Kidney Exchange*

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Abstract

Most transplanted kidneys are from cadavers, but there are also substantial numbers of transplants from live donors. Recently, there have started to be kidney *exchanges* involving two donor-patient pairs such that each donor cannot give a kidney to the intended recipient because of immunological incompatibility, but each patient can receive a kidney from the other donor. Exchanges are also made in which a donor-patient pair makes a donation to someone on the queue for a cadaver kidney, in return for the patient in the pair receiving the highest priority for a compatible cadaver kidney when one becomes available. We explore how such exchanges can be arranged efficiently and incentive compatibly. The problem resembles some of the "housing" problems studied in the mechanism design literature for indivisible goods, with the novel feature that while live donor kidneys can be assigned simultaneously, the cadaver kidneys must be transplanted immediately upon becoming available. In addition to studying the theoretical properties of the design we propose for a kidney exchange, we present simulation results suggesting that the welfare gains would be substantial, both in increased number of feasible live donation transplants, and in improved match quality of transplanted kidneys.

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1 Introduction

Transplantation is the preferred treatment for the most serious forms of kidney disease. As of this writing there are over 55,000 patients on the waiting list for cadaver kidneys in the United States, of whom almost 15,000 have been waiting more than three years. By way of comparison, in 2002 there were over 8,000 transplants of cadaver kidneys performed in the United States. In the same year, about 3,400 patients died while on the waiting list, and another 900 became too ill to be eligible for transplantation. See Table 1.[1]

As these numbers make clear, there is a considerable shortage of cadaveric kidneys, compared to the demand. (We note in passing that the substantial consensus in the medical community remains firmly opposed to allowing organs–even cadaveric organs–to be bought and sold, and in most states this is a felony under the Uniform Anatomical Gift Act of 1987.[2]) Congress views cadaveric kidneys offered for transplantation as a national resource, and the National Organ Transplant Act of 1984 established the Organ Procurement and Transplantation Network (OPTN). Run by the United Network for Organ Sharing (UNOS), it has developed a centralized priority mechanism for the allocation of cadaveric kidneys.

In addition to transplants of cadaver kidneys, in 2002 there were also somewhat over 6,000 transplants of kidneys from living donors, a number that has been increasing steadily from year to year. Transplants from live donors generally have a higher chance of success than those from cadavers. The way such transplants are typically arranged is that a patient identifies a healthy willing donor (a spouse, for example) and, if the transplant is feasible on medical grounds (i.e. appropriate blood types, and absence of "positive crossmatch" antibodies), it is carried out. If the transplant from the willing donor is not feasible, the patient typically enters (or remains on) the queue for a cadaver kidney, while the donor returns home.

Recently, however, in a relatively small number of cases, additional possibilities have been utilized when a transplant from a live donor and the intended recipient is infeasible. One of these, called a *paired exchange*, involves two patient-donor couples, for each of whom a transplant from donor to intended recipient is infeasible, but such that the patient in each couple could feasibly receive a transplant from the donor in the other

[1] http://www.optn.org/, accessed 7/31/01 and annual report of UNOS/OPTN.

[2] There is, however, a steady stream of literature both by doctors and by economists, considering how the shortage of organs might be alleviated by allowing their purchase and sale, and what effects this might have. See e.g. Nelson et al. (1993) for an argument in favor of the status quo, and e.g. Becker and Elias [2002] for an argument in favor of a market. Recent Congressional testimony endorsing the status quo but suggesting that empirical investigation of financial incentives might be in order can be found in Slade [2003]. In the present paper we will consider how welfare can be improved in the existing legal environment.

Table 1: US Kidney Transplants. The data for years 1992-2001 are constructed from the annual report of UNOS-OPTN, the data for 2002 are constructed from the national database of UNOS-OPTN. Number of registrations may have multiple counts of patients since one patient may have registered in multiple centers for the waitlist.

	Year										
	1992	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**
Cadaveric Donors	4,276	4,609	4,797	5,003	5,038	5,083	5,339	5,386	5,490	5,528	5,630
Cadaveric Transplants	7,202	7,509	7,638	7,690	7,726	7,769	8,017	8,023	8,089	8,202	8,534
Live Donors	2,535	2,851	3,009	3,377	3,649	3,912	4,361	4,552	5,324	5,974	6,233
Live Donor Transplants	2,535	2,850	3,007	3,370	3,638	3,906	4,359	4,547	5,315	5,969	6,236
Total Waitlist Patients	22,063	24,765	27,258	30,590	34,000	37,438	40,931	43,867	47,596	51,144	54,844
New Waitlist Registrations	15,224	16,090	16,538	17,903	18,328	19,067	20,191	20,986	22,269	22,349	23,494

couple (Rapaport [1986], Ross et al. [1997]). This pair of couples can then exchange donated kidneys. Compared to receiving cadaver kidneys at an unknown future time, this improves the welfare of the transplant recipients (and therefore accomplishes the aim of each patient's donor). In addition, it relieves the demand on the supply of cadaver kidneys, and thus potentially improves the welfare of those patients on the cadaver queue. A small number of these two-couple operations have been done, and the transplantation community has issued a consensus statement declaring them to be ethically acceptable (Abecassis et al. [2000]).

Another possibility is an *indirect exchange* involving an exchange between one immunologically infeasible patient-donor couple, and the cadaver queue (Ross and Woodle [2000]). In this kind of exchange, the patient in the couple becomes the highest priority person on the cadaver queue, in return for the donation of his donor's kidney to someone on the queue.[3] This improves the welfare of the patient in the couple, compared to having a long wait for a suitable cadaver kidney, and it clearly benefits the recipient of the live kidney, and others on the queue who benefit from the increase in kidney supply due to an additional living donor. However Ross and Woodle note that his may have a negative impact on type O patients already on the cadaver queue, an issue studied by Zenios, Woodle, and Ross [2001], to which we shall return.

In contrast to the system for cadaveric organs, and despite the growing interest in at least small scale exchanges involving living donors, there is no national system, or even an organized system at any level, for managing exchanges of kidneys from live

[3]Priority on the cadaver queue (which will be discussed later) is actually a bit complex, as queues are organized regionally, and actually consist of multiple queues, on which priority has to do with how well matched the available kidney is to each patient.

donors. However, individual hospitals are beginning to think about larger scale living donor exchanges. As this paper was being written, the first three-couple kidney transplant exchange in the United States was reported at Johns Hopkins Comprehensive Transplant Center in Baltimore, among three couples for whom no two-couple transfer was feasible (Olson, August 2, 2003).[4] In the present paper we will consider how such a system of exchanges might be organized, from the point of view of achieving efficiency, and providing consistent incentives to patients, donors, and doctors, and what it's welfare implications might be.[5]

Our proposals build on the literature of discrete allocation, starting from the "housing market" of Shapley and Scarf [1974]. They studied a model in which individuals are each endowed with an indivisible good (a "house") and have preferences over all the houses in the market. They considered a "top trading cycles" mechanism, proposed by David Gale, and observed that for any preferences it always exhausted the gains from trade and produced an outcome in the core of the market. It was subsequently shown that this outcome is unique when preferences are strict (Roth and Postlewaite [1977], and that the top trading cycles mechanism makes it a dominant strategy for every agent to state his true preferences (Roth [1982]). In what follows, it will sometimes be helpful to think of the traders and houses being analogous to transplant patients and their potential live donors. Of course we will have to be careful to remember that donors are not houses, they are agents with preferences, and any individually rational, incentive compatible mechanism for kidney exchange will have to accommodate their preferences also.[6]

Abdulkadiroğlu and Sönmez [1999] considered the more general problem of allocating dormitory rooms to students. In their model, some students already occupy a room, other (new) students do not have a room, and some rooms are unoccupied. They observed that the allocation schemes used in practice on many campuses do not achieve efficient allocations, because they do not allow students who already have a room to attempt to trade it for a room they prefer, without running the risk of receiving a less preferred room.

[4]Earlier multiple couple exchanges in Romania were reported by Lucan et al. [2003].

[5]While we know of no other research investigating the design of efficient live donor exchange as contemplated here, the management of the cadaver queue has inspired a substantial literature, including consideration of how it will interact with indirect exchanges. Some notable contributions are Votruba [2001], Wujciak and Opelz [1993, 1997], Zenios, Wein, and Chertow [1999] and, particularly, Zenios, Woodle and Ross [2001].

[6]It may not at first glance be obvious why strategic misrepresentation of preferences is a potentially serious problem in medical applications, since no one chooses to be ill. However existing cadaver queue procedures are frequently gamed, by physicians acting as advocates for their patients. Thus for example, on July 29, 2003 two Chicago hospitals settled a Federal lawsuit alleging that some patients had been fraudulently certified as sicker than they were to move them up on the liver transplant queue (Warmbir, 2003), while Scanlon et al. (2003) report evidence of similar "gaming" behavior on the waiting list for cadaver hearts.

Abdulkadiroğlu and Sönmez proposed a mechanism that generalizes the top trading cycles mechanism to this more general environment, and retains its desirable properties. In what follows it will be helpful to think of the students with rooms as analogous to transplant patients with potential live donors, the students without rooms as analogous to transplant patients who are on the cadaver waiting list, and the unoccupied rooms as analogous to cadaver kidneys. Once again, these analogies will not be precise. The biggest difference is that (unlike vacant rooms) cadaver kidneys cannot be placed in inventory and allocated at the same time as live donor kidneys. The supply of cadaver donors is a stochastic process (and each kidney must be transplanted within hours). But as in the case of campus housing, in order to achieve all possible gains from trade, we will have to consider not only trade among patients with donors, and not only allocation of cadaver kidneys to patients without donors, but also indirect exchanges that link the two groups of patients and sources of kidneys.

The design we propose here is intended to build on and complement the existing practices in kidney transplantation. In this respect and others it is in the modern tradition of engineering economics as applied to other problems of allocation, such as labor market clearinghouses (e.g. in medicine[7]), or auctions.[8]

This paper is organized as follows. Sections 2-4 give some necessary background on kidney disease and transplantation, including description of existing live kidney exchange programs. Section 5 reviews the most relevant economics literature on discrete allocation. Section 6 formally models the kidney exchange environment, and section 7 presents some closely related alternative designs for kidney exchange, and proves some theorems about their efficiency and incentive compatibility, which are extended in section 8 to the case in which patients may have multiple willing donors. These theoretical results make clear that there are potential welfare gains from considering such exchange, but they do not give any indication of the magnitude of such gains. For this we turn, in section 9, to the results of preliminary simulations, which suggest that the potential gains are substantial, both in increased number of feasible live-donor transplants, and in quality of kidney to patient match. Section 10 concludes. (All proofs are in the Appendix.)

2 Kidney Matching for Transplantation

End Stage Renal Disease (ESRD), also known as chronic kidney disorder, is a fatal disease unless treated with dialysis or kidney transplantation. Transplantation is the preferred

[7]Roth and Peranson [1999].

[8]See e.g. Roth [2002] and Milgrom [forthcoming] for discussions of recent practical experience in the design of mechanisms for exchange and allocation.

treatment because it enables the patients to resume normal activities.

Two genetic characteristics play key roles in the possibility and success of a kidney transplant. The first of these is the ABO blood-type: There are four blood types A, B, AB and O, representing which of two proteins, A and/or B are present. (Type O, pronounced "oh," stands for zero, i.e. neither of the proteins A or B is present.) In the absence of other complications, kidneys can be successfully transplanted if they do not contain blood proteins that are foreign to the patient. So type O kidneys can be transplanted into any patient; type A kidneys can be transplanted into type A or type AB patients; type B kidneys can be transplanted into type B or type AB patients; and type AB kidneys can only be transplanted into type AB patients.

The second genetic characteristic that affects the success of a kidney transplant is the tissue type, also known as human leukocyte antigens (HLA) type: HLA type is a combination of six proteins, two of type A, two of type B, and two of type DR. Empirical evidence shows that as the HLA mismatch between the donor and the recipient increases, the likelihood of graft survival decreases (see Opelz [1997]).[9] HLA plays another key role in transplantation through the pre-transplant "crossmatch" test. Prior to transplantation, the potential recipient is tested for the presence of preformed antibodies against HLA in the donor kidney. The presence of antibodies, also known as a *positive crossmatch*, significantly increases the likelihood of graft rejection by the recipient. Hence the transplantation is not carried out if there is a positive crossmatch.

3 The Supply of Kidneys for ESRD Patients

As we have already mentioned, kidney transplantation is the treatment of choice for patients suffering from ESRD. There are two main sources of kidneys for transplantation:

1. Cadaveric kidney donations, and

2. living kidney donations by families, spouses, and unrelated donors.

Since the gap between the demand and supply is large and increasing and since several thousand patients die every year waiting for transplantation, the importance of efficient and equitable allocation of donated kidneys is apparent. When a cadaveric kidney becomes available for transplantation, the priority of each patient on the waiting list is determined by a point system based on factors including the ABO blood-type, HLA antigen-match, time spent on the waiting list, the region the kidney is harvested, etc. and the kidney is offered to the patient with the highest priority. If the patient declines the offer, the kidney

[9] The number of HLA-mismatches is the number of donor HLA proteins that are absent in the recipient.

is offered to the patient with the second highest priority, and so on. The median waiting time for a cadaveric kidney exceeds three years and it is increasing. This, together with the superior survival rates of kidney grafts donated by living donors and the decreased risk for the donor (less than 2% major complications including a mortality rate of 3 in 10,000 transplants, see Ross et al. [1997]) have significantly increased the number of patient families, spouses and even friends who are eager to donate a kidney for their loved ones. However a significant number of donors are eliminated from consideration due to the incompatibility of the potential donor kidney with the intended recipient. The size of such incompatibilities has especially risen in recent years due to an increase in intended donations by non-blood related but emotionally related donors such as spouses and close friends. For example at the University of Chicago, between 10 and 20 percent of patients with available living donors cannot receive transplants from them because of ABO blood-type incompatibility (Ross et al. [1997]). In addition 15 percent of all potential donors are rejected based on a positive crossmatch and this rate is even higher between spouses (because wives can develop antibodies to their husbands' foreign proteins during pregnancies) (Ross and Woodle [2000]). Based on these difficulties, researchers in the transplantation community have been developing innovative kidney exchange programs to increase the utilization rate of intended donations from living donors.

4 Existing Kidney Exchange Programs

To minimize the elimination of physically eligible volunteer kidney donors on the basis of immunologic incompatibilities, Rapaport [1986] proposed the creation of a living donor pool so that kidneys obtained from the pool of intrafamilial incompatible donors can be used on an interfamilial basis. Rapaport states

> "The continued elimination of so many physically eligible and willing volunteer kidney donors on the basis of immunologic grounds is no longer acceptable, in view of the current acute need for organs. One feasible approach to this problem could be an international living-related donor and recipient exchange, whereby kidneys obtained from such a pool of intrafamilial incompatible donors would used on an interfamilial basis, following the same moral and ethical guidelines and medical criteria that currently govern living-related intrafamilial kidney donor transplantation. Such an international registry would permit the recipients with ESRD in each family, and their related volunteer donors who are incompatible with them, to be placed on a central organ-sharing registry, which would implement a direct exchange of kidneys between two given families, where the donor A, who is not compatible with

his own recipient A, but is compatible with recipient B from family B, would be the source of a kidney for recipient B, while donor B, who is incompatible with his own recipient B, but is compatible with recipient A, would give a kidney to recipient A."

Building on Rapaport's [1986] proposal, Ross et al. [1997] again proposed to increase the supply of living kidney donations by using kidneys from living ABO-incompatible donors through an exchange arrangement between two living kidney donor-recipient pairs. In year 2000, UNOS initiated pilot testing of the Rapaport [1986]-Ross et al. [1997] proposal, currently known as the *paired-kidney-exchange program*. The same year the transplantation community issued a consensus statement indicating that paired-kidney-exchange program is considered to be "ethically acceptable" (Abecassis et al. [2000]).

Ross et al. [1997] initially proposed paired-exchange among donor-recipient pairs who are ABO blood-type incompatible. Terasaki, Gjertson and Cecka [1998] indicate that while 36% of all non-blood related donor-recipient pairs are ABO-incompatible, only a sixth of this rate, i.e. 6% of all non-blood related donor recipient pairs can potentially benefit from such an exchange. This is because, the majority of ABO-incompatible donor-recipient pairs have an O type recipient (who is in need of an O type kidney), and these pairs never benefit from a paired exchange, since an O type *donor* is never ABO-incompatible with his intended recipient. Hence the only potential pairs are blood type A and B mismatches. Ross and Woodle [2000] indicate that positive crossmatches between potential donor and recipients will add to the pool of incompatible pairs that are eligible for paired-kidney-exchange programs and also propose additional kidney exchange programs that have potentially great clinical relevance. One of the exchange programs they have proposed is the *indirect exchange program*: Under this program a potential donor who is immunologically incompatible with his potential recipient donates his kidney to the cadaveric waiting list and his paired recipient will receive priority for the next ABO-compatible crossmatch-negative cadaveric kidney. Ross and Woodle carefully differentiate between two types of indirect exchanges depending on the nature of the incompatibility between the pair:

1. The incompatibility may be due to a positive crossmatch, or

2. it may be due to ABO blood-type incompatibility.

The welfare implications of the two cases are quite different, since in the first case an indirect exchange can create a Pareto improvement, while in the second an exchange that helps some patients may impose a negative externality on others . Ross and Woodle [2000] state:

> "One type of living-cadaveric donor exchange involves a living donor and recipient who are ABO compatible but have a positive crossmatch. The person on the waiting list who is offered the living kidney is the person who would otherwise receive the cadaveric kidney. For this person, the choice is between a cadaveric kidney or a living kidney. The decision of the potential recipient on the waiting list who is offered the choice between a cadaveric or a living kidney is relatively simple: given the greater likelihood of graft survival, it is rational and reasonable that the individual will accept the exchange.
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> The more likely indirect exchange, however, involves an ABO-incompatible donor recipient pair such that the person who would receive the living donor's kidney is not the individual who would get the cadaveric kidney now to be transplanted into the paired recipient. This is because the most likely living donor who cannot donate directly will be an A, B, or AB donor who cannot donate to the potential O recipient. As such, the individual on the waiting list who benefits from the exchange will have the same blood type as the living donor and individuals on the waiting list with the same blood type as the paired recipient may have longer waiting times because the paired recipient gets priority for the next cadaveric kidney of that blood type. The problem is that although the number of kidneys available for individuals on the cadaveric waiting list could be numerically increased by ABO-incompatible indirect exchanges, and the quality of kidneys for cadaveric waiting list recipients improved, changes in beneficiaries will occur."

There is widespread agreement in the transplantation community that indirect ABO-incompatible exchange can harm those O blood-type patients who have no opportunity to receive a live donation. This is because,

- on the one hand they will be losing their priority to O blood-type patients whose ABO-incompatible donors donate to the cadaveric pool, and
- on the other hand very few O blood-type living kidneys will be offered to their pool since an O blood-type donor can directly donate to his intended recipient unless there is a positive crossmatch.

Despite this widespread concern, many transplant centers have started pilot indirect exchange programs since 2000. Some examples include Johns Hopkins Comprehensive

Transplant Center, Washington Regional Voluntary Living Donor Program, Center for Organ Recovery and Education (Pittsburgh), and New England Medical Center (Boston).

The present paper explores the design and potential benefits of a more comprehensive kidney exchange mechanism that will

1. allow not only for paired-kidney-exchange and indirect exchange but also many other forms of exchange such as a three-way exchange, more elaborate forms of indirect exchange, etc.,

2. benefit not only the incompatible pairs but also the compatible ones,

3. eliminate or at least reduce the adverse affect to O blood-type patients with no living donors, and

4. organize the live kidney exchange in a way that is compatible with the existing system of cadaveric kidney allocation.

As mentioned in the introduction, we will extend recent developments in the mechanism design literature, motivated by housing allocation problems. We first quickly review these.

5 House Allocation

Shapley and Scarf modeled a "housing market" as consisting of n agents each of whom is endowed with an indivisible good, a "house." Each agent has preferences over all the houses (including his own), and there is no money in the market, trade is feasible only in houses. They attribute to David Gale the "top trading cycle" algorithm that, for any preferences, produces an allocation of houses to agents in the core of the market. The algorithm can be described as follows: Each agent points to her most preferred house (and each house points to its owner). Since the number of agents is finite and since each house has an owner, there is at least one cycle in the resulting directed graph. In each such cycle, the corresponding trades are carried out, i.e. each agent in the cycle receives the house she is pointing to, and these agents and houses are removed from the market. (A cycle may of course consist of an agent pointing to her own house.) The process continues (with each agent pointing to her most preferred house that remains on the market) until no agents and houses remain, and the final allocation is the one in which each agent receives the house with which she left the market.

