

Optimal two stage committee voting rules¹

Ian Ayres² Colin Rowat³ Nasser Zakariya⁴

November 11, 2004

¹The authors thank Toke Aidt, Steve Bellenot, Aaron Benjamin, Mayling Birney, Gautam Chinta, Jayasri Dutta, Herakles Polemarchakis, Indra Ray, Thomas Richard, Demetri Sevastopulo, Philippa Townsend, Ben Zissimos, and seminar participants at Brown University, ESEM 2004, the University of Birmingham, the University of Warwick. Alejandro Caceres' and Serge Uogintas' generous assistance with the Maple code is particularly appreciated.

²Yale Law School

³Department of Economics, University of Birmingham

⁴FAS, Harvard University

Abstract

We study option management by committee. Analysis is illustrated by tenure decisions. Our innovations are two-fold: we treat the committee's problem as one of social choice, not of information aggregation; and we endogenise the outside option: rejecting a candidate at either the probationary or tenure stage return the committee to a candidate pool. For committees with N members, we find three key results: (1) a candidate's fate depends only on the behaviour of two 'weather-vane' committee members - generalised median voters; (2) enthusiastic assessments by one of these weather-vanes may harm a candidate's chances by increasing others' thresholds for hiring him; and (3) sunk time costs may lead voters who opposed hiring a candidate to favour tenuring him, even after a poor probationary performance. We also characterise the optimal voting rule when $N = 2$. A patient or perceptive committee does best with a (weak) majority at the hiring stage and unanimity at the tenure stage. An impatient or imperceptive committee does best under a double (weak) majority rule. If particularly impatient or imperceptive, this rule implies that any hire is automatically tenured. Perversely, the performance of a patient, imperceptive committee improves as its perceptiveness further declines.

Key words: intertemporal strategic voting, real options, social choice, heterogenous priors, tenure

JEL classification numbers: C73, D71, D72, D80, G12

This file: 041108twostage.tex

1 Introduction

Many decisions involve more than one stage of decision making. When a committee is responsible for making such decisions, it is natural to ask what voting rules it should follow. This paper does so in a real options setting: the first decision purchases the right to make a later, final decision. Rejection at either stage presents the committee with a new option to consider. As the committee will then re-apply its voting rule, the value of the outside option is endogenous. When committees are comprised of two members, we determine the voting rule that maximises the value of this option.

For expositional purposes, the real option that we consider is drawn from labour economics: the right to offer an employee a permanent position. Tenure in academia and ‘up or out’ rules in law, accountancy and the military are all examples of employment decisions that fall within the scope of our analysis (Meyer, 1992).¹

Within finance or corporate governance, portfolios of both real and financial options may be managed by teams. In political economics, bills in parliamentary democracies must survive votes after their second and third readings before being enacted into law; bicameral systems often require that bills be passed by both houses in sequence; international treaties tend to be signed by States and then ratified. In law and economics, judicial decisions motivated Condorcet’s interest in committee decisions. The possibility of appeal and penalty phases introduces a second decision making stage in both cases.² In the economics of the family, a period of engagement often precedes marriage which, if unsuccessful, typically ends candidacy.

While all of these situations differ in details, we believe that our model captures the central feature of all of them, namely the management of an option by committee.

A shortlist consisting of a single candidate is considered for probationary hiring by a committee. Its members form priors over the likelihood that the

¹McPherson and Schapiro (1999) provides a recent introduction to the tenure literature, which includes Chatterjee and Marshall (2001); Chen and Ferris (1999); Ehrenberg, Pieper, and Willis (1998); Carmichael (1988) and Ito and Kahn (1986). Another academic example is the practice of refereeing articles, which are often accepted for publication only after surviving an initial round of refereeing.

²The most notorious penalty phase decision dates to Socrates’ trial. Athenian juries first voted on the defendant’s innocence or guilt; a guilty finding then led to a second jury vote over the proposed penalties. Thus, Socrates was found guilty by 280 jurists, but 360 voted for his execution.

candidate will be a good hire; these priors become common knowledge. Thus, in contrast to much of the committee literature, our committee’s problem is one of social choice, not information aggregation. We are agnostic as to whether the agents have private values or merely different perceptions of the common good.³ We also take no position on the information used in forming these priors: committee members may or may not have access to private information; they may or may not engage in deliberation.⁴

Having formed priors, committee members vote either in favour of or against hiring the candidate.⁵ If hired, the candidate emits a publicly observed performance signal. All members use this signal to update their beliefs about the candidate. They then vote either in favour of or against granting the candidate tenure.

As the signal is commonly known, committee members at the hiring stage know that their relative assessments of a candidate will remain unchanged if the candidate survives to the tenure stage. If the candidate is rejected at either the hiring or tenure stages, the committee returns to the pool of candidates to begin the process anew. Once back in the pool, each committee member expects to receive V , the value of the expected game.

Framing the committee’s problem recursively is the main technical contribution of this paper: the value of the outside option, V , is determined endogenously, as a function of the voting rule. This structure embeds a fixed point problem in the analysis: the value of the tenure process depends on committee members’ voting strategies. Those voting strategies, in turn, depend on the value that committee members ascribe to rejecting a candidate and returning to the market.

We first analyse agents’ optimal voting strategies for general committees of size N taking V as given. Then, to solve the fixed point problem and assess optimal voting rules, we specialise to the cases of $N = 1$ (to develop intuitions) and $N = 2$. Technically, each additional committee member adds a dimension of integration to the problem. While our methods apply to larger N , we as yet can only apply them on a case-by-case basis rather than for general N .

³On perceived common interests, Feddersen and Pesendorfer (1997, fn. 3) discuss ‘competence’ preferences.

⁴See Gerardi and Yariv (2003); Austen-Smith and Feddersen (2002) and Li, Rosen, and Suen (2001) on debate, deliberation and strategic reporting of one’s signals.

⁵See Ghosal and Lockwood (2003) and the references therein for an analysis of abstention.

We conduct our initial analysis in the context of a general committee of N members, instead of specialising to $N = 2$, for three reasons. First, they provide results for two stage committee problems when the value of the outside option, V , is exogenous - novel and interesting in their own right. Second, the generality facilitates extensions of the optimality analysis to larger committees. Finally, there are almost no costs of this extra generality.

For general committees, we find rich intertemporal patterns of strategic voting. Some of these are specifically intrapersonal. For example, sunk time costs can create situations in which a committee member who opposes hiring a candidate goes on to support tenure in spite of a bad probationary performance.

More of the patterns, however, are interpersonal. Once a voting rule is fixed, a tenure ‘weather-vane’ voter - the member who always votes with the winning side in the tenure vote - may be identified. Other committee members then condition their votes at the hiring stage on two factors: their own priors about a candidate, and the tenure weather-vane’s, the former influencing the payoffs assigned to particular outcomes, the latter influencing the probability of those outcomes.

As an immediate consequence, committee members vote against hiring a candidate that they personally favour if the tenure weather-vane is sufficiently opposed to ensure his defeat at the tenure stage. At the other extreme, a sufficiently enthusiastic assessment by a tenure weather-vane under some conditions will cause other committee members to apply a higher standard to the initial hiring of a candidate. We term this effect ‘inverse enthusiasm’. Otherwise, enthusiasm by the tenure weather-vane lowers other committee members’ thresholds. Insight into the conditions governing these opposing effects is gained by explicit appeal to an options interpretation.

Analogously to the tenure weather-vane, a probationary weather-vane may also be identified. Thus, a sufficient statistic for a candidate’s fate is the priors of these two committee members. Increases in the mean priors about a candidate may harm him; increases in the variance of those priors may help him.

Finally, optimal voting rules for two person committees are derived.

If the committee is either patient or perceptive, the optimal decision rule is a (weak) majority requirement at the hiring stage, rising to unanimity at the tenure stage. We refer to this as a ‘rising threshold’ rule. This seems consistent with the practice in many departments: whatever the formal rule, one wants to go to the dean with unanimity. It is also intuitively appealing:

the committee hires readily as this entails either little cost or gains good information on the candidate. The decision to grant tenure, though, is taken more seriously.

Peculiarly, the performance of patient but imperceptive committees improves - albeit only slightly - as the committee becomes less perceptive. This owes to the tendency of the more optimistic committee member to 'take a chance' on a candidate under the rising threshold rule. As the informativeness of the probationary signal declines, the optimist loses the ability to believe that the pessimist will be swayed by a good probationary performance. This leads the optimist to take fewer such chances.

The rising threshold result has the flavour of that in Meyer (1991, Section 7), in which it may be optimal to bias against the winner of an earlier contest. Here, discounting and aggregation via the committee work in opposite directions: an individual committee member lowers her standards, perhaps approving of a candidate once the time costs are sunk and he has become eligible; the committee, as a whole, however, has a higher standard for the second vote.

If, instead, the committee is both impatient and imperceptive, requiring a single vote to pass at each stage is the optimal decision rule. This rule, less demanding than the rising threshold, helps the committee by tenuring candidates more rapidly than does the rising threshold. When the committee is particularly impatient or imperceptive, this rule never yields genuine probationary periods: if hired, a candidate is certainly granted tenure, removing a further impediment to rapidly tenuring a candidate. In these cases, the committee's problem is endogenously reduced to a one stage problem.

Two voting rules - double unanimity and falling thresholds (unanimity followed by weak majority) - are dominated for all parameter values. As committee members do not pool their information, the former is dominated for reasons other than those developed in Feddersen and Pesendorfer (1998). Their jurists, who have no private interests, may convict more innocent defendants when required to reach unanimity than under majority votes: a pivotal jurist who initially believes in a defendant's innocence will allow guilty votes from the remaining jurists to overrule her private signal. Under weaker rules, the jurist does not have this incentive to vote against her private signal.

For reviews of the literature on jury decisions and information aggregation, we recommend Persico (2004) and Ghosal and Lockwood (2003). The social choice literature is also large. Much of its concern with decision rules has, however, been positively motivated. Caplin and Nalebuff (1988), for

example, show that the Condorcet paradox can always be overcome with a super-majority rule of 64% when individual preferences and their distribution are appropriately restricted. We avoid problems of cycles by restricting committee members' choice sets to two elements, allowing us to concentrate on normative questions.

To our knowledge, ours is the first recursive analysis of a committee decision problem. We are also unaware of analyses of two stage committee decisions.⁶ The question of sequential voting, in which each agent votes once, but chooses its timing, has been addressed by Dekel and Piccione (2000). Morton and Williams (1999) investigate this in the context of US presidential primary campaigns, presenting laboratory evidence. Polborn (2000) analyses an environment in which voters decide on tax reform voting rules when young (and poor); after a stochastic shock, when they are older (and wealthier), they vote on tax reform. He shows that median voters will support supermajority rules.

Section 2 presents the model more formally. Section 3 then analyses the committee members' decisions during their two decision stages, taking the voting rule and V as given. Section 4 solves the fixed point problem for V when $N = 1$; Section 5 addresses $N = 2$. Section 6 concludes and compares our model to a standard options model.

2 The model

Consider a pool of *ex ante* identical candidates.⁷ In the game's prehistory, one of them is placed on a shortlist and evaluated by a committee of N risk neutral members, indexed by i . For expositional clarity, assume the committee members to be female and the candidate male.

