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NEGOTIATING THE MEMBERSHIP[1]

ABSTRACT. In cooperative games in which the players are partitioned into groups, we study the incentives of the members of a group to leave it and become singletons. In this context, we model a non-cooperative mechanism in which each player has to decide whether to stay in his group or to exit and act as a singleton. We show that players, acting myopically, always reach a Nash equilibrium.

KEYWORDS: Cooperative games, coalition structure, Owen value, game theory, Nash equilibrium

1. INTRODUCTION

Endogenous formation of coalitions has been widely studied in the game theory literature. For example, Chatterjee et al. (1993) and Okada (1996) study coalition formation models in which players can agree on payoff division at the time they form a coalition.

[1] Latest version at: `http://webs.uvigo.es/vidalpuga/`.

In these models, the coalitions are formed along with the final payoff of their members. An alternative approach is to assume that the final payoff is given by the coalition structure. For example, Hart and Kurz (1983) and Bloch (1996) present models of endogenous formation of coalitions in two stages: in the first stage, players decide the coalition structure. In the second stage, the final payoff is given according to the chosen coalition structure. In Hart and Kurz's model, the final payoff is given by the Owen value (Owen (1977)). A similar model is given by Aumann and Myerson (1988), where players decide how to connect through a graph, and the final payoff is given by the Myerson value (Myerson (1977)) depending on the particular graph.

On the other hand, there are many situations in which the coalition structure is given *a priori*. For example, consider the members of a Parliament. Even though all have the same rights, they do not act independently, since they belong to different political parties. Other examples include wage bargaining between firms and labor unions, tariff bargaining between countries, bargaining between the member states of a federated country, etc. Broadly speaking, these coalitions negotiate among them as single agents. The fundamental feature is that the coalition structure is exogenously given by the problem, which means that players do not choose which coalition they belong to.

In this paper, we take an intermediate approach between the endogenous and the exogenous coalition structure models. We assume that there exists a prior coalition structure (exogenous), but players inside *a priori* union may have the chance to free ride and act as singletons (endogenous). For example, consider the parties with representation in the European Parliament. Some of these parties may decide, prior to the discussion of an issue, to collude and defend a common policy. By doing so, they join forces and act as a single party.

Usually, this cooperation is useful because the colluded party is stronger than the sum of the individual parties. It may happen, however, that this cooperation is not beneficial, as the "joint-bargaining paradox" of Harsanyi

(1977) shows. The paradox is that an individual can be worse off bargaining as a member of a coalition than bargaining alone. Chae and Heidhues (2004, p. 47) justify this paradox as follows: *Treatening a group as a single bargainer reduces multiple "rights to talk" to a single right and thereby benefits the outsiders.*

Supranational parties such like the EPP-ED[1] or the Socialist Group usually do not act as single agents, because its members are not committed to follow the same policies on the same issues. Instead, these supranational associations provide a common working environment is which cooperation agreements are easier to settle, but only if they are beneficial for everyone.

In this framework, we define a mechanism in two stages: in the first stage, players simultaneously announce whether they stay or exit their coalition. The decision to stay is interpreted as the agreement to act as a single player in the second stage. The players who decide to leave their coalition act as singletons. In the second stage, the final payoff is given by the Owen value.

In games with coalition structure, the Owen value is a relevant solution concept. It has been supported axiomatically (Owen (1977), Hart and Hurz (1983, 1984), Winter (1992), Calvo et al. (1996)) and also non-cooperatively (Vidal-Puga and Bergantiños (2003)). Moreover, it has been successfully applied to cost allocation problems (Vázquez-Brage et al. (1997)) and political situations (Carreras and Owen (1988, 1993), Ono and Muto (2001)). Vidal-Puga (2005) also shows that the Owen value arises in equilibrium of a non-cooperative game that models the bargaining among heterogeneous groups.

Hence, it seems justifiable to assume that, once the coalition structure is formed, the final payoff is given by the Owen value. Notice that this assumption is also made by Hart and Kurz (1983).

In Sections 2 and 3 we present the notation and the model of coalition formation. We are interested in finding the stability of the resulting coalition structure. We focus on the incentives of each player to stay or leave his group. These incentives are given by the difference between what they get by changing their strategies and what they get by not doing it. In Section 3, we show that these differences are independent of the order in which players move. As a consequence, there are no cycles. Players, acting myopically, can reach a Nash equilibrium. In Section 4, we study a possible generalization of the model.

2. PRELIMINARIES

We consider a *coalitional game* as a pair (N, v) with a finite set of *players* $N = \{1, 2, ..., n\}$ and a *characteristic function* $v : 2^N \to \mathbb{R}$ with $v(\emptyset) = 0$. Following usual practice, we often refer to "the game v" instead of "the coalitional game (N, v)".

Given two games v, w, let $v + w$ define the game $(v + w)(S) = v(S) + w(S)$ for all $S \subset N$.