The proof that this allocation is in the core is straightforward: no blocking coalition can contain members of the first cycles to leave the market, since those agents each already received their first choice house. But then no blocking coalition can form containing

members of the second set of cycles to leave the market, since the only houses they might prefer to those they received are those that left in the first set of cycles, and the owners of these houses are not in any blocking coalition. Hence there exists no coalition of agents who could have traded among themselves to achieve an outcome they prefer. It can further be shown that, when the method of top trading cycles is used to allocate houses, it is a dominant strategy for each agent to reveal his true preferences (Roth [1982]). And when all preferences are strict, the procedure yields a unique outcome (Roth and Postlewaite [1977]). [10]

Note that paired kidney exchanges similarly seek the gains from trade among patients with willing donors, but (with the recent Johns Hopkins 3-pair exchange being a notable exception) mostly among just two pairs. In the kidney exchange to be considered below, we will argue that, if we consider exchange only among patients with donors, the properties of the housing market model essentially carry over unchanged, if we assume that donors' preferences are aligned with those of their intended recipient (e.g. if both patients and their donors are primarily concerned with the probability that the intended recipient will have a successful transplant). We will also assume, as is the current practice, that all surgeries in a given cycle are carried out simultaneously, since a donor's willingness to donate a kidney might change once her intended recipient has received a successful transplant. Of course, some of the desirable incentive and efficiency properties would be lost if the cycle size were restricted. (In practice, there will be limitations on the number of simultaneous surgeries that can be conducted, and so we will keep track of cycle sizes in the simulations we report.)

However the kidney transplant environment consists not just of patients with donors, but also patients without donors, and cadaver kidneys that are not tied to any specific patient. Abdulkadiroğlu and Sönmez [1999] studied the housing allocation problems on college campuses, which are in some respects similar. In particular, Abdulkadiroğlu and Sönmez [1999] analyze what they call *house allocation problems with existing tenants*: A set of houses must be allocated to a set of students by a centralized housing office. Some of the students are existing tenants each of whom already occupies a house and the rest of the students are newcomers. In addition to occupied houses, there are vacant houses. Existing tenants are not only entitled to keep their current houses but also apply for other houses.

The mechanism known as *random serial-dictatorship with squatting rights* (RSD-SR)

[10] Abdulkadiroğlu and Sönmez [1998] consider a housing allocation model in which the houses are not initially owned, and prove the surprising result that the same probability distribution over the set of efficient allocations can be achieved by either of two allocations methods: either randomly assign the houses and find the unique core outcome, or randomly order the agents and let them choose sequentially from the stock of houses.

is common in housing allocation on college campuses.[11] This mechanism works as follows:

1. Each existing tenant decides whether he will enter the housing lottery or keep his current house. Those who prefer keeping their houses are assigned their houses. All other houses become available for re-allocation together with the vacant houses.

2. An ordering of participating students (i.e. newcomers and existing tenants who decide to go ahead with the lottery) is randomly chosen.

3. Once the agents are ordered, available houses are allocated using the induced *serial dictatorship*: The first student is assigned his top choice, the next student is assigned his top choice among the remaining houses, and so on.

While this mechanism is common in applications, it suffers from a major deficiency. Since it does not guarantee each existing tenant a house that is as good as his own, some existing tenants may choose to keep their houses even though they wish to move, and this may result in loss of potentially large gains from trade.[12] In the context of kidney transplantation, the counterpart of an existing tenant and his current house is a recipient-donor pair. Just as an occupied house may be unavailable for re-allocation unless its current owner is assured a better house, a potential donor will in most cases be unavailable for donation unless his intended recipient is assured a better kidney. The apparent similarity between the two problems suggests that the solution offered in the context of on-campus housing may be of use in the context of kidney transplantation as well.

The inefficiency of the RSD-SR is caused by the mechanism's failure to guarantee each existing tenant a house at least as good as the one he already holds. To achieve efficiency this deficiency needs to be addressed.[13] This is the motivation for the generalization of the top trading cycles (TTC) mechanism proposed by Abdulkadiroğlu and Sönmez [1999], which they called *you request my house-I get your turn* (YRMH-IGYT):

1. Each student (existing tenant or newcomer) reports his strict preferences over all houses.

2. An ordering of agents is randomly chosen.

[11] Some examples include undergraduate housing at Carnegie-Mellon, Duke, Michigan, Northwestern and Pennsylvania.

[12] See Chen and Sönmez [2002] for experimental evidence of this inefficiency.

[13] Formally speaking, one must incorporate individual rationality constraints into an implementation problem to assure voluntary participation. See Jackson and Palfrey [2001] for a general treatment of "voluntary implementation."

3. For any given preference list and ordering, the outcome is obtained using the following (YRMH-IGYT) algorithm:

   (a) Assign the first student his top choice, the second student his top choice among the remaining houses, and so on, until someone requests the house of an existing tenant.

   (b) If at that point the existing tenant whose house is requested is already assigned another house, then do not disturb the procedure. Otherwise *modify the remainder of the ordering by moving the existing tenant to the beginning of the line* and proceed with the procedure.

   (c) Similarly, move any existing tenant who has not already been assigned a house to the beginning of the line once his house is requested.

   (d) If at any point a cycle forms, it is formed by exclusively existing tenants and each of them requests the house of the tenant who is next in the cycle. (A *cycle* is an ordered list of students $(i_1, i_2, \ldots, i_k)$ where student i_1 requests the house of student i_2, student i_2 requests the house of student i_3,..., student i_k requests the house of student i_1.) In such cases remove all students in the cycle by assigning them the houses they request and proceed with the procedure.

The key innovation in the YRMH-IGYT mechanism is that an existing tenant whose current house is requested is upgraded to the first place in the line of agents remaining unassigned, before his house is allocated. As a result the YRMH-IGYT mechanism assures every existing tenant a house that is at least as good as his own (since the tenant is always free to point to his own house if all houses he prefers have already left the market). Therefore existing tenants do not have any reason to hesitate entering the market and consequently the eventual house allocation is Pareto efficient. Note that *the idea of upgrading an existing tenant whose current house is requested to the top of the line was also invented by the transplantation community in the form of an indirect exchange program*: When a potential donor donates his kidney to the highest priority patient in the waiting list, his intended recipient is upgraded to the top of the waiting list. Hence an indirect exchange is one of several possible types of exchanges that can be realized under the YRMH-IGYT mechanism, in addition to the more straightforward exchanges among couples each of whom has a donor.

By way of analogy, note once again that what prompted the need for the introduction of simple kidney exchange programs by researchers and clinicians in the transplantation community was the loss of many physically eligible volunteer kidney donors because of immunologic incompatibilities. Under these exchange programs, a potential donor

who is incompatible with his intended recipient is given the incentive to go ahead with the donation, because his donation makes it possible for his intended recipient to be transplanted a compatible kidney. Similarly, the potential efficiency loss in the campus housing problem is that some rooms might fail to be traded, even when welfare enhancing trades are possible. The YRMH-IGYT is an attempt to address that problem in the housing context. In the following sections, we will consider how it must be adapted to the problems of kidney exchange, and study its properties in the context of kidney transplantation.

6 Kidney Exchange Problem

While there are clear similarities between house allocation and kidney exchange, there are also important differences.

As already mentioned, the counterpart of an existing tenant and his house is a donor-recipient pair in the context of kidney exchange. We denote a donor-recipient pair by (k_i, t_i). We will often refer to donor k_i as kidney k_i, and recipient t_i as patient t_i.

In the context of house allocation with existing tenants, there are newcomers none of whom owns a specific house and vacant houses none of which is owned by a specific student. The counterpart of newcomers in the context of kidney exchange are patients who have no living donors and the counterpart of vacant houses are cadaveric kidneys which are not targeted for specific patients. This analogy reveals one important difference between the two models: The house allocation model is static and in particular the set of vacant houses is fixed. The kidney exchange problem, on the other hand, is dynamic and in particular it is not clear which cadaveric kidneys will be available, when they will be available, etc. *Therefore while occupied houses as well as vacant houses are jointly allocated under the YRMH-IGYT mechanism, this is not possible in the context of kidney exchange*. Instead, patients with live donors who are not themselves allocated a live donor kidney will be assigned to the cadaver queue (with a priority reflecting whether their donor's kidney was donated to someone on the queue).

Let K denote the set of living donor kidneys at a specific point in time. While patients and their doctors may define their preferences over possible kidneys as they wish, we will consider here, for specificity, the preferences that come from maximizing the probability of a successful transplant. Given any patient t, part of K is outside the consumption set due to ABO blood-type incompatibility or a positive crossmatch. Among feasible kidneys, HLA match plays a significant role in the graft survival. Using data from 1,585 transplants from non-spousal non-blood related living donors from 1986 to 1995 at 198 transplant centers, Opelz [1997] shows that the 3-year graft survival rate is 87±2% when

there is no HLA-DR mismatch between the recipient and the donor, the same rate is 81±2% when there is 1 HLA-DR mismatch, and it is 75±4% when there are 2 HLA-DR mismatches.[14] Therefore the graft failure rate in three years almost doubles from 13% to 25% when the HLA-DR mismatch increases from 0 to 2. These results are statistically highly significant. There are similar conclusive results correlating overall HLA match with graft survival (see for example Opelz [1997], Opelz [1998]). It is also known that as the donor age increases, the rate of graft survival first increases and then decreases. Therefore *patients have heterogenous preferences over compatible kidneys although there are correlations based on age and the health of the donor*. So, although current pilot kidney exchange programs do not rely on patient preferences, there are very good medical reasons to do so. Indeed Opelz [1997] states:

> "A potential implication of the results presented herein is that the transplantation of kidneys from unrelated live donors should be done more selectively so that poor HLA matches may be avoided. One could speculate that an exchange of live-donor organs might be advantageous in certain situations. For example, if a potential live donor were willing to donate a kidney to a recipient with whom he/she was a poor HLA match, that kidney could conceivably be given to another patient whose HLA antigen profile closely matched that of the donor. In return, the recipient who did not receive the designated live-donor kidney would be given priority for receiving another well-matched live-donor graft. Similar exchanges of live-donor kidneys for the purpose of obtaining ABO compatibility were proposed by Rapaport [1986] and recently again by Ross et al. [1997]."

In what follows we will consider all preferences to be strict. One appealing possibility is breaking the indifferences based on the time spent in the system, parallel to the tie-breaking rule for UNOS cadaveric kidney priority mechanism.

If only direct exchanges among donor-recipient pairs are considered, one can directly use Gale's Top Trading Cycles mechanism. However this will not allow for indirect exchanges. Other things being equal, graft survival rates of kidneys from living donors is superior to the graft survival rates of cadaveric kidneys. However a cadaveric kidney with good HLA match may be preferred by some patients to a living donor kidney with poor HLA match. Opelz [1997] states:

> "Even well-matched cadaver donor kidneys would provide an advantage over those from poorly matched live donors. For instance, for the study period from

[14] Each of these rates is 2% lower when spousal living donors are considered.

1986 to 1995, the Collaborative Transplant Study shows a 5-year graft survival rate of 77±1% for 2392 cadaver kidney grafts from donors below the age of 50 with no HLA-A, -B,-DR mismatch. This success rate compares favorably with that of poorly matched grafts from unrelated live donors."

In a very recent article, Mandal et al. [2003] provide a more detailed comparison of graft survival rates for cadaveric kidneys and living donor kidneys. They show that as patient age increases, it gets harder for cadaveric kidneys to outperform living donor kidneys and for patients older than 60 years of age cadaveric kidneys never outperform living donor kidneys based on graft survival rates.

Since the supply of specific cadaveric donor kidneys is not predictable, patients do not know which cadaveric kidneys will be available and when they will be available. Therefore a patient who wishes to trade his donor's kidney in return for a priority in the cadaveric kidney waiting list is receiving a *lottery* instead of a specific kidney. Taking this into consideration the patient, his doctor, and his donor shall decide whether this option is acceptable and if so, where it ranks in patient's preferences. Based on Mandal et al. [2003], this option will be increasingly less appealing as the patient's age increases.

Given a patient t_i, let $K_i \subset K$ denote the set of living donor kidneys that are compatible with patient t_i. Note that kidney k_i may or may not be in K_i. Let w denote the option of entering the waiting list with priority reflecting the donation of his donor's kidney k_i, and P_i denote his strict preferences over $K_i \cup \{k_i, w\}$. For our purposes the relevant part of P_i is the ranking up to kidney k_i or w, whichever ranks higher. If patient t_i ranks kidney k_i at the top of his preferences that means he does not wish to participate in an exchange. If patient t_i ranks k_i on top of w that means he (or his donor) does not consider exchanging kidney k_i with a priority in the cadaveric kidney waiting list.

We are finally ready to formalize a (static) *kidney exchange problem*. A kidney exchange problem consists of

1. a set of donor-recipient pairs $\{(k_1, t_1), \ldots, (k_n, t_n)\}$,
2. a set of compatible kidneys $K_i \subset K = \{k_1, \ldots, k_n\}$ for each patient t_i, and
3. a strict preference relation P_i over $K_i \cup \{k_i, w\}$ for each patient t_i.

Let R_i denote the weak preference relation associated with the strict preference relation P_i. Note that, since the preferences are strict, given two kidneys k, k' if kR_ik' and $k \neq k'$ then kP_ik'.

The outcome of a kidney exchange problem is a *matching* μ of kidneys/waitlist option to patients such that

1. each patient t_i is either assigned a kidney in $K_i \cup \{k_i\}$ or the waitlist option w, and

2. no kidney can be assigned to more than one patient although the waitlist option w can be assigned to several patients. (This latter possibility makes the kidney exchange problem different from the housing problems formulated above.)

Let $\mu(t_i)$ denote the assignment of patient t_i under matching μ.

Patients who have no living donors, but are on the queue for cadaver kidneys, are not explicitly included as agents in the kidney exchange problem as we model it here, although they will be affected by the kidney exchange mechanism we introduce. The main source of kidney grafts is cadaveric donors for these patients although, as in the case of an indirect exchange program, occasionally cadaveric kidneys will be traded with living donor kidneys. On the other hand, each patient who is transplanted a kidney through the exchange mechanism is dropped from the waiting list for cadaveric kidneys and therefore it is also benefitting patients with no living donors. (We will discuss this in more detail when we report simulation results.)

We also do not explicitly model the priority rules by which the cadaveric queue is managed, although these will affect the desirability of joining the queue, with and without the high priority that is obtained by donating a kidney to someone in the queue. That is, we consider the cadaver queue to be a separate process, whose effect on the kidney exchange among live donors will be reflected in where each agent ranks the alternative "w" compared to other alternatives. (A patient who ranks his own donor higher than w may be choosing to enter the cadaver queue without special priority, when his own donor has a kidney that is infeasible for him.)

7 Top Trading Cycles and Chains Mechanism for Kidney Exchange

A *kidney exchange mechanism* is a systematic procedure to select a matching for each kidney exchange problem. We are almost ready to introduce the *Top Trading Cycles and Chains (TTCC)* mechanism, a variant of TTC mechanisms, for kidney exchange. Before that we give a few definitions and observations to facilitate the description of the mechanism.

7.1 Cycles and w-Chains

The mechanism relies on an algorithm consisting of several rounds (no more than the number of the agents). In each round

- each patient t_i points either towards a kidney in $K_i \cup \{k_i\}$ or towards w, and
- each kidney k_i points to its paired recipient t_i.

A *cycle* is an ordered list of kidneys and patients $(k'_1, t'_1, k'_2, t'_2, \ldots, k'_m, t'_m)$ such that

- kidney k'_1 points to patient t'_1,
- patient t'_1 points to kidney k'_2,

$$\vdots \qquad\qquad \vdots$$

- kidney k'_m points to patient t'_m, and
- patient t'_m points to kidney k'_1.

Cycles larger than a single pair are associated with direct exchanges, very much like the paired-kidney-exchange programs, but may involve more than two pairs. Whenever a cycle is formed the corresponding trade is carried out, so that

- patient t'_1 is assigned kidney k'_2,
- patient t'_2 is assigned kidney k'_3,

$$\vdots \qquad\qquad \vdots$$

- patient t'_m is assigned kidney k'_1.

Note that each cycle is of even size and in our algorithm each kidney or patient can be part of at most one cycle. In other words *no two cycles can intersect*. That is because, each patient points to a unique kidney/waitlist option and each kidney points to a unique patient.

A *w-chain* is an ordered list of kidneys and patients $(k'_1, t'_1, k'_2, t'_2, \ldots, k'_m, t'_m)$ starting with a kidney, ending with a patient such that

- kidney k'_1 points to patient t'_1,
- patient t'_1 points to kidney k'_2,

$$\vdots \qquad\qquad \vdots$$

- kidney k'_m points to patient t'_m, and
- patient t'_m points to w.

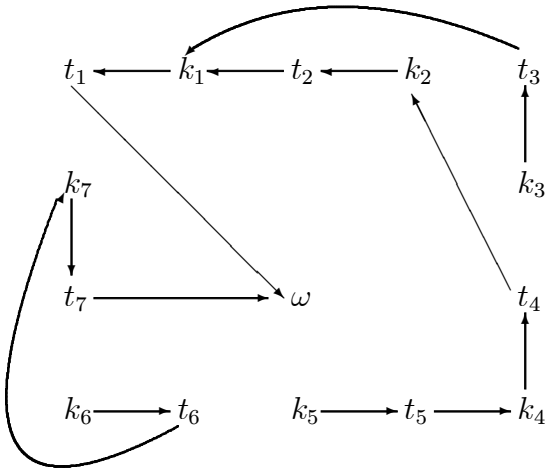

Figure 1: Example 1

We refer the last pair (k'_m, t'_m) in a w-chain $(k'_1, t'_1, k'_2, t'_2, \ldots, k'_m, t'_m)$ as the *head* and the first pair (k'_1, t'_1) as the *tail* of the w-chain. A w-chain is also of even size but, unlike in a cycle, a kidney or a patient can be part of several w-chains. In particular, note that, whenever $(k'_1, t'_1, k'_2, t'_2, \ldots, k'_m, t'_m)$ is a w-chain, $(k'_2, t'_2, \ldots, k'_m, t'_m)$ is a w-chain as well. This motivates the definition of a *maximal w-chain*. A w-chain $(k'_1, t'_1, \ldots, k'_m, t'_m)$ is maximal if there is no donor-recipient pair (k'_0, t'_0) such that $(k'_0, t'_0, k'_1, t'_1, \ldots, k'_m, t'_m)$ is also a w-chain.

It is easy to see that w-chains are associated with indirect exchanges. The current pilot indirect exchange programs in the U.S. use only *minimal* w-chains that consist of a single donor-recipient pair.

The notion of a maximal w-chain does not resolve the potential conflict between w-chains; a kidney or a patient may even be part of several maximal w-chains. Therefore the selection process of w-chains in the algorithm is not as straightforward as cycles.

Example 1: Consider the patient-kidney graph in Figure 1 with 7 pairs $(k_1, t_1), \ldots, (k_7, t_7)$:

In this graph, each kidney-patient pair initiates (i.e. is the tail of) a w-chain:

$$W_1 = (k_1, t_1), W_2 = (k_2, t_2, k_1, t_1), W_3 = (k_3, t_3, k_1, t_1), W_4 = (k_4, t_4, k_2, t_2, k_1, t_1),$$
$$W_5 = (k_5, t_5, k_4, t_4, k_2, t_2, k_1, t_1), \; W_6 = (k_6, t_6, k_7, t_7), \text{ and } W_7 = (k_7, t_7).$$

There are two minimal w-chains W_1 and W_7: All other w-chains contain either W_1 or W_7. There are three maximal w-chains W_3 (which contains W_1), W_5 (which contains W_1,W_2, and W_4) and, W_6 (which contains W_7). Note that w-chains W_1,W_2,W_3, W_4, and W_5 intersect, since they all contain W_1. Similarly w-chains W_6 and W_7 intersect since they both contain W_7. □

One practical possibility is choosing among w-chains with a well-defined *chain selection rule*, very much like the rules that establish priorities on the cadaveric waiting list. Existing indirect exchange programs choose the minimal w-chains, but this may not be efficient. Selection of longer w-chains will benefit other patients as well and therefore the choice of a chains selection rule has efficiency implications (see Theorem 1). Chain selection rules may be also used for specific policy objectives such as increasing the inflow of blood-type O living donor kidneys to the cadaveric waiting list.

Whenever w-chain $(k'_1, t'_1, \ldots, k'_m, t'_m)$ is selected in the algorithm, the following elaborate indirect exchange will be carried out:

- patient t'_1 is assigned kidney k'_2,
- patient t'_2 is assigned kidney k'_3,

  $\vdots \qquad\qquad \vdots$
- patient t'_{m-1} is assigned kidney k'_m,
- patient t'_m receives high priority for the next compatible kidney in the cadaveric waiting list, and
- kidney k'_1 is either offered to the cadaveric waiting list or to another patient with a paired donor.

