Is Batting Last an Advantage?

by Theodore L. Turocy

Department of Economics

Texas A&M University

College Station TX 77843

Phone: 979.862.8082

Fax: 979.847.8757

Version 1.0

September 4, 2003

Email: turocy@econmail.tamu.edu

Abstract

This paper applies the theory of zero-sum stochastic games to assess the validity of baseball's ancient wisdom that batting last confers a strategic advantage. Results from numerical calculation of Markov perfect equilibrium suggest that the team that bats last will have an advantage if in fact the offense has, in some sense, more useful strategic actions available than the defense. An example is provided where the advantage depends on details of the teams playing. Regardless of which team has the advantage, all calculations indicate the advantage is negligible in magnitude.

JEL classification codes: C63, C73.

Keywords: zero-sum games, Markov perfect equilibrium, baseball.

This document has been produced using GNU T_EX_{MACS} (see http://www.texmacs.org).

1 Introduction

A long-standing tenet in the received wisdom of baseball is that having "last ups" confers a strategic advantage on the team that bats last. The origins of this belief can be traced back at least as far as pioneering baseball writer Henry Chadwick, who, describing an American Association game played by the Brooklyn club in Louisville in 1888, wrote:[1]

> The Brooklyn team won their first game of their Western tour today, after a close and exciting contest, in which the rule of being last at the bat was again shown to be of conspicuous advantage... The home team had but one inning left to play, while Brooklyn - owing to being last at the bat - had two, and the confidence the knowledge of this fact gave them was inspiring, and on this occasion, as on others, it gave them the victory.

The belief is so firmly ingrained that invoking it requires no explanation today. For example, New York Yankees manager Joe Torre said after a loss to the Anaheim Angels on August 21, 2002, "It's tough to lose in an extra-inning game at home because you have the advantage of batting last, but we didn't get the hits when we needed them."[2]

Since the official rules of baseball were fully recodified in 1950, the visiting team has been designated to bat in the top halves of innings. This is set out in Rule 4.02, which pertains to how a game begins:[3]

> The players of the home team shall take their defensive positions, the first batter of the visiting team shall take his position in the batter's box, the umpire shall call "Play" and the game shall start.

Prior to 1950, however, the rules for determining which team batted first varied. Originally, the order in which the teams batted was determined by a coin toss, except in 1877,

1. *Brooklyn Eagle*, June 29, 1888.

2. `http://www.usatoday.com/sports/scores102/102233/20020821AL--NYYANKEES-0nr.htm/`

3. The official rules of baseball are available online at the website of Major League Baseball, `http://www.mlb.com`.

when the home team batted first. Starting in 1885 in the American Association and in 1887 in the National League, the home team was given the choice of batting first or last. (Note that the 1888 contest described in Chadwick's quote featured the visiting club, Brooklyn, batting last.) It quickly became the custom for the home team to bat last, although isolated instances of home clubs batting first persisted into the twentieth century.[4]

In view of Chadwick's argument, why might clubs in the early days of professional baseball have chosen at times to bat first? A comment published in the *Detroit News* in 1914 gives this reason:[5]

> Right now all clubs go to the field first when on the home grounds; the custom has become firmly rooted, and no manager ever thinks of changing.
>
> Yet, many years ago, it was equally the rule for the home team to bat first, and the argument on which the managers maintained the system was the supposed advantage of 'getting the first crack at the new ball!'
>
> When the game was played with only one ball, and was held up till that ball came back after every journey, a hard-hitting club could, very often, get a flock of runs by starting right in at the jump, taking first bat and collecting hits before the other team had any chance. By the time that ball was turned over to the other club it was black and hard to hit - hence an actual and indisputable advantage for the team first at bat. But when the statute was introduced providing a fresh white ball whenever the original ball vanished, this advantage was destroyed.

So, looking back at baseball history, while there was disagreement as to whether batting first or batting last was an advantage, it appears that, whenever clubs *did* choose to bat first, it may have been to trying to take advantage of having a more resilient ball to hit.

4. See Nemec [7] for a complete chronology of this rule.
5. *Detroit News*, September 1, 1914.

The assertion that batting last gives a strategic advantage at the end of the game seems to be largely unquestioned. Chadwick's theory, espoused frequently in his writings on baseball, suggests a belief that the extra information available to the club batting last, specifically, the realized score of the other club in the inning, is useful in determining how to play their half of the inning.

