Missing Contracts:
On the Rationality of not Signing a Prenuptial Agreement*

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Abstract

Many couples do not sign prenuptial agreements, even though this often leads to costly and inefficient litigation in case of divorce. In this paper we show that strategic reasons may prevent agents from signing a prenuptial agreement. Partners which have high productivity in marital activities wish to signal their type by running the risk of a costly divorce. Hence this contract incompleteness arises as a screening device. Moreover, the threat of costly divorce is credible since the lack of an ex-ante agreement leads to a moral hazard problem within the couple, which induces partners to reject any ex-post amicable agreement, under specific circumstances. We also investigate conditions that make this contract incompleteness an optimal form of contracting and we briefly discuss the effects of enforceable and/or mandatory premarital agreements on the rate of divorce and on the social welfare.

Finally, our model suggests that there is no major objection in making prenuptial agreements enforceable, but also that there are not good reasons to make them mandatory.

Keywords: asymmetric information, incomplete contracts, prenuptial agreement.

JEL Classification: D82, K12, D10.

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1 Introduction

In the present paper we study why economic agents may rationally choose not to sign specific kind of contracts, or equivalently, why they may omit specific clauses when signing a contract. Our leading example will be the modest diffusion of prenuptial agreements.

Prenuptial agreements usually include clauses on how partners should behave during marriage and on how they should divide the common assets in case of divorce. Even though it is quite clear that prenuptial agreements allow savings in litigation costs, they are still uncommon. "Legal commentators and practitioners estimate that only 5-10% of the (USA) population enter into prenuptial agreements, and one study suggests that only 1.5% of marriage licence applicants would consider entering into such agreement" (H. Mahar, 2003). Referring to the literature on incomplete contracts, the usual justifications for this phenomenon are the lack of enforceability, the presence of transaction costs and, finally, agents' excessively optimistic expectations.

None of these explanations survives at a closer scrutiny in the case of prenuptial agreements. For instance, many States in USA have nowadays adopted the Uniform Premarital Agreement Act (UPAA)[1]. This Act provides that courts must enforce premarital agreements, whenever they satisfy some simple formalities[2]. Similar reforms have been adopted in other countries, e.g., Australia. Therefore it is difficult to assume that in these countries (or states) prenuptial agreements are difficult to enforce.

[1] Drafted by the National Conference of Commissioners On Uniform State Law in 1983. This Act is now adopted in some form in many States.

[2] "A premarital agreement must be in writing and signed by both parties. It is enforceable without consideration." (UPAA, Section 2). Section 6 establishes substantive requirements for enforceability. Basically it is required the agreements to be voluntary executed and each party to have (before execution) an adequate knowledge of the property or financial obligations of the other party.

As far as it regards the cost of prenuptial agreements, it might be very high if the involved agents have complex activities, but for more common people many Internet sites offer kits which help to stipulate premarital agreements at a very low cost, without (the help of) any legal advisors.[3] The costs of contracting might also be a consequence of forecasting problems. Also this explanation is not satisfactory, since many marriages end within the early years of the union (one fifth of first marriages ends within 5 years, and one third ends within 10, see Bramlett and Moshler (2001)). Therefore the rate of divorce is high in the first years of marriage, making less difficult to write ex-ante a satisfactory agreement on how to divorce.

Nevertheless couples could have too optimistic estimates of the probability of divorce. For instance there could be a wrong perception of the overall divorce rate.[4] However, from interviews conducted in the US it seems that people correctly estimates the divorce rates, but that most people assume a much lower probability that they will personally divorce than the overall rate (H. Mahar, 2003). Thus there is some evidence of excessively optimistic expectations, but wrong expectations can never explain why most people do not even sign postnuptial agreements when marriage comes into a crisis, or why a significant number of couples choose an adversarial divorce and not an amicable one.

If none of the previous explanations seems convincing, we still have to find one. The already quoted contribution by H. Mahar (2003) suggests an interesting one. In the interviews there reported, 60% of the respondents considers receiving a proposal of a prenuptial agreement a bad signal. The *signaling content* of the contract will be our starting point.

[3] See, for example, www.mylawyer.com for US law, or www.canlaw.com for Canadian law.

[4] See, for instance Baker and Emery (1993).

The stream of literature more closely related to our contribution, takes into consideration the strategic contents of the contracting activity. Non-contigent contracts as a signaling/ screening device are analyzed in Aghion-Bolton (1987), Diamond (1993) Hermalin (2001), Bordignon and Brusco (2001), and especially Spier's (1992). Incomplete contracts may help in establishing the appropriate incentive in presence of imperfect verifiability (Bernheim and Whinston (1998)). However, even though the last two contributions deal with the endogenization of incomplete contracts, none of them is able to prove that the contract remains incomplete as the costs of the complete contract (included the cost of verifiability) disappear. In Spier (1992) the opposite is proved. Therefore, in these contributions strategic considerations can amplify contract incompleteness, but they cannot be their primary source.

Summarizing what said so far, a missing (premarital) agreement is an extreme form of contract incompleteness, however, we depart from the incomplete contracts literature quite substantially. In fact, this literature justifies incomplete contracts through the presence of some form of complexity costs. These can be either costs associated with the difficulty (or impossibility) to foresee the future contractually relevant contingencies (Hart and Moore (1990), (1999)), or the costs of writing ex-ante the contract (Dye (1985), Anderlini and Felli (1999) (2001), Battigalli and Maggi (2002)), or finally the costs of verification ex-post (Townsend (1979)).[5] Hence in all this literature it is assumed that complete contracts are more costly than incomplete ones. On the contrary, in our contribution, we make the opposite assumption. If we go back to prenuptial agreements, their absence can be a credible signaling and/or screening device, if (literally) incomplete contracts

[5] However, see also Maskin and Tirole (1999), Tirole (1999)) in their critique to the incomplete contract literature.

are more costly than the complete ones. It appeared quite natural to us assuming that *incomplete contracts are more costly to implement than complete ones* because they imply higher litigation costs, but quite surprisingly, to our knowledge, we are the only ones to exploit this feature. [6]

After what we said, a natural question arises: is our model an incomplete contracts one? Even though the question appears to be rather simple, it has no easy answer. On the one hand, our contracts are literally incomplete. On the other hand, literal incompleteness does not imply any unforeseen contingency. In fact, parties can predict their course of action in any future contingency and therefore in our model writing an incomplete contract is an optimal (in the second best sense) form of contracting.[7] Hence we leave to the reader the decision on whether our model deals with incomplete contracts or not. However, we are convinced that our paper can provide two minor, but important, contributions to the incomplete contract literature. In order to explain these contribution, recall that, loosely speaking, there are two kinds of incomplete contracts: flat contracts, i.e., those specifying the same action in different contingencies, and those not specifying what to do in some contingency.

Our model endogenously generates a literally incomplete contract according to the latter sense, but it is not able to generate a flat contract. This result suggests that the two form of incompleteness might have different explanations. Current literature neglects this point, because it tries to derive both forms of contract incompleteness by cost of complexity. The

[6] Focusing on litigation costs which derive from incompleteness, our paper is also linked with the literature on the breach of contracts (e.g. Rogerson (1984), Shavell (1980)). Anyway, this literature typically takes for granted that private contracts are incomplete and analyzes under which conditions clauses which specify damage for breach may approximate efficiency. On this respects the focus of our paper is different since we allow parties to sign efficient and complete contracts where all aspects of the divorce process are specified.

[7] This resembles an old idea that the absence of contigent dealings is closely related to moral hazard and imperfect information (see, for instance Arrow (1974).

second contribution to the general theory of incomplete contract is that literal incompleteness might matter. Also this point is not considered in the general literature because it is assumed that literal incompleteness has no effect when renegotiation is allowed.[8]

Let us see why this is not the case in our model, analyzing first the problems induced by renegotiation. Suppose that the absence of a prenuptial agreement is a signaling and/or screening device, and suppose that the equilibrium is separating, that is, agents more willing to spend effort to the benefit of the partnership do not sign prenuptial agreements, while the others do and marriage is between spouses of the same type. Call the former type of agents the high productivity (*high* for short) one and the latter the low productivity (*low* for short) one. Then, one should expect that after marriage high type couples sign postnuptial agreements, that is, renegotiate the prenuptial arrangements. However, if they do so, the lack of a prenuptial agreement would not be a credible signal and agents' types would not separate. In fact, the low type agents would find it optimal to imitate those of the high one by not signing pre-nuptial agreements, but signing only postnuptial ones. Said it differently, contracts would be literally incomplete, but they would be completed afterwards with no real effects on couples.

In our model couples will not necessarily sign a post-nuptial agreement. Intuitively, mutual distrust prevents real world couples to sign post-nuptial agreements when they start fighting. In order to model this "mutual distrust" we added a second stage to the signaling one, where a moral hazard problem takes place. Moreover, the absence of prenuptial agreements worsen the moral hazard problem, since it leaves undefined what has to be considered an appropriate behavior during marriage. In this setup, for some

[8] See Hermalin and Katz (1993).

parameter values and some histories of the game, the lack of a prenuptial agreement is renegotiation proof, since the proposal of a post-nuptial one is taken as a signal of unmonitored misbehavior of the proposer.

Another way to explain our model is to tell the same story backwards. Spouses usually engage in common activities but they might exert different levels of effort, which are not observable. In equilibrium, the high types can run the risk of being expropriated of the results of their effort in case of divorce, because they have a lower probability of divorcing. On the contrary, in equilibrium, the low type cannot.

Which are the substantive prediction of our model? First of all, our model exhibits a (divine Bayesian-perfect) equilibrium with the coexistence of couples who sign prenuptial agreements, who do not sign prenuptial agreements, but then divorce amicably (signing a postnuptial agreement) and finally couples who end their marriage by means of costly litigations in front of a court. This result is obtained in a model with a relatively simple structure. We think it as an important feature of our model since all these cases are observed in reality. Second, in equilibrium litigation takes place when there is some sort of asymmetry in the production activities, i.e., when one partner is more productive than the other.

Moreover, the model enables us to explore some issues about the (social) optimality of prenuptial agreements. There are two main issues. The former is about the desirability of enforceable prenuptial agreements. The latter can be phrased in Becker's (1998) wording: why don't we make prenuptial agreements mandatory, since they save considerable litigation costs?

In the paper we prove that the introduction of enforceable premarital agreement, for specific parameter values, has no effect on the rate of divorce, namely it does not increase it, and therefore it turns out to be false one of the main argument against their enforceability. Proving social optimality of enforceability is a more difficult question, since without enforceability the only possible equilibrium is a pooling one. In our model the separating equilibrium does not clearly (Pareto) dominates the pooling one, nor viceversa.

Finally, we also proved that for some parameter values the high types would like and are able to separate from the low types by not signing prenuptial agreements. Hence, if prenuptial agreements become mandatory, the high type agents will be compelled to find new costly ways to separate from the low ones. Thus mandatory agreements in our model would be at best irrelevant. Therefore, our model suggests some cautiousness in imposing the adoption of mandatory prenuptial agreements.

In summary our model suggests that there should be no major objection in making the prenuptial agreements enforceable, but also that there are not good reasons to make them mandatory.

The main limitation of our model is that, in order to solve it in a relatively easy way, we had to assume symmetry among agents. This implies that we are not able to study the effects of asymmetries between genders and/or between partners in terms of wealth, kind of assets and capabilities owned.

