Informational Cascades Elicit Private Information*

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Abstract

We introduce cheap talk in a dynamic investment model with information externalities. We first show how social learning adversely affects the credibility of cheap talk messages. Next, we show how an informational cascade makes thruthtelling incentive compatible. A separating equilibrium only exists for high surplus projects. Both an investment subsidy and an investment tax can increase welfare. The more precise the sender's information, the higher her incentives to truthfully reveal her private information.

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1 Introduction

A decision maker typically faces a lot of uncertainty when deciding over a course of action. For example, investors know they face the risk of losing all their money. Students do not know which University degree maximises their future job market prospects. Consumers do not know which product offers the best price/quality ratio... To be more specific, suppose someone has the opportunity to invest in a project whose returns are positively correlated with the "general future health of the U.S. economy". Obviously, assessing the future state of the U.S. economy is a hard task and no human being is smart enough to make an errorless prediction about it. However, investors do not live like Robinson Crusoe - isolated on an island. Instead, they realise that the economy is populated by many other potential investors who all face the same type of risk. Moreover, they know that if they were to meet and exchange opinions, this would enable them to reduce their forecasting error. But if investors really care about one another's opinions, how will this information be disseminated throughout the economy?

Casual observation of everyday life suggests there are two different channels through which investors may learn about one another's opinions: one may learn through *words* or one may learn through *actions*. With the former, we have in mind a situation in which one investor simply tells her opinion to (possibly many) other investors. For example, every now and then managing directors of important companies appear in the media and express their opinions on a wide range of issues such as future technological developments, future oil prices, future market growth, etc... Some institutions are even specialised in collecting and summarising the opinions of a large number of market participants. For example, the Munich-based IFO institute for economic research releases a quarterly index reflecting the business confidence of the average German investor. With learning through actions, we mean that if someone invests in a one-million-dollar project in the U.S., this reveals her confidence in the American business climate.

In this paper, we analyse the interaction between both communication channels. More specifically, we consider the following set-up: N players must take an investment decision and possess a private, imperfect signal concerning the future state of the world. Investment is only profitable in the good state. For the sake of simplicity, we assume that the returns of the investment project only depend on the state of the world. Hence, for efficiency reasons one would want to have all players truthfully exchanging their signals. Players can invest in two periods. In the second period, everyone observes how many agents invested at time one. One randomly drawn player (the sender) is asked to divulge her private information (i.e. her signal) to the other players (the receivers) prior to the first investment period, and we compute all monotone equilibria[1] of our game.

We first show that both communication channels do not co-exist peacefully, in the sense that there does not exist a monotone equilibrium in which the sender truthfully announces her private information and in which subsequently a lot of information is generated through actions. This tension between both communication channels manifests itself differently depending on the surplus generated by the project: for low surplus projects the unique monotone equilibrium is the pooling one[2], while for high surplus projects there also exists an equilibrium in which the sender truthfully reveals her private information but in which "little" information is transmitted through actions.

The intuition behind this result goes as follows: in our model expected payoffs are driven by the relative number of optimists in the economy (the higher the proportion of optimists in the population, the higher the probability that the world is in the good state). At time two all players observe the number of period-one investments and use this knowledge to get an "idea" of the proportion of optimists in the economy. This updating process depends on the period-one investment strategies[3] (which are affected by the sender's message). If the investment only generates a low surplus, pessimists will - independently of the sender's message - never invest in the first period. Both sender's types then want to send the message which makes the optimists invest with as large a probability as possible[4]. Thus both sender's types share the same preferences over the receivers' actions, and therefore no information can be transmitted through cheap talk. For high surplus projects, however, this intuition is incomplete. In that case all players face a positive gain of investing after receiving the message "I am an optimist". If a player then believes that everyone will invest at time one, it's optimal for her to do so too (i.e. an informational cascade[5] in which everyone invests

[1]Bluntly stated, in a monotone equilibrium we rule out the (unintuitive) possibility that pessimistic players are more likely to invest (at time one) than optimistic ones.

[2]In this equilibrium no credible information is transmitted through words, but "a lot" of information is transmitted through actions.

[3]For example, upon observing k period-one investments, players compute different posteriors if pessimists invested (at time one) with zero probability and optimists with a probability equal to one, than if pessimists invested with the same probability as the optimists.

[4]If the sender succeeds for example in making the optimistic receivers invest with probability one, she perfectly learns the proportion of optimists in the population.

[5]All players - irrespective of their private information - rely on the public information (i.e. the message of the sender) and take the same action at time one. By definition, this is an informational cascade.

is ignited by the arrival of a favourable message). In our model this informational cascade induces a pessimist to send the message "I am a pessimist": if she were to deviate and sent instead the message "I am an optimist", she wouldn't be able to learn anything about the proportion of optimists in the population and would never invest. An optimist faces a high opportunity cost of waiting, and independently of her message, invests at time one. Hence, she cannot gain by sending the message "I am a pessimist".[6]

We next argue that our analysis allows us to draw some positive and normative conclusions. In particular, we show that an investment subsidy, by artificially increasing the surplus generated by the project, promotes truthful revelation of private information. However, this does not mean that an investment subsidy always increases welfare: a social planner knows that if the subsidy induces truthful revelation, this comes at the cost of less information transmission through actions. In the paper we show that a social planner may even want to tax investments to cause information to be revealed through actions instead of words. Finally, we also show that a more able sender (i.e. a sender possessing a more precise signal) has more incentives to truthfully reveal her private information than a less able one.

This paper belongs to the literature on informational cascades (see a.o. Banerjee (1992), Bikhchandani, Hirschleifer and Welch (BHW,1992), Chamley and Gale (CG,1994), Chamley (2001),...). Those papers assume away any preplay communication and study the efficiency properties of social learning (= learning through actions). We provide a justification for this approach: for low surplus projects, no information can be transmitted through words because players want to influence their future learning capabilities. In those papers the public information is the consequence of some costly actions undertaken by the early movers: for example a second mover knows that the first mover is an optimist because she spent money to undertake a new investment project. Hence, in those papers the credibility of the public information is not an issue. In this paper it is costless to send public information, and its credibility must therefore be carefully checked. Those papers show how an informational cascade develops as a consequence of the arrival of some early (and credible) information. In this paper, we show that the causality can also be reversed: it is the informational cascade, by reducing the gain of sending the message "I am an optimist", which causes the public information to be credible.

[6]Note that in the separating equilibrium information only gets transmitted through actions when the sender announces "I am a pessimist". As will become clear below, the amount of information produced after the arrival of an unfavourable message is always lower than the one that would have been produced in the absence of cheap talk (or in the pooling equilibrium).

Obviously, this is not the first paper to investigate the credibility of cheap talk statements. In a seminal paper, Crawford and Sobel (1982) already analysed the issue of information transmission through cheap talk. However, in their model the receiver chooses an action which influences both player's payoffs after having received a message from the informed sender. In our model the sender first sends a message and then plays a (waiting) game with the receivers. Farrell (1987,1988), Farrell and Gibbons (1989) and Baliga and Morris (2000) also assume that both players play a game after having received or sent a message. However, they consider a very different game: in Farrell (1987,1988) and Baliga and Morris the communication stage is followed by a coordination game, while in Farrell and Gibbons both players engage in a bargaining game after the communication stage. As we consider a (very) different game, we also get very different results: Crawford and Sobel have shown how the credibility of cheap talk statements are undermined when the sender and the receiver have different preferences over the optimal action, Baliga and Morris argued that positive spillovers impede information exchange, while we show how social learning may destroy incentives for truthtelling (and how informational cascades help in restoring these incentives).

This paper is organised as follows. In section two, we present our two-stage game. In the third section, we take the players' posteriors as given and solve for all monotone continuation equilibria. We next compute equilibrium strategies in the sender-receiver game (section four). We first show how the credibility of cheap talk may be undermined when players can postpone their investment decisions (Proposition 2). Next, we show how this credibility can be restored by an informational cascade (Proposition 3). In section 5, we discuss some normative and positive implications of our theory. In section 6 we analyse the case in which the sender may be uninformed. Final comments are summarised in the seventh and final section.

2 The Model

Assume that a population of $N \geq 5$ risk neutral players must decide whether to invest in a risky project or not. The value V of the investment project can take two values: $V \in \{1, 0\}$, with equal probabilities . The state of the economy is described by $\Theta \in \{G, B\}$. If $\Theta = G$ the good state prevails and $V = 1$ whereas if $\Theta = B$, the economy is in a bad state and $V = 0$. The cost of the investment project is denoted by c. Each player receives a private, conditionally independent signal concerning the realised state of the world. Formally, player l's signal $s_l \in \{g, b\}$ $(l = 1, ..., N)$ where $\Pr(g|G) = \Pr(b|B) = p > \frac{1}{2}$. We assume that:

A1: $1 - p < c < p$.

A1 implies that a player who received signal g is - a priori - willing to invest ($\Pr(G|g) = p > c$), and that a player who received a signal b is a priori not willing to invest ($\Pr(G|b) = 1 - p < c$). Henceforth, we call a player who received a good (bad) signal an optimist (pessimist)[7]. If $c \leq \frac{1}{2}$ ($c > \frac{1}{2}$), we call the investment opportunity a high (low) surplus project. We analyse the stage game that unfolds as follows:

- -1 The state of nature is realised and players receive signals,
- 0 A randomly selected player i is asked to report her signal. Her message, $\hat{s}_i \in \{g, b\}$, is made public to all the other players,
- 1 All players make investment decisions,
- 2 All players observe who invested at time one, and those who haven't invested yet make new investment decisions,
- 3 All players learn the true state of the world. Payoffs are received and the game ends.

In the first stage (time zero) player i (= the sender) influences the posteriors of the remaining players (= the receivers), and thus the equilibrium strategies at the second stage (time one and two). Henceforth we call the second stage the waiting game (or the continuation game). At time one, player l must choose an action, a_l, from the set {invest, wait}. At time two all players who waited at time one must choose an action from the set {invest, not invest}. Each player only possesses one investment opportunity, so a period-one investor cannot invest in a second project at time two. Investments are irreversible. If a player does not invest in any of the two periods, she gets zero. Investment decisions at period one are represented by a N-vector x where the l-th coordinate equals 1 if player l invested at time one and zero otherwise. δ denotes the discount factor.

We let h_t ($t = 0, 1, 2$) denote the history of the game at time t. Thus $h_0 = \{\emptyset\}$, $h_1 = \hat{s}_i$ and $h_2 = (\hat{s}_i, x)$. H_t denotes the set of all possible histories at time t, and the set of histories is $H = \bigcup_{t=0}^{2} H_t$. A symmetric behavioural strategy for the receivers

[7]Observe that in our model all players are Bayesian rational: optimists (pessimists) do not overestimate (underestimate) the probability that $\Theta = G$. Hence, our definitions differ from the ones that are used by behavioural economists. However, these definitions are intuitive and should not confuse the reader.

is a function $\rho : \{g, b\} \times H \to [0,1]$ with the interpretation that $\rho(s_j, h_t)$ represents the probability of investing at date t given s_j and h_t ($j = 1, ..., N$ and $j \neq i$). For instance, $\rho(g, b)$ is the probability that an optimistic receiver invests at time one given that $\hat{s}_i = b$, and $\rho(b, g)$ is the probability that a pessimistic receiver invests at time one given that $\hat{s}_i = g$. Since each player can only invest once, $\rho(s_j, h_2) = 0$ if player j invested at time one, and $\rho(s_j, h_0) = 0$ since no one can invest at time zero. A behavioural strategy for the sender is a function $\sigma : \{g, b\} \times H \to [0,1]$. $\sigma(g, h_0)$ ($\sigma(b, h_0)$) represents the probability with which an optimistic (pessimistic) sender sends $\hat{s}_i = g$. $\sigma(\cdot, h_1)$ ($\sigma(\cdot, h_2)$) represents the probability that player i invests at date one (two). As before, $\sigma(\cdot, h_2) = 0$ if the sender invested in the first period. Let $\rho_1 \equiv (\rho(b, h_1), \rho(g, h_1))$, $\sigma_1 \equiv (\sigma(b, h_1), \sigma(g, h_1))$ and $\sigma_0 \equiv (\sigma(b, h_0), \sigma(g, h_0))$.

Suppose player j is an optimistic receiver. At time one, player j computes $q_\omega \equiv \Pr(G|s_j = g, \hat{s}_i, \sigma(g, h_0), \sigma(b, h_0))$. If $\sigma(g, h_0) = 1$ and $\sigma(b, h_0) = 0$ (in this case an optimistic sender always sends a favourable message, while a pessimist always sends an unfavourable one) then $q_\omega = \frac{p^2}{p^2+(1-p)^2} \equiv \overline{q}_\omega$ after a good message, and $q_\omega = \frac{1}{2}$ after a bad message. A simple computation shows that for all values of $\hat{s}_i, \sigma(g, h_0), \sigma(b, h_0)$, $q_\omega \in [\frac{1}{2}, \overline{q}_\omega]$, and that all values in the interval are attained for some values of $\sigma(g, h_0)$, $\sigma(b, h_0)$ and $\hat{s}_i$. Similarly, q_π denotes a pessimist's posterior probability that $\Theta = G$ and $q_\pi \in [\underline{q}_\pi, \frac{1}{2}]$ where $\underline{q}_\pi = \frac{(1-p)^2}{p^2+(1-p)^2}$. Henceforth, to save on notations, we will, in general, not include $\sigma(\cdot)$ and $\rho(\cdot)$ in our list of conditioning variables. This omission should not confuse the reader as it will be obvious which $\sigma(\cdot)$ and $\rho(\cdot)$ enter into the computation of a player's posterior.

When solving our game, we rely on four equilibrium selection criteria. First, we require a candidate equilibrium to belong to the class of the perfect Bayesian equilibria. Henceforth, $\sigma^*(\cdot)$ ($\rho^*(\cdot)$) denotes the value taken by $\sigma(\cdot)$ ($\rho(\cdot)$) in a perfect Bayesian equilibrium (PBE). In a PBE strategies and beliefs (concerning the other players' types) must be such that (i) the sender cannot gain by choosing a $\sigma \neq \sigma^*$ given her beliefs and given ρ^*, (ii) receivers cannot gain by choosing a $\rho \neq \rho^*$ given their beliefs and given σ^* and (iii) beliefs must be computed using Bayes's rule whenever possible. As usual, a pooling equilibrium is a PBE in which $\sigma^*(g, h_0) = \sigma^*(b, h_0)$. In that case the message $\hat{s}_i = g$ is as likely to come from an optimistic as from a pessimistic sender. Hence, in that case messages have no informational content and do not affect posteriors. For the sake of concreteness (and wlog), we assume that $\sigma^*(g, h_0) \geq \sigma^*(b, h_0)$. This assumption merely defines message $\hat{s}_i = g$ as the one which influences posteriors in a (weakly) favourable way. Under this assumption, a separating equilibrium is a PBE in which $\sigma^*(g, h_0) = 1$ and $\sigma^*(b, h_0) = 0$. Note that at time one q_ω can be different from p (the posterior of the receivers may differ from the sender's). Therefore, we do not impose $\sigma^*(g, h_1)$ to be equal to $\rho^*(g, h_1)$. Similarly, we allow $\sigma^*(b, h_1)$ to be different from $\rho^*(b, h_1)$.

Second, we restrict ourselves to the class of *monotone* strategies. Consider players l and l' (where l or l' may be the sender). Call q (q') the time-one posteriors of player l (l') (where q and $q' \in \{1-p, q_\pi, p, q_\omega\}$). Strategies are said to be monotone if they possess the following two properties: 1) if $q = q'$, then Pr(l invests at time one) = Pr(l' invests at time one), 2) if Pr(l invests at time one) > Pr(l' invests at time one), then $q > q'$. Remark that from the first property, monotone strategies are symmetric. Note that the first property implies that whenever the sender's message is uninformative, the sender invests at period 1 with the same probability as a receiver of the same type, which need not hold in symmetric strategies. Property two implies that the time-one investment probabilities (weakly) increase in the time-one posteriors. We do not expect "real-world" players to play non-monotone strategies, and, in that sense, we believe this to be a realistic restriction on the strategy profiles.

Third, consider a candidate equilibrium in which optimistic (pessimistic) receivers randomise at time one with probability $\hat{\rho}(g, \cdot)$ ($\hat{\rho}(b, \cdot)$). We require each PBE to be stable in the following sense: suppose player j is an optimistic (pessimistic) receiver. Suppose she anticipates all other optimistic (pessimistic) receivers to randomise with probability $\hat{\rho}(g, \cdot) - \epsilon_1$ ($\hat{\rho}(b, \cdot) - \epsilon_1$) (where ϵ_1 represents an arbitrary small, but strictly positive number). Then, it must be optimal for player j to invest at time one.

