Multigame models of innovation in evolutionary economics

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We incorporate information measures representing knowledge into an evolutionary model of coevolving firms and markets whereby the growing orderliness of firms potentiates a predictable progression of market exchange innovations which themselves become beneficial only with the growing orderliness of firms. We do this by generalizing Nelson and Winter style evolutionary models which are well suited to the study of entry, exit, and growth dynamics at the level of individual firms or entire industries. The required innovation is to use information measures to impose an order on the routines constituting a firm, and by correlating order with firm profitability, allow the preferential selection of innovations which increase order. In this viewpoint, the coherent mathematical framework provided by information and probability theory describes firm orderliness and variability, as well as all selection operations. This informational approach allows modelling the synergistic interactions between routines in a single firm and between different firms in a general but comprehensive manner, so that we can successfully model and predict innovations specifically focussed on organizational order. In particular, we can predict the coevolution over time of firm organizational complexity and of increasingly sophisticated market exchange mechanisms for routines permitting that increased organizational order. We demonstrate our approach using numerical simulations and analytic techniques exploiting a multigame player environment.

I. INTRODUCTION

Innovation and the diffusion of innovations through an industrial sector or an entire economy can be modelled using evolutionary Nelson and Winter style approaches [1]. These approaches employ stochastic random variables to generate variation in the microeconomic efficiencies of individual firms within some economic sector subject to competitive selection to model entry, exit and growth dynamics of individual firms or of entire industrial sectors. As such, these models adequately reproduce non-equilibrium growth dynamics [2–9]. Alternative approaches to evolutionary economics can examine optimal strategy mixes generating information in uncertain environments [10], adopt a nonlinear dynamics approach [11, 12], or population ecology approaches [13–16]. Various studies have sought to extend evolutionary models beyond the simple examination of growth and diffusion dynamics. Specific extensions include, for instance, modelling firm learning of novel competencies [17], the subdivision of the sequence of knowledge production into defined stages leading to formation of a market for knowledge [18], and assessments of the impact of uncertainty on firms [19]. In addition, innovation failures can be examined by, for instance, spatial lock-in and contingent path dependence arising from chance and increasing returns in economic geography [20, 21], and temporal lock-in as in the example of the QWERTY keyboard [22].

A number of authors have discussed needed generalizations to the Nelson and Winter approach, including ongoing efforts by the original exponents of these models to, for instance, more fully incorporate institutions [23]. Additionally, the need has been noted "for a theory of endogenous change of available opportunities and of their selection ... [determined by] the environment in which various forms of organisations coexist and evolve ... [requiring a relating of] micro-dynamics to macro-dynamics, in order to analyse the interplay of social and individual processes." [24]. Required models then, likely required a merger of genetic algorithms [25, 26], game theory and evolutionary game theory [27], to allow modelling an economy as an evolving network [28] implicitly incorporating learning [29]. In addressing similar goals, Potts noted the need for an evolving multi-agent framework where agents were bundles of variable sets of resources, control algorithms and schemata, and interaction tags effectively layering an evolving communication network over an economy which conditioned events and future evolution [30]. Similarly, Foster has emphasized that "economic and social systems are knowledge based and that the primary interactions within them are exchanges of information." [31]. In these latter two references, the interactions between resources and routines creates an evolving network of synergistic interactions across an economy, and consideration of this web of interactions has led Mathews to generalize the original routines of Nelson and Winter to become the resources and the resource markets of the productive economy [32]. Here, resources—any entity necessary to the production of goods and services—are produced and exchanged by firms seeking to ensure distinctiveness and generate entrepreneurial profits by maximizing the synergystic interaction between firm resources. In particular, "Resources can be specialized and bundled together in highly distinctive configurations, to lend firms special competitive advantages. Resources can be built by firms internally, and they can be traded—as described every day in the business pages of the newspaper." [32].

This paper makes a first effort at modelling these evolving and interacting synergistic networks by adopting an informational perspective. This allows representing the synergistic interactions between routines by using information measures to order firm routines and thereby correlate firm survivability with firm order, or equivalently, with a firm's synergistic routine interactions. This is a natural step to adopt as any model describing evolving populations of firms subject to variability and probabilistic selection processes, is most naturally done us-

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ing a probabilistic mathematical framework which naturally subsumes information measures. In particular, firms making economic decisions in uncertain environments are processing information to reduce variance or uncertainty. Any economic activity which reduces variance is implementing a selection operation and thus is effectively implementing a Darwinian evolutionary process. We follow [33] in emphasizing the role of selection in potentiating the appearance of apparently self-organizing ordered structures in economics, whether these operations have involved selecting cards from packs, selecting job applicants, computer systems, suppliers, and so on. All such selection operations can be treated with full generality as grist for the evolutionary mill. The mathematical description of a Darwinian evolutionary process is entirely subsumed within information and probability theory. Then, uncertain economic environments are properly described by probability distributions which are collapsed into distributions with reduced uncertainty through the processing of information. Firms which process information generate internal and external order and gain advantage by operating in a known environment rather than in a merely probable one. Cost/benefit ratios of information processing investments can be precisely quantified by comparing probable benefits returned using distributions prior to, and after collapse with these known ratios guiding innovation investment decisions. Firm investments and innovations modify the internal routines and organizational capacities of the firm and its ability to order its external environment. The demand for information and for the capacity to exploit that information creates an information market. More importantly, the evolution of information processing systems must necessarily occur in a predictable sequence from less complex to more complex allowing reliable prediction of plausible innovation sequences. The ability to predict such plausible sequences, while only general, nevertheless exists, partly refuting claims that evolutionary models must take a set of routines as a given and must necessarily fail to model the origins of innovation [2], or that a "predictive theory of novelty is simply a contradiction in terms", due to the "inherently unpredictable nature of imaginative, creative processes" [31].

Just as any Nelson and Winter style model based on routines must include a market for the exchange of routines, then so must any evolutionary model of ordered routines incorporate markets for the exchange of information about ordered routines. The new feature is that orderliness is itself a growing quantity in the model, leading to the expectation that markets will themselves evolve over time. Hence, the expectation is that the orderliness of firms and information market operations must coevolve over time. Our results about evolving market structures are necessarily general, and reflect similar observations of temporal sequencing in the evolving structures of economic ecologies. For instance, coevolving systems can create an ordered progression of economic ecologies—in caricature, young ecosystems are dominated by "r-strategists" exploiting a rapid turnover of large numbers of offspring organisms while older ecosystems are dominated by "K-strategists" who carefully nurture a much smaller number of offspring organisms [34]. This approach, as also ours, outlines a framework to understand coevolving endogenous institutions and requires not only equations describing the selection of organisms, but also linked equations describing institutional change, probably requiring numerical multi-agent models.

This paper demonstrates that in many cases, economic markets are created and destroyed by evolutionary innovation processes and that the operation of one market can naturally be expected to lead to the generation of new markets. In some cases, it is possible to make plausible predictions of evolutionary innovation sequences over a broad range of differing markets. The market evolution discussed here complements the market reproduction with variation mechanisms [35] subject to selection pressures due to their relative success in achieving a particular end [36], as well as the market structuring mechanisms permitting the coevolution of for instance, software markets and software firms [37]. Here, software firms provide initially free software versions to generate network benefits and externalities to create a market for a commercial version of the software. More generally, Metcalfe has noted "Nor are market institutions given. They have to be established and their establishment, growth, stabilisation and decline involve the investment of real resources in market making activity." [38], while also, Potts has described the economy as a coordinated system of distributed knowledge mediated by markets as knowledge restructuring mechanisms, and where the growth of knowledge leads to changes in the structure of the economic system via a continual process of recombination of interconnected rules into viable economic building blocks which replicate and diffuse through the economic population [39]. A similar emphasis on the importance of connections appears in [40], and see also [41].