The paper proceeds as follows. In Section 2 we describe the bench mark model and we present how courts rule on divorce when agents have not drawn up divorce clauses in their marriage contracts. In Section 3 we state our main result and in the following Section 4 generalize this result discussing the optimal contract, proving the uniqueness of the proposed equilibrium. In Section 5 we briefly discuss some assumptions and interpretations of the model, including its applicability to other kind of partnership, like joint venture and mergers. Section 6 concludes.

2 The Agents' Model

Population is constituted by many agents, N, who live two periods and in period 0 have a private endowment equal to 1. Each agent i can use the private endowment as an input factor, denoted by e, to produce a durable goodAlternatively i can enjoy the endowment as leisure, denoted by l. Hence: $e_i+l_i \leq 1$ for each agent $i \in N$. For simplicity, let assume that $e_i, l_i \in \{0,1\}$. We refer to the activities l_i and e_i respectively as agent i's leisure and effort, but we may think at them as money, time or abilities, etc., spent respectively for private consumption or for the production of the durable good. There exist two durable goods, G_1 and G_2, and each agent has to specialize in the production of one of them. Production of the durable good $G_i \in \{0,1\}$ is a risky activity which depends on the effort devoted by the agent i. We denote by $\mu_{e_i}^r$ the probability that $G_i = 1$ when agent i has devoted effort e_i in the production activity, with $\mu_1^r > \mu_0^r$. There are two types of agents, who differ for the probability of producing successfully the durable goods: $r = \{l, h\}$, $\mu_1^h > \mu_1^l$, but $\mu_0^h = \mu_0^l > 0$. Hence the high type has a comparative advantage with respect to the low one in producing the public goods. The total numbers of the high and low types are even numbers. We assume that:

$$\mu_1^l - \mu_0^l > \frac{1}{2} \tag{1}$$

The meaning of this assumption will be made clear shortly below. In essence, it ensures that unmatched agents prefer to exert effort rather than enjoying leisure. During their marriage, each agent benefits of both goods produced by the partners: durable goods are "public goods" within the couple.[9] There is no discounting.

Agents are matched in pairs; θ is the degree of fit of the partners and we assume that it is a random variable with $\theta \in \{\theta_b, \theta_g\}$, with $\theta_b \leq -1, \theta_g > 0$, $E(\theta) \geq 0$. The assumption that $\theta_b \leq -1$ implies that the psychological aspects of marriages, summarized by the degree of fit, are more relevant than the economic ones. More to the point, the psychological aspects can drive couples to divorce even when successful from a productive point of view. We deserve this as a reasonable assumption for relationships like marriages. Let $p = \Pr\{\theta = \theta_b\}$. We assume that the degree of fit, θ, and the production of the durable goods, G_i, are uncorrelated. Agents' expected utility function is linear and separable in all components. Given our assumptions, all agents prefer to be matched with a high-type partner rather than with a low-type one. If an agent decides to marry, the ex post utility function is:

$$u_i = G_i + \theta + G_j + l_i + \xi\left[\theta + G_i + G_j\right] \\ + (1-\xi)\left[\alpha_i G_i + (1-\alpha_j) G_j - \Phi_i\right]$$

where ξ is an indicator functions such that $\xi = 1$ if agent i has a partner at time $t = 1$ and $\xi = 0$ otherwise; α_i and $(1-\alpha_j)$ are respectively the portions of good G_i and good G_j that agent i receives according to the divorce rule (and $(1-\alpha_i)$ and α_j are the portions of j), and Φ_i is the sum of the litigation costs and monetary transfers (eventually negative) from i to j.

The information structure of the game is as follows. The agents cannot

[9] For a justification of these assumptions see for instance Weiss (2001).

observe the degree of fit θ until they marry. Agents can observe the goods eventually produced but not the level of effort. Courts (or any third party) may costlessly observe the amount of total production, but verifying the level of effort is a costly activity. This assumption will be discussed in the closing section. The cost of this activity is assumed to be equal to $2F$. Thus, going to court is a costly way to verify some pieces of information.

The timing of the game is the following. In the first stage nature selects high types with probability q and low ones with complementary probability. Then each agent announces the contract that he is going to propose ("no contract" is an admissible announcement). Writing (and reading) a contract costs c, where c is a fixed and arbitrarily small amount. Then a matching phase starts. We will be more precise on matching later on. Namely, we will introduce a simplified matching mechanism in the bench-mark case and a more sophisticated one when generalizing the model. After matching, marriage begins and each agent decides whether to devote effort in producing a durable, (public within the couple), good or to enjoy leisure. Nature determines the degree of fit of the married partners, who afterwards observe the outcome and the level of production. Spouses simultaneously decide whether to continue the marriage or to end it. Divorce occurs if at least one of the partners wishes to end the marriage. In case of divorce spouses may negotiate an ex-post agreement. We simply assume that Nature draws up one of the two partners who is entitled to propose an ex-post agreement, while the other can accept or refuse it. In order to maintain this simplified bargaining structure without generating odd results, we impose that proposals must guarantee to the partner at least an equal division of marital assets and expenses. The proposer can always propose agreements which are more (less) favorable to the partner (himself).

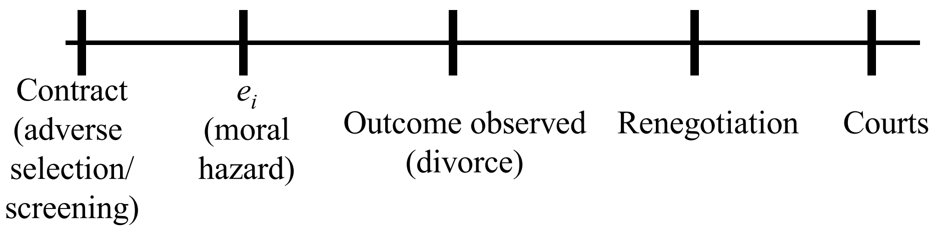

Figure 1: Timing of the Game

Proposing (and accepting) an ex-post agreement has the same cost as an ex-ante agreement, c. If the proposal is accepted the ex-post agreement is enforced. On the contrary, if the proposal is rejected, spouses go to court.

We assume that both ex-ante and ex-post marriage contracts are enforceable. A (ex-ante) contract prescribes both some specific behavior that agents have to follow during the relationship and how to divide the joint production in case of divorce, eventually conditioning the sharing rule to the prescribed actions, whenever they are verifiable. Given the structure of the game, an ex-post contract may only determine a sharing rule, while a prenuptial one can determine also the activity in which each partner has to specialize. As mentioned above, when no contract has been drawn by the parties, then a court decides how to divide the common assets.

Since the verification activity is costly, going to court implies a loss of efficiency. In our model spouses can avoid the cost of litigation by signing this (complete) contract:

Definition 1 *The simple contract:*

1. *agent* 1 *is in charge to producing good* G_1 *and agent* 2 *is in charge to produce good* G_2*;*

2. *in case of divorce each agent receives the eventually produced good.*

Partners who sign the simple contract have no cost in divorcing and therefore their decision (whether divorcing or not) is ex-post efficient; therefore they always divorce when the match is bad, $\theta = \theta_b$, since we assume that $\theta_b \leq -1$. Moreover, given condition 1, the simple contract induces agents to exert effort.[10]

Hence, in case of complete information, i.e. in the case where agents' types are known, spouses will write a complete contract in order to avoid the moral hazard problem and to attain the efficient outcome.

If agents decide to divorce in front of a court, then the following rules apply.

The Court's Rules Divorce rules adopted by courts vary among different countries and States. There are basically two regimes: (i) a "community property" regime, which essentially means that marital property belongs equally to the partners and in case of (no-fault) divorce it is equally shared; (ii) an "equitable division" regime according to which the "division of jointly owned marital property and the amount of any monetary transfer between the partners is determined by a court in order to arrive at a fair and equitable solution". [11] In this second case the marital property is not always divided equally, and many factors, such as the contribution of each party to the well-being of the family and to the acquisition of the marital property, the circumstances and factors which contributed to the dissolution of the marriage, the personal characteristics (age, mental condition) of each partner, etc., are usually taken into account by the court. In our model agents may differ for their type and the effort they exerted, but the courts

[10] Note that agents who sign the simple contract stricly prefer to marry rather than to remain single.

[11] The Code of Virginia § 20-107.3. In US a community property regime is adopted, for instance in Arizona, California, and Texas, but the majority of States provide for equitable rather than equal distribution.

may only observe this second characteristic and then equitable divorce rules should only depend upon the levels of effort. We restrict a divorce rule to select an equal division of the property (without any monetary transfer) in all cases where parties behaved identically, either exerting effort or not exerting it. In case, the parties exerted a different level of effort and an agent used his/her own endowment for personal well-being, we require that court adopts the rule which (i) awards the other party with a monetary compensation in order to offset the agent who has exerted effort for the well-being of the couple; (ii) assigns at least half of the marital asset to the agent who exerted more effort. Let $k \in \{0, 1, 2\}$ be the amount of public good produced within the couple. It follows that in case one agent exerted effort and the other did not, the shirking agent has to pay a monetary amount $m_k \in \left[\frac{1}{2}, 1\right]$ to the partner, moreover a share $s_k \in \left[\frac{1}{2}, 1\right]$ of the k public goods is assigned to the agent who exerted effort in production. Given that the public good has the same monetary value for both partners, then we may express this class of rules in a compact way. Define the transfers received by the working and shirking partner respectively as T_{ik}^{+} and T_{ik}^{-}. For what said before they are equal to:

$$
\begin{aligned}
T_{ik}^{+} &= s_k k + m_k \\
T_{ik}^{-} &= (1 - s_k)\, k - m_k
\end{aligned}
$$

In order to simplify calculations, we will also express T_{ik}^{+} and T_{ik}^{-} as:

$$
\begin{aligned}
T_{ik}^{+} &= \beta_k\,(k+1) \\
T_{ik}^{-} &= k - \beta_k\,(k+1)
\end{aligned}
$$

This second formulation is easier to use since it summarizes in a single parameter, $\beta_k = \frac{s_k k + m_k}{k+1}$, two of the first one, s_k and m_k. Therefore, the court rule can be expressed as:

$$T_i = \begin{cases} T_{ik}^{+} & \text{if} \quad e_i > e_{-i} \\ T_{ik}^{-} & \text{if} \quad e_i < e_{-i} \\ \frac{1}{2}k & \text{if} \quad e_i = e_{-i} \end{cases}$$

Finally note that $\beta_k \geq \frac{1}{2}$ for all $k = 0, 1, 2$.

3 The Bench-Mark Case

In this section we will present the simplest version of the model, which we call the bench-mark case (BMC). In the next section we will generalize the results and discuss the uniqueness of the proposed equilibrium and whether contract incompleteness is an optimal form of contracting. We first describe the matching mechanism which characterizes the bench-mark case.

The Matching Mechanism Each agent in the population proposes a contract. Agents are drawn in pairs from a ballot box containing the entire population. If both agents propose the same contract, then this contract is going to be signed (and no contract is signed whenever both agents do not want to sign any ex-ante contract). If agents propose different contracts, then agents are put again in the ballot and new pairwise extractions are made. Pairs are sequentially drawn and agents do not know how many pairs were drawn before them. Matching is in logic time and ends when all the remaining agents were already matched with all the others left in the ballot.