Shortlisting the candidate gives each committee member information about that candidate's type, in her subjective opinion: a $\tau = 1$ candidate is one that she thinks will be good for the department; a $\tau = 0$ candidate is one that she thinks will be bad for the department. The subjective probability that she assigns to the shortlisted candidate being of type $\tau = 1$ is $p_{0i} \sim [0, 1]$. Without loss of generality, let $p_{01} \geq \dots \geq p_{0N}$. Generically, these can be written as strict inequalities; we assume this in what follows.

⁶See Maug and Yilmaz (2002) for a recent article on two-class voting rules.

⁷This model differs from those that allow an agent to specify the order in which candidates are tested; q.v. Weitzman (1979).

We are agnostic at this point as to the distribution from which these signals are drawn and their independence. They may reflect deliberation, but that is unmodelled. We merely require that the expectations are common knowledge and that they be allowed to differ.

At time 0, the committee members vote on hiring. Voting is costless and mandatory: abstentions are not allowed. If strictly more than $\delta_0 N$ of committee members approve, the candidate is given probation.

If the hiring vote is successful, a public signal, $\theta \in \{0, 1\}$, is then received by all committee members over the probationary period. It is common knowledge that all committee members view the signal as accurately reflecting the candidate's type with probability σ : $P(\theta = 1|\tau = 1) = P(\theta = 0|\tau = 0) = \sigma$. Without loss of generality, $\sigma \in (\frac{1}{2}, 1]$: otherwise, the signal's complement could be used as the informative signal. Thus, the probability of a Type I error is $1 - \sigma$, the same as that of a Type II error.

Thus,

$$E_{0i}(\theta) = \sigma p_{0i} + (1 - \sigma)(1 - p_{0i}); \quad (1)$$

where E_{0i} are member i 's expectations at time 0.

After the signal's receipt, committee members calculate p_{Ti} , their posterior beliefs:

$$p_{Ti}(\theta, p_{0i}) = \frac{\iota(\theta) p_{0i}}{\iota(\theta) p_{0i} + [1 - \iota(\theta)](1 - p_{0i})}; \quad (2)$$

where

$$\iota(\theta) \equiv \sigma\theta + (1 - \sigma)(1 - \theta) \in [0, 1].$$

Thus $\iota(\theta) = P(\theta|\tau = 1)$.

A committee member's posterior belief that a candidate is of type $\tau = 1$ is therefore an increasing function of her prior belief:

$$\frac{\partial p_{Ti}}{\partial p_{0i}} = \frac{\iota(\theta)[1 - \iota(\theta)]}{[\iota(\theta) p_{0i} + [1 - \iota(\theta)](1 - p_{0i})]^2} > 0. \quad (3)$$

Thus, $p_{T1} \geq \dots \geq p_{TN}$.

At time $T > 1$, a tenure vote is taken. If strictly more than $\delta_T N$ committee members approve, the candidate is tenured.⁸ As this date cannot be brought forward, the real option is European rather than American.

⁸We develop the exposition for the fully general $T > 1$. However, the structure of the model is such that no advantages accrue to $T > 2$: no further information is gained about the candidate. Thus, the fixed point problem is addressed by setting $T = 2$.

If, at either decision stage, the candidate fails to obtain the necessary votes, he is dismissed and the department returns to the pool of candidates.⁹ In contemplating the consequences of doing so, committee members imagine that their new priors will be independently drawn from the uniform distribution over the unit interval.

To summarise:

Definition 1. Given N and T , a voting rule is a pair, $\delta = (\delta_0, \delta_T)$.

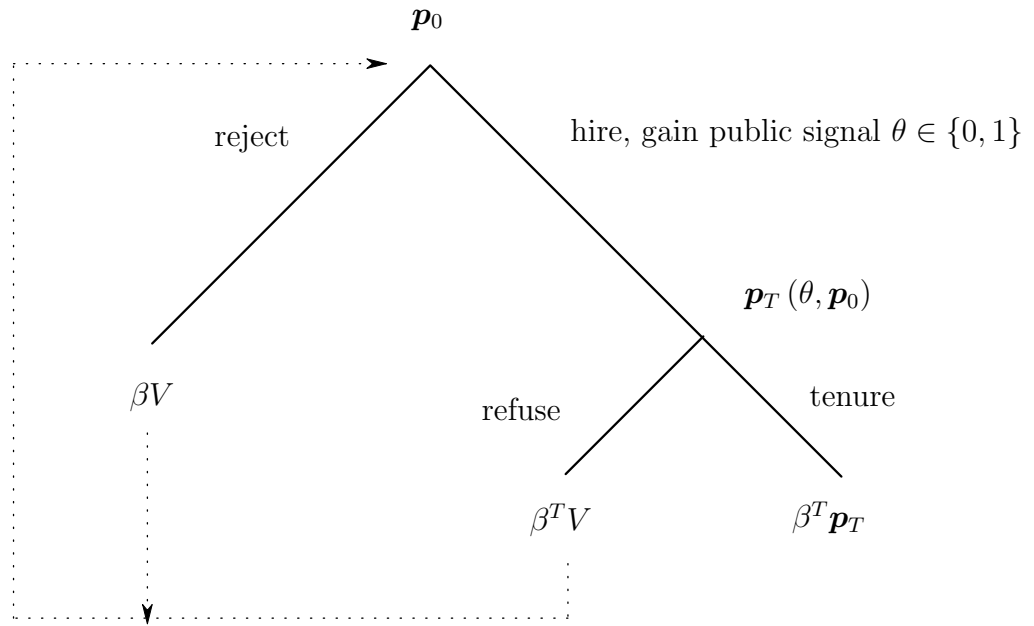


Figure 1: The game tree and its subjective expected payoffs

This pattern of play is summarised in Figure 1. Payoffs are associated with terminal nodes only: in options parlance, no dividends are paid. Payoffs are discounted to time 0. Thus:

⁹It is therefore assumed that the department cannot hire directly at the tenured level. Established and recognised lawyers are occasionally hired directly as lateral partners. Some departments, such as Yale's Comparative Literature department, only hire at the tenure level directly.

1. if a candidate is not hired, the department returns to the pool the following year. The value of restarting the hiring process is βV , where $\beta \in [0, 1]$ is the discount factor.
2. if a probationary candidate is denied tenure, the department immediately returns to the pool. It neither suffers costs nor gains benefits from having granted probation to a candidate.
3. finally, if a candidate is granted tenure, committee member i anticipates an expected payoff of $\beta^T p_{Ti}$: the actual payoff is equal to the candidate's type.

That the probationary period does not impose costs or deliver benefits in its own right may be interpreted as the probate's teaching load financing the probationary period. Granting tenure to a $\tau = 0$ candidate may be similarly interpreted. (As tenure does not convey salary guarantees, this may be thought of as adjusting a tenured, non-research productive faculty member's salary.) This structure may fit the legislative process more closely: a bill does not impose costs or benefits until it is enacted, but information about these is gained during its consideration.

Under some parameter values, the game tree in Figure 1 degenerates: some terminal nodes are dominated for committee members holding any priors. For example, were it possible that $\sigma = \frac{1}{2}$, refusing tenure to a candidate granted probation would always be dominated by not hiring the candidate. Similarly, $T = \infty$ would leave tenure refusal dominated by initial rejection. Finally, if $\beta = 1$, all candidates are hired: hiring allows no worse a payoff to be obtained than that possible by returning to the pool; a positive signal, however, convinces any committee member that the candidate is worth tenuring. In what follows, only generic cases are considered, unless otherwise specified.

Lemma 1. *When $\beta < 1$, $V(\cdot) \in [0, 1)$.*

Proof. Each committee member views the most successful hiring process as one that shortlists a $p_{0i} = 1$ candidate immediately, granting him tenure when he becomes eligible. Value, V , is bounded away from one as $\beta^T < 1$: discounting exacts some cost. As no payoffs accrue until a terminal node is reached, the least successful hiring process grants a $p_{0i} = 0$ candidate tenure at some point; this earns an outcome of zero. \square

The lemma's inequality on β eliminates the case in which a perfectly patient committee never grants a candidate tenure. This would produce the indeterminate $V = V$.

The value, V is a function of model parameters $\delta_0, \delta_T, \beta, N, \sigma$ and T . At this point, its parameters will be treated as fixed, and thus its value as constant. Fixed point arguments will later be used to compute V as a function of its parameters.

Now define strategies:

Definition 2. A strategy for committee member i is a pair of functions $v_{0i} : [0, 1]^N \rightarrow \{0, 1\}$ and $v_{Ti} : [0, 1]^N \rightarrow \{0, 1\}$.

Action 1 corresponds to a 'yes' vote, and action 0 to a 'no'. Assume that weakly dominated strategies are not played.¹⁰ At time T , either $v_{Ti} = 0$ or $v_{Ti} = 1$ is weakly dominated: when i is pivotal, she loses by voting against her beliefs; otherwise, her vote does not alter the outcome. This has three consequences. First, it reduces the relevant domain of v_{Ti} to the support of $p_{Ti}, [0, 1]$: committee members do not vote strategically at time T . Second, as dominated strategies cannot be present in the support of mixed strategies, the assumption allows concentration on pure strategies at time T . Finally, it ensures a unique equilibrium in the time T stage game (see Theorem 1), simplifying time 0 analysis. It will be seen later that this effectively makes the domain of v_{0i} two dimensional.

Therefore, the expected utility of member i , as assessed at time T and discounted to time 0, is:

$$u_{Ti}(\mathbf{v}_T(\mathbf{p}_T)) \equiv \left\{ \begin{array}{l} \beta^T p_{Ti} \text{ if } \sum_{j=1}^N v_{Tj}(\mathbf{p}_T) > \delta_T N \\ \beta^T V \text{ otherwise} \end{array} \right\}.$$

Her expected utility at time 0 is

$$u_{0i}(\mathbf{v}_T(\mathbf{p}_T), \mathbf{v}_0(\mathbf{p}_0)) \equiv \left\{ \begin{array}{l} u_{Ti}(\mathbf{v}_T(\mathbf{p}_T(\theta, \mathbf{p}_0))) \text{ if } \sum_{j=1}^N v_{0j}(\mathbf{p}_0) > \delta_0 N \\ \beta V \text{ otherwise} \end{array} \right\}.$$

Definition 3. A perfect Bayesian equilibrium is:

- $\mathbf{v}^* = \{v_{0i}^*, v_{Ti}^*\}_{i=1}^N$, a strategy profile; and

¹⁰This assumption imposes fewer informational requirements on agents, not requiring that they know each others' valuations. In this context, this is not a particular advantage as we have already assumed that priors are common knowledge.

- $\mathbf{p}_T(\mathbf{p}_0, \theta)$, a profile of posterior beliefs;

such that

- $v_{Ti}^*(\mathbf{p}_T) \in \arg \max_{\{0,1\}} \left\{ P \left(\sum_{j=1}^N v_{Tj}(\mathbf{p}_T) > \delta_T N \right) \beta^T p_{Ti} + \left[1 - P \left(\sum_{j=1}^N v_{Tj}(\mathbf{p}_T) > \delta_T N \right) \right] \beta^T V \right\};$
- $v_{0i}^*(\mathbf{p}_0) \in \arg \max_{\{0,1\}} \left\{ P \left(\sum_{j=1}^N v_{0j}(\mathbf{p}_0) > \delta_0 N \right) u_{Ti}(\mathbf{v}_T(\mathbf{p}_T(\theta, \mathbf{p}_0))) + \left[1 - P \left(\sum_{j=1}^N v_{0j}(\mathbf{p}_0) > \delta_0 N \right) \right] \beta V \right\};$

and

- $p_{Ti}(\mathbf{p}_0, \theta)$ is defined by equation 2;

$\forall i = 1, \dots, N$.