Given a scalar α and a game v, let αv define the game $(\alpha v)(S) = \alpha v(S)$ for all $S \subset N$.

Given a *coalition* $T \subset N$, we define the *unanimity game* (N, u_T) with *carrier* T as the coalitional game given by

$$u_T(S) = \begin{cases} 1 & \text{if } T \subset S \\ 0 & \text{otherwise.} \end{cases}$$

According to Harsanyi (1959), unanimity games form a basis for the space of cooperative games, i.e.

$$v = \sum_{T \subset N} \lambda_T(v) u_T$$

where the Harsanyi dividends $\lambda_T(v)$ are given by

$$\lambda_T(v) = \sum_{S \subset T} (-1)^{|T|-|S|} v(S)$$

for all $T \subset N$.

A *coalition structure* over N is a partition $P = \{S_1, ..., S_p\}$ on the set of players N. The *quotient game* of v over P is the coalitional game $(P, v/P)$ defined as follows:

$$(v/P)(B) = \sum_{S_q \in B} v(S_q)$$

for all $B \subset P$. Thus, v/P is the game played by the coalitions in P.

We denote the set of all games (N, v, P) over N with coalition structure as $CTU(N)$.

A *value* is a function $\Psi : CTU\,(N) \to \mathbb{R}^N$ that assigns to each cooperative game with coalition structure (N, v, P) a vector in $\mathbb{R}^N$, so that $\Psi_i\,(N, v, P)$ represents the payoff assigned to player $i \in N$. With a slight abuse of notation, we say that $\Psi_i\,(N, v, P)$ is the *value* of player i.

Let Π be the set of permutations of the elements of N. We say that $\pi \in \Pi$ is *compatible* with P if the members of the same coalition are together. We denote the set of all permutations compatible with P as $\Pi_P \subset \Pi$. Namely, $\pi \in \Pi_P$ if and only if it satisfies:

$$\forall i, j \in S_q \in P, \forall k \in N \qquad \pi(i) < \pi(k) < \pi(j) \Longrightarrow k \in S_q.$$

Given $\pi \in \Pi$, we define

$$Pr(i, \pi) := \{j \in N : \pi(j) < \pi(i)\}$$

as the set of *predecessors* of i with respect to π.

The *Owen value* (Owen (1977)) is defined as follows:

$$\Phi_i(N, v, P) = \frac{1}{|\Pi_P|} \sum_{\pi \in \Pi_P} \left[v(Pr(i, \pi) \cup \{i\}) - v(Pr(i, \pi)) \right].$$

When the game is clear, we use $\Phi(P)$ instead of the more cumbersome $\Phi(N, v, P)$.

We consider the Owen value as a solution of the game. A characterization of the Owen value is given by Owen (1977) as follows. The Owen value is the only function $\Phi : CTU(N) \to \mathbb{R}^N$ satisfying the following axioms:

1. Efficiency: $\sum_{i \in N} \Phi_i(P) = v(N)$ for each $(N, v, P) \in CTU(N)$.

2. Symmetry in each union:

$$v(S \cup \{i\}) = v(S \cup \{j\}), \forall S \subset N \backslash \{i,j\} \Longrightarrow \Phi_i(P) = \Phi_j(P)$$

for all $i, j \in S_q \in P$.

3. Symmetry in the quotient game:

$$(v/P)(B \cup \{S_q\}) = (v/P)(B \cup \{S_r\}), \forall B \subset P \backslash \{S_q, S_r\} \Longrightarrow \sum_{i \in S_q} \Phi_i(P) = \sum_{i \in S_r} \Phi_i(P)$$

for all $S_q, S_r \in P$.

4. Null player:

$$v(S \cup \{i\}) = v(S), \forall S \subset N \backslash \{i\} \Longrightarrow \Phi_i(P) = 0$$

for all $i \in N$.

5. Additivity:

$$\Phi(N, v + w, P) = \Phi(N, v, P) + \Phi(N, w, P)$$

for all $(N, v, P), (N, w, P) \in CTU(N)$.

Given a unanimity game u_T with carrier $T \subset N$, Property 4 implies that $\Phi_i(P) = 0$ for all $i \notin T$.

3. THE MODEL

Let (N, v, P) be a game with coalition structure. Fix $S_q \in P$. We consider the following mechanism[2] in two stages for players in S_q:

First stage Simultaneously, each player in S_q announces whether he wants to stay or to exit the coalition. Given the announcements of each player, a coalition structure is formed. The players who announced to exit act as singletons.

Second stage Each player receives his Owen value.

