The Harsanyi paradox and the "right to talk" in bargaining among coalitions*

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Abstract

We introduce a non-cooperative model of bargaining when players are divided into coalitions. The model is a modification of the mechanism in Vidal-Puga (Economic Theory, 2005) so that all the players have the same chances to make proposals. This means that players maintain their own "right to talk" when joining a coalition. We apply this model to an intriguing example presented by Krasa, Tamimi and Yannelis (Journal of Mathematical Economics, 2003) and show that the Harsanyi paradox (forming a coalition may be disadvantageous) disappears.

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1 Introduction

Many economic situations can be modelled as a set of agents or players with independent interests who may benefit from cooperation. Moreover, it is not infrequent that these agents have partitioned themselves into coalitions (such as unions, cartels, or syndicates) for the purpose of bargaining.

Assuming that cooperation is carried out, the question is how to share the benefit between the coalitions and between the members inside each coalition, i.e. which "value" best represents the expectation of each individual. The economic theory has addressed this problem from two different points of view. One of them is axiomatic, or cooperative. The other one is positive, or non-cooperative.

The axiomatic point of view focuses on finding allocations which satisfy "fair" (or at least "reasonable") properties, such as efficiency (the final outcome must be efficient), symmetry (players with the same characteristics must receive the same), etc. The non-cooperative point of view leads to the study of the allocations which arise in a given non-cooperative environment. In this paper, we follow a non-cooperative approach.

Taking an axiomatic point of view, Owen (1977) presented a value for transfer utility games with coalition structure. Further axiomatic characterizations were provided by Hart and Kurz (1983), Winter (1992), Calvo, Lasaga and Winter (1996) and Albizuri and Zarzuelo (2004), among others.

Owen assumed that this structure was exogenously given. Hart and Kurz (1983) reinterpreted the Owen value assuming that players form coalitions in order to improve their bargaining power.

Under both approaches, the main idea is that the coalitions play among themselves as individual agents in a game between coalitions, and the surplus obtained by each coalition is distributed among its members.

Recently, the Owen value has been non-cooperatively supported by Vidal-Puga and Bergantiños (2003) and Vidal-Puga (2005). In these papers, the players play a non-cooperative mechanism[1] in two stages: in the first stage,

[1] To avoid ambiguities with cooperative games, we use the term *non-cooperative mech-*

the players inside a coalition bargain among themselves the strategy to follow in the second stage, where bargaining takes place among coalitions.

Vidal-Puga (2005) generalizes a previous mechanism of Hart and Mas-Colell (1996). In Hart and Mas-Colell's model, a player is randomly chosen in order to propose a payoff. If this proposal is not accepted by all the other players, the mechanism is played again under the same conditions with probability $\rho \in [0, 1)$. With probability $1 - \rho$, the proposer leaves the game and the mechanism is repeated with the rest of the players.

In Vidal-Puga (2005), this procedure is played in two rounds. First, agreements are negotiated within coalitions and then through delegates among coalitions. In the first round, a player is randomly chosen out of each coalition and proposes a payoff. Each proposal is voted by the rest of the members of its own coalition. If one of them rejects the proposal, the mechanism is either played again under the same conditions (probability ρ), or the proposer leaves the game and the mechanism is repeated with the rest of the players (probability $1 - \rho$). If there is no rejection, the proposal of one of the coalitions is randomly chosen. If this proposal is not accepted by all other coalitions, the mechanism is played again under the same conditions (probability ρ), or the entire proposing coalition leaves the game and the mechanism is repeated with the rest of the players (probability $1 - \rho$).

Vidal-Puga (2005) shows that this mechanism in two rounds implements the Owen value in a non-restrictive class of games.

Frequently, it is interpreted that players form coalition structures in order to improve their bargaining strength (Hart and Kurz (1983)). However, as Harsanyi (1977, p. 203) points out, the bargaining strength does not improve in general. An individual can be worse off bargaining as a member of a coalition than bargaining alone. Chae and Heidhues (2004, p. 47) provide an explanation for this paradox. By merging in a coalition structure, players reduce their multiple "rights to talk" to a single right in the game between coalitions, hence improving the position of the outsiders.

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anism, or simply *mechanism*, rather than non-cooperative game.

The meaning of "rights to talk" is not clear from an axiomatic viewpoint (see for example Chae and Moulin (2004)). However, it has a clear meaning in the mechanism in Vidal-Puga (2005). The right to talk is simply the right to make a proposal. This right is dispelled as the size of the coalition increases.

