Alliances and Negotiations.

Paola Manzini*

Queen Mary, University of London and IZA

Marco Mariotti†

University of Exeter

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Abstract

A characteristic of many bargaining situations is that the negotiators represents the interests of a set of parties (trade unions, political parties, etc.) with composite interests, whose bargaining behaviour is regulated by some collective decision mechanism. In this paper we provide a natural model of such circumstances, and show how different preference aggregation procedures within the composite player affect the bargaining outcome. In particular we find that unanimity procedures lead to 'more aggressive' behaviour than majority procedures, and that procedures which introduce minimum safeguards for the members of an alliance may result in agreements that are worse than without those safeguards.

Keywords: Alliances, multiperson bargaining

*Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS (tel: 020-7882 5083; e-mail: p.manzini@qmw.ac.uk, http://www.qmw.ac.uk/~ugte172).

†Department of Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU, UK (tel: 01392-263219; e-mail: m.mariotti@exeter.ac.uk).

1 Introduction

In negotiations, the parties involved are often conglomerations of composite interests, whose bargaining behaviour is determined by some collective decision mechanism. In this paper we introduce a framework to study how such mechanisms (e.g. majority versus unanimity voting) are related to bargaining outcomes.

For example, in the diplomatic negotiations leading Nato to take military action against Yugoslavia in 1999, the alliance was often accused of taking an intransigent stance. In particular, the perception was that the 'hawks' in the alliance (mainly the British and American governments) were calling the shots despite the strong preference of a number of other members for a more accommodating stance. When faced with this accusation, Nato representatives invariably replied that Nato is an organisation based on unanimity, and that any position taken through complete consensus by so many countries could hardly be blamed for being extremist. This argument may sound convincing at first, and as far as we know it has not been seriously challenged in the public debate: but does it stand up to scrutiny? Motivated by such type of questions, and by the apparent lack of a general framework to address them, we introduce a formal definition of a (bargaining) *alliance* and analyse its behaviour.

For the moment, think of an alliance as a group of 'similarly but not identically motivated members'. We focus on the issue of how the mechanism of decision making within the alliance affects the stance taken *vis à vis* opponents in negotiations. This issue is clearly relevant for a very wide range of economic and political situations. Apart from military alliances, other leading instances of alliances are trading blocks, trade unions, political parties, public companies, business partnerships and families.

For our purposes, an alliance is a group of individual decision makers that:

1. share a common interest, yet also have heterogeneous preferences; and

2. must take a common stance in negotiations.

Our proposal is to model the two seemingly contradictory features in 1 as follows: we assume that members of an alliance share the same preference ordering over the feasible alternatives; yet, their preferences differ in 'intensity'. This is formalised by assuming that members have von Neumann-Morgenstern utility functions which, while not coinciding, are all monotonic increasing when alternatives are listed in an appropriate order, which is then the common preference ordering of the members. In thinking about this definition, one may be helped by considering the shareholders of a company: presumably all of them will agree that higher profits are better than lower ones; however, usually any two distinct shareholders will differ in the evaluation of risky prospects of future profits.

As for 2, we consider several internal decision procedures by which a collective stance is reached by the members of an alliance. We focus especially on the issue of procedures based on unanimity versus procedures based on majority. This allows us to test the seemingly common view that unanimity procedures should yield 'less extreme' negotiating stances. Moreover, we show that, intriguingly, an alliance may greatly benefit from 'rigid' procedures which do not protect some of its members from expected (out of equilibrium) losses during the course of bargaining. The reason for this is that such procedures render credible certain threats concerning out of equilibrium behaviour.

Technically, the centerpiece of our analysis is Rubinstein's Rubinstein (1982) model of alternating offer bargaining, on which we graft the various collective decision procedures. This essentially generates particular models of multiperson bargaining. In view of the fact that such models are often known to exhibit a great multiplicity of equilibria, one might initially be skeptical about the value of this enterprise. But it turns out that focussing on alliances pays off by yielding sharp predictions and by generating, in most cases (although not always), a *unique* equilibrium - *without* imposing stationarity as is often done in multiperson models of bargaining.

In the next section we describe a model in which one single agent negotiates with an alliance. In section 3 we consider a version of the model in which the objects bargained over can be expressed as a real interval. The next section briefly explains the extension to a case with a more general sets of alternatives. The concluding section states the results for the case of bilateral bargaining between two alliances, and summarises the main findings.

2 The basic model

In order to simplify the exposition we assume in the basic model that a single agent negotiates with an alliance. In section 5 we will show that the results we obtain under this simplification essentially carry over to the case of negotiations between two alliances.

2.1 Preferences

There are $N+1$ agents negotiating over alternatives $s \in S$. Specific assumptions on S will be made in later sections. The breakdown event is denoted b. Each agent i has preferences over the set of lotteries on $S \cup \{b\}$, $\mathcal{L}(S \cup \{b\})$, which are representable by a von Neumann-Morgenstern (henceforth vNM) utility function $u_i : \mathcal{L}(S \cup \{b\}) \rightarrow \mathcal{R}$. Abusing notation, we use the same symbol u_i to represent preferences over riskless alternatives.