Thus, the set of strategies for each player is $\{s, e\}$, where 's' means "to stay" and 'e' means "to exit". We work only with pure strategies. Let $\gamma(i) \in \{s, e\}$ be the strategy of player i. Let $\gamma = (\gamma(i))_{i \in S_q}$ be a strategy profile. We denote the resulting coalition structure as P_γ, namely

$$P_\gamma := \left\{ \{i\}_{i \in S_q : \gamma(i) = s} \right\} \cup \{\{i\}\}_{i \in S_q : \gamma(i) = e} \cup \{S_r\}_{r \neq q}.$$

In particular, if $\gamma(i) = s$ for all $i \in S_q$, we have $P_\gamma = P$.

The final payoff for the players is given by the Owen value under this coalition structure $\Phi(P_\gamma)$.

Example 1 *Let*[3] $P = \{123|45|6\}$ *and* $S_q = \{1, 2, 3\}$. *Assume* $\gamma(1) = \gamma(2) = s$ *and* $\gamma(3) = e$. *Then,* $P_\gamma = \{12|3|45|6\}$. *Assume* $\gamma'(1) = s$ *and* $\gamma'(3) = \gamma'(2) = e$. *Then,* $P_{\gamma'} = \{1|2|3|45|6\}$. *Assume* $\gamma''(1) = \gamma''(2) = \gamma''(3) = e$. *Then,* $P_{\gamma''} = \{1|2|3|45|6\}$.

A strategy profile γ is a *panic equilibrium* if $\gamma(i) = e$ for all $i \in S_q$. A panic equilibrium is clearly a Nash equilibrium, because the coalition structure does not change by the individual deviation of a player.

Remark 2 *Assume that players begin playing* γ *with* $\gamma(i) = s$ *for some* i, *and change their strategies myopically. This means that they sequentially change their strategies only if the payoff in the new coalition structure is larger for them. Then, it is straightforward to check that a panic equilibrium cannot be reached following this myopic behavior.*

Given a strategy profile γ, we say that P_γ *derives* from P, and it is a *derived coalition structure*. We say that two strategy profiles γ and γ' are *adjacent* through $i \in S_q$, and we write $\gamma \sim_i \gamma'$, if $\gamma(j) = \gamma'(j)$ for all $j \in S_q \backslash \{i\}$ and $\gamma(i) \neq \gamma'(i)$. We then call player i the *link* between γ and γ'. We say that γ and γ' are *adjacent*, and we write $\gamma \sim \gamma'$, if there exists a link $i \in S_q$ such that $\gamma \sim_i \gamma'$. Two derived coalition structures P_γ and $P_{\gamma'}$ are *adjacent through* i if their respective strategy profiles γ and γ' are adjacent through i. Also, P_γ and $P_{\gamma'}$ are *adjacent* if there exists a link i such that P_γ and $P_{\gamma'}$ are adjacent through i. We denote these as $P_\gamma \sim_i P_{\gamma'}$ and $P_\gamma \sim P_{\gamma'}$, respectively.

Example 3 *Let $P = \{123\}$, $P_1 = \{12|3\}$, and $P_2 = \{1|2|3\}$. Then, P, P_1 and P_2 derive from P. Moreover, P and P_1 are adjacent. Player 3 is the link between P and P_1. Similarly, P_1 and P_2 are adjacent, and they have two possible links, player 1 or player 2. However, P and P_2 are not adjacent.*

Notice that two adjacent derived coalition structures may be equal, as the next example shows.

Example 4 *Let $P = \{12\}$, $\gamma(1) = \gamma(2) = e$, $\gamma'(1) = e$, $\gamma'(2) = s$. Then, $P_\gamma \sim P_{\gamma'}$ and $P_\gamma = P_{\gamma'} = \{1|2\}$. However, $\gamma \neq \gamma'$.*

A *path* over P is an ordered list of strategy profiles $\Im = [\gamma_0, \gamma_1, ..., \gamma_m]$ such that $\gamma_{l-1} \sim \gamma_l$ for all $l = 1, ..., m$. We say that $\Im$ has *length* m. If $\gamma_m = \gamma_0$, we say that $\Im$ is a *closed path*. Let $[i_1, i_2, ..., i_m]$ be the list of links between the strategy profiles, i.e. $\gamma_{l-1} \sim_{i_l} \gamma_l$ for all $l = 1, ..., m$. Let $[P_0, P_1, ..., P_m]$ be the list of coalition structures derived from $\Im$, i.e. $P_l = P_{\gamma_l}$ for all $l = 0, 1, ..., m$.

