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SPECULATION IN SECOND-PRICE AUCTIONS WITH RESALE

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Abstract

This paper contributes to the literature on second-price auctions with resale. We add speculators—bidders with value zero—to the standard symmetric independent private values environment. There always exists a continuum of inefficient equilibria that are profitable for a speculator. With no reserve price in the initial auction, speculation can enhance the initial seller's expected revenue. On the other hand, speculation can harm the initial seller even if she chooses an optimal reserve price. Our results are valid for English auctions as well.

Keywords: Second-Price Auction, Speculation, Resale

1. Introduction

There is abounding evidence of resale following auctions. Even in markets where resale is expressly forbidden, it is common practice.[2] Despite all this, relatively little is known about the theoretical properties of auctions with resale. Meanwhile, standard auction formats continue to be used without a full understanding of the implications of resale opportunities.

Opportunities for resale change the auction environment in numerous ways. The explicit recognition of multiple periods allows for the possibility of information arrivals and changes in the bidder population that affect how the initial seller and buyers should behave. Moreover, resale presents an otherwise absent opportunity for speculators or "shill" bidders to participate

in the auction. Our main purpose in this paper is to examine the role of speculators in standard second-price or English auctions with resale. Speculators are zero-value traders whose sole motivation for participating in the auction is to make a profit by purchasing the good and then reselling it at a higher price. We construct equilibria in which speculators play an important role. In particular, we show that speculation can be socially harmful, yet profitable for a speculator and revenue enhancing for the initial seller. These results contradict the commonly held notion that in environments with private values the opportunity for resale does not distort the second-price auction.[3]

Our results correct a misconception about resale. While it is known that inefficient auction outcomes are not necessarily corrected by resale (see McAfee, 1998, or Krishna, 2002, Ch. 4.4), resale is generally perceived to be beneficial for efficiency because it provides an opportunity to improve upon "mistakes" made in the initial auction. We show that, in fact, the possibility of resale can increase inefficiency because of the role it creates for speculators.

We consider a two-stage game in which a seller offers a good via a second-price auction to a group consisting of at least one speculator and regular buyers with symmetric independent private values. The winner can either consume the good or put it up for resale. The resale environment is identical to the initial auction environment: no new bidders arrive after the initial auction, and no information becomes public beyond what is revealed via bids. Our finding that speculators can play a role in such a constant private values environment implies that speculators are important in a changing environment as well.

All our results are based on the existence of a continuum of inefficient perfect Bayesian equilibria. For any probability $q > 0$, we construct an equilibrium such that one speculator enters the initial auction and wins with probability q. This includes the case $q = 1$ in which the speculator wins the initial auction for sure. In all our equilibria, the good is resold with positive probability and the speculator has a positive expected payoff.[4] A second speculator has no incentive to enter the initial auction because the first speculator's bid is greater or equal to her expected resale revenue.

Our equilibria are valid for a large class of resale mechanisms. We first consider the case

where speculators, but not regular buyers, can offer the good for resale. In this case, our equilibria are valid for arbitrary resale mechanisms of the speculator, provided the speculator has sufficient bargaining power and a symmetry condition is satisfied. The speculator's resale mechanism may involve a single stage or many stages or may be a simple take-it-or-leave-it offer. Secondly, we consider the case where everybody can offer the good for resale. We show that our equilibria are valid no matter what resale mechanism regular buyers may use, provided the speculator's resale mechanism is a second-price auction, with or without reserve price. In particular, our equilibria are valid if every agent, buyer or speculator, can offer the good for resale via a second-price auction with an optimal reserve price.

The key to understanding our equilibria is to recognize that when resale is permitted, value-bidding is not part of a weakly dominant strategy for any bidder. In the equilibria we construct, low-value buyers prefer to pool at a bid of zero because otherwise they will either have to bid quite high to win the auction, or lose and get a bad resale offer because their bid signals a high value. High-value buyers bid their values. The speculator makes an intermediate bid such that whenever she wins she gets the good for free.

We show that, via speculation, the possibility of resale can help the initial seller, but also harm her. We compare the initial seller's expected revenue in our equilibria to her expected revenue in the dominant-strategy equilibrium of the second-price auction without resale. In one of our equilibria ($q = 1$) the initial seller's revenue equals zero. Thus, speculation can harm the initial seller in any market with at least two regular buyers. Speculation can also help the initial seller: equilibria with q sufficiently close to 0 increase seller revenue if the speculator's resale mechanism is a second-price auction with an optimal reserve price. Therefore, a revenue maximizing seller who cannot set a positive reserve price can always have an incentive to allow resale and attract a speculator. This may seem surprising because the speculator acts independently of the seller.

The impact of speculators is not eliminated if there is a reserve price in the initial auction. Provided the reserve price is not too high, equilibria with speculation still exist. We then consider the extended game where the initial seller chooses a reserve price. Assuming that the

speculator's resale mechanism is a second-price auction with optimal reserve price, and the initial seller maximizes expected revenues, an equilibrium exists in which the initial seller sets a reserve price above the price she would set in the absence of resale, and obtains a smaller revenue. This shows that the possibility of resale can not only destroy the efficiency of the second-price auction without reserve price, but can also harm a revenue maximizing seller.

Resale via a second-price auction with an optimal reserve price is an interesting case that merits special attention. If the distribution function for buyer values satisfies Myerson's (1981) regularity property then, given the posterior beliefs in our equilibria, this resale mechanism maximizes the resale revenue among all conceivable mechanisms (Myerson, 1981); i.e., this is a mechanism that the resale seller often likes to use if she is free to choose among mechanisms. In addition, the scenario where speculators buy and sell in a second-price auction mechanism captures aspects of trading in on-line auction houses such as eBay, Yahoo, and Amazon. The participation of speculators in on-line auction houses is suggested by an abundance of individuals with very high numbers of buyer and seller feedback.

Private-value auctions with resale have been examined by Haile (1999, 2000, 2003) and Gupta and Lebrun (1999) but these works do not examine the role of speculators. Moreover, in these papers resale emerges not as an alternative to the bid-your-value equilibrium, but rather as a direct consequence of changes in the environment between periods. In Haile (1999), resale results from the arrival of new buyers in the resale market. In Haile (2000, 2003) and Gupta and Lebrun (1999), resale occurs because agents receive new information about their private values or values are made public after the initial auction. Gupta and Lebrun (1999) allow for asymmetry between the two bidders in a first-price auction, but their assumptions do not permit traders with commonly known values like the speculators in our model.

The work that comes closest to modelling speculators in auctions is Bikhchandani and Huang (1989). They consider resale in an environment where *all* bidders in the initial auction bid solely for the purpose of resale. Bids are based on privately observed signals about the common value of the goods in the secondary market. The role of the bidders in the initial auction is to aggregate information and to transfer the goods to the secondary market. This is important for evaluating

different auction mechanisms for selling treasury bills. Bikhchandani and Huang do not address speculation in auctions with regular buyers.

Resale has also been incorporated into the study of optimal auctions (cf. Ausubel and Cramton (1999), Jehiel and Moldovanu (1999), Zheng (2002), and Calzolari and Pavan (2002)). Much of this work builds on the implementation results of Myerson (1981). Ausubel and Cramton show how resale possibilities can make seller-revenue maximization compatible with efficiency. Jehiel and Moldovanu examine whether the Coasian notion that the initial assignment of property rights has no effect on efficiency holds when resale occurs according to optimal mechanisms in an environment with private values (but no private information) and negative consumption externalities. Our work is more closely related to Zheng (2002) and Calzolari and Pavan (2002). Zheng identifies conditions that are sufficient to implement seller-optimal outcomes when (repeated) resale is permitted. The mechanism provided by Zheng reduces, in our case, to a second-price auction with an optimal reserve price in the first stage. Then, the bid-your-value equilibrium does in fact maximize seller revenue. However, our results indicate that other equilibria exist, that are substantially less profitable for the initial seller. Finally, Calzolari and Pavan assume, contrary to Zheng, that the distribution of bargaining power in the resale market is a function of the identity of the buyers. In a version of their model, where the set of bidders remains constant across periods, they find that resale is necessarily revenue decreasing for the initial seller. Our results show that in fact resale can cause seller revenue to increase even if all the bargaining power is preassigned to one of the bidders provided that bidder is a speculator.

Auctions without resale but with allocative externalities (see Jehiel and Moldovanu (1996, 2000), and Jehiel, Moldovanu, and Stacchetti (1999)) share certain aspects of auctions with resale. In particular, a bidder's expected payoff can be positive in the event that some other bidder wins the initial auction. However, our results show that bidding in an auction with resale has a signalling aspect that goes beyond allocative externalities. Consider a deviating bid in the initial auction that leaves the identity of the winner unchanged. In an environment with allocative externalities, such a deviation has no impact on the deviator's payoff, but in an environment with resale the deviation can be a signal about the deviator's value and therefore

can change the deviator's payoff. For example, in our equilibria buyers with a low value pool at bid 0 because a deviation to a positive bid would signal a high type.[5]

We start off in Section 2 with one buyer and one speculator, in order to illustrate the structure of the equilibria that we construct for the multiple-buyer case. Section 3 contains the general model and the definition of the equilibrium concept. Section 4 constructs the equilibria with speculation on which all our results are based. Basic properties of these equilibria are established. We show that our equilibria remain valid if the initial auction is replaced by an English auction. The results in Section 5 pertain to resale via a second-price auction with an optimal reserve price. In Section 6 we discuss how our equilibria change when the initial seller sets a reserve price. In Section 7 we discuss numerous issues including alternative off-the-equilibrium-path beliefs, equilibrium selection, and inefficient equilibria without speculators. Appendix A reviews conditional distribution functions. Appendix B contains proofs.

2. Resale With One Buyer and One Speculator

Before presenting the model and results for multiple buyers, let us consider a market with one regular buyer and a speculator. A good is offered via a second-price auction without reserve price. The winner of the auction can offer the good for resale via posting a take-it-or-leave-it price. The buyer has a private value for the good that is distributed on $[0, 1]$ according to some strictly increasing and continuous distribution function $F(\cdot)$. The speculator has value zero. Without the resale possibility, each agent's dominant strategy would be to bid her value, and an efficient allocation would be guaranteed. The same equilibrium outcome can occur if resale is possible, but there exist many other equilibria with very different properties.