The following lemma will be a key element of our algorithm.

Lemma 1 *Consider a graph in which both the patient and the kidney of each pair are distinct nodes as is the waitlist option w. Suppose each patient points either towards a kidney or w, and each kidney points to its paired recipient.*[15] *Then either there exists a cycle or each pair initiates a w-chain.*

[15]Using graph theory terminology, there is a *directed link* from each patient to a kidney or w, and a directed link from each kidney to its paired recipient.

We are finally ready to introduce the *top trading cycles and chains mechanism.*

Because the exchange mechanism interacts with many parts of the kidney transplant environment, it will clarify the discussion which follows to start by indicating which parts of the environment we take as fixed for our present purpose.

First, we take the operation of the cadaver queue as fixed, both in how it gives priority to candidates without donors, and (higher priority) to candidates who come to the queue from the kidney exchange mechanism and whose donor makes a donation to someone on the queue. The cadaver queue can be thought of as a stochastic arrival process of cadavers and patients, interacting with a scoring rule that determines which patients are offered which cadaver kidneys.

We also take as given the size of the live kidney exchange, i.e. the set of patient-donor pairs is taken to be fixed. In practice, the set of patient-donor pairs will grow as the geographic area served by the kidney exchange is increased, or as the time between exchanges is increased. A larger pool of possible exchanges will increase the potential efficiency gains that can be realized by exchange, but will also increase the size of the trading cycles that might be needed to achieve these efficiencies. We will keep track of both of these when we report simulations.

Both the operation of the cadaver queue, and the frequency and scope of the kidney exchange will influence patients' "reservation utility," i.e. how they compare various opportunities for direct or indirect exchange to the option of not making any exchange now, but waiting for a future opportunity. Patients can express this reservation utility by where they put their own donor in their preferences.

7.2 The Exchange Mechanism

For the mechanism defined below, we assume that when one among multiple chains must be selected, a fixed chain selection rule is invoked to make the choice. We will consider a number of such chain selection rules, and their implications for incentives, efficiency, and equity.

Throughout the procedure kidneys are assigned to patients through a series of exchanges. Some patients and their assigned kidneys will be immediately removed from the procedure and play no further role in exchange, while others will remain with their assignments but they will assume a *passive* role. So at any given point in the procedure, some agents may no longer be participants, while of the remaining participants, some will be active and the others passive.

At a given time and for a given kidney exchange problem $\langle \{(k_i, t_i)\}_{i=1}^n, (K_i)_{i=1}^n, (P_i)_{i=1}^n \rangle$, the TTCC mechanism determines the exchanges as follows:

1. Initially all kidneys are *available* and all agents are active. At each stage of the procedure

   - each remaining *active* patient t_i points to the best remaining unassigned kidney or to the waitlist option w, whichever is more preferred, based on his preferences P_i,
   - each remaining passive patient continues to point to his assignment, and
   - each remaining kidney k_i points to its paired recipient t_i.

2. *By Lemma 1, there is either a cycle, or a w-chain, or both.* By definition, a cycle can neither intersect with another cycle nor with a w-chain.

   (a) Proceed to Step 3 if there are no cycles. Otherwise locate each cycle and carry out the corresponding exchange. Remove all patients in a cycle together with their assignments. Note that patient t_i is removed if and only if kidney k_i is removed.

   (b) Each remaining patient points to its top choice among remaining kidneys and each kidney points to its paired recipient. By Lemma 1, there is either a cycle, or a w-chain, or both. Proceed to Step 3 if there are no cycles. Otherwise locate all cycles, carry out the corresponding exchanges, and remove them.

   (c) Repeat Step 2b until no cycle exists.

3. If there are no pairs left, then we are done. Otherwise by Lemma 1, each remaining pair initiates a w-chain. Some of these w-chains may intersect and others may not.

   (a) If each remaining w-chain is minimal, then each remaining patient points to the wait list option w. In this case carry our the basic indirect exchanges and we are done.

   (b) Otherwise select *only one* of the chains with the *chain selection rule*. The assignment is *final* for the patients in the selected w-chain. In addition to selecting a w-chain, the chain selection rule also determines

      i. whether the selected w-chain is removed and the associated exchange is immediately carried out, or

      ii. the selected w-chain remains in the procedure although each patient in it is passive henceforth.

The relevance of the last point is the following: Whenever a w-chain $(k'_1, t'_1, \ldots, k'_m, t'_m)$ is selected, even though the assignments of all patients in the w-chain are finalized, the kidney k'_1 at the tail of the w-chain can be utilized in two possible ways: It can be immediately offered to the waiting list (in which case the w-chain shall be removed) or it may be made available for the remaining patients as the process continues and hence *the selected w-chain may possibly grow later on*, although the patients already in it are not affected.

4. Each time a w-chain is selected, a new series of cycles may form. Repeat Steps 2 and 3 with the remaining active patients and unassigned kidneys until no patient is left.

At the end of the procedure, each patient with a living donor is assigned a kidney (or a high priority place on the waiting list). However that does not necessarily mean each of these patients receives a transplant. In particular cycles (k_i, t_i) that consist of a single patient-donor pair (i.e. a cycle in which the patient points to his own donor) may mean a number of things:

1. The patient t_i may be transplanted the kidney of his paired donor k_i if there is no incompatibility.

2. The cycle (k_i, t_i) may form "late" in the algorithm as a result of the lack of demand on kidney k_i and thus all preferred kidneys might have been assigned to other patients via direct or indirect exchanges. If kidney k_i is not a good match for patient t_i, he can either wait for other pairs to enter the system or consider an indirect exchange and rank the cadaveric waitlist option w before kidney k_i. (Note that this cannot be the case under the preference P_i for otherwise patient t_i would be at the head of a chain).

   It is also possible that a patient t_i with donor k_i, who is in no rush, might have a high reservation utility and have ranked k_i high with the hope of trading it with a high quality living donor kidney the next time the exchange is run, after new donors have entered the system.

We next give a detailed example in order to illustrate the dynamics of the TTCC mechanism.

Example 2: Consider a kidney exchange problem with 12 pairs $(k_1, t_1), \ldots, (k_{12}, t_{12})$ where

- patients t_1, t_{10} each has blood-type AB,
- patients t_3, t_5, t_8 each has blood-type A,
- patients t_4,t_7, t_{12} each has blood-type B,
- patients t_2, t_6, t_9, t_{11} each has blood-type O, and
- donors k_2, k_4, k_7, k_{12} each has blood-type A,
- donors k_1, k_9, k_{10} each has blood-type B,
- donors k_3,k_5,k_6,k_8, k_{11} each has blood-type O.

Note that pairs (k_2, t_2), (k_4, t_4), (k_7, t_7), (k_9, t_9), (k_{12}, t_{12}) are ABO-incompatible. Preferences of patients over compatible kidneys and the waitlist option are given as follows:

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
k_9	k_{11}	k_2	k_5	k_3	k_3	k_6	k_6	k_3	k_{11}	k_3	k_{11}
k_{10}	k_3	k_4	k_9	k_7	k_5	k_{11}	k_4	k_{11}	k_1	k_6	k_3
k_1	k_5	k_5	k_1	k_{11}	k_8	k_1	k_{11}	ω	k_4	k_5	k_5
	k_6	k_6	k_8	k_4	k_6	k_3	k_2		k_5	k_{11}	k_9
	k_2	k_7	k_{10}	k_5		k_9	k_3		k_6		k_8
		k_8	k_3			k_{10}	k_8		k_7		k_{10}
		k_{11}	k_6			k_1			k_2		k_{12}
		k_{12}	ω			ω			ω		
		ω									

Suppose that patients are ordered in a priority-list based on their indices starting with the patient with the smallest index. We will implement the TTCC mechanism using the following chain selection rule: Choose the longest w-chain. In case the longest w-chain is not unique, choose the w-chain with the highest priority patient; if the highest priority patient is part of more than one, choose the w-chain with the second highest priority patient, and so on. Keep the selected w-chains until the termination.

We next execute the TTCC mechanism:

Round 1. There is a single cycle $C_1 = (k_{11}, t_{11}, k_3, t_3, k_2, t_2)$. Remove the cycle by assigning k_{11} to t_2, k_3 to t_{11}, and k_2 to t_3.

[Figure 2]

Round 2. Upon removing cycle C_1, a new cycle $C_2 = (k_7, t_7, k_6, t_6, k_5, t_5)$ forms. Remove it by assigning k_7 to t_5, k_6 to t_7, and k_5 to t_6.

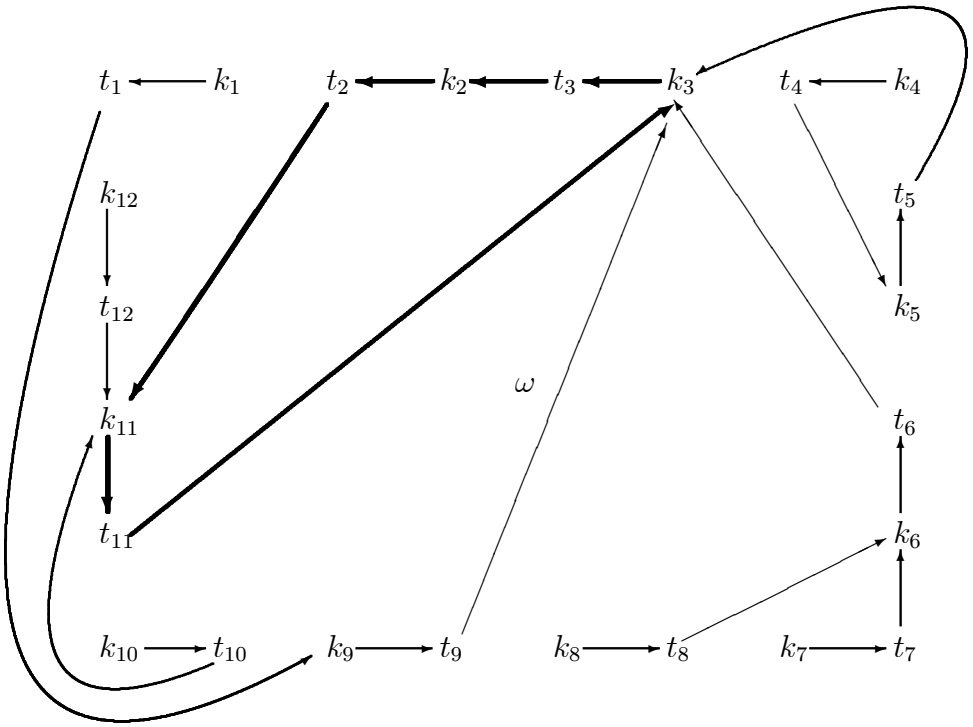

Figure 2: Example 2, Round 1

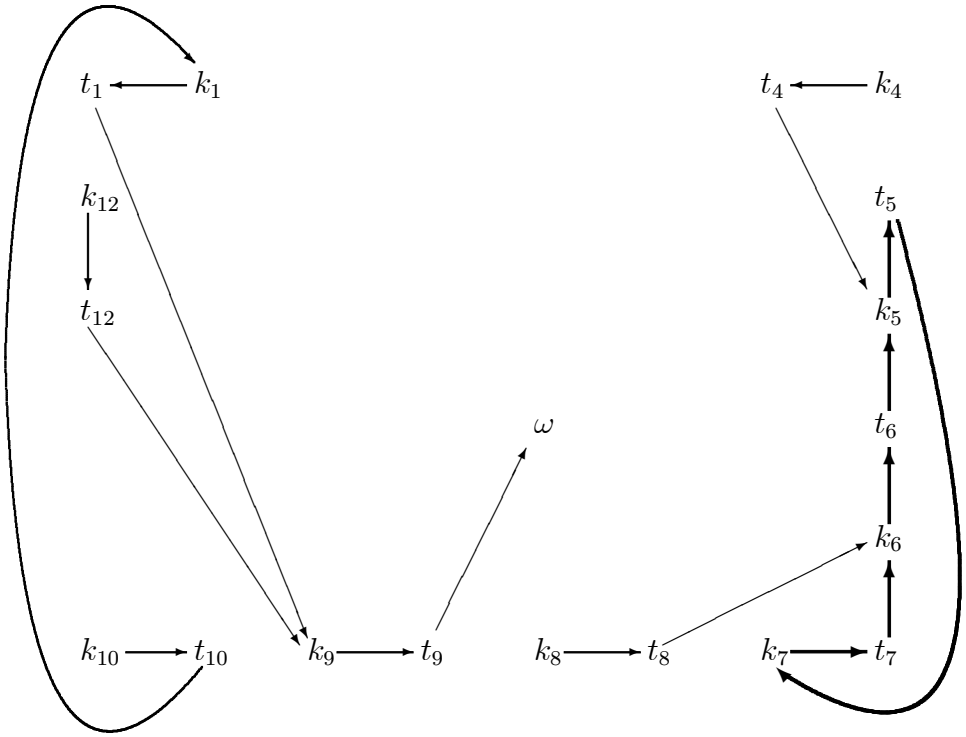

Figure 3: Example 2, Round 2

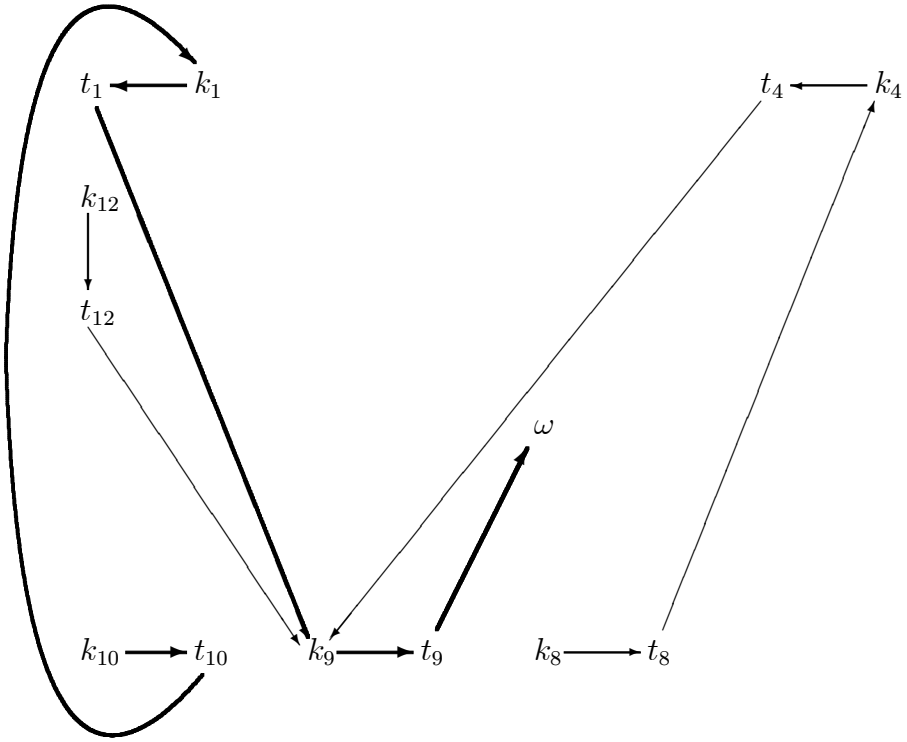

Figure 4: Example 2, Round 3

[Figure 3]

Round 3. No new cycle forms and hence each kidney-patient pair starts an w-chain. The longest w-chains are W_1=$(k_8, t_8, k_4, t_4, k_9, t_9)$ and $W_2 = (k_{10}, t_{10}, k_1, t_1, k_9, t_9)$, each with three pairs. Since t_1, the highest priority patient, is in W_2 but not in W_1, choose and fix W_2. Assign w to t_9, k_9 to t_1, and k_1 to t_{10} but do not remove them. Kidney k_{10}, the kidney at the tail of W_2, remains available for the next round.

[Figure 4]

Round 4. Upon removing the w-chain W_2, a new cycle $C_3 = (k_4, t_4, k_8, t_8)$ forms. Remove it by assigning k_4 to t_8 and k_8 to t_4.

[Figure 5]

Round 5. No new cycles form and the pair (k_{12}, t_{12}) "joins" W_2 from its tail to form the longest w-chain $W_3 = (k_{12}, t_{12}, k_{10}, t_{10}, k_1, t_1, k_9, t_9)$. Fix W_3 and assign k_{10} to t_{12}. Since no patient is left, w-chain W_3 shall be removed and kidney k_{12} at its tail shall be offered to the highest priority patient at the cadaveric waiting list.

[Figure 6]

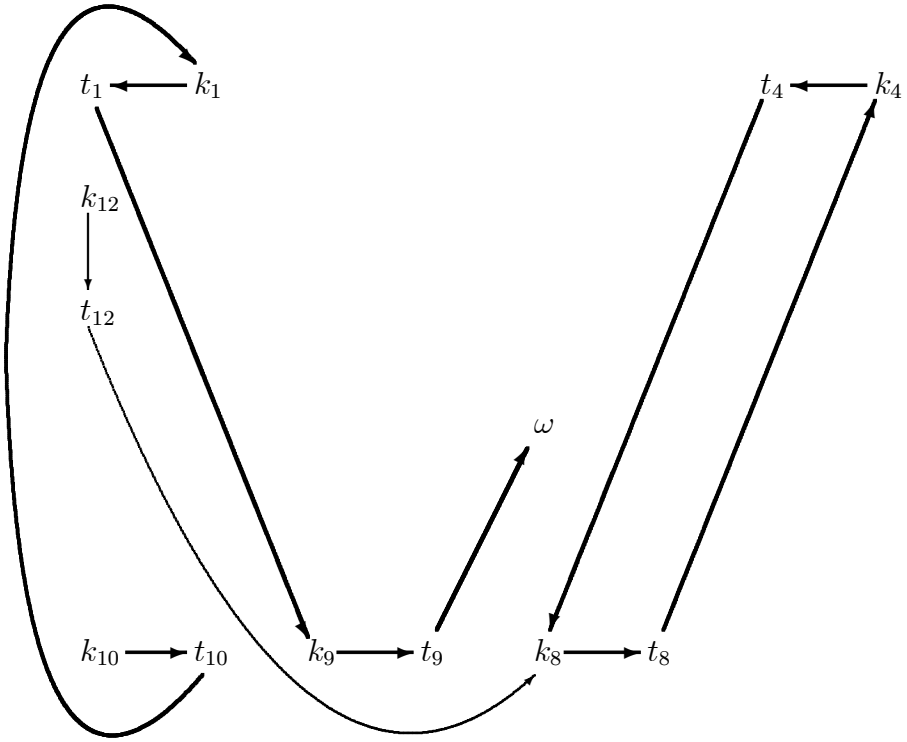

Figure 5: Example 2, Round 4

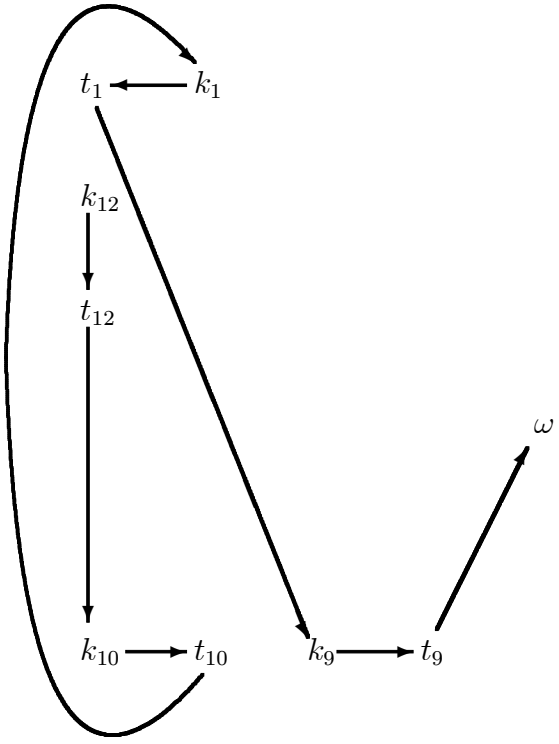

Figure 6: Example 2, Round 5

The final matching is

$$\begin{pmatrix} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 & t_9 & t_{10} & t_{11} & t_{12} \\ k_9 & k_{11} & k_2 & k_8 & k_7 & k_5 & k_6 & k_4 & \omega & k_1 & k_3 & k_{10} \end{pmatrix}.$$

□

It is worth emphasizing that the chain selection policy does not affect a patient who is at the head of a chain: Since he points to the waitlist option, he will eventually be selected regardless of the chain selection rule. However whether his intended donor's kidney is offered to the cadaveric waiting list or another patient with a living donor depends on the rule.