The coevolution of firms and markets can be modeled using a modified game theory in which the creation and destruction of resources over time create and destroy games exploiting those resources for payoffs, with the resulting time-dependent payoffs forcing participants to shift from one game to the next to maximize benefit [42]. The resulting multigame environment models the innovation of, for instance, mechanisms to create resource and payoff distributions, to find new strategies of accessing new games, to move from one game to another, and so on. This multigame environment relaxes most of the constraints usually imposed in game theory. These include the restriction to only a single game possessing a fixed number of players, each able to select strategies from an immutable set of possible strategies under the influence of a number of unchanging and known payoff functions [43]. The need to allow game players to extend their strategy sets in evolutionary economics was noted in Ref. [44]. To illustrate the difference between game theory and multigame theory: game theory is constrained to model incomplete information games by Bayesian analysis [45] for instance. In constrast, in multigame theory the absence of information creates demand for information which makes it profitable to innovate to create supply and thus generates a novel market in that information. This approach is more in accordance with observed economic behaviour.

The resulting multigame modeling environment provides an analytic generalization of the "sugarscapes" common in alife simulations [46–49], and of agent-based computational economics [50, 51], which aim to examine the spontaneous appearance of large scale order in systems of autonomous agents. In these simulations, an environment hosts agents exploiting a time-dependent resource distribution (sugar) which is consumed, concentrated and traded, perhaps with preferential selection of trading partners [52].

II. PROBABILITY AND EVOLUTION IN ECONOMICS

Firms are modeled as a single entity consisting of many component parts ideally operating towards a single common goal through the application of various routines. The total of all routines available to a firm fully characterizes its available action choices and depend contingently on the previous history of the firm, the capabilities of its staff, its cash-flow situation, its organizational structure, and so on. The innovation model developed here will consider innovations of routines by single firms, the imitation and diffusion of routines between firms and the establishment of more and more complex markets for the exchange of increasingly complex routines between firms.

As is well known, it is difficult to articulate the single common goal being pursued by the components of a firm. Firm profitability is often adopted as the goal of a firm and this is adequate as long as models focus only on interactions between firms each considered as a single entity. However, this paper seeks to consider the internal organization of firms (its routines) and then consider how innovation of routines might add or subtract to satisfaction of firm goals. In many cases it is impossible to determine how routines might add to profitability and other measures to assess routines must be used. For example, it is often impossible to directly correlate staff outputs now to subsequent firm profitability. This leads many firms to adopt routines which set staff renumeration proportional in part to firm profitability so self-interested staff will act to maximize firm profitability. Here, firm routines are selected from among the many possible to maximally correlate and organize the activities of firm components. The best routines are those which maximize the fitness between the goals of firm components and the goals of the firm itself. This paper adopts the "fitness" of a firm and of particular routines as an indicator of the contribution made to firm goals. This word carries obvious links to evolutionary biology and in both fields is poorly defined. It is not clear how the organization of a firm contributes to firm goals though some fitness measure is required for models of innovation markets in firm organization levels.

Selection processes in uncertain environments form a very large component of all the routines of a firm. Staff are selected from a range of applicants of largely unknown quality, goods of unknown quality are selected from a range of suppliers, individual staff select the proportion of their time spent on various projects without knowing future project outcomes, and so on. More importantly, firms themselves are subject to selection processes in the various uncertain markets in which the firm operates.

A selection process is mathematically identical to the use of information processing to reduce uncertainty in probability distributions. Then, the usual way to model selection in an uncertain environment is via probability and information theory. Elementary introductions to probability theory use examples such as "select two cards without replacement from a pack", or "select five marbles with replacement from an urn" to introduce selection processes. A selection process conditions an initial probability distribution (every one of 52 cards is equiprobable) to derive a new, selected probability distribution whose reduced width reflects decreased uncertainty. In such elementary probability examples, information is successively applied to manipulate probability distributions to derive desired results. A selection process is equivalent to the processing of information to manipulate probability distributions. Different words are commonly used for these simple mathematical operations including to select, condition, partition or collapse a probability distribution.

The mathematical selection operation can be illustrated using a selection process which partitions a population of firms with a fitness probability distribution $P(f)$ into fitness classes so that goods are purchased only from highly organized (and fit) and thus high quality firms, for instance. (Here, the fitness of a firm is crudely denoted by a single parameter f.) This selection process, denoted S_s, might act to eliminate from consideration all firms which have a fitness less than some value f_0 to generate a new population distribution $P_s(f) = S_s\big[P(f)\big]$ as

$$\begin{aligned} P_s(f) &= P(f|f > f_0) \\ &= \frac{P\big[f \odot (f > f_0)\big]}{P(f > f_0)} \\ &= \begin{cases} 0 & \text{if } f \leq f_0 \\ N^{-1}P(f) & \text{if } f_0 < f \end{cases}, \end{aligned} \tag{1}$$

where the notations "$\odot$" and "$\oplus$" mean logical "AND" and "OR" respectively and "$|$" indicates the conditional "given". In this equation, the normalization factor $N = P(f > f_0)$ is the proportion of the surviving firm population. (These results apply only if $N \neq 0$.) The same operation describes the selection of cards from a pack according to some criteria.

The above mathematical selection process entirely subsumes Darwinian evolutionary processes which require variation and selection (and heritability). A probability or resource distribution with non-zero variance provides variation which is selected to optimize some benefit. (The remaining criteria of heritability is subsumed within the continued functioning of the firm from one period to the next.) Firms and economies routinely exploit information to optimize selection operations and thus naturally operate in an evolutionary manner. The demand for information makes it profitable to innovate to provide supply establishing a market whenever derived benefits significantly outweigh information processing costs. It is well understood that information processing can confer real benefits.

For example, consider a typical situation where a firm of fitness f must purchase goods from one supplier selected from a range of suppliers offering goods of varying quality. The purchased goods might be of high or low quality and these goods increment firm fitness by an amount Δf denoting the difference between total derived benefits and total required costs. Final firm fitness is then $f' = f + \Delta f$. (This example is equivalent to a child selecting a card from a pack of cards of differing worth to modify the value of their own hand.) From the firms's point of view, the suppliers are described by a probability distribution $P(\Delta f)$ giving the probability that any individual supplier provides a particular incremental fitness to the firm. Suppose that suppliers consist of a large fraction $\eta_b \approx 1$ which provide bad goods and which contribute negative average incremental fitness $\Delta\bar{f}_b < 0$ to decrease firm fitness $f' < f$. (The child selects one of the numerous bad cards to decrease the value of their hand.) Conversely, suppose a small proportion of the suppliers $\eta_g = 1-\eta_b \approx 0$ offer good quality so $\Delta\bar{f}_g > 0$ and the average firm fitness is increased with $f' > f$. (The child selects a rare good card to increase the value of their hand.) The initial supplier (card) distribution breaks into a bad supplier (bad card) distribution denoted b, and a good supplier (good card) distribution denoted g, as

$$\begin{aligned} P(\Delta f|g \oplus b) &= P(\Delta f \odot g) + P(\Delta f \odot b) \\ &= P(g)P(\Delta f|g) + P(b)P(\Delta f|b) \\ &= \eta_g P_g(\Delta f) + \eta_b P_b(\Delta f). \end{aligned} \quad (2)$$

with respective means for the good and bad distributions of $\Delta\bar{f}_g > 0$ and $\Delta\bar{f}_b < 0$. If the firm does not employ information processing to select a supplier, a random selection of suppliers must be made giving an average expected return $r = \eta_g\Delta\bar{f}_g + \eta_b\Delta\bar{f}_b \approx \Delta\bar{f}_b \ll 0$, much less than zero. Conversely, when the firm is able to acquire and successfully process information about a supplier's quality and can target their purchases to good suppliers, then the probability distribution that guides their search collapses to $P(\Delta f) \to P_g(\Delta f)$ using the methodology of Eq. (1) with a much improved expected benefit $r' = \Delta\bar{f}_g \gg r \approx \Delta\bar{f}_b$. (If the child cheats to gain information to detect desired cards before making a selection, it is easier to select a good card.) Information processing can confer a real benefit in economies.