This paper addresses this question using the tools of game theory. Numerical analysis of an equilibrium model of strategic choice indicates it is unclear whether the informational advantage Chadwick cited is paramount. If the defense has sufficient strategic power, theory suggests that batting *first* may yield a higher winning percentage; in addition, whether a club would prefer to bat first or last may depend on details of the characteristics of the players, and not just the rules of the game. Regardless, the magnitude of the advantage, whichever club it favors, is miniscule; in the models to be considered here, strategic advantages are on the order of one extra victory per ten seasons of play.

The rules of baseball naturally result in action taking place in more-or-less discrete events which transform the state of the game. Thus, baseball is naturally suited to a Markov process model. The operations research literature contains several studies implementing this idea, with dynamic programming used to compute recommendations on the advisability of certain strategies. The idea of using a Markov process representation of a baseball game dates back in print at least to HOWARD's [4] book. BELLMAN [1] explicitly suggests dynamic programming as a solution concept for strategy questions, though he does not pursue specific numerical examples in his exposition. TRUEMAN [8] applied a Markov chain analysis, which modeled the outcome of batters' times at bat similarly to the present paper, to address the question of how a baseball team should sequence its batters in the batting order to maximize the production of runs, a topic revisited more recently by BUKIET ET AL [2].

In order to address the question of whether it would be advantageous to bat first or last, this paper extends Bellman's idea and explicitly models strategic choice in baseball as a two-player, zero-sum game, instead of a unilateral decision problem. Dynamic programming is thus replaced as the solution concept by Markov-perfect Nash equilibrium. By choosing a sufficiently rich definition of the state space of the game, it is possible to solve backwards in time for optimal strategies in all innings except for the ninth (final) inning of play. Since ties in baseball are resolved by playing an additional inning until the tie is broken, these "extra" innings can be viewed as equivalent to the ninth inning, and so a fixed-point problem is present. Even with a state space consisting of millions of states, this structure permits equilibrium to be computed in less than a minute on a reasonably modern computer.

The study of the advisability of individual baseball strategies was launched by LINDSEY [5]. In the absence of large-scale computer databases on baseball play and cheap computational resources for numerical calculation, Lindsey used a small sample of data to give general advice on whether certain strategies were likely to increase or decrease run scoring on average. The specific advisability of any strategy, however, often depends on details of the state of the game. The program which generates the output presented herein is capable of outputting these recommendations, but the size of the state space ensures that the output is massive. Instead of summarizing these results, this paper will instead consider the *aggregate* advantage, for example, the team that bats last enjoys by the presence of the sacrifice bunt in a typical game.[6]

The word "strategy" in baseball denotes a concept somewhat different from its formal definition in game theory as a complete contingent plan of action for a player. A "baseball strategy" is more akin in the parlance of game theory to a stage-game action, that is, a choice available to a player at (a subset of) the states of the Markov game. These

6. The computer program will be made available on the author's website, `http://econweb.tamu.edu/turocy`, under the GNU General Public License. This may be of specific interest to those readers wondering, for example, how often it makes sense to give an intentional walk to Barry Bonds.

actions are taken precisely to manipulate the state of the game in a team's favor, by modifying the transition probabilities from the current state. In fact, many "baseball strategies" employed by the offense are referred to as "one-run strategies," since they increase the probability of scoring at least one (more) run in an inning, at the cost of a decrease in the likelihood of the rest of the inning resulting in two or more runs.

Of these "baseball strategies," this paper selects three notable examples:

- The sacrifice bunt. A sacrifice bunt involves the batter deliberately striking the ball softly (a "bunt") in such a way that the defense causes him to be put out, so as to advance runners on the bases towards their ultimate goal of scoring a run.

- The intentional walk. An intentional walk is issued when the pitcher deliberately throws four unhittable pitches to the batter, allowing him to reach base safely, but preventing him from obtaining a more damaging result (such as a home run).

- The stolen base. A stolen base is an advancement of a baserunner during a pitch rather than a batted ball. For the purposes of this paper, a stolen base attempt is modeled as a simultaneous move stage game between the offense and defense. This game typically has a pursuer-evader structure, with a unique Nash equilibrium in randomized strategies when viewed as a game in isolation.

Aside from a measure of prominence these three "baseball strategies" enjoy within the sport, these have been selected as representing the possible range of actions available to baseball teams. The sacrifice bunt is well-approximated by giving the offense the option to advance a baserunner with probability one while giving up an out, also with probability one;[7] essentially, it reduces, in this conception, to a decision which may be unilaterally

7. Sacrifice bunt attempts do also result in failure for the offense, as well as failure by the defense in putting out the batter, with a nontrivial frequency. This stylized modeling can be viewed as the model of bunting which is most favorable to the team on offense.

made by the offense. Similarly, the intentional walk can be thought of as an action available to the defense also resulting in a state transition with probability one. The stolen base represents an intermediate case, where both offense and defense take actions which affect state transitions.