7.3 Examples of Plausible Chain Selection Rules

Depending on policy priorities, one may consider adopting a number of alternative chain selection rules. We next give several examples:

a. *Choose minimal w-chains and remove them.*

   For this simplest chain selection rule the outcome is not affected whether *all* minimal w-chains are removed simultaneously or they are removed one at a time.

b. *Choose the longest w-chain and remove it.* If the longest w-chain is not unique then use a tie-breaker to choose among them. One possibility is priority ordering patient-donor pairs based on factors such as waiting time and breaking the ties based on this priority order.

c. *Choose the longest w-chain and keep it.* If the longest w-chain is not unique, choose one based on a tie-breaker.

d. *Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair and remove it.*

   This chain selection rule may be used to increase the inflow of O blood-type living kidneys to the cadaveric waiting list by giving higher priorities to pairs with O blood-type donors.

e. *Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair and keep it.*

   This chain selection rule may be especially appealing: the TTCC mechanism with this chain selection rule is not only *efficient* but also *strategy-proof* (see Section 7.4).

A w-chain that is formed at an interim step of the procedure may grow at following steps unless it is removed; hence the immediate removal of w-chains has a potential efficiency cost. Therefore the following "hybrid" of the last two chain selection rules may appeal to those who wish to "minimize" the efficiency loss while increasing the inflow of O blood-type living kidneys to the cadaveric waiting list.

f. *Prioritize the patient-donor pairs so that pairs with O blood-type donor have higher priorities than those who do not. Choose the w-chain starting with the highest priority pair; remove it in case the pair has an O blood-type donor but keep it otherwise.*

7.4 Efficiency and Incentives

In what follows, we will speak of Pareto efficiency in terms of the agents in the kidney exchange problem, namely the paired patients and donors who are available to participate in the kidney exchange.[16]

Given a kidney exchange problem $\langle \{(k_i, t_i)\}_{i=1}^n, (K_i)_{i=1}^n, (P_i)_{i=1}^n \rangle$, a matching μ is *Pareto efficient* if there is no other matching ν which is weakly preferred by all patients and strictly preferred by at least one patient. A kidney exchange mechanism is *efficient* if it always selects a Pareto efficient matching at any given time. A natural question at this point is whether the TTCC mechanism is efficient. The answer depends on the choice of the chain selection rule.

Theorem 1 *Consider a chain selection rule such that any w-chain selected at a non-terminal round remains in the procedure and thus the kidney at its tail remains available for the next round. The TTCC mechanism, implemented with any such chain selection rule, is efficient.*

Two examples of such chain selection rules are

1. the rule that chooses the longest w-chain and keeps it, and
2. the priority based rule that selects the w-chain starting with the highest priority pair and keeps it.

[16] A Pareto efficient outcome with respect to this population of agents will remain Pareto efficient if we enlarge the set of agents to include those on the cadaver queue, but may not if we enlarge the set of agents to include more patient-donor pairs.

Chain selection rules that remove a selected w-chain before termination of the algorithm, on the contrary, may yield Pareto inefficient outcomes. Consider the following example.

Example 3: Suppose the TTCC mechanism is implemented with the chain selection rule that selects the longest w-chain and removes it. Suppose there are five donor-patient pairs $(k_1, t_1), \ldots, (k_5, t_5)$ and the patients have the following preferences:

t_1	t_2	t_3	t_4	t_5
ω	k_1	k_2	k_1	k_4
	ω	k_1	k_3	k_5
		ω	k_4	ω
			ω	

There is no cycle at Round 1 and the longest w-chain is $(k_3, t_3, k_2, t_2, k_1, t_1)$. Upon removal of this w-chain, there are two cycles (k_4, t_4), (k_5, t_5) and therefore the outcome is matching

$$\mu = \begin{pmatrix} t_1 & t_2 & t_3 & t_4 & t_5 \\ \omega & k_1 & k_2 & k_4 & k_5 \end{pmatrix}$$

although it is Pareto dominated by the matching

$$\nu = \begin{pmatrix} t_1 & t_2 & t_3 & t_4 & t_5 \\ \omega & k_1 & k_2 & k_3 & k_4 \end{pmatrix}.$$

□

Roth [1982] showed that the TTC mechanism is *strategy-proof* (i.e., *dominant strategy incentive compatible*) in the context of housing markets.[17] That is, *truthful preference revelation is a dominant strategy* of the preference revelation game induced by the TTC mechanism and hence an agent can *never* profit by misrepresenting his preferences. Recall that, *in the absence of indirect exchanges*, the static kidney exchange problem is a housing market model, and therefore the Roth [1982] result immediately applies. [18]

[17] See Abdulkadiroğlu and Sönmez [1999,2003] and Papai [2000] for extensions of Roth [1982] in economic and abstract domains.

[18] That is, at any specific point in time, a patient cannot receive a more preferred kidney by misrepresenting his preferences. However, given the dynamic aspect of the problem, a patient may have a high likelihood of a favorable trade in the future and thus he may be unwilling to trade his paired donor's kidney at this time unless he receives a well-matched kidney. Therefore a kidney has an *option value* for its intended recipient and this will be reflected in the stated preferences. We emphasize that of course we speak of strategy proofness in the limited strategy space–the space of stated preferences–we

What happens when indirect changes are allowed? Is the TTCC mechanism strategy-proof? As in the case of efficiency, the answer depends on the choice of the chain selection rule.

Theorem 2 *Consider the following chain selection rules:*

1. *Choose minimal w-chains and remove them.*
2. *Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair and remove it.*
3. *Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair and keep it.*
4. *Prioritize the patient-donor pairs so that pairs with O blood-type donor have higher priorities than those who do not. Choose the w-chain starting with the highest priority pair; remove it in case the pair has an O blood-type donor but keep it otherwise.*

The TTCC mechanism, implemented with any of these chain selection rules is strategy-proof.

Among these four chain selection rules, the last two are especially plausible: The third rule yields an efficient and strategy-proof mechanism whereas the fourth one gives up efficiency in order to increase the inflow of blood-type O kidneys to the cadaveric waiting list.

On the negative side, strategy-proofness of TTCC is lost if one adopts a chain selection rule that chooses among the longest w-chains. The following example makes this point.

Example 4: Consider the problem in Example 2, but suppose patient t_4 misrepresents his preferences as $P_4' = k_5, k_1, k_9, \ldots$ improving the ranking of kidney k_1. While Round 1, Round 2 remain as in Example 2, Round 3 changes and this time the longest w-chain at Round 3 is $W_4 = (k_8, t_8, k_4, t_4, k_1, t_1, k_9, t_9)$ (see Figure 7). Therefore patient t_4 is assigned kidney k_1 instead of kidney k_8, profiting form his preference misrepresentation. □

A chain selection rule which chooses among longest w-chains upsets strategy-proofness because it allows a patient to influence his assignment by influencing the lengths of w-chains via a preference misrepresentation.

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have modeled for the kidney exchange problem.

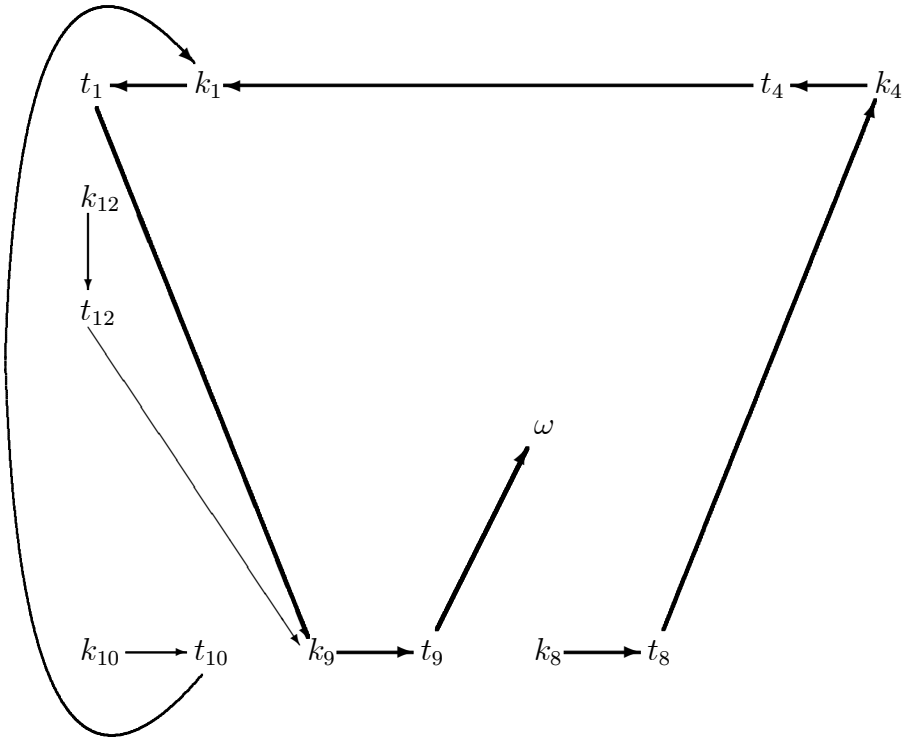

Figure 7: Example 4, Round 3

7.5 Sequencing of Transplantations, Donor Incentives, and Size of Exchange Cycles

Each potential donor is an agent with his own incentives. Clearly it will cause serious complications if a donor withdraws his consent after his intended recipient is transplanted a kidney through an exchange arrangement. In the context of paired-kidney-exchange, Ross et al. [1997] propose to perform the two transplantations simultaneously to avoid such complications, and their proposal is reiterated by the consensus statement of the transplantation community (Abecassis et al. [2000]). A similar practice should be adopted to carry out the exchanges under TTCC, and, following the final assignment of kidneys, all transplantations associated with a given cycle of the TTCC algorithm that produced the assignment should be performed simultaneously.

A similar, but slightly different, practice needs to be adopted to perform the transplantations associated with w-chains that are selected and (eventually) removed by the algorithm. The difference concerns the patient who is upgraded at the top of the cadaveric waiting list. Since the time between the procurement and the transplantation of a graft is key for the success of a cadaveric transplantation, coordinating a cadaveric transplantation with several other living-donor transplantations is essentially impossible. Therefore when a w-chain is selected by the algorithm, all the transplantations indicated

by that chain, with the exception of the cadaveric transplantation, should be performed simultaneously, and the cadaveric transplantation can be performed later on, when the cadaver kidney becomes available, without any need for coordination with the remaining transplantations. Ross and Woodle [2000] propose a similar timing of transplantations in the context of indirect exchange.

There is one last constraint on the timing of transplantations. Whenever the transplantations associated with a cycle/w-chain fails to be performed due to a last-minute difficulty, this not only affects the patients in this cycle/w-chain but potentially other patients whose assignments were finalized by the algorithm in the subsequent rounds. Therefore transplantations of a cycle/w-chain should be performed only after all transplantations called for in previous rounds are performed. There are, however, no additional timing constraints imposed by two or more distinct cycles/w-chains that formed in the same round.

Given the need for the elaborate timing of transplantations, the choice of the size of a kidney exchange "organization" is not a straightforward task. On one hand, as the number of participating pairs grows not only superior HLA matches is obtained but also more patients receive a transplant on average (see Tables 4, 5, and 6). On the other hand, as the number of participating pairs grows the average number and more importantly the average size of cycles/w-chains increase (see Tables 10, 11, and 12) making the coordination of transplantations increasingly more difficult. Moreover, since last minute changes are costly, all positive crossmatches should be determined when preferences are determined and not when a cycle/w-chain is selected. Therefore preference construction also gets increasingly difficult as the number of participating pairs grows. The simulations in section 9 explore the implications of different numbers of participating pairs.

There are currently 59 organ procurement organizations (OPOs) in the U.S., organized into 11 UNOS regions. When a cadaveric kidney becomes available, it is first offered to the highest priority patient within the OPO the kidney is harvested.[19] If the kidney cannot be allocated within the OPO, it is offered to the regional waiting list; and if it cannot be allocated within the region, it is offered to the national waiting list. One possibility may be to consider a similar structure in the context of kidney exchange. Exchanges may be carried out within an OPO; pairs who cannot be accommodated within a reasonable time may be included in a regional list; and if this does not work either the pair may be included in a national list.

[19]There is an important exception: If there is a patient with a perfect HLA match within the U.S., he is offered the kidney and in case the kidney is accepted his OPO pays back with the next kidney procured which is of the same ABO blood-type.

8 Kidney Exchange with Multiple Donors

It is not unusual for some patients to have more than one potential living donor. In this section we extend the kidney exchange problem to allow for this possibility. To simplify the presentation and avoid repeating ourselves, we only consider direct exchanges (i.e. exchanges exclusively involving live donors) and hence study the TTC mechanism. An extension allowing for indirect exchanges would involve the TTCC mechanism in a manner parallel to the discussion above.

Given a patient t_i, let $m_i \geq 1$ denote the number of his potential donors and let $k_{i1}, \ldots, k_{im_i}$ denote his potential donors. Following the terminology in the basic model, we often refer donor $k_{i\ell}$ as kidney $k_{i\ell}$. Given patient t_i, let K_i denote the set of all kidneys that are compatible with patient t_i and let P_i denote his strict preferences over $K_i \cup \{k_{i1}, \ldots, k_{im_i}\}$.

A (static) *kidney exchange problem with (possibly) multiple donors* consists of

1. a set of patients $\{t_1, \ldots, t_n\}$,
2. a set of kidneys $\{k_{i1}, \ldots, k_{im_i}\}$ *attached* to each patient t_i,
3. a set of compatible kidneys K_i for each patient t_i, and
4. a strict preference relation P_i over $K_i \cup \{k_{i1}, \ldots, k_{im_i}\}$ for each patient t_i.

Following the tradition in paired-kidney-exchange, we restrict ourselves to exchanges in which only one of the donors attached to a patient donates his kidney in exchange for a kidney transplanted to patient t_i. Equivalently there is *no free kidney*: Each patient has to provide a kidney in order to receive one. Therefore the outcome of a kidney exchange problem with multiple donors is a *matching* μ of kidneys to patients such that

1. each patient t_i is assigned a kidney in $K_i \cup \{k_{i1}, \ldots, k_{im_i}\}$, and
2. only one of the kidneys in $\{k_{i1}, \ldots, k_{im_i}\}$ is assigned to a patient, for any patient t_i.

Let $\mu(t_i)$ denote the assignment of patient t_i under matching μ.

Definitions of a *cycle*, a *mechanism*, *Pareto efficiency*, *efficiency*, and *strategy-proofness* directly carry over to the present model. A matching is *individually rational* if it assigns each patient a kidney that is at least as good as *any* of his attached kidneys. A mechanism is *individually rational*, if it always selects an individually rational matching.

The following lemma will be of use to extend the TTC mechanism to the present context.

Lemma 2 *Consider a graph which consists of patients and kidneys. Suppose each patient points to a kidney and each kidney points to a patient. Then there exists a cycle.*

At a given time, and for a given kidney exchange problem with multiple donors, the TTC mechanism determines the exchanges in several rounds as follows:

Round 1: Initially all kidneys are available. Each patient points to his most preferred kidney and each kidney points to its attached patient. By Lemma 2, there exists a cycle. Locate each cycle and carry out the corresponding exchange. *Remove all patients in a cycle together with their assignments and with their unassigned attached kidneys*. If there are any remaining patients, proceed to Round 2.

In general, at

Round s: Each patient points to his most preferred remaining kidney and each kidney points to its attached patient. By Lemma 2, there exists a cycle. Locate each cycle and carry out the corresponding exchange. Remove all patients in a cycle together with their assignments and with their unassigned attached kidneys. If there are any remaining patients, proceed to Round (s+1).

This extension of the top trading cycles mechanisms inherits the appealing properties of the basic version for housing markets. We summarize some of these properties as a theorem.

Theorem 3 *The TTC mechanism is individually rational, efficient, and strategy-proof. Moreover it is the only mechanism which satisfies each of these properties.*[20]

9 Simulations

The theoretical treatment of the TTCC mechanism makes clear that larger exchanges may yield welfare gains, but it gives us no idea of their magnitude, i.e. whether they are worth pursuing. The following simulations are meant as a first step in that direction. We use data where it exists, e.g. on the likelihood of mismatches and positive crossmatches. Where no data exists–on the willingness of patients and donors to trade a live donation for priority on the cadaver queue–we do robustness checks by simulating a wide range of preferences.[21]

[20]The original uniqueness result is due to Ma [1994] in the context of housing markets. See also Sönmez [1999] and Svensson [1999].

[21]For a discussion of how computation similarly complements theory in other practical problems of economic design, see Roth [2002].

Table 2: ABO Blood-type, gender, and age ditributions for new patients used in the simulations.

A. Patient Blood type	Frequency
O	45.6%
A	39.5%
B	11.1%
AB	3.8%

B. Patient Gender	Frequency
Female	40.9%
Male	59.1%

C. Patient Age	Frequency
<18	5.6%
18–34	13%
35–49	34.9%
50–64	38.9%
>64	7.6%

9.1 Patient and Donor Characteristics

We run our simulations using the Caucasian ESRD patient population between 18 and 79 years of age in the United States Renal Data System (USRDS). For the population blood-type and age distribution, we use the respective distributions for new ESRD waitlist patients recorded between January 1995 and April 2003 in the USRDS database. For the gender distribution we use data recorded between 1992 and 2001.[22] The blood-type, gender and age distributions are reported in Table 2. We construct the conditional distribution of patient ages given that the patient is adult (i.e. of age 18-79) using this table.

For the HLA protein characteristics of the population, we use the distribution reported in Zenios [1996] using the USRDS registration data for years between 1988 and 1991. We assume that all HLA proteins and blood-type are independently distributed following Zenios [1996].

In our simulations, for a demonstration of "possible efficiency gains" by the TTCC mechanism, we consider a scenario with unrelated donor-patient pairs. About 25.3% all living-donor transplants were in this category in 2001.[23] UNOS database distinguishes two

[22]Based on web UNOS/OPTN data and annual report as of 7/14/2003 retrieved from http://www.OPTN.org.

[23]In the medical literature, focus on living-donor transplants has been on unrelated living-donor transplants. Although superior HLA matches with related recipients are assumed to make them relatively good matches, there is no formal data analysis in this respect. Moreover, the real increase in the number of living-donor transplants has come from unrelated donors in the recent years. Hence, we focus our

Table 3: Relation among Unrelated Donors, Age, and Positive Crossmatch Distributions for Living Donors.

A. Unrelated Living Donors	**Frequency**
Spouse	53.5%
Other	46.5%

B. Living Donor Age	**Frequency**
<18	5.6%
18–34	13%
35–49	34.9%
50–64	38.9%
>64	7.6%

C. Positive Crossmatch	**Frequency**
Female Patient - Husband	33.3%
Other	11.1%

types of unrelated donors: spouses and others. The distribution of unrelated living-donor transplants based on the donor relation and donor age are given in Table 3 for the time spell 1992-2001. We assume that unrelated donors are also adults. Hence, we use this table to find the conditional distribution of the age of a non-spousal unrelated donor given that he is an adult. We assume that HLA and blood-type characteristics of the donor have the same distribution as the patients'. The characteristics of a non-spousal unrelated donor is independently distributed with the patient. The characteristics of a spouse is independently distributed with the patient except his or her age. We assume that the spouse age is the *same* as the patient age.

The positive crossmatch probabilities are assumed to be 0.333 between a female patient and her donor husband, and 0.111 for every other patient-donor pair following Zenios, Woodle and Ross [2001]. Wives, who have previously been pregnant, are documented to have more frequent positive crossmatches with their husbands than other donors.