In this paper, we study the effect that provides to maintain the "rights to talk" of the players inside a coalition. We modify the mechanism by Vidal-Puga (2005) so that players maintain their "rights to talk". Hence, the coalitions with more members have more chances to make proposals. This new mechanism is still a generalization of the mechanism of Hart and Mas-Colell (1996), in the sense that both coincide when the coalition structure is trivial (i.e. all the coalitions are singletons, or there exists a unique coalition).

In Section 2 we present the notation used throughout the paper. In Section 3 we present the formal mechanism and state our main result. In Section 4 we analyze an intriguing example presented by Krasa, Tamimi and Yannelis (2003). Finally, Section 5 is devoted to a brief discussion.

2 Preliminaries

Let U be a (maybe infinite) set of potential *players*. A *non-transferable utility game*, or *NTU game*, is a pair (N,V) where $N \subset U$ is finite and V is a correspondence which assigns to each $S \subset N$, $S \neq \emptyset$ a nonempty, closed, convex and bounded-above subset $V(S) \subset \mathbb{R}^S$ representing all the possible payoffs that the members of S can obtain for themselves when playing cooperatively. For $S \subset N$, we maintain the notation V when referring to the application V restricted to S as player set. For simplicity, we denote $V(i)$ instead of $V(\{i\})$, $S \cup i$ instead of $S \cup \{i\}$ and $N \backslash i$ instead of $N \backslash \{i\}$. We denote the set of NTU games as NTU.

For each $i \in N$, let $r_i := \max\{x : x \in V(i)\}$. We will assume that, for each $S \subset N$ and $x \in V(S)$, the vector $y \in \mathbb{R}^N$ with $y_i = x_i$ for all $i \in S$ and $y_i = r_i$ for all $i \in N \backslash S$, belongs to $V(N)$.

When

$$V(S)=\left\{x\in\mathbb{R}^S:\sum_{i\in S}x_i\leq v(S)\right\}$$

for some $v:2^N\to\mathbb{R}$ with $v(\emptyset)=0$, we say that (N,V) is a *transferable utility game* (or *TU game*) and it is represented by (N,v).

Given $N\subset U$ finite, we call *coalition structure* over N a partition of the player set, i.e. $\mathcal{C}=\{C_1,C_2,...,C_p\}\subset 2^N$ is a coalition structure if it satisfies $\bigcup_{C_q\in\mathcal{C}}C_q=N$ and $C_q\cap C_r=\emptyset$ when $q\neq r$. A coalition structure $\mathcal{C}$ over N is *trivial* if either $\mathcal{C}=\{\{i\}\}_{i\in N}$ or $\mathcal{C}=\{N\}$. For any $S\subset N$, we denote the restriction of $\mathcal{C}$ to the players in S as $\mathcal{C}_S$ (notice that this implies that $\mathcal{C}_S$ may have less or the same number of coalitions as $\mathcal{C}$).

We denote an NTU game (N,V) with coalition structure $\mathcal{C}$ over N as $(N,V,\mathcal{C})$. We denote the set of NTU games with coalition structure as $CNTU$.

Given G is a subset of NTU or $CNTU$, a *value* in G is a correspondence which assigns to each $(N,V)\in G$ or $(N,V,\mathcal{C})\in G$ a vector of $\mathbb{R}^N$. A well-known value in TU games is the *Shapley value* (Shapley (1953)). We denote the Shapley value of the TU game (N,v) as $\varphi^N\in\mathbb{R}^N$. For TU games with coalition structure, Owen (1977) proposed a single value based on Shapley's which takes into account the coalition structure $\mathcal{C}$. We call this value the *Owen coalitional value*, or simply the *Owen value*. We denote the Owen value of the TU game with coalition structure $(N,v,\mathcal{C})$ as $\phi^N\in\mathbb{R}^N$.

3 The mechanism

In this section we describe the coalitional mechanism. This mechanism is a modification of the bargaining mechanism presented in Vidal-Puga (2005).

Even though the model is defined for NTU games, we focus on TU games.

Fix $(N,V,\mathcal{C})\in CNTU$. For each $S\subset N$, we denote by Γ_S the set of applications $\gamma:\mathcal{C}_S\to S$ satisfying $\gamma\left(C'_q\right)\in C'_q$ for each $C'_q\in\mathcal{C}_S$. For simplicity, we denote $\gamma_q:=\gamma\left(C'_q\right)$.