More importantly, well known techniques exist by which organizational structure (information) can be exploited to allow the reliable prediction of plausible innovation sequences. These methods do not specify which particular innovations will appear but they do constrain the space of possible innovations and are commonly used to guide investment decisions.

For example, consider the probability of innovation P_i of a composite mechanism $M = m_1 \odot m_2$ consisting of two components m_1 and m_2 via the usual probability decomposition $P_i(M) = P_i(m_1)P_i(m_2)$. This decomposition allows the trite observation that component mechanisms m_1 and m_2 must appear before the more complex M can appear as if either of $P_i(m_1) = 0$ or $P_i(m_2) = 0$ then necessarily $P_i(M) = 0$. Thus, for instance, computer-network routers are a nonviable innovation before computers appear and are widespread and linked by networks.

Consider next the probable operational success P_s of an already existing composite mechanism $M = m_1 \odot m_2$ with components m_1 and m_2. A high quality mechanism has a high probability of successfully completing its task so $P_s \approx 1$ while a poor quality mechanism has low probability of success giving $P_s \approx 0$. The decomposition $P_s(M) = P_s(m_1)P_s(m_2)$ allows prediction of future sequences of innovations whenever there is an imbalance between the probable success of mechanisms m_1 and m_2. Suppose that mechanism m_1 is poorly optimized while m_2 is highly optimized so $P_s(m_1) \approx 0 \ll P_s(m_2) \approx 1$ giving $P_s(M) \approx P_s(m_1)$. Then, any increase or decrease in the operational efficiency of mechanism m_2 will not affect the success of the parent mechanism M so that markets are neutral to improvements in mechanism m_2. This allows the reliable prediction that market selection pressures will act to improve mechanism m_1 while decreasing the quality (and cost) of m_2 until $P_s(m_2) \approx P_s(m_1)$. The resulting outcome of this process then allows the prediction that if a composite mechanism is designed for obsolescence in some period, then few of its components will have individual lifetimes much greater than this period.

III. SIMULATING INNOVATION MARKETS

In this section, a crude simulation illustrates later analytic methods to model ordered stages in the evolution of firm routines, and in the formation of markets exchanging routines between firms.

Consider a population of recent start-up firms in a new economic sector so novel that little is known about which routines optimize firm goals. (Consider internet start-ups for instance.) In the absence of an ability to select optimal routines, firms must start-up with whatever routines they can establish themselves. Suppose each firm consist of exactly 20 routines numbered $(g_1, \ldots, g_{20})$ selected from the pool of all possible routines labeled by integers from $[1, 100]$ inclusive. Suppose further that odd numbered routines are easy to find and implement, while even numbered routines are hard to find and implement. Then, at time $t = 0$, firms are created by random selections from the pool of all 100 possible routines though even numbered routines are $2^{-5} = 1/32$ times less likely to be selected than odd numbered routines. Otherwise, selection probabilities are uniform.

To allow examination of firm organizational structure, an arbitrary definition of fitness is imposed on the firm population with the fitness of any firm being defined as

$$f(g_i) = \sum_{i=1}^{19} \begin{cases} 0 & \text{if } g_{i+1} = g_i \\ \frac{1}{g_{i+1}-g_i} & \text{if } g_{i+1} \neq g_i \end{cases}. \quad (3)$$

Then, each routine makes a fitness contribution dependent on its nearest neighbours. When nearest neighbour separation $g_{i+1}-g_i$ is zero no contribution is made, when it is unity a maximum contribution of $+1$ is made, and as the separation increases a decreasing contribution occurs. Fitness contributions are negative when $g_{i+1}-g_i$ is negative. This definition of fitness allows firms to possess a maximum fitness of $+20$ when all routines are consecutive and incrementing, and a minimum of -20 when all routines are consecutive and decrementing. The scarcity of even numbers in the routine pool means that firms are initially overwhelmingly constructed using only odd numbered routines. The possible fitness range of a firm constructed using only odd numbered routines is between -10 and $+10$ as nearest neighbours are separated by at least two units.

A typical randomly chosen initial firm is

$$\begin{aligned} &(95, 75, 99, 15, 85, 25, 89, 97, 7, 47, \\ &\ 37, 5, 93, 17, 55, 73, 95, 29, 23, 59) \end{aligned} \quad (4)$$

which has fitness -0.0279. Note the absence of even numbered routines.

It is to be noted that the initial random distribution of routines in firms, the absence of order of those routines, and the resulting minimal fitness of all firms mitigate against firms seeking to imitate or otherwise exchange routines with each other. If all firms are roughly equally unfit and it is not yet known which routines contribute to fitness, then there can be no incentive for imitating routines or purchasing routines or head-hunting staff from other firms to copy routines. This model predicts that there will be no market in firm routines in novel economic sectors.

In the absence of an exchange market, a firm can only increase its fitness by internal innovation of routines. This is simulated by allowing each firm to operate over 600 generations where each generation sees the sequential potential innovation of each and every routine in the firm. (This creates a highly accelerated simulation for display purposes.) For each routine in the firm, innovate a potential new routine from the innovation pool, evaluate the fitness of the firm both with and without the innovation, and discard the lower fitness routine.

A neutral innovation mechanism is applied in cases where the mutated and original firm have the same fitness. This can arise when routines have the structure $(\ldots, x, a, x, \ldots)$ where the total fitness is independent of routine a. (It cancels out of the total fitness expression.) Whenever innovations are entirely neutral, an allowance for neutral drift is made by letting roughly 10% of neutral innovations proceed.

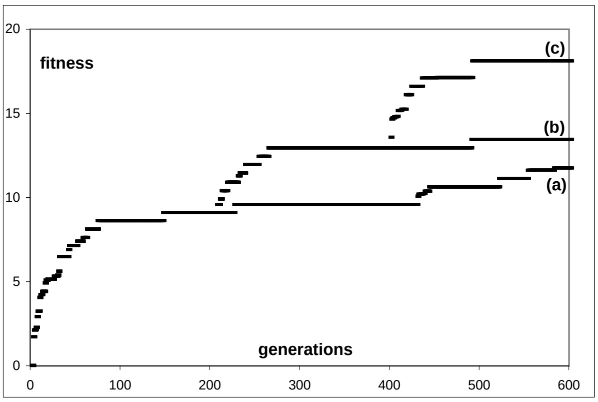

FIG. 1: The evolution of fitness of a single typical firm subject to (a) random innovation with a scarcity of even numbered routines for generations from 0 — 600, (b) exchange of routines with other firms giving access to many even numbered routines from generation 200 — 600, and (c) three-routine packet exchange with other firms from generation 400 — 600. The evolution of a single firm is followed in each case with new market strategies being introduced at generations 200 and 400 respectively.