We will establish that for any number $\theta^* > 0$, there exists a perfect Bayesian equilibrium such that if the buyer's value is not larger than θ^* then she bids 0 and waits for the resale offer, but if her value exceeds θ^* then she bids her value and consumes the good immediately if she wins. Given this, for the speculator, any bid $b_s^* \in (0, \theta^*)$ is optimal. She expects to pay 0 when she wins. After winning, the support of her posterior distribution for the buyer's value is $[0, \theta^*]$. Thus, she will make some take-it-or-leave-it offer $r^* \in (0, \theta^*)$ and have a positive expected payoff

in equilibrium. There is clearly no profitable deviation from b_s^*. The only deviations that change her expected payoff are $b_s = 0$ or any bid $b_s > \theta^*$. By bidding $b_s = 0$, the speculator might lose the tie against the buyer and thereby forego her resale revenue. A deviation to $b_s > \theta^*$ changes the speculator's realized payoff only in the event that the buyer's value is in $(\theta^*, b_s]$, but then the speculator's resale revenue will not exceed her payment in the initial auction.

Now suppose b_s^* is defined such that the buyer with the "marginal" value θ^* is indifferent between overbidding the speculator in the initial auction and waiting for resale; i.e.,

$$\theta^* - b_s^* = \delta(\theta^* - r^*),$$

where $\delta < 1$ is the discount factor. Buyers with value above θ^* will then strictly prefer to win the initial auction rather than to wait for resale, while for buyers with a value in (r^*, θ^*) the opposite is true.[6] Now suppose that the buyer deviates to a bid in $(0, b_s^*]$. Because such a bid does not occur in equilibrium, the speculator cannot use Bayesian updating to form a posterior belief about the buyer's value; we may assume that the speculator believes that the value of the deviating buyer is θ^*. As a result, the speculator will post a take-it-or-leave-it price of θ^*. Because the buyer could get a take-it-or-leave-it price of $r^* < \theta^*$ by bidding 0, she has no incentive to deviate to a bid in $(0, b_s^*]$. Finally, note that a potential second speculator has no incentive to enter the auction because she can win only by bidding at least b_s^* while her resale revenue is bounded above by $r^* < b_s^*$. Thus, the equilibrium remains valid with free entry of speculators.

In the equilibrium constructed around any $\theta^* \in [0, 1]$ the speculator wins the auction with probability $F(\theta^*)$. Hence all winning probabilities between 0 and 1 are supported by equilibria. There are two sources of inefficiency in each of these equilibria. First, if the buyer's value is in $[0, r^*)$ then the good is not consumed. Second, if the buyer's value is in $[r^*, \theta^*]$ then the consumption of the good is delayed from period 1 to period 2. Therefore, when resale is possible the second-price auction loses what has been propagated as one of its main advantages in environments with private values: to assure an efficient allocation.

In all equilibria the initial seller's revenue is positive while they would be 0 with resale not being possible; i.e., speculation boosts the initial seller's revenue. In this simple example, this is

a trivial consequence of there being only one regular buyer. We show later that with two or more regular buyers in the market, some equilibria raise initial seller's revenue and other equilibria reduce it.

With only one buyer in the market, if the initial seller sets a reserve price that is optimal without resale, no speculator will be attracted (because her resale revenue would not make up for what she has to pay in the initial auction). Thus, the possibility of resale does no harm to a seller who sets an optimal reserve price. We will show that this changes if at least two regular buyers are in the market.

3. Model

There are $n \geq 2$ risk-neutral *buyers*, who are interested in buying a single indivisible object, called the *good*. The good is initially owned by a *seller* who offers the good via a second-price auction without reserve price (positive reserve prices will be addressed in Section 6). Buyer $i = 1, \ldots, n$ has the random value $\tilde{\theta}_i \in [0, 1]$ for the good. A risk-neutral agent called *speculator* s who has no use value for the good will also be included in the model; define $\theta_s = 0$. The restriction to a single speculator simplifies the presentation; we will argue that our equilibria remain valid with free entry of speculators (see below Proposition 1). Our setup contrasts other market models with resale where speculators are not considered.

We assume the standard symmetric independent-private-values model (e.g., Krishna, 2002) where the random variables $\tilde{\theta}_1, \ldots, \tilde{\theta}_n$ representing buyer's values are stochastically independent and all have the same distribution function $F(\cdot)$ that is differentiable on $(0, 1)$ with $F' > 0$ and satisfies $F(0) = 0$ and $F(1) = 1$. We will also use the random variable $\tilde{\theta}_s = 0$, and the vector random variables $\underline{\tilde{\theta}} = (\tilde{\theta}_1, \ldots, \tilde{\theta}_n)$ and $\tilde{\theta}_{-i} = (\tilde{\theta}_1, \ldots, \tilde{\theta}_{i-1}, \tilde{\theta}_{i+1}, \ldots, \theta_n)$.

Before period 1, buyer $i = 1, \ldots, n$ privately learns the realization of her value, $\tilde{\theta}_i = \theta_i$. In period 1, a second-price auction without reserve price takes place. All buyers as well as the speculator simultaneously submit bids $b_1(\theta_1), \ldots, b_n(\theta_n), b_s = b_s(0) \geq 0$. The highest bidding agent becomes the new owner of the good. If there are several highest bids, the winner is determined by an independent and fair random draw. The trade price p^1 in period 1 equals

the second-highest bid. We assume that bids are not publicly announced, but the winner's payment, p^1, becomes public (Remark 1 concerns relaxing this assumption). All agents will use the information incorporated in p^1 in order to update their beliefs about the values of the other bidders. For all $i \neq s$ and $b_i \geq 0$, let $\Pi_i^{b_i}(\cdot \mid \cdot)$ denote a conditional distribution function[7] for $\tilde{\theta}_{-i}$ conditional on the random variable

$$\tilde{p}_i^{b_i} = \max\{b_i, \max\{b_j(\tilde{\theta}_j) \mid j \neq i, s\}\}.$$

The distribution function $\Pi_i^{b_i}(\cdot \mid p^1)$ represents buyer i's posterior belief about $\tilde{\theta}_{-i}$ after submitting the bid b_i and making the observation $p^1 = \tilde{p}_i^{b_i}$. She makes this observation when the speculator wins at price p^1. Whenever a conditional expectation based on such an observation will be considered below, we implicitly refer to the conditional distribution function $\Pi_i^{b_i}(\cdot \mid \cdot)$.

Below we construct posterior distribution functions such that for $b_i' < b_i < p^1$ we have $\Pi_i^{b_i'}(\cdot \mid p^1) = \Pi_i^{b_i}(\cdot \mid p^1)$. I.e., a buyer's posterior beliefs do not depend on her own bid as long as her bid has no impact on the outcome of the auction. On the other hand, the beliefs can depend on whether or not $b_i = p^1$. If $b_i < p^1$ then buyer i learns that there exists one other buyer who has bid p^1, and all remaining buyers have bid not more than p^1. If $b_i = p^1$ then she only learns that all other buyers have bid not more than p^1; she cannot learn anything about whether another buyer has bid precisely p^1.

Let $\Pi_s(\cdot \mid \cdot)$ denote a conditional distribution function for the vector of buyer values $\underline{\tilde{\theta}}$, conditional on the random variable

$$\tilde{p}_s^1 = \max\{b_j(\tilde{\theta}_j) \mid j \neq s\}.$$

The distribution function $\Pi_s(\cdot \mid p^1)$ represents the speculator's posterior belief about $\underline{\tilde{\theta}}$ after making the observation $p^1 = \tilde{p}_s^1$. She makes this observation when she wins at price p^1. Whenever a conditional expectation based on such an observation will be considered below, we implicitly refer to the conditional distribution function $\Pi_s(\cdot \mid \cdot)$.

In period 2 a resale market opens with the same participants as in period 1. Agents have a discount factor of $\delta \in (0, 1)$ between periods 1 and 2; in particular, in period 1 a buyer with value θ_i is willing to pay $\delta\theta_i$ for the right to consume the good in period 2. Any agent might

have an incentive to make a bid above her value in an attempt to buy the good in period 1 and sell it in period 2. We will construct equilibria where one agent, the speculator, submits a bid above her value.

We allow for the possibility that not all agents have access to the same resale mechanism. For now, let us focus on the case where buyers *cannot* resell, while the speculator can. In Proposition 2 we address the case where all agents can resell. Given that the speculator wins in period 1, a resale mechanism $g^{p^1}(\cdot)$ is played among buyers. The speculator's actions in period 2 are reflected in the dependence of the resale mechanism on p^1. For example, the resale mechanism can be an auction for which, depending on p^1, the speculator determines a reserve price or an entry fee (the implicit independence of the speculator's period-1 bid b_s is innocuous because the speculator's posterior beliefs are independent of b_s). We write

$$g^{p^1}(a) = (p_1(a), \ldots, p_n(a), q_1(a), \ldots, q_n(a)),$$

where $a = (a_1, \ldots, a_n)$ $(a_i \in A_i,\ i = 1, \ldots, n)$ lists the actions taken by the buyers in period 2, $p_i(a)$ denotes buyer i's expected payment in period 2, and $q_i(a)$ denotes the probability that buyer i obtains the good in period 2. A period-2 strategy for buyer i is a function

$$\sigma_i :\ \{(p^1, b_i) \mid 0 \leq b_i \leq p^1\} \times [0,1] \to A_i, \quad (p^1, b_i, \theta_i) \mapsto \sigma_i(p^1, b_i, \theta_i)$$

that depends on her observation p^1, her period-1 bid b_i, and her type θ_i (we allow for dependence on b_i because the buyer's posterior beliefs may depend on b_i).

For all $i \neq s$, $\theta_{-i} \in [0,1]^{n-1}$, and $p^1 \geq 0$, we use the shortcut

$$\sigma_{-i}^{p^1}(\theta_{-i}) = (\sigma_j(p^1, b_j(\theta_j), \theta_j)(j \neq i, s))$$

for the period-2 actions taken by buyers other than i who follow their bid functions $b_j(\cdot)$ $(j \neq i, s)$ in period 1. For all $\underline{\theta} = (\theta_1, \ldots, \theta_n) \in [0,1]^n$ and $p^1 \geq 0$, we use the shortcut

$$\sigma^{p^1}(\underline{\theta}) = (\sigma_j(p^1, b_j(\theta_j), \theta_j)(j \neq s))$$

for the period-2 actions taken after all buyers follow their bid functions.