Definition 5 *Given a value Ψ, we say that a closed path $\Im = [\gamma_0, \gamma_1, ..., \gamma_m]$ is a* cycle *for Ψ if $\Psi_{i_l}(P_{l-1}) < \Psi_{i_l}(P_l)$ for all $l = 1, 2, ..., m$, where $P_l = P_{\gamma_l}$ is the coalition structure derived from γ_l and i_l is the link between γ_{l-1} and γ_l, for all $l = 1, 2, ..., m$.*

Example 6 *Let $P = \{123\}$ and $v(\{1, 2, 3\}) = 30$. Let Ψ be a value such that $\Psi(P) = (10, 10, 10)$. If the coalition structure is $P_\gamma = \{12|3\}$, the players get $\Psi(P_\gamma) = (4, 11, 15)$. If $P_\gamma = \{1|23\}$, they get $\Psi(P_\gamma) = (11, 4, 15)$. If $P_\gamma = \{13|2\}$, they get $\Psi(P_\gamma) = (15, 4, 11)$. If $P_\gamma = \{1|2|3\}$, they get $\Psi(P_\gamma) = (10, 10, 10)$. Then, every coalition structure belongs to a cycle[4]. Moreover, the only Nash equilibrium is the panic equilibrium (see Figure 1).*

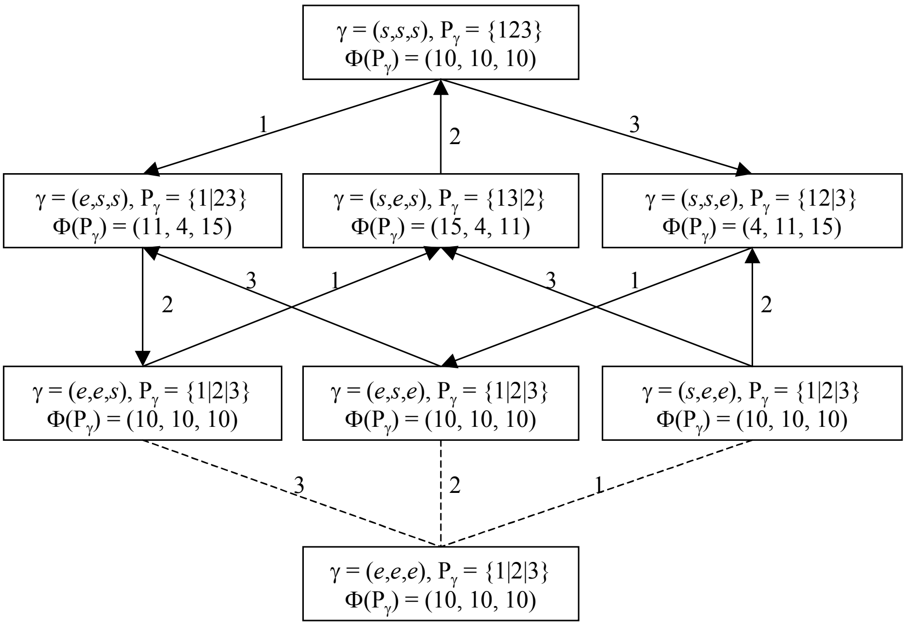

Figure 1: The arrows represent the adjacent strategy profiles. The number next to each arrow indicates the link. Each arrow points to the strategy profile that increases the payoff of the link. Notice that the panic equilibrium (e, e, e) is not reachable by the arrows (see Remark 2).

We study the existence of cycles for the Owen value. Hence, from now on, when we say cycle, we mean cycle for Φ.

The existence of cycles indicates an instability in the mechanism, as the next lemma shows:

Lemma 7 *If the only Nash equilibrium is the panic equilibrium, then there exists a cycle.*

Proof. Assume the only Nash equilibrium is the panic equilibrium and there are no cycles. Let γ_0 be a strategy profile that is not the panic equilibrium. Then, there exists a player $i_1 \in S_q$ who benefits from changing his strategy $\gamma_0(i_1)$. Let γ_1 be the adjacent strategy profile (i.e. $\gamma_0 \sim_{i_1} \gamma_1$) and let P_0 and P_1 be their respective coalition structures (i.e. $P_0 = P_{\gamma_0}$ and $P_1 = P_{\gamma_1}$). By Remark 2, γ_1 is not the panic equilibrium. Moreover, $\Phi_{i_1}(P_0) < \Phi_{i_1}(P_1)$. Since γ_1 is not a Nash equilibrium, there exists another player $i_2 \in S_q$ who benefits from changing $\gamma_1(i_2)$. Let γ_2 be the adjacent strategy profile and let P_2 be its derived coalition structure. Then, γ_2 is not the panic equilibrium, and $\Phi_{i_2}(P_1) < \Phi_{i_2}(P_2)$. We repeat the process with all the players who are willing to change their strategies. Since there exist no cycles, we cannot come back to a previous strategy profile. So, there should be a strategy profile γ_m (which is not the panic equilibrium) in which no player can improve his payoff by changing his strategy, i.e. γ_m is a Nash equilibrium. This contradiction proves the result. ■

Definition 8 *Given a path $\Im = [\gamma_0, \gamma_1, ..., \gamma_m]$, the* differential *of $\Im$ in v is the number:*

$$\delta(\Im, v) := \sum_{l=1}^{m} [\Phi_{i_l}(P_l) - \Phi_{i_l}(P_{l-1})] \tag{1}$$

where $P_l = P_{\gamma_l}$ is the coalition structure derived from γ_l, and i_l is the link between γ_{l-1} and γ_l, for all $l = 1, 2, ..., m$.