3.1 A Training Example: No-Renegotiation and No Moral Hazard.

We provide now a simplified example which may help in understanding our results. First of all, we impose that all partners will exert effort. Therefore the moral hazard problem during marriage cannot takes place. Second, we do not allow for renegotiation, namely, no postnuptial agreement can be signed. These assumptions will be removed in the general model, when we allow for renegotiation between partners. Since all partners exert effort in the production activities, then in case of divorce in front of a court, the marital asset will be always equally shared. Second, we assume that

$$\frac{1}{2} > -\theta_b - F > 0 \tag{2}$$

where F is the cost of litigation[12]. Assumption 2 implies that those partners who do not sign a prenuptial agreement (and therefore have to face a costly litigation) divorce if and only if there was a bad match and no agent produced successfully the public good.

We show that, for some parameters value, there exists a separating equilibrium where all couples are formed by agents of the same type, high-type agents do not write premarital agreements and low-type agents do sign the simple contract of Definition 1. Moreover high-type agents divorce when $\theta = \theta_b$ and no agent produced, low-type agents divorce whenever $\theta = \theta_b$, as it was argued when the simple strategy was introduced. Therefore the divorcing decision of the low type is ex-post efficient, while high-type partners suffer inefficient continuation of marriage when at least one public good is

[12] In this example F is not the cost of verification, but some other kind of legal costs, as the costs for legal advice, etc.

produced, due to the presence of the litigation costs.[13]

In equilibrium the high type agent has the following utility:

$$
\begin{gathered}
E[\theta]+2\left(\mu_1^h\right)^2+2\mu_1^h\left(1-\mu_1^h\right) \\
(1-p)\left(\theta_g+2\left(\mu_1^h\right)^2+2\mu_1^h\left(1-\mu_1^h\right)\right)+ \\
p\left(\mu_1^h\right)^2(\theta_b+2)+2p\mu_1^h\left(1-\mu_1^h\right)(\theta_b+1)-(1-\mu_1^h)^2F
\end{gathered}
$$

where the first line is the expected utility of period one, the second line is the expected utility in period two in case the match is good, and the third line is the expected utility in period two in case the match is bad.

To provide a numerical example, let assume that $p=\frac{1}{2}$, $-\theta_b=\frac{4}{3}$, $\theta_g=2$ (so that $E(\theta)=\frac{1}{3}$), $c=0$ (writing a contract is costless), (half of) the litigation costs are $F=1.2$ and finally the two expected productivities are $\mu_1^h=\frac{9}{10}$ and $\mu_1^l=\frac{3}{4}$. Substituting these numbers in the previous formula, we obtain the high type agent's equilibrium expected utility level: $\frac{1}{3}+3.928$.

If the high-type agent deviates and joins a low-type partner, then she obtains an expected utility level equal to $\frac{1}{3}+3.925$ [14], which compared to the equilibrium expected utility level guarantees that the self-selection constraint for the high-type is satisfied.

For the same parameter values and with analogous formula as those for the high type, in equilibrium the low type has an expected utility equal to:

[13] We will prove in the general case that this feature of the model disappears if we allow for renegotiation: hence there will be no inefficient divorce decision.

[14] The expression above comes from the formula of the expected utility of the deviating high-type:

$$E[\theta]+\mu_1^h+\mu_1^l+(1-p)\left(\theta_g+\mu_1^l+\mu_1^h\right)+p\mu_1^h$$

$\frac{1}{3} + 3.625$, while if he deviates joining a high-type, he obtains a utility level equal to: $\frac{1}{3} + 3.62$. It follows that also the self-selection constraint of the low-type is satisfied.

3.2 Results with Renegotiation and Moral Hazard

Let us come back to the more general formulation of the model. The lack of enforceability of postmarital agreements is hard to support both from a theoretical point of view and in practical terms, since in many legislations where premarital agreements are enforceable, ex-post marital agreement are enforceable too. If agents may write ex-post enforceable agreements, the previous equilibrium (outcome) does not survive if there is no moral hazard problem during marriage. [15]

Consider the case where the level of production is equal to one and $\theta = \theta_b$ in the example of the previous section. In equilibrium a high-type agent remains married in the second period and her utility is equal to $-\frac{1}{3}$; this agent would obtain a greater utility offering to the partner to equally divide the marital asset in order to divorce amicably. Moreover, accepting the offer is a dominant strategy for the partner.

In the general model, however, provided that some conditions are satisfied, there exists a Bayesian perfect separating equilibrium where all partnerships are formed by agents of the same type. High-type agents propose to the selected partner to sign "no contract", low-type agents propose to sign the simple contract of definition 1. In equilibrium all agents exert the efficient level of effort: low-type because the simple contract guarantees the

[15] In some legislations post-marital agreements are enforceable if they are signed some years before a partner starts the divorce procedure. In our model we do not make any distinction between postmarital agreement and amicable divorce. The only relevant distinction is whether the ex-post agreeement is proposed before or after the effort is exerted. Here we concentrate in the second case and we briefly discuss the first in the final section, given that our results do not change.

right incentive in order for the spouses to exert effort, high-type agents since the probability of adversarial divorces is positive and courts are going to punish the cheating spouse. Hence, in equilibrium the moral hazard problem is solved. Since in equilibrium all agents do exert the maximal level of effort, litigation in a world of symmetric information would never occur. Nevertheless, high-type agents choose not to complete their contracts in some states of the world. In fact, we prove that when renegotiation is admissible and both partners had the same productivity (either 0 or 1) they divorce amicably, but when one partner was productive and the other was not they end their marriage in front of a court. Inefficient litigation is the result of a signaling game. Players who propose an ex-post agreement signal something about their behavior during the relationship. When only one public good was produced, the receivers assign probability 1 that the proposer of an ex-post agreement has exerted zero effort. Given this belief it is optimal for the receiver to refuse the renegotiation proposal, because the court will force the partner to compensate for the misconduct. Finally, in our equilibrium all marriages end when the match is bad: there is no inefficient continuation of marriage.

Proposition 1 *There exist upper and lower bounds on F, the litigation costs, and on μ_1^h and μ_1^l, the types' productivities, (conditions (C) of the appendix), for which there exists a separating perfect Bayesian equilibrium such that:*

1) all marriages are formed by agents of the same type;

2) all agents exert the efficient level of effort;

3) high-type agents do not sign any contract;

4) low-type agents sign the simple contract and therefore never incur in litigation costs.

5) all couples divorce when $\theta = \theta_b$ (the degree of fit is negative);

6) there will be a postnuptial agreement when high-type partners decide to divorce and (i) no good was produced, or (ii) both goods were produced;

7) high-type partner face costly litigation if only one of the two high-type partners produced the public good (and $\theta = \theta_b$);

8) when only one good was produced, high-type agents who receive an ex-post agreement proposal will believe with probability one that the partner shirked and consequently refuse it; therefore, nobody will propose an ex-post agreement.

Proof. See the Appendix. ■

The separating equilibrium does exist since for high-type agents the gain of joining with a high-productivity partner is greater then the expected cost of facing a contentious divorce, while the opposite holds true for low-types agents. This occurs since the ex-ante probability to face a contentious divorce for a deviating low type who joins a high type is greater than the probability of a costly divorce for a high-type agent in equilibrium.

Conditions (C) is described in details in the appendix. However, the conditions fix lower and upper bounds on the litigation costs F, on the low types' productivity, μ_1^l, and a lower bound on the high types' productivity, μ_1^h. In fact, if F is very low, the threat of costly litigation when no premarital agreement is signed is not strong enough to induce low type agents to separate and hence to sign contracts containing divorce clauses. On the contrary, if litigation costs are very high, then, in case the marriage ends, all agents who did not sign divorce clauses, will write an agreement just before going to the court. In this case *contract incompleteness* cannot be a credible threat of costly litigation.

Bounds on μ_1^h and μ_1^l can be expressed as upper and lower bounds on

$\left(\mu_1^h - \mu_1^l\right)$. The difference in productivity of the two types can be thought of as the gain (loss) in separation for a high (low) type. Therefore the upper and lower bounds for $\left(\mu_1^h - \mu_1^l\right)$ are the mirror images of those on F and have the same interpretation.

The equilibrium of Proposition 1 is sustained by a specific out-of-equilibrium belief. That is, if a partner receives an ex-post negotiation proposal and the total production is equal to one, he assigns probability one that the proposer shirked. This belief does imply that agents who exerted effort do not make any proposal when the production is equal to one and the match is bad. The next goal of this section is to show that this belief is the only one satisfying the weakest of the divinity criteria: the D1 criterion.

We will not define the D1 criterion formally. Its intuition is as follows. Suppose that one player is observed deviating from the equilibrium and that there are two different types of that player, types 1 and 0. Moreover, suppose that any belief that the deviating player might held, induces 0 to deviate whenever it induces 1 to do so, but not the opposite. That is, there are beliefs that induce 0 to deviate, but not 1. Then, according to D1, we must assign probability zero that the deviating player is of type 1.

Proposition 2 *For any positive, arbitrarily small, cost of proposing an ex-post agreement, c, if*

$$\left(\mu_1^h - \mu_1^h\right) \leq \frac{1}{(2 - \frac{1}{2}p)} \tag{3}$$

then only the equilibrium beliefs of Proposition 1 satisfy the D1 criterion, that is, the counterpart infers that the proposer shirked with probability 1 if only one public good was produced.

Proof. See the Appendix ■

The intuition of this proposition is the following. Agents who have not exerted effort are more prone to renegotiate since in case of litigation in front of a court they incur in the penalty that the court inflicts to shirking agent. For this reason an agent infers that the partner has shirked in case she receives an agreement proposal. Finally, in order to assign a positive probability to the fact the proposer did not exert effort, it must be the case that shirking is not a dominated strategy, as condition (3) guarantees.

Remark 1 *We assumed that both agents simultaneously choose to divorce, if $\theta = \theta_b$, before negotiating the divorce. Our argument still holds if we assumed that the proposer may also propose to continue the marriage. In this case, the equilibrium beliefs should assign probability 1 that the partner who proposes to continue the marriage is a shirker.*

Proof. See the Appendix ■

Finally, it is worth noticing that this proposition easily extends also to the generalized version of the model, which we are going to present.

4 Optimal Incomplete Contracts

In this section we want to prove that literally incomplete contracts not only arise as an equilibrium phenomenon, but they may be an optimal form of contracting. In order to prove this result, we have to modify the structure of the game in order to allow agents to sign any type of contract, possibly with a private arbitrator who can eventually enforce a contract implementing the same outcome as that described in the previous section. Thus, if partners can find a Pareto improvement with respect to the outcome of that equilibrium, they will choose it and we should not observe literal incomplete contracts in equilibrium. To this end, we have to generalize the matching stage, since each agent must have the chance to propose any type of contract to the potential partners without bearing the risk of remaining unmarried.

The Generalized Matching Mechanism Each agent proposes a contract ("no contract" is an admissible announcement) and describes the set of acceptable contracts (which has to contain the proposed contract). Agents are drawn in pairs from a ballot box containing the entire population. If both agents propose the same contract, then this contract is going to be signed (and no contract is signed whenever both agents do not want to sign any ex-ante contract). If agents propose different contracts, but they are both acceptable for the other, then one of these contracts is randomly chosen and signed. If only one of the contracts is acceptable for the other, then it is signed. If none of the previous cases occurs, then agents are put again in the ballot and new pairwise extractions are made according to the same rules as the previous matching mechanism.

4.1 Results

We assume that in equilibrium all agents accept to sign all contracts which guarantee them the same expected utility level than that provided by the contract they sign in the proposed equilibrium. It follows that whenever there exists any (literally) complete contract which guarantees a higher-utility to high-type agents, then contract incompleteness should not arise in equilibrium. In Remark 3 of the appendix we show how the equilibrium described in Proposition 1 has to be modified in order to take into account the new set-up.