The agents $i = 2, ..., N+1$ form an alliance in the sense explained in the introduction. Let A denote the set of these agents. So, given $s, s' \in S$, $u_i(s) \geq u_i(s')$ for some $i \in A$ implies $u_j(s) \geq u_j(s')$ for all $j \in A$. Agent 1 has opposite preferences to A over riskless alternatives, that is, given $s, s' \in S$, $u_i(s) \geq u_i(s')$ for some $i \in A$ implies $u_1(s) \leq u_1(s')$. For all $i \in \{1\} \cup A$ there exists $s \in A$ such that $u_i(s) > u_i(b)$.

Note that we do not assume that, given a probability p, for all $i \in \{1\} \cup A$, for all $s \in S$ there exists $s' \in S$ such that $u_i(s') = pu_i(s) + (1-p)u_i(b)$. This means that, in the bargaining game described below, for some player some alternatives may not have a certainty equivalent in S. This way of proceeding seems appropriate in the absence of an explicit discussion of the mechanism by which the 'agenda' S is arrived at. It may well be the case, for example, that for some member of A the breakdown event is so bad that the prospect of risking a breakdown with some positive probability or otherwise getting some 'poor' alternative is worse than any alternative in S. It would certainly be inappropriate to assume, as is commonly done in models of *two-person* bargaining with risk (e.g. Binmore et al. (1986), Osborne and Rubinstein (1990), Myerson (1991), Binmore et al. (1992), Muthoo (1999)), that $u_i(b) = \min_{s \in S} u_i(s)$ or $u_i(b) \geq \min_{s \in S} u_i(s)$, since this would eliminate or limit an important source of heterogeneity between the members of A, namely the extent to which breakdown is disliked. The possible lack of a feasible certainty equivalent does not create any particular problem when $S = [0, 1]$ but requires some analysis in the general case.

2.2 Bargaining

In the bargaining process specified below, agents in A must make *collective* proposals to agent 1. Similarly,agents in A will have to *collectively* either accept or reject proposals by agent 1. We call the mechanism by which agents in A make such collective proposals, acceptances or rejections an *internal procedure,* or simply a *procedure.* Once an internal procedure $\mathcal{P}$ has been fixed, we use expressions such as 'proposal by A', 'acceptance by A' and 'rejection by A' treating A as a single player.

Given any internal procedure $\mathcal{P}$, negotiations between 1 and A proceed in an alternating offers fashion. That is, A, using the procedure $\mathcal{P}$, proposes an alternative $s \in [0, 1]$, which 1 can either accept or reject. In the former case, negotiations end with agreement on s. Otherwise, negotiations continue with probability p; 1 makes a proposal $s' \in [0, 1]$ to A, who can accept or reject, using the procedure $\mathcal{P}$. In the former case, negotiations end with agreement on s'. Otherwise, negotiations continue with probability p, with A making a proposal. Negotiations proceed in this way over an unbounded number of rounds (numbered $r = 0, 1, 2, \ldots$) until agreement is reached, with agent A proposing in even rounds and 1 proposing in odd rounds. After a rejection by A or 1, with probability $1 - p \in (0, 1)$ negotiations break down irreversibly and the breakdown event occurs.

Given a procedure $\mathcal{P}$, we denote by $\Gamma(\mathcal{P})$ the bargaining game in which procedure $\mathcal{P}$ is used.

2.3 Internal Procedures

We consider several types of internal procedures and study their impact on the outcome of negotiations. We illustrate below the types procedures analysed.

2.3.1 Unanimity procedures

Under unanimity procedures, the eventual proposal by A as well as the decision to reject or accept a proposal by 1, are arrived at through non-cooperative negotiation games. Regarding A's responses, given a proposal s by 1, each $i \in A$ either accepts (Y) or rejects (N), in an ordering to be specified later. If all $i \in A$ choose Y, then s is accepted, and the game ends. Otherwise, the game moves, with probability $p_A \in (0, 1)$, to the next round, in which a proposal by A must be made in the following way. Each $i \in A$ proposes an alternative s_i. If, and only if, $s_i = s$ for all $i \in A$ for some $s \in S$, s constitutes A's proposal. Failure to reach unanimity, both when accepting and when proposing, results with probability $1-p_A \in (0, 1)$ in irreversible breakdown of internal negotiations, in which case the outcome is for simplicity identified with b. If internal negotiations continue (with probability p_A), each $i \in A$ makes a new proposal, and so on until unanimity is reached. Regarding the order of moves, we consider two possibilities. In the *simultaneous unanimity* procedure, denoted $\mathcal{P}^{UNSIM}$, all $i \in A$ move simultaneously both when accepting and when proposing. Alternatively, fix a protocol σ, namely a sequence $(i)_{i \in A}$ of agents in A. In the *sequential unanimity procedure with protocol* σ, denoted $\mathcal{P}^{UN\sigma}$, all $i \in A$ move sequentially according to the protocol σ both when accepting and when proposing.

