Evolution and Walrasian Behavior in Market Games

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Abstract

We revisit the question of price formation in general equilibrium theory. We explore whether evolutionary forces lead to Walrasian equilibrium in the context of a market game, introduced by Shubik (1972). Market games have Pareto inferior (strict) Nash equilibria, in which some, and possibly all, markets are closed. We introduce a strong version of evolutionary stable strategies (*SESS*) for finite populations. Our concept requires stability against multiple, simultaneous mutations. We show that the introduction of a small number of "trading mutants" is sufficient for Pareto improving trade to be generated. Provided that agents lack market power, Nash equilibria corresponding to approximate Walrasian equilibria constitute the only approximate SESS.

Keywords: Walrasian Equilibrium, Market Games, Evolutionary Stability

1 Introduction

Walrasian equilibrium is a cornerstone of modern economics. It is, therefore, not surprising that the question of Walrasian price formation has been the topic of extensive study in general equilibrium theory. The tâtonnement process has been used extensively in this context.[1] The study of tâtonnement, however, has produced largely negative results, and this has led some researchers to conclude that decentralized information about prices alone is not sufficient to bring the economy to the Walrasian equilibrium. In addition, and perhaps more importantly, the tâtonnement has been criticized for lacking micro foundations since the price adjustment process is not the outcome of the individual optimization.

Even if we put the traditional stability question aside, Walrasian equilibrium may be challenged on the basis of complexity considerations. Can "unsophisticated" agents learn to behave in such a way that an outside observer of the economy will see a Walrasian equilibrium allocation? Evolutionary game theory seems to provide an appropriate framework to formulate this question. After all, competitive outcomes are often justified by appealing to the natural selection of behavior that is more "fit."[2] In this paper we explore whether evolutionary forces can lead to Walrasian equilibrium in the context of a market game, introduced by Shubik (1972).[3] Our story is not explicitly dynamic. Rather, we show that any non-Walrasian outcome can be disturbed by the introduction of a small number of "mutants," who can become better off in relative terms by choosing different trading patterns.

Market games are one of the non-cooperative structures that give rise to competitive outcomes when agents lack market power. Thus, it has served as a non-cooperative foundation for the Walrasian equilibrium. Even in large

[1] See Arrow and Hurwicz (1959) for a classic reference.

[2] See Alchian (1950) for one of the first attempts to formalize this argument.

[3] There is extensive literature on market games. Standard references include Shapley (1977), Shapley and Shubik (1977), Dubey and Shubik (1977), and Mas-Colell (1982).

economies, however, in addition to approximately Walrasian outcomes, market games obtain other, Pareto inferior (strict) Nash equilibria, in which at least some, and possibly all, markets are closed due to a coordination failure. Our study concerns a pure exchange economy with a finite number of agents and a finite number of goods. We study the limit case, as formalized by Postlewaite and Schmeidler (**PS,** 1978), where the number of agents is large. We introduce a strong version of evolutionary stable strategies (SESS) for asymmetric, finite populations. Roughly speaking, SESS requires stability against all simultaneous mutations by at most one agent per population.

We demonstrate that (partial) autarky outcomes are not SESS in an approximate sense that we make precise. A small number of suitable mutations is sufficient for Pareto improving trade to be generated and for a market to open. Thus, evolutionary forces provide an avenue through which the economy can escape situations in which some markets are closed due to a coordination failure. We demonstrate that in a replicated version of the game, as agents' market power becomes insignificant, Nash equilibria that support approximate Walrasian equilibria of the underlying economy are the only approximate SESS.[4]

We can summarize the intuition behind our main result as follows. A Pareto inferior situation, in which some markets are closed, cannot be disturbed by a single mutant. This is because a single agent cannot create beneficial trade. On the other hand, the introduction of one trading mutant *on each side of the market* is sufficient to open a market, thus leading the economy to a Pareto superior trading regime. All other states that involve trade, but not individual optimization at the given prices, can be disturbed by the introduction of a single mutant who chooses the best basket at the given prices. An important ingredient in our analysis is that the number of agents in the replicated economy under

[4] Our results are related to Dubey and Shubik (1978), who introduce an outside agency that ensures that arbitrarily small amounts of bids and asks are present in all markets. Our argument, however, does not rely on the existence of this agency. In addition, we impose minimal rationality requirements on our agents, and we explicitly consider all non-Nash outcomes.