For buyer i, the probability of obtaining the good in period 2 when taking action $a_i \in A_i$ is denoted

$$Q_i^{p^1,b_i}(a_i) = E[q_i(a_i, \sigma_{-i}^{p^1}(\tilde{\theta}_{-i})) \mid p^1 = \tilde{p}_i^{b_i}].$$

The respective expected payment is denoted

$$P_i^{p^1,b_i}(a_i) = E[p_i(a_i, \sigma_{-i}^{p^1}(\tilde{\theta}_{-i})) \mid p^1 = \tilde{p}_i^{b_i}].$$

For buyer i, the probability of obtaining the good in period 2, when acting in period 2 as if she had type $\theta_i' \in [0,1]$, after submitting the bid $b_i \geq 0$ in period 1, is denoted

$$Q_i^{p^1,b_i}(\theta_i') = Q_i^{p^1,b_i}(\sigma_i(p^1, b_i, \theta_i')). \tag{1}$$

The respective expected payment is denoted

$$P_i^{p^1,b_i}(\theta_i') = P_i^{p^1,b_i}(\sigma_i(p^1, b_i, \theta_i')). \tag{2}$$

The expected payment to the speculator in period 2 is

$$\begin{aligned} P_s^{p^1} &= E[\sum_{i=1}^{n} p_i(\sigma^{p^1}(\underline{\tilde{\theta}})) \mid p^1 = \tilde{p}_s^1] \\ &= \sum_{i=1}^{n} E[P_i^{p^1,b_i(\tilde{\theta}_i)}(\tilde{\theta}_i) \mid p^1 = \tilde{p}_s^1]. \end{aligned} \tag{3}$$

For any combination of bids $b_1, \ldots, b_n, b_s \geq 0$ in period 1, let $\tilde{w}(b_1, \ldots, b_n, b_s)$ denote the random variable that determines the winner of the auction in period 1. For all agents i (including $i = s$), let

$$\tilde{b}_{-i} = (b_j(\tilde{\theta}_j))_{j \neq i}, \ \tilde{b}_{-i}^{(1)} = \max\{b_j(\tilde{\theta}_j) \mid j \neq i\},$$

denote the list of all other agents' bids and, respectively, the highest among those bids. The expected payoff of buyer i with type θ_i as a function of her bid $b_i \geq 0$ is given by

$$\begin{aligned} u_i(b_i, \theta_i) = E\left[\left(\theta_i - \tilde{b}_{-i}^{(1)}\right) \cdot \mathbf{1}_{\tilde{w}(b_i, \tilde{b}_{-i}) = i}\right] \\ + \delta E\left[\left(Q_i^{\tilde{p}_i^{b_i}, b_i}(\theta_i)\theta_i - P_i^{\tilde{p}_i^{b_i}, b_i}(\theta_i)\right) \cdot \mathbf{1}_{\tilde{w}(b_i, \tilde{b}_{-i}) = s}\right]. \end{aligned} \tag{4}$$

The expected payoff of the speculator with bid $b_s \geq 0$ is given by

$$u_s(b_s) = E\left[\left(\delta P_s^{\tilde{p}_s^1} - \tilde{p}_s^1\right) \cdot \mathbf{1}_{\tilde{w}(b_s, \tilde{b}_{-s}) = s}\right]. \tag{5}$$

A *perfect Bayesian equilibrium* for the second-price auction with the collection of resale mechanisms $(g^{p^1}(\cdot))_{p^1 \geq 0}$ is a strategy-belief vector

$$(b_1(\cdot), \ldots, b_n(\cdot), b_s^*, \sigma_1(\cdot), \ldots, \sigma_n(\cdot), \Pi_1^{(\cdot)}(\cdot \mid \cdot), \ldots, \Pi_n^{(\cdot)}(\cdot \mid \cdot), \Pi_s(\cdot \mid \cdot))$$

such that the following conditions are satisfied:

$$\forall i = 1, \ldots, n,\ \theta_i \in [0,1],\ p^1 \geq b_i \geq 0:$$

$$\sigma_i(p^1, b_i, \theta_i) \in \arg\max_{a_i \in A_i} Q_i^{p^1, b_i}(a_i)\theta_i - P_i^{p^1, b_i}(a_i), \tag{6}$$

$$\forall i = 1, \ldots, n,\ \theta_i \in [0,1],\ p^1 \geq b_i \geq 0:$$

$$Q_i^{p^1, b_i}(\theta_i)\theta_i - P_i^{p^1, b_i}(\theta_i) \geq 0, \tag{7}$$

$$\forall p^1 \geq 0:\ P_s^{p^1} \geq 0. \tag{8}$$

$$\forall i = 1, \ldots, n,\ \theta_i \in [0,1]:\ b_i(\theta_i) \in \arg\max_{b_i \geq 0} u_i(b_i, \theta_i). \tag{9}$$

$$b_s^* \in \arg\max_{b_s \geq 0} u_s(b_s). \tag{10}$$

Condition (6) requires that the buyers choose optimal period-2 actions. A similar optimality requirement for the speculator is not introduced because it would not play any role for our results. Conditions (7) and (8) reflect voluntary participation in the resale market. Conditions (9) and (10) require that bids in period 1 are chosen optimally. Note that we have not introduced the action "stay out of the initial auction" because bidding 0 is at least as good as that.

A second speculator will have an incentive to enter the initial auction only if there exists a positive bid such that her expected revenue is positive. For the equilibria we construct we will show that a second speculator has no incentive to enter if she has access to the same resale mechanism as the first speculator.

The following properties of the outcome of the resale market are important. The proof is a standard application of the envelope theorem (see, e.g., Milgrom and Segal, 2002).

LEMMA 1 *Consider a perfect Bayesian equilibrium. Then, for all $i \neq s$ and $p^1 \geq b_i \geq 0$, the functions $Q_i^{p^1, b_i}(\cdot)$ and $P_i^{p^1, b_i}(\cdot)$ are weakly increasing. For all $0 \leq \theta_i' \leq \theta_i \leq 1$ we have*

$$Q_i^{p^1, b_i}(\theta_i)\theta_i - P_i^{p^1, b_i}(\theta_i) = Q_i^{p^1, b_i}(\theta_i')\theta_i' - P_i^{p^1, b_i}(\theta_i') + \int_{\theta_i'}^{\theta_i} Q_i^{p^1, b_i}(\hat{\theta})\, d\hat{\theta}.$$

Throughout the paper we will always assume that the speculator's resale mechanism is such that, for the buyers' period-1 bid functions and posterior beliefs that we construct, an equilibrium in the resale mechanism exists; i.e., there exist period-2 strategies $(\sigma_1(\cdot), \ldots, \sigma_n(\cdot))$ such that (6), (7), and (8) are satisfied. Furthermore, we make three assumptions on the speculator's resale mechanism. The first two essentially provide a lower bound for the speculator's bargaining power in the resale market, the third is a symmetry assumption.

Our first assumption requires that if the speculator wins at a price that reveals the highest value among buyers with certainty to her, then she can make a take-it-or-leave-it offer identical to the highest value. I.e., the speculator has full bargaining power in the case where she is certain about the available surplus.

ASSUMPTION 1 *Consider any $p^1 \geq 0$ such that the first order statistic of $\Pi_s(\cdot \mid p^1)$ has the distribution function $\hat{\theta} \mapsto \mathbf{1}_{\hat{\theta} \geq \theta'}$ for some $\theta' > 0$. Then the outcome of the resale mechanism $g^{p^1}(\cdot)$ equals the outcome of a take-it-or-leave-it offer at price θ'.*

The second assumption requires that if the speculator wins the initial auction and in expectation a positive surplus is available in period 2, the speculator captures some part of that surplus, no matter how small. Without such an assumption it is hard to see how a speculator could be attracted.

ASSUMPTION 2 *Consider any $p^1 \geq 0$ such that the posterior $\Pi_s(\cdot \mid p^1)$ puts probability less than 1 on the value profile $(0, \ldots, 0)$. Then the outcome of the resale mechanism $g^{p^1}(\cdot)$ is such that $P_s^{p^1} > 0$.*

The third assumption requires that the resale market treats symmetric buyers symmetrically. This assumption will allow us to construct equilibria of the overall game that are symmetric among buyers.

ASSUMPTION 3 *Consider any $p^1 \geq 0$ such that $\Pi_s(\cdot \mid p^1)$ induces independent and identical marginal distributions for the buyers' values, and all buyers $i = 1, \ldots, n$ submit the same bid $b^* \leq p^1$ and have the same posterior $\Pi^*(\cdot) = \Pi_i^{b^*}(\cdot \mid p^1)$. Then the functions $Q_i^{p^1, b^*}(\cdot)$ and $P_i^{p^1, b^*}(\cdot)$ are independent of i.*

Many of our results rely on the following collection of resale mechanisms. We call $(g^{p^1}(\cdot))_{p^1 \geq 0}$ a *second-price auction with optimal reserve price* if for all $p^1 \geq 0$ the mechanism $g^{p^1}(\cdot)$ is a second-price auction with a reserve price that is optimal given the posterior $\Pi_s(\cdot \mid p^1)$, provided all buyers bid their values in period 2. Whenever we consider this collection of resale mechanisms, we will focus on equilibria where buyers bid their values in period 2.

Assumptions 1 to 3 are satisfied if the speculator's resale mechanism is a second-price auction with optimal reserve price. Similarly, the assumptions are satisfied if the speculator's resale mechanism is a first-price auction with optimal reserve price, is restricted to an optimal take-it-or-leave-it offer, or is a second-price auction without a reserve price. Alternatively, the speculator's resale mechanism may include multiple stages (here, agents may discount payments and probabilities depending on the stage within period 2). For example, a resale mechanism in which the speculator makes multiple consecutive take-it-or-leave-it offers until one is accepted,[8] also satisfies Assumptions 1 to 3.

4. Speculation with General Resale Mechanisms

In this section, we construct the equilibria on which all our results are based. These equilibria are such that the speculator wins the auction in period 1 with positive probability, and bids in period 1 generally differ from values. This contrasts the environment without resale possibility where it is a dominant strategy for each bidder to bid her value. With a resale possibility, there still exists an equilibrium with value-bidding and no resale occurrence, but no agent has a dominant strategy.

Our purpose is to construct, for all $q \in (0, 1]$, a perfect Bayesian equilibrium

$$\mathcal{E}(q) = (b_1(\cdot), \ldots, b_n(\cdot), b_s^*, \sigma_1(\cdot), \ldots, \sigma_n(\cdot), \Pi_1^{\cdot}(\cdot \mid \cdot), \ldots, \Pi_n^{\cdot}(\cdot \mid \cdot), \Pi_s(\cdot \mid \cdot))$$

such that the speculator wins with probability q in period 1. Define $\theta^* \in (0, 1]$ by $F^n(\theta^*) = q$, and define buyers' bid functions such that any buyer with a value above θ^* bids her value, while

all other buyers bid 0. I.e., for all $i \neq s$ and $\theta_i \in [0,1]$, let

$$b_i(\theta_i) = \begin{cases} 0 & \text{if } \theta_i \leq \theta^*, \\ \theta_i & \text{if } \theta_i > \theta^*. \end{cases}$$

Let us now construct the conditional distribution functions on which the agents' posterior beliefs after period 1 are based. To define these functions, we make use of the distribution functions $\hat{F}_b(\cdot) = \min\{F(\cdot)/F(b), 1\}$ for all $b \in (0,1]$. For all $\underline{\theta} = (\theta_1, \ldots, \theta_n) \in [0,1]^n$ and $p^1 \in [0,1]$, let

$$\Pi_s(\underline{\theta} \mid p^1) = \begin{cases} \prod_{i \neq s} \hat{F}_{\theta^*}(\theta_i) & \text{if } p^1 = 0, \\ \frac{1}{n} \sum_{i \neq s} \mathbf{1}_{\theta_i \geq \theta^*} \prod_{j \notin \{s,i\}} \hat{F}_{\theta^*}(\theta_j) & \text{if } p^1 \in (0, \theta^*], \\ \frac{1}{n} \sum_{i \neq s} \mathbf{1}_{\theta_i \geq p^1} \prod_{j \notin \{s,i\}} \hat{F}_{p^1}(\theta_j) & \text{if } p^1 > \theta^*. \end{cases}$$

In words, if the speculator pays 0 then she concludes that every buyer's value is distributed according to $\hat{F}_{\theta^*}(\cdot)$; i.e., she learns that everybody's value is at most θ^*. If she wins at a price in $(0, \theta^*]$—an out-of-equilibrium event—then she believes that the highest value among buyers is θ^* and this value might come with equal probability from every buyer (see Remark 2 for alternative off-equilibrium-path beliefs). Similarly, if she pays more than θ^* then she concludes that the highest value equals her payment, and the highest value belongs to each of the buyers with the same probability.