2.3.2 Majority procedures

Under majority procedures, the eventual proposal by A as well as the decision to reject or accept a proposal by 1, are arrived at by playing the same non-cooperative negotiation game illustrated for the majority procedures, but with the outcomes changed in the obvious way, namely: if and only if the majority of $i \in A$ choose Y, is a proposal by 1 accepted; if and only if $s_i = s$ for the majority of $i \in A$ for some $s \in S$, s constitutes A's proposal. The *simultaneous majority procedure* is denoted $\mathcal{P}^{MAJSIM}$ and the *sequential majority procedure with protocol* σ is denoted $\mathcal{P}^{MAJ\sigma}$.

2.3.3 Procedures with safeguard

As we shall see, for the procedures illustrated above there are equilibria where at some nodes A must take an action which is worth to some $i \in A$ less than the worst feasible alternative. This occurs in particular when a proposal by 1 is rejected. Especially in view of the fact that any member of A bargaining on its own with 1 could always obtain an immediate agreement by accepting the worst alternative, this feature could be thought to be conducive to instability in the alliance. It is interesting therefore to consider procedures built from a given procedure $\mathcal{P}^X$ by constraining it to yield each $i \in A$ at least $\min_{s \in S} u_i(s)$ at any node where A is accepting or rejecting an offer by 1: at those nodes, a rejection cannot be made by A if it yields less than $\min_{s \in S} u_i(s)$ to some $i \in A$. For any procedure $\mathcal{P}^X$, let $\mathcal{P}^X_{SAFE}$ denote the 'procedure with safeguard' in which the acceptance rule of $\mathcal{P}^X$ has been so constrained.

2.3.4 Remarks on the formal definition of a procedure

To avoid cumbersome notation, and since the meaning should be clear from the above explanations, we have not defined a procedure rigorously. A procedure is formally a complex object. We view it not merely as a set of ex-ante rules on 'how to play the game': a procedure also restricts the actions that can be taken by members of A *at an equilibrium*. That is, we consider a given equilibrium concept as a benchmark of the basic rationality requirements on players, to which a procedure adds the further 'institutional' structure which we want to single out for analysis. For instance, in this perspective the concept of strong equilibrium[1] could be viewed as a procedure adopted to play a game with Nash equilibrium as a benchmark.

To illustrate this point with more precision, an equilibrium notion E can be viewed as defining the set $E_\sigma(n)$ of all allowable deviations[2] at each node n from any specified strategy profile σ (e.g. in a subgame perfect equilibrium any agent i can deviate at any node if he improves; in a strong equilibrium any coalition can deviate if every agent in it improves, and so on). An $E-$equilibrium is a strategy profile σ such that $E_\sigma(n)$ is empty for all nodes n.

So, fix an equilibrium notion E. The game $\Gamma(\mathcal{P}^X_{SAFE})$ has the same extensive form of

[1]Recall that a strong Nash equilibrium is a Nash equilibrium which is invulnerable to feasible joint deviations by any subset of players, keeping the strategies of the other (non deviating) players fixed.

[2]Among the feasible ones.

$\Gamma(\mathcal{P}^X)$. We define the $E-$equilibria of the game $\Gamma(\mathcal{P}^X_{SAFE})$ as the set Σ_{ES} of strategies profiles of $\Gamma(\mathcal{P}^X)$ such that for all $\sigma \in \Sigma_{ES}$:

1. for all nodes n for which $E_\sigma(n)$ is nonempty, it is the case that any deviation in $E_\sigma(n)$ would push some agent $i \in A$ below his safeguard level.

2. at any node n the action taken yields, given σ, an expected payoffs above the safeguard level for all players $i \in A$.

Point 1 above justifies as equilibria situations in which agent $i \in A$ is protected from being driven below his safeguard level by profitable agreements to deviate among other members of A (or by profitable individual deviations of other members of A).

Point 2 instead disallows as equilibria situations in which agent $i \in A$ is forced below his safeguard payoff by the actions of the rest of A; it is i in effect that can force on the other members a collective action by A that protects his safeguard level.

Note that Σ_{ES} is not a refinement of Σ_E, where Σ_E is the set of $E-$equilibria; and depending on agents' preferences $\Sigma_E \cap \Sigma_{ES}$ may be empty or not empty.

We have chosen to introduce the single term 'procedure' in this way because, although it lumps together formally distinct objects such as rules of the game and equilibrium notion, its interpretation seems to us close to that given to it in the natural language. Clearly, this is just a matter of presentation, which does not affect the substance of any of the results. r

2.4 Equilibrium notion

We now discuss the equilibrium concepts we use. The basic notion for procedures of the sequential type ($\mathcal{P}^{UN\sigma}$, $\mathcal{P}^{MAJ\sigma}$, $\mathcal{P}^{UN\sigma}_{SAFE}$, $\mathcal{P}^{MAJ\sigma}_{SAFE}$) is that of subgame perfect equilibrium (s.p.e.). However, we also need to model a situation in which agents $i \in A$ act non-cooperatively but, being members of an alliance, can openly negotiate and sign binding agreements to *coordinate* their strategies if needed. With procedures of the simultaneous type ($\mathcal{P}^{UNSIM}$, $\mathcal{P}^{MAJSIM}$, $\mathcal{P}^{UNSIM}_{SAFE}$, $\mathcal{P}^{MAJSIM}_{SAFE}$) we thus require the actions in the internal negotiation games to be stable not only with respect to *individual* deviations of each