Notice that each term in (1) represents the amount by which a player i_l improves his payoff when the strategy profile changes from γ_{l-1} to γ_l, which is the change that he is capable to do.

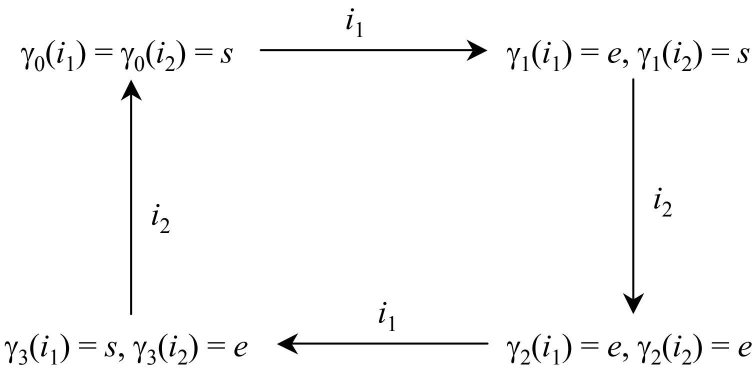

Figure 2: $\mathfrak{T} = [\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_0]$ is a closed path of length 4.

Lemma 9 *The differential* $\delta(\Im, v)$ *is additive on* v*, i.e.*

$$\delta(\Im, v + w) = \delta(\Im, v) + \delta(\Im, w)$$

for all $\Im$ *and all games* v, w*.*

Proof. Immediate from the additivity of the Owen value. ■

Proposition 10 *The differential of any closed path is* 0*.*

Proof. Let $\Im = [\gamma_0, \gamma_1, ..., \gamma_m]$ be a closed path with links $[i_1, ..., i_m]$. Let $[P_0, P_1, ..., P_m]$ be their associated coalition structures. We proceed by induction on m. First, we note that m should be an even number, because each link i_l should change his strategy $\gamma(i_l)$ an even number of times, so that the strategy profile goes back to its original position, i.e. $\gamma_0 = \gamma_m$.

For $m = 2$, the result is trivial, because $i_1 = i_2$ and $\phi_{i_2}(P_1) - \phi_{i_2}(P_0) = -\left(\phi_{i_1}(P_0) - \phi_{i_1}(P_1)\right)$.

For $m = 4$, we have $\Im = [\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4]$ and three cases: a) $i_1 = i_2$, $i_3 = i_4$; b) $i_1 = i_3$, $i_2 = i_4$; and c) $i_1 = i_4$, $i_2 = i_3$. In cases a) and c), we have two closed paths of length 2, so the differential is 0. We prove the result for case b) (Figure 2). We can assume without loss of generality that in γ_0 both players play 's'.

Assume we are in a unanimity game u_T, and both players belong to the carrier T. In particular, this implies $|S_q \cap T| \geq 2$. Let p_0 be the number of coalitions in P_0 with nonempty intersection with T. Then, it is well-known (Owen (1995, p. 307)) that the Owen values for i_1 and i_2 in P_0 are

$$\Phi_{i_1}(P_0) = \Phi_{i_2}(P_0) = \frac{1}{p_0|S_q \cap T|}.$$

Analogously, we have

$$\begin{aligned}
\Phi_{i_1}(P_1) &= \frac{1}{p_0+1} & \Phi_{i_2}(P_1) &= \frac{1}{(p_0+1)(|S_q \cap T| - 1)} \\
\Phi_{i_1}(P_2) &= \frac{1}{p_0+2} & \Phi_{i_2}(P_2) &= \frac{1}{p_0+2} \\
\Phi_{i_1}(P_3) &= \frac{1}{(p_0+1)(|S_q \cap T| - 1)} & \Phi_{i_2}(P_3) &= \frac{1}{p_0+1}.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\delta(\Im, u_T) &= \left[\phi_{i_1}(P_1) - \phi_{i_1}(P_0)\right] + \left[\phi_{i_2}(P_2) - \phi_{i_2}(P_1)\right] \\
&\quad + \left[\phi_{i_1}(P_3) - \phi_{i_1}(P_2)\right] + \left[\phi_{i_2}(P_0) - \phi_{i_2}(P_3)\right] \\
&= 0.
\end{aligned}$$

When one of the players does not belong to the carrier (say, player i_1), then $\Phi_{i_1}(P_\gamma) = 0$ for any γ and

$$\begin{aligned}
\Phi_{i_2}(P_0) &= \Phi_{i_2}(P_1) = \frac{1}{p_0|S_q \cap T|} \\
\Phi_{i_2}(P_2) &= \Phi_{i_2}(P_3) = \frac{1}{p_0+1}
\end{aligned}$$

from where it is not difficult to check that $\delta(\Im, u_T) = 0$.