study is much greater than the number of possible mutants. Consequently, while mutations can change certain agents' baskets, they only have a negligible effect on prices. As a result, no mutations can lead to improvements, in an approximate sense, if the economy is at a Walrasian equilibrium. Thus, consistent with the traditionally held view, our findings provide support for the popular belief that evolutionary forces lead to competitive outcomes, but only when individual agents are small compared to the market.[5]

The paper proceeds as follows. Section 2 reviews some concepts from evolutionary game theory, introduces our solution concept and presents an example. In Section 3, we apply the solution concept to the market game and discuss the main result. A brief conclusion follows.

2 The Solution Concept

We start by stating some related existing definitions of the evolutionary stability. First, consider a population consisting of a continuum of agents. We assume that N agents are selected from this population to play a normal-form game $\Gamma = (N, X, u)$. The standard definition of an Evolutionary Stable Strategy (ESS) for two-player symmetric games is as follows (see Weibull, 1995):

Definition 1 *$x \in \Delta$ is an* **ESS** *if, for every strategy $y \neq x$, there exists $\varepsilon_y > 0$ such that*

$$u\left(x, \left(1-\varepsilon\right)x + \varepsilon y\right) > u\left(y, \left(1-\varepsilon\right)x + \varepsilon y\right)$$

for all $\varepsilon \in (0, \varepsilon_y)$, where Δ is a set of mixed strategies.

Next, consider a finite population of size N. The definition of ESS for N-player symmetric games is as follows (see Schaffer, 1988, 1989):

[5]This is in contrast to some recent papers in the literature, notably Vega-Redondo (1997). See our conclusion section for further discussion.

Definition 2 *$x \in X$ is an* **ESS** *if, for any strategy $y \neq x$,*

$$u\left(x \mid y, x, x, ...\right) \geq u\left(y \mid x, x, ...\right).$$

An equivalent definition is that x is an ESS if it solves the following maximization problem:

$$\max_{y \in X} \; u\left(y \mid x, x, ...\right) - u\left(x \mid y, x, x, ...\right). \tag{1}$$

We amend Schaffer's (1988) definition in two ways. First, we extend the definition of an ESS from one finite population to multiple finite populations. Second, we build a strong version of the evolutionary stability, one that requires stability against a simultaneous invasion of multiple mutants from different populations.

We will first present the concept in the context of an example. In the next section, we will apply it to a market game. Suppose that there are K finite populations. Each population, i, contains $n_i \geq 2$ agents. We assume that all agents from population i play an N-player game, Γ, where $N = n_1 + ... + n_K$. Γ is assumed to have the following symmetry property. All players from population i have the same set of strategies, X^i, and the same payoff function, u^i. In other words, if two players (from the same population) play the same strategy, they will obtain the same payoffs. Hence, we can write

$$\Gamma = \left(\{n_1, ..., n_K\}; \times_{n_1} X^1 \times ... \times_{n_K} X^K; \left(u^1, ..., u^1; ...; u^K, ..., u^K\right)\right). \tag{2}$$

Suppose that one player from population i plays strategy y^i, while all other players from population i play strategy x^i. If at most one player in each population plays a strategy which is different from the one chosen by every other player in the population, the payoff function of a player from population i can be written as:

$$u^i\left((y^i, \overline{x^i}); (y^1, \overline{x^1}); ...; (y^K, \overline{x^K})\right), \tag{3}$$

where $\left(y^i, \overline{x^i}\right)$ denotes the case where one player from population i plays strategy y^i, while all other players from population i play strategy x^i.

We are now ready to define our main concept.