$i \in A$ but also with respect to *joint* deviations by *groups* of agents in A. That is, we consider s.p.e. in which at no information set can a subset of agents $A' \subset A$ improve the payoff of each agent in A' by jointly changing their actions, given the equilibrium continuation. We define a subgame perfect equilibrium with this property a *jointly stable* subgame perfect equilibrium (j.s.s.p.e.). The requirement of joint stability merely serves the function of excluding artificial equilibria which are created by the use of a simultaneous move internal negotiation game[3]. For example, without this restriction, there would be (stationary) s.p.e. where A always proposes an alternative s such that $u_1(s) \geq u_1(s')$ for all $s' \in S$ even if the continuation payoff for player 1 by rejecting and counterproposing is strictly less than $pu_1(s)$; this would happen as no $i \in A$ could individually deviate to a better proposal.

3 Negotiating over a parameter

We will focus mainly on the case in which the space of alternatives is the unit interval, $S = [0,1]$. This parameter could measure for instance the level of a tariff, or more abstractly a 'stance' between two extremes. We will show later, in section 4, that this restriction of the space of alternatives does not essentially alter the nature of the results. However, the analysis carried out in this context allows one to use and extend the technique introduced in Rubinstein (1982)[4] which proves to be quite illuminating, especially in the study of procedures with safeguard of section 3.3.

For $i \in A$, u_i is increasing and concave on $[0,1]$, while u_1 is decreasing and concave on $[0,1]$. For $i \in A$, for all $s \in [0,1]$ let the 'certainty equivalent' functions[5] $d_i : [0,1] \times \{0, 1, ...\infty\} \rightarrow [0,1]$ be defined by :

$$d_i(s,r) = \begin{cases} s' \in S \text{ such that } u_i(s') = p^r u_i(s) + (1-p^r)\, u_i(d) & \text{if such an } s' \text{ exists} \\ \\ 0 & \text{otherwise} \end{cases}$$

[3]Hence we do not regard this requirement as sufficiently interesting to be part of a specific procedure.

[4]And simplified by Shaked and Sutton (1984).

[5]This corresponds to the 'present value' functions in Osborne and Rubinstein (1990), p. 34.

Similarly,

$$d_1(s,r) = \begin{cases} s' \in S \text{ such that } u_1(s') = p^r u_1(s) + (1-p^r)\, u_1(d) & \text{if such an } s' \text{ exists} \\ \\ 1 & \text{otherwise} \end{cases}$$

To simplify notation we write throughout $d_i(s)$ instead of $d_i(s,1)$ for all $i \in \{1\} \cup A$.

3.1 Unanimity

We consider first the game $\Gamma\left(\mathcal{P}^{UNSIM}\right)$. Let $\overline{s}_{Ai}$ (resp. $\underline{s}_{Ai}$) be the supremum (resp. infimum) of the set of certainty equivalents[6] for player $i \in A$ of the j.s.s.p.e. outcomes (alternative-round pairs) in subgames where A makes a proposal. Similarly, let $\overline{s}_1$ (resp. $\underline{s}_1$) be the supremum (resp. infimum) of the set of certainty equivalents of j.s.s.p.e. outcomes in subgames where agent 1 makes a proposal. Note that $u_i(\overline{s}_{Ai}) \geq u_i(\underline{s}_{Ai})$ for all $i \in A$, whereas $u_1(\overline{s}_1) \leq u_1(\underline{s}_1)$. Let G^A denote subgames where A is the proposer and let G^1 denote subgames where 1 is the proposer.

Theorem 1 *In negotiations over a parameter, the game $\Gamma\left(\mathcal{P}^{UNSIM}\right)$ has a unique j.s.s.p.e., which is stationary and with immediate agreement on the alternative s_A^* characterised as follows. Let s_i^* be the unique equilibrium alternative which would be agreed upon in a two-person bargaining game between agent i alone and agent 1. Then s_A^* is preferred by all agents in A to all s_i^*, or $s_A^* \geq s_i^*$ for all $i \in A$.*

Proof: The values $\overline{s}_{Ai}$, $\underline{s}_{Ai}$, $\overline{s}_1$ and. $\underline{s}_1$can be bounded by several inequalities. For any proposal s by agent 1 such that $s < d_i(\underline{s}_{Ai})$ for some $i \in A$, it is optimal, if feasible, for this agent i to induce a subgame of type G^A. With the procedure $\mathcal{P}^{UNSIM}$ it is feasible for agent i to induce G^A by rejecting s. Thus the certainty equivalent of any equilibrium outcome in subgames where agent 1 acts as proposer must not be smaller than the certainty equivalent of $\underline{s}_{Ai}$ for each $i \in A$:

$$\underline{s}_1 \geq d_i(\underline{s}_{Ai}) \text{ for all } i \in A \tag{1}$$

[6] As we show below, this set is not empty.