The paper proceeds as follows. Section 2 presents a model of a baseball game as a two-player, zero-sum Markov game, and describes the variations and parameterizations of interest. Section 3 describes numerical results quantifying the advantage in equilibrium to batting first or last under a number of strategic treatments. Section 4 concludes with discussion and interpretation of the results.

2 The Model

The foundation of the model is a Markov chain approach to the progress of a baseball game similar to that in BUKIET et al [2]. The game is conceptualized as a sequence of transitions, each transition representing the net effect of a play on the state of the game. The state is described by a 7-vector $(\iota, \tau, \lambda, \omega, \theta, \beta_1, \beta_2) \in \Sigma$, with components

- ι: the current inning (1 through 9; extra innings to break ties are treated as the ninth inning);
- τ: the half inning, denoting whether the team that bats first or second is currently on offense;
- λ: the lead (in runs) currently enjoyed by the batting team (if positive; their deficit in runs if negative);
- ω: the number of outs (0, 1 or 2) recorded so far in the half inning;
- θ: the set of bases currently occupied by baserunners (a subset of $\{1, 2, 3\}$);

- β_t: the position in the batting order (one through nine) occupied by the next batter scheduled to bat for each team t (the first batter follows the ninth in the rotation).

Transitions between states are governed by matrices of transition probabilities which are assumed to depend only on β_τ, the identity of the batter. In other words, batters may be heterogeneous, with different batting abilities, but their performance is otherwise independent of the game situation. The transition probabilities also obey all the rules of baseball. This results in a sparse transition matrix; the number of states which can be reached with positive probability in one transition ranges from 5 to 25.[8]

The two teams are modeled as unitary actors, each with the goal of maximizing the probability the team eventually wins the game. The baseline model is a game only in a trivial sense, in which the teams have at each state in Σ only a single action. It is easy to see that in this trivial game, if the teams are identical, each team will win with probability exactly one-half.

To this baseline model, additional actions representing "baseball strategies" are added. The "baseball strategies" considered in this paper are the sacrifice bunt, the intentional walk, and the stolen base; the presence or absence of these three features of the game gives a total of eight possible treatments (including the baseline where all are absent). Consideration of the equilibrium values (which are winning percentages) of the seven variants compared to the baseline of one-half then measures which team is favored by the presence of the strategy, and by how much.

At each state, decisions are made in the following order. Where not explicitly noted, state transitions occur in accordance with baseball rules.

- If intentional walks are permitted, in any state the defense may choose to intentionally walk the batter. If this choice is made, the stage game ends, and the state transitions with probability one.

8. The extreme cases are given by any state with $\theta = \emptyset$, and states with $\theta = \{1, 2, 3\}$ and $\omega = 0$, respectively.

- If sacrifice bunts are permitted, and $\theta \in \{\{1\}, \{2\}, \{1, 2\}\}$,[9] the offense may choose to bunt. If they do so, the stage game ends, and the state transitions with probability one reflecting a successful bunt: the number of outs ω increments by one, and baserunners each advance one base.

- If stolen bases are permitted, and $\theta = \{1\}$, a simultaneous-move game is played. The offense chooses between actions S, attempting a stolen base, and N, not attempting a stolen base. The defense chooses between actions B, focusing attention on the batter, and R, focusing attention on the baserunner. The effect of action R is to reduce the probability that an attempt to steal the base is successful; this is operationalized by assuming the probability of success is π_R if the offense plays S and the defense R, which is strictly smaller than the probability of success π_B which obtains if the defense plays B. On the other hand, playing R against N results in an increase in the frequency with which the batter hits safely, changing the transition probabilities favorably for the offense.[10] In most cases, and for plausible parameter values, this simultaneous-move game will have a unique equilibrium in which both players randomize between their actions.

The transition probabilities between states generated by the outcome of batters' times at bat are generated using a two-step process, similar to the methods in Trueman [8] and Bukiet et al [2]. The basic outcome of each time at bat is modeled as a multinomial random variable, with seven possible outcomes: *single*, *double*, *triple*, *home run*, *walk*, *strikeout*, and *generic out* (which includes all other events where the batter is put out after hitting the ball). The frequencies of these outcomes form a vector $(\phi^{ti}_{1B}$,

9. Thus, the "squeeze" bunt, which attempts to score a runner from third base, is not considered, in particular because modeling this play as being successful with probability one is unrealistic.