We proceed now in the following way. First we show that there is no separating complete contract which guarantees to high-type agents a higher payoff than that of the BMC equilibrium. Hence, we are able to prove that a contract with missing clauses is the optimal separating contract, because the high types are in their first best contract provided they are separated. Finally, we provide sufficient conditions for our equilibrium to guarantee the highest utility to high-type agents, when we consider pooling equilibria too.

4.1.1 Separating Contracts without Third Parties

As before, we restrict our attention to symmetric contracts. Moreover, since we look at contracts without any arbitration and we allow for renegotiation, in equilibrium we cannot have contracts where there is waste of resources. To be more specific, we do not allow, for instance, contracts which contain clauses imposing to destroy part of the joint production in case of divorce. Hence, the only complete contracts which may succeed in separating types are those which "punish" the agent that was ex-post not productive. Since the productivity of the two types is different, an appropriate punishment can deter low-type agents from signing contracts with high-type partners. To summarize, a symmetric complete separating contract without arbitration:

a. specifies which good each agent has to produce and

b. imposes a penalty $P \geq 0$ to the non-productive agent which has to be paid to the counterpart, when only one agent produced and spouses decide to divorce.

Proposition 3 *If Conditions* (C) *of the appendix hold and*

$$\frac{4p\beta_1 - 1}{2\,(1-p)} \leq \theta_g \leq \frac{(2-p)}{p}$$

then high-type agents prefer the equilibrium described in Proposition 1 to any separating contract without arbitration.

Proof. See appendix ∎

The intuition of the restrictions on θ_g in previous Proposition 3 is the following. If $P \leq \theta_g$, then the contract would provide the first best outcome for high-types. However, when $P \leq \theta_g \leq \frac{(2-p)}{p}$ there is no "punishment" which may induce low-type to separate, because the expected gain in having a more productive partner is greater than the expected loss of paying this partner when he only had success in producing. If $P > \theta_g$ punishment may be large enough to deter low types from signing these contracts. In this case, however, the productive partner wishes to divorce even when the match is good, if she is the only productive agent, in order to receive P. Therefore, when $\theta_g \geq \frac{4p\beta_1 - 1}{2(1-p)}$ the expected loss due to inefficient divorce for a pair of high-partners who sign a complete contract, is larger than their expected cost of litigation when they do not write any contract.

4.1.2 Separating Contracts with Third Parties

We consider here contracts where a third party (an arbitrator) is involved. Contracts with arbitrator are used in order to ascertain the state of the world and therefore it is natural to assume that cost of verification for the arbitrator is at least $2F$, the state verification cost for the court. Moreover, we assume that any contract can be renegotiated by the two parties before the actual decision of divorcing. This assumption seems to us consistent with the framework used in the bench-mark case, where partners can sign ex-post agreement before divorcing. The assumption of renegotiability simply implies that the amount of transfers to the arbitrator in case both agents exerted effort (as it occurs in equilibrium!) has to be at most equal to $2F$. Any contract that provides for a higher payment to the arbitrator will be renegotiate before divorcing. Therefore, as in the case without arbitrator, parties cannot destroy resources in equilibrium. Nevertheless, we allow parties to write premarital agreements which provide for specific actions within the marriage (and not only in case of divorce). In particular, we allow contracts with costly state verification without divorcing. Under these assumptions we can prove the following proposition.

Proposition 4 *If conditions* (C) *of the appendix hold, then the high-type agents prefer the equilibrium described in Proposition 1 to any other separating contract with arbitrator, provided that costs of state verification for the latter are at least* $2F$*,* c *is arbitrarily small and* $0.36 \leq p \leq 0.5$*.*

Proof. See Appendix. ■

The intuition of Proposition 4 is the following. Contracts which call for state verification by means of a third party are more costly than the incomplete contract, except in one case, that is when the contract with arbitrator calls for state verification only when no good were produced. Nevertheless, in this case if the probability of divorcing is higher than 0.36, then the transfer that should be paid to the arbitrator in order to induce the low type to not sign this contract, has to be higher than the upper bound on F, $\bar{F}$, allowed according to conditions (C). However, renegotiation proofness forces the payment to be equal F, thus inducing the result that no separating equilibrium does exist for the relevant set of parameter values. Finally, the condition $p \leq \frac{1}{2}$ guarantees that all contracts which call for state verification when the match is good or for both levels of matches are more costly that the contracts which call for state verification when the match is bad.

4.1.3 The Pooling Equilibria.

Let us assume that a pooling equilibrium does exist. The utility level for each agent in this pooling equilibrium depends upon the probability to marry a high-type partner. If q tends to zero, i.e., there are few high-type agents in the population, then the high-type agent utility level tends exactly to his payoff when he deviates and marries a low-type partner in the separating equilibrium of Proposition 1. It follows that, at least for q sufficiently small or equivalently for a sufficiently high ratio, $1-q$, of the low-type, high-type payoff in the (efficient) pooling equilibrium is lower than their payoff in the separating equilibrium. But then it is easy to show that high-type agents are able to separate from low-type agents drawing up different contracts, namely choosing a complete contract which mimics the outcome of the BMC equilibrium described in the previous sections.

Corollary 1 *If conditions* (C) *of the Appendix hold,* $\frac{4p\beta_1-1}{2(1-p)} \leq \theta_g \leq \frac{(2-p)}{p}$*,* $0.36 \leq p \leq 0.5$*,* q *is not too big, state verification is at least as costly for any arbitrator than for courts, then there exist no equilibrium which ensures a higher expected utility to high type than the separating equilibrium with no premarital contracts.*

Proof. It is a direct consequence of the above discussion and all previous propositions. ∎

4.1.4 Enforceability of Premarital Contracts

In this paper we assumed that premarital agreements were enforceable. It might be argued that allowing couples to sign enforceable prenuptial contracts would increase the rate of divorce.[16] We are able to prove that in our setting couples who sign a premarital agreement divorce at the same rate as couples who do not sign any agreement.

[16] This increase could bring an efficiency loss whenever the social costs of divorces are higher than the private gains from separation.

In our model there does not exist any other costs of divorcing apart from the litigation costs. Therefore couples who sign a premarital agreement, setting to zero the litigation costs, always divorce efficiently. If $\theta = \theta_b < -1$ and partners did not divorce in equilibrium, then we would observe inefficient divorcing.[17]

Inefficient divorcing may occurs only if both the two following conditions hold: (i) the cost of litigations are higher than the costs of remaining married; (ii) any proposal to reach an ex-post agreement in order to avoid the litigation costs would be rejected by the partner. Let focus on condition (ii). Condition (ii) implies that a partner who receives the proposal of an amicable agreement assigns positive probability to the fact that the proposer did not exert effort. We prove that there exists at least a proposal, namely the "fair" proposal (each partner receives $\frac{1}{2}k$), such that the unique beliefs which satisfy the D1 criterion assign zero probability to the fact that the proposer has shirked. Intuitively, if no proposal is made, partners continue their relationship. Hence, if such an equilibrium existed, the equilibrium payoff is the same both for shirkers and non-shirkers. On the contrary, the risk of deviating and making a proposal is higher for the agent who did not exert effort, since in case of rejection the court is going to punish the shirking proposer.

Proposition 5 *All perfect Bayesian equilibria which satisfy the D1 criterion induce efficient divorcing in equilibrium, if c is sufficiently small.*

Proof. See Appendix. ■

This proposition implies the following remark.

Remark 2 *If ex-post amicable agreements are allowed and the cost of writ-*

[17] If $-\theta_b = 1$ and both partners have produced, then to remain married is Pareto efficient.

ing a contract, c, is small, then the introduction in the legislation of enforceable premarital agreements has no effect on the rate of divorce.

5 Discussion and Interpretation

Existence of other type of equilibria. We prove the existence of the separating equilibrium where partners who did not sign a premarital agreement litigate in front of a court only when the total production is equal to one. It may be proved that there are other separating equilibria in which high-type partners do not sign premarital agreeements. Namely, there exist a separating equilibrium where partners litigate if and only if the total production is less than two, and another one where they litigate only when the total production is equal to zero. In the proof of Proposition 4 we compare these equilibria with the equilibrium of Proposition 1: the first equilibrium is clearly Pareto dominated, while the second equilibrium may be proved to exist for a different set of parameters values. We chose to focus only on one equilibrium to keep the exposition as simple as we could. We picked the equilibrium in which we observe litigation when the production is asymmetric simply because we believe it more realistic.

Joint Production. In our model there is no joint production, but only joint consumption. We do not assume joint production in order to make the complete efficient contract as simple as possible; so that in our model the lack of prenuptial agreements clearly does not depend on any form of complexity costs. Notice, however, that in our model the crucial assumptions are that partners have different activities and that those activities result in different products. For instance if partners were involved with different roles in two joint production processes with two different outputs, our model would need minor modifications (the most important being that the endowment should be used in three different activities: leisure, production process 1 and production process 2).

Other Forms of Partnerships. The field of application of this model is mainly (pre- and post-) nuptial agreements. There are area where we observe similar phenomena. For instance, in the merger between Time-Warner and AOL a few analysts were concerned about the absence of de-merger clauses. It is possible that this absence could be explained by similar arguments as those developed in this paper, however we feel that in this case signaling consideration to third parties (i.e. the financial market operators) might be at least as important as the former. In fact, it is possible that the inclusion of de-merger clauses could be interpreted by the market as low level of trust on the economic efficiency of merger by the parties themselves, thus inducing higher financial costs. However, we think that relevant examples in the research joint venture literature can be found.

Matching. We restricted ourselves to matching rules which prevent agents from remaining unmatched. In fact, we were interested in analyzing how partners contract over their divorce and how different contracts affect the ex-post probability of divorce, while we were not concerned about how the divorcing decision affects the ex-ante probability to marry. However, we conjecture that generalizing the model would not change the qualitative results.

Renegotiation. In the model agents may complete their contracts after having observed the outcome, and therefore when they already exerted the effort. High-type agents reject any proposal in the renegotiation stage since they believe that the partner who makes a proposal did not exert effort. What does it happen if we allow agents to complete the contract at time 0, before effort is exerted? Let us consider a game where at time zero an agent may ask to complete the contract to the partner, but, consistently with this assumption, the partner may accept or reject the proposal. If she rejects, she can then decide whether to continue the marriage or to divorce in order to marry with a different partner. It is easy to show that our main argument still holds. There exists a separating equilibrium where high-type agents reject the proposal to complete the contract since they believe with probability one that the partner who makes the proposal is a low-type. Moreover, (after rejection) they divorce in order to find a new partner. One can also check that these beliefs are the only one to satisfy the D1 criterion. In fact, a deviating low-type is going to face litigation with higher probability than a high type in equilibrium. Therefore he is more prone to renegotiate the incomplete contract which is costly in case of divorce.

Bargaining. The bargaining structure deserves some justifications, because that on the ex-ante agreement is simultaneous, while that about the ex-post one is sequential. First of all, we feel that sequential bargaining is the correct stylized way of modeling an informal bargaining. Second, it is rather easy to prove that even if we assumed sequential bargaining in the ex-ante stage, nothing substantial would change to the proposed equilibrium. In the renegotiation stage we use a sequential bargaining with restrictions on the admissible proposals in order to avoid that the proposer has all the bargaining power. Alternatively we could assume that agents bargain sequentially and iteratively (*à la* Rubinstein) in the renegotiation stage. We think that this assumption may considerably increase the complexity of the model without adding fundamental elements to our analysis.