3 The meetings

3.1 The tenure committee meeting

Perfection allows v_{Ti} to be considered independently of v_{0i} :

Theorem 1.

$$v_{Ti}^* = \left\{ \begin{array}{l} 1 \text{ if } p_{Ti} \geq V \\ 0 \text{ otherwise} \end{array} \right\} \forall i \in N.$$

Proof. When committee member i is pivotal, she chooses the terminal payoff that she judges higher. When she is not, her choice is without consequence. Voting against her preferred terminal payoff is weakly dominated. \square

Therefore:

Definition 4. Member k is a weather-vane voter at time T for beliefs \mathbf{p}_T and voting rule δ_T when

$$k = \left\{ \begin{array}{l} \lceil \delta_T N \rceil \text{ if } \delta_T N \text{ is not an integer;} \\ \lceil \delta_T N \rceil + 1 \text{ otherwise;} \end{array} \right. \quad (4)$$

where $\lceil \cdot \rceil$ is the least integer function.

Generically, the weather-vane is unique.

A weather-vane voter is therefore an δ_T -percentile voter, a generalised median voter. If $\delta_T N$ is not an integer, then $\lceil \delta_T N \rceil > \delta_T N$ and there are enough votes for passage. If $\delta_T N$ is an integer, then $\lceil \delta_T N \rceil = \delta_T N$ and, since we use strict inequality, this is not enough for passage; $\delta_T N + 1$ will be enough.

Thus, the committee's decision coincides with the weather-vane's vote. A weather-vane differs from a dictator in that the aggregation rule does not privilege her *ex ante* or independently of the beliefs of others. To see how a weather-vane and a pivotal voter differ, consider:

Example 1. *Suppose that $N = 5$, $\delta_T = \frac{1}{2}$. Therefore, $k = 3$. Suppose further that the priors are such that even committee member 5, the most skeptical, is willing to vote for tenure. In this case, k is not pivotal: withholding her vote would leave a majority in place. Now suppose, instead, that the priors are such that members 4 and 5 will vote against tenure. In this case, the remaining members are all pivotal.*

In what follows, the weather-vane voter at time T is denoted by k .

3.2 The hiring committee meeting

Now consider behaviour at the initial meeting, that of the hiring committee. Analogously to the tenure committee meeting, each committee member compares her beliefs to threshold levels, functions of V . We derive and present the four relevant threshold levels. The first two divide the tenure weather-vane's priors into three regions: one in which she is already convinced that the candidate should not be tenured, even if he performs well during probation; one in which she is convinced that he should be tenured, even if he performs badly; and an intermediate region in which she will allow his probationary performance to determine her vote. In this latter two regions, thresholds are then established for the generic committee member's priors, below which she opposes hiring and, above which, she favours it.

Define

$$\bar{p} \equiv \frac{\sigma V}{\sigma V + (1 - \sigma)(1 - V)}; \quad (5)$$

$$\underline{p} \equiv \frac{(1 - \sigma)V}{(1 - \sigma)V + \sigma(1 - V)}. \quad (6)$$

Thus, \bar{p} (*resp.* \underline{p}) leaves committee member i indifferent between returning to the candidate pool next year and granting the existing candidate tenure if he produces a bad (*resp.* good) performance signal. If $p_{0i} \geq \bar{p}$ (*resp.* $p_{0i} \leq \underline{p}$) then committee member i will vote for (*resp.* against) tenure even when the candidate emits the bad (*resp.* good) signal during probation:

Lemma 2.

$$\begin{aligned} p_{0i} \leq \underline{p} &\Leftrightarrow p_{Ti}(1, p_{0i}) \leq V; \\ p_{0i} \geq \bar{p} &\Leftrightarrow p_{Ti}(0, p_{0i}) \geq V. \end{aligned}$$

Proof. By equation 2 and definitions 5 and 6,

$$p_{Tk}(1, \underline{p}) = p_{Tk}(0, \bar{p}) = V. \quad (7)$$

The result follows from the monotonicity of p_{Ti} in p_{0i} , shown in equation 3. \square

As we would expect, a committee member must have a higher prior if she is to support tenure for a candidate who has performed badly than she must have for one who has performed well:

Lemma 3. $1 > \bar{p} > \underline{p}$.

Proof. By equation 3, p_{Ti} is strictly increasing in p_{0i} . Thus, $p_{Ti}(1, p_{0i})$ and $p_{Ti}(0, p_{0i})$ have well defined inverse functions; denote them by h_1 and h_0 respectively. Thus, by equation 7,

$$(V) \equiv \underline{p}, h_0(V) \equiv \bar{p}.$$

Treating the first argument of p_{Ti} as continuous allows:

$$\frac{\partial p_{Ti}}{\partial \theta} = \frac{(1 - p_{0i}) p_{0i} \iota'}{[\iota p_{0i} + (1 - \iota)(1 - p_0)]^2};$$

where $\iota' = 2\sigma - 1$,

$$\sigma > \frac{1}{2} \Rightarrow p_{Ti}(1, p_{0i}) > p_{Ti}(0, p_{0i}) \forall p_{0i} \in (0, 1).$$

Thus, if $p_{Ti}(1, p) = p_{Ti}(0, q)$, then $q > p$. Hence, $h_0(V) > h_1(V)$.

To complete the proof, note that $p_{Ti}(0, \cdot)$ maps the interval $[0, 1]$ to itself. As this must also then hold true for its inverse function, h_0 , $\bar{p} = h_0(V) \leq 1$. \square

Corollary 1. *Given weather-vane k ,*

1. $p_{0k} \geq \bar{p} \Leftrightarrow E_{0i}(v_{Tk}^*) = 1$;
2. $p_{0k} < \underline{p} \Leftrightarrow E_{0i}(v_{Tk}^*) = 0$; and
3. $p_{0k} \in [\underline{p}, \bar{p}) \Rightarrow E_{0i}(v_{Tk}^*) = \sigma p_{0i} + (1 - \sigma)(1 - p_{0i})$.

Proof. The first two cases follow directly from Lemmata 2 and 3. When p_{0k} is in the range specified, the third is agent k 's expectation that $\theta = 1$ will be observed. By Theorem 1, this is equivalent to $v_{Tk} = 1$. \square

The ability to predict k 's behaviour allows committee members also to predict whether, when hiring a candidate, he is facing a true probationary period or not:

Definition 5. *If a candidate is hired and $p_{0k} \in [\underline{p}, \bar{p})$ then the candidate is a probationary hire.*

Definition 6. *If a candidate is hired and $p_{0k} \geq \bar{p}$ then the candidate is a permanent hire.*

For reasons that will later be apparent, we ignore the possibility that the candidate will be hired when $p_{0k} < \underline{p}$.

Now define

$$\tilde{p} = \beta^{1-T}V;$$

the initial belief that leaves i indifferent between returning to the pool and hiring the candidate when she knows that the weather-vane will support the candidate's tenure ($p_{0k} \geq \bar{p}$) regardless of his performance. Thus, \tilde{p} is independent of σ . The definition is derived from

$$E_{0i}(\beta^T p_{Ti}(\theta, \tilde{p})) = \beta V.$$

As the committee members are rational and \mathbf{p}_0 is common knowledge, $E_{0i}(\cdot) = E_0(\cdot) \forall i \in N$.

Now define the expected payoff to hiring a candidate if $p_{0k} \in [\underline{p}, \bar{p})$, so that the weather-vane only votes for tenure if $\theta = 1$:

$$\begin{aligned} f(p_0) &\equiv P(\theta = 1) \beta^T p_T(1, p_0) + P(\theta = 0) \beta^T V \\ &= \{[\sigma p_0 + (1 - \sigma)(1 - p_0)] p_T(1, p_0) + [(1 - \sigma)p_0 + \sigma(1 - p_0)] V\} \beta^T \\ &= \{\sigma p_0 + [(1 - \sigma)p_0 + \sigma(1 - p_0)] V\} \beta^T. \end{aligned} \tag{8}$$

When $p_0 \notin [\underline{p}, \bar{p})$, $f(p_0)$ does not have the meaning indicated above.

Lemma 4. *The function $f(p_0)$ is strictly increasing over $(0, 1)$.*

Proof.

$$f' = [\sigma + (1 - 2\sigma)V] \beta^T;$$

so that $f' > 0 \Leftrightarrow V < \frac{\sigma}{2\sigma-1}$. Consider the function $\frac{x}{2x-1}$: at $x = \frac{1}{2}$, it is infinite; at $x = 1$, it equals 1; its derivative is $\frac{-1}{(2x-1)^2}$. As $\frac{\sigma}{2\sigma-1}$ monotonically decreases over $x \in (\frac{1}{2}, 1)$ to $1 \geq V$, the result follows. \square

Let \hat{p} be the initial belief that leaves i indifferent between returning to the pool and probationarily hiring the candidate. As the performance signal is now used, we expect \hat{p} to be a function of its accuracy:

Lemma 5. *When $p_{0k} \in [\underline{p}, \bar{p})$,*

$$\hat{p} \equiv \frac{\beta^{1-T} - \sigma}{(1 - \sigma)V + \sigma(1 - V)}.$$

Proof. The value of \hat{p} is found by solving $f(\hat{p}) = \beta V$. \square

Note that \hat{p} can be defined outside the interval $p_{0k} \in [\underline{p}, \bar{p})$ but will no longer have the indifference interpretation. The analysis of $\frac{x}{2x-1}$ in the proof of Lemma 4 shows that \hat{p} is finite.

We expect $\hat{p} \geq \underline{p}$ as there would otherwise be a range of priors in which members were willing to hire even if the candidate stands no chance of obtaining tenure:

Lemma 6. $\hat{p} \geq \underline{p}$.

Proof. Calculation shows that $f(\underline{p}) = \beta^T V \leq \beta V = f(\hat{p})$. By Lemma 4, this establishes the result. \square

The inequality reduces to an equality as $\beta \rightarrow 1$.

These results allow us to express the optimal behaviour of voter i as a function of p_{0k} . More specifically, voter i sets a threshold for each level of conviction held by the tenure weather-vane. Figure 2 outlines the results:

- I Member i knows that, if hired, the candidate will fail tenure. As this delays the department's ability to return to the pool, she opposes hiring. When $p_{0i} > \bar{p}$, she votes against a candidate, even though he has already convinced her that he should be granted tenure.