10. This model is discussed in more detail in a companion paper, Turocy [9], including a calibration of model parameters to Major League Baseball data.

ϕ^{ti}_{2B},ϕ^{ti}_{3B}, ϕ^{ti}_{HR}, ϕ^{ti}_{BB}, ϕ^{ti}_{SO}, ϕ^{ti}_{OUT}) of probabilities representing, essentially, the batting abilities of batter i on team t. Conditional on the realization of this outcome, state transitions occur according to the aggregate empirical frequencies observed in Major League Baseball during the seasons 1973 through 1992, inclusive. For example, during the period, there were 111,017 singles hit with no outs ($\omega = 0$) and no runners on base ($\theta = \emptyset$). Of these, 108,434 (97.67%) resulted in a state with $\omega = 0$ and $\theta = \{1\}$; 1436 (1.29%) in $\omega = 0$ and $\theta = \{2\}$; 261 (0.24%) in $\omega = 0$ and $\theta = \{3\}$; 873 (0.79%) in $\omega = 1$ and $\theta = \emptyset$; and 13 in $\omega = 0$, $\theta = \emptyset$, and a run scoring. These frequencies then determine the state transition probabilities in the model conditional on a single being hit in that situation.[11]

The solution concept for the game is Markov perfect equilibrium. The state space of the game is in principle infinite, since there is no bound on the lead a team may achieve. For computational tractability, the game is modified such that there is an upper bound Λ such that a team achieving a lead of at least Λ wins the game with probability one.[12] The value of Λ should be chosen sufficiently large that solving the baseline game results in a game value that is satisfactorily close to one-half; the results reported in the next section use a value of $\Lambda = 30$.[13]

With this modification, the stochastic game has a finite number of states, and a finite number of actions, and so Markov perfect equilibrium does exist (see, for example, Fudenberg and Tirole [3], Theorem 13.1). From a theoretical perspective, the question of whether a minimax theorem holds for this game, thus guaranteeing that the con-

11. The effects of errors and baserunners being put out attempting advancement are thus incorporated in the model. Events that result in a state change but do not result in the termination of a batter's time at the plate, including wild pitches, passed balls, and balks, are infrequent enough to have a small effect and are omitted for simplicity.

12. This implements a "mercy rule" similar to those often used to terminate lopsided games in youth and recreational baseball.

13. No Major League Baseball team has even scored 30 runs in a single game since the nineteenth century.

cept of the value of the game is well-defined, is open. The game is similar to the binary Markov game of WALKER AND WOODERS [10], for which a minimax theorem holds. However, the theorem does not apply here since each state can transition directly into more than two states.

Because players bat in a fixed order, and because each player who comes to bat will eventually be put out, score a run, or be on base when the defense records the third out, there is a partial ordering $\succ$ of the states in innings one through eight such that $s_1 \succ s_2$ if and only if state s_1 can occur before state s_2. Therefore, for these innings, it is possible to solve for equilibrium backwards in time, starting with the end of the eighth inning and ending with the beginning of the game. The process of computing the equilibrium thus spends most of its time solving the fixed-point problem involving the ninth inning, as the value of a tie game at the end of the ninth inning is equal to the value of a tie game at the beginning of the ninth inning, everything else being equal. Value-function iteration on the ninth inning converges very rapidly for the examples presented. Iteration was stopped when the value of a tie game at the beginning of the ninth inning changed by less than 10^{-4} in successive iterations; this threshold was generally reached within 15-20 iterations. Therefore, equilibrium can be computed accurately and efficiently, even though the game as specified features a state space with 2,134,512 elements. This iterative process was initiated with different initial conditions, each resulting in equilibrium winning percentages within the given tolerance. As such, numerical analysis suggests the notion of the value of the game is well-defined in the versions of the games reported.

3 Results

3.1 Games played between identical teams

To investigate how the presence or absence of the "baseball strategies" under consideration affect the equilibrium value, this section considers equilibrium in games played between two typical teams. The notion of "typical" is operationalized by creating a lineup where the vector ϕ^{ti} is given by the empirical frequency of each of the seven outcomes of a batter's time at bat, given that the batter occupied the ith position in his team's batting order. While an individual player may, and does often, occupy different positions in the batting order in different games in a season, the general structure of a team's batting order does not change substantially, and the structure of batting orders across teams follows some general customs. So, these aggregate constructed lineups share most of the general structure one would find in a typical lineup of a randomly-selected team.