Conversely, any proposal s by agent 1 such that $s > d_i(\overline{s}_{Ai})$ for *each* agent $i \in A$ must be accepted in an equilibrium. This bounds from above the certainty equivalent for agent 1 of equilibrium outcomes when he proposes, as follows[7]:

$$\overline{s}_1 \leq \max_{i \in A} d_i(\overline{s}_{Ai}) \tag{2}$$

Next, observe that in any s.p.e. agent 1 must accept any proposal s by A such that $s < d_1(\underline{s}_1)$. So agents in A can Pareto improve (by avoiding delays in the internal proposal game and by ensuring immediate acceptance by agent 1) on any set of actions yielding any $i \in A$ less than $u_i(d_1(\underline{s}_1))$[8]. Thus:

$$\underline{s}_{Ai} \geq d_1(\underline{s}_1) \text{ for all } i \in A \tag{3}$$

The next inequality simply follows from the fact that agent 1 in any s.p.e. must reject any proposal s by A such that $s > d_1(\overline{s}_1)$

$$\overline{s}_{Ai} \leq d_1(\overline{s}_1) \text{ for all } i \in A \tag{4}$$

The above $3N+1$ inequalities characterising a j.s.s.p.e. can now be considerably simplified. Clearly $\underline{s}_{Ai}$ can be attained in a j.s.s.p.e. with immediate agreement (given $d_1(\underline{s}_1)$, choose the stationary j.s.s.p.e. in which all $i \in A$ always propose $d_1(\underline{s}_1)$). Therefore, since all $i \in A$ agree on the ranking of alternatives in the current round, it must be that there exists $\underline{s}_A \in [0,1]$ such that, for all $i \in A$, $\underline{s}_{Ai} = \underline{s}_A$. A similar reasoning establishes that for all $i \in A$ it must be $\overline{s}_{Ai} = \overline{s}_A$ for some $\overline{s}_A \in [0,1]$. Then the following four inequalities are implied:

$$\underline{s}_1 \geq d_i(\underline{s}_A) \text{ for all } i \in A \tag{5}$$

$$\overline{s}_1 \leq \max_{i \in A} d_i(\overline{s}_A) \tag{6}$$

$$\underline{s}_A \geq d_1(\underline{s}_1) \tag{7}$$

$$\overline{s}_A \leq d_1(\overline{s}_1) \tag{8}$$

[7] Rcall that this corresponds to a *lower bound* on the expected utility agent 1 can get when a proposer.

[8] Of course, there may be s.p.e. in which agents coordinate in $\mathcal{P}^{UNSIM}$ on a proposal strictly lower than $d_1(\underline{s}_1)$. See Remark 1 below.

Since 5 can be written as

$$\underline{s}_1 \geq \max_{i \in A} d_i \left(\underline{s}_A \right)$$

inequalities 5-8 also characterise the stationary s.p.e. of a fictitious two player alternating offers bargaining game, Γ^{1R}, in which the players are agent 1 and a 'representative' agent R whose utility function u_R is defined stepwise as follows. Let the function $k : [0,1] \rightarrow \{2, ..., N+1\}$ be defined by

$$k(s) = \min \left\{ \arg \max_{i \in A} d_i(s) \right\}$$

Then

$$u_R(s) = u_{k(s)} \text{ for all } s \in [0,1]$$

so that by definition R's certainty equivalent function d_R is given by

$$d_R(s) = \max_{i \in A} d_i(s)$$

That is, d_R corresponds to the upper envelope of the d_i functions (see example in figure 1). Although u_R is typically not concave or even continuous, some crucial properties of the d_i are inherited by d_R. Firstly, the continuity of the d_i implies that d_R is continuous.

Moreover, a straightforward extension of a standard result (see e.g. Osborne and Rubinstein (1990) pp. 74-75) shows that concavity of the functions u_i implies *increasing loss to delay* of an additional round[9]: that is, $\phi_i(s) = s - d_i(s)$ is an increasing function. This implies immediately that $\phi_R(s) = s - d_R(s)$ is also an increasing function. This fact, by the analysis in Rubinstein (1982), guarantees existence and uniqueness of the s.p.e. agreements of the fictitious game Γ^{1R}, and hence of the original game $\Gamma\left(\mathcal{P}^{UNSIM}\right)$. Note for future reference that, as usual, these values are geometrically characterised by the unique intersection (s_A^*, s_1^*) between the graphs of the functions d_1 and d_R, that $d_1(d_R(s_A^*)) = s_A^*$ and that, in addition, $d_1(d_R(s)) < s$ for $s > s_A^*$.

Finally, we can characterise the equilibrium of Γ^{1R} in terms of pairwise bargaining games between agent 1 and each agent $i \in A$ individually. All such bargaining games have a unique s.p.e. (which is stationary). Denote by s_i^* the equilibrium proposals by agent $i \in A$ in the game with agent 1. These values again correspond to the (unique)

[9] Osborne and Rubinstein (1990) show this for the case where $u_i(b) = u_i(0)$.