The Role of the Court. A particularly relevant issue is whether this contract incompleteness depends crucially on the court's rule. Courts, in fact, could implement very different (and odd) rules. Consider the following: in case partners exerted different level of effort, assign all the assets to the partner who shirked and impose to the keen partner the payment of the entire amount of litigation costs $2F$. This rule would generate a (pooling) equilibrium with complete contracts. Alternatively, as some authors suggested (See Becker (1998)), premarital agreements could be mandatory. Nevertheless, we proved in the previous section that if high-type agents are "sophisticated" enough, for specific parameter values, they would separate by writing a premarital agreement with an arbitrator which mimics the separating contract. Therefore, there is no gain in efficiency in having a court which applies different rules with respect to those we stated, or in making premarital agreements mandatory.

Moreover, we assume that courts may verify how agents use their initial endowment (effort). This seems to us a reasonable assumption, since courts have coercive and mandatory power and they can inspect bank accounts, call a third party as a witness , etc.. On the contrary, we do not assume that courts may verify which good each agent actually produced. In this paper, the production technology is a "black box" for courts, which can only observe the individual inputs and the total output. We think that this feature of the model is a relevant one and that it would be even more so, if we assumed some form of joint production.

In this model, using courts instead of private arbitrators saves writing costs of contracts. However, there might be other reasons to prefer courts to private arbitrators. For instance, the verification technology (which in our setting determines the litigations costs) may present increasing return to scale, so that a centralized institution (a court) is more efficient than private arbitrators[18]; another justification is that private arbitrators may be more prone to collusive agreements with one of the party than a court. Such consideration is even more relevant if we introduce some form of asymmetry between the parties, which seems an important issue, since it is reasonable to assume that a court protects the weakest party more efficiently than a private arbitrator.

[18] It worths noticing that in our framework courts have to be efficient, as far as it regards the cost of verification, but not too much. In fact if courts are "too efficient" and are able to verify agents' effort without no costs, high-type agents will prefer to draw up contracts which provide for private coslty arbitration, in order to maintain positive costs of divorce.

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6 Appendix[19]

We present formally the conditions for which Proposition 1 holds, which were denoted in the text as conditions (C).The set (C) satisfies the following conditions:
$C.1$: $\mu_1^h \geq 2\frac{\beta_1}{4\beta_1-1}$,
$C.2$: $\mu_1^l \in [\underline{m}, \bar{m}]$ where
$\underline{m} = \frac{2(1-\beta_1 p)\mu_1^h + \left(1-p\left(1-\mu_1^h\right)\right)c}{2(1-\beta_1 p)+(4\beta_1-1)p\left(1-\mu_1^h\right)+\left(2\mu_1^h-1\right)cp}$
$\bar{m} = \frac{\left(4\mu_1^h - p\mu_1^h - 2\right)\mu_1^h - \left(2\mu_1^h-1\right)(1-p)c}{\left(2\mu_1^h-1\right)(2-p)}$
$C.3$ $F \in \left[\underline{F}, \bar{F}\right]$ *where*
$\underline{F} = \frac{\left(2-\frac{1}{2}p\right)\left(\mu_1^h-\mu_1^l\right)+(1-p)c}{p\left(\mu_1^h+\mu_1^l-2\mu_1^l\mu_1^h\right)} + c$
$\bar{F} = \min\left(\frac{(2-p)\left(\mu_1^h-\mu_1^l\right)+(1-p)c}{2p\mu_1^h\left(1-\mu_1^h\right)} + c, 2\beta_1 - \frac{1}{2}\right)$
Now we state a precise version of the perfect Bayesian equilibrium whose main features are briefly mentioned in Proposition 1.

Proposition 1b: *If conditions C hold, then the following perfect Bayesian equilibrium exists:*

High-type agents:

1) announce to sign no contract;

2) exert the efficient level of effort $e_i = 1$*;*

3) choose to divorce if $\theta = \theta_b$*;*

4) in case of divorce and the partner is the proposer of the ex-post agreement:
4.1) if they exerted effort they accept only proposals such that they receive a total transfer of at least $2\beta_1$*, if only one good was produced;*
4.2) if they exerted effort they accept any equal division proposal when either both goods were produced or no good was produced;
4.3) if they shirked they accept any proposal such that they pay at most a total transfer of $2\beta_1$ *to the partner;*
5) in case of divorce and if they are the proposers of the ex-post agreement:
5.1) they propose an equal division if either both goods were produced or no good were produced;
5.2) if they exerted effort they do not make any proposal if only one good was produced;
5.3) if they shirked they offer to pay a total transfer of $2\beta_1$ *to the partner if only one good was produced;*

Low-type agents:
1) announce to sign the simple contract as from Definition 1;

[19] **Note for the referees**. In a final version the appendix can be shortened considerably, skipping algebraic passages.

2) exert the efficient level of effort $e_i = 1$;

3) behave as high type agents both in the decision about divorcing and in the renegotiation stage, whenever no complete contract was drawn.

If an agent proposes a premarital contract, the partner assumes with probability 1 that the proposer is a low-type. In case of divorce and only one good was produced, if partner j proposes an ex-post agreement, then agent i has beliefs which assign probability 1 that agent j exerted zero effort. If no good was produced or two goods were produced and if partner j proposes an ex-post agreement, then i believes that agent j exerted effort. These are the only relevant beliefs.

Proof: The high type has the following utility in equilibrium:

$$E[\theta] + 2\left(\mu_1^h\right)^2 + 2\mu_1^h\left(1-\mu_1^h\right) +$$
$$(1-p)\left(\theta_g + 2\left(\mu_1^h\right)^2 + 2\mu_1^h\left(1-\mu_1^h\right)\right) +$$
$$p\left(\mu_1^h\right)^2 + p\left(1-\mu_1^h\right)\mu_1^h - 2p\left(1-\mu_1^h\right)\mu_1^h F$$
$$-cp\left(\mu_1^h\right)^2 - cp\left(1-\mu_1^h\right)^2$$

which is equivalent to:

$$E[\theta] + (4-p)\mu_1^h + \theta_g(1-p) - 2p\left(1-\mu_1^h\right)\mu_1^h F - cp\left(\mu_1^h\right)^2 - cp\left(1-\mu_1^h\right)^2 \tag{4}$$

while deviating and joining a low-type the utility is:

$$E[\theta] + 2\mu_1^h\mu_1^l + \mu_1^h\left(1-\mu_1^l\right) + \mu_1^l\left(1-\mu_1^h\right) +$$
$$(1-p)\left(\theta_g + 2\mu_1^h\mu_1^l + \mu_1^h\left(1-\mu_1^l\right) + \mu_1^l\left(1-\mu_1^h\right)\right) +$$
$$p\mu_1^h\mu_1^l + p\left(1-\mu_1^l\right)\mu_1^h - c$$

which may be simplified:

$$E[\theta] + 2\left(\mu_1^h + \mu_1^l\right) + \theta_g(1-p) - p\mu_1^l - c \tag{5}$$

Note that a high-type agent who joins a low-type one will surely choose $e_i = 1$, since by assumption $\mu_1^l - \mu_0^l > \frac{1}{2}$, $\mu_1^h > \mu_1^l$ and $\mu_0^h = \mu_0^l$. Hence the self-selection constraint for the high type is satisfied whenever (4) $\geq$ (5), or

$$F \leq \frac{(2-p)\left(\mu_1^h - \mu_1^l\right) + (1-p)c}{2p\mu_1^h\left(1-\mu_1^h\right)} + c$$

A high-type which joins a high-type partner, but who deviates and chooses $e_i = 0$, obtains:

$$\begin{gathered} E[\theta] + 2\mu_0^h\mu_1^h + \mu_0^h\left(1-\mu_1^h\right) + \mu_1^h\left(1-\mu_0^h\right) + 1 + \\ (1-p)\left(\theta_g + 2\left(\mu_0^h\mu_1^h\right) + \mu_0^h\left(1-\mu_1^h\right) + \mu_1^h\left(1-\mu_0^h\right)\right) + \\ p\mu_0^h\mu_1^h - p\left(\mu_0^h\left(1-\mu_1^h\right) + \mu_1^h\left(1-\mu_0^h\right)\right)(2F + 2\beta_1 - 1) \\ -cp\mu_0^h\mu_1^h - cp\left(1-\mu_0^h\right)\left(1-\mu_1^h\right) \end{gathered}$$

which may be simplified in:

$$\begin{gathered} E[\theta] + 1 + 2\left(\mu_0^h + \mu_1^h\right) - p\mu_0^h\mu_1^h + \theta_g(1-p) - \\ 2p\left(\mu_0^h - 2\mu_0^h\mu_1^h + \mu_1^h\right)(\beta_1 + F) - cp\mu_0^h\mu_1^h - cp\left(1-\mu_0^h\right)\left(1-\mu_1^h\right) \end{gathered} \tag{6}$$

Hence, the incentive compatibility constraint for the high-type is satisfied whenever (4) $\geq$ (6), or:

$$\begin{gathered} 2\left(\mu_1^h - \mu_0^h\right) - 1 - p\mu_1^h\left(1-\mu_0^h\right) + 2p\left(\mu_0^h - 2\mu_0^h\mu_1^h + \mu_1^h\right)\beta_1 \\ +2p\mu_1^h\left(1-\mu_1^h\right)F + p(2F-c)\left(2\mu_1^h - 1\right)\left(\mu_1^h - \mu_0^h\right) \geq 0 \end{gathered}$$

Since $\mu_1^h - \mu_0^h \geq \frac{1}{2}$ by assumption (1) and given that c is arbitrarily small we certainly have $2F > c$, a sufficient condition is:

$$2p\left(\mu_0^h - 2\mu_0^h\mu_1^h + \mu_1^h\right)\beta_1 - p\mu_1^h\left(1-\mu_0^h\right) \geq 0$$

which is equivalent to:

$$\beta_1 \geq \frac{1}{2}\frac{1}{1 + \frac{\mu_0^h\left(1-\mu_1^h\right)}{\mu_1^h\left(1-\mu_0^h\right)}}$$

which is certainly satisfied since $\beta_1 \geq \frac{1}{2}$.
The low type has the following utility in equilibrium:

$$\begin{gathered} E[\theta] + 2\left(\mu_1^l\right)^2 + 2\mu_1^l\left(1-\mu_1^l\right) + \\ (1-p)\left(\theta_g + 2\left(\mu_1^l\right)^2 + 2\mu_1^l\left(1-\mu_1^l\right)\right) + \\ p\left(\mu_1^l\right)^2 + p\mu_1^l(1-\mu_1^l) - c \end{gathered}$$

which after easy calculation becomes:

$$E[\theta] + (4-p)\mu_1^l + \theta_g(1-p) - c \tag{7}$$

The low-type agent's utility if she joins a high-type and chooses $e_i = 1$, is:

$$E[\theta] + 2\mu_1^h\mu_1^l + \mu_1^h\left(1-\mu_1^l\right) + \mu_1^l\left(1-\mu_1^h\right) +$$
$$(1-p)\left(\theta_g + 2\mu_1^h\mu_1^l + \mu_1^h\left(1-\mu_1^l\right) + \mu_1^l\left(1-\mu_1^h\right)\right) + p\mu_1^h\mu_1^l +$$
$$\frac{1}{2}p\left(\left(1-\mu_1^h\right)\mu_1^l + \left(1-\mu_1^l\right)\mu_1^h\right) - p\left(\mu_1^h\left(1-\mu_1^l\right) + \mu_1^l\left(1-\mu_1^h\right)\right)F$$
$$-cp\mu_1^h\mu_1^l - cp\left(1-\mu_1^h\right)\left(1-\mu_1^l\right)$$