Definition 3 $x = \left(\overline{x^1}, ..., \overline{x^K}\right) \in X$ *is a* **Strong ESS** *(***SESS***) if, for any strategy* $y^i \neq x^i$*,*

$$u^i\left(x^i, \overline{x^i}; \gamma^1, ..., \gamma^K\right) \geq u^i\left(y^i, \overline{x^i}; \gamma^1, ..., \gamma^K\right), \textit{ for all } i, \tag{4}$$

and for all γ^j*, where* $\gamma^j = \left(x^j, \overline{x^j}\right)$*, or* $\gamma^j = \left(y^j, \overline{x^j}\right)$*.*

Condition (4) is equivalent to saying that

$$u^i\left(y^i, \overline{x^i}; \gamma^1, ..., \gamma^K\right) - u^i\left(x^i, \overline{x^i}; \gamma^1, ..., \gamma^K\right), \tag{5}$$

as a function of y^i, reaches its maximum value of zero when $y^i = x^i$, for all γ^j. That is, x^i is a solution to the following maximization problem:

$$\max_{y^i}\left[u^i\left(y^i, \overline{x^i}; \gamma^1, ..., \gamma^K\right) - u^i\left(x^i, \overline{x^i}; \gamma^1, ..., \gamma^K\right)\right], \tag{6}$$

for all γ^j, where $\gamma^j = \left(x^j, \overline{x^j}\right)$, or $\gamma^j = \left(y^j, \overline{x^j}\right)$, $j \neq i$.

A notable feature of the SESS is that it requires stability against up to K simultaneous mutations (one per population). Clearly, this is a stronger concept than Schaffer's ESS. Thus, SESS will not exist in general either. An important feature of our concept is that it can be applied to asymmetric games. Another difference from standard evolutionary models is that instead of a continuum, we shall assume a finite number of agents. Below, we give an example of a 4-player coordination game in which SESS uniquely selects the Pareto efficient Nash equilibrium even though there is another ESS.

Example 1. Suppose that there are two populations (I and II), each consisting of two players. Each player has two available actions (a and b). Let $\theta_{I(II)}$ stand for the number of a-players in population $I(II)$. Payoffs are defined as follows.

$$\begin{array}{cccc} \underline{u_I(a,\theta_I,\theta_{II})} & \underline{u_I(b,\theta_I,\theta_{II})} & \underline{u_{II}(a,\theta_I,\theta_{II})} & \underline{u_{II}(b,\theta_I,\theta_{II})} \\ u_I(a,1,0)=0 & u_I(b,0,0)=2 & u_{II}(a,0,1)=0 & u_{II}(b,0,0)=2 \\ u_I(a,2,0)=0 & u_I(b,1,0)=2 & u_{II}(a,0,2)=0 & u_{II}(b,0,1)=2 \\ u_I(a,1,1)=3 & u_I(b,0,1)=1 & u_{II}(a,1,1)=3 & u_{II}(b,1,0)=1 \\ u_I(a,2,1)=3 & u_I(b,1,1)=1 & u_{II}(a,1,2)=3 & u_{II}(b,1,1)=1 \\ u_I(a,1,2)=4 & u_I(b,0,2)=0 & u_{II}(a,2,1)=4 & u_{II}(b,2,0)=0 \\ u_I(a,2,2)=4 & u_I(b,1,2)=0 & u_{II}(a,2,2)=4 & u_{II}(b,2,1)=0 \end{array} \tag{7}$$

For example, $u_I(a,1,0)=0$ means that the payoff of the player in population I who plays action a, when all other players (one player in population I and two players in population II) play action b, is zero. Clearly, this is a coordination game. It has two symmetric strict Nash equilibria in which all agents play a and all play b, respectively. The a-equilibrium is an SESS. Notice, however, that the b-equilibrium is not an SESS since one mutation per population (type) to playing strategy a will result in a payoff of 3 for each of the two mutants (instead of 1 for the b-players).

Later, we shall need to make use of the following approximate notion of an SESS.

Definition 4 *$x \in X$ is an ϵ-**SESS** if, for any $y^i \neq x^i$,*

$$u^i\left(x^i, \overline{x^i}; \gamma^1, ..., \gamma^K\right) \geq u^i\left(y^i, \overline{x^i}; \gamma^1, ..., \gamma^K\right) - \epsilon, \text{ for all } i, \tag{8}$$

and for all γ^j, where $\gamma^j = \left(x^j, \overline{x^j}\right)$, or $\gamma^j = \left(y^j, \overline{x^j}\right)$, $j \neq i$.

Thus, an ϵ-SESS requires that no mutant can be better off by more than a small amount, ϵ. In the next section we motivate and use both the SESS and the ϵ-SESS concepts in the context of our main topic of study, a strategic market game.