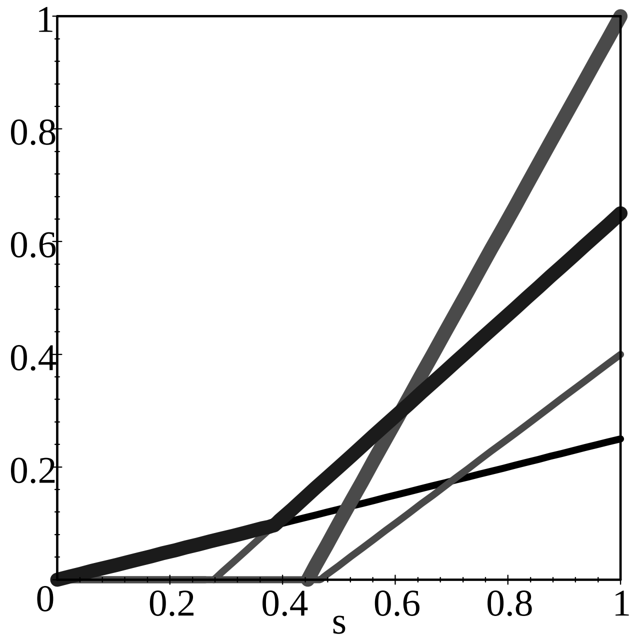

Figure 1: Equilibrium in the $\Gamma\left(\mathcal{P}^{UNSIM}\right)$ game ($A = \{2, 3, 4\}$)

intersections of the graph of the function d_1 with that of each function d_i, and $d_1\left(d_i\left(s_i^*\right)\right) = s_i^*$ for all $i \in A$.

The point of intersection between the graph of d_1 and the graph of d_R is a point of intersection between the graph of d_1 and the graph of some d_i, $i \in A$. Since the intersection (x^i, y^i) between the graph of d_1 and the graph of each d_i is unique and has $x^i = s_i^*$, it follows that $s_A^* = s_i^*$ for some $i \in A$. Suppose now that there exists $j \in A$ for which $s_A^* < s_j^*$. Because the graph of d_1 intersects the graph of d_R only once, it cannot be the case that $d_R(s_j^*) = d_j(s_j^*)$; hence from $d_R(s) \geq d_i(s)$ for all $s \in [0, 1]$ we have $d_R(s_j^*) > d_j(s_j^*)$. By the fact that d_1 is nondecreasing this implies $d_1\left(d_R(s_j^*)\right) \geq d_1\left(d_j(s_j^*)\right)$. But, since $d_1\left(d_j(s_j^*)\right) = s_j^*$, this contradicts the fact that $d_1\left(d_R(s)\right) < s$ for $s > s_A^*$. We can conclude that $s_A^* \geq s_i^*$ for all $i \in A$.

A stationary j.s.s.p.e. of $\Gamma\left(\mathcal{P}^{UNSIM}\right)$ can now be constructed in the obvious way. ■

We see then that the unanimity procedure leads an alliance to adopt a negotiating stance which coincides with that of the agent which is, in terms of the outcome achievable in a hypothetical pairwise bargaining with agent 1, the 'most aggressive' in the alliance.

Furthermore, the solution is highly insensitive to the preferences of the less aggressive members. Any change in such preferences which does not make one of the less aggressive members the most aggressive will not have any impact on the equilibrium agreement.

Finally, note that while the less aggressive members clearly benefit from their membership in the alliance (in the sense that they obtain a higher payoff than they would in individual bargaining), still *within* the game they may forced at some (out of equilibrium) nodes by the most aggressive member to take actions which they consider suboptimal: in particular, there will be out of equilibrium nodes in which the most aggressive member forces a rejection of offers that seem perfectly acceptable to other members.

The fact that there is no equilibrium other than the one 'most unfavourable' to agent 1 is quite remarkable. For instance, one might have expected agent 1 to condition his actions on the identity of rejectors to prevent such an outcome to obtain. Namely, following a rejection by the most aggressive member of the alliance, agent 1 could threaten to make a very unfavourable counteroffer (from the point of view of A). Our result shows that such tactics can never be credible.

Remark 1 *The requirement of joint stability has considerable bite. In its absence, many other s.p.e. exist. For example, consider the following strategies: all $i \in A$ always propose 0 and accept any proposal by agent 1; agent 1 always proposes 0 and accepts only proposals not greater than $d_1(0)$. These strategies constitute an s.p.e. of $\Gamma\left(\mathcal{P}^{UNSIM}\right)$ because although each individual member of A has the power to disrupt the proposal game within A, this would only delay by one round the attainment of the same outcome.*

We now turn to the procedures $\mathcal{P}^{UN\sigma}$.