Finally, we require every candidate equilibrium to be robust to the introduction of an ϵ-reputational cost. More specifically, we assume that with probability ϵ_2 receivers detect any "lie" (i.e. the optimistic sender who sends message $\hat{s}_i = b$, or the pessimistic sender who sends message $\hat{s}_i = g$) from the sender, in which case she suffers a reputational cost equal to ϵ_3. It is important to note that ϵ_2 is unrelated to the sender's behaviour in the continuation game. This assumption ensures that the sender's behaviour in the continuation game is only driven by informational reasons (and not by her desire to "mask" a past lie). Let $\epsilon \equiv \epsilon_2.\epsilon_3$ and we assume that ϵ represents an arbitrary small, but strictly positive, number. With this reputational cost, an optimistic sender prefers to send a favourable to an unfavourable message (as will become clear below, in the absence of this ϵ, she would be indifferent between the two messages).

3 Strategic Waiting

Before proving the existence of a PBE in our game, we restrict our attention to *monotone continuation equilibria*. Henceforth, $\tilde{\sigma}(\cdot)$ ($\tilde{\rho}(\cdot)$) denotes the value taken by

$\sigma(\cdot)$ ($\rho(\cdot)$) in a monotone continuation equilibrium (MCE). A MCE is identical to a PBE except that we do not require the sender to choose $\tilde{\sigma}_0$ optimally given her beliefs and given equilibrium behaviour in the continuation game. Stated differently, in a MCE we do not endogenise the receivers' time-one posteriors. Instead, we just treat them as if they were exogenous and compute all monotone continuation equilibria. Note that every PBE is a MCE, while the contrary need not hold.

Our model is void of any competition effects or positive network externalities. Hence, a player's expected gain of investing is solely determined by the relative number of optimists (as compared to the number of pessimists) in the population. Call n the random number of optimists in our population. The higher n (for any fixed N), the higher $\Pr(G|n)$ and the higher the expected gain from investing. Unfortunately, by postponing one's investment decision, players observe x instead of n. Hence, at time two all players who waited at time one face an inference problem: on the basis of x they must try to get "as precise an idea" about n.

As we only consider symmetric strategies, player i does not care about *who* invests, but rather in *how many* players invest. Therefore, from the sender's point of view all information contained in x can be summarised by k^s (= the number of receivers who invest at time one).[8] Similarly, from a receiver's point of view all information contained in x can be summarised by k (= the number of remaining receivers who invest at time one) and a_i (= the time-one action of the sender). Note that $k = k^s$ or $k = k^s - 1$.

We thus continue our analysis by working with k, k^s and a_i. Let $q \in \{q_\pi, q_\omega, 1-p, p\}$. If player j waits, she observes k and a_i and invests if $\Pr(G|q,k,a_i) \geq c$. Hence, for a given k and a_i player j's payoff equals $\max\{0, \Pr(G|q,k,a_i) - c\}$. Of course, player j cannot ex ante know the realization of k and a_i. Therefore, player j's ex ante gain of waiting (net of discounting costs), $W(q, \sigma_1, \rho_1)$, equals

$$(1) \qquad W(q,\sigma_1,\rho_1) = \sum_{a_i}\sum_{k} \max\{0, \Pr(G|q,k,a_i) - c\} \Pr(k|q,a_i) \Pr(a_i|q).$$

Similarly, player i's gain of waiting, $W(q, \rho_1)$, equals

$$(2) \qquad W(q,\rho_1) = \sum_{k^s} \max\{0, \Pr(G|q,k^s) - c\} \Pr(k^s|q).$$

We know enough to start analysing equilibrium behaviour in the continuation game. We first investigate the case in which the project is a low surplus one (i.e. $c > \frac{1}{2}$) and in which the sender truthfully announces that she's an optimist (i.e. $q_\pi = \frac{1}{2}$

[8]In mathematical terms, we mean that $\Pr(n|x, s_i) = \Pr(n|k^s, s_i)$, $\forall n$.

and $q_\omega = \overline{q}_\omega$). This case is both simple and rich enough to capture many important mechanisms of our model. We show that in this case there exists a unique MCE in which $\tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = 0$ and $\tilde{\rho}(g,g) \in (0,1)$. As $1 - p < \frac{1}{2} = \Pr(G|s_j = b, \hat{s}_i = g) < c$, it trivially follows that $\tilde{\sigma}(b,g)$ and $\tilde{\rho}(b,g)$ equal zero. To understand why $\tilde{\sigma}(g,g)$ also equals zero, we first must understand how $\tilde{\rho}(g,g)$ is determined and how it varies with changes in q. As all receivers know that $s_i = g$ and that $\tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = 0$, equation (1) can be rewritten as

$$(3) \quad W(\overline{q}_\omega, (0,0), (0, \rho(g,g))) = \sum_k \max\{0, \Pr(G|\overline{q}_\omega, k, \text{wait}) - c\} \Pr(k|\overline{q}_\omega, \text{ wait}).$$

To gain some insight behind equations (1), (2) and (3), it is useful to contrast the polar case in which equation (3) is evaluated at $\rho(g,g) = 0$ with the other one in which (3) is evaluated at $\rho(g,g) = 1$. Thus, suppose that $\rho(g,g) = 0$. Then,

$$\Pr(k = 0, |\overline{q}_\omega, a_i = \text{ wait}, \tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = \rho(g,g) = 0) = 1.$$

At time two, player j computes $\Pr(G|\overline{q}_\omega, 0, \text{ wait}) = \overline{q}_\omega$. This is intuitive: player j, independently of n, always observes zero period-one investments. Stated differently, if $\rho(g,g) = 0$, it's as if she doesn't receive any additional information concerning the realised state of the world. Therefore she has no reason to change her posterior and $\Pr(G|\overline{q}_\omega, 0, \text{ wait}) = \overline{q}_\omega$. Hence, $W(\overline{q}_\omega, (0,0), (0,0)) = \overline{q}_\omega - c$. Suppose now that $\rho(g,g) = 1$. Then, in the next period player j learns how many optimists are present in the economy (i.e. $n = k+2$)[9]. At time two player j computes $\Pr(G|n)$, and invests if $\Pr(G|n) \geq c$. As before, player j cannot ex ante know how many optimists are present in the economy, and therefore:

$$(4) \qquad W(\overline{q}_\omega, (0,0), (0,1)) = \sum_n \max\{0, \Pr(G|n) - c\} \Pr(n|\overline{q}_\omega)$$

Lemma 1 *$\forall N \geq 5$, $W(q, \sigma_1, (0,1)) > q - c$.*

Proof: See Appendix.
To understand Lemma 1, in this paragraph we intuitively explain why $\forall N \geq 5$, $W(\overline{q}_\omega, (0,0), (0,1)) > \overline{q}_\omega - c$. We can rewrite player j's gain of investing as follows:

$$\overline{q}_\omega - c = \sum_n \Pr(G|n) \Pr(n|\overline{q}_\omega) - c.$$

[9]By assumption, player j is an optimist who waited at time one. Moreover, we analyse a case in which player j learned (through the sender's message) that $s_i = g$. Therefore, $n = k + 2$.

Suppose $\rho(g,g) = 1$ and assume that player j decides to wait at time one and then to invest unconditionally (i.e. to invest at time two independently of n). The above equality merely states that investing at time one is payoff-equivalent (net of discounting costs) to *unconditionally* investing at time two. Equation (4) learns us that waiting (when $\rho(g,g) = 1$) is equivalent to making an optimal *conditional* second-period investment decision. Observe that n cannot take a value lower than two because both players j and i are assumed to be optimists. If $\Pr(G|n = 2)$ is higher or equal than c, then the optimal conditional second-period investment decision always coincides with unconditionally investing at time two. This means that $\overline{q}_\omega - c$ is equal to $W(\overline{q}_\omega, (0,0), (0,1))$. Hence, $W(\overline{q}_\omega, (0,0), (0,1))$ is strictly greater than $\overline{q}_\omega - c$ if (and only if) $\Pr(G|n = 2) < c$. In this model all players possess a signal of the same precision and $\Pr(\Theta = G) = \frac{1}{2}$. Therefore, $\forall\, c \in (1-p, p)$ it takes three pessimistic receivers to refrain an optimist, who learned through the sender's message that $s_i = g$, from investing (and therefore N must be greater or equal than five).

Lemma 1 holds $\forall q$. This is intuitive: suppose for instance that player j anticipates that $\Theta = G$ with probability $q_\omega < \overline{q}_\omega$. this means that player j puts a (strictly) positive probability on the event that $s_i = b$ (and thus that $n = 1$). As $\Pr(G|n = 1) < \Pr(G|n = 2)$, it follows that $q_\omega - c$ is also strictly lower than $W(q_\omega, (0,0), (0,1))$. Lemma 1 holds $\forall \sigma_1$. This is also intuitive: whenever $\sigma(g, h_1) \neq \sigma(b, h_1)$, the sender's time-one action conveys some information about her type. Obviously, this cannot decrease player j's gain of waiting. To focus on the interesting parameter range, we assume that:

A2: $\frac{\overline{q}_\omega - c}{W(\overline{q}_\omega, (0,0), (0,1))} < \delta < 1$

The first inequality of A2 puts a lower bound on the discount factor δ such that an optimistic receiver, who learned (through the sender's message) that $s_i = g$, faces a positive option value of waiting (i.e. if player j expects all the optimistic receivers to invest and all the other players to wait, then she rather waits). The first inequality ensures thus that $\tilde{\rho}(g,g) < 1$. The second inequality ensures that $\tilde{\rho}(g,g) > 0$.

Lemma 2 *Under A2, $q - c < \delta W(q, (0,0), (0,1))$.*

Proof: See Appendix.

In words, Lemma 2 states that if a player who possesses the highest possible posterior faces a positive option value of waiting, then this will also be true for all less optimistic ones. The intuition behind Lemma 2 will be explained on the basis of Graph 2 below.

Equation (3) is increasing in $\rho(g,g)$. To see this, compare the following two "scenarios". In scenario one all optimistic receivers randomise with probability $\rho'(g,g)$, in scenario two all optimistic receivers randomise with probability $\rho(g,g) < \rho'(g,g)$. Call n^r the number of optimistic receivers.[10] Call k' (k) the number of players investing at time one when $n^r - 1$ optimistic receivers invest with probability $\rho'(g,g)$ ($\rho(g,g)$). Now, having $n^r - 1$ players investing with probability $\rho(g,g)$ is ex ante equivalent to the following two-stage experiment: first let all $n^r - 1$ players invest with probability $\rho'(g,g)$. Next let all k' investors re-randomise with probability $\frac{\rho(g,g)}{\rho'(g,g)}$. Therefore the statistic k is generated by adding noise to the statistic k'. Therefore k' is a sufficient statistic for k. From Blackwell's value of information theorem (1951) we know that this implies that $W(\overline{q}_\omega,(0,0),(0,\rho'(g,g))) \geq W(\overline{q}_\omega,(0,0),(0,\rho(g,g)))$. Moreover, Chamley and Gale (1994, Proposition 2) have shown that

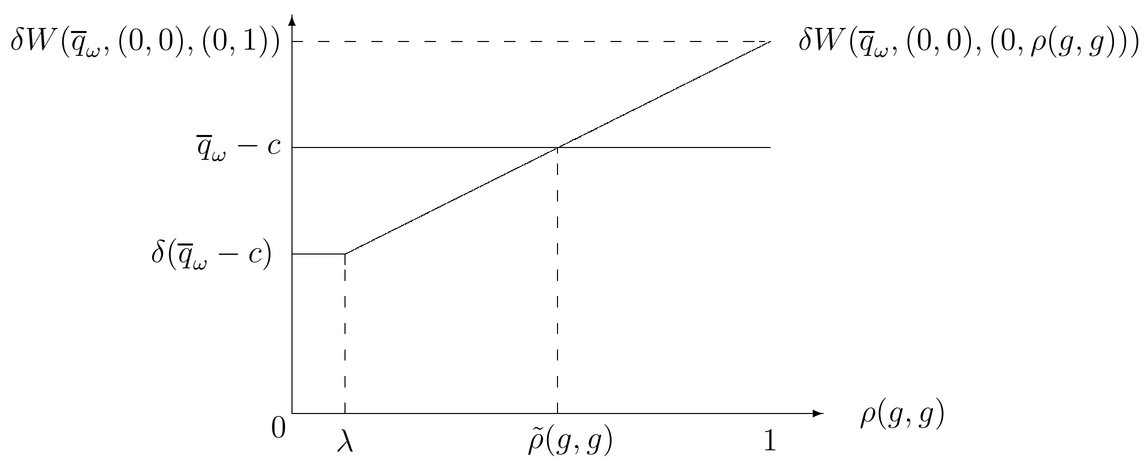
$$\forall \rho(g,g), \rho'(g,g) \in [0,\lambda], W(\overline{q}_\omega,(0,0),(0,\rho(g,g))) = W(\overline{q}_\omega,(0,0),(0,\rho'(g,g))), \text{ while}$$

$$\forall \rho'(g,g) > \lambda, W(\overline{q}_\omega,(0,0),(0,\rho(g,g))) < W(\overline{q}_\omega,(0,0),(0,\rho'(g,g))).$$

This line of reasoning also extends to equations (1) and (2)[11].

Intuitively, $\rho(g,g)$ captures the ex ante amount of information produced by the optimistic receivers. The higher $\rho(g,g)$, the easier one can infer n out of k (this can best be seen by comparing the two polar cases where $\rho(g,g) = 0$ and $\rho(g,g) = 1$ (see above)) and thus the higher the ex ante gain of waiting. Graphically one has:

Graph 1: Existence of a MCE in which $\tilde{\rho}(g,g) \in (0,1)$.



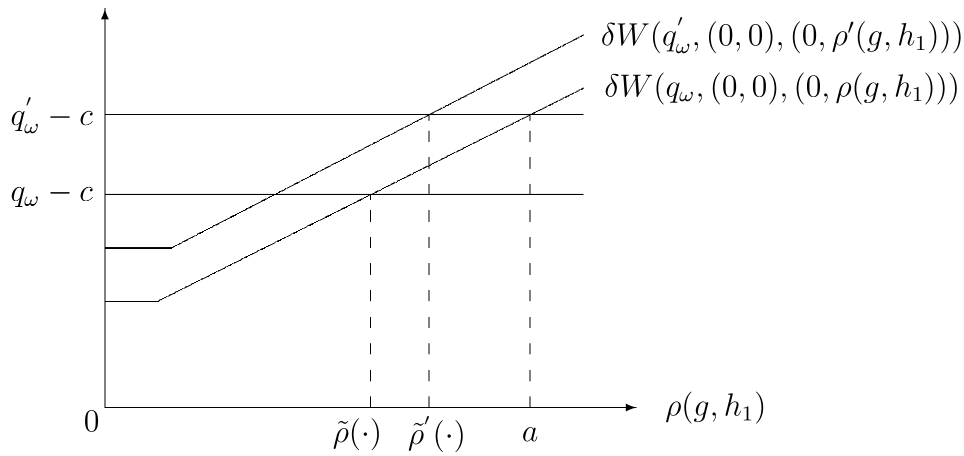
In equilibrium the gain of waiting must be equal to the gain of investing, i.e. $\overline{q}_\omega - c = \delta W(\overline{q}_\omega,(0,0),(0,\tilde{\rho}(g,g))$. If $\rho(g,g) = \lambda$, $\delta W(\overline{q}_\omega,(0,0),(0,\lambda)) = \delta[\overline{q}_\omega - c] < \overline{q}_\omega - c$. If

[10] Note that $n^r = n$ if $s_i = b$, otherwise $n^r = n - 1$.

[11] For a formal proof, see the Appendix.

$\rho(g,g) = 1$, by A2, $\delta W(\overline{q}_\omega, (0,0), (0,1)) > \overline{q}_\omega - c$. By monotonicity, as $\overline{q}_\omega > c$ there exists a unique $\tilde{\rho}(g,g)$ which makes the optimists indifferent between investing and waiting. So far, we assumed that $\tilde{\sigma}(g,g) = 0$ and showed that $\tilde{\rho}(g,g) \in (0,1)$. We still must explain why the gain of waiting of the optimistic sender exceeds her gain of investing. Consider therefore the following graph.