9.2 Preference Construction

The preferences of patients over available kidneys (or equivalently living-donors) in the sample are determined using the survival analysis of grafts reported in Mandal et al. [2003]. This analysis uses data obtained from first-time kidney-only transplants between 1995 and 1998 in the USRDS database. We assume that the utility function of each patient

simulation analysis on unrelated living-donor transplants to make it comparable with relevant medical studies. In related living-donor transplants, compatibility problems with regard to ABO-incompatibility and positive crossmatch still continue to exist. Hence, implementation of the TTCC mechanism will benefit these patients, as well.

depends on the donor age and the number of HLA mismatches throughout our simulations. Mandal et al. [2003] derive the conditional risk of graft failure for several factors such as patient race, patient age, patient health condition (more specifically previous history of diabetes), patient gender, donor type (cadaver or live), donor age, HLA mismatches, and cadaveric organ cold waiting time by taking a baseline patient's graft failure risk to be 1 (a baseline patient is a non-diabetic white female patient receiving a perfectly matched organ from a live donor). Due to their estimation technique, the relative risks of failure for each transplant type is constant at each time t after graft transplant with respect to this baseline patient given that both organs survived until time t (i.e., using a Cox proportional hazards estimation).[24]

Using this analysis, and assuming that a patient's utility from a graft is a monotone decreasing function of its relative risk of failure determined by

1. the age of the living-donor and
2. the HLA mismatch number,

we derive the preferences of patients. Following Mandal et al. [2003], we assume that the graft failure risk of a patient from a live organ transplant is in functional form $a^x b^{\frac{y}{10}}$ where x is the number of HLA proteins in A, B and DR loci of the donor that do not exist in the recipient, also known as the *HLA mismatch* number (changes in the range $\{0, 1, 2..., 6\}$), y is the age of the living-donor (changing in the open interval $(18, 80)$), a is the relative graft failure risk per HLA mismatch, and b is the relative graft failure risk per 10 years of increase in the donor age. Therefore, since a patient's utility over grafts is a monotone decreasing function of its relative risk of failure, the utility function

$$u(x, y) = -\ln(a)\, x - \ln(b)\, \frac{y}{10}$$

represents patient preferences. Marginal rate of substitution, $-\frac{10\ln(a)}{\ln(b)}$, is determined as

- 5.14 years of younger donor age per each additional HLA mismatch for patients younger than 60, and
- 5.10 years of younger donor age per each additional HLA mismatch for patients older than 59.

[24] Although we use statistics derived in different studies using data from different time spells, it is natural to assume that distributions of new patient characteristics are the same and independent across years for the same population, nemely the Caucasian ESRD patients in the United States.

using the Mandal et al. [2003] estimation. Using this utility function, we construct patient preferences over compatible donors. We use the index of the donor as a tie-breaker whenever a patient receives the same utility from two kidneys. Hence, given a sample of compatible donors who are randomly drawn, we construct a strict preference ordering for each patient.

We also consider an alternative preference construction in subsequent simulations. Under this alternative treatment, we assume that patient t_i prefers another donor k_j to his own donor k_i if and only if

a. kidney k_i is not compatible with him, or

b. although kidney k_i is compatible with him, it has *more than* an equivalent of one additional HLA mismatch than kidney k_j has.

Hence, this preference construction illustrates a scenario in which patients are rather "cautious" towards other donors' kidneys. This assumption may be realistic especially when patients do not know much about the previous health conditions of other donors.

Under both preference scenarios, waitlist option may or may not be considered acceptable by a patient. Indirect exchanges are rare in the United States and are still considered to be experimental procedures. Currently very few transplant centers offer this option. Hence national data on the number of indirect exchanges is uninformative. When a patient participates in an indirect exchange and is given priority at the cadaveric waiting list, the expected HLA match quality is very low. Moreover cadaveric grafts are documented to have higher graft failure risks than living-donor kidneys. Mandal et al. [2003] report that this risk is 1.49 times higher than living donor grafts for patients younger than 60 and 1.67 times higher for patients older than 59. Therefore many patients, who can continue their lives with dialyses, may not consider the waitlist option acceptable, even if they are incompatible with their paired donor. Instead, they may want to keep their own donor for his option value for potential future exchanges with future donor-patient pairs. On the other hand, waitlist option may be considered acceptable by patients who have urgency for a kidney transplant. Since the median waiting time in the waitlist exceeds 3 years (it was 1134 days for a female and 1023 days for a male in 1998), the life quality is quite low for many patients who go through continuous dialysis.

Since the expected quality of HLA match is very low when a patient is given priority in the waiting list and since the graft failure rates are significantly higher for cadaveric kidneys than living-donor kidneys, we assume that a patient considers the waitlist option acceptable *only if* his donor is not compatible with him. We also assume that the patients who consider this option acceptable prefer any compatible living-donor kidney to this option. Because there is no reliable data available on the rate of patients who consider

this option acceptable, we use different assumptions in our simulations. We consider four treatments, in which respectively 0%, 10%, 20% and 40% of the patients with incompatible donors prefer the waitlist option to their own donors.

9.3 Outline of the Simulations

We consider five different regimes of kidney exchange mechanisms to contrast with the no-exchange regime. The *no-exchange regime* is a mechanism where patients can only receive kidneys from their own donors if they are compatible. Otherwise, they wait in the waiting list for a cadaveric kidney and they cannot trade their donor's kidney with another patient or with a priority in the waitlist. This is the traditional system that is currently in use in many countries.

We consider the following five alternative exchange mechanisms:

- The first exchange regime is the *paired-kidney-exchange mechanism*. Under this mechanism, each patient is assigned his paired donor's kidney whenever they are compatible. Incompatible pairs are considered for paired-kidney-exchange. An incompatible pair is randomly selected and randomly matched with another incompatible pair such that each of the two patients are compatible with the other one's paired donor. The same matching process is repeated until no additional paired-kidney exchanges are possible. This is meant to roughly approximate the paired exchange program in use in several regional transplantation centers.

- The second exchange regime is the *top trading cycles mechanism*.

- The third exchange regime is the *paired and indirect exchange mechanism*. Under this regime, we first run the paired-kidney-exchange mechanism. At the end of the algorithm, patients who have listed the waitlist option to be acceptable and who have not been matched with a living-donor kidney, are upgraded to the top of the waiting list, and make their donors' kidneys available for the waitlist patients.

- The fourth and fifth exchange regimes are two variants of the *top trading cycles and chains mechanism*. As the chain selection rules of the TTCC mechanism, we use Rule e and Rule f (introduced in Section 7.3). In *Rule e*, we prioritize patient-donor pairs in a single list; we choose the w-chain with the highest priority pair and keep it. In *Rule f*, we prioritize the patient-donor pairs so that pairs with O blood-type donor have higher priorities than those who do not; we choose the w-chain starting with the highest priority pair; we remove it in case the pair has an O blood-type donor but keep it otherwise.

In our simulations, we randomly simulate a sample of n donor-patient pairs using the population characteristics explained above. Then, we determine the preferences of patients over kidneys in the sample: For each patient t_i, we first check whether a donor k_j is ABO-compatible. If k_j is ABO-compatible, then we check whether there is a positive crossmatch between t_i and k_j. If they test negative for crossmatch, then k_j is in the compatible donor set K_i of patient t_i. After finding the set of compatible kidneys for each patient, we obtain a preference ordering on this set, using the utility functions described above. We construct 8 sets of preferences for each patient using the rational or cautious preference construction methods discussed above and assuming that 0%, 10%, 20%, or 40% of patients with incompatible donors consider the waitlist option acceptable. We simulate each of the six mechanisms under these eight preference scenarios. We use a Monte-Carlo simulation size of 100 trials for three different population sizes of 30, 100, and 300 pairs.

Note that no-exchange regime yields the same outcome for all 8 preference profiles, and the same is true for the paired-kidney-exchange mechanism as well. Moreover the TTC regime yields the same outcome for the rational preferences regardless of the rate of patients considering the waitlist option acceptable and the same is also true for cautious preferences. Finally note that when no patient considers the waitlist option acceptable, each version of the TTCC mechanism reduces to the TTC mechanism, and paired and indirect exchange mechanism reduces to the paired-kidney-exchange mechanism.

9.4 Discussion of Results

The simulation results suggest that very substantial gains in the number and match quality of transplanted kidneys might result from adoption of the TTCC mechanism.

We report the details of this analysis in 9 tables, 3 for each population size. The rows of the tables refer to different regimes under different preference profiles constructed: the top four rows refer to the outcomes under mechanisms without the waitlist option (and to their counterparts, which permit the waitlist option in preferences, when nobody lists the waitlist option). The other rows are divided into three groups, where each group reports outcomes of mechanisms with the waitlist option when 10%, 20%, or 40% of the patients with incompatible donors list the waitlist option over their donors in their stated preferences. We also report outcomes of the mechanisms under cautious and rational preference statements separately when two outcomes differ from each other.

The first table of each population size (Table 4 for n=30, Table 5 for n=100, and Table 6 for n=300) reports the general patient statistics under each regime in the columns. The first column in these tables reports the *total live donor transplants* as percentage of the population size, which is the sum of next two columns, *transplants from own*

compatible donor (i.e., by cycles of 1 pair length involving a compatible donor-patient pair) and *transplants from trades* (i.e., by cycles and w-chains with multiple pairs). The third column is the percentage of *patients upgraded to the top of the waitlist* as heads of w-chains. The fourth and fifth columns report the quality of matches in the live donor transplants: the *risk of graft failure relative to the risk under no-exchange mechanism with population size 30* is reported in the fourth column and the *number of HLA mismatches* for an average transplant is reported in the fifth column. The relative risk is calculated for each transplant using the Mandal et al. [2003] analysis: the failure risk of a transplant is given as $a^x b^{\frac{y}{10}}$ where $a = 1.06$ for younger patients (i.e., younger than 60) and 1.05 for older patients (i.e., older than 59) is the risk associated with each HLA mismatch and x is the HLA mismatch number; $b = 1.12$ for younger patients and 1.10 for older patients is the risk associated with each 10 year increase in the donor age;[25] then we find the average risk in the population; finally, we normalize the average risk associated with one regime with the risk associated with the baseline, no-exchange regime for population size 30.[26] Standard errors of the estimates are reported in parentheses below them.

In the next set of tables, we report the effect of each regime on the waitlist additions for each blood type for each population size (Table 7 for n=30, Table 8 for n=100, and Table 9 for n=300). The columns are separated into two main groups. The first group reports the *net* percentage of patients sent to the top of the waitlist using the waitlist option for each blood type (the percentages are taken with respect to the population size). This is a *net upgrade burden*, because it considers the difference between the patients added at the top of the list and the kidneys made available for the waitlist patients. The second part of the columns in these tables, for each blood type we report the percentage (of the population size) that do not receive any transplants and do not trade their donors for a spot at the top of the waitlist. The total percentage is also reported. These patients continue to remain in the waitlist with their current priorities.

In the final set of tables, we report the sizes of cycles and w-chains under the mechanisms for each population size (Table 10 for n=30, Table 11 for n=100, and Table 12 for n=300). The columns of these tables are divided into two for cycles and w-chains, respectively. Each part reports the number, the average length, the maximum length of cycles/w-chains. The last column of each part reports the length of the *longest* cycle/w-chain encountered in 100 trials that we simulated. The lengths of cycles/w-chains

[25] Mandal et. al. do not find that patient gender and patient age have significant effects on risk of failure.

[26] For example in the third row of Table 4, under the TTC mechanism with rational preferences, the relative graft failure risk is 0.95, meaning that for every 100 grafts allocated by the no-exchange regime that fail at any time t after the transplant, we expect only 95 grafts allocated by the TTC regime to fail (conditional on their survival until time t).

are measured in pairs. Standard errors of the estimates are reported in parentheses below them.

Next we highlight a number of these results:

1. A transition to TTCC mechanism will significantly improve the utilization rate of potential unrelated living-donor kidneys: Assuming a population size of 100 pairs (see Table 5), while approximately 55% of potential living-donor kidneys are utilized under the no-exchange regime, this rate increases to 73.5% under the paired-kidney-exchange, and to 88-89.5% under the TTCC mechanism. The efficiency gain gets larger as the population size grows further: For example, for a population of 300 pairs the utilization rate increases to 91-92% under the TTCC mechanism.[27]

2. A transition to TTCC significantly decreases the HLA mismatch (and hence significantly increases the HLA match quality), especially for the large populations: Assuming a population size of 100 pairs (see Table 5), the average HLA mismatch is 4.8 under the no-exchange regime as well as the paired-kidney-exchange regime, but it reduces to 3.7-3.9 under the TTCC mechanism. The average HLA mismatch remains the same for the former two regimes even when the population grows, but it further reduces under the TTCC mechanism. For example it reduces to 3.3-3.4 for a population size of 300.[28] With the increase in HLA match quality, the risk of graft failure decreases substantially by a transition to the TTCC mechanism.

3. Under the TTCC mechanism, average/maximal sizes of cycles/w-chains increase as the population grows. However the increase is less than proportional: For a population size of 30 pairs, the average cycle size is 2.5-3 pairs, average size of the longest cycle is 5.4-5.7 pairs, and the longest cycle ever observed is 10 pairs. The same statistics are 1.8-2 pairs, 2-2.5 pairs, 7 pairs respectively for w-chains. For a population size of 300 pairs, the corresponding statistics are 4.9-6.3 pairs,

[27] We test whether the medians of the distributions of average percentage of live donor transplantations under different regimes are significantly different from each other using a (paired-sample) Wilcoxen signed rank test for population size 100: between paired-kidney-exchange regime and no-exchange regime the difference is highly significant with a p-value less than 10^{-17}; between various TTCC regimes and paired-kidney-exchange regime the difference is highly significant with p-values less than 10^{-17}. Similar significance results apply for population sizes 30 and 300, as well.

[28] We test whether the medians of the average HLA mismatch distributions under different regimes are significantly different from each other using a (paired-sample) Wilcoxen signed rank test for population size 100: between paired-kidney-exchange regime and no-exchange regime the difference is not significant with a p-value 0.87; between various TTCC regimes and paired-kidney-exchange regime the difference is highly significant with p-values less than 10^{-17}. Similar significance results apply for population sizes 30 and 300, as well.

16-17 pairs, 26 pairs respectively for cycles, and 3.3-4.3 pairs, 5.7-6.5 pairs, 16 pairs respectively for w-chains (see Table 10 for n=30, Table 11 for n=100, and Table 12 for n=300).

4. O blood-type patients *without* living donors *benefit* from a transition to TTCC mechanism. That is achieved by significantly reducing the rate of O blood-type patients with potential donors who are forced to rely on the cadaveric waiting list because of an incompatibility. Consider population size of 100 pairs (see Table 8): The O blood type patients, who join the cadaveric waiting list under the no-exchange regime, are 27.6% of all patients with living donors. This rate reduces to 21.9% under the paired-kidney-exchange and further to 5.5-8.7% under the TTCC mechanism. That means out of 100 patients with living donors, 13.2-16.4 patients with O blood types drop from "competition" at the waiting list from a transition from paired-kidney-exchange to TTCC mechanism. The cost of the transition to O blood-type patients with no living donors is the priority upgrade for a number of O blood-type patients, who are only 0.9-4.2% of all patients with living donors, in the cadaveric waiting list. This is a very plausible tradeoff given the very high weight given to HLA match in cadaveric kidney allocation: Any O blood-type patient who joins the waiting list is a real "competitor" regardless of when he joins the waitlist.[29] The benefit to O blood-type patients with no living donors increases as the population further grows.

10 TTCC vs. Current Kidney Exchange Programs, and Some Concluding Remarks

TTCC mechanism is motivated by the present kidney exchange programs but it has a number of major advantages over them. In this section we discuss some of these advantages:

1. *Only the incompatible pairs can benefit from the current kidney exchange programs whereas all pairs can potentially benefit from the TTCC mechanism*: Current kidney

[29] We test whether the medians of the distributions of average *percentage of all patients joining the O blood type queue* (i.e. the sum of net waitlist upgrades from first part of the table and other waitlist additions from the second part of the table) under different regimes are significantly different from each other using a (paired-sample) Wilcoxen signed rank test for population size 100: between paired-kidney-exchange regime and no-exchange regime the difference is highly significant with a p-value less than 10^{-17}; between various TTCC regimes and paired-kidney-exchange regime the difference is highly significant with p-values less than 10^{-17}. Similar significance results apply for population sizes 30 and 300, as well.

exchange programs are designed for pairs for which the donor kidney is incompatible with the intended recipient. Part of such incompatibilities are overcome by these exchange programs and the loss of physically eligible and willing volunteer kidney donors is reduced. However even if a kidney is compatible with a patient it may not be a good match. The TTCC mechanism takes patient preferences into consideration and hence its implementation improves the match quality between patients and kidneys, consistent with the suggestions of Opelz [1997].[30]

2. *The current paired-kidney-exchange programs almost exclusively utilize exchanges between only two donor-recipient pairs whereas the TTCC mechanism utilizes exchanges among three or more pairs as well*: The current paired-kidney-exchange programs are designed to avoid the potential donor loss arising from immunologic incompatibilities between donors and their intended recipients. If one only considers ABO blood-type incompatibilities, basic exchanges involving only two pairs is without loss of generality because, as Terasaki, Gjertson and Cecka [1998] indicate, the only eligible pairs are

   - blood-type A patients with blood-type B donors, and
   - blood-type B patients with blood-type A donors.

   However there are two major reasons why basic exchanges involving only two pairs will not be sufficient in general:

   (a) The incompatibility may also be due to a positive crossmatch and hence additional pairs may be needed even if the only objective is merely obtaining a compatible match. Lucan et al. [2003] indicate that this recently was the case at a transplant center in Romania.[31] They state

   "In the transplant session involving three pairs, two displayed ABO incompatibility (A vs B in the first pair and B vs A in the second pair). Although regarding ABO matching between the two pairs, transplantation was feasible, the direct exchange was not possible because the A donor had a positive cross-match with the A recipient of the other pair. A third pair with A blood group but a positive cross-match was used to solve the problem. The

[30] Gerhard Opelz, along with Thomas Wujciak are the designers of X-COMB mechanism for the allocation of cadaveric kidneys and since 1996 this mechanism is used in the Eurotransplant (current members: Belgium, Germany, Luxembourg, the Netherlands, Austria, and Slovenia) region. The HLA match plays a key role in X-COMB mechanism.

[31] And recall the recent three-pair exchange at Johns Hopkins.

kidney from the first pair went to the second pair; the second, to the third, and the first pair received the kidney from the third."

(b) Moreover when one considers patient preferences and tries to obtain not only compatible but also good matches, it is no longer clear that exchanges between merely two pairs account for a majority of desired exchanges. In addition to the three-way exchange mentioned above, Lucan et al. [2003] also directed an exchange between four pairs, and one of the participating pairs were compatible but a poor match; they state

"In the transplant session with four pairs: the first pair had an O blood group donor with an A blood group recipient and a positive crossmatch. In the second pair, both donor-recipient had A blood group but six mismatches. The third pair had an A blood group donor with a B blood group recipient, and the forth pair a B blood group donor and O blood group recipient. The exchange of kidneys among these for pairs, was performed as follows: first to second, second to third, third to forth, and fourth to first."[32]

It is worthwhile to emphasize that the above mentioned exchange is not the only one that results in the assignment of a compatible kidney for each of the four patients. Donor two could have directly donated to its paired recipient and a three-way exchange is feasible among the remaining three pairs. Nevertheless the more elaborate exchange was preferred by the transplant team presumably to assure a superior match.[33]

3. *Under the TTCC mechanism, not only the patient who is willing to exchange his donor's kidney with a priority in the cadaveric waiting list benefits, but other patients with paired donors also benefit from his presence; this is not the case under the present indirect exchange programs*: Under the current indirect exchange programs the minimal w-chains are selected. Whenever a patient t_i is willing to exchange his donor's kidney k_i for a priority at the waiting list, his donor's kidney is offered to the waiting list and only patient t_i benefits from this arrangement among patients with living donors. However the presence of such a patient might also benefit other pairs as in the case of the TTCC mechanism. For example when patient t_i agrees to exchange kidney k_i for a priority at the waiting list, kidney k_i may be offered for another patient t_j (instead of the waiting list) and the paired donor of patient

[32]There is a minor mistake in the last sentence of this statement. Clearly what is intended is "first to forth, forth to third, third to second, and second to first."

[33]Lucan et al. [2003] indicate that their experience with multiple pairs (three and four) kidney exchange is unique in Europe.

t_j might in return offer his kidney k_j to the cadaveric pool. In this way two pairs benefit from the indirect exchange instead of one. In general several pairs, depending on the size of the eventually selected w-chain, may benefit from the presence of just one patient who is willing to accept a cadaveric kidney in exchange for his donor's kidney. Therefore the TTCC mechanism is a more efficient way of implementing indirect exchanges.

4. *The adverse affect of the present indirect exchange programs on blood-type O patients with no living donors will definitely be reduced and probably reversed under the TTCC mechanism*: While O blood-type donors are ABO-compatible with all patients, O blood-type patients are ABO-compatible with only O blood-type donors. That is, using the basic microeconomics jargon, while O blood-type kidneys are potentially demanded by all patients they are supplied by only O blood-type donors. This asymmetry on ABO-compatibility is behind the vulnerability of O blood type patients with no living donors under the current indirect exchange programs. Zenios, Woodle and Ross [2001] state

   "One of the major concerns is that ABO-incompatible indirect exchanges will tend to involve living donors with blood types A, B and AB but not O whereas a large proportion of their paired recipients will be of blood type O. If so, the waiting time for potential blood type O recipients on the cadaveric list who do not have a living donor may increase despite the increased number of kidneys transplanted. Since blood type O individuals already have the longest mean waiting time on the cadaveric list, such an exchange would not be justifiable as it harms an already vulnerable population."