Theorem 2 *In negotiations over a parameter, for any protocol σ the game $\Gamma\left(\mathcal{P}^{UN\sigma}\right)$ has a unique s.p.e. outcome, which coincides with the unique j.s.s.p.e. outcome of the game $\Gamma\left(\mathcal{P}^{UNSIM}\right)$.*

Proof. Fix σ. We will show that the s.p.e. is characterised by inequalities 1-4, which characterise the j.s.s.p.e. outcome of $\Gamma\left(\mathcal{P}^{UNSIM}\right)$ and ensure uniqueness. That

1, 2 and 4 must hold at an s.p.e. is obvious. Consider now inequality 3, and suppose by contradiction that instead $\underline{s}_{Ai} < d_1(\underline{s}_1)$ for some[10] $i \in A$. However, if members in A coordinated on an offer of $s' = d_1(\underline{s}_1) - \varepsilon > \underline{s}_{Ai}$ for all $i \in A$ (with $\varepsilon > 0$ sufficiently small), this offer would surely be accepted by agent 1 since it is smaller (that is, better) than the certainty equivalent of the any possible s.p.e. continuation. The corresponding certainty equivalent to each agent $i \in A$ would be $s' > \underline{s}_{Ai}$.

Take the first agent in the protocol not to offer s', and index him by σ_1. His proposal can only be optimal if, according to the equilibrium strategies, in the subgame that follows σ_1's out of equilibrium choice of s' some other agent $i \in A$ would himself make a proposal different from s'. Let σ_2 be the first such an agent. In turn, this can only be optimal for agent σ_2 if, according to the equilibrium strategies, in the subgame that follows this agent's (and all previous others') out of equilibrium choice of s', some other agent $i \in A$ would make a proposal different from s', and so on. This argument shows that the existence a subgame perfect equilibrium in which agents in A do not coordinate on s' implies the existence of an infinite sequence $\sigma_1, \sigma_2 \ldots$of distinct agents $i \in A$, contradicting the finiteness of A. This contradiction concludes the proof. ■

The above result shows two separate things, both interesting in their own respect: the invariance of the s.p.e. outcome with respect to changes in the protocol, and its coincidence with the j.s.s.p.e. outcome of $\Gamma\left(\mathcal{P}^{UN\sigma}\right)$. With *any* sequential procedure subgame perfection is enough to guarantee that the requirement of joint stability is also satisfied. We shall see presently that this feature is not peculiar to procedures based on unanimity.

3.2 Majority

The analysis of majority procedures follows closely that of unanimity procedures, so it is only sketched. Consider the game $\Gamma\left(\mathcal{P}^{MAJSIM}\right)$ first. Let the function $m : [0,1] \to \{2, ..., N+1\}$ be defined, for any $s \in [0,1]$, as the lowest number for which:

$$\text{For } N \text{ even} \quad : \quad \#\{i \in A | d_i(s) \leq d_{m(s)}\} = \frac{N}{2}$$

[10] In fact, if this holds for one $i \in A$, it holds for all $i \in A$.

$$\text{For } N \text{ odd} \quad : \quad \#\{i \in A | d_i(s) \leq d_{m(s)}\} = \frac{N+1}{2}$$

That is, $m(s)$ is that agent $j \in A$ for which, given $s \in [0,1]$, his d_j function assumes the median value in the set of values assumed at s by all d_i (if there is more than one agent with this property, we choose the one with the lowest index). By a reasoning analogous to that of the previous subsection, the j.s.s.p.e.e is characterised by the following four inequalities[11]:

$$\underline{s}_1 \geq d_{m(\underline{s}_A)}(\underline{s}_A) \tag{9}$$

$$\overline{s}_1 \leq d_{m(\overline{s}_A)}(\overline{s}_A) \tag{10}$$

$$\underline{s}_A \geq d_1(\underline{s}_1) \tag{11}$$

$$\overline{s}_A \leq d_1(\overline{s}_1) \tag{12}$$

The previous analysis can now be carried through with the function $d_M : [0,1] \to [0,1]$ defined by $d_M(s) = d_{m(s)}$ (which inherits from the d_i exactly the same properties as d_R) in place of d_R, yielding:

Theorem 3 *In negotiations over a parameter, the game $\Gamma\left(\mathcal{P}^{MAJSIM}\right)$ has a unique j.s.s.p.e., which is stationary and with immediate agreement on the alternative s_A^* characterised as follows. Let s_i^* be the unique equilibrium alternative which would be agreed upon in a two-person bargaining game between agent i alone and agent 1. Then s_A^* is the median of the set $\{s_i^*\}_{i \in A}$.*

The equilibrium can be visualised as the intersection of the d_1 function with the 'median envelope' of the d_i functions, d_M (see example in figure 2). The majority procedure leads an alliance to adopt a negotiating stance which coincides with that of the agent which is median in terms of the outcome achieved in hypothetical pairwise bargaining with 1.

For the game $\Gamma\left(\mathcal{P}^{MAJ\sigma}\right)$ by using arguments analogous to those in the proof of theorem 2 one can again establish the invariance of the s.p.e. outcome with respect to the protocol and the implication of joint stability:

[11] Note that in order to write the inequalities corresponding to 1, the requirement of joint stability is now needed.