Buyer i's posterior beliefs are defined as follows. For all $\theta_{-i} \in [0,1]^{n-1}$, $p^1 \in [0,1]$, and $b_i \geq 0$, let

$$\Pi_i^{b_i}(\theta_{-i} \mid p^1)$$

$$= \begin{cases} \prod_{j \notin \{s,i\}} \hat{F}_{\theta^*}(\theta_j) & \text{if } p^1 = 0 \text{ or } p^1 = b_i \in (0, \theta^*], \\ \frac{1}{n-1} \sum_{j \notin \{s,i\}} \mathbf{1}_{\theta_j \geq \theta^*} \prod_{k \notin \{s,i,j\}} \hat{F}_{\theta^*}(\theta_k) & \text{if } p^1 > b_i \text{ and } p^1 \in (0, \theta^*], \\ \frac{1}{n-1} \sum_{j \notin \{s,i\}} \mathbf{1}_{\theta_j \geq p^1} \prod_{k \notin \{s,i,j\}} \hat{F}_{p^1}(\theta_k) & \text{if } p^1 > b_i \text{ and } p^1 > \theta^*, \\ \prod_{j \notin \{s,i\}} \hat{F}_{p^1}(\theta_j) & \text{if } p^1 = b_i \text{ and } p^1 > \theta^*, \\ \text{any distribution function} & \text{if } p^1 < b_i. \end{cases}$$

Let us first understand these beliefs in the case $p^1 > b_i$; i.e., some other buyer has bid higher than buyer i. Buyer i then conditions on the observation that the highest bid among other

buyers equals p^1. Any such bid between 0 and θ^* is out-of-equilibrium and buyer i then believes that the highest value among other buyers is θ^*; note that this is consistent with the speculator's beliefs. If $p^1 > \theta^*$ then buyer i concludes that the highest value among other buyers equals p^1 and this value belongs to each of the other buyers with the same probability.

Now consider the case $p^1 = b_i$. Here, buyer i conditions on the observation that the highest bid among other buyers is not larger than p^1. If $p^1 = 0$ then buyer i only concludes that no other buyer has a value above θ^*. She has the same belief if $p^1 \in (0, \theta^*]$ because then she has no reason to believe that anybody but herself has deviated from equilibrium. If $p^1 > \theta^*$ then the only thing she can conclude is that nobody has a value above p^1.

The beliefs in the remaining case $p^1 < b_i$ are irrelevant because the event $\tilde{p}_i^{b_i} < b_i$ is empty, by definition of $\tilde{p}_i^{b_i}$. In Appendix B we prove the following.

LEMMA 2 *For all $i \neq s$ and $b_i \geq 0$, the function $\Pi_i^{b_i}(\cdot \mid \cdot)$ is a conditional distribution function. The function $\Pi_s(\cdot \mid \cdot)$ is a conditional distribution function.*

Given the buyers' period-1 bid functions and the posterior beliefs, let $\sigma_1(\cdot), \ldots, \sigma_n(\cdot)$ be any profile of period-2 strategies such that (6), (7), and (8) are satisfied.

By Assumption 3, there exist functions $P(\cdot)$ and $Q(\cdot)$ such that $P(\cdot) = P_i^{0,0}$ and $Q(\cdot) = Q_i^{0,0}$ for all buyers i. We define the speculator's bid by

$$b_s^* = \theta^* - \delta \left(\theta^* Q(\theta^*) - P(\theta^*)\right). \tag{11}$$

The following lemma, proved in Appendix B, determines the speculator's winning probability and payoff. The speculator submits a bid between 0 and θ^*. Thus she wins in period 1 if and only if no buyer's value exceeds θ^*. In particular, if she wins then she does so at price $p^1 = 0$. Together with Assumption 2 this implies that her expected payoff is positive.

LEMMA 3 *Given the construction above, $b_s^* \in (0, \theta^*)$. Moreover, the speculator wins with probability q in period 1 and her expected payoff is given by*

$$u_s(b_s^*) = q\delta P_s^0 > 0, \tag{12}$$

where

$$P_s^0 = nE[P(\theta_i) \mid \theta_i \leq \theta^*] > 0. \tag{13}$$

It remains to be shown that the agents' bids in period 1 are optimal. The expected payoff of the speculator from bid $b_s \geq 0$ is given by

$$u_s(b_s) = \begin{cases} q\delta P_s^0/2 & \text{if } b_s = 0, \\ q\delta P_s^0 & \text{if } b_s \in (0, \theta^*], \\ q\delta P_s^0 + E\left[(\delta\tilde{\theta}_{-i}^{(1)} - \tilde{\theta}_{-i}^{(1)})\mathbf{1}_{\theta^* \leq \tilde{\theta}_{-i}^{(1)} \leq b_s}\right] & \text{if } b_s > \theta^*, \end{cases}$$

where $\tilde{\theta}_{-i}^{(1)}$ denotes the random variable for the highest value among buyers other than i. From the payoff function $u_s(\cdot)$, equilibrium condition (10) is immediate. The speculator is indifferent between bids between 0 and θ^* because no buyer submits such a bid. Bidding 0 is not optimal because it reduces the speculator's chances to win and make a period-2 profit. Bidding more than θ^* is not optimal because in the event that she needs such a high bid in order to win, her payment in period 1 equals the highest value among all buyers.

The expected payoff for buyer i with type $\theta_i \in [0, 1]$ when she bids $b_i \geq 0$ is given by

$$u_i(b_i, \theta_i) = \begin{cases} F^{n-1}(\theta^*)\left(\theta_i Q(\theta_i) - P(\theta_i)\right) & \text{if } b_i = 0, \\ F^{n-1}(\theta^*)\max\{0, \theta_i - \theta^*\} & \text{if } b_i \in (0, b_s^*), \\ F^{n-1}(\theta^*)\left(\frac{\max\{0,\theta_i-\theta^*\}}{2} + \frac{\theta_i - b_s^*}{2}\right) & \text{if } b_i = b_s^*, \\ F^{n-1}(\theta^*)(\theta_i - b_s^*) & \text{if } b_i \in (b_s^*, \theta^*), \\ F^{n-1}(\theta^*)(\theta_i - b_s^*) + E\left[(\theta_i - \tilde{\theta}_{-i}^{(1)})\mathbf{1}_{\theta^* \leq \tilde{\theta}_{-i}^{(1)} \leq b_i}\right] & \text{if } b_i \geq \theta^*. \end{cases} \tag{14}$$

The crucial step in the verification of equilibrium condition (9) is that buyers with value $\theta_i < \theta^*$ prefer to bid 0 and wait for resale rather than bid θ^*, while for buyers with $\theta_i > \theta^*$ the opposite is true, and type θ^* is indifferent between bidding her value and waiting for resale. This is shown in Lemma 4, the proof of which can be found in Appendix B.

LEMMA 4 *Given the construction above, for all $i \neq s$ and $\theta_i \in [0, 1]$, buyer i's expected payoff*

satisfies

$$u_i(\theta^*, \theta_i) \begin{cases} \geq u_i(0, \theta_i) & \text{if } \theta_i > \theta^*, \\ = u_i(0, \theta_i) & \text{if } \theta_i = \theta^*, \\ \leq u_i(0, \theta_i) & \text{if } \theta_i < \theta^*. \end{cases}$$

Once we have this, verifying equilibrium condition (9) is straightforward. In particular, types below θ^* pool at bid 0 rather than bid their value because otherwise the speculator gets too optimistic about the deviator's value. Types above θ^* find it optimal to bid their value for the same reasons as in a second-price auction without resale.

The arguments so far show that $\mathcal{E}(p)$ is a perfect Bayesian equilibrium. Finally, let us show that in equilibrium the speculator's resale revenue is bounded above by her period-1 bid. The proof can be found in Appendix B.

LEMMA 5 *Given the construction above,* $P_s^0 \leq b_s^*$.

The proof uses the fact that the speculator's resale revenue is bounded above by the average resale payment of the highest type, θ^*, who participates in the resale mechanism. To keep type θ^* indifferent between waiting for resale and buying the good at price b_s^* in period 1, her average resale payment must be below b_s^*.

Summarizing the above results, we now have the following.

PROPOSITION 1 *Consider a second-price auction with resale. For all* $q \in (0, 1]$*, a perfect Bayesian equilibrium exists in which the speculator bids* $b_s^* > 0$ *in period 1 and wins the initial auction with probability* q*. The speculator's payment when she wins is 0, and her expected resale revenue conditional on winning in period 1,* P_s^0*, satisfies*

$$0 < P_s^0 \leq b_s^*. \tag{15}$$

The speculator's expected payoff is given by

$$u_s(b_s^*) = q\delta P_s^0 > 0. \tag{16}$$

An important property of these equilibria is (15): the speculator would make losses if she had to pay her own bid in the initial auction. A potential second speculator must at least match

the first speculator's bid in order to win with positive probability in period 1. Equation (15) shows that by doing so she would make losses, provided the outcome of the resale market does not change due to the presence of the second speculator.[9] In this sense, our equilibria remain valid with free entry of speculators.

Because the speculator wins the initial auction with positive probability, a delayed allocation sometimes occurs. Therefore, none of the equilibria are efficient. Even ignoring inefficiencies due to delay, typical resale markets will induce an inefficient allocation in the sense that the speculator keeps the good with positive probability (e.g., when the speculator's resale mechanism is a second-price auction with optimal reserve price), or a buyer who does not have the highest value can end up with the good (e.g., when the speculator's resale mechanism is an optimal take-it-or-leave-it offer).