The coalitional bargaining mechanism associated with $(N, V, \mathcal{C})$ and $\rho \in [0, 1)$ is defined as follows:

> In each round there is a set $S \subset N$ of active players. In the first round, $S = N$. Each round has one or two stages. In the first stage, a *proposer* is randomly chosen from each coalition. Namely, a function $\gamma \in \Gamma_S$ is randomly chosen, being each γ equally likely to be chosen. The coalitions play sequentially (say, for example, in the order $\left(C'_1, C'_2, ..., C'_p\right)$) in the following way: γ_1 proposes a feasible payoff, i.e. a vector in $V(S)$. The members of $C'_1 \backslash \gamma_1$ are then asked in some prespecified order to accept or reject the proposal. If one of them rejects the proposal, then we move to the next round where the set of active players is S with probability ρ and $S \backslash \gamma_1$ with probability $1 - \rho$. In the latter case, player γ_1 gets r_{γ_1}. If all the players accept the proposal, we move on to the next coalition, C'_2. Then, players of C'_2 proceed to repeat the process under the same conditions, and so on. If all the proposals are accepted in each coalition, the proposers are called *representatives*. We denote the proposal of γ_q as $a\left(S, \gamma_q\right) \in V\left(S\right)$.
>
> In the second stage, a proposal is randomly chosen. *The probability of* $a\left(S, \gamma_r\right)$ *being chosen is proportional to the size of* C'_r*, i.e.* $\frac{|C'_r|}{|S|}$. Assume $a\left(S, \gamma_q\right)$ is chosen. We call player γ_q the *representative-proposer*, or simply *r.p.* If all the members of $S \backslash C'_q$ accept $a\left(S, \gamma_q\right)$– they are asked in some prespecified order – then the game ends with these payoffs. If it is rejected by at least one member of $S \backslash C'_q$, then we move to the next round where, with probability ρ, the set of active players is again S and, with probability $1 - \rho$, the entire coalition C'_q *drops out* and the set of active players becomes $S \backslash C'_q$. In the latter case each $i \in C'_q$ gets r_i.

Clearly, given any set of strategies, this mechanism finishes in a finite number of rounds with probability 1.

This mechanism coincides with the mechanism in Vidal-Puga (2005) except that the probability of a coalition to be chosen is proportional of its size[2]. With this modification, when there is no rejection *each player has the same probability to be chosen r.p.* Hence, players do not loose their "right to talk" when joining a coalition.

The mechanism also generalizes Hart and Mas-Colell's (1996) for trivial coalition structures. For $\mathcal{C} = \{N\}$, the second stage is trivial, since there is a single representative and a single proposal. Moreover, the first stage coincides with Hart and Mas-Colell's mechanism. For $\mathcal{C} = \{\{i\}\}_{i\in N}$, the first stage is trivial. Each player states a proposal, and in the second stage a proposal is randomly selected with equal probability and voted by the rest of the players/coalitions.

As usual, we consider stationary subgame perfect equilibria. In this context, an equilibrium is stationary if the players' strategies depend only on the set S of active players. They do not depend, however, on the previous history or the number of played rounds.

Let S denote the set of active players. Given a set of stationary strategies, we denote by $a(S,i)^\gamma \in V(S)$ the payoff proposed by $i \in C'_q \in \mathcal{C}_S$ when the set of proposers is determined by some $\gamma \in \Gamma_S$ with $\gamma_q = i$. Thus, for a given $\gamma \in \Gamma_S$,

$$a(S)^\gamma := \sum_{C'_q \in \mathcal{C}_S} \frac{|C'_q|}{|S|} a\left(S, \gamma_q\right)^\gamma \in V\left(S\right) \tag{1}$$

is the expected final payoff when all the proposals are accepted and γ determines the set of proposers (or representatives).

We denote

$$a(S) := \sum_{\gamma \in \Gamma_S} \frac{1}{|\Gamma_S|} a\left(S\right)^\gamma \in V(S)$$

as the expected final payoff when all the proposals are accepted.

[2]In Vidal-Puga (2005) each coalition is chosen with the same probability.

Given $i \in C'_q \in \mathcal{C}_S$, let $\Gamma_{S,i}$ be the subset of functions $\gamma \in \Gamma_S$ such that $\gamma_q = i$. Notice that $|\Gamma_S| = |\Gamma_{S,i}| \left|C'_q\right|$ for all $i \in C'_q \in \mathcal{C}_S$.