and after easy calculations we obtain:

$$\begin{aligned} &E[\theta] + \left(\mu_1^h + \mu_1^l\right)\left(2 - \frac{1}{2}p\right) + \theta_g(1-p) - \\ &p\left(\mu_1^h\left(1-\mu_1^l\right) + \mu_1^l\left(1-\mu_1^h\right)\right)F \\ &-cp\mu_1^h\mu_1^l - cp\left(1-\mu_1^h\right)\left(1-\mu_1^l\right) \end{aligned} \tag{8}$$

while that of joining high type, if he chooses $e_i = 0$, is:

$$E[\theta] + 2\mu_1^h\mu_0^l + \mu_1^h\left(1-\mu_0^l\right) + \mu_0^l\left(1-\mu_1^h\right) + 1 +$$
$$(1-p)\left(\theta_g + 2\mu_1^h\mu_0^l + \mu_1^h\left(1-\mu_0^l\right) + \mu_0^l\left(1-\mu_1^h\right)\right) +$$
$$p\mu_1^h\mu_0^l - p\left(\mu_1^h\left(1-\mu_0^l\right) + \mu_0^l\left(1-\mu_1^h\right)\right)(2F + 2\beta_1 - 1)$$
$$-c\mu_1^h\mu_0^l - c\left(1-\mu_1^h\right)\left(1-\mu_0^l\right)$$

which may be simplified in:

$$\begin{aligned} &E[\theta] + 2\left(\mu_1^h + \mu_0^l\right) + 1 + \theta_g(1-p) - p\mu_1^h\mu_0^l - \\ &2p\left(\mu_1^h - 2\mu_1^h\mu_0^l + \mu_0^l\right)(\beta_1 + F) \\ &-cp\mu_1^h\mu_0^l - cp\left(1-\mu_1^h\right)\left(1-\mu_0^l\right) \end{aligned} \tag{9}$$

Now we have to check which is the best deviation for a low-type agent. After some calculations we have that (8) $\geq$ (9) is equivalent to:

$$\begin{aligned} &\left(2 - \frac{1}{2}p\right)\left(\mu_1^l - \mu_0^l\right) - 1 + p\left(2\beta_1 - \frac{1}{2}\right)\left(\mu_1^h + \mu_0^l - 2\mu_1^h\mu_0^l\right) \\ &+p\left(2\mu_1^h - 1\right)\left(\mu_1^l - \mu_0^l\right)(F - c) + \left(\mu_1^h - 2\mu_1^h\mu_0^l + \mu_0^l\right)pF \geq 0 \end{aligned} \tag{10}$$

Since we assumed that $\mu_1^l - \mu_0^l \geq \frac{1}{2}$, and given tht c is small we can assume

$F > c$, it is surely satisfied if:

$$p\left(2\beta_1 - \frac{1}{2}\right)\left(\mu_1^h - 2\mu_1^h\mu_0^l + \mu_0^l\right) - \frac{1}{2}p\left(\mu_1^l - \mu_0^l\right) \geq 0$$

which is equivalent to:

$$\beta_1 \geq \frac{1}{4} + \frac{1}{4}\frac{\mu_1^l - \mu_0^l}{\mu_1^h - 2\mu_1^h\mu_0^l + \mu_0^l}$$

which is always satisfied since $\beta_1 \geq \frac{1}{2}$ and:

$$\frac{\mu_1^l - \mu_0^l}{\mu_1^h - 2\mu_1^h\mu_0^l + \mu_0^l} \leq 1$$

In fact:

$$\begin{aligned} \mu_1^h - 2\mu_1^h\mu_0^l + \mu_0^l - \left(\mu_1^l - \mu_0^l\right) &= \\ \mu_1^h - 2\mu_1^h\mu_0^l + 2\mu_0^l - \mu_1^l &\geq \\ \mu_1^h - 2\mu_1^h\mu_0^l + 2\mu_0^l - \mu_1^h &= \\ 2\mu_0^l\left(1 - \mu_1^h\right) &\geq 0 \end{aligned}$$

It follows that the self-selection constraint for the low type is satisfied if (7) $\geq$ (8), or:

$$F \geq \frac{\left(2 - \frac{1}{2}p\right)\left(\mu_1^h - \mu_1^l\right) + (1-p)\,c}{p\left(\mu_1^h + \mu_1^l - 2\mu_1^l\mu_1^h\right)} + c$$

We can now state two necessary conditions for the separating equilibrium to exist:

$$\begin{aligned} \frac{\left(2 - \frac{1}{2}p\right)\left(\mu_1^h - \mu_1^l\right) + (1-p)\,c}{p\left(\mu_1^h + \mu_1^l - 2\mu_1^l\mu_1^h\right)} + c \leq F \leq \\ \frac{(2-p)\left(\mu_1^h - \mu_1^l\right) + (1-p)\,c}{2p\mu_1^h\left(1 - \mu_1^h\right)} + c \end{aligned} \tag{11}$$

and

$$\frac{\left(2 - \frac{1}{2}p\right)\left(\mu_1^h - \mu_1^l\right) + (1-p)\,c}{p\left(\mu_1^h + \mu_1^l - 2\mu_1^l\mu_1^h\right)} + c \leq 2\beta_1 - \frac{1}{2} \tag{12}$$

(11) and (12) prove condition $C.3$ Note that Condition (12) guarantees that the lower bound of Condition (11) is not greater than $2\beta_1 - \frac{1}{2}$. In fact, when only one good was produced and the level of match is bad each partner obtains an equilibrium utility level equal to $\frac{1}{2} - F$. The receiver will accept proposals which assign to him at least $2\beta_1$. If making such a proposal would guarantee to the proposer at least the equilibrium utility, he would make

such a proposal; hence, we must impose: $1 - 2\beta_1 < \frac{1}{2} - F$, which implies (12). A necessary condition for satisfying inequalities (11) is:

$$\frac{(2-p)\left(\mu_1^h - \mu_1^l\right) + (1-p)\,c}{2\mu_1^h\left(1-\mu_1^h\right)} \geq \frac{\left(2-\frac{1}{2}p\right)\left(\mu_1^h - \mu_1^l\right) + (1-p)\,c}{\mu_1^h + \mu_1^l - 2\mu_1^l\mu_1^h}$$

which is equivalent to:

$$-\left(\mu_1^h - \mu_1^l\right)\frac{\left(2\mu_1^h-1\right)(2-p)\mu_1^l-\left(4\mu_1^h-p\mu_1^h-2\right)\mu_1^h-\left(2\mu_1^h-1\right)(1-p)c}{2\mu_1^h\left(1-\mu_1^h\right)\left(\mu_1^h-2\mu_1^h\mu_1^l+\mu_1^l\right)} \geq 0$$

or:

$$\mu_1^l \leq \frac{\left(4\mu_1^h - p\mu_1^h - 2\right)\mu_1^h - \left(2\mu_1^h - 1\right)(1-p)\,c}{\left(2\mu_1^h - 1\right)(2-p)}$$

Provided that the numerator is positive, which is the case iff:

$$\mu_1^h \geq \frac{1 + (1-p)\,c + \sqrt{(1-p)^2\,c^2 + \left(3p - 2 - p^2\right)c + 1}}{(4-p)}$$

Condition (12) can be rewritten as:

$$\left(2\left(1-\beta_1 p\right) + \left(4\beta_1 - 1\right)p\left(1-\mu_1^h\right) + \left(2\mu_1^h - 1\right)cp\right)\mu_1^l$$
$$-2\left(1-\beta_1 p\right)\mu_1^h - \left(1 - p\left(1-\mu_1^h\right)\right)c \geq 0$$

or:

$$\mu_1^l \geq \frac{2\left(1-\beta_1 p\right)\mu_1^h + \left(1 - p\left(1-\mu_1^h\right)\right)c}{2\left(1-\beta_1 p\right) + \left(4\beta_1 - 1\right)p\left(1-\mu_1^h\right) + \left(2\mu_1^h - 1\right)cp}$$

Therefore a necessary condition for our equilibrium to exist is

$$\frac{2\left(1-\beta_1 p\right)\mu_1^h + \left(1 - p\left(1-\mu_1^h\right)\right)c}{2\left(1-\beta_1 p\right) + \left(4\beta_1 - 1\right)p\left(1-\mu_1^h\right) + \left(2\mu_1^h - 1\right)cp} \leq \mu_1^l \leq \frac{\left(4\mu_1^h - p\mu_1^h - 2\right)\mu_1^h - \left(2\mu_1^h - 1\right)(1-p)\,c}{\left(2\mu_1^h - 1\right)(2-p)}$$

this proves $C.2$ and it implies:

$$\frac{\left(4\mu_1^h - p\mu_1^h - 2\right)\mu_1^h - \left(2\mu_1^h - 1\right)(1-p)\,c}{\left(2\mu_1^h - 1\right)(2-p)} \geq \frac{2\left(1-\beta_1 p\right)\mu_1^h + \left(1 - p\left(1-\mu_1^h\right)\right)c}{2\left(1-\beta_1 p\right) + \left(4\beta_1 - 1\right)p\left(1-\mu_1^h\right) + \left(2\mu_1^h - 1\right)cp} \tag{13}$$

Notice that the derivative of the rhs with respect to c is:

$$\frac{2(1-p)\left(1-\left(2\mu_1^h-1\right)p\beta_1\right)+\left(4\mu_1^h+p\left(1-\mu_1^h\right)-1\right)p\left(1-\mu_1^h\right)}{\left(2(1-\beta_1 p)+(4\beta_1-1)p\left(1-\mu_1^h\right)+\left(2\mu_1^h-1\right)cp\right)^2}>0$$

Note that the derivative is positive since $1>\mu_1^h\geq\frac{1}{2}$ and $\frac{1}{2}\leq\beta_1\leq 1$. Hence (13) implies:

$$\begin{gathered}\frac{\left(4\mu_1^h-p\mu_1^h-2\right)\mu_1^h-\left(2\mu_1^h-1\right)(1-p)c}{\left(2\mu_1^h-1\right)(2-p)}-\frac{2(1-\beta_1 p)\mu_1^h+\left(1-p\left(1-\mu_1^h\right)\right)c}{2(1-\beta_1 p)+(4\beta_1-1)p\left(1-\mu_1^h\right)+\left(2\mu_1^h-1\right)cp}\geq\\ \frac{\left(4\mu_1^h-p\mu_1^h-2\right)\mu_1^h-\left(2\mu_1^h-1\right)(1-p)c}{\left(2\mu_1^h-1\right)(2-p)}-\frac{2(1-\beta_1 p)\mu_1^h}{2(1-\beta_1 p)+(4\beta_1-1)p\left(1-\mu_1^h\right)}\geq\\ \frac{\left(4\mu_1^h-p\mu_1^h-2\right)\mu_1^h}{\left(2\mu_1^h-1\right)(2-p)}-\frac{2(1-\beta_1 p)\mu_1^h}{2(1-\beta_1 p)+(4\beta_1-1)p\left(1-\mu_1^h\right)}=\\ \frac{(4-p)\left(1-\mu_1^h\right)\left(4\beta_1\mu_1^h-\mu_1^h-2\beta_1\right)p\mu_1^h}{(2-p)\left(2\mu_1^h-1\right)\left(2(1-\beta_1 p)+(4\beta_1-1)p\left(1-\mu_1^h\right)\right)}\geq 0\end{gathered}$$

or:

$$\mu_1^h\geq\frac{2\beta_1}{4\beta_1-1}$$

which is also necessary for (13). Finally notice that for $c\leq\frac{1}{2}$:

$$\begin{gathered}\frac{2\beta_1}{4\beta_1-1}\geq\\ \frac{2}{4-p}\geq\\ \frac{1+(1-p)c+\sqrt{(1-p)^2c^2+(3p-2-p^2)c+1}}{4-p}\end{gathered}$$

