SIMULATION OF BEHAVIOR AND INTELLIGENCE

Equilibrium in a Distribution Network

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Abstract

A network consisting of suppliers, agents, and distributors is considered. The flow of orders and deliveries between the different elements are determined. A monotonic game [1] for the customers in the network is described.

Keywords: suppliers, distributors, game, monotonic, network

1. Introduction

The article attempts to develop a numerical description of the supply and demand structure for the deliveries of commodities in the network. We suppose that orders and deliveries be met with conditions of uncertain overhead expenses. In certain situations, the orders and deliveries do not match for a given supply and demand structure. In such cases, individual participants in the network are assumed to act rationally with the object of maximizing their profit.

Numerical analysis of such situations reveals, however, that the allegedly rational behavior of the participants is not always such that they attempt to enter certain losing transactions so as to additionally increase the profit from already profitable transactions (after offsetting the negative effect of the former, of course). On the other hand, we claim that, if the participants avoid all losing transactions, their behavior is certainly rational and the network in such cases will be in a Nash equilibrium [2] .

All this suggested to allows us a series of computer simulations to perform. First, in order to determine the possible response of the network participants to different supply and demand structures. Second, in order to identify the participants where the executive efforts might be applied to prevent distinctive actions that may misbalance the equilibrium in the network. With this object, we used a model to construct an “elasticity” measure for the choice of customers; this measure is represented by the overhead expense interval for which the network remains in equilibrium.

2. Description of a distribution network: the chain model

The distribution of commodities in the network is characterized by sales figures that may be expressed as one of the following three alternative numbers [3]: a) a demand h which is disclosed to the particular participant either externally or by other participant in the network; b) a capable supply x calculated at the cost of all commodities produced by the participant for delivery outside the network or to the other participants; c) actual sales g calculated at the prices actually paid by the customers for the delivered commodities.

Let us first consider the simplest case of distribution in a chain: this elementary model is used at this stage solely as a convenient means of simplifying the presentation.

An order is thus defined as a certain quantity of a particular commodity ordered by one of the participant's from another participant in the network; a delivery is similarly defined as a certain quantity of a commodity delivered by one of the participant's to another participant in the network. We assume that the network includes suppliers who are only capable of making deliveries – the produces; participants, who both issue orders and make deliveries – the agents; and the distributors, who only order commodities from other participants. [1]

In what follows we always consider the flow of orders and deliveries for the case of "pipeline" distribution without "closed circuits." Therefor, we can always identify a unique direction of "flow" of orders from the distributors to the produces via agents and a "flow" of deliveries in the reverse direction.

Let us consider in more detail this particular flow of orders and deliveries of commodities in the network. The direction of the flow of orders (deliveries) is defined by assigning serial numbers – the indexes $1,2$ and 3 – to the producer, to the agent, and to the distributor, respectively. The producer and the agent act as suppliers, the agent and the distributor act as customers. The agent thus has the dual role of a supplier and a customer, whereas the producer only acts as a supplier and the distributor only acts as a customer.

The flow of orders to the produces from the customers is characterized by two numbers h_{23} and h_{12} . The number h_{wj} $(w=1,2; j=2,3)$ is the demand h_{wj} disclosed by the customer j to the supplier w . The flow of deliveries to the distributor is similarly characterized by two numbers x_{12} and x_{23} , which are interpreted as the corresponding capable sales. We assume that sales equal the distribution in this network.

Suppose that the demand of the distributor to the external customers is fixed at the level of d bank notes. The capable sales of the producer are s bank notes. In other words, d is the estimated level of orders from the external customers and it plays the same role as the number h for the customers in the network. Similarly, s is the intrastate level of estimated deliveries by the producer, and it has the same role as x for the customers.

[1] Note that in subsequent sections distributors naturally also act as suppliers to external customers.

Let us now consider the exact situation in a chain. To make deliveries at a demand level of d bank notes, the distributor have to place orders with the agent in the amount of $h_{23} = n_{23} d$ bank notes, where n_{23} are the distributor's cost of commodities sold (the cost per 1 bank note of sales). The agent, having received an order from the distributor, will in turn place an order with the supplier in the amount $n_{12} \cdot h_{23}$, where n_{12} is the agent's cost per 1 bank note of sales. On the other hand, the estimated sales of the producer are x_{12} bank notes, $x_{12} = s$. Assuming that all the transactions between the suppliers and the customers in the network are materialized in amounts not less than those indicated in the purchase orders, the actual sales of the producer to the agent are given by $g'_{12} = min\{x_{12}, h_{12}\}$.