Let

$$a(S,i) := \sum_{\gamma \in \Gamma_{S,i}} \frac{1}{|\Gamma_{S,i}|} a(S,i)^\gamma \tag{2}$$

be the expected payoff proposed by $i \in C'_q \in \mathcal{C}_S$ when he is a proposer.

Proposition 3.1 *For all $S \subset N$,*

$$a(S) = \sum_{i \in S} \frac{1}{|S|} a(S,i). \tag{3}$$

Proof. Given $S \subset N$,

$$\begin{aligned} a(S) &= \sum_{\gamma \in \Gamma_S} \frac{1}{|\Gamma_S|} a(S)^\gamma \\ &= \sum_{\gamma \in \Gamma_S} \frac{1}{|\Gamma_S|} \sum_{C'_q \in \mathcal{C}_S} \frac{\left|C'_q\right|}{|S|} a\left(S, \gamma_q\right)^\gamma \\ &= \sum_{C'_q \in C_S} \frac{\left|C'_q\right|}{|S|} \sum_{\gamma \in \Gamma_S} \frac{1}{|\Gamma_S|} a\left(S, \gamma_q\right)^\gamma \\ &= \sum_{C'_q \in C_S} \frac{\left|C'_q\right|}{|S|} \sum_{i \in C'_q} \frac{1}{\left|C'_q\right|} \sum_{\gamma \in \Gamma_{S,i}} \frac{1}{|\Gamma_{S,i}|} a\left(S, \gamma_q\right)^\gamma \end{aligned}$$

since $a\left(S, \gamma_q\right)^\gamma = a(S,i)^\gamma$ for all $i \in C'_q$, $\gamma \in \Gamma_{S,i}$:

$$a(S) = \sum_{C'_q \in C_S} \frac{\left|C'_q\right|}{|S|} \sum_{i \in C'_q} \frac{1}{\left|C'_q\right|} \sum_{\gamma \in \Gamma_{S,i}} \frac{1}{|\Gamma_{S,i}|} a(S,i)^\gamma$$

under (2):

$$a(S) = \sum_{C'_q \in C_S} \frac{\left|C'_q\right|}{|S|} \sum_{i \in C'_q} \frac{1}{\left|C'_q\right|} a(S,i) = \sum_{C'_q \in C_S} \frac{1}{|S|} \sum_{i \in C'_q} a(S,i) = \sum_{i \in S} \frac{1}{|S|} a(S,i).$$

■

Proposition 3.1 states that the probability that the final proposal comes from a particular player (when all the proposals are accepted) is equal for all the players, i.e. they maintain their respective "rights to talk".

Theorem 3.1 *Let $(N, v, \mathcal{C})$ be a TU game with coalition structure. Assume there is a stationary subgame perfect equilibrium in which all the proposals $\left(a(S,i)^{\gamma}_{i\in S,\gamma\in\Gamma_{S,i}}\right)_{S\subset N}$ are accepted. Then, $a(S) = \zeta^S$ for all $S \subset N$, where $\left(\zeta^S\right)_{S\subset N}$ is inductively defined as follows: $\zeta_i^{\{i\}} = r_i$ for all $i \in N$. Assume we know $\zeta^T \in \mathbb{R}^T$ for all $T \subsetneq S$. Then, $\zeta_i^S =$*

$$\frac{1}{|S|}\left[v(S) + \sum_{j\in C'_q\setminus i}\frac{|S|}{\left|C'_q\right|}\left(\zeta_i^{S\setminus j} - \zeta_j^{S\setminus i}\right) + \sum_{C'_r\in\mathcal{C}_S\setminus C'_q}\left(\frac{\left|C'_r\right|}{\left|C'_q\right|}\sum_{j\in C'_q}\zeta_j^{S\setminus C'_r} - \sum_{j\in C'_r}\zeta_j^{S\setminus C'_q}\right)\right]$$

for all $i \in C'_q \in \mathcal{C}_S$.

Proof. Assume the set of active players is S. Since all the proposals are accepted, the final expected payoff is $a(S)$. Moreover, the final expected payoff when $\gamma \in \Gamma_S$ determines the set of proposers is $a(S)^{\gamma}$.

- Assume we are in the second stage and the r.p. is $\gamma_q \in C'_q \in \mathcal{C}_S$. If all the players in $S\setminus C'_q$ accept γ_q's proposal, the final payoff will be $a\left(S,\gamma_q\right)^{\gamma}$. If at least a player in $S\setminus C'_q$ rejects γ_q's proposal, the final expected payoff will be $\rho a(S) + (1-\rho)\,a\left(S\setminus C'_q\right)$. It is well-known that, in equilibrium, acceptable proposals would leave the responders indifferent to accepting or rejecting. Hence,

$$a_i\left(S,\gamma_q\right)^{\gamma} = \rho a_i(S) + (1-\rho)\,a_i\left(S\setminus C'_q\right) \tag{4}$$

  for all $i \in S\setminus C'_q$.