A typical simulation over 600 generations starting from the initial firm of Eq. (4) is shown as curve (a) of Fig. 1. This curve shows an initial rapid increase in fitness as the firm innovates new routines conferring higher fitness. The scarcity of even numbers in the pool of possible routines means that it is difficult for firms to innovate to fitness levels higher than +10. Towards the end of the simulation, the number of even numbered routines in the firm is slowly increasing allowing some increase in fitness. The probability of any given innovation is constant in time, though the increase in firm organization means that the probability of a good innovation decreases in proportion to the increase in firm organization. This generates long periods of stasis in the simulation for long times. After 600 generations, the firm has innovated to

$$(27, 29, 31, 33, 95, 96, 9, 10, 11, 13, \\ 14, 15, 87, 91, 92, 93, 11, 12, 13, 15) \qquad (5)$$

with fitness 11.7563. Note that innovations have fixed 5 even numbered routines into the firm.

At around generation 200, about one quarter of the routines in firms are even numbered, and this greatly exceeds the number of evens available from the routine pool (with proportion about 3%). The evolution of firms creates a new resource of even numbered routines within the environment. If firms could access the routines of other firms through imitation, purchase, theft or whatever, they gain access to a far higher proportion of even numbered routines than are readily available from innovations. The formal details of this evolutionary step are discussed in subsequent sections. The present simulation merely illustrates the effects of this step.

At generation 200, the initial firm of Eq. (4) has evolved to

$$(27, 29, 31, 7, 9, 15, 17, 10, 11, 13, \\ 14, 15, 23, 25, 91, 93, 11, 12, 13, 15) \qquad (6)$$

with fitness 9.1101 and containing 3 even numbers. At this stage, the potential benefit from exchanging routines with other firms to access even numbers creates a demand for the innovation of single routine exchange mechanisms. (For display purposes, this new innovation is introduced at a fixed generation 200 and this new routine is not identified by any particular number.)

The effect of single routine exchange between firms possessing many even numbers is shown in the fitness curve (b) of Fig. 1. Starting at generation 200, a very steep increase in fitness is manifest as firms become evenly populated by even and odd numbers. The steep growth gradually tails off into a long period of stasis as firm organization becomes locked into inefficient arrangements. For instance, by generation 400, the above firm evolves to

$$(27, 28, 29, 8, 9, 47, 9, 10, 11, 13, \\ 14, 15, 16, 17, 92, 93, 11, 12, 13, 15) \qquad (7)$$

with fitness 12.9535, while by generation 600 it becomes

$$(27, 28, 29, 8, 9, 84, 9, 10, 11, 13, \\ 14, 15, 16, 17, 92, 93, 11, 12, 13, 14) \qquad (8)$$

with fitness 13.4535. Here, the fitness combination $(\ldots, 8, 9, 47, 9, 10, \ldots)$ is locked to change by single routine innovation or exchange as all possible changes are deleterious. Only neutral drift can change this sequence though this has no effect on fitness values. Stasis then exists under single routine innovations and under single routine exchange processes.

By generation 400, average firm organization levels have increased and firms now contain large islands of consecutively numbered routines conferring high fitness.

This novel feature of the environment did not exist before about generation 400, and thus, no market could possibly exist for multiple routine exchange. As before, the novel appearance of these ordered islands creates a potential benefit to any firm innovating routines allowing the exchange of ordered routine packets with other firms. When the exchange of single routines confers no benefits, the exchange of ordered packets of routines does provide potential fitness benefits.

For display purposes, a new strategy allowing the ordered exchange of packets of three routines between different firms is enacted after generation 400. The initial firm chosen is the generation 400 firm of Eq. (7) with fitness evolution shown as curve (c) of Fig. 1. As usual, the innovation of a new strategy accessing a novel resource leads to a rapid increase in fitness with an eventual tail off into stasis as increasing order makes the exchange of 3-routine packets deleterious. At generation 600, the firm becomes

$$(4,5,6,7,8,9,10,11,12,13, \\ 14,15,16,17,25,26,27,28,29,30) \quad (9)$$

with fitness 18.125. Note the persistence of the boundary $(\ldots,16,17,25,26,\ldots)$ between consecutive ordered domains. This boundary is relatively impervious to three-routine packet exchanges.

All of the above exchange processes have been with randomly selected exchange partners. However, as discussed in Eq. (2) benefits exist for firms using information to target exchange practices. Suppose that instead of exchange with random partners, firms had innovated mechanisms to target their exchanges to only those firms offering high fitness exchanges. Thus, if a firm consisted of routines numbered in the 20s, exchanges are rejected with firms whose routines are numbered in the 80s. The effects of information-based targeted exchanges are shown in Fig. 2 offering a comparison of the rates of fitness growth for firms making exchanges with randomly selected other firms [Curves (b) and (c) copied from Fig. 1] and firms using information to target their exchange partners to maximize benefit [curves (b') and (c')]. Curve (b') shows the effect of targeted single routine exchange introduced at generation 200 using the initial firm of Eq. (6). By generation 600, the firm becomes

$$(29,30,31,8,9,4,9,10,11,13, \\ 14,15,89,90,91,10,11,12,13,14) \quad (10)$$

with fitness 13.458. Note the initially higher rates of fitness growth and higher resulting fitness due to targeted exchange policies, and the continued existence of the $(\ldots,9,4,9,\ldots)$ trap impervious to even targeted single routine exchange. Curve (c') shows the effect of targeted 3-routine packet exchange implemented at generation 400 using the initial firm of Eq. (7). By generation 600 the firm is

$$(38,39,40,41,42,43,10,11,12,13, \\ 14,15,16,17,9,10,11,12,13,14) \quad (11)$$

with fitness 16.845. Here, targeted packet exchange had a slightly higher initial rate of fitness growth which resulted in three ordered domains of routines and lower overall fitness compared to untargeted packet exchange which had resulted in two ordered domains of routines. This displays the common observation that an overly rapid local optimization can lead to poor global optimization as commonly observed in simulated annealing in neural network learning.

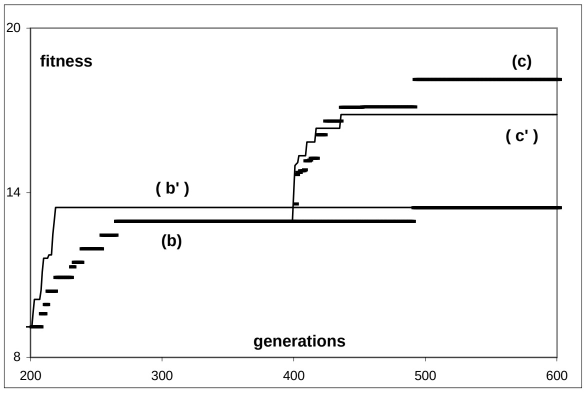

FIG. 2: A comparison of the rates of fitness growth for firms making exchanges with randomly selected other firms and firms using information to target their exchange partners to maximize benefit. Curves (b) and (c) are from Fig. 1 and show random selections. Curves (b') and (c') show the effects of targeted exchange of single routines and packets of routines respectively. Targeted exchanges increase initial rates of fitness growth but this overly rapid local optimization can lead to poor global optimization with the fitness of curve (c') substantially below that of curve (c) by generation 600.

The important point about this simulation is that single routine exchanges cannot arise as long as the routine distribution in firms is identical to the routine distribution available from innovations. If these distributions are identical then there is no benefit and many costs involved in innovating mechanisms to exchange routines, and such mechanisms would not arise. It is only when firms evolve to contain a desired resource that is not readily available from innovations, that the benefits of exchange of routines begins to outweigh costs. (Conversely, in economic sectors in which innovation costs outweigh imitation and exchange costs, imitation markets will rapidly appear and must be constrained if these impact adversely on innovation investment decisions.) The evolution of routine probability distributions governs the benefit payoffs of new strategies and thus governs their evolution. Similarly, routine packet exchange processes cannot evolve until firms have evolved islands of order. There are no benefits and many costs associated with the exchange of disordered routine packets. This leads to the expectation that packet exchange processes can only arise after the creation of significant order via single routine exchange processes. Thus, packet exchange processes must follow single routine exchange processes which in turn, must follow lengthy periods of random innovations with selection. Market evolution must proceed in ordered sequences and economic models must be able to predict such sequences.