Graph 2: The effect of a change in q_ω on $\tilde{\rho}(g,h_1)$.



The Graph above addresses the question: "What happens with $\tilde{\rho}(g,h_1)$ if q_ω increases?" Suppose player j first anticipates that $\Theta = G$ with probability q_ω. As before, graph two shows the existence of a unique $\tilde{\rho}(g,h_1)$ where the gain of investing equals the gain of waiting. Assume now that for some exogenous reason player j becomes "more optimistic" in the sense that she now anticipates that $\Theta = G$ with probability $q_\omega^{'} > q_\omega$. Graph two shows that the comparison between $\tilde{\rho}(\cdot)$ and $\tilde{\rho}^{'}(\cdot)$ depends on the relative strength of two opposing effects. On the one hand, an increase in q_ω increases an optimist's gain of investing, which, were $W(\cdot)$ independent of q_ω, would increase $\rho(g,h_1)$ from $\tilde{\rho}(g,h_1)$ to point a in graph two. On the other hand, an increase in q_ω also leads to an increase in the gain of waiting. This second effect decreases $\rho(g,h_1)$ from point a until the point $\tilde{\rho}^{'}(g,h_1)$. The relative strength of both effects ultimately depends on how the shift of the gain of waiting compares to the one of the gain of investing. One can show that the first effect always dominates the second one and thus that $\tilde{\rho}^{'}(g,h_1) > \tilde{\rho}(g,h_1)$.

The intuition behind this result mainly lies in the presence of a discount factor in the model. An increase in q_ω increases $W(q_\omega, \cdot)$ for two different reasons: (i) it increases the likelihood that $\Pr(G|q_\omega, k, a_i) > c$ and thus that player j will get a non-zero expected utility and (ii) it increases her expected gain of investing whenever player j does so. However, the presence of δ in front of $W(q_\omega, \cdot)$ (and not in front of $q_\omega - c$) dampens this increase in $W(q_\omega, \cdot)$. Note that in this and our previous paragraph, our reasoning did not rely on the fact that in our limit case $\sigma_1 = (0,0)$ and $\sigma(b, h_0) = 0$. Actually, one can show that this positive correlation between $\tilde{\rho}(\cdot)$ and q_ω is robust in the sense that it holds $\forall \sigma_1$. Graph 2 also provides the intuition behind Lemma 2. To see this, suppose that $q_\omega^{'} = \overline{q}_\omega$ and that $\rho_1 = (0,1)$. From A2, we know that $q_\omega^{'} - c < \delta W(q_\omega^{'}, (0,0), (0,1))$. From Graph 2 follows that $\forall q$, $\delta W(q, (0,0), (0,1)) > q - c$ (the downward shift in the gain of investing overcompensates the one in the gain of waiting).

There are two different reasons why $\tilde{\sigma}(g, g) = 0$: the first one is due to the fact that the sender observes k^s and not k, the second one is due to the fact that $p < \overline{q}_\omega$. To illustrate the first reason suppose the sender's posterior probability that $\Theta = G$ equals the one of the optimistic receivers. One can think about the statistics k and k^s as follows. Let the n^r optimistic receivers invest with probability $\tilde{\rho}(\cdot)$. Next, construct k as follows: if player j invested[12], $k = k^s - 1$, otherwise $k = k^s$. Hence, k^s is a sufficient statistic for k and, thus, player i's gain of waiting cannot be lower than player j's. To illustrate the second reason, suppose that if the sender waits, she observes k instead of k^s. Call a the probability with which the optimistic receivers must invest to make an optimistic sender indifferent between investing and waiting. From Graph 1, we know that $a \in (0,1)$. As $\overline{q}_\omega > p$, from the explanation of Graph 2 we know that $\tilde{\rho}(g, g) > a$. From Proposition 2 of Chamley and Gale (1994) we know that this implies that $p - c < \delta W(p, (0, \tilde{\rho}(g, g)))$. We know enough to state:

Proposition 1 *(Characterisation of all MCE's)*
1) If $q_\pi < 1 - p < c < q_\omega < p$, $\exists$ a unique MCE in which $\tilde{\rho}(b,b) = \tilde{\sigma}(b,b) = 0$ and $\tilde{\rho}(g,b) \in [0,1), \tilde{\sigma}(g,b) = 1$.
2) If $q_\pi < 1 - p < q_\omega \leq c < p$, $\exists$ a unique MCE in which $\tilde{\rho}(b,b) = \tilde{\sigma}(b,b) = \tilde{\rho}(g,b) = 0$ and $\tilde{\sigma}(g,b) = 1$.
3) If $1 - p < q_\pi < c < p < q_\omega$, $\exists$ a unique MCE in which $\tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = 0$ and $\tilde{\rho}(g,g) \in (0,1)$.
4) If $1 - p < c \leq q_\pi < \frac{1}{2} < p < q_\omega$, $\exists$ an MCE in which $\tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = 0$ and $\tilde{\rho}(g,g) \in (0,1)$. Depending on the values of our exogenous parameters, there may also exist one (and only one) other MCE in which $\tilde{\sigma}(b,g) = 0$ and $\tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$.
5) If $1 - p < c \leq q_\pi = \frac{1}{2} < p < q_\omega = \overline{q}_\omega$, $\exists$ two MCE's. In the first one $\tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = 0$ and $\tilde{\rho}(g,g) \in (0,1)$. In the second one $\tilde{\sigma}(b,g) = 0$ and $\tilde{\rho}(b,g) =$

[12] Remind that player j is an optimistic receiver who is indifferent between investing and waiting and who, therefore, invests with probability $\tilde{\rho}(\cdot)$.

$\tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$.
6) If $q_\pi = 1-p < c < q_\omega = p$, $\exists$ *a unique MCE in which* $\tilde{\sigma}(b,h_1) = \tilde{\rho}(b,h_1) = 0$ *and* $\tilde{\sigma}(g,h_1) = \tilde{\rho}(g,h_1) \in (0,1)$.

Proof: See Appendix.
In cases 1) and 2) we characterise all MCE's when $\sigma(b,h_0) < \sigma(g,h_0)$ and when the sender sent $\hat{s}_i = b$. In cases 3), 4) and 5) we characterise all MCE's when $\sigma(b,h_0) < \sigma(g,h_0)$ and when the sender sent $\hat{s}_i = g$. Case 6) considers the case in which the sender's message did not affect the receiver's posteriors (i.e. $\sigma(b,h_0) = \sigma(g,h_0)$).

In case 1), $q_\omega < p$. As we are focusing on monotone strategies we assume that $\rho(g,b) \leq \sigma(g,b)$. There does not exist a MCE in which both the optimistic receivers and the optimistic sender randomise as this contradicts (a.o.) the insight summarised in Graph 2. There are two possibilities: (i) $\delta W(q_\omega,(0,1),(0,0)) \geq q_\omega - c$ or (ii) $\delta W(q_\omega,(0,1),(0,0)) < q_\omega - c$. To understand the important distinction between (i) and (ii), suppose that $s_j = g$ and that player j anticipates all receivers to wait at time one. As already argued above, k does then not contain any information about the realisation of n^r. As $\tilde{\sigma}_1 = (0,1)$, a_i perfectly reveals the sender's type. In possibility (i), the informational gain of observing a_i exceeds the discounting cost, and, thus, player j prefers to wait. Similarly, an optimistic sender, anticipating that $\tilde{\rho}_1 = (0,0)$, faces no informational gain of waiting while its discounting cost is positive. Hence, it's in her best interest to invest at time one and, thus, there exists a MCE in which $\tilde{\rho}_1 = (0,0)$ and $\tilde{\sigma}_1 = (0,1)$. We now explain why in possibility (ii) the continuation game is characterised by a unique MCE in which $\tilde{\rho}(g,b) \in (0,1)$ and $\tilde{\sigma}(g,b) = 1$. In possibility (ii), the additional information (about the sender's type) does not compensate the discounting cost. Hence, there does not exist a MCE in which $\tilde{\rho}_1 = (0,0)$. From Lemma 2, we know that $q_\omega - c < \delta W(q_\omega,(0,0),(0,1))$. Obviously,

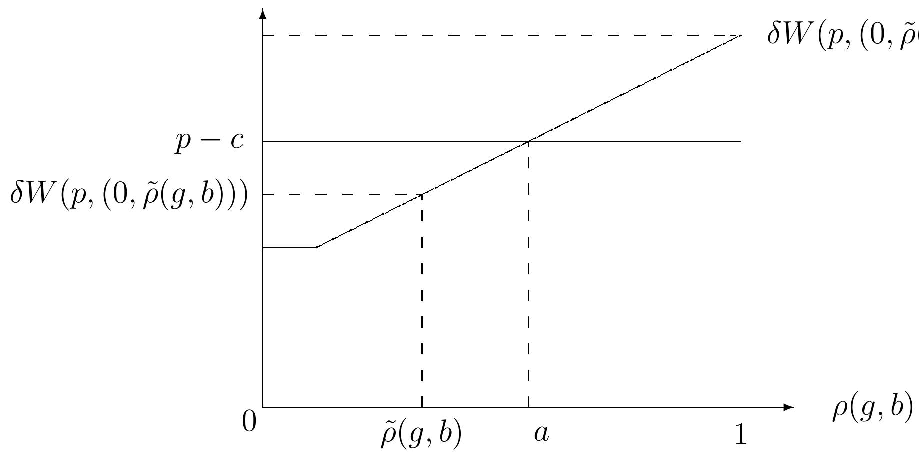
$$W(q_\omega,(0,0),(0,1)) \leq W(q_\omega,(0,1),(0,1)),$$

because in the former case no information (about the sender's type) is revealed through a_i, while in the latter case a_i perfectly reveals her type. Thus,

$$\delta W(q_\omega,(0,1),(0,0)) < q_\omega - c < \delta W(q_\omega,(0,1),(0,1)),$$

and from CG's analysis (see Graph 1) we know there exists a unique $\tilde{\rho}(g,b)$ which equates an optimist's gain of investing with her gain of waiting. We are left to intuitively explain why the optimistic sender, knowing that $\tilde{\rho}_1 = (0,\tilde{\rho}(g,b))$, prefers to invest with probability one. This can best be illustrated on the basis of the following Graph:

Graph 3: An optimist's optimal time 1 action after sending $\hat{s}_i = b$.



To understand graph 3, first note that an optimist's payoff of sending an unfavourable message equals $\max\{p-c, \delta W(p,(0,\tilde{\rho}(g,b)))\}$. $p-c$ denotes her payoff of investing at time one, given that she sent an unfavourable message. $\delta W(p,(0,\tilde{\rho}(g,b)))$ denotes her payoff of waiting given that she sent an unfavourable message. Call a the probability with which optimists must invest to make player i indifferent between investing and waiting. As the optimists received an unfavourable message, they anticipate that $\Theta = G$ with a probability equal to $q_\omega < p$. From the insight summarised in Graph 2, we know that $\tilde{\rho}(g,b) < a$ (because $q_\omega < p$). From CG's analysis follows that $\delta W(p,(0,\tilde{\rho}(g,b))) < p - c$. Therefore, if an optimist were to sent message b, it would be optimal for her to invest at time one.[13]

Case 2) is identical to case 1) except that $q_\omega \leq c$. Unsurprisingly, in this case no one (except the optimistic sender) invests at time one. The intuition why cases 3), 4) and 5) are characterised by a MCE in which only the optimistic receivers invest, is based on the insights summarised in Graphs 1 and 2. In case 4) we must make a distinction between the following two possibilities: (i) $\delta W(q_\pi,(0,1),(1,1)) > q_\pi - c$ or (ii) $\delta W(q_\pi,(0,1),(1,1)) \leq q_\pi - c$. Observe that in both (i) and (ii), the pessimistic receivers invest with the same probability as the optimistic ones. As k does then not contain any information about the realisation of n^r, a receiver only wants to wait to learn the sender's type. As in case 1), in (i) the informational gain of observing a_i exceeds the discounting cost and there cannot exist a MCE in which all receivers invest at time one. The contrary situation applies in possibility (ii). Case 5) is identical to case 4) except that the sender truthfully announced that she's an optimist. Hence, observing a_i = invest does not yield any additional information about the sender's type. Therefore, a pessimistic receiver who anticipates everyone to invest at time one

[13]Note that in this paragraph, we abstracted from the fact that the sender observes k^s, while the receivers "only" observe k. As proven in the Appendix, as long as we focus on the class of monotone strategies, this is without loss of generality.

(i.e. $\tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$), cannot gain by waiting. Case 6 corresponds to the case originally analysed by CG.

Consider the MCE in which all receivers invest at time one (see cases 4) and 5)). Note that all receivers possess some public (i.e. the favourable message sent by player i) and some private information (i.e. their signals). All players, independently of their signals, rely on the public information by investing at time one. This behaviour is identical to the one followed by the players inside an informational cascade in BHW's and Banerjee's (1992) model. In those models all players also possess some public (i.e. the action(s) of the first mover(s)) and private information (i.e. their signals) and they, independently of their signals, all adopt the same action. Therefore, we call the MCE in which $\rho^*(b,g) = \rho^*(g,g) = 1$ an informational cascade. Chamley (2001) has shown that this informational cascade does not hinge on our use of a binomial distribution. Rather, it can be recovered under a wide range of distributional assumptions.

4 Cheap Talk

We now analyse player i's incentives to truthfully reveal her private information at time zero. In our opinion one may think about player i in two ways. First, one may interpret player i as a "guru" whose opinion concerning investment matters is often asked by the media. Second, given our assumptions one would want to introduce an opinion poll (instead of just interviewing one player) at time zero. Unfortunately, analytical results are harder to get when one introduces other players at time zero. Therefore one can also interpret our model as one explaining "the economics of opinion polls" under the simplifying assumption that the size of the opinion poll equals one. We first state and prove the following "negative" result.

PROPOSITION 2 *For low surplus projects, there exists a unique monotone PBE in which* $\sigma^*(b,h_0) = \sigma^*(g,h_0) = 1$. *This PBE is supported by the out-of-equilibrium belief that if* $\hat{s}_i = b$*, the sender is a pessimist.*

Proof: See Appendix.

Proposition 2 basically states that for projects with low surplus, no information can be transmitted through cheap talk: as the message $\hat{s}_i = g$ is as likely to come from an optimistic sender as from a pessimistic one, posteriors are unaffected by the sender's message. We explain the intuition behind Propositon 2 in two paragraphs. First, we explain why $\sigma^*(b,h_0)$ must be equal to $\sigma^*(g,h_0)$. Next, we explain why $\sigma^*(b,h_0) = \sigma^*(g,h_0) = 1$. This permits us to better highlight the role played by the ϵ-reputational cost in our model.

Player i only possesses a noisy signal concerning the realised state of the world and is primarily interested in knowing n (and this is true for the optimistic as for the pessimistic sender). From the insight summarised in graph 2, we know that if player i succeeds to increase q_ω, this will enable her (whenever $\rho(b, h_1)$ remains equal to zero) to get a "better idea" of n after observing k. Stated differently, the higher q_ω, the higher player i's gain of waiting (provided that $\rho(b, h_1)$ remains equal to zero). If $c > \frac{1}{2} = \overline{q}_\pi$, then $\rho^*(b, h_1)$ will -independently of $\sigma(g, h_0)$, $\sigma(b, h_0)$ and $\hat{s}_i$ - always be equal to zero. Both sender's types thus want to send the message which yields the largest increase in q_ω and therefore the pessimist loses if she were to reveal her negative private information. Hence, in the absence of an ϵ-reputational cost, $\sigma^*(b, h_0)$ cannot be different from $\sigma^*(g, h_0)$.

The reason why $\sigma^*(b, h_0) = \sigma^*(g, h_0) = 1$ is based on our ϵ-reputational cost. As messages do not affect posteriors, the optimistic sender cannot influence her gain of waiting. To avoid paying ϵ, she thus strictly prefers to send $\hat{s}_i = g$. The pessimistic sender knows that $\sigma^*(g, h_0) = 1$. As argued above, if she sends $\hat{s}_i = g$, she learns more (about the receivers' types) than by sending $\hat{s}_i = b$ (note, however, that this will be at the expense of her reputation). As $\epsilon \to 0$, she also strictly prefers to send $\hat{s}_i = g$ instead of $\hat{s}_i = b$.