- Let $\gamma \in \Gamma_S$ be the function which determines the set of proposers. Given $C'_q \in \mathcal{C}_S$, assume we are in the subgame which begins after player γ_q makes his proposal $a\left(S,\gamma_q\right)^{\gamma}$. If all the players in $C'_q\setminus\gamma_q$ accept γ_q's proposal, the expected final payoff will be $a(S)^{\gamma}$. If at least a player in $C'_q\setminus\gamma_q$ rejects γ_q's proposal, the expected final payoff will be $\rho a(S) + (1-\rho)\,a\left(S\setminus\gamma_q\right)$. By the same argument as above, we conclude that

$$a_i(S)^{\gamma} = \rho a_i(S) + (1-\rho)\,a_i\left(S\setminus\gamma_q\right) \tag{5}$$

  for all $i \in C'_q\setminus\gamma_q$.

The following claim states that the proposals are always Pareto efficient:

Claim 3.1 *Let $\gamma \in \Gamma_S$ be the function which determines the set of proposers. Assuming $\gamma \in \Gamma_{S,i}$ for some $i \in S$, we have*

$$\sum_{j \in S} a_j (S,i)^\gamma = v(S). \tag{6}$$

Assume Claim 3.1 does not hold, i.e. $\sum_{j\in S} a_j (S,i)^\gamma < v(S)$. Let $\varepsilon > 0$ be such that $\sum_{j\in S} a_j (S,i)^\gamma + |S| \varepsilon \leq v(S)$. Suppose player i changes his strategy and proposes $b(S,i)^\gamma$ with $b_j (S,i)^\gamma := a_j (S,i)^\gamma + \varepsilon$ for all $j \in S$. By a similar argument as before, it is straightforward to check that this new proposal is bound to be accepted in both the first and the second stages. Hence, the expected final payoff for player i increases by $\frac{|C'_q|}{|S|}\varepsilon$. This contraction proves Claim 3.1.

From Claim 3.1, it is easily checked that

$$\sum_{i \in S} a_i (S) = v(S). \tag{7}$$

Fix $i \in C'_q \in \mathcal{C}_S$. From (1) it is readily checked that, for any $j \in C'_q \backslash i$, $\gamma \in \Gamma_{S,i}$:

$$a_j (S,i)^\gamma = \frac{|S|}{|C'_q|} a_j (S)^\gamma - \sum_{C'_r \in \mathcal{C}_S \backslash C'_q} \frac{|C'_r|}{|C'_q|} a_j (S, \gamma_r)^\gamma$$

under (5) and (4), $a_j (S,i)^\gamma =$

$$\begin{aligned} & \frac{|S|}{|C'_q|} \left[\rho a_j(S) + (1-\rho) a_j(S\backslash i)\right] - \sum_{C'_r \in \mathcal{C}_S \backslash C'_q} \frac{|C'_r|}{|C'_q|} \left[\rho a_j(S) + (1-\rho) a_j(S\backslash C'_r)\right] \\ = \; & \rho a_j(S) + (1-\rho) \left[\frac{|S|}{|C'_q|} a_j(S\backslash i) - \sum_{C'_r \in \mathcal{C}_S \backslash C'_q} \frac{|C'_r|}{|C'_q|} a_j(S\backslash C'_r) \right]. \end{aligned} \tag{8}$$

Under (3) and (2),

$$\begin{aligned} & |S| \, a_i (S) \overset{(3)}{=} \sum_{j\in S} a_i (S,j) \overset{(2)}{=} \sum_{j \in S} \sum_{\gamma \in \Gamma_{S,j}} \frac{1}{|\Gamma_{S,j}|} a_i (S,j)^\gamma \\ = \; & \sum_{\gamma \in \Gamma_{S,i}} \frac{1}{|\Gamma_{S,i}|} a_i (S,i)^\gamma + \sum_{j \in C'_q \backslash i} \sum_{\gamma \in \Gamma_{S,j}} \frac{1}{|\Gamma_{S,j}|} a_i (S,j)^\gamma + \sum_{j \in S\backslash C'_q} \sum_{\gamma \in \Gamma_{S,j}} \frac{1}{|\Gamma_{S,j}|} a_i (S,j)^\gamma . \end{aligned}$$

