

Quantum Solution of Coordination Problems

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Abstract

We present a quantum solution to coordination problems that can be implemented with existing technologies. It provides an alternative to existing approaches, which rely on explicit communication, prior commitment or trusted third parties. This quantum mechanism applies to a variety of scenarios for which existing approaches are not feasible.

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The existence of multiple equilibria in economic systems can lead to coordination failures and consequently to inefficient outcomes. Examples that have been extensively studied range from firms having to decide whether or not to enter a competitive market and how to position their offerings, to the coordinated resolution of social dilemmas involved in the provision of public goods.

Coordination problems have long been studied in the context of game theory [1, 2, 3], where the coordination game is specified by a payoff matrix which yields several Nash equilibria. These equilibria can at times give the same payoff to all players, in which case the problem is for them to agree on which one to coordinate, or different payoffs, leading to a competitive coordination game in which players prefer different equilibria.

A simple example of a cooperative coordination game is that of two people having to choose driving on the left or the right side of the road, for which the payoff matrix is shown in Table 1. As can be seen, there are two Nash equilibria, with equal payoffs, corresponding to both drivers choosing the same side of the road. The coordination problem consists in both drivers finding a way to agree on which side of the road to drive.

choice	L	R
L	2, 2	-3, -3
R	-3, -3	2, 2

Table 1: Payoff structure for two driving choices: left (L), right (R). Each row and column corresponds to choices made by the first and second players, respectively, and their corresponding payoffs.

This and many other instances of coordination problems can be solved in several ways. A first solution resorts to a trusted third party who knows the preferences of the participants and is given the authority to pick an equilibrium which is then broadcasted to the players. In the case of competitive coordination problems the trusted third party may also have enforcement powers, since some players may wish to move the group to a more preferable equilibrium.

Another solution to coordination problems involves communication among players so that they can negotiate a choice. In the case of cooperative games even one player flipping a coin and broadcasting the result as the corresponding choice provides an effective solution. In a competitive setting, the negotiation might be such that the players wish to choose their equilibria at random as it would then be perceived as a fair choice. This case would require a trusted mechanism of coin flipping over a communication line, which can be enforced through cryptographic protocols.

A third mechanism for solving coordination problems invokes social norms, in which common knowledge of the participants' preferences can distinguish a given equilibrium from the others, as in the case of choosing the largest river as a boundary between two countries. Such distinguished equilibria are often called focal points [1, 4].

While these mechanisms can solve in principle coordination problems, there are times when none of these options are available, either because they are too expensive, slow or difficult to implement, or because privacy worries prevent the participants from using any of these options. Even worse, a constraint from a larger context, such as the need to use a mixed strategy, might make it disadvantageous for players to have their choices revealed in advance. Furthermore in cases where communication between parties is asynchronous there is the additional problem of achieving common knowledge [2], i.e. all parties know that the others know how to act. A simple example is that of using email to coordinate a meeting when one is not sure that recipients have read their emails and acknowledgements before the start of the meeting [5, 6].

It would appear that in these instances the only choice left for the participants is to choose at random which strategy to pursue, which would lead to many instances of coordination failures and a consequent reduction in their respective payoffs. Nevertheless, as we now show, there is an alternative and superior solution, which resorts to quantum mechanics to solve coordination problems without communication, trusted third parties or prearranged strategies. Moreover, this quantum solution is implementable in the real world, thus

making coordination problems easier to solve.

The way quantum dynamics allows for the practical solution of a coordination problem is via the generation of particles in entangled states. Quantum entanglement results in the appearance of specific quantum correlations between parts of a composite system, which can be exploited for quantum information processing [7]. In particular, parametric-down conversion techniques can produce twin photons which are perfectly quantum correlated in time, space and often in polarization [8]. These photons can then be physically separated at arbitrary distances so that each participant gets one of the entangled particles.

In the simplest case, where players face two choices, 0, and 1, they can use entangled particles with two physically observable states, such as their spin or polarization. As shown in Fig. 1, the corresponding state is then given by a superposition of the two correlated possibilities, denoted as $(|0, 0\rangle + |1, 1\rangle)/\sqrt{2}$. At a time of their choosing, each participant observes the state of their particle, resulting into a 0 or a 1 state, and makes the corresponding choice. The key aspect that makes this technique different from random choices is that entanglement implies a definite correlation between the two measurements, i.e. both players get either a 0 or a 1, irrespective of the spatial separation between them, and without communication.

One could argue that this correlation could also be achieved classically by flipping two coins in advance, hiding them into two boxes given to the players which they then open at some later time. However, unlike the entangled quantum solution we just described, this procedure predetermines the outcome, which may not be desirable if the players wish to defer the choices as long as possible. In this case an adversary might learn how the players will choose long before they actually do, and thus adjust its strategy accordingly.

To illustrate this consider the case of two players trying to coordinate on a mixed strategy against a third one without resorting to previous agreement or communication, as in the case of a coordinated attack on a rival or enemy. For the sake of example, consider the game of rock, paper, scissors, in which the two allied players must make the same choice to have any chance of winning.

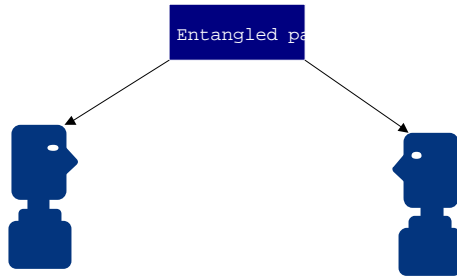


Figure 1: A source of entangled photons sends one each to the participants of a coordination game.