   This vulnerability will be diminished and probably reversed under our proposed TTCC mechanism for a number of reasons:

   (a) More O blood-type patients with living donors will be matched with living donors and thus dropped from the cadaveric waiting list.

   (b) The current indirect exchange programs are designed for patients who are immunologically incompatible with their living donors, and based on the asymmetry on ABO-compatibility the recipient of a participating pair is much more likely to be blood-type O than the donor of the pair. The main idea underlying the TTCC mechanism is quite different: Since it strives to increase the match quality based on the preferences, the extreme asymmetry dictating the choice of qualified pairs is greatly reduced. That is because, pairs with O blood-type donors can still trade under the TTCC mechanism, even if they are compatible, in order to achieve a superior match.

(c) Under the present indirect exchange programs, whenever a patient with an incompatible donor participates, the priority list is adjusted. Hence the priority for one cadaveric kidney is modified, in order to increase the total kidney supply by one. The TTCC mechanism, on the other hand, utilizes larger w-chains as well. For example if an eventually selected w-chain involves three donor-recipient pairs, then the priority for one cadaveric kidney is modified, in order to increase the total kidney supply by three. Therefore the average number of kidneys gained per indirect exchange is higher under the TTCC mechanism than the present indirect exchange programs and hence the average number of modifications in the cadaveric waiting list is smaller under TTCC for each gained kidney.

(d) The flexibility on chain selection can be used to increase the inflow of O blood-type living kidneys to the cadaveric kidney waiting list. For example if (k_1, t_1, k_2, t_2) is an w-chain where kidney k_1 is blood-type A and kidney k_2 is blood-type O, the smaller chain (k_2, t_2) may be selected and removed so that the living kidney offered to the waiting list is of type O.[34] This, however, will have an efficiency cost since it benefits fewer patients.

Ross and Woodle [2000], inventors of the indirect exchange program, state

"If mechanisms can be developed to avoid increasing the waiting time for blood group O recipients, we would support the implementation of the indirect ABO-incompatible exchange."

The TTCC mechanism is a likely solution for this challenging task.

A Appendix: Proofs

Proof of Lemma 1: Consider a graph where each patient points either towards a kidney or w, and each kidney points to its paired recipient. Suppose there is no cycle. Consider an arbitrary pair (k_i, t_i). Start with kidney k_i and follow the path in the graph.

[34] In order to eliminate or reduce the adverse affect of indirect exchange programs on patients with no living donors, Zenios, Woodle and Ross [2001] propose preferential selection of O blood-type paired donors for patients with multiple potential donors who wish to participate in indirect exchange programs. Their proposal is consistent with a direct extension of the TTCC mechanism to the model with multiple potential donors discussed in Section 8 when the flexibility on chain selection is used to increase the inflow of O blood-type kidneys to the cadaveric waiting list: Suppose at any stage of the algorithm patient t points to w and he has two potential donors k (of blood-type O) and k' (of blood-type A). Two intersecting chains are (k, t), (k', t) and selecting the former based on the potential donor blood-type is consistent with Zenios, Woodle and Ross [2001] proposal.

Since there are no cycles, no kidney or patient can be encountered twice. Hence by the finiteness of pairs, the path shall terminate at w. This is the w-chain initiated by pair (k_i, t_i) completing the proof. $\diamondsuit$

***Proof of Lemma 2*:** Consider a graph where each patient points towards a kidney and each kidney points to a patient. Consider an arbitrary pair (k, t) such that kidney k points to patient t. Start with kidney k and follow the path in the graph. Since each node points to another, the path never terminates and since there are finite number of kidneys, a kidney shall be encountered twice. Let kidney k' be any such kidney and let t' be the patient on the path who points to kidney k'. The path starting with kidney k' and ending with patient t' is a cycle. $\diamondsuit$

Proof of Theorem 1: Let TTCC mechanism be implemented with a chain selection rule where any w-chain selected at a non-terminal round remains in the procedure and the kidney at its tail remains available for the next round. Fix the time and consider the algorithm. Any patient whose assignment is finalized in Round 1 has received his top choice and cannot be made better off. Any patient whose assignment is finalized in Round 2 has received his top choice among remaining choices and cannot be made better off without hurting a patient whose assignment is finalized in Round 1. Proceeding in a similar way, no patient can be made better off without hurting a patient whose assignment is finalized in an earlier round. Therefore TTCC mechanism selects a Pareto efficient matching at any given time provided that w-chains are removed at the termination. $\diamondsuit$

Before proving Theorem 2, we show that the three priority based chain selection rules described in its statement are members of a wider class of chain selection rules. Under this class each ordering of patient-donor pairs together with a fixed pair defines a chain selection rule, and it is given as follows: Priority order donor-patient pairs in a single list. Fix a pair (k_j, t_j). Whenever an w-chain is to be selected,

- select the w-chain starting with the highest priority pair (k_i, t_i), and
- remove the w-chain if the pair (k_i, t_i) has *strictly* higher priority than the fixed pair (k_j, t_j) and keep it until termination otherwise.

This class covers the following three smaller classes of chain selection rules:

1. Priority order donor-patient pairs in a single list and let pair (k_j, t_j) be the *lowest* priority pair. Then by definition any selected w-chain is immediately removed unless it starts with the pair (k_j, t_j). But only at the final round the selected w-chain can possibly start with (k_j, t_j), because it is the lowest priority pair, and hence in this case as well the selected w-chain is removed. Therefore this case corresponds

to priority based chain selection rules which immediately remove w-chains upon selection.

2. Priority order donor-patient pairs in a single list and let pair (k_j, t_j) be the *highest* priority pair. Then by definition any selected w-chain remains until termination and hence it corresponds to priority based chain selection rules which keep the selected w-chains until termination.

3. Finally if each pair with an O blood-type donor is given higher priority than each of the other pairs, and if (k_j, t_j) is fixed to be the *highest priority pair among pairs which do not have an O blood-type donor*, then

   (a) a selected w-chain is removed if it has an O blood-type donor, and

   (b) it is kept until termination otherwise.

First, we prove the following lemma which will be useful for the proof of Theorem 2.

Lemma 3 *Consider the TTCC mechanism implemented with a priority based chain selection rule. Fix the stated preferences of all patients except patient t_i at P_{-i}. Suppose that in the algorithm the assignment of patient t_i is finalized at Round s under P_i and at Round s' under P_i'. Suppose $s \leq s'$. Then the remaining active patients and unassigned kidneys at the beginning of Round s are the same, whether patient t_i announces P_i or P_i'.*

Proof of Lemma 3: Patient t_i fails to participate in a cycle or a selected w-chain prior to Round s under either preferences. Therefore at any round prior to Round s

1. not only the highest priority active patient is the same, whether patient t_i announces P_i or P_i',

2. but also the same cycles/w-chains form, and in case there are no cycles, the same w-chain is selected, whether patient t_i announces P_i or P_i'.

Hence the remaining active patients and unassigned kidneys at the beginning of Round s are the same, whether patient t_i announces P_i or P_i'. ♢

We are ready to prove Theorem 2.

Proof of Theorem 2: We first consider the chain selection rule that chooses the minimal w-chains and removes them. Recall that for each patient t_i, the relevant part of preference P_i is the ranking up to k_i or w, whichever is more preferred. Given the preference profile $(P_i)_{i=1}^n$, construct a new preference profile $(P_i')_{i=1}^n$ as follows: For each patient t_i with $k_i P_i w$, let $P_i' = P_i$. For each patient t_i with $w P_i k_i$, construct P_i' from P_i by swapping the

ranking of k_i and w. Note that $k_i P_i' w$ for each patient t_i and because the relevant part of preferences are the more preferred of k_i and w, $\langle \{(k_i, t_i)\}_{i=1}^n, (P_i')_{i=1}^n \rangle$, is a housing market. Let μ denote the outcome of TTC mechanism for this housing market and construct matching ν from matching μ as follows: If $P_i' \neq P_i$ and $\mu(t_i) = k_i$ then $\nu(t_i) = w$, otherwise $\nu(t_i) = \mu(t_i)$. The key observation is that ν is the outcome of the TTCC mechanism when it is implemented with the minimal w-chain selecting chain selection rule. Therefore by Roth [1982], a patient can never receive a more preferred kidney by a preference misrepresentation. He can receive the waitlist option w by a misrepresentation but cannot profit from it. That is because, TTCC mechanism never assigns a patient, a kidney that is inferior to w. Hence TTCC is strategy-proof with this choice of chain selection rule.

Next consider any of the priority based chain selection rules. Consider a patient t_i with true preferences P_i. Fix an announced preference profile P_{-i} for all other patients. We want to show that revealing his true preferences P_i is at least as good as announcing any other preferences P_i' under the TTCC mechanism. Let s and s' be the rounds at which patient t_i leaves the algorithm under P_i and P_i' respectively. We have two cases to consider.

Case 1: $s < s'$.

By Lemma 3, the same kidneys remain in the algorithm at the beginning of Round s whether patient t_i announces P_i or P_i'. Moreover, patient t_i is assigned his top choice remaining at Round s under P_i. Therefore his assignment under P_i is at least as good as his assignment under P_i'.

Case 2: $s \geq s'$.

Upon announcing P_i', the assignment of patient t_i is finalized either by joining a cycle, or by joining a selected w-chain. We will consider the two cases separately.

Case 2a: The assignment of patient t_i is finalized by joining a cycle under P_i'.

Let $(k^1, t^1, k^2, \ldots, k^r, t_i)$ be the cycle patient t_i joins, and thus k^1 be the kidney he is assigned under P_i'. Next suppose he reveals his true preferences P_i. Consider Round s'. By Lemma 3, the same active patients and available kidneys remain at the beginning of this round whether patient t_i announces P_i' or P_i. Therefore at Round s', kidney k^1 points to patient t^1, patient t^1 points to kidney k^2, ..., kidney k^r points to patient t_i. Moreover, they keep on doing so as long as patient t_i remains. Since patient t_i truthfully points to his best remaining choice at each round, he either receives a kidney better than kidney k^1 or eventually points to kidney k^1, completes the formation of cycle $(k^1, t^1, k^2, \ldots, k^r, t_i)$, and gets assigned kidney k^1.

Case 2b: The assignment of patient t_i is finalized by joining a selected w-chain under P_i'.

Let $(k^1, t^1, k^2, \ldots, k^r, t_i = t^r, k^{r+1}, \ldots, k^{r+m}, t^{r+m})$ be the selected w-chain patient t_i

joins, where $r \geq 1$ and $m \geq 0$, under P_i'. Therefore, under P_i', patient t_i is assigned the kidney k^{r+1} if $m \geq 1$ and the waitlist option w if $m = 0$. Also note that, given the considered class of priority based chain selection rules, pair (k^1, t^1) is the highest priority pair in Round s'. Next suppose patient t_i reveals his true preferences P_i. Consider Round s'. By Lemma 3, the same active patients and available kidneys remain at the beginning of this round whether patient t_i announces P_i' or P_i. We will complete the proof by showing that, upon announcing his truthful preferences P_i, the assignment of patient t_i is finalized in Round s' and thus he is assigned his top choice available at the beginning of Round s': Recall that for this case there is no cycle in Round s' when patient t_i announces P_i'. Therefore when he announces his true preferences P_i, either there is no cycle in Round s' or there is one cycle which includes him. If it is the latter, then his assignment is finalized in Round s' and we are done. Otherwise, each pair initiates an w-chain by Lemma 1 and one of these w-chains has to be selected. By the choice of a priority based chain selection rule, this will be the w-chain that starts with the highest priority pair (k^1, t^1). But the path starting with kidney k^1 passes through patient t_i and therefore the selected w-chain includes patient t_i. Hence in this case as well his assignment is finalized in Round s' completing the proof. ♢

The following lemma will be useful for the proof of Theorem 3.

Lemma 4 *Fix the stated preferences of all patients except patient t_i at P_{-i}. Suppose that in the algorithm patient t_i is removed at Round s under P_i and at Round s' under P_i'. Suppose $s \leq s'$. Then the remaining patients and kidneys at the beginning of Round s are the same whether patient t_i announces P_i or P_i'.*

***Proof of Lemma 4*:** Since patient t_i fails to participate in a cycle prior to Round s in either case, the same cycles form and therefore the same patients and kidneys are removed before Round s. ♢

***Proof of Theorem 3*:** We will first show that TTC mechanism satisfies *individual rationality, efficiency*, and *strategy-proofness*, and conclude the proof by showing that TTC is the only mechanism to satisfy each of these properties. Relabel the kidneys so that patient t_i prefers kidney k_{i1} to any other kidney attached to him.

***Individual rationality*:** Consider any patient t_i and relabel the kidneys so that kidney k_{i1} is his most preferred kidney attached to him. As the algorithm proceeds, patient t_i will either be assigned a better kidney or eventually form the cycle (k_{i1}, t_i) and be assigned kidney k_{i1}.

***Efficiency*:** Proof of *efficiency* is similar to the proof of Theorem 1. Fix the time and consider the algorithm. Any patient whose assignment is finalized in Round 1 has received

his top choice and cannot be made better off. Any patient whose assignment is finalized in Round 2 has received his top choice among remaining choices and cannot be made better off without hurting a patient whose assignment is finalized in Round 1. Proceeding in a similar way, no patient can be made better off without hurting a patient whose assignment is finalized in an earlier round. Therefore TTC mechanism selects a *Pareto efficient* matching at any given time.

Strategy-Proofness:[35] Consider a patient t_i with true preferences P_i. Fix an announced preference profile P_{-i} for all other patients. We want to show that revealing his true preferences P_i is at least as good as announcing any other preferences P_i' under TTC mechanism. Let s' be the round at which patient t_i leaves the algorithm under P_i', $(k^1, t^1, k^2, \ldots, k^r, t_i)$ be the cycle he joins, and thus k^1 be the kidney he is assigned. Let s be the round at which he leaves under his true preferences P_i. We want show that his assignment under P_i is at least as good as kidney k^1. We have two cases to consider.

Case 1: $s \geq s'$.

Suppose patient t_i announces her true preferences P_i. Consider Round s'. By Lemma 4, the same patients and kidneys remain at the beginning of this round whether patient t_i announces P_i' or P_i. Therefore at Round s', kidney k^1 points to patient t^1, patient t^1 points to kidney k^2, ..., kidney k^r points to patient t_i. Moreover, they keep doing so as long as patient t_i remains. Since patient t_i truthfully points to his best remaining choice at each round, he either receives a kidney better than kidney k^1 or eventually points to kidney k^1, completes the formation of cycle $(k^1, t^1, k^2, \ldots, k^r, t_i)$ and gets assigned kidney k^1.

Case 2: $s < s'$.

By Lemma 4, the same kidneys remain in the algorithm at the beginning of Round s whether patient t_i announces P_i' or P_i. Moreover, patient t_i is assigned his top choice remaining at Round s under P_i. Therefore in this case as well his assignment is at least as good as kidney k^1.

***Uniqueness*:** Fix the time, the set of patients $\{t_{1,\ldots,}t_n\}$ and the set of kidneys K. Let φ be a mechanism that is *individually rational, efficient,* and *strategy-proof*. By definition, mechanism φ selects a *Pareto efficient* matching at each given time. We shall show that $\varphi(P) = TTC(P)$, for any preference profile P. Given any preference profile P, let $\varphi_i(P)$ denote the assignment of patient t_i under mechanism φ for the preference profile P.

Fix a preference profile P and let $\mu = TTC(P)$. Consider the algorithm and let T_s denote the set of patients whose assignments are finalized in Round s. For any patient

[35] Proof of *strategy-proofness* is similar to proofs of analogous results in Roth [1982], Abdulkadiroğlu and Sönmez [1999,2003].

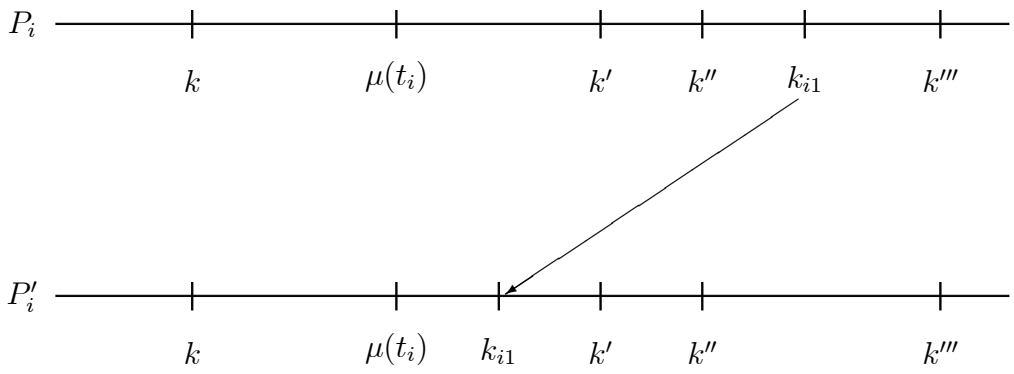

Figure 8: Construction of Preference P_i'.

t_i, relabel the kidneys so that kidney k_{i1} is his most preferred kidney attached to him. By *individual rationality*, $\mu(t_i)R_ik_{i1}$ for any patient t_i. For any patient t_i, construct a (possibly) new preference relation P_i' as follows:

1. If $\mu(t_i) = k_{i1}$ then $P_i' = P_i$,
2. otherwise $\mu(t_i)P_ik_{i1}$ and construct P_i' from P_i by simply inserting kidney k_{i1} right after kidney $\mu(t_i)$ and keeping the relative ranking of all other kidneys in K_i exactly the same (see Figure 8).

Let $P' = (P_i')_{i=1}^n$. We will show that $\varphi(P) = \mu$ via two claims.

Claim 1: $\varphi(P') = \mu$.
Proof of Claim 1: We will show that $\varphi_i(P') = \mu(t_i)$ for any patient t_i by induction on the round patient t_i leaves the algorithm under P.

Consider the set of patients in T_1. Each patient $t_i \in T_1$ is assigned his top choice under P_i by matching μ. Therefore by construction of P_i' and *individual rationality* of φ, either $\varphi(P') = \mu(t_i)$ or $\varphi(P') = k_{i1}$. Hence, under matching $\varphi(P')$ each patient $t_i \in T_1$ is assigned a kidney who is attached to a patient in T_1 and since only one kidney attached to each patient is eventually assigned to any patient, no patient outside T_1 is assigned a kidney that is attached to a patient in T_1. Moreover, since $\mu(t_i)R_ik_{i1}$ for any patient t_i, *Pareto efficiency* of $\varphi(P')$ implies $\varphi_i(P') = \mu(t_i)$ for any patient $t_i \in T_1$.

Next suppose that for any patient t_i who has left the algorithm before Round s, $\varphi_i(P') = \mu(t_i)$; we will show that $\varphi_i(P') = \mu(t_i)$ for any patient $t_i \in T_s$ as well. For any Round u, let K^u denote the the set of kidneys each of which is attached to a patient who has left the algorithm at Round u under P. Recall that only one kidney attached

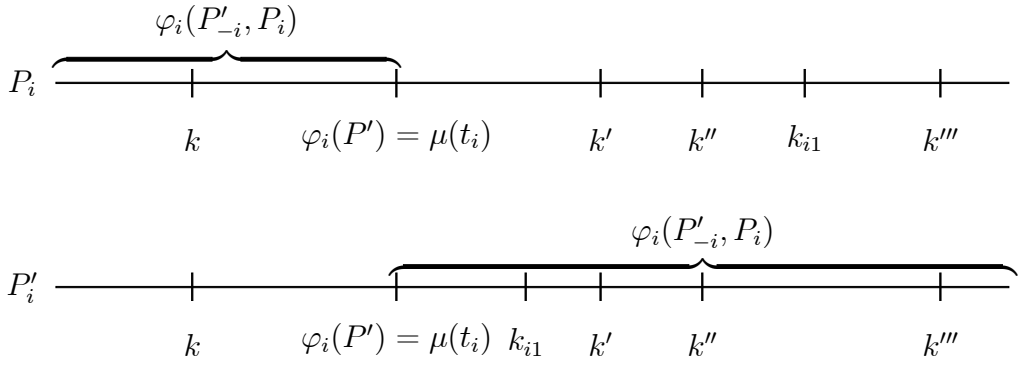

Figure 9: $\varphi(P'_{-i}, P_i) = \mu(t_i)$ by *strategy-proofness*.

to each patient is eventually assigned to any patient; therefore $\varphi_i(P') \notin \cup_{u=1}^{s-1} K^u$ for any patient $t_i \in T_s$ by the induction hypothesis. Moreover each patient $t_i \in T_s$ is assigned his top choice in $K \setminus \cup_{u=1}^{s-1} K^u$ under $\varphi(P)$. Therefore by construction of P'_i and *individual rationality* of φ, either $\varphi_i(P') = \mu(t_i)$ or $\varphi_i(P') = k_{i1}$. Hence only patients in T_s are assigned kidneys that are attached to patients in T_s under $\varphi(P')$ and therefore, since $\varphi(P')$ is *Pareto efficient*, $\varphi_i(P') = \mu(t_i)$ for any patient $t_i \in T_s$.