We study the three terms one by one. For the first term:

$$\begin{aligned}
&\sum_{\gamma\in\Gamma_{S,i}}\frac{1}{|\Gamma_{S,i}|}a_i(S,i)^{\gamma}\overset{(6)}{=}v(S)-\sum_{\gamma\in\Gamma_{S,i}}\frac{1}{|\Gamma_{S,i}|}\sum_{j\in S\setminus i}a_j(S,i)^{\gamma}\\
=\;& v(S)-\sum_{\gamma\in\Gamma_{S,i}}\frac{1}{|\Gamma_{S,i}|}\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}\sum_{j\in C'_r}a_j(S,i)^{\gamma}-\sum_{\gamma\in\Gamma_{S,i}}\frac{1}{|\Gamma_{S,i}|}\sum_{j\in C'_q\setminus i}a_j(S,i)^{\gamma}\\
&\overset{(4)(8)}{=}v(S)-\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}\sum_{j\in C'_r}\left[\rho a_j(S)+(1-\rho)a_j\left(S\setminus C'_q\right)\right]\\
&-\sum_{j\in C'_q\setminus i}\left[\rho a_j(S)+(1-\rho)\left[\frac{|S|}{\left|C'_q\right|}a_j(S\setminus i)-\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}\frac{|C'_r|}{\left|C'_q\right|}a_j(S\setminus C'_r)\right]\right]
\end{aligned}$$

under (7), $\sum_{j\in S\setminus i}\rho a_j(S)=\rho(v(S)-a_i(S))$ and thus

$$\begin{aligned}
=\;& v(S)-\rho(v(S)-a_i(S))-(1-\rho)\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}\sum_{j\in C'_r}a_j\left(S\setminus C'_q\right)\\
&-(1-\rho)\sum_{j\in C'_q\setminus i}\left[\frac{|S|}{\left|C'_q\right|}a_j(S\setminus i)-\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}\frac{|C'_r|}{\left|C'_q\right|}a_j(S\setminus C'_r)\right].
\end{aligned}$$

For the second term:

$$\begin{aligned}
&\sum_{j\in C'_q\setminus i}\sum_{\gamma\in\Gamma_{S,j}}\frac{1}{|\Gamma_{S,j}|}a_i(S,j)^{\gamma}\overset{(8)}{=}\\
&\sum_{j\in C'_q\setminus i}\left[\rho a_i(S)+(1-\rho)\left[\frac{|S|}{\left|C'_q\right|}a_i(S\setminus j)-\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}\frac{|C'_r|}{\left|C'_q\right|}a_i(S\setminus C'_r)\right]\right]\\
=\;&\rho\left(\left|C'_q\right|-1\right)a_i(S)\\
&+(1-\rho)\left[\sum_{j\in C'_q\setminus i}\frac{|S|}{\left|C'_q\right|}a_i(S\setminus j)-\left(1-\frac{1}{\left|C'_q\right|}\right)\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}|C'_r|\,a_i(S\setminus C'_r)\right].
\end{aligned}$$

For the third term:

$$\begin{aligned}
&\sum_{j\in S\setminus C'_q}\sum_{\gamma\in\Gamma_{S,j}}\frac{1}{|\Gamma_{S,j}|}a_i(S,j)^{\gamma}\overset{(4)}{=}\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}\sum_{j\in C'_r}\left[\rho a_i(S)+(1-\rho)a_i(S\setminus C'_r)\right]\\
=\;&\rho\left(|S|-\left|C'_q\right|\right)a_i(S)+(1-\rho)\sum_{C'_r\in\mathcal{C}_S\setminus C'_q}|C'_r|\,a_i(S\setminus C'_r).
\end{aligned}$$

Hence, adding terms, $|S|\, a_i(S) =$

$$v(S) - \rho v(S) - (1-\rho) \sum_{C'_r \in \mathcal{C}_S \backslash C'_q} \sum_{j \in C'_r} a_j\left(S \backslash C'_q\right)$$
$$- \sum_{j \in C'_q \backslash i} (1-\rho) \left[\frac{|S|}{|C'_q|} a_j(S \backslash i) - \sum_{C'_r \in \mathcal{C}_S \backslash C'_q} \frac{|C'_r|}{|C'_q|} a_j(S \backslash C'_r) \right]$$
$$+ (1-\rho) \left[\sum_{j \in C'_q \backslash i} \frac{|S|}{|C'_q|} a_i(S \backslash j) + \sum_{C'_r \in \mathcal{C}_S \backslash C'_q} \frac{|C'_r|}{|C'_q|} a_i(S \backslash C'_r) \right]$$
$$+ \rho\, |S|\, a_i(S).$$