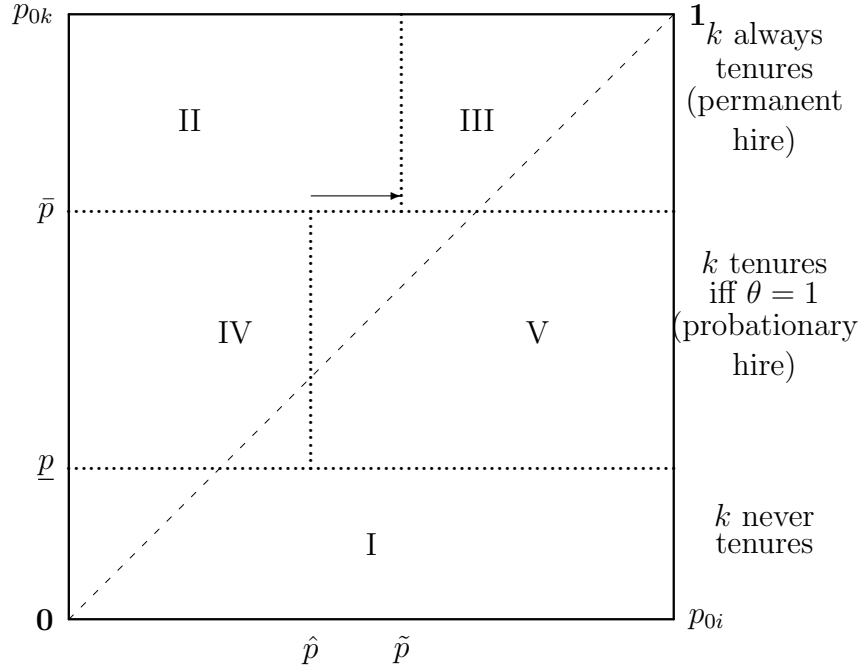


Figure 2: Voter i 's optimal behaviour at time 0 given weather-vane k at time T when $V < L$

- II Member i knows that, if hired, the candidate will gain tenure, regardless of his performance. Since she is sufficiently skeptical about the candidate, she opposes the hire. When $p_{0i} \in (\hat{p}, \tilde{p})$ she votes against the candidate even though she would vote for him were the tenure weather-vane less enthusiastic. We term this effect, indicated by the arrow, ‘inverse enthusiasm’.
- III Again, member i knows that the candidate will gain tenure if hired. Now, however, she is sufficiently confident in the candidate to take the chance of hiring him.
- IV Member i is skeptical of the candidate. She therefore votes against hiring the candidate, rather than allowing k the opportunity to resolve her uncertainty.

V Member i votes in favour of the candidate as she is willing to allow the weather-vane to resolve her uncertainty.

As yet, we have not determined the relative sizes of \tilde{p} , \hat{p} and \bar{p} . These depend, in part, on the following object:

$$L(\beta, \sigma, T) \equiv \frac{\sigma\beta^{T-1} - (1 - \sigma)}{(2\sigma - 1)}.$$

We defer a proper discussion of the interpretation of L until Section 6, confining ourselves to noting now that:

Lemma 7. $L \leq 1$

Proof.

$$\begin{aligned}\beta^{T-1} &\leq 1 \\ \sigma\beta^{T-1} &\leq \sigma \\ \sigma\beta^{T-1} - (1 - \sigma) &\leq 2\sigma - 1.\end{aligned}$$

The result then follows from $2\sigma > 1$. □

We now present two lemmata. The first provides a condition that determines whether i 's threshold for hiring is higher or lower when weather-vane k will grant tenure regardless of performance. The second shows that this same condition governs whether i or k applies a higher standard when k will grant tenure regardless of performance.

Lemma 8. $\hat{p} \leq \tilde{p} \Leftrightarrow V \leq L(\beta, \sigma, T)$.

Proof. By definition of \hat{p} and \tilde{p} , the lemma's first inequality may be expressed as

$$\frac{\beta^{1-T} - \sigma}{\sigma + (1 - 2\sigma)V}V \leq \beta^{1-T}V.$$

As, by Lemma 4, the left hand side of this is positive for $\sigma \in (\frac{1}{2}, 1]$, this may be manipulated for the result. □

Similarly, then:

Lemma 9. $\bar{p} \geq \tilde{p} \Leftrightarrow V \leq L(\beta, \sigma, T)$.

Thus,

$$V \leq L(\beta, \sigma, T) \Leftrightarrow \hat{p} \leq \tilde{p} \leq \bar{p}. \quad (9)$$

The sign of this inequality in V and L will be seen to divide the committee's parameter space into two regions.

Lemma 8 examines how the standard that committee member i applies depends on her perception of the weather-vane's support for a candidate. When $V \leq L$, an enthusiastic tenure weather-vane ($p_{0k} \geq \bar{p}$) discourages committee members from hiring a candidate ($\tilde{p} \geq \hat{p}$). When $V \geq L$, the opposite holds: an enthusiastic tenure weather-vane ($p_{0k} \geq \bar{p}$) now encourages committee members to hire a candidate ($\tilde{p} \leq \hat{p}$). The first, inverse enthusiasm, has already been introduced. Analogously, we term the second 'pro-enthusiasm'.

To understand these effects, think of $L(\beta, \sigma, T)$ as akin to the strike price of the option and V as related to the return to buying the market. This interpretation is more fully developed in Section 6.

Thus, when the strike price is high relative to the return to the market ($V \leq L$), i requires a higher standard to favour hiring if she knows that k will grant tenure ($p_{0k} \geq \bar{p}$): hiring exercises the option at the relatively high price. If, however, the weather-vane is not convinced, then hiring will only exercise the option in the good state of nature, a proposal more appealing to i .

On the other hand, when the return to the market is high relative to the strike price ($V \geq L$), i is more easily convinced to favour hiring if she knows that k will grant tenure ($p_{0k} \geq \bar{p}$): hiring exercises the option at the relatively low price. If k is not convinced, i is more easily convinced to return to the market, with its relatively high rate.

Thus, a committee member will demand a higher prior to support initial hiring if she wants to hire on a different basis (e.g. probationary or not) than does the tenure weather-vane voter.

Now consider Lemma 9. When $V \leq L$, the prior at which k is indifferent between probationary and permanent hire is larger than that which leaves i indifferent between rejection and permanent hire. Consider this in the special case in which i and k are the same committee member. That individual's priors lie along the dotted diagonal line in Figure 2.

Seen this way, $\bar{p} \geq \tilde{p}$ divides the member's priors into three zones: those in which she will not tenure at $t = T$, regardless of performance; probationary hiring; and permanent hiring. The relatively high strike price, L , thus seems

to attach value to retaining all three possible outcomes.

When, on the other hand, $V \geq L$, the probationary hiring zone disappears: the committee member opposes hiring unless she is convinced from the outset that the candidate should be granted tenure. This case presents an apparent paradox: the diagonal now passes through region II. In this region, $p_{0k} \in (\bar{p}, \tilde{p})$, the tenure weather-vane opposes hiring but will vote in favour of tenure even after a bad probationary performance. Her behaviour can be explained by the time costs that are sunk *ex post*, once the tenure meeting is reached.

When inequality 9 holds, a weaker version of this reasoning applies in region IV: $p_{0k} \in (\underline{p}, \hat{p})$ implies that the tenure weather-vane votes against hiring, although she would vote in favour of tenure if the candidate was hired and generated $\theta = 1$.

The lemmata above clearly give tenure weather-vanes an incentive to dissemble, abstain or delegate their later tenure votes to others. While clearly plausible phenomena, they are not allowed by the present model.

As every agent i compares her p_{0i} to the same cut points, voting ‘yes’ when above it and ‘no’ otherwise, a hiring weather-vane at $t = 0$ may be defined analogously to the tenure weather-vane at $t = T$:

Definition 7. *Member j is a weather-vane voter at time 0 for beliefs \mathbf{p}_0 and voting rule δ_0 when*

$$j = \begin{cases} \lceil \delta_0 N \rceil & \text{if } \delta_0 N \text{ is not an integer;} \\ \lceil \delta_0 N \rceil + 1 & \text{otherwise;} \end{cases} \quad (10)$$

Thus, the outcome of the hiring process is a function of the beliefs of weather-vanes j and k , as shown in Figure 3. The panel on the right refers to the case when the inequalities in equation 9 do not hold. Refer to this two dimensional space as weather-vane space.

In general, only half of the weather-vane space is accessible. Which half is depends on the voting rule adopted. When, for example, $j < k$, the tenure weather-vane will, by definition, have at least as high a level of conviction about a candidate as will the hiring weather-vane. In this case, the lower triangles are accessible. When $j = k$, this reduces to the special case of the diagonal line.

Candidates’ success therefore depends on the beliefs of no more than two committee members, who are determined on the basis of their ordinal beliefs and the voting rule. This ordinality means that a candidate’s success cannot

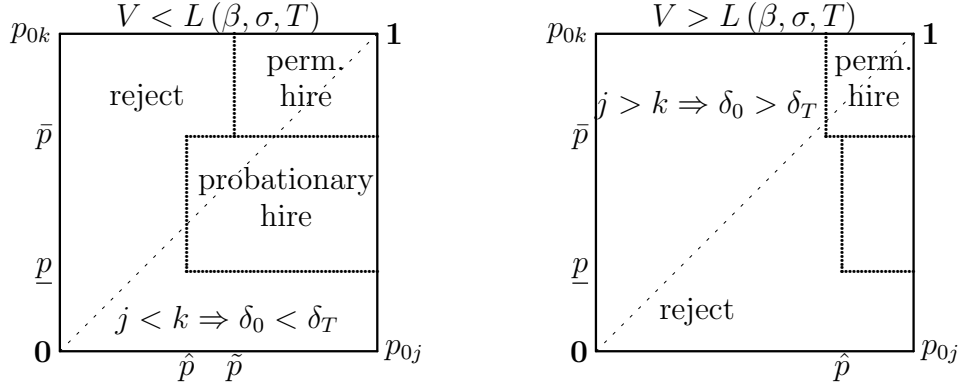


Figure 3: Outcomes given weather-vanes j and k

easily be expressed in terms of the usual moments of $\{p_{0i}\}$. Clearly, examples can be constructed in which higher mean priors help a candidate. We present examples of three less obvious effects.

First, higher mean priors may harm a candidate through the inverse enthusiasm effect mentioned above:

Example 2. Suppose $N = 5$, $\delta_0 = 1$, $\delta_T = \frac{1}{2}$ and that $V < L$. Then $j = 5$ and $k = 3$. Let $1 > p_{01} - 5\varepsilon = p_{02} - 4\varepsilon = p_{03} - 2\varepsilon = p_{04} - \varepsilon = p_{05} > \hat{p}$, where $\varepsilon > 0$, $p_{03} \in (\bar{p} - \varepsilon, \bar{p})$ and $p_{04} < \tilde{p}$. Thus, the candidate is hired unanimously on a probationary basis. Now leave the priors unchanged other than to add ε to p_{03} so that $p_{03} > \bar{p}$. Thus, the mean prior has increased; the variance has remained unchanged. The tenure weather-vane's increased enthusiasm means that hiring would be permanent. Thus, the two less convinced committee members prevent the hire.

Next consider an example in which increased variance hurts hiring possibilities:

Example 3. Suppose $N = 5$, $\delta_0 = \frac{1}{2}$, $\delta_T = \frac{2}{3}$ and that $V < L$. Then $j = 3$ and $k = 4$. Let $p_{01} = p_{02} = p_{03} = \hat{p} + \varepsilon < 1$ and $\underline{p} < p_{04} = p_{05} = \hat{p} - \varepsilon < \bar{p}$ where $\varepsilon > 0$. Hence the candidate is hired with an average prior of $\hat{p} + \frac{1}{5}\varepsilon$; the sample variance is $\frac{30}{25}\varepsilon^2$. Now increase the most optimistic beliefs to $p_{01} = p_{02} = \hat{p} + 2\varepsilon$, but reduce the hiring weather-vane's to $p_{03} = p_{04} = p_{05} = \hat{p} - \varepsilon$: the average prior remains unchanged, the variance more than doubles, and the candidate is not hired.