The vectors ϕ are derived from batting performance over twenty seasons of Major League Baseball, from 1973 to 1992 inclusive. The vectors ϕ are computed separately for the American League and National League. During the period of the sample, the American League made use of the designated hitter, which permits teams to nominate a player, generally a talented batter, to bat instead of the pitcher; the National League required pitchers to bat. Since pitchers are selected for their pitching, and not batting, abilities, pitchers are as a group by far the worst batters, and so equilibrium strategies are different when the pitcher is at bat or scheduled to bat soon.[14]

The two teams are taken to be identical in all respects; that is, $\phi^{1i} = \phi^{2i}$ for all batters i. Table 1 presents the probability the team batting last (the home team under current rules) wins the game, in equilibrium, for each of the eight strategic environments under consideration.[15]

14. For example, in the calculated equilibria, by far most of the situations where the sacrifice bunt is advisable occur when the pitcher is batting. Similarly, some of the most opportune situations for the defense to issue the intentional walk arise when the pitcher is scheduled to bat next. Both equilibrium features concur with general baseball practice.

15. For the environments including the stolen base, the parameters $\pi_B = .9$ and $\pi_R = .1$ are used, and the improvement in performance enjoyed by the batter in the contingency where (N, R) is played is operationalized by doubling the frequency of hitting a single. These parameters are chosen as representative based on Turocy [9], as resulting in equilibrium predictions that approximate aggregate observed data. The qualitative results survive across other parameterizations.

SB	IBB	SH	American League	National League
no	no	no	.50000	.50000
yes	no	no	.49990	.49989
no	yes	no	.49983	.49974
yes	yes	no	.49972	.49961
no	no	yes	.50056	.50068
yes	no	yes	.50044	.50048
no	yes	yes	.50017	.50015
yes	yes	yes	.50005	.49995

Table 1. Probability the last-batting team wins a game between identical teams, given the set of stage-game actions available (SB=stolen base, IBB=intentional walk, SH=sacrifice bunt).

Result 1. *The magnitude of the strategic advantage is small.*

The winning percentages in Table 1 suggest that for teams of approximately equal strength, which team bats last has only a small effect on the relative likelihoods of eventual victory. For a sense of scale in interpreting these and subsequent winning percentages, note that the schedule of a Major League Baseball team consists of 162 games, 81 of which are played at the team's home park. So, a difference in winning percentage of .001 corresponds to one extra victory or defeat per six seasons overall. Restricting consideration to only the home portion of a team's schedule, imagining, perhaps, a club asking permission to "volunteer" to bat first even when at home, that winning percentage margin of .001 would amount to one extra victory or defeat in *twelve* seasons of play. The percentages quoted in Table 1 differ from the baseline winning percentage of one-half by less than that margin.

Chadwick's arguments in favor of an advantage for batting last are based on the ability of the team batting last to use the information of how many runs were scored in the top of the inning. The most pivotal case is the bottom of the ninth inning, in which the game is tied; in this situation, the batting team knows that scoring one run will result in victory with probability one. Equilibrium calculations, however, indicate this knowledge is not that valuable; the equilibrium strategies do not differ greatly in the top and bottom of the same inning (that is, they do not vary greatly with τ), holding the other aspects of the state vector constant. While the team batting last in a tie game in the ninth may know one run will win the game with probability one, the team batting first in the same inning can be sure one run will win the game with high probability (around .85 for these parameterizations).

Note that the fact that the equilibrium winning percentages stay close to one-half does not imply that strategy is irrelevant to success in baseball. For example, results in Turocy [9] indicate that unilateral commitment to never stealing a base (that is, strategy N) would cost a typical team about two victories in a 162-game season. Similar results can be obtained for intentional walks and sacrifice bunts. So, the small advantages afforded in equilibrium by these "baseball strategies" does not arise because the strategies are useless; nonoptimal play (for example, by choosing to commit to never sacrificing, giving an intentional walk, or stealing a base) would result in a noticeable number of foregone wins in even a season.

Result 2. *The direction of the advantage is related to the relative strategic strengths of offense and defense; batting last is generally better if the offense has more "strategic power."*

The discussion so far, motivated by Chadwick's hypothesis, has made no mention of a role for the defense. The treatment most favorable to the foregoing argument is the one featuring only the sacrifice bunt, insofar as this is the only game considered where only the team on offense makes any nontrivial choices.