The above simulation can be endlessly generalized. For instance, innovations might develop entirely new routines with fractional number values (20.3 say) whose incorporation greatly increases fitness. Alternatively, firms might innovate the organizational capacity to handle larger units allowing mergers giving access to maximum fitness levels of +40 say. A different innovation in organizational capacity might modify the fitness formula of Eq. (3) to $1/(g_1 - g_N)$ which depends only on the first and last routines of the firm. All other routines are rendered redundant leading to firm size reductions.

The next sections seek to develop the capability to reproduce this simulation analytically. An analytic approach can increase understanding and comprehension in ways that particular and contingent simulations cannot. An analytic treatment of the above crude simulation might be applied to any number of evolving information processing systems including the growth of order in stamp collections or in stock market portfolios, or as here, in exchange markets.

IV. MULTIGAME THEORY

This section develops a pseudo analytic approach to multigame theory and for convenience groups together a number of equations, very briefly and almost in table format, which are properly introduced and repetitively used later in this paper.

Game theory follows the dynamics of a time dependent environment containing players or firms able to enact different strategies or routines with varying probabilities. The environment

$$E(t) \equiv \{G_i(f,t), P_m(\Delta f|i)\} \tag{12}$$

consists of all firm records G_i for firm i with fitness f together with a resource pool $P_m(\Delta f|i)$ which is conditioned by firm index i and specifies the probability of an incremental change in fitness $f(t+1) = f(t)+\Delta f$ for any firm accessing this resource pool at time t. Each firm's record is

$$G_i = \left\{ \begin{array}{l} P_i(f,t), \\ \epsilon_a = 1 \ : \ \{p_I, p_m, S_I, S_m\} \\ \epsilon_j = 0 \ : \ \{p_j, S_j\} \ \forall j \neq a \end{array} \right\} \tag{13}$$

which specifies each firm's necessarily probabilistic fitness distribution $P_i(f)$ (as strategy choice is probabilistic). The use of this distribution avoids the need to track the details of every routine in every firm and makes an analytic evolutionary treatment possible. Firm records also list available strategy sets S and the probabilities p with which strategies are chosen. Game theory generally considers only a single game so firms must participate with unit probability $\epsilon_a = 1$ in a single game denoted a, and can participate in all other games $j \neq a$ with probability $\epsilon_j = 0$. At time t, firms implement probabilistic strategies

$$S_r = \epsilon_a(p_I S_I + p_m S_m) + \sum_j \epsilon_j p_j S_j. \tag{14}$$

At every step, firms are also subject to some selection operation S_s

$$P_i(f, t+k) = (S_s S_r)^k \, P_i(f,t). \tag{15}$$

causing inefficient firms to be deleted from the game. Here, k time steps each consist of implementing chosen strategies to maximize their fitness probability distributions followed by selection operations. Operations S_r and S_s do not commute. The above dynamical regime is sufficient to model the dynamics of a single game and is expected to generate changes in the environment due to firm activities.

The dynamical evolution of the environment can generate novel resource pools which can be accessed by innovating novel strategies to define new games in the environment. The innovation of new games is modeled by providing mechanisms to allow firms to access new games with probability $\epsilon_j > 0$ for some j. At every time step k, the probability that any given firm innovates a new strategy to access a new resource pool is, provided $\epsilon_j(t+k) = 0$,

$$\begin{aligned} P[\epsilon_j(t+k+1) > 0|\epsilon_j(t+k) = 0] &= \eta_1(\Delta F) \\ P[\epsilon_j(t+k+1) = 0|\epsilon_j(t+k) = 0] &= \eta_0(\Delta F) \end{aligned} \tag{16}$$

where probabilities are determined in terms of functions $\eta_1(\Delta F)$ and $\eta_0(\Delta F) = 1 - \eta_1(\Delta F)$. As the probability of innovating a new strategy is low, typically $\eta_1(\Delta F)$ is close to zero and $\eta_0(\Delta F)$ is close to one.

These probabilities are dependent on parameter ΔF which measures the relative benefits and costs of accessing novel resource pools created in the dynamically evolving environment. Reasonable though heuristic results can be obtained by taking $\Delta F \propto B/C$ to be proportional to the benefits B to be derived from accessing a new resource pool and taking the costs of innovation C to increase exponentially with benefit, so $C \propto \exp[\Delta F]$. Consequently, the evolution probabilities of Eq. (16) to be proportional to the benefit / cost ratio gives

$$\begin{aligned} \eta_1(\Delta F) &= \bar{\eta}_1 \Delta F e^{-\Delta F} \\ \eta_0(\Delta F) &= 1 - \eta_1(\Delta F). \end{aligned} \tag{17}$$

Setting $\bar{\eta}_1 \approx 0$ gives the required probabilities with η_0 being close to one and η_1 being close to zero. These probabilities determine the approximate number of time steps required to innovate a new strategy as

$$N \approx \frac{1}{\eta_1(\Delta F)} \approx \frac{1}{\bar{\eta}_1 \Delta F} e^{\Delta F}. \tag{18}$$

When benefits are zero ($\Delta F = 0$) or when costs are high ($e^{-\Delta F} = 0$), then the probability of evolving a new strategy $P[\epsilon_j(t+k+1) > 0|\epsilon_j(t+k) = 0] \to 0$ and an infinite number of time steps are required. When benefits and costs are in balance ($\Delta F \exp[-\Delta F] \approx O(1)$ when $\Delta F = 1$) then the average number of time steps required to innovate a new strategy is $N \approx 1/\bar{\eta}_1$ in this model.

The ability to generate novel strategy sets and new games in response to environmental dynamics creates a multigame environment. Evolution of this environment causes the appearance and disappearance of novel resources which are accessed by newly innovated strategies in novel games. Game theory normally considers situations where players have no choice about which game they might use to optimize their payoffs, though in economics, such choices play a typically large part of any individual's efforts to maximizing profits.

V. INNOVATION MARKETS

This section models evolving economic systems in which firm innovation over time creates novel supply and novel demand to establish novel markets. In turn, the appearance of these new markets modifies the further evolution of firms to generate yet more new markets. These sequences of changes can occur in predictable evolutionary sequences in some circumstances.

Suppose that some recently produced technology has established a new economic sector and that a large number of new firms have rushed to enter this market. (Consider for instance, the effect of the internet.) We ignore the possible entry of large existing firms and treat all new entrants as small. The recent entry of firms into this novel economic sector means that firms are poorly organized and their routines are not optimal though firms are unsure about how exactly to optimize their routines. Further, the absence of long-time established firms means that firms cannot optimize routines by the common practice of imitating market leaders. In this situation, all firms are organized sub-optimally with different firms typically employing highly divergent strategies accompanied by considerable debate about which routines and strategies are optimal. The divergence of strategies employed by different firms means that there are few benefits to be derived from exchange between firms (including imitation). Thus, mechanisms allowing the exchange of routines between firms are presumed not to exist initially and the evolution of such mechanisms is a desired outcome of the model.

Consider an environment $E(t)$ containing firms i each with fitness denoted f reproducing identically from year to year using strategy S_I implemented with probability p_I. Firms which cannot improve their operations by imitation or by exchange must seek to optimize activities via internally sourced innovation. Innovations are assumed to occur in an essentially random manner with this strategy S_m implemented with probability p_m. The absence of other strategies implies $p_I + p_m = 1$.