If the allies make different choices their payoffs are zero and the third player gets a payoff of 1. When the two allies make the same choice the payoff to the allies and the third player are given the payoff matrix of the rock, paper scissors game, which is shown in Table 2.

choice	rock	paper	scissors
rock	0, 0	0, 1	1, 0
paper	1, 0	0, 0	0, 1
scissors	0, 1	1, 0	0, 0

Table 2: Payoff structure for the rock, paper, scissors for the pair of allied players against the third player. Each row and column corresponds to choices made by the pair (assuming that they are the same) and their opponent, respectively, and their corresponding payoffs. For example, the entry of the first column, second row corresponds to the allies both choosing paper and the third player choosing rock.

This game has the feature that no single choice is best, i.e. there is no pure strategy Nash equilibrium. Instead, the best strategy for rational players is to make the choices randomly and with equal probability, which gives it a mixed

strategy Nash equilibrium with expected payoff of $1/3$.

For the full game without coordination the pair of allied players only has $1/3$ chance of making the same choice, and another $1/3$ to win against their opponent, leading to an expected payoff of $1/9$. If they can be perfectly coordinated their payoff would be $1/3$. In this example it is necessary to play random choices because any a priori commitment between the allied pair would no longer be a random strategy, and therefore discoverable by observation. If they instead use a pseudorandom number generator with a common seed, it could be compromised by the opponent discovering either the random pattern or the seed. On the other hand, if one of the allied pairs were to use a perfect private coin toss and communicated it to the other, it would risk being detected or jammed, leading to loss.

Using the quantum mechanism the players both can have undetectable randomness in their choices, no communication between them and still maintain complete correlation at every period of the game. Entanglement thus offers a way for the players to get correlated random bits they can use in addition to any public, broadcast information, without communication or prior agreement.

Furthermore, this quantum solution cannot be achieved via a classical simulation since we are requiring the absence of any communication among the participants. This is unlike the situation with other quantum games proposals [9].

Thus, quantum information processing, which already offers the potential for improved computation, cryptography and economic mechanisms [10, 11, 12, 13, 14] can lead to a perfect solution of complex coordination problems without resorting to the complex signaling procedures that have been discussed since they were first studied systematically. In particular it solves the problem of achieving common knowledge in the presence of asynchronous communication.

This quantum solution of a coordination problem is not just a theoretical construct, as it can be implemented over relatively large distances. It has been recently shown that it is possible to produce pairs of entangled photons using parametric down conversion, that can be sent separately over distances over

many kilometers. If the lifetime of the entangled state is long, each participant can then receive an entangled photon and perform a polarization measurement later, thus not having to communicate with each other during the whole procedure. On the other hand, if the lifetime of the entangled state is shorter than the period of the game, photons can be regenerated periodically, thereby requiring a transmission channel from the source to the participants (but not between the participants). In this case the advantage lies not in avoiding the possibility of blocked communication by an adversary, but in avoiding the detection of a coordinated solution. This makes for a feasible quantum solution to coordination problems that can be implemented with current technology, in contrast with most schemes for arbitrary quantum computation.

This scheme can also be extended to situations involving many participants. If the problem can be decomposed into independent pairwise coordination that can be then coordinated at a higher level (hierarchy) then the solution we described above can be applied to each pair. More interestingly, it has been recently shown [15] that it is possible to create entangled states of many particles in a single step and on demand, which implies that coordination problems involving many participants can also be solved using the scheme proposed in this paper.

Finally, this quantum approach to coordination games is more general than it may first appear, as it can also be applied to a variety of economic situations that involve achieving some or partial coordination among members of a group.

One example is several groups participating in an auction in which the value of an item to a person depends on what others in their group get. For example imagine bidding for construction tools that members of a group share and the auction is for each item separately. In this case, the valuation depends on the complementarity of the goods that the whole group gets, rather than who in the group gets each item. While more complicated than the pure coordination game we discussed because, this problem also involves a bidding strategy, and thus the need for the group to coordinate without signaling to other groups. The coordination part of this problem can be solved by having a source produce a

quantum state $|S\rangle$ given by an entangled state that for the case of two particles could be written as

$$|S\rangle = a|AA\rangle + a|BB\rangle + b|AB\rangle + b|BA\rangle \quad (1)$$

with the constants, a, b , chosen to favor a particular outcome and subject to the normalization condition $2|a|^2 + 2|b|^2 = 1$. Notice that these coefficients allow balancing the two parts of the utilities involved in this game, the desire for coordination and for low cost to the participants. For example, as cost difference increases, one could reduce a , and increase b . In other words, given a cost difference between items, one can pick a suitable superposition of states.

Another situation where our quantum mechanism could be useful is a coordination problem in which each player does not know others involved in the game, or they wish to remain mutually anonymous and avoid communication. If the interested players are known to be members of a larger group [16]), and entangled states are easily distributed among members of the larger group, those players interested in coordinating their activities can use the entangled states to ensure all players make the same choice.

As we have shown, the utilization of simple properties of quantum states gives a solution to coordination problems that does not require communication, trusted third parties involved in the decision making or prior commitment. Equally interesting, this solution is achievable with today's technology and opens the practical use of quantum entanglement in real world problems.

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