In Table 1, every pairwise comparison of a treatment with the sacrifice bunt, an offensive strategy, versus the one without, holding the availability of other strategies equal, results in an increase of the winning percentage of the team batting last. On the other hand, making pairwise comparisons between treatments with and without the intentional walk, a defensive strategy, results in an increase of the winning percentage of the team batting first.

This observation indicates a modification of Chadwick's theory: the team that gets to "go last" has the advantage. Since many choices are made within each half-inning of a baseball game, "going last" is intended to mean "playing the side with more strategic options." If the defense has more, or more valuable, strategic options at its disposal, then being on defense in the bottom of the last inning would be advantageous.

This reinterpretation of Chadwick's ideas may not be unreasonable. The rules of baseball evolved substantially during the nineteenth century; in fact, the game played by "major league" teams in the early 1880s would resemble a modern game of medium-pitch softball in a good amateur tournament.[16] In this environment, the balance of strategy may have been very different; indeed, it might have been a reasonable approximation of the game to think of only the offense as having any choices of consequence to make.

The early results of LINDSEY [5] suggested that the sacrifice bunt is advisable only within a very limited number of situations. As such, it would be unexpected for the advantage of the sacrifice bunt to be extremely large. The inclusion of basestealing is intended to mitigate this by including another "baseball strategy" often thought to favor the team batting last. However, the presence of basestealing on average favors the team batting *first*. Basestealing is modeled with both offense and defense playing active strategic roles. It turns out that, in a tie game in the top of the last inning (that is, $\iota = 9$, $\tau = 1$, and $\lambda = 0$) the team batting first is able to use the stolen base to increase their chances of getting a lead; when they do so, the stolen base becomes less attractive to the team batting in the bottom of the inning. The defense, in that situation, is able to exploit its lead strategically.

Result 3. *The direction of the advantage is not solely a function of the rules; it may also depend on characteristics of the players.*

Consider the winning probabilities of the last-batting team in the full model containing all three "baseball strategies." When the model is calibrated to a typical American League lineup, the team batting first wins in equilibrium slightly less than one-half of the games; on the other hand, when calibrated for the National League in the same period, the team batting first enjoys a small advantage in equilibrium. This can be understood by remembering the rule difference between the leagues: in the National League, the pitcher, a very weak hitter, takes a turn at bat. This means that it becomes more attractive to give an intentional walk to the batter who bats before the pitcher. So, when the team batting

16. For example, until 1884 the motion the pitcher could use to deliver a pitch was very restricted; even in 1884, the pitcher still was required to throw underhand, and from a distance closer to the plate (and similar to that in softball today).

first holds a small lead in the last inning, the intentional walk is a more valuable strategic weapon in the National League than the American.

This specific example highlights an important assumption in the model: no substitutions are permitted. In practice, a club would always put in a substitute batter for the pitcher in such a situation. The model can be extended in principle to accommodate this; the complications involved are discussed in Section 4. However, this example illustrates that it may not be possible to answer the question of whether a club would prefer to bat first independent of knowledge about the details of the teams involved; the answer may differ from team to team, or opponent to opponent.

Result 4. *The advantages in Table 1 arise from endgame effects.*

To verify that the deviations from winning percentages of one-half are effects created by the finite length of the game, as opposed to a systematic, inning-by-inning advantage, one can imagine modifying the rules of baseball to have more than nine innings. Considering the full model with all strategies available, if the game were 20 innings long instead of nine, the team batting last would win with probability .50003 in the American League, and .49996 in the National. For games of 100 innings, the probabilities are .50001 and .49999, respectively.

3.2 Games with a home field advantage

What about games between unequal opponents? In fact, the "median" game of baseball takes place between opponents who can be expected to perform differently. Empirically, offensive statistics for home teams in baseball are better than those of visitors, taken in aggregate. Over the course of the seasons 1973-1992, the home team outperformed the visitors in most categories (singles, doubles, and so forth) by about 3 percent (that is, the home team hits singles at a rate 3% higher than the visitors, etc.). Here the model is distinguishing between a home field advantage, which is improved performance due to better rest, familiar surroundings, or fan support, and an advantage to batting last, which is strategic in nature.