Now, since the agent paid the producer g'_{12} for the commodities ordered, the agent's revenue is $x_{23} = g'_{12} / n_{12}$, where clearly $x_{23} \geq g'_{12}$. The difference between the revenue x_{23} and the costs g'_{12} is defined as $p_{12} = g'_{12}(1 - n_{12}) / n_{12}$.

From the same considerations, $g'_{23} = min\{x_{23}, h_{23}\}$ [2] give the actual sales of the agent to the distributor. We similarly define the difference $p_{23} = g'_{23} \cdot (1 - n_{23}) / n_{23}$. The numbers p_{12}, p_{23} represent the profit of the customers in the network.

In conclusion of this section, let us consider the numbers p_{12} , p_{23} more closely. We see from the above discussion that the material costs are the only component of the costs of commodities sold for the customers in the network; no other producing costs and no overhead expenses are considered. And yet in Section 4 the numbers p_{12} , p_{23} are used as the admissible bounds on overhead expenses, which are assumed to be unknown. It is in this sense we construct a model of a monotone game of customers.

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[2] In subsequent sections, g'_{wj} is replaced by $g_{wj} = g'_{wj} / n_{wj}$. The numbers g and g' differ in the units of measurement of the commodities delivered to the user j . While g' represents the sales at the cost, g represents the same sales at actual selling prices.

3. Description of a distribution network: the general form

Consider now a distribution network consisting of n participants indexed w , $j = 1,2,...,n$. The state of a supplier w is characterized by a $(m+1)$ - component vector [3] $\langle d_w, y_w \rangle = \langle d_w, h_{wk+1,...,} h_{wn} \rangle$ $(n-k=m)$, the state of a customer j by a $(v+1)$ - component vector $\langle s_j, x_j \rangle = \langle s_j, g_{1j},...,g_{vj} \rangle$. The components of the $\langle d_w, y_w \rangle$ and $\langle s_j, x_j \rangle$ vectors are interpreted as follows: d_w is the total orders amount of the supplier w acting as a customer; s_j is the capable sales total amount of the customer j acting as a supplier; h_{wj} is the cost of orders placed by the customer j with the supplier w ; g_{wj} are actual sales (deliveries) to customer j from the supplier w . As indicated in the footnote, g_{wj} represents the deliveries valued at the selling prices of the customer j acting as a supplier. The vectors $\langle d_w, y_w \rangle$, $\langle s_j, x_j \rangle$ are the order and the delivery vectors, respectively.

With each participant in the network we associate certain domains in the nonnegative orthants of the $(m+1)$ – and the $(v+1)$ – dimensional space. These domains are the regions of feasible values of vectors $\langle d_w, y_w \rangle$, $\langle s_j, x_j \rangle$ in the $(m+v+2)$ – dimensional space.

For some of the participants vectors with $g_{wj} > 0$ are inadmissible, and for some participants vectors with $h_{wj} > 0$ are inadmissible. Participants having the former property will be called produces and those having the latter property will be called distributors; all other participants in the network will be called agents. In what follows the numbers s_w $(w = 1,2,...,k)$ characterize the k produces; the number s_w represents the capable sales controlled by the participant w . The numbers d_j $(j = v+1, v+2,...,n)$ correspondingly characterize the r distributors: the number d_j represents the demand to the external customers $(n-v=r)$.

Let us now impose certain constrains on the admissible vectors in this network. The following constrains are strictly "local," i.e., they apply to the individual participants in the network.

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[3] k is the number of produces, see below.

The admissible network states are constrained by balance conditions equating the actual sales from all the suppliers to a particular customer to capable sales of that customer acting as a supplier:

$$s_j = \sum_{w=1}^{v} g_{wj} \quad (j = k+1, k+2, ..., n) \ . \tag{1}$$

We also require balance conditions between the cost of orders placed by all the customers with a particular supplier and the demand figure of that supplier acing as a customer:

$$d_w = \sum_{j=i+1}^{n} h_{wj} \quad (w = 1, 2, ..., v) \ . \tag{2}$$

As we have noted above, the distribution network considered in this article does not allow "closed-circuit motion" of orders or deliveries until a particular order reaches a producer or the delivery reaches a distributor. The indexes labeling the participants in such networks are ordered so [4] that if w is a supplier and j is a customer, then $w < j$ $(w = 1, 2, ..., v; j = v+1, v+2, ..., n)$. Such networks are called a-cyclic, and their description requires certain additional assumptions.