Note that Proposition 2 fundamentally rests on the assumption that players can wait and observe the period-one investment decisions. If players were not allowed to observe past investment decisions, our game would be characterised by a unique PBE in which $\sigma^*(g, h_0) = 1$ and $\sigma^*(b, h_0) = 0$. The intuition is simple: if the sender is optimistic she will, independently of her message, invest in the first period. If she is pessimistic she will, independently of her message, not invest. Hence, to save on the ϵ-reputational cost, a sender strictly prefers to truthfully report her type. Hence, Proposition 2 shows how the credibility of cheap talk statements can be adversely affected when players can learn through actions. As we mentioned in our introduction, the literature on social learning (see a.o. Banerjee (1992), Bikhchandani, Hirschleifer and Welch (BHW,1992), Chamley and Gale (CG,1994), Chamley (2001),...) assumes that information only gets revealed through actions. As those models are void of any competition effects, some economists wonder why information should not be revealed through words.[14] Proposition 2 thus provides a justification for the "ad-hoc" omission of a cheap-talk communication channel in many herding models. This paper also

[14]For example, Zwiebel (1995,p.16) wrote:

> Relative performance evaluation also justify agents' unwillingness to share information, an issue that is problematic in many herding models.

possesses a more "positive" result which is summarised below.

PROPOSITION 3 *For high surplus projects our game is characterised by two monotone PBE's: a pooling and a separating one. In the separating equilibrium,* $\rho^*(b,g) = \rho^*(g,g) = 1$. *The pooling equilibrium is supported by the out-of-equilibrium belief that if* $\hat{s}_i = b$, *the sender is a pessimist.*

Proof: See Appendix.
The intuition behind our pooling equilibrium (in which both sender's types send the message $\hat{s}_i = g$) is identical to the one we explained above. We are left to explain the intuition behind our separating equilibrium. Suppose the investment project is a high surplus one (i.e. $c \leq \frac{1}{2}$) and that all receivers revise their posteriors under the assumption that $\sigma^*(b, h_0) = 0$ and that $\sigma^*(g, h_0) = 1$. Consider first the optimistic sender. From the insight summarised in graph 3, we know that if she deviates and sends $\hat{s}_i = b$, it's optimal for her to invest at time one. Similarly, if she sends $\hat{s}_i = g$, from point 5 of Proposition 1 we know that it's optimal for her to invest at time one along with all the other receivers. Hence, *absent the ϵ-reputational cost*, an optimistic sender is indifferent between the two messages. If she prefers not to be caught "lying", she strictly prefers to truthfully report her signal. Consider now the pessimistic sender. If she sends $\hat{s}_i = b$, $q_\pi < 1 - p < c < q_\omega = \frac{1}{2}$. From points 1 and 2 of proposition 1 we know that $\rho^*(g, b) \in [0, 1)$. We now argue that $\rho^*(g, b) > 0$ if $c < \frac{1}{2}$. As all receivers know s_i at time one, no additional information (about the sender's type) can be learned through the observation of a_i. Therefore, a receiver's gain of waiting is independent of σ_1.[15] Hence, if $q_\omega = \frac{1}{2} > c$,

$$\delta W(\frac{1}{2}, (0, 1), (0, 0)) = \delta W(\frac{1}{2}, (0, 0), (0, 0)) = \delta(\frac{1}{2} - c) < \frac{1}{2} - c.$$

From Graph 1, we know there exists then a unique $\rho^*(g, b) > 0$ such that an optimistic receiver is indifferent between investing and waiting. Hence,

$$E(U_i | s_i = b, \hat{s}_i = b) = \delta W(1 - p, (0, \rho^*(g, b))) > 0, \forall c < \frac{1}{2}.$$

If $\hat{s}_i = g$, $c \leq q_\pi = \frac{1}{2} < p < q_\omega$. From point 5 of Proposition 1, we know there exists a MCE in which everyone invests at time one, and thus $E(U_i | s_i = b, \hat{s}_i = g) = 0$. As $E(U_i | s_i = b, \hat{s}_i = b) > E(U_i | s_i = b, \hat{s}_i = g)$ (whenever $c < \frac{1}{2}$), a pessimist strictly prefers to reveal her unfavourable information.

[15] See the Appendix for a formal proof.

In words, a separating equilibrium is fundamentally driven because: (i) both sender's types face different opportunity costs of waiting and (ii) sending a favourable message creates an informational cascade. An optimist believes the investment project is good. For her "time is money" and she is only willing to postpone her investment plans (with probability one) if pessimists don't invest *and* if optimists invest with a probability higher than a (see Graph 3). Unfortunately these two aims cannot be simultaneously achieved by none of the two messages. Therefore, in the presence of an ϵ-reputational cost, she strictly prefers to send $\hat{s}_i = g$. A pessimist believes the investment project is bad. She is unwilling to invest unless she observes "relatively many" optimists investing at time one. If the pessimist were to deviate and sent a favourable message, an informational cascade would occur, she wouldn't receive any payoff-relevant information and she would get zero. Hence, it is the informational cascade which ultimately induces a pessimist to send an unfavourable message. If $\rho^*(b, h_1)$ would always be equal to zero (as is the case for low surplus projects), a pessimist would never want to send a negative message because - if this message were to be believed - this would reduce $\rho^*(g, h_1)$.

Observe that Proposition 3 also stresses the importance of the informational cascade to elicit private information. There only exist two monotone PBE's. There does thus not exist a monotone PBE in which $\sigma^*(b, h_0) < \sigma^*(g, h_0)$ and in which $(\rho^*(b, g), \rho^*(g, g)) \neq (1, 1)$.

5 Some normative and positive implications of our theory

5.1 Should we subsidise investments?

Denote by sub an investment subsidy granted to each period-one investor. Call $c' \equiv c - sub$. A social planner can, by appropriately choosing sub, alter the amount of learning in two different ways. First, by making it relatively more attractive to invest at time one, she can influence all players' gain of waiting in a favourable way. Second, by setting sub such that $c' \leq \frac{1}{2} < c$, she changes the sender's incentives to truthfully reveal her private information (and thus the nature (separating vs pooling) of the equilibrium played in our game). In a full-fledged welfare study, one should compute the value of sub which maximises expected welfare. This exercise, however, is lengthy and outside the scope of this paper. Rather, in this subsection we assume that $sub \in [-\epsilon, \overline{sub})$ and highlight some advantages and disadvantages of setting $sub \neq 0$. If $sub = -\epsilon$ (where, as above, ϵ represents an arbitrary small, but strictly positive number) this means that the social planner taxes first-period investments. Note that we only allow for a "low" subsidy[16] in the sense that

$$sub < \overline{sub} \equiv \min\{\overline{sub}_1, \overline{sub}_2\}, \text{ where}$$

$$\overline{sub}_1 \equiv \delta W(\overline{q}_\omega, (0,0), (0,1)) - (\overline{q}_\omega - c) \text{ and}$$
$$\overline{sub}_2 \equiv c + p - 1.$$

If $sub < \overline{sub}_1$, this means that the most optimistic type in our model still faces a positive option value of waiting. If $sub < \overline{sub}_2$, this means that $1 - p < c'$. In the Appendix, we show that $\forall sub \in [-\epsilon, \overline{sub})$, Propositions 2 and 3 are unaffected by the introduction of a first-period subsidy, i.e. if $c' > \frac{1}{2}$, the unique monotone PBE is the pooling one, if $c' \leq \frac{1}{2}$ there exists a separating and a pooling equilibrium.

We first analyse the case in which the first-period subsidy does not change the nature of the played equilibrium. To illustrate our way of working, suppose the investment project is a high surplus one and that players always focus on the separating equilibrium. As mentioned above, in this equilibrium the message of the sender reveals her type, and strategies of period one are given by: after a good message, everyone invests in period 1, after a bad message, optimistic receivers invest with probability $\rho^*(g, b)$, and the remaining players do not invest.

Lemma 3 $\forall sub \in [0, \overline{sub})$, *$\rho^*(g, b)$ is strictly increasing in sub and $\rho^*(g, b) < 1$.*

Proof: See Appendix.
The intuition behind Lemma 3 is straightforward. We are considering a separating equilibrium. Thus, after the arrival of an unfavourable message, optimistic receivers know they are the only players in the economy who face a positive gain of investing. If an optimistic receiver waits, she forfeits the investment subsidy. Hence, the higher *sub*, the higher a player's cost of waiting. However, in equilibrium the gain of waiting must equal the cost of waiting, and, thus, the higher *sub*, the higher a player's gain of waiting (and from Graph 1 we know that this requires a higher $\rho^*(g, b)$).

$Wel(g, sub, sep)$ ($Wel(b, sub, sep)$) denotes the expected payoffs (net of the subsidies received) of the optimistic (pessimistic) players given the first-period subsidy

[16] We consider an investment subsidy which may be paid to a potentially very large number of firms. In comparison to the investment cost, it is then unlikely that the subsidy would be very important. We do not have in mind a situation in which a government offers a generous subsidy to attract an important investment project (e.g. the subsidy offered by the French Government to attract Eurodisney).

and given that all players focus on the separating equilibrium. For the optimistic players, one has

$$Wel(g, sub, sep) = \frac{N}{2}(p - c + sub) - (\frac{1}{2}2p(1-p)(N-1)\rho^*(g, b)$$
$$+\frac{1}{2}[(p^2 + (1-p)^2)(N-1) + 1])sub.$$

The first term is given by the expected number of optimists multiplied by their expected utilities. The second is the expected number of optimistic players who invest in period one[17] times the subsidy which is paid to them. Note that this last expression simplifies to

$$(5) \qquad Wel(g, sub, sep) = \frac{N}{2}(p - c) + (N-1)p(1-p)(1 - \rho^*(g, b))sub.$$

From Lemma 3 we know that $(1-\rho^*(g, b))sub$ (and thus also $Wel(g, sub, sep)$) need not be monotonic in sub. This is intuitive: an increase in sub increases an optimist's gain of waiting, but also reduces the probability that an optimist will wait and effectively benefit from a more informative signal. For pessimists, one has

$$(6) \qquad Wel(b, sub, sep) = (N-1)p(1-p)(\frac{1}{2} - c)$$
$$+\frac{1}{2}[(p^2+(1-p)^2)(N-1)\delta W(\frac{(1-p)^2}{p^2 + (1-p)^2}, (0, 1), (0, \rho^*(g, b))) + \delta W(1-p, (0, \rho^*(g, b)))].$$

The first term corresponds to the expected welfare for pessimistic receivers given an optimistic sender. Similarly, the first term between square brackets corresponds to the expected welfare of all pessimistic receivers given a pessimistic sender. The second term between square brackets corresponds to the expected utility of the pessimistic sender. In the Appendix, we prove that $Wel(b, sub, sep)$ is strictly increasing in sub. This is also intuitive: the higher sub, the higher $\rho^*(g, b)$ and the higher a pessimist's gain of waiting. Total social welfare equals

$$Wel(sub, sep) = Wel(g, sub, sep) + Wel(b, sub, sep).$$

Suppose now all players focus on the pooling equilibrium. From above, we know that both sender's types then send the message $\hat{s}_i = g$, that optimists invest with probability $\rho^*(g, g)$ and that pessimists do not invest. Note that receiving the message

[17] With probability $\frac{1}{2}$, the sender is pessimistic, in which case $2p(1-p)(N-1)$ optimistic receivers invest at time one with probability $\rho^*(g, b)$; with probability $\frac{1}{2}$, the sender is optimistic, in which case $(p^2 + (1-p)^2)(N-1) + 1$ optimistic players (= conditional expected number of optimistic receivers plus the optimistic sender) invest at time one with probability one.

$\hat{s}_i = g$ in the pooling equilibrium is informationally different from receiving the same message in the separating one (and, more importantly, leads to a different behaviour in the continuation game). To avoid confusion, in this subsection we denote by $\rho^*(g, h_1)$ ($\rho^*(g, g)$) the probability with which all optimists invest at time one in the pooling (separating) equilibrium. Here again, we estimate the social welfare separately for optimists and for pessimists (total welfare is denoted by $Wel(sub, pool)$). For optimists, this writes:

$$Wel(g, sub, pool) = \frac{N}{2}(p - c) + \frac{N}{2}(1 - \rho^*(g, h_1))sub. \tag{7}$$

For pessimists, we have:

$$Wel(b, sub, pool) = \frac{N}{2}\delta W(1 - p, (0, \rho^*(g, h_1))). \tag{8}$$

LEMMA 4 *$\forall sub \in [0, \overline{sub})$, $\rho^*(g, h_1)$ is strictly increasing in sub and $\rho^*(g, h_1) < 1$.*

Proof: See Appendix.
The intuition is similar to the one behind Lemma 3. As above, $Wel(g, sub, pool)$ need not be monotonic in sub, while $Wel(b, sub, pool)$ strictly increases in sub. Our main result is summarised below.

PROPOSITION 4 *If the subsidy does not alter the nature of the played equilibrium, any $sub \in (0, \overline{sub})$ is (strictly) better (for welfare) than no subsidy at all. The relationship between welfare and sub need, however, not be monotonic.*

Proof: From Lemmas 3 and 4 follows that $\forall sub \in (0, \overline{sub})$ $(1 - \rho^*(g, b))sub$ and $(1 - \rho^*(g, h_1))sub$ are both strictly positive. This result, combined with our earlier insight (proven in the Appendix) that equations (6) and (8) are strictly increasing in sub, shows that $Wel(sub, sep) > Wel(0, sep)$ and $Wel(sub, pool) > Wel(0, pool)$. Q.E.D.

Proposition 4 is not very surprising: because of the information externality the social benefit of investing at time one exceeds the private one. Hence, a social planner fixes $sub > 0$ to close the gap between both benefits. However, it would be premature to conclude that - in the presence of information externalities - investments must always be subsidised as the example below suggests.

Suppose $c = \frac{1}{2}$ and that our players focus on the separating equilibrium. We now show that the social planner can increase welfare by imposing an arbitrarily small, but strictly positive, investment tax (i.e. $sub = -\epsilon$). We first compute $Wel(0, sep)$.

Observe that in the separating equilibrium $\Pr(G|s_j = g, \hat{s}_i = b) = \frac{1}{2} = c$, and thus there exists a PBE in which $\rho^*(g,b) = 0$. Hence, from equation (5) follows that

$$Wel(g, 0, sep) = \frac{N}{2}(p - c). \tag{9}$$

As $\rho^*(g,b) = 0$,

$$\delta W(\frac{(1-p)^2}{p^2 + (1-p)^2}, (0,1), (0,0)) = \delta W(1-p, (0,0)) = 0,$$

and from equation (6) we know that

$$Wel(b, 0, sep) = (N-1)p(1-p)(\frac{1}{2} - c) = 0. \tag{10}$$

Adding (9) and (10), one has

$$Wel(0, sep) = \frac{N}{2}(p - c). \tag{11}$$

This is intuitive: if $\hat{s}_i = g$, pessimists invest at time one and get a zero payoff. If $\hat{s}_i = b$, $\rho^*(g,b) = 0$ and our pessimistic players also get a zero payoff. Hence, if $c = \frac{1}{2}$ total welfare is only determined by the expected utilities of the optimistic players. If $\hat{s}_i = g$, all optimists invest at time one. If $\hat{s}_i = b$, optimistic receivers do not invest, but nonetheless obtain the same payoff (i.e. zero) as the one they would obtain if they were to invest at time one. Stated differently, unconditionally investing at time one is - for an optimist - payoff equivalent to the alternative strategy in which she only invests if $\hat{s}_i = g$. Thus, an optimist gets $p - c$ and, in expected terms, half of the population is optimistic. Thus, welfare equals $\frac{N}{2}(p - c)$.

If $sub = -\epsilon$, $c' > \frac{1}{2}$ and the unique monotone PBE is the pooling one. As $\epsilon \to 0$,

$$Wel(g, -\epsilon, pool) \to \frac{N}{2}(p - c) \text{ and } Wel(b, -\epsilon, pool) = \delta W(1-p, (0, \rho^*(g, h_1))).$$

As $\rho^*(g, h_1) > \rho^*(g, b) = 0$, pessimists benefit from a more informative statistic in the pooling equilibrium and thus $Wel(0, sep) < Wel(-\epsilon, pool)$. Our main insight is summarised below.