Innovations are implemented in the hope that they improve firm outcomes but this is uncertain and innovations can increment or decrement firm fitness by an amount Δf with probability $P_m(\Delta f|i)$ conditioned on firm index i. Implementing a single bad innovation can destroy a firm and, as might be expected, there are far more bad innovations than good so the distribution is weighted to negative values and has mean $\Delta\bar{f}_m \ll 0$. In these circumstances firms take steps to prevent innovation and maintain stasis by setting innovation probabilities to zero, $p_m = 0$. Alternatively, firms can enact a strategy of using internal vetting processes to select hopefully good innovations. Using the techniques of Eqs. (1) and (2), this results in an innovation benefit probability distribution centered around zero with positive and negative innovations roughly equally weighted. In this case the distribution mean is approximately $\Delta\bar{f}_m \approx 0$. In turn, the reasonable probability of obtaining a positive benefit allows firms to set innovation probabilities greater than zero, $p_m > 0$. As is well known, innovation vetting processes are important to preserve firm viability. In this paper, all firms use internal selection processes to maximize innovation benefits though we subsume these processes within the strategy S_m. (Explicit models of these vetting processes could be constructed.)

Innovation benefits must be conditioned on firm index i as a good innovation for one firm is a bad innovation for another firm with different established routines . Further, as company organization fitness levels increase and companies become more highly organized, beneficial innovations become more and more hard to come by. This was observed in the previous simulation of Fig. 1. Thus, this paper assumes that as the average fitness of firm i

$$\bar{f}_i = \int_{-\infty}^{\infty} P_i(f,t)\, df \tag{19}$$

rises, the distribution $P_m(\Delta f|i)$ migrates to the left

$$P_m(\Delta f|i) = P_m(\Delta f + \bar{f}_i) \tag{20}$$

for example so that the probability of beneficial innovations decreases

$$\int_0^{\infty} P_m(\Delta f|i)\, d(\Delta f) \to 0. \tag{21}$$

This models the inability to find ever increasingly beneficial innovations so that firms relying solely on innovation eventually cease to increase fitness. This mechanism generates stasis. (See Fig. 3.)

For simplicity, the innovation benefit distribution is considered to be time independent and conditioning events which can create macroscopic changes in the innovation incremental fitness distribution P_m are ignored. For example, a very bad innovation implemented at one time makes almost any other innovation at subsequent times highly beneficial. (If the firm adopts some procedure leading to bankruptcy, any change at all to that procedure might generate better outcomes.)

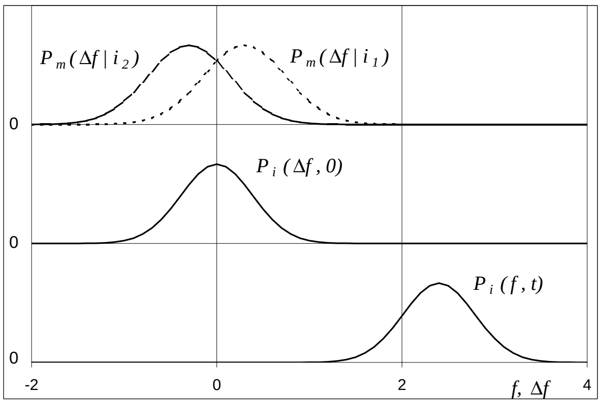

FIG. 3: $P_m(\Delta f|i)$ specifies the probable incremental return Δf from innovation conditioned on firm index i. Firm i_2 has higher fitness than firm i_1 so $P_m(\Delta f|i_2)$ has decreased probability of beneficial innovations compared to $P_m(\Delta f|i_1)$. $P_i(f,0)$ denotes the initial probable return distribution for firms i with fitness denoted f, while $P_i(f,t)$ is the evolving firm fitness distribution. (Vertical scales are arbitrary.)

At time $t = 0$, firm i possesses a probable fitness distribution $P_i(f,0)$ weighted to low fitness values as, by assumption, firms are recent entrants to a new sector and are ignorant of how to best optimize routines. (See Fig. 3.) The low fitness values of all firms precludes exchange markets.

Fitness is improved by judicious selection of strategy mix $S_r = \epsilon_a(p_I S_I + p_m S_m)$ to maximally increase fitness in a selective environment. Natural selection S_s operates on all firms simultaneously to partition the entire population into high and low fitness classes with elimination of the low fitness class. Firms which can't compete are eliminated. A typical selection operation might be modeled as

$$S_s P_i(f,t) = \begin{cases} 0 & i \in \text{ low fitness class,} \\ & \\ P_i(f,t) & i \in \text{ high fitness class.} \end{cases} \quad (22)$$

The evolution of each firm is then enacted using Eq. (15). Repeated application of chosen firm strategies generates variation which is combined with selection to implement a Darwinian selection process. The expected outcome is a population of highly fit firms possessing widely divergent organizational structures employing a large range of differing routines as the occasional good innovation is incorporated into different firms. This process will tend to increase fitness returns, initially quite rapidly and then more slowly as more and more high fitness innovations are incorporated into firms. The fitness increase is expected to asymptote to some maximum as it takes longer and longer to find better innovations as modeled by Eq. (21). The generated dynamics is expected to be similar to be that shown in the simulation of Fig. 1.

The need to maximize fitness growth rates in a competitive environment allows predictions about strategy selection probabilities. Successful firms initially optimize their rates of fitness growth by setting $p_m \approx 1$ as initial fitness levels are very low, and innovations are relatively beneficial. As fitness increases beneficial innovations become increasingly hard to find and firms are expected to minimize innovations $p_m \approx 0$ and associated expences. A novel feature of this approach is that the optimum strategy mix changes over time in predictable ways as population fitness levels change.

A. Exchange of single routines

The evolving environment $E(t)$ now surrounds any individual firm with many other firms offering examples of other organizational styles and newly innovated routines. The ongoing survival of these neighbouring firms means that their organization and previously adopted innovations offer high fitness and this leads to the evolution of acquisition and exchange mechanisms. A high fitness routine is just as likely to be found in any firm so exchange mechanisms will probably be untargeted. A firm can acquire innovations from other firms by either imitation, acquisition of staff, purchase of firm components, or by other exchange mechanisms. The equivalent point in the previous simulation occurs when evolution by random innovation gives the firm population the even numbers which are scarce in the innovation pool creating rewards for the evolution of exchange mechanisms.

If a firm obtains innovations from its neighbouring firms, it gains access to innovations which have been subject to many periods of selection and which carry high fitness. Denote the probable incremental fitness returns to firm i when an exchange occurs with firm j as $P_e(i,j,\Delta f|i)$ which, as usual, is conditioned on firm index i. Then, increasing fitness of firm i decreases the probability of a beneficial exchange taking place. (See Fig. 4.) This distribution offers high average fitness returns with mean $\Delta\bar{f}_e$ much greater than those available from the innovation pool, $\Delta\bar{f}_m \approx 0$. With the appearance of this new resource pool, the environment evolves to be

$$E(t) \equiv \{G_i(f,t), P_m(\Delta f|i), P_e(i,j,\Delta f|i)\}. \quad (23)$$

This evolved environment offers two different games for firms even though no firms have yet innovated novel exchange strategies. This expanded array of strategies is explicitly recognized using expanded firm records

$$G_i = \left\{ \begin{array}{l} P_i(f,t), \\ \epsilon_a = 1 \;:\; \{p_I, p_m, S_I, S_m\} \\ \epsilon_e = 0 \;:\; \{p_e, S_e\} \\ \epsilon_j = 0 \;:\; \{p_j, S_j\} \;\; \forall j \neq a, e \end{array} \right\}. \quad (24)$$

recognizing that firms can enact two strategies accessing strategy mix $\{p_I, p_m, S_I, S_m\}$ with probability $\epsilon_a = 1$ (certainty), and an as-yet-unevolved exchange strategy $\{p_e, S_e\}$ accessed with probability $\epsilon_e = 0$ (not innovated yet). Each firm's chosen strategy is then

$$S_r = \epsilon_a(p_I S_I + p_m S_m) + \epsilon_e p_e S_e, \quad (25)$$

where $\epsilon_a + \epsilon_e = 1$, $p_I + p_m = 1$ and $p_e = 1$. Natural selection continues to operate as in Eq. (22) making firm survival contingent on continually improving fitness.