Consider the constants $a_{wj} \geq 0$ and $b_{wj} \geq 0$ satisfying the following constraints:

$$\sum_j a_{wj} \leq 1 \ (j > w; w = 1, 2, ..., v) \ , \ \sum_w b_{wj} \leq 1 \ (w < j; j = k+1, ..., n) \tag{3}$$

For the supplier w, the number a_{wj} is the fractional cost of orders made to the customer j. For customer j, the number b_{wj} is the fractional cost of the deliveries from supplier w which are necessary for meeting the sales target.

Suppose that purchase of orders in the distribution network move from distributors through agents to suppliers. This flow is conducted at the wholesale prices. The deliveries (also conducted at the wholesale prices) flow in the opposite direction. We express the effective wholesale prices by a set of constants n_{wj} $(w = 1, 2, ..., v; j = k+1, k+2, ..., n)$, which represent the participant's cost per one bank note of sales for a customer acting as a supplier.

[4] The term topological sorting is used in [4] to describe the ordering of indexes having this property.

The set of constants a_{wj} , b_{wj} and n_{wj} make it possible to uniquely determine the level of orders and deliveries in a given transaction. Indeed, the level of orders to the supplier w from the customer j is given by $h_{wj} = b_{wj} d_j n_{wj}$. The relation (see Section 2) determines the level of deliveries $g'_{wj} = min\{x_{wj}, h_{wj}\}$, where $x_{wj} = s_w a_{wj}$ are the capable sales values at cost prices. Considering the difference in revenue from sales of customer j acting as a supplier, we conclude that the deliveries from the supplier w to the customer j are given by $g_{wj} = g'_{wj} / n_{wj}$.

In conclusion of this section, let us consider one computational aspect of order and delivery vectors in an a-cyclic distribution network. [5] It is easily seen that the components d_j , s_w , h_{wj} and g_{wj} $(w = 1,2,...,v; j = k+1, k+2,...,n)$ as obtained from (1) and (2) are given by

$$d_w = \sum_j b_{wj} d_j n_{wj} \quad (j > w; w = 1,2,...,v) \tag{4}$$

$$s_j = \sum_w min\{s_w a_{wj}; b_{wj} d_j n_{wj}\} / n_{wj} \quad (w < j; j = k+1,...,n) \tag{5}$$

The starting data in (4) is the demand of the distributors to external customers, i.e., the numbers $d_{v+1}, d_{v+2},...,d_n$. The starting data in (5) are the capable sales levels $s_1, s_2,...,s_k$ of the produces, which together with the numbers $d_1, d_2,...,d_v$ from (4) are used in (5) to compute the actual sales of the customers.

4. A monotonic game of customers in the distribution network

In the previous section we considered an a-cyclic distribution network with participants indexed by $w = 1,2,...,v; j = k+1, k+2,...,n$. The index j identifiers a customer; the index w identifiers a supplier.

Let us interpret the activity of the network as a monotone game [1] , in which the customers need to decide from what supplier to order a particular commodity.

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[5] Here we need only consider the principles of the computational procedure.

Suppose that in addition to the cost of materials, the customers bear uncertain overhead costs in their transactions with the suppliers. Because of the uncertainty of overheads, it is quite possible that in some transactions the overheads will exceed the gross profit from sales. In this case, the potentially feasible transactions will not take place.

Let the set R_j represents all the potential transactions corresponding to the set of suppliers from which the customer j is to make his choice. The choice of the customer j $(j = k+1, k+2, ..., n)$ is a subset A^j of the set R_j : $A^j \subseteq R_j$; the case $A^q = Æ$ is not excluded: it requires the customer's refusal to choose. The collection $\langle A^{k+1}, A^{k+2}, ..., A^n \rangle$ represents the customer's joint choice. It is readily seen that the sets R_j are finite and nonintersecting; their union corresponds to set W : $W = R_{k+1} \cup R_{k+1} \cup ... \cup R_n$.

In what follows, we focus on the criterion by which the customer j chooses his suppliers A^j . In distinction from the standard monotone game [1] , which is based on a coalition of players, we will consider the strategy of individual customers whose objective is to maximize the profit from the actual sales revenues. We will thus essentially deal with a coalition-less *m* players game, $m = n - k$.