There always exists an equilibrium, $\mathcal{E}(1)$, where the expected revenue of the initial seller is zero. I.e., speculation can harm the initial seller whenever there are at least two buyers.

English Auction

The equilibria that we have constructed remain valid if the second-price auction in period 1 is replaced by an English auction. The main difference is that in an English auction the losing bids become public during the auction, so that bidders revise their beliefs each time a bidder drops out. In the spirit of our equilibria, the speculator believes that she definitely will not win the auction as soon as a positive standing high bid is reached (because then she believes that some buyer's value is greater than θ^*). It is therefore optimal for her to drop out at price b_s^*.[10] If a buyer with value less than or equal to θ^* deviates and drops out at some price between 0 and b_s^*, the speculator revises her belief and believes the buyer's value is θ^* (or above θ^*, which would also support our equilibrium).

5. Resale via a Second-Price Auction with an Optimal Reserve Price

In this section, we focus on the case where the speculator's resale mechanism is a second-price auction with optimal reserve price. This mechanism is particularly attractive because, in

our equilibria, it maximizes resale revenue among all conceivable mechanisms if $F(\cdot)$ satisfies a regularity property (Myerson, 1981). We have two results.

Proposition 2 shows that the equilibria that we constructed in the previous section remain valid if any buyer who wins in period 1 can offer the good for resale herself; i.e., in equilibrium no buyer will attempt to resell even if she can. This holds true no matter what resale mechanism the winning buyer uses. In particular, our equilibria remain valid if every agent, buyer or speculator, can offer the good for resale via a second-price auction with an optimal reserve price.

Proposition 3 shows that there always exists an equilibrium such that the initial seller's expected revenue is higher than in an environment without resale. This implies that a revenue maximizing seller who cannot set a positive reserve price can always have an incentive to allow resale and attract a speculator.

PROPOSITION 2 *For any $q \in (0,1]$, consider the profile $\mathcal{E}(q)$ constructed in the previous section for the case where the speculator's resale mechanism is a second-price auction with optimal reserve price.*

Then, $\mathcal{E}(q)$ remains an equilibrium if any buyer who wins the initial auction can offer the good for resale and can obtain the total surplus that is available in the resale market.

The proof can be found in Appendix B. It is sufficient to consider a deviation of a buyer with a value in $[0, \theta^*]$ to a bid in $(b_s^*, \theta^*]$. Bidding more than θ^* cannot be optimal because in the event that the buyer needs such a high bid in order to win, her payment in period 1 equals the highest value among the other bidders. The proof makes no use of the assumption that reserve prices are chosen optimally. In fact, the result holds for any reserve price. In particular, it holds even if the speculator cannot set any positive reserve price.

The next result shows that speculation can enhance the initial seller's expected revenue. This happens in equilibria with a small winning probability for the speculator. This holds no matter how many buyers are in the market, although in a market with many buyers and value-bidding the initial seller already appropriates almost all of the available surplus. The proof, in Appendix B, requires that the distribution function for buyer values is sufficiently smooth.

PROPOSITION 3 *Assume that $F(\cdot)$ is differentiable $n+1$ times. Let $\pi(q)$ denote the initial seller's expected revenue in the equilibrium $\mathcal{E}(q)$ constructed in the previous section, for the case where the speculator's resale mechanism is a second-price auction with optimal reserve price. Let $\pi(0)$ denote the expected revenue of the initial seller when every agent bids her value in period 1 and consumes the good immediately if she wins.*

Then we have $\pi(q) > \pi(0)$ for all q sufficiently close to 0.

Note that there exist two distinct events that cause the revenue in an equilibrium with speculation to differ from the revenue that arises from value-bidding and immediate consumption. Event (i) is that θ^* is between the highest and the second-highest value, in which case revenue rises to θ^*, and event (ii) is that θ^* is above the highest value, in which case revenue falls to 0. If q (and thus θ^*) is close to 1 then the probability of (i) is small and the probability of (ii) is large, allowing expected revenue to be reduced. If q (and thus θ^*) is close to 0 then both probabilities are small; in fact, it turns out that the kth order effect of introducing a small θ^* is zero for all $k < n$, but the nth order effect is positive. Therefore, if θ^* is close to 0 then speculation enhances the initial seller's expected revenue.

6. Speculation When the Initial Seller Can Set a Reserve Price

So far we have considered an environment where the initial seller does not use a reserve price. In this section, we show that equilibria with speculation can exist even if there is a reserve price in the initial auction (Proposition 4). Moreover, in Proposition 5 we consider the game where the initial seller chooses a reserve price that maximizes her expected revenue, under the assumption that the speculator's resale mechanism is a second-price auction with optimal reserve price. We show that an equilibrium exists in which the initial seller sets a reserve price above the price she would set in the absence of resale, and obtains a smaller revenue.

We assume that before the initial auction begins the initial seller announces some reserve price $r \geq 0$, and she is committed to not offer the good in period 2 if she does not sell it in period 1. The auction rules are as before, except that any bid below r is to be identified with

non-participation. It is straightforward to adapt the definition of perfect Bayesian equilibrium to the environment with reserve price r. Any equilibrium $\mathcal{E}(q)$ with $b_s^* \geq r$ naturally corresponds to a strategy-belief vector $\mathcal{E}(r, q)$ in the game with reserve price r.

The proof of the following result is a straightforward adaptation from Proposition 1.

PROPOSITION 4 *Suppose the initial seller commits to a reserve price r such that $r \leq \delta P_s^0$ for some $q \in (0, 1]$. Then $\mathcal{E}(r, q)$ is a perfect Bayesian equilibrium and the speculator's resulting expected payoff is $q(\delta P_s^0 - r)$.*

This result shows in particular that even a substantial reserve price might not prevent speculation if there exist many buyers, discounting is small, and the speculator's resale mechanism is a second-price auction with an optimal reserve price. This is because for large n, the resale revenue P_s^0 tends to θ^*. In particular, for any given reserve $r < 1$, the vector $\mathcal{E}(r, 1)$ is an equilibrium if n is sufficiently large and δ is sufficiently close to 1.

Now we consider the game where the initial seller can choose any reserve price before the initial auction starts. We assume that the initial seller's payoff equals her expected revenue. Proposition 5 then shows that the possibility of resale can harm the initial seller (given our assumption that there are at least two buyers in the market).[11]

PROPOSITION 5 *Consider the game where the initial seller's objective is to maximize expected revenues, and she can choose a reserve price before the initial auction starts. Suppose the speculator's resale mechanism is a second-price auction with an optimal reserve price.*

Let r^1 denote the largest among the reserve prices that are optimal for the initial seller if all agents bid their values in period 1 and the winner consumes the good immediately; let π^* denote the resulting expected revenue of the initial seller.

If δ is sufficiently close to 1 then there exists an equilibrium such that the initial seller sets a reserve price $r > r^1$ and her expected revenue is smaller than her no-resale revenue π^*.

Proof. Observe first that $n \geq 2$ implies $r^1 < P_s^0$ for $q = 1$ (because r^1 is an optimal reserve price for the speculator in period 2 and there is positive probability that at least 2

buyers' values are above r^1). Therefore, $r^1 < \delta P_s^0$ for all δ is sufficiently close to 1. Define $\underline{r} = (r^1 + \delta P_s^0)/2$. Now suppose that equilibrium $\mathcal{E}(\hat{r}, 1)$ is played following any reserve price $\hat{r} < \underline{r}$, but value-bidding and immediate consumption occurs following any reserve price $\hat{r} > \underline{r}$. Following the reserve price $\hat{r} = \underline{r}$, suppose that either $\mathcal{E}(\hat{r}, 1)$ is played or value-bidding and immediate consumption occurs, depending on which of these two leads to a higher expected revenue for the initial seller. Given these strategies, every reserve price $\hat{r} < \underline{r}$ results in an expected revenue of $\hat{r}$ for the initial seller. Thus, some reserve price $r \geq \underline{r}$ is optimal. No reserve price $\hat{r} > \underline{r}$ can result in an expected revenue of π^* because otherwise $\hat{r}$ would be optimal in the absence of resale, contradicting our assumption that r^1 is maximal. To complete the proof, we have to exclude the possibility that the reserve price $\hat{r} = \underline{r}$ results in an expected revenue of $\pi^* = \underline{r}$. But this would imply $\pi^* = \underline{r} < P_s^0$, which is impossible because in period 2 the speculator faces the same market as the initial seller in period 1 if resale is impossible.

7. Remarks and Extensions

1. Our equilibria are quite robust with respect to the initial seller's bid announcement policy.[12] Whatever the policy, the winner learns the second-highest bid from her payment p^1, and this information is sufficient to make the inferences that support our equilibria. Moreover, in equilibrium the third-highest and lower bids (as well as the identity of the respective bidders) reveal no relevant information that is not already revealed by the second-highest bid; the same is true after any single-agent deviation from her equilibrium bid in period 1.

2. Our equilibria are quite robust with respect to the specification of the speculator's off-equilibrium-path beliefs (i.e., her beliefs after she wins at a price in $(0, \theta^*]$). For concreteness, suppose the speculator's resale mechanism is a second-price auction with optimal reserve. For example, the off-equilibrium-path beliefs might be identical to the beliefs after winning at 0; i.e., the speculator simply believes that the deviator is a buyer type who was supposed to bid 0.[13] Given such beliefs, the buyer's deviation has no impact on

the resale market outcome, and thus the deviation is not profitable. Generally, any off-equilibrium-path beliefs such that the deviation leads to a weakly increased resale reserve price support our equilibria. To give an example of beliefs that do not generally support our equilibria, suppose that the speculator believes that the deviator is a buyer who, incorrectly, plays her part in a bid-your-value equilibrium. I.e., for all $p^1 \in (0, b_s^*]$, after winning at price p^1 the speculator believes that one buyer's value equals p^1 and everybody else's value is distributed on $[0, \theta^*]$. In the case of a market with a single buyer ($n = 1$), the resulting resale price is p^1, and thus for types close to θ^* it is profitable to deviate to a small positive bid.

3. Assumption 1, that the speculator has full bargaining power in the case where she is certain about the available surplus, can easily be relaxed in the case where there is only one buyer and one speculator in the market. Suppose that after the speculator wins, with probability $\lambda \in (0, 1)$ the speculator makes a take-it-or-leave-it-offer and with probability $1 - \lambda$ the buyer makes a take-it-or-leave-it-offer. Consider a buyer with value θ^*. Her expected payoff from waiting for resale is $\delta(\theta^* - \lambda r^*)$, while her expected payoff from bidding her value is $\theta^* - b_s^*$. These payoffs are equal for some $b_s^* \in (0, \theta^*)$. A buyer with value $\theta_i < \theta^*$ who deviates to a bid in $(0, b_s^*)$ obtains the payoff $\delta(1 - \lambda)\theta_i$, given the speculator's belief that her type is θ^*. Therefore, she prefers to bid 0.