Finally consider the opposite:

Example 4. Consider the committee and voting rule of Example 3 and let $V < L$. Now let $p_{01} = p_{02} = \hat{p} + \varepsilon < 1$ and $\underline{p} < p_{03} = p_{04} = p_{05} = \hat{p} - \varepsilon < \bar{p}$, where again $\varepsilon > 0$. The priors' average is now $\hat{p} - \frac{1}{5}\varepsilon$; the variance is as in the initial case considered in Example 3, but now the candidate is not hired. Now let the hiring weather-vane join the optimists, so that $p_{03} = \hat{p} + \varepsilon$, but make the pessimists gloomier, so that $p_{04} = p_{05} = \hat{p} - 2\varepsilon$: the sample average prior remains the same, the variance again more than doubles, but the candidate is hired.

Thus, the role of variance can be reduced to how it influences the weather-vane: losing the hiring weather-vane loses the candidate the job; winning the hiring weather-vane wins the job. Insofar as the priors' variance reflects the beliefs of all committee members, it contains richer information than is necessary to analyse the candidate's survival.

Recasting analysis in terms of the two weather-vanes allows us to restate our discussion of Lemmata 8 and 9 in terms of them:

Lemma 10. When $V \geq L, j \geq k$ implies that there do not exist priors, \mathbf{p}_0 , that allow probationary hiring. When $V \geq L$ and $j \leq k$, and when $V \leq L$, such priors exist.

This follows from inspection of Figure 3. Thus, if a voting rule that sets the candidate a rising threshold over time ($j > k$) generates a high value, V , it does so by discarding the possibility of probationary hire. In other words, it does so by discarding the possibility of buying options on candidates. This suggests that V in this region should be independent of σ . For $j < k$ and $V > L(\beta, \sigma, T)$, on the other hand, the value function should depend on σ . These observations are confirmed in our explicit computations with $N = 1$ and $N = 2$, below. The phenomenon whereby a committee chooses not to use informative signals is further explored in our discussion of optimal voting rules (Section 5.5).

The initial discussion of degeneracy was based on the extensive form representation in Figure 1. Viewing Figures 2 and 3 as strategic form representations allows this discussion to be revisited. All of the conclusions presented initially can now be stated in terms of threshold beliefs, V and L .

Setting $\sigma = \frac{1}{2}$ implies $\underline{p} = \bar{p} = V > L = -\infty$. Thus, the possibility of probationary hiring disappears. Setting $T = \infty$ sets \hat{p} and \tilde{p} to infinity as well and $L < 0$. Thus, the committee always rejects candidates.

Finally, when $\beta = 1$ so does L , so that $L > V$. Thus, probationary hiring is retained. As $\sigma \rightarrow 1, \underline{p} \rightarrow 0, \bar{p} \rightarrow 1$ and $\hat{p} \rightarrow 0$. In the limit, all candidates are probationarily hired.

Up to this point, V has been treated as a positive parameter. In fact, V depends on the committee's voting behaviour. This presents a fixed point problem: the cut points $\hat{p}, \tilde{p}, \underline{p}$ and \bar{p} depend on V which, in turn, depends on the cut points. The next two sections address this problem. To build intuition, we first address this problem in the context of a single committee member.

4 A single committee member: $N = 1$

When $N = 1$, a generic expression for V may be constructed from the diagonal in Figure 3. Depending on the inequalities in equation 9, this contains either two or three relevant intervals. We consider the simpler case first.

When the inequalities in equation 9 do not hold, V is bounded below. Thus, as displayed in the right panel of Figure 3, there are only two relevant intervals. In the first, the candidate is not hired; in the second, he is hired on a permanent basis. Probationary hire is excluded.

We assume, for tractability's sake, that the committee member imagines her priors over the as yet unknown next candidate, $p_0 \in [0, 1]$, to be drawn from the uniform distribution. Therefore,

$$V = \int_0^{\bar{p}} \beta V dp_0 + \int_{\tilde{p}}^1 \beta^T p_0 dp_0 \Rightarrow V = \beta^{T-2} \left[1 \pm \sqrt{1 - \beta^2} \right] \geq L(\beta, \sigma, T). \quad (11)$$

One of these solutions may be eliminated by the following lemma, which applies more generally to $N > 1$:

Lemma 11. *When $\beta < 1, \tilde{p} > 1 \Rightarrow V = 0$.*

Proof. By Lemma 3, $\tilde{p} > 1 \Rightarrow \tilde{p} > \bar{p}$. This, by inequalities 9, in turn implies that $\hat{p} \geq \tilde{p} > 1$. Thus, the committee never hires a candidate. As it is impatient, it therefore expects no return. \square

Corollary 2. *$V > \beta^{T-1}$ is inadmissible.*

Proof. $V > \beta^{T-1} \Rightarrow \tilde{p} > 1$. By Lemma 11, this implies $V = 0 < \beta^{T-1}$, a contradiction. \square

The lemma thus eliminates the larger solution to equation 11, which sets $\tilde{p} = \frac{1}{\beta} \left[1 + \sqrt{1 - \beta^2} \right] \geq 1$. Thus, only the smaller one is admissible.

As predicted by Lemma 10, the solution is a function of β and T alone: when probationary hire is discarded, so is the value function's dependence on σ .

Now consider the case in which the inequalities in expression 9 hold, bounding V above. This involves three relevant intervals. First, over $p_0 \in [0, \hat{p})$, the candidate is not hired. The difference between \underline{p} and \hat{p} reflects that between *ex ante* and *ex post* calculations: having reached the tenure meeting, the member's impatience about the costs associated with probation are irrelevant; at the hiring meeting, she requires a higher level of confidence in a candidate.

Second, over $p_0 \in [\hat{p}, \bar{p})$, the candidate is hired probationarily. Finally, when $p_0 \in [\bar{p}, 1]$, the candidate is permanently hired. Thus,

$$V = \hat{p}\beta V + \int_{\hat{p}}^{\bar{p}} \{[(1 - \sigma)p_0 + \sigma(1 - p_0)]V + \sigma p_0\} \beta^T dp_0 + \int_{\bar{p}}^1 \beta^T p_0 dp_0 \leq L. \quad (12)$$

This expression yields an unwieldy polynomial in V , hindering analytical progress. Thus, projections of V onto the (β, σ) plane are derived computationally for $T = 2$.¹¹ We focus on the $T = 2$ case from now on.¹²

The results are combined with those from equation 11 and displayed in Figure 4. The rightmost contour corresponds to $V = \frac{9}{10}$.

The curve bisecting the panels represents the intersection of the V and L surfaces. Above it, $V < L$, producing equation 12; below it, $V > L$, producing equation 11 - the region without probationary hire. As β and σ increase, the costs of probationary hire decrease: mistakes are less costly, probationary periods more informative. Thus, the region in which the committee discards probationary hire shrinks.

The other statics are also appealing. Value strictly increases in β . It increases strictly in σ when equation 12 holds, but is otherwise insensitive to σ , as noted above. This may be restated to note that higher values are attainable when $V < L$ than when $V > L$: there is value in being able to

¹¹The Maple code used is available at www.economics.bham.ac.uk/rowat/research/ARZ-maple.zip. Corollary 2 is used to eliminate solutions setting $V > \beta$ in this and the $N = 2$ cases.

¹²The results for $T > 2$ are as expected: the contours become compressed toward $\beta = 1$.

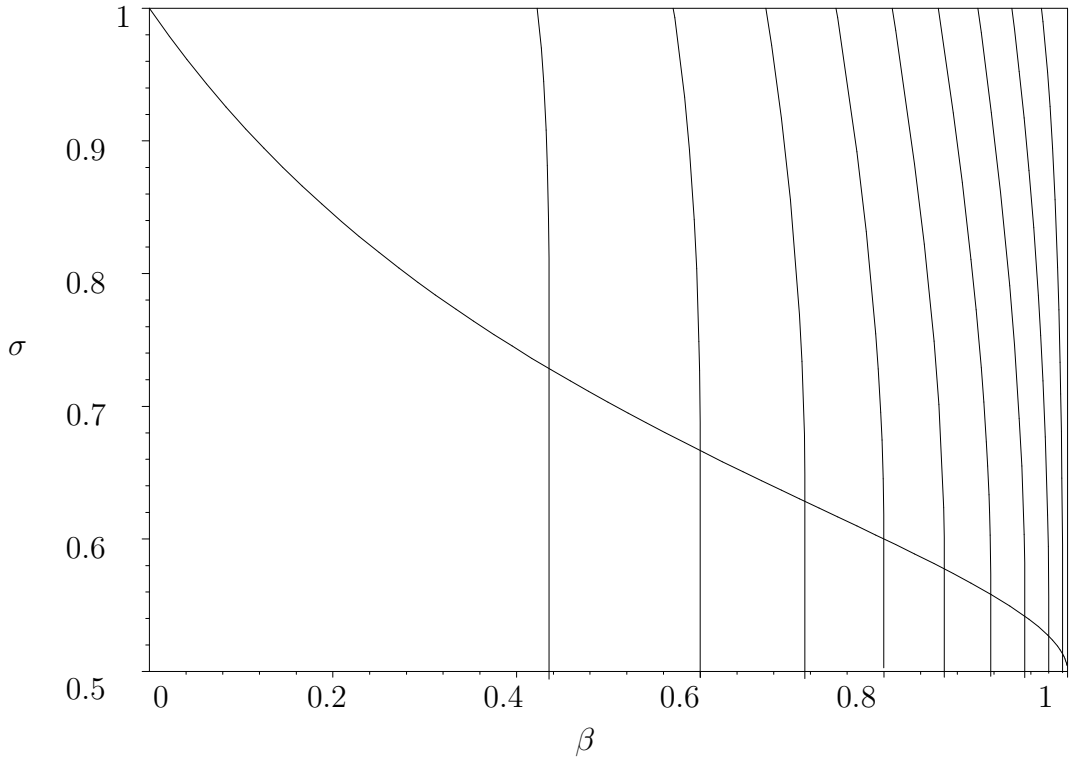


Figure 4: Value function contours $V = \{\frac{1}{10}, \dots, \frac{9}{10}\}$ when $N = 1$

use an informative signal. In larger committees, we shall see that this is not always so.

5 Committees of two: $N = 2$

Now consider $N = 2$. This is the simplest example in which voting rules may be compared. The value of the game now generally depends on both committee members' priors, (p_{01}, p_{02}) . This implies two dimensional integration. We therefore generalise the assumption in the previous section so that $\mathbf{p}_0 \sim UID[0, 1]^2$. As committee members 1 and 2 can only be identified as such once their priors have been realised, refer to them as a and b *ex ante*.

This generates four separate cases to consider, corresponding to the possible combinations of weather-vanes, (j, k) . Without loss of generality, as

committee members are identical before their priors are realised, we concentrate on the payoffs expected by b . To indicate that value varies by voting rule, let V_{jk} be the value function under weather-vanes j and k .

5.1 Double (weak) majority: $(j, k) = (1, 1)$

The case of $N = 2$ and $(j, k) = (1, 1)$ is that of (weak) majority rule at both stages. This corresponds to any voting rule, (δ_0, δ_T) , in which $\delta_0, \delta_T \in (0, \frac{1}{2})$. The more optimistic committee member is the weather-vane at both stages.