Proposition 5 *An investment tax can - by altering the nature of the played equilibrium - (strictly) increase welfare.*

5.2 How does the sender's ability influence her incentives for truthful revelation?

So far we assumed that the sender was "as able" as the receivers in the sense that all players possess a signal of the same precision. One may find it more natural to endow player i with a more precise signal. After all, in our model she can be interpreted as a guru and people typically think of them as being better informed (that's the reason why they appear in the media). There is a straightforward way to allow for a better informed sender. Let's assume that player i's signal is drawn from the distribution: $\Pr(g|G) = \Pr(b|B) = r$ and $\Pr(b|G) = \Pr(g|B) = 1-r$ (where $1 > r > p$). The higher r, the "smarter" or the better informed the sender. Our main result is summarised below:

PROPOSITION 6 $\forall c \in (1-p, \min\{p, \frac{(1-p)r}{(1-p)r+p(1-r)}\})$, $\exists$ *a separating equilibrium. This range of parameter values cannot decrease in the precision of the sender's signal.*

Proof: A MCE in which $\tilde{\rho}(b,g) = \tilde{\rho}(g,g) = 1$ exists only if $\Pr(G|b, \hat{s}_i = g) \geq c$. This posterior probability is now computed as:

$$\Pr(G|b, \hat{s}_i = g) = \frac{\Pr(G, \hat{s}_i = g|b)}{\Pr(\hat{s}_i = g|b)} = \frac{(1-p)r}{(1-p)r + p(1-r)} > \frac{1}{2}.$$

Using a reasoning identical to the one we outlined above, one can check that, if $c \in (1-p, \frac{(1-p)r}{(1-p)r+p(1-r)})$, there exists a separating equilibrium. Q.E.D.

The intuition behind proposition 6 is simple. As we showed in Proposition 3, a separating equilibrium exists if $\rho^*(b,g) = 1$. In other words, a separating equilibrium only exists if the sender can make the pessimists change their minds. Proposition 6 therefore rests on the intuitive idea that the "smarter" the sender (or the more precise her private information), the "easier" it will be for her to make the pessimists change their minds. If the sender cannot convince the remaining pessimists to invest at time one (either because the sender is commonly perceived to be "stupid" or because the investment project only generates a low surplus) then she doesn't want to reveal any unfavourable information because this will worsen her second-period inference problem.

6 The case of an uninformed sender

So far, we did not allow the sender to be uninformed. One may find this a restrictive assumption. However in this section we argue that the central result of our paper also holds with an uninformed sender.

Suppose that with some probability ϵ (where, as above, ϵ represents an arbitrarily small, but strictly positive number) player i does not possess any private information.[18] More specifically, assume that $\Pr(s_i = g|G) = \Pr(s_i = b|B) = (1-\epsilon)p$, $\Pr(s_i = \phi|G) = \Pr(s_i = \phi|B) = \epsilon$ and $\Pr(s_i = b|G) = \Pr(s_i = g|B) = (1-\epsilon)(1-p)$. Player i's message is now $\in \{b, \phi, g\}$. Throughout this subsection, we assume that $c \in (1-p, \frac{1}{2}]$.

In this set-up there exists a semi-separating equilibrium in which the b-type and the ϕ-type both send the same message (say, message $\hat{s}_i = \phi$) and the g-type sends a different message (say, message $\hat{s}_i = g$). To understand this, we first explain how in equilibrium player j computes her posteriors given the different sender's strategies. First, assume player j is an optimist. Upon receiving the message $\hat{s}_i = \phi$, she computes:

$$\Pr(G|s_j = g, \hat{s}_i = \phi, \text{only } b\text{-type and } \phi\text{-type send message } \phi) > \frac{1}{2}$$

Next, assume player j is a pessimist, she computes:

$$\Pr(G|s_j = b, \hat{s}_i = \phi, \text{only } b\text{-type and } \phi\text{-type send message } \phi) < 1-p \tag{12}$$

Similarly, if player i sends $\hat{s}_i = g$, a pessimist computes:

$$\Pr(G|s_j = b, \hat{s}_i = g, \text{only } g\text{-type sends message } g) = \frac{1}{2}$$

From the previous section we know that a pessimistic sender strictly prefers to send message $\hat{s}_i = \phi$ rather than message $\hat{s}_i = g$. Consider now a sender who doesn't possess any information. What is her expected gain of sending message $\hat{s}_i = g$? In that case from above we know that there exists a continuation equilibrium in which everyone invests at time one. As player i faces a positive gain of investing, she gets $\frac{1}{2} - c \geq c$. What is her expected gain of sending message $\hat{s}_i = \phi$? Upon receiving message $\hat{s}_i = \phi$, from (12) follows that pessimists do not invest at time one. Optimists compute $\Pr(G|s_j = g, \hat{s}_i = \phi)$ and invest with probability $\rho^*(g, \phi)$. If player i invests she gets $\frac{1}{2} - c$. If she waits, she gets $\delta W(\frac{1}{2}, \rho^*(g, \phi))$. From our previous section we know that the following equalities and inequality are satisfied:

$$\text{gain send } g = \frac{1}{2} - c = \delta W(\frac{1}{2}, a) < \delta W(\frac{1}{2}, \rho^*(g, \phi)) = \text{gain send } \phi$$

[18]Note that, for simplifying reasons, we still assume that $s_j \in \{g, b\}$ $(j \neq i)$, i.e. only the sender may be uninformed.

Therefore it's optimal for her to wait at time one and she strictly prefers to send message $\hat{s}_i = \phi$.[19]

Finally, from the previous section we also know that an optimistic sender cannot gain by deviating neither. The proposition below summarises the insight present in this subsection:

PROPOSITION 7 *If there exists an arbitrarily small probability of player i being uninformed, then* $\forall c \in (1-p, \frac{1}{2}]$*, there exists a semi-separating equilibrium. In that equilibrium* $\rho^*(g,g) = \rho^*(b,g) = 1$.

Proof: See Appendix.

Two conclusions can be drawn out of our last proposition : (i) the separating equilibrium highlighted in Proposition (3) is driven by the assumption that the sender can either be an optimist or a pessimist, (ii) however this does not mean that the insight present in Proposition (3) is worthless. After all, the occurence of an informational cascade along the equilibrium path is also stressed in Proposition (7). Our last Proposition shows that one should not interpret Proposition (3) as follows: "Informational cascades induce all possible types of players to truthfully reveal their private information". Instead, Proposition (3) should be interpreted as: "Informational cascades put an upper limit above which some types of players don't want to misrepresent their information".

7 Conclusions

In this paper we introduced cheap talk in an investment model with information externalities. We first showed that for low surplus projects, the unique monotone PBE is the pooling one. This is because a pessimist is reluctant to divulge her bad information as this worsens her second-period inference problem. For high surplus projects, however, there exists a separating equilibrium: as a pessimist doesn't learn anything upon observing an informational cascade (which occurs whenever the sender sends a favourable message) revelation of bad information is compatible with maximising behaviour. A subsidy on low-surplus projects increases welfare, *provided the subsidy*

[19] In the equation above $a \in [0,1)$ denotes the probability with which optimists must invest at time one to make an uninformed sender indifferent between investing and waiting. As $\epsilon > 0$, $\Pr(G|s_j = g, \hat{s}_i = \phi) > \frac{1}{2}$. From the insight presented in Graph 2 we know that $a < \rho^*(g,\phi)$. Note that, when explaining the intuition behind our semi-separating equilibrium, we abstracted from the fact that the sender observes k^s and not k. As shown in the Appendix, this is without loss of generality.

does not turn a low-surplus project into a high-surplus one. Without an adequate equilibrium selection theory, one cannot appraise the welfare consequences of a policy aimed at subsidising high-surplus projects. Finally, we argued that in our context "smart" people have more incentives to truthfully reveal their private information than "stupid" ones.

The reader must bear in mind that we only introduced cheap talk in an endogenous-queue set-up. More research is thus needed to check the robustness of exogenous-queue herding models to the introduction of cheap talk. In our model one should think about the sender as a famous investor who's being interviewed by the media. We believe it would be equally interesting to consider a set-up in which many players have access to the communication channel through words. In particular, we have two interpretations in mind. First, one could model "the economics of opinion polls" in which a subset of the population is asked to simultaneously send a message to all players in the economy. Second, one could model "the economics of business lunches" in which a subset of the population meet and discuss the investment climate prior to the first investment date (the outcome of the discussion is not divulged to the other players in the economy). We also believe this to constitute an interesting topic for future research.

Appendix

1 Some Definitions and Useful Lemmas

Let $q \in \{q_\omega, q_\pi, 1-p, p\}$.

$$\rho_1 \equiv (\rho(b,h_1), \rho(g,h_1)), \tilde{\rho}_1 \equiv (\tilde{\rho}(b,h_1), \tilde{\rho}(g,h_1)), \text{ and}$$

$$\sigma_1 \equiv (\sigma(b,h_1), \sigma(g,h_1)), \tilde{\sigma}_1 \equiv (\tilde{\sigma}(b,h_1), \tilde{\sigma}(g,h_1)).$$

$$\Delta^r(q, \sigma_1, \rho_1) \equiv \delta W(q, \sigma_1, \rho_1) - (q - c'), \tag{13}$$

where $c' = c - sub$, and

$$W(q, \sigma_1, \rho_1) = \sum_{a_i} \sum_{k} \max\{0, \Pr(G|q,k,a_i) - c\} \Pr(k|q,a_i) \Pr(a_i|q). \tag{14}$$

Similarly,

$$\Delta^s(q, \rho_1) \equiv \delta W(q, \rho_1) - (q - c'),$$

where,

$$W(q, \rho_1) \equiv \sum_{k^s} \max\{0, \Pr(G|s_i, k^s) - c\} \Pr(k^s|s_i).$$

In words, $\Delta^r(q, \sigma_1, \rho_1)$ denotes a receiver's difference between her gain of waiting and her gain of investing given her posterior, σ_1, ρ_1 and sub. $\Delta^s(p, \rho_1)$ denotes the difference between an optimistic sender's gain of waiting and her gain of investing. Note that the sender, when observing k investments, computes her posterior by explicitly taking into account the fact that $N-1$ (and not $N-2$) players were investing with probability $\rho(b,h_1)$ if they were pessimists and with probability $\rho(g,h_1)$ if they were optimists. Observe that, as $sub \in [-\epsilon, \overline{sub})$ ($\epsilon > 0$ and $\epsilon \to 0$ and the definition of $\overline{sub}$ can be found in the body of our paper), $1 - p < c' < p$.

Lemma 5 *$\Delta^r(q, \sigma_1, \rho_1)$ is (weakly) increasing in $(\sigma(g,h_1) - \sigma(b,h_1))$.*

Proof: As we are focusing on monotone strategies $\sigma(g,h_1) - \sigma(b,h_1) \geq 0$. We prove the Lemma in two different steps. First, we show that $\Delta^r(\cdot)$ is weakly increasing in $\sigma(g,h_1)$ for any given $\sigma(b,h_1) \leq \sigma(g,h_1)$. Next, we show that $\Delta^r(\cdot)$ is weakly decreasing in $\sigma(b,h_1)$ for any given $\sigma(b,h_1) \leq \sigma(g,h_1)$.

Step 1: Fix an arbitrary $\sigma(b,h_1) \leq \sigma(g,h_1)$, and consider two investment probabilities $\sigma(g,h_1) < \sigma'(g,h_1)$. Call a_i (a'_i) the time-one action taken by the sender when $\sigma_1 = (\sigma(b,h_1), \sigma(g,h_1))$ ($\sigma_1 = (\sigma(b,h_1), \sigma'(g,h_1))$). Having the optimistic sender randomize with probability $\sigma(g,h_1)$ is ex ante identical to the following two-stage experiment: let the optimistic sender invest with probability $\sigma'(g, h_1)$. Construct a_i then in the following way:

$$\text{if } a_i' = \text{ invest}, \quad \begin{cases} a_i = \text{ invest} & \text{with probability } \frac{\sigma(g,h_1)}{\sigma'(g,h_1)}, \\ a_i = \text{ wait} & \text{with probability } 1 - \frac{\sigma(g,h_1)}{\sigma'(g,h_1)}, \end{cases}$$

$$\text{if } a_i' = \text{ wait}, \qquad a_i = \text{ wait} \quad \text{with probability } 1.$$

Hence, a_i' is a sufficient statistic for a_i and from Blackwell's theorem follows that $\forall \sigma(b, h_1) \le \sigma(g, h_1)$, $W(q, (\sigma(b, h_1), \sigma(g, h_1)), \rho_1) \le W(q, (\sigma(b, h_1), \sigma'(g, h_1)), \rho_1)$.

Step 2: Fix an arbitrary $\sigma(g, h_1) \ge \sigma(b, h_1)$, and consider two investment probabilities $\sigma'(b, h_1) < \sigma(b, h_1)$. Call a_i (a_i') the time-one action taken by the sender when $\sigma_1 = (\sigma(b, h_1), \sigma(g, h_1))$ ($\sigma_1 = (\sigma'(b, h_1), \sigma(g, h_1))$). As above, one can construct a_i on the basis of a_i' in the following way: let the pessimistic sender wait with probability $1 - \sigma'(b, h_1)$.

$$\text{If } a_i' = \text{ wait}, \quad \begin{cases} a_i = \text{ wait} & \text{with probability } \frac{1-\sigma(b,h_1)}{1-\sigma'(b,h_1)}, \\ a_i = \text{ invest} & \text{with probability } 1 - \frac{1-\sigma(b,h_1)}{1-\sigma'(b,h_1)}, \end{cases}$$

$$\text{if } a_i' = \text{ invest}, \qquad a_i = \text{ invest} \quad \text{with probability } 1.$$

As before, a_i' is a sufficient statistic for a_i and from Blackwell's theorem follows that $\forall \sigma(b, h_1) \le \sigma(g, h_1)$, $W(q, (\sigma(b, h_1), \sigma(g, h_1)), \rho_1) \le W(q, (\sigma'(b, h_1), \sigma(g, h_1)), \rho_1)$. Q.E.D.

LEMMA 6 *$\Delta^r(q, \sigma_1, \rho_1)$ is strictly decreasing in q, $\forall \rho_1$, $\forall \sigma_1$.*

Proof: Consider player l and player l'. Both players received the same message from the sender but player l anticipates that $\Theta = G$ with probability q, while player l' anticipates that $\Theta = G$ with probability q'. Suppose, wlog, that $q' > q$. Observe that equation (14) can be rewritten as:

$$W(q, \sigma_1, \rho_1) = q \sum_x \Pr(x|G, \hat{s}_i)(1 - c) I_{\{\Pr(g|q,x) \ge c\}} + (1 - q) \sum_x \Pr(x|B, \hat{s}_i)(-c) I_{\{\Pr(g|q,x) \ge c\}}, \tag{15}$$

where $I_{\{.\}}$ represents the indicator function. Remind that x denotes a $(1 \times N)$ vector where the l-th element equals one if player l invested at time one and zero otherwise. We start by proving the following inequality:

$$q' - q \ge W(q', \sigma_1, \rho_1) - W(q, \sigma_1, \rho_1). \tag{16}$$

Note that

$$W(q', \sigma_1, \rho_1) - W(q, \sigma_1, \rho_1) \leq W(q', \sigma_1, \rho_1) - W'(q, \sigma_1, \rho_1),$$

where,

$$W'(q, \sigma_1, \rho_1) \equiv q \sum_x \Pr(x|G, \hat{s}_i)(1-c) I_{\{\Pr(g|q',x) \geq c\}}$$
$$+(1-q) \sum_x \Pr(x|B, \hat{s}_i)(-c) I_{\{\Pr(g|q',x) \geq c\}}.$$

Hence, a sufficient condition for (16) to hold is that

$$q' - q \geq W(q', \sigma_1, \rho_1) - W'(q, \sigma_1, \rho_1). \tag{17}$$

Note that the RHS of (17) can be written as:

$$W(q', \sigma_1, \rho_1) - W'(q, \sigma_1, \rho_1) = (q'-q) \sum_x \Pr(x|G, \hat{s}_i)(1-c) I_{\{\Pr(g|q',x) \geq c\}} \tag{18}$$
$$-(q'-q) \sum_x \Pr(x|B, \hat{s}_i)(-c) I_{\{\Pr(g|q',x) \geq c\}}.$$

Note also that the LHS of (17) can be rewritten as:

$$q' - q = (q'-q) \sum_x \Pr(x|G, \hat{s}_i)(1-c) - (q'-q) \sum_x \Pr(x|B, \hat{s}_i)(-c). \tag{19}$$

Using (18) and (19), inequality (17) can be rewritten as

$$(q'-q) \sum_x \Pr(x|G, \hat{s}_i)(1-c)(1 - I_{\{\Pr(g|q',x) \geq c\}})$$
$$+(q'-q) \sum_x \Pr(x|B, \hat{s}_i) c (1 - I_{\{\Pr(g|q',x) \geq c\}}) \geq 0,$$

which is obviously satisfied. Using (13), one has

$$\Delta^r(q', \sigma_1, \rho_1) - \Delta^r(q, \sigma_1, \rho_1) = \delta(W(q', \sigma_1, \rho_1) - W(q, \sigma_1, \rho_1)) - (q'-q).$$

From above (+ using the fact that $\delta < 1$), it follows that

$$\Delta^r(q', \sigma_1, \rho_1) < \Delta^r(q, \sigma_1, \rho_1),$$

which proves the Lemma. Q.E.D.