3 The Market Game

3.1 Preliminaries

We consider a pure exchange economy with L consumption goods. The economy is described by $\mathcal{E} = \langle I, w^i, u^i \rangle_{i \in I}$, where I is a finite set of agents belonging to K different populations (or types); $u^i : \mathbb{R}_+^L \to \mathbb{R}$ is the utility function of agent i, and $w^i \in \mathbb{R}_+^L$ is the endowment vector of agent i. We assume that u^i is continuous, strictly increasing in all its variables, and strictly quasi-concave on $\mathbb{R}_+^L$. Agents participate in an n-player market game related to the one in Shapley and Shubik (1977). In what follows, we largely rely on **PS** in defining the market game corresponding to $\mathcal{E}$.[6]

Let $X^i = \{x^i = (b^i, q^i) \in \mathbb{R}_+^L \times \mathbb{R}_+^L : q^i \leq w^i\}$ be the set of strategies of player i. Here, b^i denotes the vector of bids or "goods requested" by agent i, measured in abstract units of account, while q^i denotes the vector of goods offered by agent i. Individual agents have to satisfy a *balance or bankruptcy condition,* which requires that the total value of an agent's bid has to be less than, or equal to, the total "receipts" from their goods sales. More precisely, the individual balance condition is given by

$$\sum_{l \in L} b_l^i \leq \sum_{l \in L} \frac{q_l^i}{\sum_{j \in I} q_l^j} \sum_{j \in I} b_l^j. \tag{9}$$

One issue is what happens to agents who violate the balance condition. This is particularly important in our case for two reasons. First, unlike **PS**, we explicitly consider non-Nash states in which this constraint might be violated. Second, since agents in our model are concerned with *relative* performance, they might wish to take an action that will make them worse off in absolute terms if this would lead to other agents of their type becoming further worse off. This could occur if an action by a single agent would lead to other agents' becoming

[6] We believe that our main argument will apply under alternative specifications of the market game provided that they allow for a Nash equilibrium of a replicated game to approximate a Walrasian equilibrium of the underlying economy.

bankrupt. This possibility arises under the **PS** specification since they assume that agents who violate the balance condition have all their resources confiscated. With these considerations in mind, we impose the milder assumption that an agent whose total value of goods requested exceeds his total receipt value has his bid vector "shaved" by an amount that is proportional to his overbidding. More precisely, let

$$\alpha^i = \frac{\sum_{l\in L} \frac{q_l^i}{\sum_{j\in I} q_l^j} \sum_{j\in I} b_l^j}{\sum_{l\in L} b_l^i} \tag{10}$$

and let

$$\widetilde{b}_l^i = \begin{cases} \alpha^i b_l^i, & \text{if } \sum_{l\in L} b_l^i > \sum_{l\in L} \frac{q_l^i}{\sum_{j\in I} q_l^j} \sum_{j\in I} b_l^j \\ b_l^i, & \text{otherwise.} \end{cases} \tag{11}$$

The determination of the agents' resulting consumption baskets operates as follows. For all $i \in I$, and $l \in L$, let c_l^i be the consumption of good l by agent i. This is determined by

$$c_l^i = w_l^i - q_l^i + \frac{\widetilde{b}_l^i}{\sum_{j\in I} \widetilde{b}_l^j} \sum_{j\in I} q_l^j. \tag{12}$$

As usual, a strategy profile $\widehat{x}$ is a *Nash equilibrium* if

$$u^i(\widehat{x}^i, \widehat{x}^{-i}) \geq u^i(x^i, \widehat{x}^{-i}),\ \forall i,\ \forall x^i \in X^i. \tag{13}$$

A Nash equilibrium is *full* if all markets are open. A (feasible) allocation in the economy is an L-list of commodity bundles, $\left(z^i\right)_{i\in I}$, such that $\sum_{i\in I} z^i \leq \sum_{i\in I} w^i$. Proceeding as in **PS**, we say that for any $\epsilon > 0$, an allocation $\left(\overline{z}^i\right)_{i\in I}$ is ϵ-efficient (or ϵ-Pareto efficient) if for any L-list of commodity bundles $\left(z^i\right)_{i\in I}$ we have that if $u^i\left(z^i\right) \geq u^i\left(\overline{z}^i\right)$ holds for all $i \in I$, then $\sum_{i\in I} z^i > (1-\epsilon)\sum_{i\in I} w_i$.