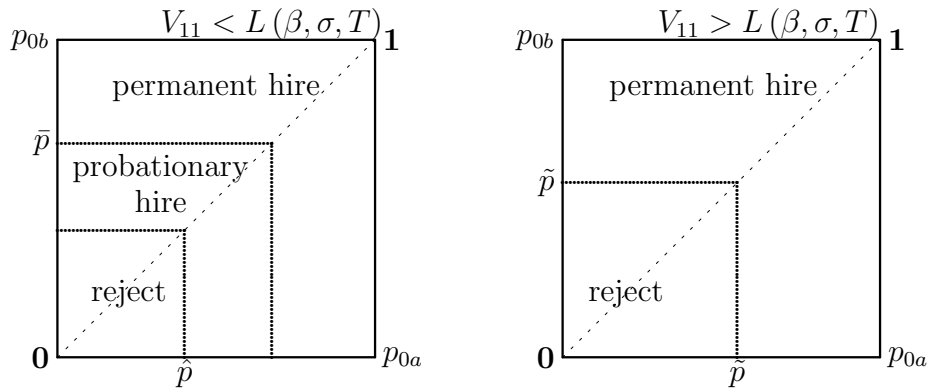


Figure 5: Payoffs expected by member b when $N = 2$, $(j, k) = (1, 1)$

Again, first consider the case of $V_{11} \geq L(\beta, \sigma, T)$. The payoffs expected by committee member b are displayed in the right panel of Figure 5. Below its diagonal, a is the more optimistic member, so that $p_{01} = p_{0a}$; above the diagonal, the roles are reversed and $p_{01} = p_{0b}$.

In the lower inner triangle, weather-vane a rejects the candidate, so that b expects a payoff of βV_{11} . In the lower quadrilateral, weather-vane a hires and tenures the candidate; the payoff expected by b now depends on her prior. The situation is symmetric in the upper triangle. In this case, while b still expects a payoff of $\beta^T p_{0b}$ if the candidate is tenured, this is higher than it would be were a the weather-vane.

Formally,

$$\begin{aligned} V_{11} &= \tilde{p}^2 \beta V_{11} + \int_{\tilde{p}}^1 \int_0^{p_{0a}} \beta^T p_{0b} dp_{0b} dp_{0a} + \int_0^1 \int_{\max\{\tilde{p}, p_{0a}\}}^1 \beta^T p_{0b} dp_{0b} dp_{0a} \\ &= \tilde{p}^2 \beta V_{11} + \frac{1}{2} \beta^T (1 - \tilde{p}^3) \geq L(\beta, \sigma, T). \end{aligned}$$

Consistent with Lemma 10, this is again independent of σ .

Now consider the case of $V_{11} \leq L(\beta, \sigma, T)$. The payoffs expected by committee member b are displayed in the left panel of Figure 5. The interpretation is the same as previously: the lower square corresponds to beliefs in which the candidate is rejected; the band above it corresponds to those beliefs in which the candidate is hired probationarily; the upmost band corresponds to those beliefs in which the candidate is hired permanently.

Our formal expression of this is written slightly counter-intuitively, but simplifies the terms in the limits of integration:

$$\begin{aligned} V_{11} &= \hat{p}^2 \beta V_{11} + \int_0^{\hat{p}} \int_{\hat{p}}^{\bar{p}} f(p_{0b}) dp_{0b} dp_{0a} + \int_{\hat{p}}^{\bar{p}} \int_0^{\bar{p}} f(p_{0b}) dp_{0b} dp_{0a} \\ &\quad + \int_0^{\bar{p}} \int_{\bar{p}}^1 \beta^T p_{0b} dp_{0b} dp_{0a} + \int_{\bar{p}}^1 \int_0^1 \beta^T p_{0b} dp_{0b} dp_{0a} \\ &= \hat{p}^2 \beta V_{11} + \beta^T \sigma V_{11} (\bar{p} + \hat{p}) (\bar{p} - \hat{p}) + \frac{1}{2} \beta^T \{ [(1 - 2\sigma) V_{11} + \sigma] (\bar{p}^3 - \hat{p}^3) + 1 - \bar{p}^3 \} \\ &\leq L(\beta, \sigma, T). \end{aligned}$$

Qualitatively, the results are similar to those in Figure 4.

5.2 Double unanimity: $(j, k) = (2, 2)$

The expected payoffs when $j = k = 2$ and $V_{22} > L(\beta, \sigma, T)$ are depicted in Figure 6. Its space is divided into the same regions as those in Figure 5, and by the same cut point, \tilde{p} . The difference between the two is that the boundaries in Figure 5 are demarcated by the beliefs of the more optimistic agent; the present ones are demarcated by the less optimistic agent. Thus,

$$V_{22} = [1 - (1 - \tilde{p})^2] \beta V_{22} + \frac{1}{2} (1 + \tilde{p}) (1 - \tilde{p})^2 \beta^T \geq L(\beta, \sigma, T). \quad (13)$$

Again, this is σ independent.

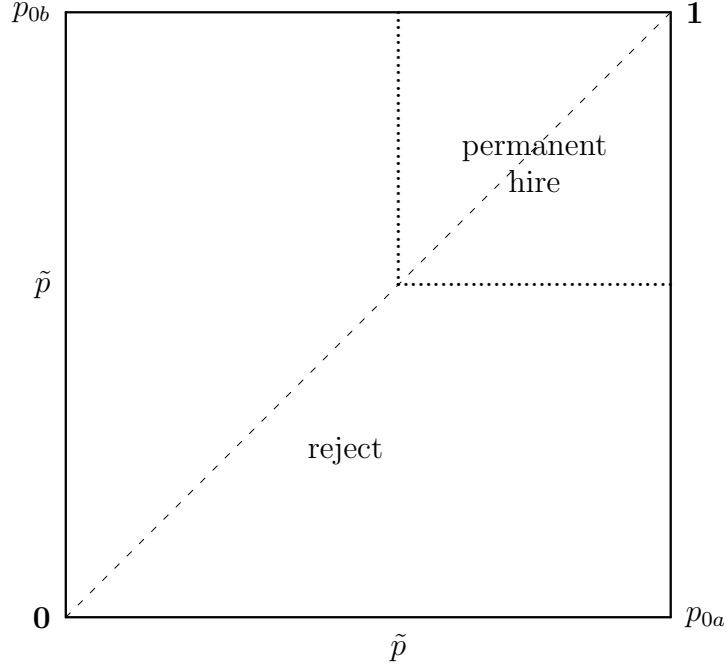


Figure 6: Payoffs expected by member b when $N = 2, (j, k) = (2, 2), V_{22} > L(\beta, \sigma, T)$

When $V_{22} < L(\beta, \sigma, T)$ the map of expected payoffs resembles that in the left panel of Figure 5, but with the boundaries demarcated by the beliefs of the less optimistic agent. Thus:

$$V_{22} = [1 - (1 - \hat{p})^2] \beta V_{22} + \frac{1}{2} (1 + \bar{p}) (1 - \bar{p})^2 \beta^T + \beta^T \sigma V_{22} [2 - (\bar{p} + \hat{p})] (\bar{p} - \hat{p}) \\ + \frac{1}{2} [(1 - 2\sigma) V_{22} + \sigma] \beta^T [(\bar{p} + \bar{p}^2 - \bar{p}^3) - (\hat{p} + \hat{p}^2 - \hat{p}^3)] \leq L(\beta, \sigma, T).$$

Again, the resulting contours are qualitatively similar to those found in the $j = k = 1$ case.

5.3 Falling threshold: $(j, k) = (2, 1)$

When $(j, k) = (2, 1)$ and $V_{21} > L(\beta, \sigma, T)$, the map of expected payoffs is the same as that in Figure 6. The value function over this range is therefore

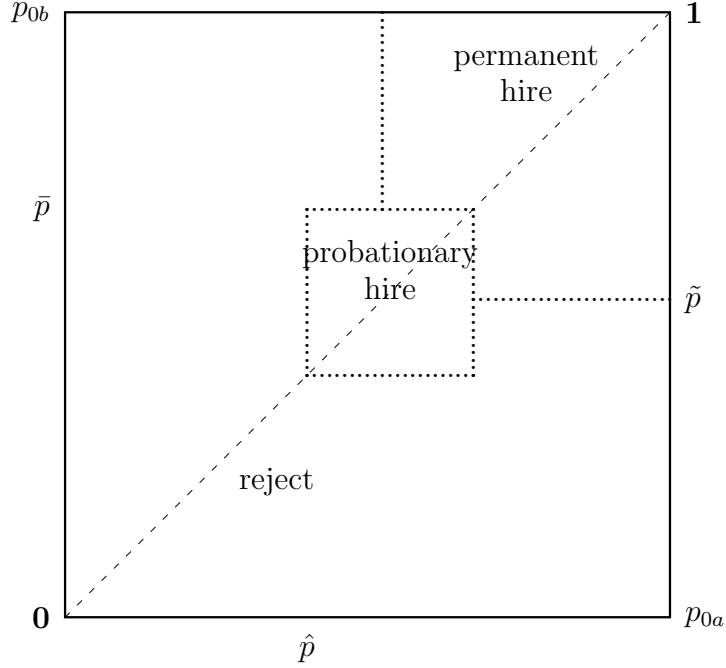


Figure 7: Payoffs expected by member b when $N = 2, (j, k) = (2, 1), V_{12} < L(\beta, \sigma, T)$

defined as in equation 13.

When $V_{21} < L(\beta, \sigma, T)$, expected payoffs are as displayed in Figure 7; they are symmetric about $p_{0a} = p_{0b}$. Now, the central square corresponds to probationary hiring; the object to its upper right corresponds to permanently hiring; and the remaining one to rejection at the initial stage.

In this case,

$$\begin{aligned}
 V_{21} = & [\hat{p}^2 + 2(\bar{p} - \hat{p})\hat{p} + 2(1 - \bar{p})\tilde{p}] \beta V_{21} \\
 & + \beta^T \left\{ \sigma V_{12} (\bar{p} - \hat{p})^2 + \frac{1}{2} [(1 - 2\sigma) V_{12} + \sigma] (\bar{p} - \hat{p})^2 (\bar{p} + \hat{p}) \right\} \\
 & + \frac{1}{2} \beta^T (1 - \bar{p}) [(1 + \bar{p})(1 - \tilde{p}) + (\bar{p}^2 - \tilde{p}^2)] \leq L(\beta, \sigma, T).
 \end{aligned}$$

As the contours of V are qualitatively similar to those already derived, they are not displayed.

5.4 Rising threshold: $(j, k) = (1, 2)$

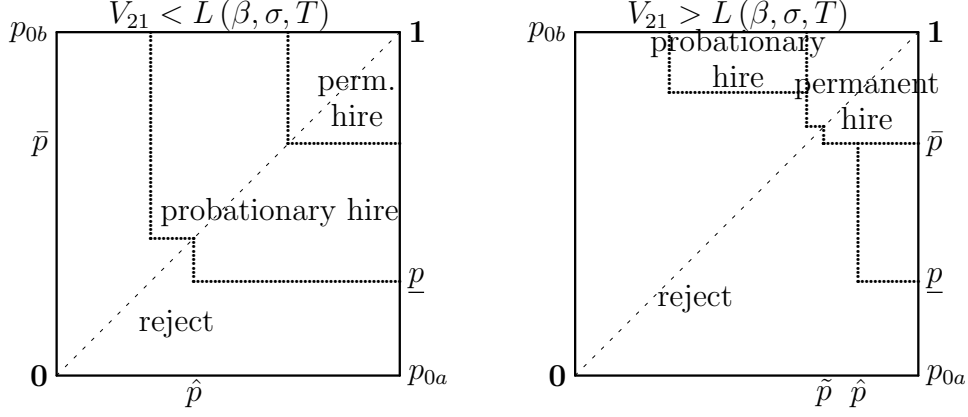


Figure 8: Payoffs expected by member b when $N = 2$, $(j, k) = (1, 2)$

When $(j, k) = (1, 2)$, expected payoffs are as depicted in Figure 8; the left panel corresponds to $V_{12} \leq L(\beta, \sigma, T)$ and the right to its complement.