Lemma 7 *$\Delta^s(p, \rho_1) = \Delta^r(p, \rho_1, \rho_1)$ and $\Delta^s(1-p, \rho_1) = \Delta^r(1-p, \rho_1, \rho_1)$.*

Proof: Suppose $s_j = g$ (the argument if $s_j = b$ is fully symmetric). Observe that, as $q_\omega = p$, player j did not learn anything about the sender's type after the communication stage. Observe also that the sender invests with the same probability as the receivers. Both observations imply that observing $a_i =$ invest is informationally equivalent to observing $a_l =$ invest (where $l \neq j$ and $l \neq i$). Hence, if player j waits she has access to an information service that is ex ante identical to the one of the optimistic sender. Thus, player j and the optimistic sender face the same gain of waiting and the same gain of investing, which implies the Lemma. Q.E.D.

LEMMA 8 *$\Delta^s(p, \rho_1)$ is strictly decreasing in p, $\forall \rho_1$.*

Proof: From Lemma 7, we know that $\Delta^s(p, \rho_1) = \Delta^r(p, \rho_1, \rho_1)$. But then it follows from Lemma 6 that $\Delta^r(p, \rho_1, \rho_1)$ is strictly decreasing in p. Q.E.D.

LEMMA 9 *$\forall \rho'(g, h_1) > \rho(g, h_1)$, $\Delta^r(q, \sigma_1, (0, \rho(g, h_1)) \leq \Delta^r(q, \sigma_1, (0, \rho'(g, h_1))$, where the inequality becomes strict whenever $\rho'(g, h_1) > \rho^c \geq 0$.*

Proof: First observe that whenever $\Pr(G|q, k, a_i)$ is well defined, one has:

Remark 1: $\Pr(G|q, k = 0, a_i) < \Pr(G|q, k = 1, a_i) < ... < \Pr(G|q, k = N - 2, a_i)$.

Remark 2: $\Pr(G|q, k = 0, a_i)$ is strictly decreasing in $\rho(g, h_1)$.

Remark 3: $\Pr(G|q, k = 0, a_i = \text{wait}) \leq \Pr(G|q, k = 0, a_i = \text{invest})$.

Remark 3 rests on the observation that, as $1 - p < c'$, $\sigma^*(b, h_1) = 0$. Before defining ρ^c we must make a distinction between the following two cases: (1) $\Pr(G|q, 0, wait)$ is well defined and (2) $\Pr(G|q, 0, wait)$ is not well defined. Observe that whenever $\rho'(g, h_1) > 0$, (2) only happens if - after the communication stage - all players learned that $s_i = g$ and that $\sigma(g, g) = 1$. In (1) we must make the following distinction: (a) $\Pr(G|q, wait) > c$ and (b) $\Pr(G|q, wait) \leq c$. In (a) we define ρ^c as the probability with which $N - 2$ receivers must invest (if they are optimists) such that $\Pr(G|q, 0, wait) = c$. Observe that in (a)

$$\Pr(G|q, 0, wait, \rho(g, h_1) = 1) < c < \Pr(G|q, wait) = \Pr(G|q, 0, wait, \rho(g, h_1) = 0),$$

and, thus, in (a) $0 < \rho^c < 1$. In (b) there does not exist a $\rho(g, h_1) > 0$ such that $\Pr(G|q, 0, wait) = c$. Hence, in (b) we define ρ^c as being equal to zero. In (2) we make the following distinction: (c) $\Pr(G|q, invest) > c$ and (d) $\Pr(G|q, invest) \leq c$. As before, in (c) we define ρ^c as the probability with which the $N - 2$ receivers must invest (if they are optimists) such that $\Pr(G|q, 0, invest) = c$. In this case $0 < \rho^c < 1$. In (d) we define ρ^c as being equal to zero.

Call k' (k) the number of time-one investors when $N-2$ receivers invest with probability $\rho'(g, h_1)$ ($\rho(g, h_1)$)if they are optimists, and with probability zero if they are pessimists. From the explanation given in the text we know that k' is a sufficient statistic for k. Consider two receivers: player 1 and player 2. Both players anticipate that $\Theta = G$ with probability q. If player 1 (2) waits, she observes statistic k' (k).

If $\rho(g, h_1) < \rho'(g, h_1) \leq \rho^c$, from Remarks 1, 2 and 3 we know that both players always invest at time two and $\Delta^r(q, \sigma_1, (0, \rho(g, h_1)) = \Delta^r(q, \sigma_1, (0, \rho'(g, h_1))$. If $\rho^c \leq \rho(g, h_1) < \rho'(g, h_1)$, with strictly positive probability

$$\Pr(G|q, k = 0, a_i) \leq c < \Pr(G|q, k' = N-2, a_i),$$

in which case player two (wrongly) doesn't invest and loses $\Pr(G|q, k' = N-2, a_i) - c > 0$. Hence, whenever $\rho'(g, h_1) > \rho(g, h_1) \geq \rho^c$,

$$\Delta^r(q, \sigma_1, (0, \rho(g, h_1)) < \Delta^r(q, \sigma_1, (0, \rho'(g, h_1)).$$

Q.E.D.
Lemma 9 gives rise to the following Corollary.

Corollary 1 $\forall \rho'(g, h_1) > \rho(g, h_1)$,
1) $\Delta^s(p, (0, \rho'(g, h_1))) \geq \Delta^s(p, (0, \rho(g, h_1)))$ *where the inequality becomes strict whenever* $W(p, (0, \rho(g, h_1))) > p - c$,
2) $\Delta^s(1-p, (0, \rho'(g, h_1))) > \Delta^s(1-p, (0, \rho(g, h_1)))$.

Proof: This Corollary was already proven in Chamley and Gale (1994) (see their Proposition 2). In our set-up the Corollary follows from our previous Lemmas as the argument below shows.

Suppose that $q \in \{1-p, p\}$ and that $\sigma_1 = \rho_1$. From Lemma 7, we know that player j's gain of waiting is then identical to player i's. Define ρ^c in a similar way as in the proof of Proposition 9. Observe that $0 < \rho^c < 1 \Leftrightarrow W(p, \rho_1, \rho_1) > p - c$. The Corollary then follows from the proof of Lemma 9. Q.E.D.

Lemma 10 $\Delta^r(\frac{1}{2}, \sigma_1, \rho_1)$ *and* $\Delta^r(\overline{q}_\omega, \sigma_1, \rho_1)$ *are independent of* σ_1.

Proof: Observe that $W(q, \sigma_1, \rho_1)$ can also be rewritten as

$$W(q, \sigma_1, \rho_1) = \Pr(a_i = \text{invest}|s_j, \hat{s}_i) W^r(q', \rho_1) + \Pr(a_i = \text{wait}|s_j, \hat{s}_i) W^r(q'', \rho_1), \text{ where} \tag{20}$$

$q' = \Pr(G|s_j, \hat{s}_i, a_i = \text{invest})$, $q'' = \Pr(G|s_j, \hat{s}_i, a_i = \text{wait})$,

$$W^r(q', \rho_1) = \sum_k \max\{0, \Pr(G|s_j, \hat{s}_i, k, \text{invest}) - c\} \Pr(k|s_j, \hat{s}_i, a_i = \text{invest}) \text{ and}$$

$$W^r(q'', \rho_1) = \sum_k \max\{0, \Pr(G|s_j, \hat{s}_i, k, \text{wait}) - c\} \Pr(k|s_j, \hat{s}_i, a_i = \text{wait}).$$

If $q = \frac{1}{2}$ or if $q = \overline{q}_\omega$, this means that the receivers learned s_i through the sender's message. Hence,

$$W(q, \sigma_1, \rho_1) = W^r(q', \rho_1) = W^r(q'', \rho_1) = \sum_k \max\{0, \Pr(G|s_j, k, s_i) - c\} \Pr(k|s_j, s_i),$$

which is independent of σ_1. Q.E.D.

Lemma 11 $\forall \rho(b, h_1) < \rho'(b, h_1)$, $\Delta^r(q, \sigma_1, (\rho(b, h_1), 1)) \geq \Delta^r(q, \sigma_1, (\rho'(b, h_1), 1))$, *where the inequality becomes strict whenever* $\rho(b, h_1) < \rho_c \leq 1$.

Proof: The proof mirrors the one we outlined in Proposition 9. Whenever $\rho(b, h_1) < 1$ and $\rho(g, h_1) = 1$, the act of waiting becomes informative and the probability with which each pessimist decides to take the informative action equals $(1 - \rho(b, h_1))$. Take any two waiting probabilities $1 - \rho(b, h_1) > 1 - \rho'(b, h_1)$. Call z (z') the number of players who waited when pessimistic receivers randomised with probability $1 - \rho(b, h_1)$ ($1 - \rho'(b, h_1)$) and optimistic receivers with probability zero. Having $N - 2$ players randomising with probability $\rho(b, h_1)$ (if they are pessimists) is ex ante identical to the following two-stage experiment: take $N - 2$ players and let them wait (if they are pessimists) with probability $(1 - \rho(b, h_1))$. Next, take the z non-investors and let them invest with probability $\frac{1-\rho'(b,h_1)}{1-\rho(b,h_1)}$. Hence, the statistic z' can be constructed by adding noise to the statistic z. In the rest of the proof we always assume that $\rho(b, h_1) < 1$. Whenever $\Pr(G|q, z, a_i)$ is well defined one has:

Remark 1: $\Pr(G|q, z = 0, a_i) > \Pr(G|q, z = 1, a_i) > ... > \Pr(G|q, z = N - 2, a_i)$.

Remark 2: $\Pr(G|q, z = 0, a_i)$ is strictly decreasing in $\rho(b, h_1)$.

Remark 3: $\Pr(G|q, z, \text{wait}) \leq \Pr(G|q, z, \text{invest})$.

As above, we must distinguish among different cases. If $\Pr(G|q, z = 0, \text{invest})$ is well defined and if $\Pr(G|q, \text{invest}) < c$, we define ρ_c as the probability with which $N - 2$ receivers must invest (if they are pessimists) such that $\Pr(G|q, 0, \text{invest}) = c$.

If $\Pr(G|q, 0, \text{invest})$ is not well defined and if $\Pr(G|q, \text{wait}) < c$, we define ρ_c as the probability with which $N-2$ receivers must invest (if they are pessimists) such that $\Pr(G|q, 0, \text{wait}) = c$. In all the other cases we define ρ_c as being equal to one.

If $\rho_c \leq \rho(b, h_1) < \rho'(b, h_1)$ from Remarks 1, 2 and 3 we know that both players never invest at time two and $\Delta^r(q, \sigma_1, (\rho(b, h_1), 1)) = \Delta^r(q, \sigma_1, (\rho'(b, h_1), 1))$. If $\rho(b, h_1) < \rho'(b, h_1) \leq \rho_c$ with a strictly positive probability

$$\Pr(G|q, z = N-2, a_i) < c \leq \Pr(G|q, z' = 0, a_i),$$

in which case player 2 wrongly invests (at time two) and loses $c - \Pr(G|q, z = N-2, a_i) > 0$. Hence, $\forall \rho(b, h_1) < \rho'(b, h_1) \leq \rho_c$,

$$\Delta^r(q, \sigma_1, (\rho(b, h_1), 1)) > \Delta^r(q, \sigma_1, (\rho'(b, h_1), 1)).$$

Q.E.D.

Lemma 12 $\Delta^r(\overline{q}_\omega, (0,0), (0,0)) < 0 < \Delta^r(\overline{q}_\omega, (0,0), (0,1))$.

Proof: The fact that $\Delta^r(\overline{q}_\omega, (0,0), (0,0)) < 0$ trivially follows from our assumption that $\delta < 1$. The second inequality rests on A2 and on the fcat that $sub < sub_1$. Q.E.D.

Lemma 13 $\Delta^r(q, (0,0), (0,1)) > 0$, $\forall q$ *and* $\forall sub \in [-\epsilon, \overline{sub})$.

Proof: From Lemmas 12 and 6 follows that $\forall q$ and $\forall sub \in [-\epsilon, \overline{sub})$,

$$0 < \Delta^r(\overline{q}_\omega, (0,0), (0,1)) < \Delta^r(q, (0,0), (0,1)).$$

Q.E.D.

2 Proof of all Lemmas and Propositions in our Paper

Proof of Lemma 1

Call n^r the number of optimistic receivers in the economy. Observe that $\Pr(G|q, n^r)$ is increasing in n^r. As explained in the paper if $\Pr(G|\overline{q}_\omega, n^r = 1) = \Pr(G|n = 2) < c$, then $\Pr(G|q, n^r = 1) < c$ and $W(q, \sigma_1, (0,1)) > q - c\ \forall q$. Hence, we just focus on the question: "How high must N be such that $\Pr(G|\overline{q}_\omega, n^r = 1) < c$?" The posterior $q_\omega = \overline{q}_\omega$ can only be generated if (i) player i sent a favourable message and (ii) $\sigma(g, h_0) = 1$ and $\sigma(b, h_0) = 0$. Therefore if $q_\omega = \overline{q}_\omega$, n cannot take a value lower than two. Now:

$$\Pr(G|n = 2) = \frac{C_N^2 p^2 (1-p)^{N-2}}{C_N^2 p^2 (1-p)^{N-2} + C_N^2 (1-p)^2 p^{N-2}}$$

where C_N^2 represents the number of possible combinations of two players out of a population of N players. It can easily be shown that $\forall N_1 > N_2 \geq 2$:

$$\frac{p^2(1-p)^{N_1-2}}{p^2(1-p)^{N_1-2} + (1-p)^2 p^{N_1-2}} < \frac{p^2(1-p)^{N_2-2}}{p^2(1-p)^{N_2-2} + (1-p)^2 p^{N_2-2}}$$

From statistical textbooks (see e.g. De Groot (1970)) we know that in our set-up $\Pr(G|n)$ is driven by the difference between the good and the bad signals in the population.[20] Therefore if $N \geq 5$, $\Pr(G|n=2) \leq 1-p$ which is strictly lower than c by A1. Q.E.D.

Proof of Lemma 2

Lemma 2 only considers the case in which $c' = c$. In Lemma 13 we already proved that the inequality holds $\forall c'$. Q.E.D.

Proof of Proposition 1

Proposition 1 only considers the case in which $c' = c$, while we prove the Proposition $\forall c'$.

Proof of Point 1: If $q_\pi < 1-p < c' < q_\omega < p$, $\exists$ a unique MCE in which $\tilde{\rho}(b,b) = \tilde{\sigma}(b,b) = 0$ and $\tilde{\rho}(g,b) \in [0,1), \tilde{\sigma}(g,b) = 1$.

The reader should bear in mind that when we claim uniqueness, we mean that (i) the MCE must be stable (see our third equilibrium selection criterion explained in the body of our paper) and (ii) we only focus on the class of monotone strategies (see our second equilibrium selection criterion explained in the body of our paper).

Observe that $q_\pi < 1-p$, which means that the sender sent message $\hat{s}_i = b$. As $q_\pi < 1-p < c'$, this implies that $\tilde{\rho}(b,b) = \tilde{\sigma}(b,b) = 0$. We first show that there does not exist a monotone continuation equilibrium in which $0 < \tilde{\rho}(g,b) \leq \tilde{\sigma}(g,b) < 1$. As both types are willing to randomise this means that

$$\Delta^r(q_\omega, (0, \tilde{\sigma}(g,b)), \tilde{\rho}_1) = 0,$$

$$\Delta^s(p, \tilde{\rho}_1) = 0.$$

[20]For example, $\Pr(G|n=1, N=3) = \Pr(G|n=2, N=5) = 1-p$. In both cases: #pessimists $-$# optimists $= N-n-n = 1$.

Both equalities cannot be simultaneously satisfied as we can successively apply Lemmas 5, 6 and 7 to construct the following contradiction:

$$0 = \Delta^r(q_\omega, (0, \tilde{\sigma}(g, b)), \tilde{\rho}_1)) \geq \Delta^r(q_\omega, \tilde{\rho}_1, \tilde{\rho}_1) > \Delta^r(p, \tilde{\rho}_1, \tilde{\rho}_1) = \Delta^s(p, \tilde{\rho}_1) = 0.$$

Next, observe that there does not exist a monotone continuation equilibrium in which $\tilde{\sigma}(g, b) < 1$ and $\tilde{\rho}(g, b) = 0$, because the optimistic sender, knowing that $\tilde{\rho}(g, b) = 0$, then strictly prefers to invest at time one with probability one.