For a general game $v = \sum_{T \subset N} \lambda_T(v)\, u_T$, we apply the additivity property of the differential:

$$\delta(\Im, v) = \sum_{T \subset N} \lambda_T(v)\, \delta(\Im, u_T) = 0.$$

Assume now the result holds for closed paths with less than m strategy profiles, $m \geq 6$. We can assume without loss of generality that i_1 changes

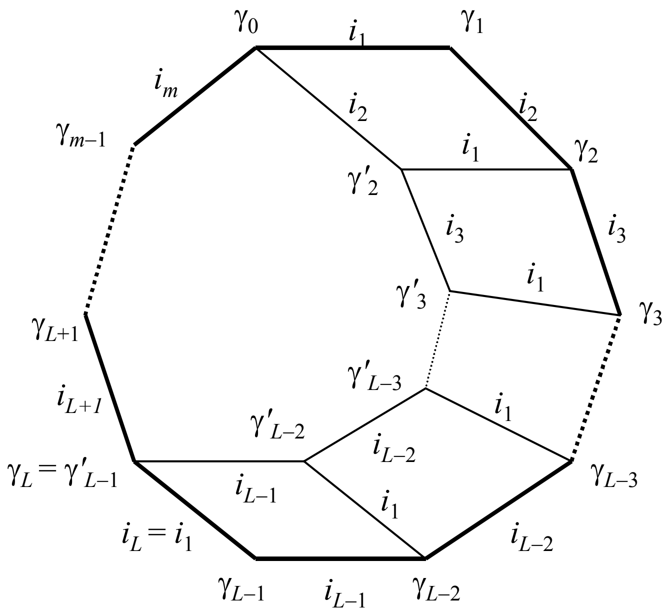

Figure 3: Each node represents a strategy profile. Each arc represents a link between two adjacent strategy profiles.

his strategy from $\gamma_0(i_1) = s$ to $\gamma_1(i_1) = e$. Since $\Im$ is a closed path, player i_1 should eventually change his strategy from e to s. Namely, there exists $L \in \{2, ..., m\}$ such that $i_1 = i_L$, $\gamma_l(i_1) = e$ for all $l = 2, ..., L-1$, and $\gamma_L(i_1) = s$.

If $L = 2$, then $\gamma_2 = \gamma_0$.

If $L > 2$, we consider the strategy profile γ'_2 which arises from γ_0 when player i_2 makes his change before i_1, i.e. $\gamma'_2(i) = \gamma_0(i)$ for all $i \neq i_2$, $\gamma'_2(i_2) = \gamma_2(i_2)$. Now, it is not difficult to check that γ'_2 is adjacent to both γ_0 and γ_2. Player i_2 is the link between γ_0 and γ'_2. Player i_1 is the link between γ_2 and γ'_2. Let γ'_3 be the strategy profile which arises from γ'_2 when player i_3 changes his strategy, and so on. We repeat the process until we reach γ'_{L-1}, which equals γ_L (see Figure 3).

Formally, we define γ'_l for $l = 2, ..., L-1$ as follows: $\gamma'_l(i) = \gamma_l(i)$ if $i \neq i_1$ and $\gamma'_l(i_1) = s$. It is straightforward to check that $\gamma'_l \sim_{i_{l+1}} \gamma'_{l+1}$ and $\gamma'_l \sim_{i_1} \gamma_l$

for all l. Moreover, $\gamma'_{L-1} = \gamma_L$.

We decompose the closed path $\Im$ in two smaller ones

$$\Im' = \begin{cases} [\gamma_0, \gamma_1, \gamma_2] & \text{if } L = 2 \\ [\gamma_0, \gamma_1, ..., \gamma_L, \gamma'_{L-2}, \gamma'_{L-1}, ..., \gamma'_2, \gamma_0] & \text{if } L > 2 \end{cases}$$

and

$$\Im'' = \begin{cases} [\gamma_2, \gamma_3, ..., \gamma_{m-1}, \gamma_0] & \text{if } L = 2 \\ [\gamma_0, \gamma'_2, \gamma'_3, ..., \gamma'_{L-2}, \gamma_L, \gamma_{L+1}, ..., \gamma_{m-1}, \gamma_0] & \text{if } L > 2. \end{cases}$$

Notice that $\delta(\Im, v) = \delta(\Im', v) + \delta(\Im'', v)$. Moreover, $\Im''$ has two less strategy profiles than $\Im$. Hence, by the induction hypothesis $\delta(\Im'', v) = 0$.