Thus,

$$\begin{aligned}
 V_{12} &= [\hat{p}^2 + 2(\hat{p} - \tilde{p})\bar{p} + 2(1 - \hat{p})\underline{p}] \beta V_{12} \\
 &+ \beta^T \left\{ \frac{1}{2} [(1 - 2\sigma) V_{12} + \sigma] (1 - \hat{p}) (\bar{p} - \underline{p}) (1 + \hat{p} + \bar{p} + \underline{p}) + 2\sigma V_{12} (\bar{p} - \underline{p}) (1 - \hat{p}) \right\} \\
 &+ \frac{1}{2} \beta^T (1 - \tilde{p}) [(1 + \tilde{p})(1 - \bar{p}) + \tilde{p}^2 - \bar{p}^2] \geq L(\beta, \sigma, T);
 \end{aligned}$$

and

$$\begin{aligned}
 V_{12} &= [\hat{p}^2 + 2(1 - \hat{p})\underline{p}] \beta V_{12} + \frac{1}{2} \beta^T (1 + \bar{p})(1 - \bar{p})^2 \\
 &+ \frac{1}{2} \beta^T [(1 - 2\sigma) V_{12} + \sigma] [\bar{p}(1 + \bar{p} - \bar{p}^2) - (\underline{p} + \underline{p}^2) + \hat{p}(\underline{p}^2 + \underline{p}\hat{p} - \hat{p}^2)] \\
 &+ \beta^T \sigma V_{12} [(1 - \hat{p})(\bar{p} + \hat{p} - 2\underline{p}) + (1 - \bar{p})(\bar{p} - \hat{p})] \leq L(\beta, \sigma, T).
 \end{aligned}$$

Thus, this is the first in which the value function contains a term in σ when $V > L$.

The contours of V_{12} are displayed in Figure 9. The most significant feature of this diagram is the behaviour of the contours when $V_{12} \geq L(\beta, \sigma, T)$. In

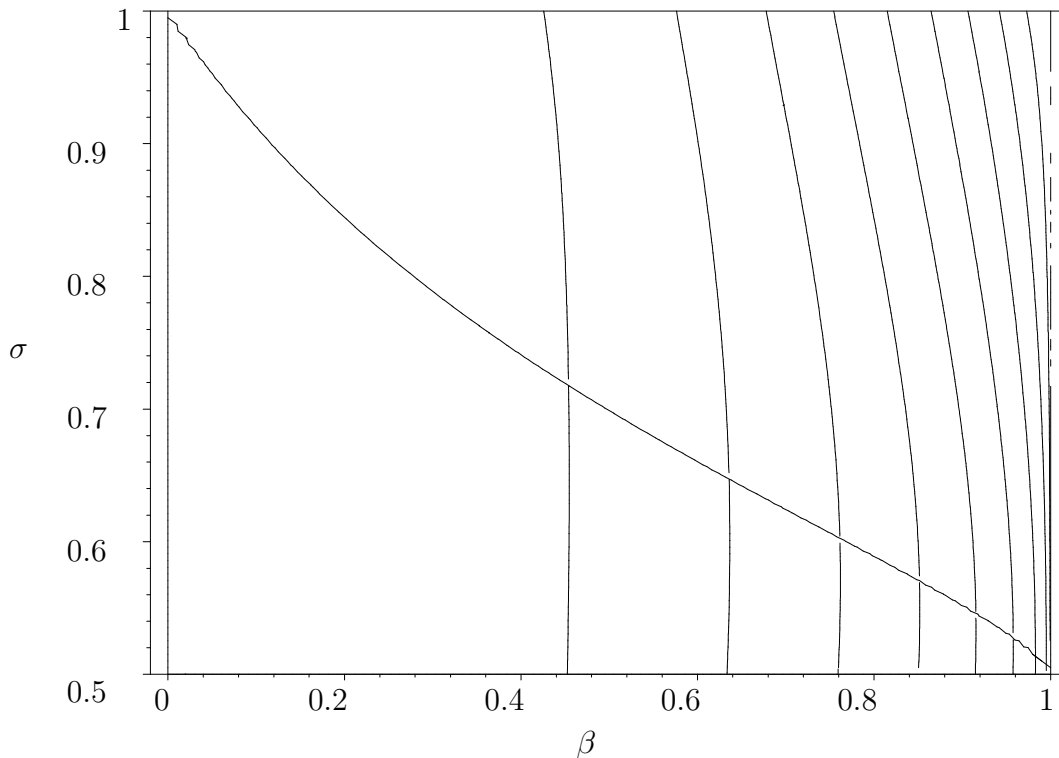


Figure 9: Value function contours $V_{12} = \{\frac{1}{10}, \dots, \frac{9}{10}\}$ when $N = 2, j = 1, k = 2$

the $N = 1$ case, and under the other voting rules, value was insensitive to the signal quality: as predicted by Lemma 10, a failure of inequality 9 corresponded to a rejection of probationary hiring in these cases.

In contrast, not only is the signal taken into account when $V_{12} \geq L(\beta, \sigma, T)$, but decreased signal quality improves the value of the problem to the committee. The effect is very weak: that the relevant contours in Figure 9 curve backward near $\sigma = \frac{1}{2}$ only becomes obvious under magnification. Technically, as $\sigma \rightarrow \frac{1}{2}, \underline{p} \rightarrow \bar{p}$, causing probationary hiring to disappear. Intuitively, as signal quality decreases, an optimistic hiring weather-vane ceases to hope that a good performance by a marginal candidate will convince the skeptics with his probationary performance.

5.5 The optimal voting rule

The optimal voting rule for $N = 2$ is determined by pairwise comparison of the voting rules analysed above. Each pair of value functions is implicitly plotted in (β, σ, V) space. This is implemented by a root-finding technique on a $(50, 50, 50)$ grid.¹³ Projection onto the (β, σ) plane then allows identification of the function with the greater value for any (β, σ) combination.

Comparison reveals $(1, 2)$ to at least weakly dominate $(2, 2)$ for all (β, σ) . Similarly, $(2, 1)$ is at least weakly dominated by $(1, 2)$ for all (β, σ) . Before discarding the dominated voting rules, we suggest intuitions for these results.

Consider first the dominance of $(2, 2)$ by $(1, 2)$. The two rules agree at the tenure meeting: unanimity is required. The first rule is, however, more restrictive at the hiring meeting. When a committee is perceptive, this more restrictive rule gives it fewer opportunities to use its perception: fewer candidates are hired. When a committee is imperceptive, its tenure decision largely reflects its priors; a good rule should therefore get candidates to that decision point quickly - which $(2, 2)$ does less well than does $(1, 2)$.

As to the dominance of $(2, 1)$ by $(1, 2)$ for all (β, σ) , neither rule is more stringent than the other in the sense of requiring the candidate to garner more votes over the two periods. However, the dominated rule imposes its stricter vote at the outset, when priors have not yet been informed by probationary performance. The preferred rule allows its stricter vote to condition on this performance.

The only remaining pairwise comparison is between $(1, 1)$ and $(1, 2)$. Figure 10 displays the result, which may be partitioned into four regions:¹⁴

I in this region, $L > V_{12} > V_{11}$. This corresponds to the left panel in Figure 8. Thus, it allows probationary hiring.

II in this region, $V_{12} > V_{11}$ and $V_{12} > L$. This corresponds to the right panel in Figure 8. Probationary hiring is again possible, and is subject to the perverse effect of decreased signal quality noted above.

¹³Maple 9.5 code available at www.economics.bham.ac.uk/rowat/research/ARZ-maple.zip.

¹⁴As some of the plotted values reflect poor numerical approximation, we ignore two features. First, the effects along $\beta = 0$ and in the $\beta = \sigma = 1$ corner. Second, the ragged $V = L$ boundary. We expect that these reflect difficulties associated with poor conditioning.

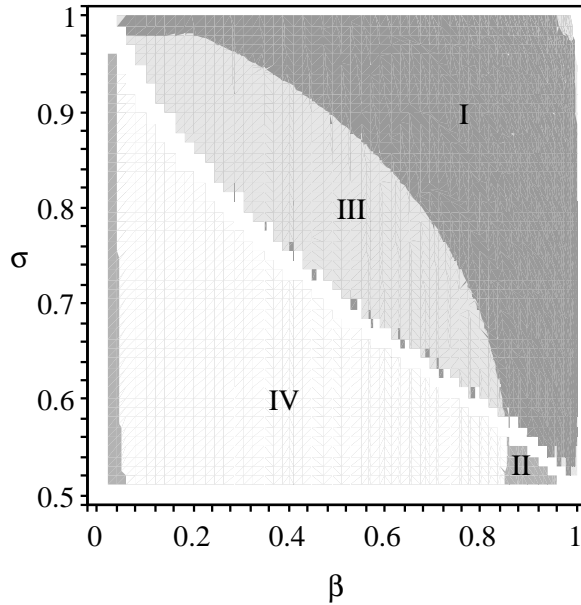


Figure 10: Optimal voting rules

III in this region, $L > V_{11} > V_{12}$. This corresponds to the left panel in Figure 5. It also allows probationary hiring.

IV finally, $V_{11} > V_{12}$ and $V_{11} > L$. This corresponds to the right panel in Figure 5. It does not allow probationary hiring.

In regions I and II, the optimal rule is to allow the optimists to ‘buy the option’ on a candidate, but the skeptics to choose whether or not to exercise it. For a patient committee, the mistake to avoid is not a delayed hire, but a bad hire. With an informative signal, even a skeptic will be convinced by a good performance. Signal quality deterioration into region II reduces the use of probationary hiring.

In regions III and IV, the double (weak) majority rule is preferred. Intuitively, skepticism about candidates when a committee is impatient and imperceptive is very costly, offering the possibility of repeated returns to the pool. Thus, optimists are made pivotal in both meetings, quickly hiring and tenuring. When the committee is particularly impatient or imperceptive - region IV - it even rejects probationary hiring, reducing its problem to a one stage problem.

Thus, as a committee becomes more impatient or imperceptive, its optimal rule progressively speeds the expected time to tenure a candidate: as patience or perceptiveness decrease, the majority requirement at tenure is eased, and then the committee gives up the ability to engage in probationary hiring.

Similar pairwise comparisons also show that the two optimal voting rules for $N = 2$ are dominated by the $N = 1$ committee. This comparison is the wrong one, however: the problem here is not one of aggregating information about a candidate, but one of social choice. A single committee member does not have less relevant information, but does not need to compromise.