4. It is natural to ask whether the speculator's off-equilibrium-path beliefs that support our equilibria are reasonable. One way of addressing this issue is to reduce our game to a standard signaling game and show that our equilibria do not fail the *intuitive criterion* of Cho and Kreps (1987).

   Let us construct a reduced game based on some equilibrium $\mathcal{E}(q)$, for the case where the speculator's resale mechanism is a second-price auction with optimal reserve. Consider a buyer $i = 1, \ldots, n$, fix all other buyers' equilibrium strategies, and fix the speculator's period-1 equilibrium bid b_s^*. Then the auction with resale is reduced to a signaling game in which buyer i's bid b_i is a message, and the speculator as the receiver responds with a period-2 reserve price r (in the event that the speculator does not win in period 1, the

reserve price is not payoff-relevant).

The intuitive criterion is implicitly concerned with posterior beliefs generated by bids that are off the equilibrium path. Our assignment of positive probability to type θ^* given any $b_i \in (0, \theta^*]$ conforms with the intuitive criterion because there exists an undominated reserve price r such that bidding b_i is at least as good for type θ^* as the equilibrium bid $b_i(\theta^*) = 0$. Any r less than or equal to the equilibrium resale reserve price works. First, any $r \in [0, 1]$ is undominated. Second, in the case $b_i \in (b_s^*, \theta^*]$, buyer i still wins against the speculator in period 1. Third, in case $b_i \in (0, b_s^*)$, the speculator wins against buyer i and the reserve price r that i faces in period 2 is not larger than in equilibrium. Fourth, buyer i's expected payoff from $b_i = b_s^*$ is a convex combination of the payoffs from the previous two cases.

5. We may also ask whether there is a natural way to select among the equilibria of the second price auction with resale. We have computed a continuum of equilibria $\mathcal{E}(q)$ $(q \in (0, 1])$, and there is the equilibrium outcome where buyers always bid their values and the good is consumed in period 1. Let us focus on the case where the speculator's resale mechanism is a second-price auction with optimal reserve.

   Consider Pareto domination. No equilibrium dominates another even though we do not consider the seller's payoff. First consider our equilibria versus the value-bidding outcome. The speculator is better off in our equilibria because with value-bidding she makes no profit. On the other hand, one can show that buyers are weakly worse off in our equilibria compared to value-bidding, no matter what their values are, and buyers with small values are strictly worse off because their payoff drops to 0. Now compare our equilibria to one another. The speculator's payoff is strictly increasing in her winning probability q. The equilibrium reserve price in period 2, r^*, is also strictly increasing in q (this can be shown using the strict monotone comparative statics techniques of Edlin and Shannon, 1998). Therefore, any buyer with a value in $(r^*, \theta^*]$ is strictly worse off when q is increased, showing that none of our equilibria Pareto dominates another even though we do not consider the seller's payoff.

A selection criterion that has bite is *strictness on the equilibrium path*. This criterion can be justified via a stability condition in a dynamic evolutionary context (Binmore and Samuelson, 1999). The criterion favors value-bidding to our equilibria. For equilibria based on value-bidding, every agent of any type is worse off when deviating from the equilibrium path. This is not the case for our equilibria. For example, buyers with low values are indifferent between bidding their value and bidding 0 in period 1. However, our equilibria become strict on the equilibrium path for all buyer types if we allow non-participation as a possible action and, keeping the period-2 outcome fixed, any buyer strictly prefers non-participation in the initial auction to the outcome "participate and win with probability 0." The speculator, while being indifferent between all bids in $(0, \theta^*)$, must bid b_s^* in order to assure optimality of the buyers' bid functions. Therefore, assuming the speculator is forward looking, our equilibria might have good stability properties in dynamic contexts. This question must be left for future research.

6. Suppose that speculators are excluded. Inefficient equilibria can exist in this case as well, but are more difficult to construct. Assume there exist 2 buyers with values independently and uniformly distributed on $[0, 1]$, and there is no discounting ($\delta = 1$). The auction winner can make a take-it-or-leave-it offer to the loser in period 2.

   Suppose that buyer 1 uses a bid function with some threshold θ^* as in the equilibria that we have constructed. For buyer 2, it is then optimal to bid $b_2^* = (3/4)\theta^*$ if her value is below θ^*, and otherwise bid her value. Buyer 2 with type $\theta_2 < \theta^*$ makes the resale offer $(\theta^* + \theta_2)/2$ if she wins at price 0 (and consumes the good in period 2 if her offer if rejected) and makes the resale offer θ^* if she wins at a price in $(0, \theta^*)$. An explicit computation now shows that all types $\theta_1 \in [0, \theta^*)$ prefer to wait for resale rather than overbid b_2^* and offer the good for resale themselves.

   Generalizing this equilibrium construction appears difficult. Even the case of a uniform distribution with discounting quickly gets complicated because all types θ_2 sufficiently close to θ^* would not offer the good for resale, but instead consume the good in period 1; i.e., we would have different thresholds $\theta_2^* < \theta_1^*$. In case $\delta \approx 0$ the equilibrium construction

definitely breaks down because then $\theta_2^* \approx 0$ and thus type θ_1^* cannot be made indifferent between bidding her value and waiting for resale.

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Appendix A: Conditional distribution functions

Let us review the defining conditions for the conditional distribution functions that we defined on p. 9 (for more see Billingsley, 1995, Ch. 33). These conditions generalize Bayes rule from discrete to arbitrary probability distributions.

For all buyers i and $b_i \geq 0$, the function

$$\Pi_i^{b_i}(\cdot \mid \cdot) : \ [0,1]^{n-1} \times [0,\infty) \to [0,1]$$

is a conditional distribution function if (i) for all $p^1 \geq 0$, the function $\Pi_i(\cdot, p^1)$ is a probability distribution function, (ii) for all $\theta_{-i} = \theta_{-i-s} \in [0,1]^{n-1}$, the function $\Pi(\theta_{-i}, \cdot)$ is Borel measurable, and (iii)

$$\forall p^1 \geq 0, \theta_{-i} \in [0,1]^{n-1} : \ \int_0^{p^1} \Pi_i(\theta_{-i} \mid b) \mathrm{d}G_i(b) = \Pr[\tilde{\theta}_{-i} \leq \theta_{-i}, \tilde{p}_i^{b_i} \leq p^1],$$

where $G_i(\cdot)$ denotes the probability distribution function for $\tilde{p}_i^{b_i}$.

Similarly, the function

$$\Pi_s(\cdot \mid \cdot) : \ [0,1]^{n} \times [0,\infty) \to [0,1]$$

is a conditional distribution function if (i) for all $p^1 \geq 0$, the function $\Pi_s(\cdot, p^1)$ is a probability distribution function, (ii) for all $\underline{\theta} \in [0,1]^n$, the function $\Pi(\underline{\theta}, \cdot)$ is Borel measurable, and (iii)

$$\forall p^1 \geq 0, \underline{\theta} \in [0,1]^{n} : \ \int_0^{p^1} \Pi_i(\underline{\theta} \mid b) \mathrm{d}G_s(b) = \Pr[\tilde{\underline{\theta}} \leq \underline{\theta}, \tilde{p}_s^1 \leq p^1],$$

where $G_s(\cdot)$ denotes the probability distribution function for $\tilde{p}_s^1$.

It is important to note that conditional distribution functions are not uniquely determined by the underlying random variables. Consider any buyer i. If $P \subseteq [0,\infty)$ is Borel measurable and $\Pr[\tilde{p}_i^{b_i} \in P] = 0$ then for all $p^1 \in P$ the posterior distribution $\Pi_i^{b_i}(\cdot, p^1)$ can be changed without violating the conditions above. I.e., after a probability-0 event the posterior is essentially left unrestricted by the conditions. Up to this degree of freedom, conditional distribution functions are uniquely determined by the underlying random variables. More precisely, if $\Pi(\cdot \mid \cdot)$ and $\Lambda(\cdot \mid \cdot)$ are both conditional distribution functions then, for any given $\theta_{-i} \in [0,1]^{n-1}$, we have $\Pi(\theta_{-i} \mid \tilde{p}_i^{b_i}) = \Lambda(\theta_{-i} \mid \tilde{p}_i^{b_i})$ with probability 1.

Appendix B: Proofs

Proof of Lemma 2. Let us first show that $\Pi_s(\cdot \mid \cdot)$ is a conditional distribution function for $\underline{\tilde{\theta}}$ conditional on $\tilde{p}_s^1$. We have to check conditions (i), (ii), and (iii) from Appendix A. Conditions (i) and (ii) are immediate. The distribution function for $\tilde{p}_s^1$ is given by $G_s(b) = F(\max\{b, \theta^*\})^n$. To verify the integral condition (iii), consider any vector $\underline{\theta} = (\theta_1, \ldots, \theta_n) \in [0,1]^n$. Whenever $p^1 \leq \theta^*$ condition (iii) is satisfied because

$$\int_0^{p^1} \Pi_s(\underline{\theta} \mid b) \mathrm{d}G_s(b) = \Pi_s(\underline{\theta} \mid 0) G_s(0) = \prod_{i \neq s} F(\min\{\theta_i, \theta^*\}) = \Pr[\underline{\tilde{\theta}} \leq \underline{\theta}, \tilde{p}_s^1 = 0].$$