Rearranging terms and dividing by $1-\rho$, $|S|\, a_i(S) =$

$$= v(S) + \sum_{j \in C'_q \backslash i} \frac{|S|}{|C'_q|} \left(a_i(S \backslash j) - a_j(S \backslash i)\right) + \sum_{C'_r \in \mathcal{C}_S \backslash C'_q} \left(\frac{|C'_r|}{|C'_q|} \sum_{j \in C'_q} a_j(S \backslash C'_r) - \sum_{j \in C'_r} a_j\left(S \backslash C'_q\right) \right)$$

from where the result is easily deduced following a standard induction argument. ■

4 An eloquent example

Krasa, Temimi and Yannelis (2003) propose a three-person economy with differential information where two players bargain as one unit against the third one. When there is complete information, the economy can be expressed as a TU game (N, v) where $N = \{1, 2, 3\}$ and v is given by

$$\begin{aligned} v(\{1\}) &= v(\{2\}) = 1 \\ v(\{3\}) &= \frac{43}{16} \\ v(\{1,2\}) &= \frac{5}{2} \\ v(\{1,3\}) &= v(\{2,3\}) = \frac{31}{8} \\ v(N) &= \frac{83}{16}. \end{aligned}$$

When there is differential information, due to incentive incompatibility, 1 and 2 are only able to achieve $v(\{1,2\}) = 2$ by themselves. For any other $S \subset N$, $v(S)$ is the same as under complete information.

Krasa, Temimi and Yannelis take the Owen value ϕ^N as a measure of players' expectations when 1 and 2 join forces. Their result is that bargaining as one unit is advantageous if and only if information is complete, as the next table shows:

ϕ^N	complete information	differential information
$\mathcal{C} = \{\{1\},\{2\},\{3\}\}$	$\left(\frac{39}{32}, \frac{39}{32}, \frac{88}{32}\right)$	$\left(\frac{109}{96}, \frac{109}{96}, \frac{280}{96}\right)$
$\mathcal{C} = \{\{1,2\},\{3\}\}$	$\left(\frac{40}{32}, \frac{40}{32}, \frac{86}{32}\right)$	$\left(\frac{108}{96}, \frac{108}{96}, \frac{282}{96}\right)$.

Consider now that we take ζ^N as a measure of players' expectations when 1 and 2 join forces. Then, bargaining as one unit is advantageous in any case, as the next table shows:

ζ^N	complete information	differential information
$\mathcal{C} = \{\{1\},\{2\},\{3\}\}$	$\left(\frac{39}{32}, \frac{39}{32}, \frac{88}{32}\right)$	$\left(\frac{109}{96}, \frac{109}{96}, \frac{280}{96}\right)$
$\mathcal{C} = \{\{1,2\},\{3\}\}$	$\left(\frac{40}{32}, \frac{40}{32}, \frac{86}{32}\right)$	$\left(\frac{112}{96}, \frac{112}{96}, \frac{274}{96}\right)$.

This last situation corresponds to the assumption that players, by joining, do not loose their respective "rights to talk". Note also that the benefit from cooperation is $\frac{1}{32}$ for each player in both cases.

5 Discussion

The Owen value seems to be a good measure of players' expectations when the coalition structure is exogenously given. For example, wage bargaining between firms and labor unions, tariff bargaining between countries, bargaining between the member states of a federated country, etc. In these situations, players do not have to wonder whether they would do it better bargaining as a unit, because it is something out of their control.

On the other hand, Hart and Kurz (1983) followed the idea that players form coalition structures in order to improve their bargaining strength. They studied four reasonable properties, or axioms, that determine uniquely the Owen value. The only property that is not satisfied by ζ is Carrier (p. 1051), which states that moving null players[3] does not affect the outcome of the rest of the agents. We will contest this property[4].

In bargaining problems, asymmetries in the final outcome may be due to the players' different bargaining powers. As Binmore (1998, p. 80) points out: "Bargaining powers are determined by the strategic advantages conferred on players by the circumstances under which they bargain." In our case, the coalition structure. Assume for example a game in which all the players are mutually substitutes[5]. Since no asymmetries are introduced in the model, the expectation *a priori* should be the same for substitute players, i.e. all players are supposed to have equal bargaining powers. In general games, however, *nothing is said about the bargaining power of the null players!* If we admit that null players do have bargaining power, then this fact can somehow increase the aggregate power of the coalition they join.

