_{kk} = 1$, $P^T = P^{T\cup\{m\}}$. Therefore, by Independence, $f_m(P^T) = f_m(P^{T\cup\{m\}})$. If both are 0, then Exclusive Self-determination is contradicted at the profile $P^{T\cup\{m\}}$, as k is qualified in this profile, m is not, and yet m's change of vote changes k qualification. If they are both 1, then this axiom is similarly contradicted at the profile $\overline{P^T}$, by Duality. ∎

Proof of Theorem 3. It is easy to see that the Liberal Rule satisfies the axioms. Assume now that f satisfies the axioms. We show that for any profile P and each individual k, $f_k(P) = P_{kk}$. By Duality it is enough to consider the case $P_{kk} = 1$. Let Q be a matrix such that for all i, j, $Q_{ik} = P_{ik}$ and $Q_{kj} = 1$. Denote by 1_{NxN} the matrix of ones. Then, $f_k(P) = f_k(Q) = f_k(Q^t) = f_k(1_{NxN})$, where the first and last equality hold by Independence, and the second equality holds by Affirmative Self-determination. Clearly, Duality and Monotonicity imply that $f_k(1_{N\times N}) = 1$. ∎

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