Now suppose $p^1 > \theta^*$. Let us first observe that for all $i \neq s$ with $\theta_i > \theta^*$ we have

$$\begin{aligned}
\int_{\theta^*}^{p^1} \mathbf{1}_{\theta_i \geq b} \prod_{j \notin \{s,i\}} \hat{F}_b(\theta_j) \, \mathrm{d}F^n(b) &= \int_{\theta^*}^{\min\{p^1, \theta_i\}} \left(\prod_{j \notin \{s,i\}} \hat{F}_b(\theta_j) \right) n F^{n-1}(b) \, \mathrm{d}F(b) \\
&= n \int_{\theta^*}^{\min\{p^1, \theta_i\}} \left(\prod_{j \notin \{s,i\}} F(\min\{\theta_j, b\}) \right) \mathrm{d}F(b) \\
&= n \Pr[\underline{\tilde{\theta}} \leq \underline{\theta}, \ \tilde{\theta}_i \in (\theta^*, p^1], \tilde{\theta}^{(1)} = \tilde{\theta}_i],
\end{aligned}$$

where $\tilde{\theta}^{(1)}$ denotes the random variable for the highest value among all buyers. Note that the first and the last expression in above equality chain are identical even if $\theta_i \leq \theta^*$ because then both expressions are 0. Summing up over all $i \neq s$ and using the fact that $G_s(b) = F^n(b)$ for $b \geq \theta^*$, we obtain the result

$$\begin{aligned}
\int_{\theta^*}^{p^1} \Pi_s(\underline{\theta} \mid b) \, \mathrm{d}G_s(b) &= \frac{1}{n} \sum_{i \neq s} \int_{\theta^*}^{p^1} \mathbf{1}_{\theta_i \geq b} \prod_{j \notin \{s,i\}} \hat{F}_b(\theta_j) \, \mathrm{d}G_s(b) \\
&= \Pr[\underline{\tilde{\theta}} \leq \underline{\theta}, \ \tilde{\theta}^{(1)} \in (\theta^*, p^1]]
\end{aligned}$$

for all $\underline{\theta} \in [0,1]^n$ and all $b > \theta^*$. Using this, we can verify condition (iii) for the case $p^1 > \theta^*$, as follows:

$$\begin{aligned}
\int_0^{p^1} \Pi_s(\underline{\theta} \mid b) \mathrm{d}G_s(b) &= \Pi_s(\underline{\theta} \mid 0) G_s(0) + \Pr[\underline{\tilde{\theta}} \leq \underline{\theta}, \tilde{\theta}^{(1)} \in (\theta^*, p^1]] \\
&= \Pr[\underline{\tilde{\theta}} \leq \underline{\theta}, \tilde{p}_s^1 = 0] + \Pr[\underline{\tilde{\theta}} \leq \underline{\theta}, \tilde{p}_s^1 \in (\theta^*, p^1]] \\
&= \Pr[\underline{\tilde{\theta}} \leq \underline{\theta}, \tilde{p}_s^1 \leq p^1].
\end{aligned}$$

For the buyers' posterior beliefs, conditions (i), (ii), and (iii) can be verified in a similar way.

PROOF OF LEMMA 3. Condition (13) holds by Assumption 2. From Lemma 1 we know that $P(\cdot)$ is increasing. Together with (13) this implies that $P(\theta_i) > 0$ for all θ_i sufficiently close to θ^*. Hence, (7) implies $Q(\theta_i) > 0$ for such θ_i. Lemma 1 now implies that

$$\theta^* Q(\theta^*) - P(\theta^*) = \int_0^{\theta^*} Q(\theta_i') \mathrm{d}\theta_i' + P(0) > 0.$$

Together with $P(\theta^*) > 0$ and because $Q(\theta^*) < 1$ this yields $0 < \theta^* Q(\theta^*) - P(\theta^*) < \theta^*$. Therefore, $b_s^* \in (0, \theta^*)$. This shows that the speculator wins in period 1 with probability $F^n(\theta^*) = q$ and if she wins then it is at price 0. Therefore, her expected payoff is given by (12).

PROOF OF LEMMA 4. Define $\Delta(\theta_i) = (u_i(\theta^*, \theta_i) - u_i(0, \theta_i))/F^{n-1}(\theta^*)$. Using (11) and (14) we find

$$\Delta(\theta^*) = \theta^* - b_s^* - (\theta^* Q(\theta^*) - P(\theta^*)) = 0.$$

Lemma 1 implies

$$\Delta(\theta_i) = \theta_i - b_s^* - \int_0^{\theta_i} \delta Q(\theta_i') \mathrm{d}\theta_i' - \delta P(0) = \int_0^{\theta_i} (1 - \delta Q(\theta_i')) \mathrm{d}\theta_i' - b_s^* - \delta P(0).$$

Hence, $\Delta(\theta_i)$ is increasing in θ_i, which implies the result.

PROOF OF LEMMA 5. Let $\underline{\theta} = \inf_{Q(\theta)>0} \theta$. By Lemma 1 we have $P(\theta_1) = 0$ for all $\theta_1 < \underline{\theta}$. Therefore, Lemma 3 implies

$$P_s^0 = nE[P(\theta_1) \mid \underline{\theta} < \theta_1 \leq \theta^*]. \tag{17}$$

From (11) we get

$$\theta^* - b_s^* = \underbrace{\delta Q(\theta^*)}_{\leq 1} (\theta^* - \frac{P(\theta^*)}{Q(\theta^*)}),$$

which implies

$$b_s^* \geq \frac{P(\theta^*)}{Q(\theta^*)}. \tag{18}$$

Now let us show that

$$\forall \theta_1, \theta_1' \in (\underline{\theta}, \theta^*] : \text{ if } \theta_1' < \theta_1 \text{ then } \frac{P(\theta_1')}{Q(\theta_1')} \leq \frac{P(\theta_1)}{Q(\theta_1)}. \tag{19}$$

From incentive compatibility (6), we have $P(\theta_1) - P(\theta_1') \geq \theta_1'(Q(\theta_1) - Q(\theta_1'))$. Multiplying by $Q(\theta_1)$ yields

$$P(\theta_1)Q(\theta_1) - P(\theta_1')Q(\theta_1) \geq \theta_1'(Q(\theta_1)^2 - Q(\theta_1)Q(\theta_1')). \tag{20}$$

From individual rationality (7) we get $\theta_1 Q(\theta_1) - P(\theta_1) \geq 0$. Multiplying by $Q(\theta_1) - Q(\theta_1')$ yields

$$\theta_1(Q(\theta_1)^2 - Q(\theta_1)Q(\theta_1')) - P(\theta_1)Q(\theta_1) + P(\theta_1)Q(\theta_1') \geq 0, \tag{21}$$

Adding up (20) and (21) yields $P(\theta_1)Q(\theta_1') - P(\theta_1')Q(\theta_1) \geq 0$, implying (19). Using (17), (19), and (18), we obtain

$$\begin{aligned} P_s^0 &= nE[P(\theta_1) \mid \underline{\theta} < \theta_1 \leq \theta^*] = nE[\frac{P(\theta_1)}{Q(\theta_1)}Q(\theta_1) \mid \underline{\theta} < \theta_1 \leq \theta^*] \\ &\leq \frac{P(\theta^*)}{Q(\theta^*)} n \underbrace{E[Q(\theta_1) \mid \underline{\theta} < \theta_1 \leq \theta^*]}_{\leq 1/n \text{ by symmetry}} \leq \frac{P(\theta^*)}{Q(\theta^*)} \leq b_s^*, \end{aligned}$$

as was to be shown.

Proof of Proposition 2. At first let us compute the speculator's equilibrium bid. Using Lemma 1, we find that

$$b_s^* = \theta^* - \delta \int_{r^*}^{\theta^*} \frac{F^{n-1}(\theta_i')}{F^{n-1}(\theta^*)} \mathrm{d}\theta_i', \tag{22}$$

where $r^* \in (0, \theta^*)$ is the speculator's optimal reserve price given her posterior $\Pi_s(\cdot \mid 0)$.

Now suppose that buyer i with type $\theta_i \leq \theta^*$ deviates to a bid $b_i \in (b_s^*, \theta^*]$ and offers the good for resale if she wins (once we have shown that this is not profitable, it follows that no other deviation is profitable either). Suppose that if she wins she expects to obtain the total surplus that is available in the resale market. In cases where another buyer's value is larger than θ^*, her deviation has no payoff consequences. So let us consider her expected payoff conditional on no buyer's value being larger than θ^*. When she deviates this payoff is given by

$$u^d(\theta_i) = \delta \int_0^{\theta^*} \max\{\theta_i', \theta_i\} \mathrm{d}\frac{F^{n-1}(\theta_i')}{F^{n-1}(\theta^*)} - b_s^*.$$

The respective equilibrium payoff is given by

$$u^{eq}(\theta_i) = \begin{cases} \delta \int_{r^*}^{\theta_i} \frac{F^{n-1}(\theta_i')}{F^{n-1}(\theta^*)} \mathrm{d}\theta_i' & \text{if } \theta_i \geq r^*, \\ 0 & \text{if } \theta_i < r^*. \end{cases}$$

Therefore, her payoff gain from the deviation is given by

$$\begin{aligned} & u^d(\theta_i) - u^{eq}(\theta_i) \\ & \leq \delta \int_0^{\theta^*} \max\{\theta_i', \theta_i\} \mathrm{d}\tfrac{F^{n-1}(\theta_i')}{F^{n-1}(\theta^*)} - \theta^* + \delta \int_{\theta_i}^{\theta^*} \tfrac{F^{n-1}(\theta_i')}{F^{n-1}(\theta^*)} \mathrm{d}\theta_i' \\ & = \delta \tfrac{F^{n-1}(\theta_i)}{F^{n-1}(\theta^*)} \theta_i - \theta^* + \delta \left(\int_{\theta_i}^{\theta^*} \theta_i' \mathrm{d}\tfrac{F^{n-1}(\theta_i')}{F^{n-1}(\theta^*)} + \int_{\theta_i}^{\theta^*} \tfrac{F^{n-1}(\theta_i')}{F^{n-1}(\theta^*)} \mathrm{d}\theta_i' \right) \\ & = \delta \tfrac{F^{n-1}(\theta_i)}{F^{n-1}(\theta^*)} \theta_i - \theta^* + \delta \left(\theta^* - \tfrac{F^{n-1}(\theta_i)}{F^{n-1}(\theta^*)} \theta_i \right) \\ & = -(1-\delta)\theta^* \leq 0. \end{aligned} \tag{23}$$

Proof of Proposition 3. Let $\tilde{\theta}^{(1)}$ and $\tilde{\theta}^{(2)}$ denote the highest and second highest value among the buyers. Defining $T(\theta^*) = \pi(q) - \pi(0)$ for $F^n(\theta^*) = q$, we have

$$\begin{aligned} T(\theta^*) \;=\; & \Pr[\tilde{\theta}^{(2)} < \theta^* < \tilde{\theta}^{(1)}](b_s^* - E[\tilde{\theta}^{(2)} \mid \tilde{\theta}^{(2)} < \theta^* < \tilde{\theta}^{(1)}]) \\ & + \Pr[\tilde{\theta}^{(1)} < \theta^*](0 - E[\tilde{\theta}^{(2)} \mid \tilde{\theta}^{(1)} < \theta^*]). \end{aligned}$$

The distribution function for $\tilde{\theta}^{(2)}$ conditional on the event $\tilde{\theta}^{(2)} < \theta^* < \tilde{\theta}^{(1)}$, can be computed as follows (for all $\theta_i \leq \theta^*$):

$$\begin{aligned} \Pr[\tilde{\theta}^{(2)} \leq \theta_i \mid \tilde{\theta}^{(2)} < \theta^* < \tilde{\theta}^{(1)}] &= \frac{\Pr[\tilde{\theta}^{(2)} \leq \theta_i, \theta^* < \tilde{\theta}^{(1)}]}{\Pr[\tilde{\theta}^{(2)} < \theta^* < \tilde{\theta}^{(1)}]} \\ &= \frac{nF(\theta_i)^{n-1}(1 - F(\theta^*))}{nF(\theta^*)^{n-1}(1 - F(\theta^*))} = \frac{F(\theta_i)^{n-1}}{F(\theta^*)^{n-1}}. \end{aligned}$$