Next, we state the two main results of **PS**. They establish the connection between full Nash equilibria of the market game and Pareto optimal states as well as Walrasian equilibria of the underlying economy.

Proposition 1 *(**PS:** Approximate Efficiency Theorem): For any positive numbers δ, β, and ϵ, any allocation resulting from a full Nash equilibrium in an economy $\mathcal{E} = \langle I, w^i, u^i \rangle_{i \in I}$ with $w^i < \beta(1, ...1)$ for all $i \in I$, $\sum_{i \in I} w^i > \#I\delta(1, ..., 1)$ and $\#I > 16\,(\#L)\,\beta/\delta\epsilon^2$ is ϵ-efficient.*

Let $B_l = \sum_{j \in I} b_l^j$, and $Q_l = \sum_{j \in I} q_l^j$, and define $p_l = B_l/Q_l$ to denote the average price of commodity l (provided that the denominator of this expression is strictly positive). Define an allocation $\widehat{x}$ resulting from a full Nash equilibrium to be ϵ-Walrasian if all markets are open and there exists $\widehat{p}$ such that for all $i \in I$, $\widehat{p}\widehat{x}^i = \widehat{p}w^i$, and

$$\#\{i \in I : \forall \widetilde{x}^i,\ \widetilde{x}^i \succ_i \widehat{x}^i \Rightarrow \widehat{p}\widetilde{x}^i > \widehat{p}(1-\epsilon)\widehat{p}w^i\} > (1-\epsilon)\#I, \tag{14}$$

where, as stated above, prices correspond to ratios of aggregate bids. **PS** showed that a full Nash equilibrium of a large enough market game is approximately efficient and, in addition, it corresponds to an approximate Walrasian equilibrium of the underlying economy.

Proposition 2 *(**PS:** Approximate Walrasian Allocation Theorem): Under the conditions of the Approximate Efficiency Theorem, full Nash equilibrium allocations are ϵ-Walrasian.*

This completes the discussion of the market game. Henceforth, we will concentrate on the evolutionary stability of the full Nash equilibria that support approximate Walrasian allocations.

3.2 Evolutionary Stability

Before we analyze the market game from an evolutionary point of view, we wish to introduce the main argument in an informal way. This will also serve as a motivating discussion for the concepts we introduced in the previous section. First, notice that no Nash equilibrium in which some markets are closed can be disturbed by the introduction of a single trading mutant. This is because at least one agent *on each side of the market* is necessary for any trade. While the existence of such (partial) autarky Nash outcomes is plausible, it is also insightful to study under what conditions evolutionary forces will result in the "opening of markets," leading to a Pareto superior outcome. To our knowledge, ours is the first example to demonstrate that evolutionary pressure can lead to the opening of new markets. The fact that this requires the simultaneous introduction of mutations from each side of the market is exactly what SESS is designed to capture.

A separate issue from whether all markets will be open is whether evolution will give rise to an efficient or, more restrictively, to a Walrasian outcome. Having established that no state in which some or all markets are closed corresponds to an SESS, we turn to the question whether states that correspond to Walrasian equilibria are SESS. Here, a difficulty arises. The fact that we deal with a finite game implies that each individual agent has some market power. Of course, this market power vanishes as the number of agents increases. This suggests that in the case where the economy is large enough, we can expect that the above question will be answered in the affirmative, but only in an approximate sense.

To see this, let us suppose that the economy is at a full Nash equilibrium. Suppose that an agent mutates to a different bid/offer. Clearly, since the previous situation was a Nash equilibrium, the mutant will be worse off. However, this does *not* imply the evolutionary stability of the full Nash equilibrium. The reason is as follows. Since there is a finite number of agents, the mutation will result in slightly different prices for at least some agents. While the mutant is worse off under the new prices, it could be that other agents of his type are even more worse off or, in other words, the mutant could be better off *in relative terms*. Thus, the evolutionary stability of the Walrasian equilibrium is not automatic. A continuity argument, however, guarantees that if the economy is large enough, a small number of mutations cannot make the mutants better off by more than an arbitrarily small amount. Thus, the full Nash equilibrium, which **PS** have shown to be approximately Walrasian, is also an approximate SESS. Formalizing the details of this argument is the main purpose of this section. In what follows, we follow Debreu and Scarf (1963)[7] in formalizing the notion of a large economy. In particular, given the economy $\mathcal{E}$, we will consider a *replica economy* $\mathcal{E}_r$ resulting from replicating economy $\mathcal{E}$ r times. The new economy, $\mathcal{E}_r$, contains rI agents, and every population increases r times. We have the following result.