To approximate this in the model, Table 2 reports winning percentages for the home team when batting first or last, where the home team produces better offensive statistics than the visitors. For these data, this is accomplished by increasing the home team's productivity by 1.5%, and decreasing the visiting team's by the same percentage.[17]

17. Since the model deals in observed frequencies of events, this difference may arise due to better offense at home, inferior defense on the road, or a combination of the two. The origin of the difference in performance is not important for the conclusions of the model.

			American League		National League	
SB	IBB	SH	last	first	last	first
no	no	no	.52891	.52891	.52789	.52789
yes	no	no	.53141	**.53161**	.53046	**.53068**
no	yes	no	.52874	**.52907**	.52763	**.52816**
yes	yes	no	.53123	**.53179**	.53018	**.53096**
no	no	yes	**.52939**	.52827	**.52846**	.52711
yes	no	yes	**.53188**	.53099	**.53096**	.52999
no	yes	yes	**.52899**	.52866	**.52793**	.52763
yes	yes	yes	**.53148**	.53138	.53042	**.53052**

Table 2. Winning percentage of home team when batting last or first, respectively, for each of the combinations of stage-game actions available. Cells in bold indicate the higher winning percentage.

Result 5. *The assumption of symmetric teams is not crucial to the results of section 3.1.*

In Table 2, cells in boldface indicate the higher winning percentage for the home team for each treatment. These cells follow the same pattern of advantages as in Table 1, indicating that the preferred choice of batting first or last is robust to the introduction of asymmetries between the teams.

The winning percentages for the home team are noticeably higher in the treatments with stolen bases. This arises because the advantage to the offense in the contingency where (N, R) is played is operationalized by a *percentage* increase in the batter's performance. Since the home team batters are already in this model hitting singles at a higher rate than the visitors' batters, the percentage increase is larger, ceteris paribus, for the home team. A model with an additive increment to this frequency would preserve the qualitative results, while resulting in winning percentages closer to the baseline treatment with no strategy.

4 Conclusions

The game of baseball involves far more strategic choice than the rather simple model studied in this paper. The type and location of each pitch thrown (high or low, fastball or curve), the batter's preparation for the pitch (expect fastball, expect curve), and the positioning of fielders (bring the infield in, play the third baseman close to the foul line) are all important parts of the game. It would be computationally challenging to analyze a model rich enough to encompass these choices explicitly. But, importantly, it would be difficult to even specify such models in a satisfactory way, since doing so would require an understanding of how each of these choices affect performance. While the present model by necessity abstracts from many of these details, the results are instructive, if not in giving a definitive answer to the question in the title, at least in understanding what characteristics of the game are likely to be relevant in determining the answer.

For it to matter in a measurable way whether a team bats first or last, it will be necessary that optimal strategy in the top of an inning differ substantively from optimal strategy in the bottom of the same inning. However, for the models in this paper, this does not hold: the risks and rewards of choices are in approximately the same proportion. Even with strategies that result in state transitions with certainty, the overall effect on winning percentage is small. Most of the unmodeled choices enumerated in the previous paragraph only modify state transitions in a cruder, probabilistic fashion; taken individually, they each would have only a tiny effect on the equilibrium value.

Also, the games studied here suggest that the offense must have, in some sense, a richer strategic portfolio in order for batting last to be advantageous. In any event, in order for the overall advantage (or disadvantage) to batting last to be measurable, the unmodeled strategic choices must overwhelmingly favor either the offense or defense. While an answer to this question may not be in principle obtainable, it is worth noting that throughout baseball history, the general policy has been to institute changes in the rules to more or less keep offense and defense balanced, at least in a subjective sense. Insofar as this balancing process tightens or relaxes restrictions on play as offense or defense, this is suggestive that a balance of strategic power may exist between offense and defense as the game is played today.

There is one missing, yet important, source of choices in the model. The concept of "strategy" in this model has been strictly one of strategy on the field. Specifically, any modeling of player substitution is not included. Yet, as noted, there are situations where player substitution would clearly be indicated. The magnitude of this omission is not clear. As noted, one reason the effect of the modeled actions on winning percentage is small is that there is an almost-symmetry in optimal choices in the top and bottom of innings. However, player substitutions in baseball have an asymmetric aspect. The strategic substitution of players in the later innings of a game generally involves the insertion of players with specialized talents suited for the game situation at hand. A very visible example is substituting a better batter in place of the pitcher in the National League. To pick up the discussion from section 3, if the pitcher is due to bat in the bottom of the ninth of a tie game, he will likely be removed and replaced with a superior batter. In the top half of the inning, it is likely the same will occur. But, since baseball's rules do not permit a player to re-enter a game once removed, the team that bats first will need to insert a new pitcher into the game for the bottom of the ninth inning, while the team that bats last, should they win the game, would not need to insert a substitute.