With this we prove $C.1$. ■

Proof of Proposition 2: Suppose i has to make a proposal and suppose that $e_i=1$. Denoted with π the probability that the partner will accept the proposal and with α being the net transfer received by i, i will propose an ex-post agreement if the following holds:

$$\pi\alpha+(1-\pi)\left(\frac{1}{2}-F\right)-c\geq\frac{1}{2}-F \tag{14}$$

Note first that the previous inequality cannot be satisfied if $\alpha\leq c+\frac{1}{2}-F$. To propose such an agreement is, in fact, a dominated strategy for any agent who exerted effort. If instead $e_i=0$ and denoted with π' the probability that the partner will accept the proposal, i will propose an ex-post agreement if the following holds:

$$\pi'\alpha + \left(1-\pi'\right)\left(-2F-2\beta_1+1\right) - c \geq -2F-2\beta_1+1 \tag{15}$$

Notice that (15) can be satisfied even for $\alpha \leq c+\frac{1}{2}-F$. Hence, for this set of contract proposals the proposition is already proved, since the intuitive criterion is sufficient. If $\alpha > c+\frac{1}{2}-F$, after simple manipulation we can prove that (14) is satisfied if:

$$\pi \geq \frac{c}{\alpha-\frac{1}{2}+F} = \underline{\pi}$$

while (15) is satisfied if:

$$\pi' \geq \frac{c}{\alpha+2F+2\beta_1-1} = \underline{\pi}'$$

Noticing that $2\beta_1 - 1 \geq 0$, it is easy to check that $\underline{\pi}' < \underline{\pi}$ for any value of $\alpha > c+\frac{1}{2}-F$. Hence there exists a larger set of conjectures that i might hold that induces i to deviate when $e_i = 0$ with respect to when $e_i = 1$. Hence for the D1 criterion the partner should assign 0 probability to the fact that $e_i = 1$.

Finally, we want to provide a condition such that shirking is not a (weakly) dominated strategy, otherwise, by the intuitive criterion, any agent should assign zero probability that a proposer of an amicable agreement shirked. Suppose that partners never go to courts, and they always agree on equally dividing their assets in case of divorce. The utility of a high-type agent who does not exert effort in the production of the public good is

$$\begin{aligned} &E\left[\theta\right] + \left(\mu_1^h+\mu_0^h\right)\left(2-\frac{1}{2}p\right) + \left(1-p\right)\left(\theta_g\right) + \\ &1 - c\mu_1^h\mu_0^h - c\left(1-\mu_1^h\right)\left(1-\mu_0^h\right) \end{aligned} \tag{16}$$

while the utility of a high-type agent who exerts effort in the production of the public good is

$$E\left[\theta\right] + 2\mu_1^h\left(2-\frac{1}{2}p\right) + \left(1-p\right)\left(\theta_g\right) - c\left(\mu_1^h\right)^2 - c\left(1-\mu_1^h\right)^2 \tag{17}$$

It is easy to check that (16) $\geq$ (17) if and only if

$$\mu_1^h - \mu_0^h \leq \frac{1}{2-\frac{1}{2}p-\left(2\mu_1^h-1\right)c}$$

which is implied by condition (3) in the text. ■

Proof of Remark 1: Beliefs which assign probability 1 that the partner who proposes to continue the marriage is a shirker are the only one satisfying the D1 criterion, as it can be easily checked by setting $\alpha = \theta_b + 1$ in equations (14) and (15).

Remark 3 *The statement of Proposition 1b has to be changed slightly when considering the matching mechanism of section 4. In fact both to the high type and to the low type we have to add the following move:*

2b) in the matching stage they accept any contract which guarantees them at least the same expected utility as they receive in equilibrium.

Moreover, we have to modify the equilibrium beliefs in the following way:

In the matching stage if an agent proposes a contract which does not guarantee to the partner the same expected utility than no-contracting, the partner assumes with probability 1 that the proposer is a low-type.

All the rest of the equilibrium description remains unchanged.

Proof of Proposition 3. In order to prove the statement we consider two different cases: the first when $P \leq \theta_g$, the second when the opposite holds. Case 1. $(P \leq \theta_g)$. In this case all marriages end if and only if $\theta = \theta_b$. In this case, the expected payoff for the high-type agents is the same as under complete information and complete contract. In fact, the expected payoff for high-type agents signing this contract is:

$$E[\theta] + 2\left(\mu_1^h\right)^2 + 2\mu_1^h\left(1 - \mu_1^h\right) + (1-p)\left(\theta_g + 2\left(\mu_1^h\right)^2 + 2\mu_1^h\left(1 - \mu_1^h\right)\right) + p\left[\left(\mu_1^h\right)^2 + \mu_1^h\left(1 - \mu_1^h\right)(1+P) + \mu_1^h\left(1 - \mu_1^h\right)(-P)\right] - c$$

which is equivalent to:

$$E[\theta] + 2\mu_1^h + (1-p)\left(\theta_g + 2\mu_1^h\right) + p\mu_1^h - c$$

Note, in fact, that P does not affect the utility level of high-type agents, due to the symmetric structure of the contract. It follows that whenever this contract is able to sustain a separating equilibrium, high-type agents strictly prefer to sign this contract than not signing any contract. We show under which conditions this contract cannot sustain a separating equilibrium. The expected payoff of a low-type agent who joins a high-type agent and sign this contract is:

$$\begin{aligned}&E[\theta] + 2\mu_1^h\mu_1^l + \mu_1^h\left(1 - \mu_1^l\right) + \mu_1^l\left(1 - \mu_1^h\right) + \\&(1-p)\left[\theta_g + 2\mu_1^h\mu_1^l + \mu_1^h\left(1 - \mu_1^l\right) + \mu_1^l\left(1 - \mu_1^h\right)\right] + \\&p\left[\mu_1^h\mu_1^l + \mu_1^l\left(1 - \mu_1^h\right)(1+P) + \mu_1^h\left(1 - \mu_1^l\right)(-P)\right] - c\end{aligned}$$

which is equivalent to:

$$E[\theta] + (1-p)\,\theta_g + (2-p)\left(\mu_1^h + \mu_1^l\right) + \\ p\left[\mu_1^l - \left(\mu_1^h - \mu_1^l\right)P\right] - c \tag{18}$$

The expected payoff for a low-type who joins a low-type (and sign a simple contract) is:

$$E[\theta] + (4-p)\,\mu_1^l + \theta_g\,(1-p) - c \tag{19}$$

It follows that a contract without costly verification cannot sustain a separating equilibrium if (18) $\geq$ (19), which implies:

$$(2-p)\left(\mu_1^h + \mu_1^l\right) + \\ p\left[\mu_1^l - \left(\mu_1^h - \mu_1^l\right)P\right] - (4-p)\,\mu_1^l \geq 0$$

Since P does not enter in the expected payoff of the high-type agents who signs the complete separating contract but it represents the punishment for a deviating low-type, then it is optimal for high-type agents to fix it at the maximal level. Substituting $P = \theta_g$ in the previous inequality we obtain:

$$(2-p)\left(\mu_1^h + \mu_1^l\right) + \\ p\left[\mu_1^l - \left(\mu_1^h - \mu_1^l\right)\theta_g\right] - (4-p)\,\mu_1^l \geq 0$$

or,

$$\theta_g \leq \frac{(2-p)}{p} \tag{20}$$

Case 2. ($P > \theta_g$). In this case high type agents divorce (i) when $\theta = \theta_b$, (ii) when $\theta = \theta_g$ and only one partner produced the good. In fact, in the last case, the productive partner prefers to receive P by the partner than continuing the marriage. The utility level of a high-type agent is

$$E[\theta] + 2\mu_1^h + (1-p)\left[\left(\mu_1^h\right)^2(\theta_g+2) + \left(1-\mu_1^h\right)^2\theta_g + \mu_1^h\left(1-\mu_1^h\right)\right] + \\ p\left[\left(\mu_1^h\right)^2 + \mu_1^h\left(1-\mu_1^h\right)\right] - c$$

which is equivalent to

$$E[\theta] + 2\mu_1^h + (2-p)\left(\mu_1^h\right)^2 + (1-p)\,\theta_g\left(1 - 2\mu_1^h + 2\left(\mu_1^h\right)^2\right) \qquad (21)$$
$$+\mu_1^h\left(1-\mu_1^h\right) - c$$

As we previously calculated, the utility of a high-type agent who does not sign any contract is:

$$E[\theta] + (4-p)\,\mu_1^h + \theta_g(1-p) - 2p\left(1-\mu_1^h\right)\mu_1^h F \qquad (22)$$

Therefore to sign no contract guarantees a higher utility to high-type agents than signing a contract without costly state verification whenever (22) $\geq$ (21), which implies:

$$\mu_1^h\left(1-\mu_1^h\right)\left((1-p)(1+2\theta_g) - 2pF\right) + c \geq 0$$

Since conditions (C) guarantee that $F \leq 2\beta_1 - \frac{1}{2}$, a sufficient condition for the previous inequality to hold is:

$$\theta_g \geq \frac{4p\beta_1 - 1}{2(1-p)} \qquad (23)$$

Conditions (20) and (23) together prove the proposition. Since $\frac{1}{2} \leq \beta_1 \leq 1$, it is easy to check that the set defined by (20) and (23) is non-empty iff:

$$0 \leq p \leq \frac{\sqrt{64\beta_1 - 7} - 5}{4(2\beta_1 - 1)} < \frac{\sqrt{64-7} - 5}{4(2-1)} \simeq 0.637\,46$$

■

Proof of Proposition 4. The more general form of complete contract should impose to partners different payments for different levels of production. Therefore, in equilibrium we should observe the following levels of transfers: T_2, T_{10}, T_{01} and T_0, where T_2 is the transfer of the player (to the partner and/or to the arbitrator), which may be negative, when both partners produced, T_{10} is the transfer paid by the productive partner when the other did not produced, T_{01} is the transfer of the unproductive partner when the other produced and T_0 is the transfer when nobody produced.[20] All equilibrium transfers assume that both partners exerted effort, since no-effort should not be observed in equilibrium. In fact, the contract may call

[20] We impose that in no state of the world the arbitrator makes (positive) transfer to the couples, i.e., the sum of the partners' transfers has to be positive. Otherwise, we might observe collusive behavior of the partners, in particular strategic divorcing. Therefore we implicitely assume that the arbitrator cannot monitor partners after divorce and that partners are able to apparently separate, still both benefiting of their joint production.

for transfers in case one or both partners did not exert effort, but these transfers are not going to affect the agents' payoff in equilibrium. A contract may imply costly state verification for all levels of production, or only when the production is below a certain level; moreover, it can call for state verification if the level of match is bad, good, or in both cases.