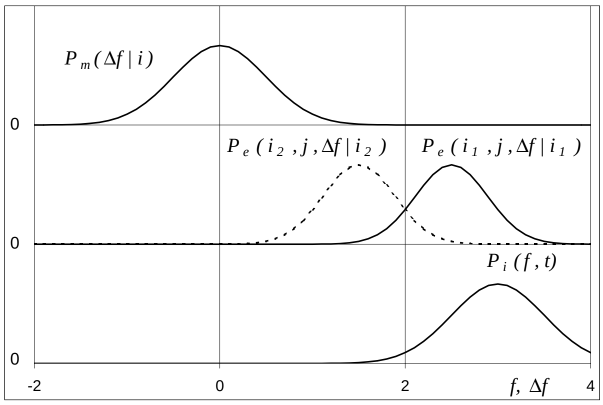

FIG. 4: $P_m(\Delta f|i)$ gives the incremental fitness probability distribution for random innovations for firm i, $P_e(i,j,\Delta f|i)$ determines the probable incremental fitness distribution from exchange between firm i and j where the average fitness of firm i_2 is much greater than that for firm i_1, and $P_i(f,t)$ is the fitness distribution of firm i with fitness denoted f. (Vertical scales are arbitrary.)

Firms accessing only the innovation pool derive an average fitness increase of $p_m\Delta\bar{f}_m \approx 0$. In contrast, any firm able to access routines from other firms by evolving a new strategy S_e can obtain an average benefit $\Delta\bar{f}_e \gg 0$. This creates a potential return difference $\Delta F = \Delta\bar{f}_e - \Delta\bar{f}_m \gg 0$, and while not yet realized, this potential difference tilts internal firm selection processes towards the innovation of mechanisms realizing and exploiting this potential return difference. The probability that strategies are innovated to access benefits ΔF from the novel resource is given by Eq. (16).

Prior to the evolution of high fitness firms in the environment, the benefit derived from the exchange of routines is $\Delta F = 0$ ensuring that exchange processes can not evolve. However, after the appearance of high fitness firms in the environment the potential benefit becomes $\Delta F > 0$ so every firm throws a dice at each time step to determine if it has innovated a novel exchange mechanism giving access to the routines of other firms. The average number of time steps required is given in Eq. (18). It is to be noted that strategies to provide the exchange of single routines might evolve as they confer greater benefits than random innovations. However, the exchange of a packet of many interlinked routines cannot yet evolve as firms lack sufficient order to make this worthwhile and $\Delta F = 0$ for packet exchange.

This approach is able to predict that exchange processes will only evolve after routines available from neighbouring firms become significantly better than those available from internal innovation. However, this model is not able to predict the specific exchange mechanism used. Basically, a supply and demand situation is created which defines a market, but the specific mechanisms used to operate this market are not specified in this approach.

Once the innovation of new strategies has occurred, routine exchanging firms re-apportion their participation between all existing games. For a given firm, if proportion ϵ_a is devoted to stasis and internal innovation, and proportion ϵ_e is spent on exchange processes, then the expected average incremental return is approximately $\epsilon_m \Delta \bar{f}_m + \epsilon_e \Delta \bar{f}_e$ when $p_m = 1$ and $p_e = 1$. As this average return is maximized by setting $\epsilon_e \approx 1$ there will be strong selection pressures for firms to minimize internal innovations and to obtain new routines only from the pre-selected routines of proven high fitness in other firms. The firm population then rapidly bifurcates into a class of firms described by $(\epsilon_a, \epsilon_e) \approx (0, 1)$ with high fitness rising from exchange mechanisms, and a class of non-exchanging firms with low fitness described by $(\epsilon_a, \epsilon_e) \approx (1, 0)$. If the component populations remain in the same market, the less-fit non-exchanging population will be rapidly eliminated.

This model then predicts that a new economic sector will initially be distinguished by innovation and will later be dominated by imitation and other firm exchange processes.

B. The exchange of packets of routines

It is expected that the exchange of single routines one at a time eventually become deleterious as consistently high fitness firms are highly ordered. Introducing a particular routine into a firm might only be beneficial if a number of other routines are present, and deliterious otherwise. Then, the exchange of randomly selected single routines even if sourced from high fitness firms will eventually destroy sufficient order to negate the benefits of new routine acquisition. At this stage, firm fitness levels stagnate and stasis is approached. If the exchange of single routines becomes deleterious because of ordered interconnections between the routines, then the exchange of ordered packets of interconnected routines becomes beneficial. The equivalent point in the simulation of Fig. 1 occurs when single routine exchange leads to firms populated by islands of order but trapped against further fitness increases. The growth of the islands of order potentiates the exchange of ordered routine packets. This cannot occur until single routine exchange processes have created such ordered islands.

In practical terms, the firms in the maturing economic sector now have sufficient information to know which routines are compatible and which conflict with each other. This information is itself a tradable commodity and might be marketed by a senior manager seeking a new position.

Potential benefits then exist for any firm innovating a strategy S_p allowing the exchange of ordered packets of routines which tilts selection pressures towards the innovation of this strategy. The methodology straightforwardly follows that above. On the appearance of such strategies, the population bifurcates into a class $(\epsilon_a, \epsilon_e, \epsilon_p) \approx (1, 0, 0)$ relying on no exchange processes, a class relying on single routine exchanges $(\epsilon_a, \epsilon_e, \epsilon_p) \approx (0, 1, 0)$ and a new highly organized class relying on ordered routine packet exchange $(\epsilon_a, \epsilon_e, \epsilon_p) \approx (0, 0, 1)$.

C. Information processing innovation systems

As seen in the simulation of Fig. 1, the exchange of routine packets allows a rapid increase in firm fitness. However, a simulated company exploiting routines in the range $\{10 — 30\}$ derives little benefit from exchanges with a firm exploiting routines in the range $\{70 — 90\}$. Exchanges between such firms are deleterious and become increasingly deleterious over time as companies become more highly organized. This suggests that there are benefits to evolving mechanisms to distinguish classes of companies so firms can optimize their packet exchange processes.

While poorly understood, it is well accepted that firms are highly ordered both temporally and spatially, and over both short and long ranges. Shuffling routines via exchange will by chance create some well ordered firms with high fitness which serve as sources of imitation for other firms. At this stage routine packet exchange mechanisms with randomly selected firms becomes increasingly deleterious compared to targeted exchanges. If exchanges are targeted to that class of firms offering high fitness packets, fitness growth rates can be maximized.