We show now that $\delta(\Im', v) = 0$. If $L = 2$, the result is trivial by induction hypothesis. If $L > 0$, we take the closed paths $\Im_0 = [\gamma_0, \gamma_1, \gamma_2, \gamma'_2, \gamma_0]$, $\Im_1 = [\gamma'_2, \gamma_2, \gamma_3, \gamma'_3, \gamma'_2]$, and so on. In general, $\Im_l = [\gamma'_l, \gamma_l, \gamma_{l+1}, \gamma'_{l+1}, \gamma'_l]$ for all $l = 2, 3, ..., L-2$. Since they are closed paths of length 4, we have $\delta(\Im_l, v) = 0$ for all $l = 1, ..., L-2$. Adding all these equations, we obtain:

$$\delta(\Im', v) = \sum_{l=1}^{L-2} \delta(\Im_l, v) = 0.$$

Hence, $\delta(\Im, v) = \delta(\Im', v) + \delta(\Im'', v) = 0$. ■

An important consequence of Proposition 10 is that there are no cycles for Φ.

Corollary 11 *There are no cycles for Φ.*

Proof. Assume there is a cycle $\Im$. Then, $\delta(\Im, v)$ is positive, which contradicts Proposition 10. ■

As another consequence of Proposition 10, we have the following definition:

Definition 12 *Given two strategy profiles γ,γ', the* differential of γ' with respect to γ *is the differential of any path from γ to γ'.*

This differential is well-defined: Assume there are two paths from γ to γ', i.e. $\Im = \{\gamma, \gamma_1, \gamma_2, ..., \gamma_m = \gamma'\}$ and $\Im' = \{\gamma, \gamma'_1, \gamma'_2, ..., \gamma'_{m'} = \gamma'\}$. Then, the closed path $\Im'' = \{\gamma, \gamma_1, \gamma_2, ..., \gamma_m, \gamma'_{m'-1}, ..., \gamma'_1, \gamma\}$ has its differential 0 and

$$0 = \delta(\Im'', v) = \delta(\Im, v) - \delta(\Im', v).$$

Thus, $\delta(\Im, v) = \delta(\Im', v)$.

Theorem 13 *Players, acting myopically, always reach a Nash equilibrium.*

Proof. We start from a strategy profile γ. Suppose that there exists a player $i \in S_q$ who benefits from changing his strategy $\gamma(i)$. Let γ' be the adjacent strategy profile (i.e. $\gamma \sim_i \gamma'$) and let P_γ and $P_{\gamma'}$ be their respective coalition structures. Then, $\phi_i(P_\gamma) < \phi_i(P_{\gamma'})$ and we deduce that the differential of γ' with respect to γ is positive. Suppose in the new strategy profile there exists another player $j \in N$ who benefits from changing his strategy $\gamma'(j)$. Let γ'' be the adjacent strategy profile and let $P_{\gamma''}$ be its respective coalition structure. Then, $\phi_j(P_{\gamma'}) < \phi_j(P_{\gamma''})$ and the differential of γ'' with respect to γ is again positive. We repeat the process with all the players who are willing to change their strategy. Since the differential is always positive, we cannot come back to a previous strategy profile. So, there should be a strategy profile γ_m in which no player can improve his payoff by changing his strategy, i.e. γ_m is a Nash equilibrium. ■

Theorem 14 *There exists a non-panic Nash equilibrium.*

Proof. It is an immediate consequence of Lemma 7 and Corollary 11. ■

4. THE MECHANISM WITH ALL THE COALITIONS

In the previous section, it was assumed that only the players of a fixed coalition S_q have the chance to exit the coalition. When a coalition negotiate a common behavior among their members (i.e. decide which of them act as a single player), it is natural to assume that the players do so independently of the other coalitions.

However, one may wonder what happens when all the coalitions play simultaneously. Thus, we study the following modification of the mechanism:

First stage Simultaneously, each player in N announces whether he wants to stay or to exit his coalition. Given the announcements of each player, a coalition structure is formed. The players who announced to exit act as singletons.

Second stage Each player receives his Owen value.

Thus, the set of strategies for each player i is again $\gamma(i) \in \{s, e\}$. Let $\gamma = (\gamma(i))_{i \in N}$ be a strategy profile. The derived coalition structure P_γ is given by

$$P_\gamma := \bigcup_{S_q \in P} \left\{ \{i\}_{i \in S_q : \gamma(i) = s} \right\} \cup \{\{i\}\}_{\gamma(i) = e} .$$

The definitions of a path, a closed path, a link, and the differential of a closed path are analogous to those of Section 3. Let γ be a Nash equilibrium. Then, γ is a *panic equilibrium* if there exists a coalition $S_q \in P$ such that $\gamma(i) = e$ for all $i \in S_q$. Notice that, in this case, there are more than one possible panic equilibrium.