Theorem 1 *Consider economy $\mathcal{E}$ for which the conditions of the Approximate Efficiency Theorem hold. For any $\epsilon > 0$, there exists $\overline{r} \in \mathbb{N}$ such that all full Nash equilibrium allocations are ϵ-SESS for all replicas, $\mathcal{E}_r$, of economy $\mathcal{E}$ with $r \geq \overline{r}$. For any allocation different from a full Nash equilibrium allocation, there exists $\epsilon_0 > 0$ and $\overline{r}_0 > 0$ such that this allocation is not an ϵ-SESS, where $\epsilon \in (0, \epsilon_0)$.*

Proof. Since the conditions of the Approximate Efficiency Theorem hold, the full Nash equilibrium allocation for the economy $\mathcal{E}$ is ϵ-efficient. We assume that all agents who have the same endowment and the same strategy choices are from the same population. Agent i's utility can be written in the following form:

$$u_r^i(x) = u_r^i\left(x^i; B; Q\right), \tag{15}$$

where $B_l = \sum_{j \in I} b_l^j$ and $Q_l = \sum_{j \in I} q_l^j$ are the aggregate bids and offers, for all $l \in L$, while $B = (B_1, ..., B_L)$, and $Q = (Q_1, ..., Q_L)$. Note that the function u_r^i is continuous in all its variables. Since in the full Nash equilibrium all markets are open, we have that $B_l > 0$ and $Q_l > 0$ for all $l \in L$.

[7] See, for example, Jehle and Reny (2001) for a more modern reference.

It is easy to check that all conditions of the Approximate Efficiency Theorem hold for economy $\mathcal{E}_r$, $r > 1$. Consider a coalition of agents, $C \neq \emptyset$, consisting of at most one agent per type, who mutate to y^i. Let $(\widetilde{B}; \widetilde{Q})$ be the resulting aggregate bids and offers. Note that, excluding all agents $i \in C$ in the new economy, $\mathcal{E}_r$, the aggregate bids and asks increase by a factor of r for all goods in comparison with economy $\mathcal{E}$. Therefore, there exists $\overline{r} \in \mathbb{N}$ such that, for any $r \geq \overline{r}$,

$$\left| u_r^i \left(x^i; B; Q \right) - u_r^i(y^i; \widetilde{B}; \widetilde{Q}) \right| < \frac{\epsilon}{2}, \text{ for all } i \in C, \tag{16}$$

and

$$\left| u_r^i \left(x^i; B; Q \right) - u_r^i(x^i; \widetilde{B}; \widetilde{Q}) \right| < \frac{\epsilon}{2}, \text{ for all } i \in I/C, \tag{17}$$

where $y^i \in X^i$, for all i, and $\widetilde{B}_l(\widetilde{Q}_l)$ differ(s) from $B_l(Q_l)$ by the choice of at most one agent per population, for all $l \in L$. Inequalities (16) and (17) follow from the continuity of functions u^i and the definition of $\widetilde{b}_l^i$ in expression (11). Now, it follows immediately that

$$\begin{aligned} &\left| u_r^i(x^i; \widetilde{B}; \widetilde{Q}) - u_r^i(y^i; \widetilde{B}; \widetilde{Q}) \right| \leq \\ &\leq \left| u_r^i \left(x^i; B; Q \right) - u_r^i(x^i; \widetilde{B}; \widetilde{Q}) \right| + \\ &+ \left| u_r^i \left(x^i; B; Q \right) - u_r^i(y^i; \widetilde{B}; \widetilde{Q}) \right| < \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon. \end{aligned} \tag{18}$$

In other words, a full Nash equilibrium allocation for the economy $\mathcal{E}_r$ is an ϵ-SESS.