We now prove the existence of a monotone continuation equilibrium in which $\tilde{\sigma}(g, b) = 1$ and $\tilde{\rho}(g, b) \in [0, 1)$. Consider the optimistic receiver. She knows that $\tilde{\sigma}(g, b) = 1$. There are then two possibilities: (i) $\Delta^r(q_\omega, (0, 1), (0, 0)) \geq 0$ and (ii) $\Delta^r(q_\omega, (0, 1), (0, 0)) < 0$. In case (i), $\tilde{\rho}(g, b) = 0$. The optimistic sender knows that $\tilde{\rho}(g, b) = 0$ and thus stictly prefers to invest at time one with probability one (i.e. $\tilde{\sigma}(g, b) = 1$). In case (ii), from Lemmas 5 and 13, one has

$$\Delta^r(q_\omega, (0, 1), (0, 1)) \geq \Delta^r(q_\omega, (0, 0), (0, 1)) > 0.$$

From Lemma 9, there exists a unique $\tilde{\rho}(g, b) \in (0, 1)$ such that $\Delta^r(q_\omega, (0, 1), (0, \tilde{\rho}(g, b))) = 0$. Successively applying Lemmas 6, 5 and 7, one has

$$0 = \Delta^r(q_\omega, (0, 1), \tilde{\rho}_1) > \Delta^r(p, (0, 1), \tilde{\rho}_1) \geq \Delta^r(p, (0, \tilde{\rho}(g, b)), \tilde{\rho}_1) = \Delta^s(p, \tilde{\rho}_1),$$

and the optimistic sender, knowing that $\tilde{\rho}(g, b)$ is fixed such that $\Delta^r(q_\omega, (0, 1), \tilde{\rho}_1) = 0$, strictly prefers to invest at time one (i.e. $\tilde{\sigma}(g, b) = 1$). Q.E.D.

Proof of Point 2: If $q_\pi < 1 - p < q_\omega \leq c' < p$, $\exists$ a unique MCE in which $\tilde{\rho}(b, b) = \tilde{\sigma}(b, b) = \tilde{\rho}(g, b) = 0$ and $\tilde{\sigma}(g, b) = 1$.

In this case the sender also sent message $\hat{s}_i = b$. As $q_\pi < 1 - p < c'$, $\tilde{\rho}(b, b) = \tilde{\sigma}(b, b) = 0$. Observe also that if $q_\omega \leq c'$, $\forall \rho(g, b) > 0$, $\Delta^r(q_\omega, \sigma_1, (0, \rho(g, b))) > 0$. Hence, $\tilde{\rho}(g, b) = 0$. The optimistic sender, knowing that $\tilde{\rho}(b, b) = \tilde{\rho}(g, b) = 0$, strictly prefers to invest at time one with probability one. Q.E.D.

Proof of Point 3: If $1 - p < q_\pi < c' < p < q_\omega$, $\exists$ a unique MCE in which $\tilde{\sigma}(b, g) = \tilde{\rho}(b, g) = \tilde{\sigma}(g, g) = 0$ and $\tilde{\rho}(g, g) \in (0, 1)$.

In this case the sender sent message $\hat{s}_i = g$. As $1 - p < q_\pi < c'$, $\tilde{\sigma}(b, g) = \tilde{\rho}(b, g) = 0$. Suppose there exists a continuation equilibrium in which $0 < \tilde{\sigma}(g, g) \leq \tilde{\rho}(g, g) < 1$. As both types of players are willing to randomize, this means that

$$\Delta^r(q_\omega, (0, \tilde{\sigma}(g, g)), (0, \tilde{\rho}(g, g))) = 0,$$

$$\Delta^s(p,(0,\tilde{\rho}(g,g)))=0.$$

Both equalities cannot be simultaneously satisfied as we can successively apply Lemmas 5, 6 and 7 to construct the following contradiction:

$$0=\Delta^r(q_\omega,(0,\tilde{\sigma}(g,g)),\tilde{\rho}_1)\leq\Delta^r(q_\omega,\tilde{\rho}_1,\tilde{\rho}_1)<\Delta^r(p,\tilde{\rho}_1,\tilde{\rho}_1)=\Delta^s(p,\tilde{\rho}_1)=0.$$

Note also that there cannot exist continuation equilibria in which $\tilde{\sigma}(g,g)=\tilde{\rho}(g,g)=0$ or in which $\tilde{\sigma}(g,g)=\tilde{\rho}(g,g)=1$ (both candidate continuation equilibria contradict our assumption that $\delta<1$ and Lemma 13).

Suppose $\tilde{\sigma}(g,g)=0$. From Chamley and Gale, we know that there exists then a unique $\tilde{\rho}(g,g)\in(0,1)$ such that $\Delta^r(q_\omega,(0,0),(0,\tilde{\rho}(g,g)))=0$. Successively applying Lemmas 6, 5 and 7, one has

$$0=\Delta^r(q_\omega,(0,0),\tilde{\rho}_1)<\Delta^r(p,(0,0),\tilde{\rho}_1)\leq\Delta^r(p,(0,\tilde{\rho}(g,g)),\tilde{\rho}_1)=\Delta^s(p,\tilde{\rho}_1),$$

and the pessimistic sender, knowing that $\tilde{\rho}(g,g)$ is fixed such that $\Delta^r(q_\omega,(0,0),\tilde{\rho}_1)=0$, strictly prefers to wait at time one (i.e. $\tilde{\sigma}(g,g)=0$). Q.E.D.

Proof of Point 4: If $1-p<c'\leq q_\pi<\frac{1}{2}<p<q_\omega$, $\exists$ an MCE in which $\tilde{\sigma}(b,g)=\tilde{\rho}(b,g)=\tilde{\sigma}(g,g)=0$ and $\tilde{\rho}(g,g)\in(0,1)$. Depending on the values of our exogenous parameters, there may also exist one (and only one) other MCE in which $\tilde{\sigma}(b,g)=0$ and $\tilde{\rho}(b,g)=\tilde{\sigma}(g,g)=\tilde{\rho}(g,g)=1$.

In this case the sender sent message $\hat{s}_i=g$. As $1-p<c'$, $\tilde{\sigma}(b,g)=0$. We prove this point in seven different steps. Steps 1, 2 and 3 show that there does not exist a monotone continuation equilibrium in which more than one type of player randomizes. Steps 4, 5 and 6 show that there exists a unique monotone continuation equilibrium in which only one type of player (i.e. the optimistic receiver) randomises (while the optimistic sender and the pessimistic receiver wait with probability 1). Step 7 investigates the existence of monotone continuation equilibria in which none of our players randomize.

Step 1: There does not exist a monotone continuation equilibrium in which $0<\tilde{\rho}(b,g)\leq\tilde{\sigma}(g,g)\leq\tilde{\rho}(g,g)<1$. Suppose the statement is true. Then one can apply Lemma 6 to construct the following contradiction

$$0=\Delta^r(q_\pi,\tilde{\sigma}_1,\tilde{\rho}_1)>\Delta^r(q_\omega,\tilde{\sigma}_1,\tilde{\rho}_1)=0.$$

Step 2: There does not exist a monotone continuation equilibrium in which $0 = \tilde{\rho}(b,g) < \tilde{\sigma}(g,g) \leq \tilde{\rho}(g,g) < 1$. Suppose the statement is true. Successively applying Lemmas 7, 6 and 5 we can construct then the following contradiction

$$0 = \Delta^s(p,(0,\tilde{\rho}(g,g))) = \Delta^r(p,(0,\tilde{\rho}(g,g)),\tilde{\rho}_1) >$$

$$\Delta^r(q_\omega,(0,\tilde{\rho}(g,g)),\tilde{\rho}_1) \geq \Delta^r(q_\omega,(0,\tilde{\sigma}(g,g)),\tilde{\rho}_1) = 0.$$

Step 3: There does not exist a monotone continuation equilibrium in which $0 < \tilde{\rho}(b,g) \leq \tilde{\sigma}(g,g) < 1 = \tilde{\rho}(g,g)$. Suppose the statement is true. This implies that

$$\Delta^r(q_\pi,(0,\tilde{\sigma}(g,g)),(\tilde{\rho}(b,g),1)) = 0, \tag{21}$$

$$\Delta^s(p,(\tilde{\rho}(b,g),1)) = 0. \tag{22}$$

Applying Lemmas 6 and 10 to equality (21), one has

$$0 = \Delta^r(q_\pi,\tilde{\sigma}_1,\tilde{\rho}_1) \geq \Delta^r(\frac{1}{2},\tilde{\sigma}_1,\tilde{\rho}_1) = \Delta^r(\frac{1}{2},(0,1),\tilde{\rho}_1). \tag{23}$$

Applying Lemmas 7 and 5 to equality (22), one has

$$0 = \Delta^s(p,\tilde{\rho}_1) = \Delta^r(p,(\tilde{\rho}(b,g),1),(\tilde{\rho}(b,g),1)) \leq \Delta^r(p,(0,1),\tilde{\rho}_1). \tag{24}$$

Inequalities (23) and (24) cannot be simultaneously satisfied as we run into the following contradiction (after applying Lemma 6)

$$0 \geq \Delta^r(\frac{1}{2},(0,1),\tilde{\rho}_1) > \Delta^r(p,(0,1),\tilde{\rho}_1) \geq 0.$$

Step 4: There does not exist a monotone continuation equilibrium in which $0 = \tilde{\rho}(b,g) < \tilde{\sigma}(g,g) < 1 = \tilde{\rho}(g,g)$. This is easy to see: if $\tilde{\rho}_1 = (0,1)$, from Lemmas 2, 5 and 7, follows that

$$0 < \Delta^r(p,(0,0),(0,1)) \leq \Delta^r(p,(0,1),\tilde{\rho}_1) = \Delta^s(p,\tilde{\rho}_1),$$

and thus the optimistic sender is not indifferent between investing and waiting.

Step 5: There does not exist a monotone continuation equilibrium in which $0 < \tilde{\rho}(b,g) < \tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$. Consider a pessimistic receiver. There are two different possibilities: (i) $\Delta^r(q_\pi,(0,1),(1,1)) \geq 0$ or (ii) $\Delta^r(q_\pi,(0,1),(1,1)) < 0$. In case (i), a pessimistic receiver, knowing that by waiting she will perfectly learn the sender's type, prefers to wait and is thus unwilling to randomize. In case (ii) from Lemmas 5 and 2 we know that

$$\Delta^r(q_\pi, (0,1), (0,1)) \geq \Delta^r(q_\pi, (0,0), (0,1)) > 0.$$

From Lemma 11 we know that there exists a unique $\tilde{\rho}(b, g)$ such that

$$\Delta^r(q_\pi, (0,1), (\tilde{\rho}(b,g), 1)) = 0.$$

In this case $c' < \frac{1}{2}$ and thus $\forall sub \in [-\epsilon, \overline{sub})$, $c \in (1-p, \frac{1}{2})$. In particular this implies that $\Pr(G|q_\pi,\text{invest}) = \frac{1}{2} > c$ and thus that $\rho_c = 1$ (for the definition of ρ_c, see Lemma 11). From Lemma 11 we know that $W(q_\pi, (0,1), (\rho(b,g), 1))$ is strictly decreasing in $\rho(b,g)$: this implies that a pessimistic receiver's best response is increasing in $\rho(b,g)$: if $\rho(b,g) > (<) \tilde{\rho}(b,g)$, player j strictly prefers to invest (wait). It is well-known that this implies that the candidate continuation equilibrium in which $0 < \tilde{\rho}(b,g) < \tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$ is unstable.

Step 6: There exists a unique monotone continuation equilibrium in which $0 = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) < \tilde{\rho}(g,g) < 1$. From Lemma 13, we know that $\Delta^r(q_\omega, (0,0), (0,0)) < 0 < \Delta^r(q_\omega, (0,0), (0,1))$. From Chamley and Gale we know that there exists a unique $\tilde{\rho}(g,g) \in (0,1)$ such that $\Delta^r(q_\omega, (0,0), (0, \tilde{\rho}(g,g))) = 0$. As $q_\pi < q_\omega$, from Lemma 6 follows that

$$0 = \Delta^r(q_\omega, (0,0), (0, \tilde{\rho}(g,g))) < \Delta^r(q_\pi, (0,0), (0, \tilde{\rho}(g,g))),$$

and thus $\tilde{\rho}(b,g) = 0$. Similarly, using Lemmas 6, 5 and 7, one has

$$0 = \Delta^r(q_\omega, (0,0), (0, \tilde{\rho}(g,g))) < \Delta^r(p, (0,0), \tilde{\rho}_1) \leq \Delta^r(p, \tilde{\rho}_1, \tilde{\rho}_1) = \Delta^s(p, \tilde{\rho}_1),$$

and thus $\tilde{\sigma}(g,g) = 0$.

Step 7: A continuation equilibrium in which $0 = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = \tilde{\rho}(g,g)$ or in which $0 = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) < 1 = \tilde{\rho}(g,g)$ or in which $0 = \tilde{\rho}(b,g) < 1 = \tilde{\sigma}(g,g) = \tilde{\rho}(g,g)$ cannot exist because they contradict A2. As $q_\pi < \frac{1}{2}$, this means that the receivers, upon receiving the message $\hat{s}_i = g$, still face some uncertainty concerning the sender's type. Depending on the values of our exogenous parameters there are two possibilities: (i) $\Delta^r(q_\pi, (0,1), (1,1)) > 0$ and (ii) $\Delta^r(q_\pi, (0,1), (1,1)) \leq 0$. In case (i) a pessimistic receiver, knowing that by waiting she learns the sender's type, strictly prefers to wait and, hence, there does not exist a continuation equilibrium in which $\tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$. In case (ii) using Lemmma 6 we know that

$$\Delta^r(q_\omega, (0,1), (1,1)) < \Delta^r(q_\pi, (0,1), (1,1)) \leq 0,$$

and thus $\tilde{\rho}(b,g) = \tilde{\rho}(g,g) = 1$. The optimistic sender, knowing that $\tilde{\rho}(b,g) = \tilde{\rho}(g,g) = 1$, strictly prefers to invest as well and thus $\tilde{\sigma}(g,g) = 1$. Q.E.D.

Proof of Point 5: If $1 - p < c' \leq q_\pi = \frac{1}{2} < p < q_\omega$, $\exists$ two MCE's. In the first one $\tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = 0$ and $\tilde{\rho}(g,g) \in (0,1)$. In the second one $\tilde{\sigma}(b,g) = 0$ and $\tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$.

In this proof $q \in \{q_\pi, q_\omega\}$. Observe that point 5 is identical to point 4, except that $q_\pi = \frac{1}{2}$, which means that the receivers perfectly inferred the sender's type out of her message. From the analysis in point 4, we know that there exists a stable monotone continuation equililbrium in which $\tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = 0$ and $\tilde{\rho}(g,g) \in (0,1)$. From Lemma 10 we know that $\Delta^r(\frac{1}{2},(0,1),(1,1)) = \Delta^r(\frac{1}{2},(1,1),(1,1))$ and that $\Delta^r(\overline{q}_\omega,(0,1),(1,1)) = \Delta^r(\overline{q}_\omega,(1,1),(1,1))$. Consider a receiver who anticipates that $\tilde{\sigma}(b,g) = \tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$. In that case there is no informational gain of waiting. As $\delta < 1$, $\delta W(q,(1,1),(1,1)) < q - c$. Hence, there exists an $\epsilon > 0$ such that $\forall sub \in [-\epsilon, \overline{sub})$, $\delta W(q,(1,1),(1,1)) < q - c'$, and all receivers prefer to invest with probability one. Similarly, the optimistic sender, knowing that $\tilde{\rho}(b,g) = \tilde{\rho}(g,g) = 1$, strictly prefers to invest at time one. Hence, in case 5 there always exists a monotone continuation equilibrium in which $\tilde{\sigma}(b,g) = 0$ and $\tilde{\rho}(b,g) = \tilde{\sigma}(g,g) = \tilde{\rho}(g,g) = 1$. Q.E.D.

Proof of Point 6: If $q_\pi = 1 - p < c' < q_\omega = p$, $\exists$ a unique MCE in which $\tilde{\sigma}(b,h_1) = \tilde{\rho}(b,h_1) = 0$ and $\tilde{\sigma}(g,h_1) = \tilde{\rho}(g,h_1) \in (0,1)$.