At this stage, firms can be naturally partitioned into classes with exchange of routines within a particular class being beneficial, while the exchange of routines outside that class is deleterious. The incremental exchange benefit probability distribution is then conditioned by firm index and class membership $P_e(i, j, \Delta f, n, m)$ describing exchanges between firm $i \in n^{\text{th}}$ class and firm $j \in m^{\text{th}}$ class. (Conditioning is ignored here.) When $n = m$, exchange benefits are high with $P_e(i, j, \Delta f, n, n) = P_{e,nn}(\Delta f)$ having mean $\Delta \bar{f}_{nn} \gg 0$. Conversely, if firms occupy different classes so $n \neq m$ then exchange benefits are low with $P_e(i, j, \Delta f, n, m) = P_{e,nm}(\Delta f)$ having mean $\Delta \bar{f}_{nm} \ll 0$. At this stage, the environment evolves to

$$E(t) \equiv \{G_i, P_m, P_e(i, j, \Delta f, n, m), I(i)\}, \qquad (26)$$

where $I(i)$ denotes a possible information distribution within the environment carrying class marker information. For environments where the information distribution does not exist to be exploited, necessarily the benefits of information processing are identically zero, $\Delta F = 0$, so the innovation of information processing mechanisms has zero probability. Each routine exchanging firm's record becomes

$$G_i = \left\{ \begin{array}{l} P_i(f,t), \\ \epsilon_a \approx 0 \ : \ \{p_I, p_m, S_I, S_m\} \\ \epsilon_e \approx 1 \ : \ \{p_e, S_e\} \\ \epsilon_c = 0 \ : \ \{p_c, S_c\} \end{array} \right\} \tag{27}$$

where $\epsilon_c = 0$ defines a new potential game where firms target only their own class using strategy S_c to access the information distribution $I(i)$ with probability p_c. [All exchange processes are subsumed within game (ϵ_e, p_e, S_e) here.]

Provided the information exists within the environment, firms acquiring and processing information about class membership can realize substantial benefits. Following the approach of Eq. (2), for firm $i \in n^{\text{th}}$ class the potential exchange population can be partitioned into low and high return classes

$$P_e(i,j,\Delta f,n,m) = \eta_{nn} P_{e,nn}(\Delta f) + \eta_{nm} P_{e,nm}(\Delta f) \tag{28}$$

where η_{nn} and η_{nm} are the population proportions of high and low fitness classes respectively. It is usually the case that $\eta_{nm} \approx 1$ and $\eta_{nn} = 1 - \eta_{nm} \approx 0$ giving an average incremental benefit obtained from exchanging with a randomly selected firm of $\Delta \bar{f}_r \approx \Delta \bar{f}_{nm} \ll 0$. Conversely, any firm exploiting environmental information to target exchanges to their own class forces a collapse $P_e(i,j,\Delta f,n,m) \to P_{e,nn}(\Delta f)$ using the methodology of Eq. (1). This collapse provides an average incremental benefit of $\Delta \bar{f}_{nn}$ much greater than that obtained from random selections $\Delta \bar{f}_{nm}$.

As previously discussed, a potential incremental fitness difference $\Delta F = \Delta \bar{f}_{nn} - \Delta \bar{f}_{nm} \gg 0$ exists which creates selection pressures for the innovation of mechanisms S_c able to recognize and exploit only high fitness classes. Once such mechanisms appear, it is expected that firms maximize their rate of fitness increase by setting $\epsilon_c \approx 1$ leading to the usual population bifurcation. One portion of this bifurcated population will exchange routines with any other firms $(\epsilon_a, \epsilon_e, \epsilon_c) \approx (0,1,0)$ while another portion of the population will be able to exchange routines only within their own class $(\epsilon_a, \epsilon_e, \epsilon_c) \approx (0,0,1)$.

VI. GENERALIZED INNOVATION SEQUENCES

The preceding examples establish that plausible predictions of evolutionary economic sequences can be made by exploiting the order and information present within the distributions describing an economic arena. This methodology can be extended.

Consider the innovation environment in interacting firms whose fitness depends contingently on the performance of other firms. For example, mining firms do well when car companies are booming, or one electronics firm suffers when another firm's system becomes an industry standard.

Consider two different populations A and B obtaining benefit from each other and where, at any time, each population can be partitioned into distinct classes. Write the population fitness distribution for firm i of population A currently occupying the n^{th} class and firm j of population B currently occupying the m^{th} class as $A_i(f,n)$ and $B_j(f,m)$ respectively.

Suppose the population interaction is such that the fitness of firms in each population depends on the current population classes such that

$$\begin{aligned} A_i(f,n) &= \delta_{nm} \tilde{A}_{i1}(f) + (1-\delta_{nm}) \tilde{A}_{i2}(f) \\ B_j(f,m) &= \delta_{nm} \tilde{B}_{j1}(f) + (1-\delta_{nm}) \tilde{B}_{j2}(f). \end{aligned} \tag{29}$$

In the case where $n = m$, $\delta_{nm} = 1$ and populations A and B have fitness distributions $\tilde{A}_{i1}(f)$ and $\tilde{B}_{j1}(f)$ respectively, while if $n \neq m$, $\delta_{nm} = 0$ giving these populations fitness distributions $\tilde{A}_{i2}(f)$ and $\tilde{B}_{j2}(f)$ respectively. These fitness distributions have been written to explicitly show the partitioning effect of class information.

A cooperative symbiosis between the two populations can be modeled by ensuring that populations occupying complementary classes $n = m$ have high average fitness so $\tilde{A}_{i1}(f)$ and $\tilde{B}_{j1}(f)$ are heavily weighted to positive fitness values. Conversely, populations having different class $n \neq m$ suffer low fitness implying $\tilde{A}_{i2}(f)$ and $\tilde{B}_{j2}(f)$ are weighted to negative fitness values.

Competitive Red Queen type arms races between the populations are modeled by simply adjusting the constituent probability distributions. Then, if A population firms occupy the same class as population B firms, then population A firms have high fitness and population B firms have low fitness. Thus, when $n = m$, $\tilde{A}_{i1}(f)$ is weighted to positive fitness levels while $\tilde{B}_{j1}(f)$ is weighted to negative fitness levels. When the firms occupy different classes $n \neq m$, population A firms have low fitness with $\tilde{A}_{i2}(f)$ being weighted to negative fitness levels while population B firms have high fitness levels with $\tilde{B}_{j2}(f)$ being weighted to positive fitness levels.

Given these class differentiated fitness distributions, there exists potential benefits for any firms with low fitness to innovate new strategies to effect a class change to realize potential benefits and to maximize fitness. As usual, the existence of potential benefits tilts innovation towards class targeting or class changing strategies.

In symbiosis, strong selection pressures ensure that both populations occupy the same class, and innovations allowing either population to enter a new class creates strong potential benefits for further innovations of either population to make the classes congruent. For Red Queen arms races, strong potential fitness benefits drive population B to innovate into any new class away from population A and for population A to innovate to track population B into this new class.

More general innovation models can be formulated. In some of the above examples, firms have processed market information for benefit. In turn, this practice makes it beneficial for firms to distort market information for their own ends, leading to distortions such as falsified annual reports and accounts, advertising and marketing, and so on. Subsequently, the existence of false information makes it beneficial for firms to innovate methods to assess the value of information. This potentiates innovations such as independent assessors, credit-raters and so on.

Interesting games result when payoffs are inherently risky with irreducible uncertainties and variances. If variances or risk cannot be reduced by processing information then this potentiates a market in the exchange of risk itself. A player can sell their risk in conducting some activity to another for a known return, while the buyer gains access to guaranteed supplies, or potentially large profits or to minimized risk due to hedged uncertainties. This market in risk itself leads to the innovation of futures markets.

VII. CONCLUSION

This paper generalizes game theory to develop a multi-game environment featuring uncertainty which is described by probability distributions. Game players process information to reduce uncertainty and to maximize benefit. Evolution of the environment occurs over time as players trade, manipulate and generate new resource distributions. Innovation occurs as game players seek to maximize benefit by changing games, accessing novel resources and trading information and novel goods. The focus on firm organization and the processing of information allows the prediction of plausible innovation sequences. This paper predicts the ordered appearance of markets in the stages of single routine exchange, to routine packet exchange, to selected packet exchange, and so on. These predictions were made on the basis of analytic work. A simple simulation was presented showing the expected punctuated equilibria dynamics.

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