Proposition 15 *The differential of a closed path is not always zero.*

Proof. Let $N = \{1, 2, 3, 4, 5\}$ and consider the unanimity game (N, u_N). Let $P = \{123|45\}$ and let $\gamma_0 = (s, s, s, s, s)$, $\gamma_1 = (e, s, s, s, s)$, $\gamma_2 = (e, s, s, e, s)$, $\gamma_3 = (s, s, s, e, s)$, and $\gamma_4 = \gamma_0$. The associated coalition structures are $P_0 = P$, $P_1 = \{1|23|45\}$, $P_2 = \{1|23|4|5\}$, $P_3 = \{123|4|5\}$ and $P_4 = P$, respectively. Then, it is straightforward to check that:

$$\begin{aligned}
\Phi_1(P_0) &= \frac{1}{6}, \Phi_4(P_0) = \frac{1}{4} \\
\Phi_1(P_1) &= \frac{1}{3}, \Phi_4(P_1) = \frac{1}{6} \\
\Phi_1(P_2) &= \frac{1}{4}, \Phi_4(P_2) = \frac{1}{4} \\
\Phi_1(P_3) &= \frac{1}{9}, \Phi_4(P_3) = \frac{1}{3} \\
\Phi_1(P_4) &= \frac{1}{6}, \Phi_4(P_4) = \frac{1}{4}
\end{aligned}$$

Let $\mathfrak{T} = [\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4]$ be a closed path. Then, $\delta(\mathfrak{T}, v) = \frac{1}{36} \neq 0$. ■

As the differential is not zero, we wonder whether there exist non-panic equilibria. The next example shows that there exist games whose unique Nash equilibria are the panic equilibria.

Example 16 *Let $n = 6$ and let v be given by the following table* [5]*:*

S	$v(S)$
$1, 2, 3, 4, 5, 6, 13, 14, 16, 23, 24, 34$	0
$46, 146$	1
$12, 25, 35, 123, 134, 234$	3
$15, 124, 125, 135, 235, 1234$	4
$26, 36, 45, 56, 126, 136, 145, 156, 236, 245, 246, 345, 346, 356, 456, 1246, 1346$	5
$1235, 1345, 2345, 2346$	6
$1236, 1245, 12345$	8
$1256, 1356, 1456, 2356, 12346, 12356$	9
$2456, 3456, 12456, 23456$	10
N	13

This game in monotonic and superadditive [6]*. Moreover, all Nash equilibria are panic equilibria. For six players, it is tedious to write all the possible strategy profiles. In Figure 4, four of these strategy profiles (which form a cycle) are represented.*

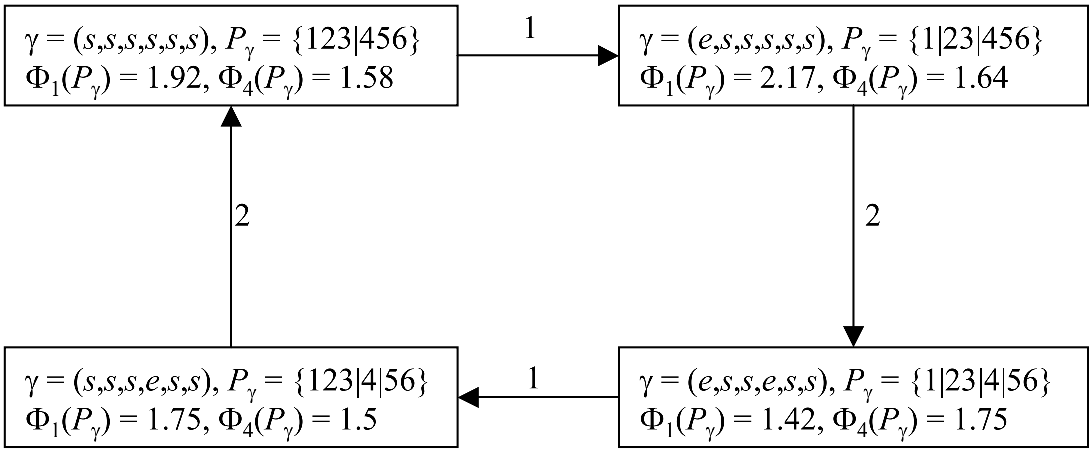

Figure 4: A cycle of length 4.

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NOTES

[1] European People's Party (Christian Democrats) and European Democrats.

[2] We use the term *mechanism* instead of *non-cooperative game* to avoid confusion with cooperative games.

[3] For simplicity, we write $\{123|45|6\}$ instead of $\{\{1,2,3\},\{4,5\},\{6\}\}$, and so on.

[4] I thank María Montero for proposing this example.

[5] We write 146 instead of $\{1,4,6\}$, and so on.

[6] A game v is *monotonic* if $v(S) \leq v(T)$ for all $S \subset T$, and *superadditive* if $v(S) + v(T) \leq v(S \cup T)$ for all S, T with $S \cap T = \emptyset$.

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