So, the respective density is given by $(n-1)F(\theta_i)^{n-2}/F(\theta^*)^{n-1} f(\theta_i)$, where $f = F'$. Similarly, the distribution function for $\tilde{\theta}^{(2)}$ conditional on the event $\tilde{\theta}^{(1)} < \theta^*$, can be computed as follows (for all $\theta \leq \theta^*$):

$$\begin{aligned} \Pr[\tilde{\theta}^{(2)} \leq \theta_i \mid \tilde{\theta}^{(1)} < \theta^*] &= \frac{nF(\theta_i)^{n-1}(F(\theta^*) - F(\theta_i)) + F(\theta_i)^n}{F(\theta^*)^n} \\ &= \frac{nF(\theta_i)^{n-1}F(\theta^*) - (n-1)F(\theta_i)^n}{F(\theta^*)^n} \end{aligned}$$

So, the respective density is given by

$$\frac{n(n-1)(F(\theta_i)^{n-2}F(\theta^*) - F(\theta_i)^{n-1})}{F(\theta^*)^n} f(\theta_i).$$

Using this, we get

$$
\begin{aligned}
T(\theta^*) &= n(1-F(\theta^*))F(\theta^*)^{n-1} \\
&\quad \cdot \left(\theta^* - \delta\theta^* + \delta \left(\frac{F(r^*)^{n-1}}{F(\theta^*)^{n-1}} r^* + \frac{n-1}{F(\theta^*)^{n-1}} \int_{r^*}^{\theta^*} \theta_i F(\theta_i)^{n-2} f(\theta_i) \mathrm{d}\theta_i \right) \right. \\
&\qquad \left. - \frac{n-1}{F(\theta^*)^{n-1}} \int_0^{\theta^*} \theta_i F(\theta_i)^{n-2} f(\theta_i) \mathrm{d}\theta_i \right) \\
&\qquad -n(n-1) \int_0^{\theta^*} \theta_i \left(F(\theta_i)^{n-2} F(\theta^*) - F(\theta_i)^{n-1} \right) f(\theta_i) \mathrm{d}\theta_i \\
&= n(1-F(\theta^*))F(\theta^*)^{n-1}(1-\delta)\theta^* + n(1-F(\theta^*))\delta F(r^*)^{n-1} r^* \\
&\quad + n(1-F(\theta^*))(n-1) \\
&\qquad \cdot \left(\delta \int_{r^*}^{\theta^*} \theta_i F(\theta_i)^{n-2} f(\theta_i) \mathrm{d}\theta_i - \int_0^{\theta^*} \theta_i F(\theta_i)^{n-2} f(\theta_i) \mathrm{d}\theta_i \right) \\
&\quad -n(n-1) \int_0^{\theta^*} \theta_i \left(F(\theta_i)^{n-2} F(\theta^*) - F(\theta_i)^{n-1} \right) f(\theta_i) \mathrm{d}\theta_i,
\end{aligned}
$$

where $r^* \in (0, \theta^*)$ denotes the speculator's optimal reserve price given her posterior $\Pi_s(\cdot \mid 0)$. Integration by parts implies

$$
\int_{r^*}^{\theta^*} \theta_i F(\theta_i)^{n-2} f(\theta_i) \mathrm{d}\theta_i = \frac{1}{n-1} (\theta^* F(\theta^*)^{n-1} - r^* F(r^*)^{n-1}) - \int_{r^*}^{\theta^*} \frac{F(\theta_i)^{n-1}}{n-1} \mathrm{d}\theta_i.
$$

Using this and three other integrations by parts, we find

$$
\begin{aligned}
T(\theta^*) &= n(1-F(\theta^*)) \left(-\delta \int_{r^*}^{\theta^*} F(\theta_i)^{n-1} \mathrm{d}\theta_i + \int_0^{\theta^*} F(\theta_i)^{n-1} \mathrm{d}\theta_i \right) \\
&\quad -n(n-1)F(\theta^*) \left(\theta^* \frac{F(\theta^*)^{n-1}}{n-1} - \int_0^{\theta^*} \frac{F(\theta_i)^{n-1}}{n-1} \mathrm{d}\theta_i \right) \\
&\quad +n(n-1) \left(\theta^* \frac{F(\theta^*)^n}{n} - \int_0^{\theta^*} \frac{F(\theta_i)^n}{n} \mathrm{d}\theta_i \right) \\
&= n(1-F(\theta^*)) \left(-\delta \int_{r^*}^{\theta^*} F(\theta_i)^{n-1} \mathrm{d}\theta_i \right) + n \int_0^{\theta^*} F(\theta_i)^{n-1} \mathrm{d}\theta_i \\
&\quad -n\theta^* F(\theta^*)^n + (n-1) \left(\theta^* F(\theta^*)^n - \int_0^{\theta^*} F(\theta_i)^n \mathrm{d}\theta_i \right) \\
&= -n(1-F(\theta^*))\delta \int_{r^*}^{\theta^*} F(\theta_i)^{n-1} \mathrm{d}\theta_i + n \int_0^{\theta^*} F(\theta_i)^{n-1} \mathrm{d}\theta_i \\
&\quad -\theta^* F(\theta^*)^n - (n-1) \int_0^{\theta^*} F(\theta_i)^n \mathrm{d}\theta_i.
\end{aligned}
$$

The implicit functions theorem implies that there exists a differentiable function $g(\cdot)$ on $[0, \epsilon)$

for some $\epsilon > 0$ such that

$$g(0) = 0,\ \forall \theta^* \in [0, \epsilon) :\ g(\theta^*) = \frac{F(\theta^*) - F(g(\theta^*))}{f(g(\theta^*))}. \tag{24}$$

Moreover,

$$g'(\theta^*) = \frac{f(\theta^*)}{2f(g(\theta^*)) + g(\theta^*)f'(g(\theta^*))}.$$

Using standard calculus methods, one sees that $g(\cdot)$ is differentiable n times. Well known results (Myerson, 1981) imply that $g(\theta^*)$ is an optimal reserve price in a second-price auction with n buyers with values independently distributed according to $\hat{F}_{\theta^*}(\cdot)$ when the seller has 0 value; i.e., we can assume that $r^* = g(\theta^*)$ for all $\theta^* \in [0, \epsilon)$. Using this, we find for $\theta^* \in [0, \epsilon)$ that

$$\begin{aligned}
\frac{\mathrm{d}T}{\mathrm{d}\theta^*} &= nf(\theta^*)\delta \int_{r^*}^{\theta^*} F(\theta_i)^{n-1}\mathrm{d}\theta_i \\
&\quad -n(1 - F(\theta^*))\delta \left(F(\theta^*)^{n-1} - F(r^*)^{n-1}g'(\theta^*)\right) + nF(\theta^*)^{n-1} \\
&\quad -F(\theta^*)^n - n\theta^* F(\theta^*)^{n-1}f(\theta^*) - (n-1)F(\theta^*)^n.
\end{aligned}$$

Differentiating $n - 1$ more times yields

$$\begin{aligned}
\frac{\mathrm{d}^n T}{\mathrm{d}(\theta^*)^n} &= n\delta f^{(n-1)}(\theta^*) \int_{r^*}^{\theta^*} F(\theta_i)^{n-1}\mathrm{d}\theta_i \\
&\quad -n(1 - F(\theta^*))\delta(n-1)!f(\theta^*)^{n-1}\left(1 - g'(\theta^*)^n\right) \\
&\quad +n!f(\theta^*)^{n-1} + (\dots)F(\theta^*) + (\dots)F(r^*) + (\dots)\theta^*.
\end{aligned}$$

Observing that $\theta^* = 0$ implies $r^* = 0$, and $g'(0) = 1/2$, we get

$$\left.\frac{\mathrm{d}^n T}{\mathrm{d}(\theta^*)^n}\right|_{\theta^*=0} = n!f(0)^{n-1}\left(1 - \delta + \delta\frac{1}{2^n}\right) > 0.$$

By the same methods, one sees that

$$\forall k = 0, \dots, n-1 :\ \left.\frac{\mathrm{d}^k T}{\mathrm{d}(\theta^*)^k}\right|_{\theta^*=0} = 0.$$

Therefore, a Taylor expansion of $T(\cdot)$ around 0 shows that $T(\theta^*) > 0$ if θ^* is sufficiently close to 0. *QED*

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Footnotes

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[2] Spectrum is allocated by auction in many countries. Some impose "use-it-or-lose-it" conditions designed to prevent resale. However, these restrictions are easy to circumvent and there are reported instances where "shell" companies were formed for the purpose of acquiring spectrum licences and then sold.

[3] This view comes from intuition derived from symmetric equilibria in symmetric environments. See, for example, results in Ausubel and Cramton (1999) and Haile (1999).

[4] Tröger (2003) shows that the situation is rather different in a first-price auction. There, speculation is not profitable for any resale mechanism, at least if only one regular buyer is in the market.

[5] The feature of pooling at bid zero resembles Jehiel and Moldovanu's (2000) result that in second-price auctions with positive externalities and a reserve price, bidders must be pooling at the reserve price. Their result, however, does not rely on signalling.

[6] Note that a buyer with value below r^* does not expect to be able to get an acceptable resale offer. So, she is indifferent between bidding and not bidding in period 1. However, in contrast to a second-price auction without resale, bidding 0 is *not* weakly dominated by any positive bid.

[7] Conditional distribution functions generalize Bayes rule to arbitrary probability distributions. See Appendix A for a brief review.

[8] Such a market makes particular sense if the good is durable. The Coase conjecture then suggests equilibria where the take-it-or-leave-it price drops quickly, but there can also exist other equilibria where the price drops slowly (Ausubel and Deneckere, 1989).

[9] Formally including multiple speculators into the model would complicate the presentation. Assumptions about posterior beliefs in the presence of multiple speculators would have to be made. We expect that equilibria with multiple active speculators playing mixed strategies exist

as well.

[10] This assumes the absence of bidding costs. On the other hand, in a dynamic context there might be a benefit from bidding up to b_s^*, because the speculator wants everybody to believe that she is willing to bid up to b_s^* in future auctions.

[11] Using similar techniques, one can also construct equilibria such that a seller who maximizes *social surplus* sets a positive reserve price—rather than using no reserve price which would be optimal in the absence of resale possibilities. Hence, positive reserve prices are not necessarily detrimental to efficiency.

[12] See Calzolari and Pavan (2002) for an analysis of bid announcement policies in optimal auctions with resale.

[13] We thank Bill Zame for suggesting these beliefs.