By induction $\varphi(P') = \mu$ completing the proof of Claim 1. Note that we have also shown that μ is the only matching that is *Pareto efficient* and *individually rational* under P'.

Claim 2: $\varphi(P) = \mu$.

Proof of Claim 2:[36] We will show that

$$\varphi(P'_{-T}, P_T) = \mu \qquad \text{for all } T \subseteq \{t_{1,\ldots,} t_n\}$$

by induction on the cardinality of T.

First show this for $|T| = 1$. Fix a patient t_i. Consider the preference profile (P'_{-i}, P_i). By *strategy-proofness*

$$\varphi_i(P') R'_i \varphi(P'_{-i}, P_i) \qquad \text{and} \qquad \varphi_i(P'_{-i}, P_i) R_i \varphi_i(P'),$$

and therefore $\varphi_i(P'_{-i}, P_i) = \varphi_i(P') = \mu(t_i)$ by Claim 1 together with the construction of P'_i. (See Figure 9.) But μ is the only matching that is *Pareto efficient* and *individually rational* under (P'_{-i}, P_i) such that $\varphi_i(P'_{-i}, P_i) = \mu(t_i)$ (for otherwise μ cannot be the

[36]Proof of Claim 2 is similar to the proof of Lemma 5.1 in Sönmez [1995].

unique *Pareto efficient* and *individually rational* matching under P'); therefore

$$\varphi(P'_{-i}, P_i) = \mu \qquad \text{for any } t_i \in T.$$

Next suppose

$$\varphi(P'_{-T}, P_T) = \mu \qquad \text{for all } T \subset \{t_{1,\ldots,}t_n\} \text{ with } |T| = \ell < n.$$

We will show that $\varphi(P'_{-T}, P_T) = \mu$ for all $T \subseteq \{t_{1,\ldots,}t_n\}$ with $|T| = \ell + 1$ as well. Let $T \subseteq \{t_{1,\ldots,}t_n\}$ be such that $|T| = \ell + 1$. Let $t_i \in T$. Consider the preference profile (P'_{-T}, P_T). By *strategy-proofness* we have

$$\varphi_i(P'_{-(T\setminus\{t_i\})}, P_{T\setminus\{t_i\}})R'_i\varphi_i(P'_{-T}, P_T) \qquad \text{and} \qquad \varphi_i(P'_{-T}, P_T)R_i\varphi_i(P'_{-(T\setminus\{t_i\})}, P_{T\setminus\{t_i\}}),$$

and therefore $\varphi_i(P'_{-T}, P_T) = \varphi_i(P'_{-(T\setminus\{t_i\})}, P_{T\setminus\{t_i\}}) = \mu(t_i)$ by the induction hypothesis (note that $|T \setminus \{t_i\}| = \ell$) and the construction of P'_i. Since $t_i \in T$ is arbitrary

$$\varphi_i(P'_{-T}, P_T) = \mu(t_i) \qquad \text{for all } t_i \in T.$$

But μ is the only matching that is *Pareto efficient* and *individually rational* under (P'_{-T}, P_T) such that this relation holds (for otherwise μ cannot be the unique *Pareto efficient* and *individually rational* matching under P'); therefore

$$\varphi(P'_{-T}, P_T) = \mu \qquad \text{for all } T \subseteq \{t_{1,\ldots,}t_n\} \text{ with } |T| = \ell + 1.$$

Hence $\varphi(P) = \mu$ by induction completing the proofs of Claim 2, the uniqueness and Theorem 3. ♢

References

[1] M. Abecassis, M. Adams, P. Adams, R. M. Arnold, C. R. Atkins, M. L. Barr, W. M. Bennett, M. Bia, D. M. Briscoe, J. Burdick, R. J. Corry, J. Davis, F. L. Delmonico, R. S. Gaston, W. Harmon, C. L. Jacobs, J. Kahn, A. Leichtman, C. Miller, D. Moss, J. M. Newmann, L. S. Rosen, L. Siminoff, A Spital, V. A. Starnes, C. Thomas, L. S. Tyler, L. Williams, F. H. Wright, and S. Youngner [2000] Consensus statement on the live organ donor. *The Journal of the American Medical Association* 284, 2919-2926.

[2] A. Abdulkadiroğlu and T. Sönmez [1998] Random Serial Dictatorship and the Core from Random Endowments in House Allocation Problems. *Econometrica*, 66, 3,

689-701.

[3] A. Abdulkadiroğlu and T. Sönmez [1999] House allocation with existing tenants. *Journal of Economic Theory* 88, 233-260.

[4] A. Abdulkadiroğlu and T. Sönmez [2003] School choice: A mechanism design approach. *American Economic Review* 93, 729-747.

[5] G. S. Becker and J. J. Elias [2002] Introducing Incentives in the Market for Living Organ Donations. Working paper, University of Chicago.

[6] Y. Chen and T. Sönmez [2002] Improving efficiency of on-campus housing: an experimental study. *American Economic Review* 92, 1669-1689.

[7] M. Jackson and T. Palfrey [2001] Voluntary implementation, *Journal of Economic Theory* 98, 1-25.

[8] M. Lucan, P. Rotariu, D. Neculoiu, and G. Iacob [2003] Kidney exchange program: A viable alternative in countries with low rate of cadaver harvesting. *Transplantation Proceedings* 35, 933-934.

[9] J. Ma [1994] Strategy-proofness and strict core in a market with indivisibilities. *International Journal of Game Theory* 23, 75-83.

[10] A. K. Mandal, J. J. Snyder, D. T. Gilbertson, A. J. Collins, and J. R. Silkensen [2003] Does cadavaric donor renal transplantation ever provide better outcomes than live-donor renal transplantation? *Transplantation* 75, 494-500.

[11] P. Milgrom [forthcoming] Putting Auction Theory to Work, Cambridge University Press.

[12] E. W. Nelson, J. E. Childress, J. Perryman, V. Robards, A. Rowan, M. S. Seely, S. Sterioff, M. R. Swanson, [1993] Financial incentives for organ donation: A report of the payment subcommittee, United Network for Organ Sharing Ethics Committee, http://www.unos.org/Resources/bioethics.asp?index=3

[13] E. Olson [2003] A Tale of 6 Kidneys, or She Gave Him Her Love, He Gave Her His Kidney. *New York Times* Saturday, August 2, A7.

[14] S. Papai [2000] Strategyproof assignment by hierarchical exchange. *Econometrica* 68, 1403-1433.

[15] F. T. Rapaport [1986] The case for a living emotionally related international kidney donor exchange registry. *Transplantation Proceedings* 18, 5-9.

[16] L. F. Ross, D. T. Rubin, M. Siegler, M. A. Josephson, J. R. Thistlethwaite, Jr., and E S. Woodle [1997] Ethics of a paired-kidney-exchange program. *The New England Journal of Medicine* 336, 1752-1755.

[17] L. F. Ross and E. S. Woodle [2000].Ethical issues in increasing living kidney donations by expanding kidney paired exchange programs. *Transplantation* 69, 1539-1543.

[18] A.E. Roth [1982] Incentive compatibility in a market with indivisibilities. *Economics Letters* 9, 127-132.

[19] A. E. Roth [2002] The Economist as Engineer: Game Theory, Experimentation, and Computation as Tools for Design Economics. *Econometrica* 70, 1341-1378.

[20] A. E. Roth and E. Peranson [1999] The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design. *American Economic Review* 89, 748-780.

[21] A.E. Roth and A. Postlewaite [1977] Weak versus strong domination in a market with indivisible goods. *Journal of Mathematical Economics* 4, 131-137.

[22] G. Opelz [1997] Impact of HLA compatibility on survival of kidney transplants from unrelated live donors. *Transplantation* 64, 1473-1475.

[23] G. Opelz, for the Collaborative Transplant Study [1998] HLA compatibility and kidney grafts from unrelated live donors. *Transplantation Proceedings* 30, 704-705.

[24] R. M. Sade [2003] Assessing Initiatives to Increase Organ Donations, Prepared Witness Testimony, The House Committee on Energy and Commerce, W.J. "Billy" Tauzin, Chairman, Subcommittee on Oversight and Investigations, June 3, http://energycommerce.house.gov/108/Hearings/06032003hearing946/Sade1500.htm

[25] D. P. Scanlon, C. S. Hollenbeak, W. Lee, E. Loh, P. A. Ubel [2003] Does Competition for Transplantable Hearts Encourage Gaming of the Waiting List? Presented at the 8th Northeast Regional Health Economics Research Symposium, Boston University, http://world.std.com/~mgtsci/conferences/downloads/NEHE_Rsrch_Symp_2002/Scanlon_Hollenbeak_Lee_ppr.pdf

[26] L. Shapley and H. Scarf [1974] On cores and indivisibility. *Journal of Mathematical Economics* 1, 23-28.

[27] T. Sönmez [1995] Strategy-proofness and implementation in matching problems. Ph.D. Thesis, University of Rochester.

[28] T. Sönmez [1999] Strategy-proofness and essentially single-valued cores. *Econometrica* 67, 677-689.

[29] L.-G. Svensson [1999], Strategy-proof allocation of indivisible goods. *Social Choice and Welfare* 16, 557-567.

[30] P. I. Terasaki, D. W. Gjertson, and J. M. Cecka [1998] Paired kidney exchange is not a solution to ABO incompatibility. *Transplantation* 65, 291.

[31] M. E. Votruba [2001] Efficiency-equity tradeoffs in the allocation of cadaveric kidneys. Working paper, Princeton University.

[32] S. Warmbir [2003] UIC hospital sued for Medicare fraud, Chicago Sun Times, July 29, http://www.suntimes.com/output/news/cst-nws-hosps29.html

[33] T. Wujciak and G. Opelz [1993] Computer analysis of cadaver kidney allocation procedures. *Transplantation* 55, 516-521.

[34] T. Wujciak and G. Opelz [1997] Matchability as an important factor for kidney allocation according to the HLA match. Transplantation Proceedings 29, 1403-1405.

[35] S. Zenios [1996] Health care applications of optimal control theory. Ph.D. Thesis, Massachusetts Institute of Technology.

[36] S. Zenios, L. M. Wein, and G. M. Chertow [1999] Evidence-based organ allocation. *The American Journal of Medicine*107, 52-61.

[37] S. Zenios, E. S. Woodle, and L. F. Ross [2001] Primum non nocere: avoiding increased waiting times for individual racial and blood-type subsets of kidney wait list candidates in a living donor/cadaveric donor exchange program. *Transplantation* 66, 406-407.

Table 4: Number of Transplants and Quality of Match for n=30

Pref.	**Mechanism**	**Total Trans. %**	**Own Donor Trans. %**	**Trade%**	**Waitlist Upgrade %**	**Rel. Risk of Failure**	**HLA Mis.**
All	**No Exchange**	54.83 (8.96)	54.83 (8.96)	0 (0)	0 (0)	1 (0.04)	4.79 (0.25)
All	**Paired**	68.50 (9.90)	54.83 (8.96)	13.67 (9.40)	0 (0)	1.00 (0.03)	4.78 (0.24)
Rational	**TTC**	82.47 (10.14)	23.03 (9.44)	59.43 (13.57)	0 (0)	0.95 (0.03)	4.16 (0.22)
Cautious	**TTC**	81.07 (10.02)	34.17 (11.27)	46.90 (13.96)	0 (0)	0.96 (0.03)	4.29 (0.23)
Waitlist 10%							
All	**Paired/Indirect**	68.50 (9.90)	54.83 (8.96)	13.67 (9.40)	2.80 (3.31)	1.00 (0.03)	4.78 (0.24)
Rational	**TTCC e**	82.97 (9.75)	22.63 (9.40)	60.33 (13.20)	1.30 (2.11)	0.95 (0.03)	4.16 (0.22)
Rational	**TTCC f**	82.97 (9.75)	22.73 (9.38)	60.23 (13.09)	1.30 (2.11)	0.95 (0.03)	4.16 (0.22)
Cautious	**TTCC e**	81.63 (9.60)	33.73 (11.33)	47.90 (13.52)	1.53 (2.29)	0.96 (0.03)	4.28 (0.22)
Cautious	**TTCC f**	81.63 (9.60)	33.73 (11.33)	47.90 (13.52)	1.53 (2.29)	0.96 (0.03)	4.28 (0.22)
Waitlist 20%							
All	**Paired/Indirect**	68.50 (9.90)	54.83 (8.96)	13.67 (9.40)	6.17 (4.30)	1.00 (0.03)	4.78 (0.24)
Rational	**TTCC e**	83.60 (9.65)	22.13 (9.30)	61.47 (13.02)	3.23 (3.09)	0.95 (0.03)	4.16 (0.22)
Rational	**TTCC f**	83.57 (9.65)	22.23 (9.30)	61.33 (12.91)	3.23 (3.09)	0.95 (0.03)	4.16 (0.22)
Cautious	**TTCC e**	82.43 (9.28)	33.40 (11.15)	49.03 (13.06)	3.10 (3.42)	0.96 (0.03)	4.29 (0.23)
Cautious	**TTCC f**	82.30 (9.32)	33.40 (11.15)	48.90 (12.99)	3.10 (3.42)	0.96 (0.03)	4.28 (0.23)
Waitlist 40%							
All	**Paired/Indirect**	68.50 (9.90)	54.83 (8.96)	13.67 (9.40)	13.20 (6.73)	1.00 (0.03)	4.78 (0.24)
Rational	**TTCC e**	84.70 (8.49	21.23 9.60)	63.47 (12.39)	6.37 (4.88)	0.95 (0.03)	4.17 (0.22)
Rational	**TTCC f**	84.43 (8.81)	21.43 (9.61)	63.00 (12.58)	6.40 (4.89)	0.95 (0.03)	4.17 (0.22)
Cautious	**TTCC e**	83.57 (8.53)	32.93 (10.98)	50.63 (12.54)	6.13 (4.39)	0.96 (0.03)	4.29 (0.22)
Cautious	**TTCC f**	83.30 (8.70)	32.93 (10.98)	50.37 (12.53)	6.27 (4.43)	0.96 (0.03)	4.29 (0.22)

Table 5: Number of Transplants and Quality of Match for n=100

Pref.	Mechanism	Total Trans. %	Own Donor Trans. %	Trade%	Waitlist Upgrade %	Rel. Risk of Failure	HLA Mis.
All	**No-Exchange**	54.79	54.79	0	0	1.00	4.83
		(4.48)	(4.48)	(0)	(0)	(0.02)	(0.14)
All	**Paired**	73.59	54.79	18.80	0	1.00	4.82
		(4.97)	(4.48)	(3.81)	(0)	(0.02)	(0.11)
Rational	**TTC**	87.85	11.51	76.34	0	0.93	3.72
		(4.54)	(3.44)	(5.45)	(0)	(0.01)	(0.10)
Cautious	**TTC**	87.23	24.01	63.22	0	0.93	3.86
		(4.73)	(4.48)	(5.46)	(0)	(0.01)	(0.11)
Waitlist 10%							
All	**Paired/Indirect**	73.59	54.79	18.80	2.31	1.00	4.82
		(4.97)	(4.48)	(3.81)	(1.38)	(0.02)	(0.11)
Rational	**TTCC e**	88.47	11.04	77.43	1.05	0.93	3.72
		(4.23)	(3.30)	(4.77)	(0.99)	(0.01)	(0.10)
Rational	**TTCC f**	88.42	11.04	77.38	1.05	0.93	3.72
		(4.21)	(3.30)	(4.73)	(0.99)	(0.01)	(0.10)
Cautious	**TTCC e**	87.86	23.68	64.18	1.22	0.94	3.87
		(4.49)	(4.56)	(5.23)	(1.03)	(0.01)	(0.11)
Cautious	**TTCC f**	87.83	23.69	64.14	1.22	0.94	3.87
		(4.49)	(4.56)	(5.23)	(1.03)	(0.01)	(0.11)
Waitlist 20%							
All	**Paired/Indirect**	73.59	54.79	18.80	4.94	1.00	4.82
		(4.97)	(4.48)	(3.81)	(2.22)	(0.02)	(0.11)
Rational	**TTCC e**	88.81	10.69	78.12	2.05	0.93	3.72
		(4.05)	(3.22)	(4.50)	(1.30)	(0.01)	(0.10)
Rational	**TTCC f**	88.72	10.70	78.02	2.06	0.93	3.71
		(4.02)	(3.21)	(4.43)	(1.32)	(0.01)	(0.10)
Cautious	**TTCC e**	88.37	23.13	65.24	2.56	0.93	3.86
		(4.23)	(4.80)	(4.87)	(1.77)	(0.02)	(0.11)
Cautious	**TTCC f**	88.31	23.14	65.17	2.57	0.93	3.86
		(4.23)	(4.80)	(4.87)	(1.77)	(0.02)	(0.11)
Waitlist 40%							
All	**Paired/Indirect**	73.59	54.79	18.80	10.24	1.00	4.82
		(4.97)	(4.48)	(3.81)	(3.07)	(0.02)	(0.11)
Rational	**TTCC e**	89.44	10.29	79.15	3.96	0.93	3.71
		(3.85)	(3.26)	(4.40)	(1.97)	(0.01)	(0.10)
Rational	**TTCC f**	89.28	10.29	78.99	4.07	0.93	3.71
		(3.88)	(3.23)	(4.42)	(2.04)	(0.01)	(0.10)
Cautious	**TTCC e**	88.97	22.81	66.16	4.72	0.93	3.85
		(4.17)	(4.83)	(4.79)	(2.60)	(0.02)	(0.11)
Cautious	**TTCC f**	88.87	22.83	66.04	4.75	0.93	3.85
		(4.18)	(4.84)	(4.83)	(2.62)	(0.02)	(0.11)

Table 6: Number of Transplants and Quality of Match for n=300

Pref.	**Mechanism**	**Total Trans. %**	**Own Donor Trans. %**	**Trade%**	**Waitlist Upgrade %**	**Rel. Risk of Failure**	**HLA Mis.**
All	**No-Exchange**	53.92 (2.82)	53.92 (2.82)	0 (0)	0 (0)	1.00 (0.01)	4.81 (0.08)
All	**Paired**	75.03 (2.72)	53.92 (2.82)	21.11 (2.51)	0 (0)	1.00 (0.01)	4.81 (0.07)
Rational	**TTC**	91.05 (3.35)	5.72 (1.28)	85.32 (3.61)	0 (0)	0.91 (0.01)	3.29 (0.06)
Cautious	**TTC**	90.86 (3.31)	15.36 (2.20)	75.51 (4.07)	0 (0)	0.91 (0.01)	3.40 (0.06)
Waitlist 10%							
All	**Paired/Indirect**	75.03 (2.72)	53.92 (2.82)	21.11 (2.51)	2.49 (0.88)	1.00 (0.01)	4.81 (0.07)
Rational	**TTCC e**	91.57 (3.16)	5.42 (1.32)	86.15 (3.33)	0.79 (0.46)	0.91 (0.01)	3.29 (0.06)
Rational	**TTCC f**	91.57 (3.16)	5.42 (1.32)	86.15 (3.33)	0.79 (0.46)	0.91 (0.01)	3.29 (0.06)
Cautious	**TTCC e**	91.43 (3.12)	15.07 (2.13)	76.36 (3.78)	0.84 (0.65)	0.92 (0.01)	3.40 (0.06)
Cautious	**TTCC f**	91.41 (3.11)	15.08 (2.13)	76.33 (3.76)	0.84 (0.65)	0.92 (0.01)	3.40 (0.06)
Waitlist 20%							
All	**Paired/Indirect**	75.03 (2.72)	53.92 (2.82)	21.11 (2.51)	4.96 (1.22)	1.00 (0.01)	4.81 (0.07)
Rational	**TTCC e**	91.92 (3.05)	5.29 (1.29)	86.63 (3.18)	1.52 (0.76)	0.91 (0.01)	3.29 (0.06)
Rational	**TTCC f**	91.91 (3.04)	5.29 (1.29)	86.62 (3.17)	1.52 (0.76)	0.91 (0.01)	3.29 (0.06)
Cautious	**TTCC e**	91.76 (2.98)	14.84 (2.12)	76.92 (3.51)	1.62 (0.93)	0.91 (0.01)	3.40 (0.06)
Cautious	**TTCC f**	91.74 (2.98)	14.85 (2.12)	76.89 (3.49)	1.62 (0.93)	0.91 (0.01)	3.40 (0.06)
Waitlist 40%							
All	**Paired/Indirect**	75.03 (2.72)	53.92 (2.82)	21.11 (2.51)	9.77 (1.73)	1.00 (0.01)	4.81 (0.07)
Rational	**TTCC e**	92.29 (2.98)	5.00 (1.29)	87.29 (3.05)	3.02 (1.36)	0.91 (0.01)	3.29 (0.06)
Rational	**TTCC f**	92.26 (2.96)	5.01 (1.29)	87.25 (3.01)	3.02 (1.35)	0.91 (0.01)	3.29 (0.06)
Cautious	**TTCC e**	92.17 (2.93)	14.42 (2.10)	77.75 (3.26)	3.19 (1.40)	0.91 (0.01)	3.39 (0.06)
Cautious	**TTCC f**	92.15 (2.93)	14.43 (2.10)	77.72 (3.24)	3.19 (1.40)	0.91 (0.01)	3.39 (0.06)

Table 7: ABO Composition of Net Waitlist Upgrades and Other Waitlist Patients as Percentage of n=30. Net Waitlist Upgrades is the difference between the number of upgraded patients of certain blood type and the number of live donor kidneys sent to the waitlist of the same blood type.