In this case $q_\pi = 1 - p$, which means that the receivers did not learn anything about the sender's type through her message. As $q_\pi = 1 - p < c'$, $\tilde{\sigma}(b,h_1) = \tilde{\rho}(b,h_1) = 0$. As explained in our paper, in this case we impose the restriction that $\tilde{\sigma}(g,h_1) = \tilde{\rho}(g,h_1)$. But then from Proposition 2 of Chamley and Gale follows that there exists a unique $\tilde{\rho}(g,h_1)$ such that $\Delta^r(p,(0,\tilde{\rho}(g,h_1)),(0,\tilde{\rho}(g,h_1))) = \Delta^s(p,(0,\tilde{\rho}(g,h_1))) = 0$. Q.E.D.

Proof of Proposition 2

Proposition 2 only considers the case in which $c' = c$, while we provide a proof $\forall c'$. In particular, we prove that $\forall c' > \frac{1}{2}$, there exists a unique monotone PBE in which $\sigma^*(b,h_0) = \sigma^*(g,h_0) = 1$. This PBE is supported by the out-of-equilibrium belief that if $\hat{s}_i = b$, the sender is a pessimist.

First we show that $\sigma^*(g,h_0) = 1$. Suppose there exists a monotone PBE in which $0 \leq \sigma^*(b,h_0) \leq \sigma^*(g,h_0) < 1$. $\sigma^*(g,h_0)$ can only be strictly lower than one if

$E(U_i|s_i = g, \hat{s}_i = b) \geq E(U_i|s_i = g, \hat{s}_i = g)$. As $\sigma^*(b, h_0) \leq \sigma^*(g, h_0)$, this means that if the optimistic sender "lies" and sends $\hat{s}_i = b$, $q_\omega \leq p$. From points 1,2 and 6 of Proposition 1, we know that her payoff (net of the ϵ-reputational cost) can then not exceed $p - c'$. Hence,

$$E(U_i|s_i = g, \hat{s}_i = b) = p - c' - \epsilon < E(U_i|s_i = g, \hat{s}_i = g) = \max\{p - c', \delta W(\cdot)\},$$

a contradiction.

As $\sigma^*(g, h_0) = 1$, the message $\hat{s}_i = b$ can only come from a pessimistic sender (if $\sigma^*(b, h_0)$ also equals one, then we assume that in the out-of-equilibrium event that $\hat{s}_i = b$, receivers believe with probability one that the sender is a pessimist). Hence, $\Pr(G|s_j = g, \hat{s}_i = b) = \frac{1}{2}$. Suppose $\hat{s}_i = b$. Then, $q_\pi < 1 - p < q_\omega = \frac{1}{2} < c' < p$ and from point 2 of Proposition 1, we know that $\rho^*(b, b) = \rho^*(g, b) = 0$. Suppose that $\hat{s}_i = g$. Then, $1 - p < q_\pi \leq \frac{1}{2} < c' < p \leq q_\omega$ and from points 3 and 6 of Proposition 1, we know that $\rho^*(b, g) = 0$ and that $\rho^*(g, g) \in (0, 1)$. Hence,

$$E(U_i|s_i = b, \hat{s}_i = b) = 0,$$

$$E(U_i|s_i = b, \hat{s}_i = g) = \delta W(1 - p, (0, \rho^*(g, g))) - \epsilon.$$

As $\rho^*(g, g) > 0$, this means that $\Pr(k = N - 1|s_i = b) > 0$, in which case the sender invests and gets a strictly positive payoff. Hence, $\delta W(1 - p, (0, \rho^*(g, g))) > 0$. As $\epsilon \to 0$, it follows that $E(U_i|s_i = b, \hat{s}_i = b) < E(U_i|s_i = b, \hat{s}_i = g)$, and thus $\sigma^*(b, h_0) = 1$. Q.E.D.

Proof of Proposition 3

Proposition 3 only considers the case in which $c' = c$, while we provide a proof $\forall c'$. In particular, we prove that $\forall c' \leq \frac{1}{2}$, our game is characterised by two monotone PBE's: a pooling and a separating one. In the separating equilibrium, $\rho^*(b, g) = \rho^*(g, g) = 1$. The pooling equilibrium is supported by the out-of-equilibrium belief that if $\hat{s}_i = b$, the sender is a pessimist.

From the proof of Proposition 2, we know that $\sigma^*(g, h_0) = 1$. We prove the Proposition in three different steps. First we show that there does not exist a monotone PBE in which $0 < \sigma^*(b, h_0) < \sigma^*(g, h_0) = 1$. Next, we show that there exists a separating equilibrium. Finally, we show that there exists a pooling equilibrium in which $\sigma^*(b, h_0) = \sigma^*(g, h_0) = 1$.

Step 1: Suppose there exists a monotone PBE in which $0 < \sigma^*(b, h_0) < \sigma^*(g, h_0) = 1$. $\sigma^*(b, h_0)$ can only be $\in (0, 1)$ if $E(U_i|s_i = b, \hat{s}_i = b) = E(U_i|s_i = b, \hat{s}_i = g)$. If the pessimistic sender sends $\hat{s}_i = b$, $q_\pi < 1 - p < c' \leq q_\omega = \frac{1}{2} < p$, and from points 1 and 2 of Proposition 1, we know that $\rho^*(b, b) = 0$ and that $\rho^*(g, b) \in [0, 1)$. If she sends $\hat{s}_i = g$, there are two possibilities: (a) $1 - p < q_\pi < c' < p < q_\omega$ and (b) $1 - p < c' \leq q_\pi < \frac{1}{2} < p < q_\omega$.

In case (a), from point 3 of Proposition 1 we know that $\rho^*(b, g) = 0$ and $\rho^*(g, g) \in (0, 1)$. Hence,

$$E(U_i|s_i = b, \hat{s}_i = b) = \delta W(1 - p, (0, \rho^*(g, b))), \text{ and}$$

$$E(U_i|s_i = b, \hat{s}_i = g) = \delta W(1 - p, (0, \rho^*(g, g))) - \epsilon.$$

We now prove that $\rho^*(g, g) > \rho^*(g, b)$. If $\rho^*(g, b) = 0$, it trivially follows that $\rho^*(g, g) > \rho^*(g, b)$. Therefore, suppose that $\rho^*(g, b) > 0$. In that case from points 1, 2 and 3 of Proposition 1 we know that $\rho^*(g, b)$ and $\rho^*(g, g)$ were "generated" by the following two equalities:

$$\Delta^r(\Pr(G|g, b), (0, 1), (0, \rho^*(g, b)) = 0, \tag{25}$$

$$\Delta^r(\Pr(G|g, g), (0, 0), (0, \rho^*(g, g)) = 0.$$

As $\Pr(G|g, b) = \frac{1}{2}$, from Lemma 10 we know that

$$\Delta^r(\Pr(G|g, b), (0, 0), (0, \rho^*(g, b)) = \Delta^r(\Pr(G|g, b), (0, 1), (0, \rho^*(g, b)).$$

As $\Pr(G|g, b) < \Pr(g, g)$, from Lemma 6 we know that

$$\Delta^r(\Pr(G|g, g), (0, 0), (0, \rho^*(g, b)) < \Delta^r(\Pr(G|g, b), (0, 0), (0, \rho^*(g, b)) = 0.$$

Hence, for equality 25 to be respected it follows from Lemma 9 that $\rho^*(g, g) > \rho^*(g, b)$. But then it follows from Corollary 1 that $\delta W(1 - p, (0, \rho^*(g, g))) > \delta W(1 - p, (0, \rho^*(g, b)))$. As $\epsilon \to 0$, it follows that in case (a) $E(U_i|s_i = b, \hat{s}_i = b) < E(U_i|s_i = b, \hat{s}_i = g)$, a contradiction.

In case (b), from point 4 of Proposition 1 we know that there always exists a monotone continuation equilibrium in which $\rho^*(b, g) = 0$ and $\rho^*(g, g) \in (0, 1)$. Depending on the values of the exogenous parameters there may also exist another monotone continuation equilibrium in which $\rho^*(b, g) = \rho^*(g, g) = 1$. If players focus on the continuation equilibrium in which $\rho^*(b, g) = 0$ and $\rho^*(g, g) \in (0, 1)$, using a reasoning identical to the one of the paragraph above, we know that the pessimistic sender cannot be indifferent between the two messages. Therefore, suppose players focus on the continuation equilibrium in which $\rho^*(b,g) = \rho^*(g,g) = 1$ (provided this continuation equilibrium exists). In that case,

$$E(U_i|s_i = b, \hat{s}_i = b) = \delta W(1-p, (0, \rho^*(g,b))), \text{ and}$$

$$E(U_i|s_i = b, \hat{s}_i = g) = -\epsilon.$$

As $\delta W(1-p,(0,\rho^*(g,b))) \geq 0 > -\epsilon$, in case (b) the sender cannot be indifferent between the two messages.

Step 2: If the pessimistic sender deviates and sends $\hat{s}_i = g$, $1-p < c' \leq q_\pi = \frac{1}{2} < p < q_\omega$. From point 5 of Proposition 1, we know that there exists two monotone continuation equilibria. If players focus on the one in which $\rho^*(b,g) = 0$ and $\rho^*(g,g) \in (0,1)$, using a reasoning identical to the one of two paragraphs above, the pessimistic sender strictly prefers to send $\hat{s}_i = g$ instead of $\hat{s}_i = b$. If players focus on the one in which $\rho^*(b,g) = \rho^*(g,g) = 1$,

$$E(U_i|s_i = b, \hat{s}_i = g) = -\epsilon < 0 \leq E(U_i|s_i = b, \hat{s}_i = b) = \delta W(1-p,(0,\rho^*(g,b))),$$

where the second inequality becomes strict whenever $c' < \frac{1}{2}$. Hence, there exists a monotone PBE in which $\sigma^*(b,h_0) = 0$ and $\sigma^*(g,h_0) = 1$.

Step 3: Suppose receivers update their posteriors under the assumption that $\sigma^*(b,h_0) = \sigma^*(g,h_0) = 1$. In the out-of-equilibrium event that $\hat{s}_i = b$, we assume that receivers believe that the sender is a pessimist (with probability one). Therefore,

$$E(U_i|s_i = b, \hat{s}_i = b) = \delta W(1-p,(0,\rho^*(g,b))).$$

If she sends $\hat{s}_i = g$, $q_\pi = 1-p < c < q_\omega = p$, and from point 6 of Proposition 1 we know that $\rho^*(b,g) = 0$ and $\rho^*(g,g) \in (0,1)$. Using a reasoning identical to the one we outlined in step 1, $\rho^*(g,g) > \rho^*(g,b)$. From Corollary 1 (+ the fact that $\epsilon \to 0$) follows that the pessimistic sender strictly prefers to "lie" and send $\hat{s}_i = g$. Q.E.D.

Proof of Lemma 3

Define $\rho^*(g,b,sub)$ as the probability which ensures the following equality

$$\frac{1}{2} - c + sub = \delta W(\frac{1}{2},(0,1),(0,\rho^*(g,b,sub))).$$

From the paper we know that

$$sub < \delta W(\overline{q}_\omega,(0,0),(0,1)) - (\overline{q}_\omega - c). \tag{26}$$

We now show that $\forall sub \in [0, \overline{sub})$, $\rho^*(g, b, sub) < 1$. $\rho^*(g, b, sub) = 1$ only if

$$\frac{1}{2} - c + sub \geq \delta W(\frac{1}{2}, (0,1), (0,1)),$$

$$\Leftrightarrow \qquad sub \geq \delta W(\frac{1}{2}, (0,1), (0,1)) - (\frac{1}{2} - c). \tag{27}$$

Inequalities 26 and 27 cannot both be satisfied as we can use Lemmas 6 and 10 to construct the following contradiction

$$sub \geq \delta W(\frac{1}{2}, (0,1), (0,1)) - (\frac{1}{2} - c) > \delta W(\overline{q}_\omega, (0,1), (0,1)) - (\overline{q}_\omega - c)$$

$$= \delta W(\overline{q}_\omega, (0,0), (0,1)) - (\overline{q}_\omega - c) > sub.$$

As $\rho^*(g, b, sub) < 1$ it trivially follows from Lemma 9 that $\rho^*(g, b, sub)$ is strictly increasing in sub. Q.E.D.

Proof of Lemma 4

The proof is similar to the one of Lemma 3. Define $\rho^*(g, h_1, sub)$ as the probability which ensures the following equality

$$p - c + sub = \delta W(p, (0, \rho^*(g, h_1, sub)), (0, \rho^*(g, h_1, sub))).$$

$\forall sub \in [0, \overline{sub})$, $\rho^*(g, h_1, sub) < 1$ as we otherwise run into the following contradiction

$$sub \geq \delta W(p, (0,1), (0,1)) - (p - c) > \delta W(\overline{q}_\omega, (0,1), (0,1)) - (\overline{q}_\omega - c)$$

$$= \delta W(\overline{q}_\omega, (0,0), (0,1)) - (\overline{q}_\omega - c) > sub.$$

As $\rho^*(g, h_1, sub)$ is always strictly lower than one, it trivially follows from Lemma 9 that $\rho^*(g, h_1, sub)$ is strictly increasing in sub. Q.E.D.

Proof of Proposition 4

From Corollary 1, we know that $\delta W(1 - p, (0, \rho^*(\cdot)))$ is strictly increasing in $\rho^*(\cdot)$. If $q_\pi = \frac{(1-p)^2}{p^2 + (1-p)^2}$, this means that the pessimistic receivers learned that $s_i = b$. Hence, $\Pr(G|q_\pi,\text{wait}) < c$ and $\rho^c = 0$ (for the definition of ρ^c, see Lemma 9). From Lemma 9 then follows that $\delta W(\frac{(1-p)^2}{p^2+(1-p)^2}, (0,1), (0, \rho^*(g, b)))$ is also strictly increasing in $\rho^*(\cdot)$. This insight - combined with our results summarised in Lemmas 3 and 4 - allows us to conclude that equations 6 and 8 are strictly increasing in sub. The remainder of the proof can be found in the body of our paper. Q.E.D.

Proof of Proposition 7

We redefine σ_1 as $\sigma_1 = (\sigma(b, h_1), \sigma(\phi, h_1), \sigma(g, h_1))$, where $\sigma(\phi, h_1)$ represents the probability with which the uninformed sender invests at time one given the message she sent at time zero. From the insight summarised in Proposition 3 it should be obvious that the pessimistic sender strictly prefers to send $\hat{s}_i = \phi$ to $\hat{s}_i = g$. One has,

$$0 = \Delta^r(q_\omega, (0, 0, 1), (0, \rho^*(g, \phi))) = \Delta^r(q_\omega, (0, 0, 0), (0, \rho^*(g, \phi)))$$

$$\leq \Delta^s(q_\omega, (0, \rho^*(g, \phi))) < \Delta^s(\frac{1}{2}, (0, \rho^*(g, \phi))).$$

The first equality sign states that optimists - after having received the message $\hat{s}_i = \phi$ - must fix $\rho^*(g, \phi)$ such that they are indifferent between investing and waiting. Upon receiving message $\hat{s}_i = \phi$, player j knows that $\Pr(s_i = g | s_j, \hat{s}_i = \phi) = 0$ and thus the receiver's gain of waiting is independent of $\sigma(g, \phi)$ (which explains the second equality sign). The first inequality sign is based on the insight, explained in the body of our paper, that k^s is a sufficient statistic for k. The second inequality sign is based on Lemma 6 (in the proof of Lemma 6, we do not rely on the fact that the sender can only send two messages). The strings of equalities and inequalities presented above prove that an uninformed sender strictly prefers to send $\hat{s}_i = \phi$ and wait, instead of sending $\hat{s}_i = g$ and invest at time one. One also has,

$$0 = \Delta^r(q_\omega, (0, 0, 1), (0, \rho^*(g, \phi))) \geq \Delta^r(q_\omega, (0, 0, \rho^*(g, \phi)), (0, \rho^*(g, \phi)))$$

$$> \Delta^r(p, (0, 0, \rho^*(g, \phi)), (0, \rho^*(g, \phi))) = \Delta^s(p, (0, \rho^*(g, \phi))).$$

The first and the second inequality signs rely on Lemmas 5 and 6. Note that the last equality sign only holds when $\epsilon \to 0$. Hence, the optimist - independently of her message - invests at time one and she cannot gain by deviating. Q.E.D.

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