Take for example the unanimity game (N, v) where $N = \{1, 2\}$ and $v(N) = 1$, $v(\{1\}) = v(\{2\}) = 0$. By a symmetry argument, the value of each player should be $\frac{1}{2}$, i.e. the expectation of each player before any implementation of the game is the same.

Assume now we add a null player 3. We get the game (N', v') with $N' = \{1, 2, 3\}$ and $v'(S) = 1$ if $\{1, 2\} \subset S$ and $v'(S) = 0$ otherwise. What would the players' expectation be in this new game?

It can be argued that the situation does not change with the presence of a player that does not contribute anything to any coalition. Hence, the value of (N', v') should be $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$. However, the situation may significantly

[3] A *null player* is a player i with $v(S \cup i) = v(S)$ for all S.

[4] The Carrier axiom in Hart and Kurz has two parts, one of them (i) can also be split into two properties: efficiency (the value is efficient for all coalition structures) and *dummy* (null players get zero). ζ satisfies efficiency, but not *dummy*.

[5] Two players i, j are *substitutes* if $v(S \cup i) = v(S \cup j)$ for all S with $i, j \notin S$.

change if we assume that player 3 joins forces with player 2. In this case, the symmetry argument used to assign the value $\left(\frac{1}{2}, \frac{1}{2}\right)$ in the previous game (N, v) vanishes. Player 1 and coalition $\{2, 3\}$ are substitutes in the game between coalitions, but not completely symmetric. The fact that $\{2, 3\}$ has two members introduces an endogenous asymmetry. Hart and Kurz (p. 1048) describe this situation as follows:

> As an everyday example of such a situation, "I will have to check this with my wife/husband" may (but not necessarily) lead to a better bargaining position, due to the fact that the other party has to convince *both* the player and the spouse.

If we accept that player 2 may benefit from the support of player 3, one may wonder how to quantify this benefit. The value ζ provides a possible answer, by assigning an allocation $\zeta^{N'} = \left(\frac{4}{12}, \frac{7}{12}, \frac{1}{12}\right)$ when the coalition structure is $\mathcal{C} = \{\{1\}, \{2, 3\}\}$.

References

[1] Albizuri M.J. and J.M. Zarzuelo, 2004, On coalitional semivalues, Games and Economic Behavior 49, 221-243.

[2] Binmore K., 1998, Just playing, Game theory and the social contract, Volume 2, MIT Press.

[3] Calvo E., J. Lasaga and E. Winter, 1996, The principle of balanced contributions and hierarchies of cooperation, Mathematical Social Sciences 31, 171-182.

[4] Chae S. and P. Heidhues, 2004, A group bargaining solution, Mathematical Social Sciences 48(1), 37-53.

[5] Chae S. and H. Moulin, 2004, Bargaining among groups: an axiomatic viewpoint, Mimeo.

[6] Harsanyi J.C., 1977, Rational behavior and bargaining equilibrium in games and social situations, Cambridge University Press.

[7] Hart S. and M. Kurz, 1983, Endogenous formation of coalitions, Econometrica 51(4), 1047-1064.

[8] Hart S. and A. Mas-Colell, 1996, Bargaining and value, Econometrica 64, 357-380.

[9] Krasa S., A. Temimi and N.C. Yannelis, 2003, Coalition structure values in differential information economies: is unity a strength? Journal of Mathematical Economics 39, 51-62.

[10] Owen G., 1977, Values of games with a priori unions, in: R. Henn and O. Moeschlin, eds., Essays in mathematical economics and game theory (Berlin), Springer-Verlag, 76-88.

[11] Shapley L.S., 1953, A value for n-person games, in: H.W. Kuhn and A.W. Tucker, eds., Contributions to the Theory of Games II (Princeton NJ), Princeton University Press, 307-317.

[12] Vidal-Puga J.J., 2005, A bargaining approach to the Owen value and the Nash solution with coalition structure, Economic Theory 25(3), 679-701.

[13] Vidal-Puga J.J. and G. Bergantiños, 2003, An implementation of the Owen value, Games and Economic Behavior 44, 412-427.

[14] Winter E., 1992, The consistency and potential for values of games with coalition structure, Games and Economic Behavior 4, 132-144.