Next, consider any strategy profile x leading to $(B; Q)$ and resulting in a state other than a Walrasian equilibrium. Then, there exists a coalition of agents, $C' \neq \emptyset$, (consisting of at most one agent per type), $\epsilon_0 > 0$, $\overline{r}_0 > 0$, and strategies y^i for all $i \in C'$, such that for any $\epsilon \in (0, \epsilon_0)$ and any $r > \overline{r}_0$,

$$u_r^i(y^i; \widetilde{B}; \widetilde{Q}) - u_r^i(x^i; B; Q) > \frac{3\epsilon}{2}, \text{ for all } i \in C', \tag{19}$$

and

$$\left| u_r^i(x^i;\widetilde{B};\widetilde{Q}) - u_r^i(x^i;B;Q) \right| < \frac{\epsilon}{2}, \text{ for all } i \in I/C'. \tag{20}$$

The last inequality (20) follows from the continuity of the utility function, u_r^i. Inequality (19) follows from the following observation. Consider any allocation x in which not all markets are open. This implies that there exists a coalition of agents, C', such that every member of the coalition can gain by using a deviant trade strategy y^i. Choose ϵ_0 to be smaller than the minimal gain over the members of C'. Similarly, if the initial allocation x is such that all markets are open but different from a Walrasian allocation, then there exists a population i such that an agent from this population can deviate from x^i to playing y^i (the best reply given strategy choices of all other agents). Such an agent will obtain a strictly higher payoff. Inequality (19) follows.

Finally, this implies that

$$\begin{aligned} & u_r^i(y^i;\widetilde{B};\widetilde{Q}) - u_r^i(x^i;\widetilde{B};\widetilde{Q}) > \\ & > \left| u_r^i(y^i;\widetilde{B};\widetilde{Q}) - u_r^i(x^i;B;Q) \right| - \\ & - \left| u_r^i\left(x^i;B;Q\right) - u_r^i(x^i;\widetilde{B};\widetilde{Q}) \right| > \epsilon. \end{aligned} \tag{21}$$

■

The next corollary connects our solution concept to Walrasian equilibrium.

Corollary 1 *Suppose that the conditions of the Approximate Efficiency Theorem hold. Then there exists $\overline{r} \in \mathbb{N}$ such that ϵ-Walrasian equilibria are the only ϵ-SESS for or all replicas, $\mathcal{E}_r$, of economy $\mathcal{E}$ with $r \geq \overline{r}$.*

Proof. This follows from Theorem 1 and from the Approximate Walrasian Allocation Theorem. ■

It is worth mentioning that the above results will *not* hold in general if the economy is populated by a small number of agents. In that case, by having a non-negligible effect on the price, an agent mutating from the full Nash equilibrium allocation may be able to make himself better off relative to the other agents of his type. Therefore, full Nash equilibria may not correspond to any ϵ-SESS if agents have significant market power. While this observation is consistent with the traditionally held view that competitive outcomes arise when individual agents are of insignificant size, it is distinct from Vega-Redondo (1997), in which a competitive outcome is shown to be evolutionary stable in the context of a Cournot oligopoly model where agents have significant market power. This suggests that whether a partial or a general equilibrium framework is assumed matters when determining the evolutionary stability of Walrasian outcomes.

4 Conclusion

We studied the evolutionary stability of the Walrasian equilibrium in the context of the strategic market game, introduced by Shubik (1972). We introduced a strong version of evolutionary stable strategies, SESS, for asymmetric games played by finite populations. SESS requires stability against multiple, simultaneous mutations. The introduction of a small number of "mutants" is sufficient for Pareto improving trade to be generated. Thus, Pareto inferior strict Nash equilibria where some or all markets are closed due to a coordination failure do not constitute SESS. Provided that agents lack market power, approximate Walrasian equilibrium outcomes are shown to be the only ϵ-SESS. While our specification of the market game closely follows the one in **PS**, we believe that our analysis holds under alternative specifications provided that they allow for a Nash equilibrium of a replicated game to approximate a Walrasian equilibrium of the underlying economy.

An important extension of our analysis concerns the relation between our static SESS concept and the asymptotically stable points of a suitably defined dynamic system describing the learning process. Such a dynamic system must be able to distinguish between Walrasian outcomes and other strict Nash equilibria involving (partial) autarky outcomes. This extension is left to future research.

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