Let us first introduce a measure of the utility of a transaction between customer j and supplier $w \in A^j$ $(j = k+1, k+2, ..., n)$. The utility of a transaction between customer j and supplier w is expressed by the corresponding profit $p_{wj} = g_{wj} \cdot (1 - n_{wj})$.

The utility of a transaction with a supplier $w \in A^j$ is a function $p_{wj}(X_{k+1}, X_{k+2}, ..., X_n)$ of many variables: the value of the variable X_j is the choice A^j of the customer j , the number of variables is $m = n - k$. To establish this fact, it is sufficient to show how to compute the components of the order and delivery vectors from the joint choice $\langle X_{k+1}, X_{k+2}, ..., X_n \rangle$. Indeed, according to our description, an a-cyclic distribution network requires defining the constants $a_{wj} \geq 0$ and $b_{wj} \geq 0$ $(w = 1, 2, ..., v; j = k+1, ..., n)$ that satisfy the constrains (3) . A pair of constants a_{wj} and b_{wj} can be assigned in a one-to-one correspondence to a supplier $w \in R_j$, rewriting (3) in the form

$$\sum_{w \in R_j} a_{wj} \le 1 \ (w = 1,2,...,v) \ , \ \sum_{w \in R_j} b_{wj} \le 1 \ (j = k+1,...,n) \qquad (6)$$

If the constrains (6) are satisfied, then the same constrains are of necessity satisfied on the subsets A^j of the set R_j . Thus, if we restrict (4) and (5) to the sets $X_j \subseteq R_j$, the numbers g_{wj} can be uniquely calculated for every joint choice $\langle X_{k+1}, X_{k+2}, ..., X_n \rangle$.

Finally, let us define the individual utility criterion of the customer j in the form

$$P_j = \sum_{w \in A^j} (p_{wj} - u_{wj}) \ , \qquad (7)$$

where u_{wj} are the customer's overhead expenses allocable to his transaction with the supplier $w \in A^j$; we define $P_j = 0$ if the customer refused to choose – $A^j = Æ$.

The function $p_{wj}(X_{k+1}, X_{k+2}, ..., X_n)$ has the obvious property of monotone utility, so that for every pair of joint choices of customers $\langle L^{k+1}, L^{k+2}, ..., L^n \rangle$ and $\langle G^{k+1}, G^{k+2}, ..., G^n \rangle$ such that $L^j \subseteq G^j \ (j = k+1,...,n)$ we have

$$p_{wj}(L^{k+1}, L^{k+2}, ..., L^n) \le p_{wj}(G^{k+1}, G^{k+2}, ..., G^n) \ . \qquad (8)$$

The property of monotone utility leads to certain conclusions concerning the behavior of customers depending on the individual utility criterion. Under certain conditions, rational behavior of customer j (i.e., maximization of the profit P_j) is equivalent to loss avoidance in every individual transaction with the supplier $w \in A^j$. This aspect is not made explicit in [1] , although it is quite obvious. Thus, using the lemma in [1] , we can easily show that if the utilities $p_{wj}(..., X_j, ...)$ are independent of the choice X_j , the customer j maximizes his profit P_j by extending his choice to the set-theoretically largest choice. In what follows we will show that this result also applies under a weaker assumption.

First, a few reservations about the proposed condition – see (9) below. This condition has a simple economic meaning.: the customer j entering into loading transactions cannot achieve a net increase in his utility of the losses. For example, if for fixed choices of all other customers in the network, the utilities $p_{wj}(...,X_j,...)$ for $w \in X_j$ are independent of the choice X_j , the condition (9) hold as a strict inequalities. These conditions are also reduces to strict inequalities when, for instance, the capable sales x_{wj} in each transaction between customer j and supplier $w \in A^j$ is not less than the demand h_{wj} so that every customer can receive the entire quantity ordered from his suppliers. In particular, by increasing the producers supply $s_1,s_2,...,s_k$ with unlimited manufacturing capacity, we can always increase the capable sales to such an extent that it exceeds the demand, so that the conditions (9) are satisfied.