		Net Waitlist Upgrades%					*Other Waitlist Additions%*				
Pref.	**Mechanism**	**O**	**A**	**B**	**AB**	**Total**	**O**	**A**	**B**	**AB**	**Total**
All	**No-Exchange**	0	0	0	0	0	28.83	10.13	5.77	0.43	45.17
All	**Paired**	0	0	0	0	0	24.50	4.07	2.80	0.13	31.50
Rational	**TTC**	0	0	0	0	0	12.47	3.17	1.77	0.13	17.53
Cautious	**TTC**	0	0	0	0	0	13.77	3.17	1.87	0.13	18.93
Waitlist 10%											
All	**Paired/Indirect**	2.23	-1.60	-0.50	-0.13	0	22.17	3.83	2.57	0.13	28.70
Rational	**TTCC e**	1.13	-0.50	-0.33	-0.30	0	11.33	2.73	1.53	0.13	15.73
Rational	**TTCC f**	1.10	-0.47	-0.33	-0.30	0	11.33	2.73	1.53	0.13	15.73
Cautious	**TTCC e**	1.07	-0.17	-0.43	-0.47	0	12.70	2.40	1.63	0.10	16.83
Cautious	**TTCC f**	1.07	-0.17	-0.43	-0.47	0	12.70	2.40	1.63	0.10	16.83
Waitlist 20%											
All	**Paired/Indirect**	4.60	-3.27	-0.83	-0.50	0	19.53	3.50	2.17	0.13	25.33
Rational	**TTCC e**	2.50	-1.07	-0.53	-0.90	0	9.93	2.10	1.03	0.10	13.17
Rational	**TTCC f**	2.43	-1.00	-0.53	-0.90	0	9.97	2.10	1.03	0.10	13.20
Cautious	**TTCC e**	2.37	-1.00	-0.67	-0.70	0	11.30	1.87	1.23	0.07	14.47
Cautious	**TTCC f**	2.27	-0.97	-0.60	-0.70	0	11.40	1.90	1.23	0.07	14.60
Waitlist 40%											
All	**Paired/Indirect**	9.10	-6.33	-1.23	-1.53	0	14.67	2.20	1.37	0.07	18.30
Rational	**TTCC e**	5.13	-2.47	-1.07	-1.60	0	7.13	1.17	0.63	0.00	8.93
Rational	**TTCC f**	4.90	-2.27	-1.03	-1.60	0	7.33	1.17	0.67	0.00	9.17
Cautious	**TTCC e**	4.90	-2.23	-1.23	-1.43	0	8.70	0.83	0.77	0.00	10.30
Cautious	**TTCC f**	4.77	-2.13	-1.23	-1.40	0	8.80	0.87	0.77	0.00	10.43

Table 8: ABO Composition of Net Waitlist Upgrades and Other Waitlist Patients as Percentage of n=100. Net Waitlist Upgrades is the difference between the number of upgraded patients of certain blood type and the number of live donor kidneys sent to the waitlist of the same blood type.

		Net Waitlist Upgrades%					*Other Waitlist Additions%*				
Pref.	**Mechanism**	**O**	**A**	**B**	**AB**	**Total**	**O**	**A**	**B**	**AB**	**Total**
All	**No-Exchange**	0	0	0	0	0	27.64	11.18	5.86	0.53	45.21
All	**Paired**	0	0	0	0	0	21.89	2.90	1.57	0.05	26.41
Rational	**TTC**	0	0	0	0	0	9.30	1.90	0.91	0.04	12.15
Cautious	**TTC**	0	0	0	0	0	9.78	2.00	0.95	0.04	12.77
Waitlist 10%											
All	**Paired/Indirect**	2.02	-1.27	-0.45	-0.30	0	19.83	2.71	1.51	0.05	24.10
Rational	**TTCC e**	0.90	-0.23	-0.22	-0.45	0	8.37	1.36	0.72	0.03	10.48
Rational	**TTCC f**	0.86	-0.23	-0.21	-0.42	0	8.41	1.37	0.72	0.03	10.53
Cautious	**TTCC e**	1.05	-0.21	-0.24	-0.60	0	8.72	1.39	0.80	0.01	10.92
Cautious	**TTCC f**	1.03	-0.21	-0.24	-0.58	0	8.74	1.40	0.80	0.01	10.95
Waitlist 20%											
All	**Paired/Indirect**	4.10	-2.51	-0.98	-0.61	0	17.70	2.33	1.39	0.05	21.47
Rational	**TTCC e**	1.74	-0.47	-0.44	-0.83	0	7.53	1.01	0.58	0.02	9.14
Rational	**TTCC f**	1.69	-0.47	-0.42	-0.80	0	7.58	1.04	0.58	0.02	9.22
Cautious	**TTCC e**	2.24	-0.67	-0.58	-0.99	0	7.51	0.94	0.61	0.01	9.07
Cautious	**TTCC f**	2.20	-0.65	-0.58	-0.97	0	7.55	0.95	0.61	0.01	9.12
Waitlist 40%											
All	**Paired/Indirect**	8.25	-5.27	-1.70	-1.28	0	13.42	1.62	1.08	0.05	16.17
Rational	**TTCC e**	3.49	-1.17	-1.07	-1.25	0	5.71	0.46	0.42	0.01	6.60
Rational	**TTCC f**	3.45	-1.16	-1.05	-1.24	0	5.75	0.47	0.42	0.01	6.65
Cautious	**TTCC e**	4.21	-1.56	-1.12	-1.53	0	5.50	0.41	0.40	0.00	6.31
Cautious	**TTCC f**	4.14	-1.52	-1.11	-1.51	0	5.57	0.41	0.40	0.00	6.38

Table 9: ABO Composition of Net Waitlist Upgrades and Other Waitlist Patients as Percentage of n=300. Net Waitlist Upgrades is the difference between the number of upgraded patients of certain blood type and the number of live donor kidneys sent to the waitlist of the same blood type.

		Net Waitlist Upgrades%					*Other Waitlist Additions%*				
Pref.	**Mechanism**	**O**	**A**	**B**	**AB**	**Total**	**O**	**A**	**B**	**AB**	**Total**
All	**No-Exchange**	0	0	0	0	0	28.32	11.42	5.79	0.55	46.08
All	**Paired**	0	0	0	0	0	21.70	2.19	1.06	0.02	24.97
Rational	**TTC**	0	0	0	0	0	7.07	1.34	0.52	0.01	8.95
Cautious	**TTC**	0	0	0	0	0	7.23	1.37	0.52	0.01	9.14
Waitlist 10%											
All	**Paired/Indirect**	2.10	-1.37	-0.41	-0.32	0	19.59	1.93	0.94	0.02	22.48
Rational	**TTCC e**	0.64	0.02	-0.16	-0.50	0	6.43	0.83	0.37	0.01	7.63
Rational	**TTCC f**	0.64	0.02	-0.16	-0.50	0	6.43	0.83	0.37	0.01	7.63
Cautious	**TTCC e**	0.72	-0.02	-0.21	-0.50	0	6.50	0.84	0.38	0.00	7.73
Cautious	**TTCC f**	0.72	-0.02	-0.21	-0.49	0	6.51	0.85	0.38	0.00	7.75
Waitlist 20%											
All	**Paired/Indirect**	4.24	-2.92	-0.73	-0.59	0	17.41	1.76	0.83	0.02	20.01
Rational	**TTCC e**	1.34	-0.22	-0.31	-0.81	0	5.73	0.55	0.27	0.00	6.56
Rational	**TTCC f**	1.33	-0.22	-0.30	-0.81	0	5.74	0.55	0.27	0.00	6.57
Cautious	**TTCC e**	1.41	-0.22	-0.38	-0.82	0	5.81	0.52	0.29	0.00	6.62
Cautious	**TTCC f**	1.41	-0.22	-0.38	-0.81	0	5.82	0.53	0.29	0.00	6.64
Waitlist 40%											
All	**Paired/Indirect**	8.40	-5.65	-1.42	-1.32	0	13.21	1.36	0.63	0.01	15.21
Rational	**TTCC e**	2.77	-0.87	-0.63	-1.27	0	4.28	0.23	0.18	0.00	4.69
Rational	**TTCC f**	2.75	-0.87	-0.63	-1.26	0	4.30	0.24	0.18	0.00	4.72
Cautious	**TTCC e**	2.94	-0.96	-0.63	-1.34	0	4.28	0.21	0.15	0.00	4.64
Cautious	**TTCC f**	2.93	-0.96	-0.63	-1.34	0	4.29	0.22	0.15	0.00	4.66

Table 10: Properties of Cycles and W-chains for n=30. Cycles of 1 pair length with an incompatible donor-patient pair are not counted.

Pref.	**Mechanism**	**Cycle No**	**Cycle Length (as pairs)**			**W-chain No**	**W-chain Length (as pairs)**		
			Mean	Max.	Longest		Mean	Max	Longest
All	**No-Exchange**	16.45 (2.69)	1 (0)	1 (0)	1	0 (0)	-	-	-
All	**Paired**	18.50 (2.46)	1.11 (0.08)	1.90 (0.30)	2	0 (0)	-	-	-
Rational	**TTC**	14.65 (2.22)	2.95 (0.43)	5.65 (1.15)	9	0 (0)	-	-	-
Cautious	**TTC**	16.79 (2.71)	2.59 (0.41)	5.42 (1.17)	10	0 (0)	-	-	-
Waitlist 10%									
All	**Paired/Indirect**	18.50 (2.46)	1.11 (0.08)	1.90 (0.30)	2	0.84 (0.99)	1 (0)	1 (0)	1
Rational	**TTCC e**	14.35 (2.21)	2.96 (0.43)	5.65 (1.15)	9	0.39 (0.63)	2.05 (1.35)	2.16 (1.37)	6
Rational	**TTCC f**	14.37 (2.21)	2.96 (0.43)	5.65 (1.15)	9	0.39 (0.63)	2.02 (1.29)	2.13 (1.31)	6
Cautious	**TTCC e**	16.50 (2.85)	2.60 (0.42)	5.42 (1.17)	10	0.46 (0.69)	1.82 (0.88)	2.00 (1.01)	4
Cautious	**TTCC f**	16.50 (2.85)	2.60 (0.42)	5.42 (1.17)	10	0.46 (0.69)	1.82 (0.88)	2.00 (1.01)	4
Waitlist 20%									
All	**Paired/Indirect**	18.50 (2.46)	1.11 (0.08)	1.90 (0.30)	2	1.85 (1.29)	1 (0)	1 (0)	1
Rational	**TTCC e**	14.05 (2.31)	2.98 (0.43)	5.65 (1.15)	9	0.97 (0.93)	1.83 (0.94)	2.13 (1.13)	5
Rational	**TTCC f**	14.07 (2.31)	2.98 (0.44)	5.65 (1.15)	9	0.97 (0.93)	1.80 (0.90)	2.09 (1.09)	5
Cautious	**TTCC e**	16.25 (2.82)	2.61 (0.42)	5.42 (1.17)	10	0.93 (1.03)	1.89 (0.90)	2.19 (1.12)	6
Cautious	**TTCC f**	16.25 (2.82)	2.61 (0.42)	5.42 (1.17)	10	0.93 (1.03)	1.85 (0.92)	2.12 (1.15)	6
Waitlist 40%									
All	**Paired/Indirect**	18.50 (2.46)	1.11 (0.08)	1.90 (0.30)	2	3.96 (2.02)	1 (0)	1 (0)	1
Rational	**TTCC e**	13.37 (2.50)	3.04 (0.47)	5.65 (1.15)	9	1.91 (1.46)	1.86 (0.83)	2.55 (1.35)	7
Rational	**TTCC f**	13.47 (2.48)	3.04 (0.47)	5.65 (1.15)	9	1.92 (1.47)	1.78 (0.76)	2.40 (1.17)	5
Cautious	**TTCC e**	15.74 (2.75)	2.62 (0.44)	5.42 (1.17)	10	1.84 (1.32)	1.81 (0.68)	2.26 (1.01)	6
Cautious	**TTCC f**	15.76 (2.72)	2.62 (0.44)	5.42 (1.17)	10	1.88 (1.33)	1.75 (0.67)	2.17 (0.97)	6

Table 11: Properties of Cycles and W-chains for n=100. Cycles of 1 pair length with an incompatible donor-patient pair are not counted.

Pref.	**Mechanism**	**Cycle No**	**Cycle Length (as pairs)**			**W-chain No**	**W-chain Length (as pairs)**		
			Mean	Max.	Longest		Mean	Max.	Longest
All	**No-Exchange**	54.79 (4.48)	1 (0)	1 (0)	1	0 (0)	-	-	-
All	**Paired**	64.19 (4.33)	1.15 (0.03)	2.00 (0.00)	2	0 (0)	-	-	-
Rational	**TTC**	36.53 (3.44)	4.22 (0.41)	10.14 (1.65)	15	0 (0)	-	-	-
Cautious	**TTC**	45.53 (4.29)	3.51 (0.33)	9.82 (1.81)	17	0 (0)	-	-	-
Waitlist 10%									
All	**Paired/Indirect**	64.19 (4.33)	1.15 (0.03)	2.00 (0.00)	2	2.31 (1.38)	1 (0)	1 (0)	1
Rational	**TTCC e**	35.20 (3.85)	4.28 (0.41)	10.14 (1.65)	15	1.05 (0.99)	3.02 (1.51)	3.54 (2.15)	14
Rational	**TTCC f**	35.21 (3.84)	4.28 (0.41)	10.14 (1.65)	15	1.05 (0.99)	2.95 (1.56)	3.47 (2.20)	14
Cautious	**TTCC e**	44.38 (4.84)	3.54 (0.34)	9.82 (1.81)	17	1.22 (1.03)	2.64 (1.38)	3.24 (1.96)	10
Cautious	**TTCC f**	44.39 (4.84)	3.54 (0.35)	9.82 (1.81)	17	1.22 (1.03)	2.64 (1.38)	3.26 (1.96)	10
Waitlist 20%									
All	**Paired/Indirect**	64.19 (4.33)	1.15 (0.03)	2.00 (0.00)	2	4.94 (2.22)	1 (0)	1 (0)	1
Rational	**TTCC e**	34.18 (4.02)	4.34 (0.43)	10.14 (1.65)	15	2.05 (1.30)	2.74 (1.26)	3.85 (2.28)	14
Rational	**TTCC f**	34.22 (3.99)	4.33 (0.43)	10.14 (1.65)	15	2.06 (1.32)	2.68 (1.28)	3.73 (2.25)	14
Cautious	**TTCC e**	43.07 (5.31)	3.58 (0.37)	9.82 (1.81)	17	2.56 (1.77)	2.71 (1.34)	3.71 (1.84)	10
Cautious	**TTCC f**	43.11 (5.37)	3.58 (0.38)	9.82 (1.81)	17	2.57 (1.77)	2.65 (1.26)	3.67 (1.85)	10
Waitlist 40%									
All	**Paired/Indirect**	64.19 (4.33)	1.15 (0.03)	2.00 (0.00)	2	10.24 (3.07)	1 (0)	1 (0)	1
Rational	**TTCC e**	32.91 (4.16)	4.41 (0.43)	10.14 (1.65)	15	3.96 (1.97)	2.40 (0.93)	3.96 (1.97)	11
Rational	**TTCC f**	32.92 (4.09)	4.41 (0.44)	10.14 (1.65)	15	4.07 (2.04)	2.29 (0.82)	3.78 (1.79)	10
Cautious	**TTCC e**	42.00 (5.43)	3.61 (0.39)	9.82 (1.81)	17	4.72 (2.60)	2.17 (0.65)	3.76 (1.64)	10
Cautious	**TTCC f**	42.07 (5.47)	3.61 (0.39)	9.82 (1.81)	17	4.75 (2.62)	2.14 (0.64)	3.71 (1.61)	10

Table 12: Properties of Cycles and W-chains for n=300. Cycles of 1 pair length with an incompatible donor-patient pair are not counted.

Pref.	**Mechanism**	**Cycle No**	**Cycle Length (as pairs)**			**W-chain No**	**W-chain Length (as pairs)**		
			Mean	Max.	Longest		Mean	Max.	Longest
All	**No-Exchange**	161.76 (8.47)	1 (0)	1 (0)	1	0 (0)	-	-	-
All	**Paired**	193.42 (7.42)	1.16 (0.02)	2.00 (0.00)	2	0 (0)	-	-	-
Rational	**TTC**	79.54 (4.91)	5.98 (0.43)	16.84 (2.41)	26	0 (0)	-	-	-
Cautious	**TTC**	102.14 (7.00)	4.89 (0.33)	16.00 (2.00)	22	0 (0)	-	-	-
Waitlist 10%									
All	**Paired/Indirect**	193.42 (7.42)	1.16 (0.02)	2.00 (0.00)	2	7.48 (2.63)	1 (0)	1 (0)	1
Rational	**TTCC e**	76.11 (5.52)	6.10 (0.44)	16.84 (2.41)	26	2.38 (1.38)	4.26 (2.27)	6.01 (3.06)	16
Rational	**TTCC f**	76.11 (5.52)	6.10 (0.44)	16.84 (2.41)	26	2.38 (1.38)	4.26 (2.27)	6.01 (3.06)	16
Cautious	**TTCC e**	99.02 (6.92)	4.94 (0.32)	16.00 (2.00)	22	2.53 (1.95)	3.82 (1.98)	5.69 (2.95)	16
Cautious	**TTCC f**	99.07 (6.86)	4.94 (0.32)	16.00 (2.00)	22	2.53 (1.95)	3.78 (1.98)	5.58 (2.89)	16
Waitlist 20%									
All	**Paired/Indirect**	193.42 (7.42)	1.16 (0.02)	2.00 (0.00)	2	14.88 (3.66)	1 (0)	1 (0)	1
Rational	**TTCC e**	74.42 (5.38)	6.15 (0.44)	16.84 (2.41)	26	4.56 (2.27)	3.46 (1.25)	6.32 (2.89)	16
Rational	**TTCC f**	74.44 (5.37)	6.15 (0.44)	16.84 (2.41)	26	4.57 (2.27)	3.44 (1.24)	6.27 (2.91)	16
Cautious	**TTCC e**	97.00 (7.23)	4.98 (0.33)	16.00 (2.00)	22	4.85 (2.79)	3.28 (1.31)	6.07 (2.64)	13
Cautious	**TTCC f**	97.04 (7.17)	4.98 (0.33)	16.00 (2.00)	22	4.86 (2.80)	3.26 (1.32)	6.01 (2.63)	13
Waitlist 40%									
All	**Paired/Indirect**	193.42 (7.42)	1.16 (0.02)	2.00 (0.00)	2	29.30 (5.18)	1 (0)	1 (0)	1
Rational	**TTCC e**	71.17 (5.88)	6.30 (0.46)	16.84 (2.41)	26	9.05 (4.07)	2.88 (0.78)	6.47 (2.40)	15
Rational	**TTCC f**	71.21 (5.85)	6.30 (0.46)	16.84 (2.41)	26	9.06 (4.06)	2.86 (0.79)	6.46 (2.50)	15
Cautious	**TTCC e**	93.76 (7.51)	5.05 (0.33)	16.00 (2.00)	22	9.57 (4.20)	2.65 (0.72)	5.99 (2.23)	12
Cautious	**TTCC f**	93.81 (7.48)	5.05 (0.33)	16.00 (2.00)	22	9.58 (4.20)	2.64 (0.73)	5.96 (2.23)	12