An important observation that we are going to use frequently in the proof is the following. Suppose that we are able to prove that a contract which provides for state verification for some levels of production only when a bad match did occurs is more costly than not signing a contract. The same result, then, follows for a contract which provides for state verification for the same levels of production and when a good match (or both levels of match) did realize. In fact, imposing state verification when the match is good or for both levels of match, makes state verification more likely (and therefore the contract more costly) when $p \leq \frac{1}{2}$.

Case 1: we consider first the case when state verification is required for all levels of production (and $\theta = \theta_b$). The high-type agent utility in equilibrium is:

$$\begin{aligned} &E[\theta] + (4-p)\,\mu_1^h + \theta_g (1-p) - \\ &p\left(\mu_h (1-\mu_h)\, T_{10} + (1-\mu_h)\,\mu_h T_{01} + (1-\mu_h)^2 T_0 + \mu_h^2, T_2\right) - c \end{aligned} \tag{24}$$

Notice that the participation constraint of the arbitrator in a symmetric scheme implies $T_k \geq F$, with $k = 0, 2$ and $T_{10} + T_{01} \geq 2F$, whenever state verification is required.

Comparing (4) with (24) of the Appendix, we have that the utility for the high types would be higher than in BMC equilibrium iff:

$$\begin{aligned} \left(\mu_1^h\right)^2 T_2 + \mu_1^h \left(1-\mu_1^h\right) T_{10} + \left(1-\mu_1^h\right) \mu_1^h T_{01} + \left(1-\mu_1^h\right)^2 T_0 + \frac{c}{p} < \\ 2F\mu_1^h \left(1-\mu_1^h\right) \end{aligned} \tag{25}$$

Notice that:

$$\begin{aligned} \left(\mu_1^h\right)^2 T_2 + \mu_1^h \left(1-\mu_1^h\right) T_{10} + \left(1-\mu_1^h\right) \mu_1^h T_{01} + \left(1-\mu_1^h\right)^2 T_0 \geq \\ \left(1-\mu_1^h\right) \mu_1^h \left(T_{10} + T_{01}\right) \geq 2F\left(1-\mu_1^h\right)\mu_1^h \end{aligned}$$

where the first inequality is algebra and the second comes from the assumption that partners have to pay the state verification costs to the arbitrator. But this last inequality contradicts (25).

Case 2: Now we consider contracts where partners ask for state verification only in some state of the world. Clearly, a contract which provides for costly

state verification when only one agent produced successfully is equivalent to the BMC equilibrium except for the cost of writing the contract. Moreover all contracts which call for costly verification when the level of total production is zero and one, or one and two, are even more costly.
Consider now contracts which call for state verification if the production level is zero and two (and $\theta = \theta_b$). High-type agents prefers the BMC equilibrium if

$$\left(\mu_1^h\right)^2 T_2 + \left(1 - \mu_1^h\right)^2 T_0 \geq 2F\mu_1^h \left(1 - \mu_1^h\right)$$

Notice that renegotiation implies that parties would renegotiate any $T_k > F$, $k = 0, 2$. In fact state verification is a way to solve the moral hazard problem. Therefore, partners will minimize expenses when both provided effort. Thus, $T_k = F$, $k = 0, 2$, if partners ask for state verification, otherwise $T_k = 0$. It is easy to check that

$$\left(\mu_1^h\right)^2 F + \left(1 - \mu_1^h\right)^2 F > 2F\mu_1^h \left(1 - \mu_1^h\right)$$

since by assumption $\mu_1^h > \frac{1}{2}$. Therefore we may focus on contracts which call for costly state verification only if either both partners produced or no partner produced successfully.
Let consider first the case when costly state verification only when both partners were productive (and $\theta = \theta_b$, where we exclude verification for θ_g, because more costly). The utility for the high types is higher in the BMC equilibrium than in signing this contract iff:

$$\left(\mu_1^h\right)^2 F + \frac{c}{p} \geq 2F\mu_1^h \left(1 - \mu_1^h\right)$$

It is easy to check that a sufficient condition is $\mu_1^h \geq \frac{2}{3}$ which is guaranteed by conditions C (in fact we have $\underline{M} \geq \frac{2}{3}$ for $\beta_1 \leq 1$).
Let finally consider the case when state verification occurs if no partner was productive. We will prove that in (C) the self selection constraint of the low type cannot be satisfied. To this end it is sufficient to prove that it is not satisfied when state verification occurs for all levels of match, since any other level of verification will implement separation among types for a smaller set of parameter values. First, we assume that partners sign a contract with the same clauses as the simple contract, except in the case when there is no production. In this case each agent pay T_0 if both partner have exerted effort, something different when at least one did not provide effort. Second, we allow parties to write contracts where costly state verification may occur without divorcing.[21] The utility of the low type in the case we are assuming

[21] This assumption implies that, when the match is good, parties may verify by means of an arbitrator the effort exerted by the partners, without loosing the benefit of being married.

is the same as in the separating equilibrium and expressed in (7). The utility deviating and joining a high type is instead:

$$E[\theta] + 2\mu_1^h \mu_1^l + \mu_1^h \left(1 - \mu_1^l\right) + \mu_1^l \left(1 - \mu_1^h\right) +$$
$$(1-p)\left(\theta_g + 2\mu_1^h \mu_1^l + \mu_1^h \left(1 - \mu_1^l\right) + \mu_1^l \left(1 - \mu_1^h\right)\right) + p\mu_1^h \mu_1^l$$
$$+p\left(1 - \mu_1^h\right)\mu_1^l - \left(1 - \mu_1^l\right)\left(1 - \mu_1^h\right) T_0 - c$$

which after simplification becomes:

$$E[\theta] + 2\left(\mu_1^h + \mu_1^l\right) - p\mu_1^h + \theta_g (1-p) - p\left(1 - \mu_1^l\right)\left(1 - \mu_1^h\right) T_0 - c$$

and together with (7) yields the following self-selection constraint:

$$T_0 \geq \frac{(2-p)\left(\mu_1^h - \mu_1^l\right)}{\left(1 - \mu_1^h\right)\left(1 - \mu_1^l\right)}$$

Recalling that for renegotiation proofness: $T_0 \leq F$, the suggested contract allows for separation only if:

$$\underline{F}' = \frac{(2-p)\left(\mu_1^h - \mu_1^l\right)}{\left(1 - \mu_1^h\right)\left(1 - \mu_1^l\right)} \leq F$$

However, it is easy to prove that $\underline{F}' > \bar{F}$, where $\bar{F}$ is defined in conditions (C). By condition $C.3$ we have that

$$\bar{F} \leq \frac{(2-p)\left(\mu_1^h - \mu_1^l\right) + (1-p)c}{2p\mu_1^h\left(1 - \mu_1^h\right)} + c$$

Hence, we want to prove that

$$\frac{(2-p)\left(\mu_1^h - \mu_1^l\right)}{\left(1 - \mu_1^h\right)\left(1 - \mu_1^l\right)} \geq \frac{(2-p)\left(\mu_1^h - \mu_1^l\right) + (1-p)c}{2p\mu_1^h\left(1 - \mu_1^h\right)} + c \tag{26}$$

If we take an arbitrarily small c, a sufficient condition is:

$$\frac{(2-p)\left(\mu_1^h - \mu_1^l\right)}{\left(1 - \mu_1^h\right)\left(1 - \mu_1^l\right)} \geq \frac{(2-p)\left(\mu_1^h - \mu_1^l\right)}{2p\mu_1^h\left(1 - \mu_1^h\right)}$$

or:

$$2p\mu_1^h - 1 + \mu_1^l \geq 0$$

Notice that for condition $C.2.$

$$\begin{aligned}
\mu_1^l &\geq \\
\frac{2(1-\beta_1 p)\mu_1^h}{2(1-\beta_1 p)+(4\beta_1-1)p(1-\mu_1^h)} &\geq \\
\frac{2(1-p)\mu_1^h}{2(1-p)+3p(1-\mu_1^h)} &\geq \\
\frac{2(1-p)\frac{2}{3}}{2(1-p)+3p\left(1-\frac{2}{3}\right)} &= \frac{4(1-p)}{3(2-p)}
\end{aligned}$$

where the second inequality is a consequence of the fact that the second line expression is decreasing in β_1, while the third comes from the fact that the third line is increasing in μ_1^h and the fact that

$$\mu_1^h \geq 2\frac{\beta_1}{4\beta_1-1} \geq \frac{2}{3}$$

Therefore we have that:

$$2p\mu_1^h - 1 + \mu_1^l \geq \frac{4}{3}p - 1 + \frac{4(1-p)}{3(2-p)} = \frac{7p-4p^2-2}{3(2-p)} \geq 0$$

where the last inequality holds iff:

$$p \geq \frac{7}{8} - \frac{1}{8}\sqrt{17} \simeq 0.359\,61$$

Therefore, if we take $p \geq 0.36$, there always exist an arbitrarily small c such that (26) is satisfied. ■

Proof of Proposition 5: Let consider any perfect Bayesian equilibrium such that, for some state of the world, $\theta = \theta_b < -1$ and agents do not divorce. It follows that in this state of the world either the proposer makes a proposal but this proposal is rejected, or the proposer does not make any proposal. The first case cannot be an equilibrium, since the proposer can increase her payoff just not making any proposal. Let consider the second case when in equilibrium the proposer does not make any proposal. This equilibrium strategy has to be sustained by out of equilibrium beliefs of the partner which assigns positive probability to the fact that the proposer has not exerted effort (only these beliefs may induce rejection). We show that there always exists at least a proposal, namely the "fair" proposal, such that these beliefs violate the D1 criterion. Let agent i be the proposer and suppose that $e_i = 1$. Denoted with π the probability that the partner will accept the proposal and with αk the net transfer assigned to i by the agreement. Agent i will propose an ex-post agreement instead of continuing the marriage if the following holds:

$$\pi\alpha k + (1-\pi)\left(\frac{1}{2}k - F\right) - c \geq \theta_b + k \tag{27}$$

If instead $e_i = 0$ and denoted with π' the probability that the partner will accept the proposal, i will propose an ex-post agreement if the following holds:

$$\pi'\alpha k + \left(1-\pi'\right)\left(-2F - \frac{1}{2}\right) - c \geq \theta_b + k \tag{28}$$

Let be $\alpha = \frac{1}{2}$ according to the proposal. After simple manipulation we can prove that (27) is satisfied if:

$$\pi \geq \frac{2\theta_b + k + 2F + 2c}{2F} = \underline{\pi} \tag{29}$$

while (28) is satisfied if:

$$\pi' \geq \frac{2\theta_b + 2k + 4F + 1 + 2c}{k + 4F + 1} = \underline{\pi}'$$

It is easy to check that $\underline{\pi}' > \underline{\pi}$, for a small c. First of all, note that $\underline{\pi}' > \underline{\pi}$ is equivalent to:

$$\frac{2\theta_b + 2k + 4F + 1 + 2c}{k + 4F + 1} - \frac{2\theta_b + k + 2F + 2c}{2F} = -\frac{(2F + k + 1)(2\theta_b + k + 2c)}{2(k + 4F + 1)F} > 0$$

which holds if and only if

$$(2\theta_b + k + 2c) < 0$$

which holds for any $-\theta_b > 1$, given that $k \leq 2$ and c sufficiently small. It follows that there exists a larger set of conjectures that i might hold that induces i to deviate when $e_i = 1$ with respect to when $e_i = 0$. Hence for the D1 criterion the partner should assign zero probability to the fact that $e_i = 0$. ■