We can now formulate the final conclusion: the following lemma suggests that each customer will make his choice so as to maximize the profit P_j , providing all the other customers keep their choices fixed .[6]

Let the suppliers not entering the set A_j be assigned indexes $q=1,2,...$. Then the profit P_j of customer j is represented by a many-variable function $P_j(t_{1j},t_{2j},...)$ with variables t_{qj} varying on $[0,b_{qj}]$.[7] The value of the function $P_j(t_{1j},t_{2j},...)$ is the customer's profit for the case when the customer has extended the choice by placing orders in the amounts of $t_{qj} d_j n_{qj}$ with the suppliers $q=1,2,...$ outside the choice A_j . The set of variables t_{qj} identifiers the suppliers $q=1,2,...$ augmenting the customers choice A_j . If all $t_{qj}=0$, the choice A_j is not augmented and the profit $P_j(0,0,...)$ coincides with (7) .

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6 The joint choice of users having this property is generally interpreted in the sense of Nash equilibrium [2] .

7 We recall that b_{qj} is the fractional cost of all the orders placed with supplier q .

The profit function $P_j(t_{1j}, t_{2j}, ...)$ thus has to satisfy the following constraint: for every t_{qj} in $[0, b_{qj}]$ $q = 1, 2, ...$

$$P_j(t_{1j}, t_{2j}, ...) \leq P_j(0, 0, ...) \quad (9)$$

Definition. *A joint choice* $\langle A_0^{k+1}, ..., A_o^n \rangle$ *of the network customers is said to be rational with the threshold* u^o *if, given a level of overhead expenses not less than* $u^o > 0$ *, the utility measure* $p_{wj} \geq u^o$ *in every transaction of customer* j *with the supplier* $w \in A_o^j$ $(j = k + 1, ..., n)$ *.*

Lemma. *The set-theoretically largest choice* $S^o = \langle A_o^{k+1}, ..., A_o^n \rangle$ *among all the joint choices rational with threshold* $u^o > 0$ *ensures that the a-cyclic distribution network is in equilibrium relative to the individual profit criterion* P_j *under the following conditions: a) the overhead expenses* u_{wj} for $w \in S^o$ *do not exceed* $min\, p_{wj}$ *over* $w \in S^o \cap R_j$ *; b) inequality (9) holds.*

The proof is given in the appendix.

In conclusion, we would like to consider yet another point. With uncertain overhead expenses, the refusal to enter into any transaction may lead to an undesirable “snowballing” of refusals by customers to choose their suppliers, see [1] . It therefore seems that customers will attempt at least to conclude transactions with $p_{wj} \geq u^o$, even when there is some risk that the overhead expenses will exceed the utility p_{wj} . Thus, without exaggeration, we may apparently state that the size of the interval $[u^o, min\, p_{wj}]$ reflects the elasticity of the customer’s choice: the number $min\, p_{wj} - u^o$ is thus a measure of a “risk” that the customer will get into non-equilibrium situation. Clearly, a customer with a small interval will have grater difficulties to maintain the equilibrium than a customer with a wide interval.

APPENDIX

Proof of the Lemma. Let S^o be a set-theoretically largest choice among all the joint choices rational with the threshold u^o , i.e., S^o is the largest choice H among all the choices such that $p_{wj}(H \cap R_{k+1},...,H \cap R_n) \geq u^o$.

Suppose that some customer p achieves a profit higher than P_p by making the choice $A^p \subseteq R_p$ which is different from $S^o \cap R_p$; $P'_p = \sum_{w \in A^p} (p_{wp}(...,A^p,...) - u_{wp}) > P_p$, subject to $u^o \leq u_{wp} \leq min_{w \in A^p} p_{wp}$.

Clearly, the choice A^p is not a subset of S^o , since this would contradict the monotone property (8) , so that $A^p \setminus S^o \neq Æ$. By the same monotone property, the customer making the choice $A^p \cup (S^o \cap R_p)$ will achieve a profit not less than P'_p . On the other hand, all transactions in $A^p \setminus S^o$ are losing transactions for this customer, since S^o is the set-theoretically largest set of non-losing transactions for an overhead threshold $u^o > 0$. For the customer p making the choice $A^p \cup (S^o \cap R_p)$ the profit P'_p does not decrease only if the total increase in utility due to the contribution p_{wp} of the transactions $w \in S^o \cap R_p$ exceeds the total negative utility due to the transactions in $A^p \setminus S^o$. Clearly, because of the constraint (9) , the customer p has no such opportunity. This contradiction establishes the truth of the lemma. ■

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