The model can in principle be extended to cover substitutions by including in the description of the Markov state the status of all players on rosters - available, currently in the game, or already removed. Such an analysis is impractical for rosters containing more than two or three substitutes at most. It may also be difficult to obtain general conclusions for the value of substitution, since the value likely depends on the specific profile of talents possessed by the available substitutes. Further, while the previous paragraph discussed substitutions on offense, the defense too may engage in tactical substitutions of pitchers. While analysis of such an expanded model falls outside the scope of the present paper, the principles suggested by the results indicate the answer depends on whether substitutions on offense or defense are more effective, and whether the optimal timing of substitutions differs greatly between the top and the bottom of an ininng.

Finally, in revisiting Chadwick's quote that opened this paper, note the use of words like 'confidence' and 'inspiring.' Not included in these models are any considerations of psychology. Despite the analysis presented here, the belief that batting last is an advantage is deeply ingrained in the beliefs of baseball players and managers at every level.[18]

18. In the spirit of full disclosure, this includes the author. In testing the program used to compute equilibrium for this model, the author became interested in the advantage the stolen base yielded to the home team. After obtaining the conclusion presented here, that the visiting team actually won more frequently when stolen bases were included in the game, the author was convinced the program was incorrect. In fact, the program is correct, but it took several weeks to become convinced.

As an empirical regularity, baseball teams playing at home, on aggregate, perform about three percent better in every measurable category. Some, perhaps much, of this effect is physiological. Baseball teams tend to stay at home for a week or more at a time, affording a more regular schedule, dining habits, and the comforts of home, and therefore are able to simply perform better. It is conceivable, however, that some of the performance improvement could derive from confidence in the belief that having the "last up" always gives them one more chance to win the game. Since the natural experiment of randomly assigning the order in which teams bat is not feasible, the question remains unanswerable.[19] However, such an advantage could not be reasonably said to derive from strategic considerations, as the advantage would not survive the "publication-proof" test (e.g., McKelvey and Riddihough [6]).

Acknowledgements

The author gratefully acknowledges valuable comments and feedback from Ken Judd, John Van Huyck, and seminar participants at Texas A&M University and the Stanford Institute for Theoretical Economics. Clifford Blau and Peter Morris were instrumental in identifying relevant quotations from the early days of baseball, and the readability of this draft was greatly enhanced by comments from Phil Birnbaum, Clem Comly, and Brian Wood. The Major League Baseball play-by-play data used in the model is available from Retrosheet (`http://www.retrosheet.org`). The author also acknowledges the Economic Research Laboratory at Texas A&M University for use of the SERF Economic Research Farm for some of the calculations presented in this work. All errors remain the sole responsibility of the author.

19. Another experiment is possible in principle. If all players and managers were presented with and convinced of the results in this paper, would the observed performance of the home teams decrease? Alas, this experiment is even more implausible in practice than the one mentioned in the text.

Bibliography

[1] R. Bellman. Dynamic programming and Markovian decision processes, with application to baseball. In S. P. Ladany and R. E. Machol, editors, *Optimal Strategies in Sports*, pages 77–85. Elsevier-North Holland, New York, 1977.

[2] B. Bukiet, E. R. Harold, and J. L. Palacios. A Markov chain approach to baseball. *Operations Research*, 25:14–23, 1997.

[3] D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, Cambridge MA, 1991.

[4] R. A. Howard. *Dynamic Programming and Markov Processes*. MIT Press and Wiley, 1960.

[5] G. R. Lindsey. An investigation of strategies in baseball. *Operations Research*, 11:477–501, 1963.

[6] R. D. McKelvey and G. Riddihough. The *hard* sciences. *Proceedings of the National Academy of Sciences*, 96:10549, September 1999.

[7] D. Nemec. *The Rules of Baseball: An Anecdotal Look at the Rules of Baseball and How They Came to Be*. The Lyons Press, 1994.

[8] R. E. Trueman. Analysis of baseball as a Markov process. In S. P. Ladany and R. E. Machol, editors, *Optimal Strategies in Sports*, pages 63–76. Elsevier-North Holland, New York, 1977.

[9] T. L. Turocy. The economic value of the stolen base. Mimeo, Texas A&M University, July 2003.

[10] M. Walker and J. Wooders. Equilibrium play in matches: Binary Markov games. University of Arizona Working Paper #00-12, July 2000.