Thus, the more appropriate comparison is between the optimal voting rules presented above, and another one in which $j = k \forall (\beta, \sigma)$: one committee member making the decisions, but on behalf of both. We have shown that $(j, k) = (1, 1)$ is optimal for some values of (β, σ) , but not others.

6 Discussion

The options interpretation suggested above is outlined more explicitly by reference to Cox, Ross, and Rubinstein (1979) (CRR). That presented a discrete-time model for valuing European options managed by risk neutral unitary investors.

In CRR, the current value of an asset on which a call is written either moves up or down to p^* by an exogenously determined amount in the next period. In our case, priors - expected values - are exogenous, and the extent of their updating to posteriors determined by the variance parameter, σ . Their time to maturity is our T . Their valuation formula does not depend on “the probability that the stock price will rise or fall”; our V_{jk} is independent of \mathbf{p}_0 and the realisation of θ .

There are, however, two principle differences between our model and the standard model, as exemplified by CRR. First is the way in which it is closed.

In CRR, the interest rate - the return to the market portfolio - is exogenous. With K , the option's strike price, the underlying asset's prices, and a no-arbitrage condition, this allows calculation of the cost of the call option.

In our case, V , the expected return to the 'market', is endogenous. Endogenising this requires giving something up to avoid over-determining the system. This explains the absence of the no-arbitrage assumption in our model: the market for candidates is not perfectly competitive. We could free the return to a 'good' candidate, currently set at 1, if we wished to both use a no-arbitrage condition and not over-determine the system.

The second difference is that our option is managed by a committee. Thus, 'rational exercise policy' gives way to a 'strategic exercise policy' in our environment. In CRR, the option's exercise depends on whether the option is in or out of the money:

$$\max \{0, p^* - K\}. \quad (14)$$

Versions of this appear in our more complicated environment. First, the committee's decision to exercise the option on a probationary candidate, granting him tenure, is governed by

$$\max \{0, p_{Tk} - V_{jk}\}.$$

The condition on when to buy the option - hire a candidate - is more complicated:

$$\max \{0, p_{0j} - \min \{\hat{p}, \tilde{p}\}\}. \quad (15)$$

Nevertheless, its interpretation is the same: when expression 15 is 'in the money', a candidate is hired. Which of the terms in the min operator is smaller depends, in turn, on

$$\max \{0, V_{jk} - L\}. \quad (16)$$

As a function of exogenous parameters, but not of the voting rule, L seems to parallel K in CRR. This offers an interpretation of the upper bound of unity on L : a strike price above the underlying asset's maximum price does not make sense.

Remembering, from expression 9, that $V_{jk} > L \Leftrightarrow \hat{p} > \tilde{p} > \bar{p}$, we may obtain an options interpretation from expression 16 as well. The committee's decision when the expression is 'in the money' is somewhat complicated, and depends on the voting rule. Generally, however, it may be interpreted as

an option on probationary hiring: hiring with the possibility of not granting tenure.

We now discuss some of our assumptions. First, consider the heterogeneous priors assumption. Theoretically it is a weaker one than that of common priors Heifetz (2001); nevertheless, it is a less common one.

Feinberg (2000) demonstrates that, under the assumptions applicable here, common priors do not exist if and only if there is a bet that risk neutral individuals are commonly known to be willing to accept against each other. This situation does not seem uncommon: trade does occur.

Morris (1995) posits a number of “common sense preconditions for explanations involving heterogeneous prior beliefs”, including the presence of limited opportunity for learning. While we have assumed that committee members learn a great deal about each other, our environment seems consistent with Morris’s example:

heterogeneous prior beliefs might persist . . . where individuals put positive probability on the truth, but learning is endogenous, so it is costly to discover the truth. Thus an employer might believe (with high probability) that a certain class of workers is of low ability. If the probability was sufficiently high, it would not even be worthwhile to hire a worker in order to discover if his belief was correct.

Now consider the assumption of Bayesian committee members. A weaker form of updating suffices for the results: committee members’ only need to be able to predict their future relative assessments of a candidate and form expectations over the weather-vane’s posteriors. This departure from rationality, however, might call into question the otherwise rational play. A second weakening is also possible: members could also receive private signals, perhaps informing the non-commonly held elements of their values. In this case, analysis would be similar, but committee members would be forced to vote on the basis of ‘expected weather-vanes’. This complicates analysis without clearly adding insight.

Tighter empirical tests may also be posed. Variance in δ and T across ranked institutions would allow estimation of the share of their ranking that can be explained by the discrepancy between institutions’ voting rules and the optimal ones.¹⁵

¹⁵Empirical testing may not be straightforward: a higher ranked institution will attract more highly qualified candidates, and possibly have more perceptive judges.

A second empirical observation may be made. Given any voting rule, δ , and knowledge of a committee's β and σ parameters, we can calculate the areas forming Figure 3. This, in turn, allows statements about the probabilities of the possible outcomes: the likelihood that a generic candidate is rejected; is hired but then refused tenure; and is hired, and then granted tenure.

Conversely, knowing these probabilities and the voting rule, may allow inference about the committee's β and σ parameters. The promotion rate to tenure in law schools, for example, is substantially higher than that of other departments in major research universities (compare Siow (1998) with Chused (1988)). Thus, law schools engage in less probationary hiring. Assuming that discount factors do not vary significantly by discipline, this difference could be accounted for if σ was lower in law schools: $\sigma \rightarrow \frac{1}{2}$ causes the zone of probationary hiring disappears.

Finally, we turn to possible extensions to the model. Without altering its analytical structure at all, that of most obvious interest is to extend the analysis of optimal rules to committees with more than two members. The techniques applied above can be extended to $N > 2$ on a case-by-case basis, but it would be preferable to have a technique that allowed assessment for a general N .

A second, more involved extension would be to make T a meaningful variable. At present, $T = 2$ is optimal for all committees: the signal that they receive on a probationary candidate is no worse over a short probationary period than it is over a long one; further, larger values of T delay the committee's ability to return to the pool if a probationary hire emits a bad signal. A more satisfactory treatment might address two issues.

First, longer probationary periods could allow the committee to receive more realisations of θ ; alternatively, the accuracy of σ could increase in T . Second, payoffs could be allowed to accrue over the probationary period, rather than just upon granting tenure. This approach alters the interpretation of the p_i : once payoffs are actually observed, they are no longer expected payoffs.

A third extension would be to consider a European option version of the model, wherein the committee has the right to 'exercise' at every point up to T . Setting T to a high value would allow analysis of jobs subject to regular review.

A fourth, deeper extension, would specify a richer environment within which the candidate presented himself to the committee. A candidate is a

rival good, and thus may face bids from other institutions. Additionally, our candidates are inert, not even, for example, choosing effort levels. This abstraction may not have important analytical consequences as the signal may represent a reduced form of the effort choice. Perhaps more importantly, a candidate might seek to alter the distribution of committee members' priors.

Finally, we may consider deepening as well as extension. Two stage decision processes may be seen as forms of incomplete contracts. While these processes are observed in practice, no motives for them from, for example, Tirole (1999), have been presented here. Indeed, as the model is structured, they seem precluded: if agents know that they all are updating their priors on the basis of a common signal, they could write a contract at the probationary decision offering permanent employment, contingent upon satisfying performance targets. Rent seeking by committee members might be one reason for retaining discretion, but that is not modelled.

References

- David Austen-Smith and Tim Feddersen. Deliberation and voting rules. mimeo, November 2002.
- Andrew Caplin and Barry Nalebuff. On 64%-majority rule. *Econometrica*, 56(4):787 – 814, July 1988.
- H. Lorne Carmichael. Incentives in academics: Why is there tenure? *Journal of Political Economy*, 96(3):453 – 472, 1988.
- Kalyan Chatterjee and Robert C. Marshall. A model of academic tenure. mimeo, 11 December 2001.
- Zhiqi Chen and J. Stephen Ferris. A theory of tenure for the teaching university. *Australian Economic Papers*, pages 9 – 25, March 1999.
- Richard H. Chused. The hiring and retention of minorities and women on American law school faculties. *University of Pennsylvania Law Review*, 137:537 –, 1988.
- Marie-Jean Antoine Nicolas de Caritat Condorcet. *Essais sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*. Paris, 1785.

- John C. Cox, Stephen A. Ross, and Mark Rubinstein. Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3):229 – 263, September 1979.
- Eddie Dekel and Michele Piccione. Sequential voting procedures in symmetric binary elections. *Journal of Political Economy*, 108(1):34 – 55, 2000.
- Ronald G. Ehrenberg, Paul J. Pieper, and Rachel A. Willis. Do economics departments with lower tenure probabilities pay higher faculty salaries? *Review of Economics and Statistics*, 80(4):503 – 512, November 1998.
- Timothy Feddersen and Wolfgang Pesendorfer. Voting behavior and information aggregation in elections with private information. *Econometrica*, 65(5):1029 – 1058, September 1997.
- Timothy Feddersen and Wolfgang Pesendorfer. Convicting the innocent: the inferiority of unanimous jury verdicts under strategic voting. *American Political Science Review*, 92(1):23 – 35, March 1998.
- Yossi Feinberg. Characterizing common priors in the form of posteriors. *Journal of Economic Theory*, 91:127 – 179, 2000.
- Dino Gerardi and Leeat Yariv. Putting your ballot where your mouth is: An analysis of collective choice with communication. mimeo, February 2003.
- Sayantana Ghosal and Ben Lockwood. Information aggregation, costly voting and common values. mimeo, January 2003.
- Aviad Heifetz. The positive foundation of the common prior assumption. Social Sciences Working Paper 1125, CalTech, July 2001.
- Takatoshi Ito and Charles Kahn. Why is there tenure? Discussion Paper 228, Center for Economic Research, University of Minnesota, February 1986.
- Hao Li, Sherwin Rosen, and Wing Suen. Conflicts and common interests in committees. *American Economic Review*, 91(5):1478 – 1497, December 2001.
- Ernst Maug and Bilge Yilmaz. Two-class voting: a mechanism for conflict resolution. *American Economic Review*, 49(5):1448 – 1471, December 2002.

- Michael S. McPherson and Morton Owen Schapiro. Tenure issues in higher education. *Journal of Economic Perspectives*, 13(1):85 – 98, Winter 1999.
- Margaret A. Meyer. Learning from coarse information: Biased contests and career profiles. *Review of Economic Studies*, 58:15 – 41, 1991.
- Margaret A. Meyer. Biased contests and moral hazard: Implications for career profiles. *Annales d'Économie et de Statistique*, (25 - 26):165 – 187, 1992.
- Stephen Morris. The common prior assumption in economic theory. *Economics and Philosophy*, 11(2):227 – 253, 1995.
- Rebecca B. Morton and Kenneth C. Williams. Information asymmetries and simultaneous versus sequential voting. *American Political Science Review*, 93(1):51 – 67, March 1999.
- Nicola Persico. Committee design with endogenous information. *Review of Economic Studies*, 71(1):165 – 192, January 2004.
- Mattias K. Polborn. Endogenous majority rules with changing preferences. mimeo, December 2000.
- Aloysius Siow. Tenure and other unusual personnel practices in academia. *Journal of Law, Economics and Organization*, 14:152 –, 1998.
- Jean Tirole. Incomplete contracts: Where do we stand? *Econometrica*, 67 (4):741 – 781, July 1999.
- Martin L. Weitzman. Optimal search for the best alternative. *Econometrica*, 47(3):641 – 654, May 1979.