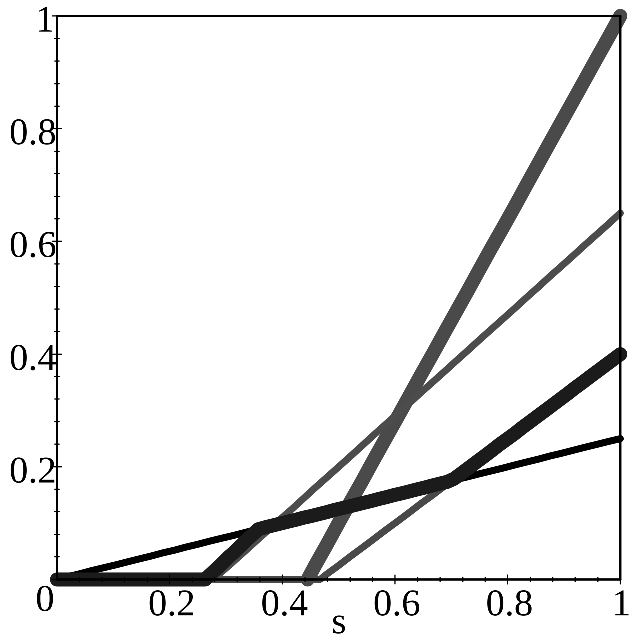

Figure 2: Equilibrium in the $\Gamma\left(\mathcal{P}^{MAJSIM}\right)$ game ($A = \{2, 3, 4\}$)

Theorem 4 *In negotiations over a parameter, for any protocol σ the game $\Gamma\left(\mathcal{P}^{MAJ\sigma}\right)$ has a unique s.p.e. outcome, which coincides with the unique j.s.s.p.e. outcome of the game $\Gamma\left(\mathcal{P}^{MAJSIM}\right)$.*

3.3 Procedures with Safeguard

It is by now clear that all the equilibria derived so far have the following feature: At out of equilibrium nodes where A rejects a proposal s by 1 to counterpropose an alternative s', it may happen that, for some $i \in A$, $pu_i(s') + (1-p)\,u_i(b) < u_i(s)$. The procedures considered essentially force this member i to enter, at some nodes, a subgame which he regards as worse than *any* possible alternative. The results derived so far change substantially when safeguards to avoid this are introduced in the procedures. In particular, the uniqueness of the equilibria which characterised the other cases is lost. More remarkably, it turns out that safeguards can be harmful in the extreme to members of the alliance. To illustrate the logic of the equilibrium in these cases we focus on $\Gamma\left(\mathcal{P}_{SAFE}^{UNSIM}\right)$.

The supremum and infimum of j.s.s.p.e. outcomes to all agents can be bounded as follows. Suppose agent 1 made a proposal s less than $d_i\left(\underline{s}_A\right)$ for some i. Then by rejecting, because of the safeguard requirement, this agent could induce a rejection by A only if $d_j\left(\underline{s}_A\right) > 0$ for all $j \in A$. Thus for s to be accepted, it must be no less than $d_i\left(\underline{s}_A\right)$ for all agents in A if $d_i\left(\underline{s}_A\right) > 0$ for all $i \in A$, and it can be 0 otherwise. So we have

$$\underline{s}_1 \geq \max_{i\in A} d_i\left(\underline{s}_A\right) \text{ if } \min_{i\in A} d_i\left(s\right) > 0$$

and

$$\underline{s}_1 \geq 0 \text{ if } \min_{i\in A} d_i\left(s\right) = 0$$

By a similar reasoning we can establish the analogous upper bound on $\overline{s}_1$. The bounds on $\underline{s}_A$ and $\overline{s}_A$ are as those derived for $\Gamma\left(\mathcal{P}^{UNSIM}\right)$.

Defining $d_G : [0,1] \to [0,1]$ as

$$d_G(s) = \begin{cases} \max_{i\in A} d_i\left(s\right) & \text{if } \min_{i\in A} d_i\left(s\right) > 0 \\ \\ 0 & \text{if } \min_{i\in A} d_i\left(s\right) = 0 \end{cases}$$

the bounds found above can therefore be written as

$$\underline{s}_1 \geq d_G\left(\underline{s}_A\right) \quad (13)$$

$$\overline{s}_1 \leq d_G\left(\overline{s}_A\right) \quad (14)$$

$$\underline{s}_A \geq d_1\left(\underline{s}_1\right) \quad (15)$$

$$\overline{s}_A \leq d_1\left(\overline{s}_1\right) \quad (16)$$

Consider now the set E of points $(s, s') \in [0,1] \times [0,1]$ satisfying

$$s = d_G\left(s'\right) \quad (17)$$

$$s' = d_1\left(s\right) \quad (18)$$

The set E is nonempty if and only if the function $g : [0,1] \to [0,1]$ given by $g(s) = d_1\left(d_G\left(s\right)\right)$ has a fixed point.

Lemma 5 *There exists $s^* \in [0,1]$ such that $g(s^*) = s^*$*

Proof: The (possibly discontinuous) function g is clearly non decreasing given the definition of d_G and the assumptions on the functions d_i. It maps the complete lattice $([0,1], \geq)$ to itself. Therefore Tarsky's fixed point theorem implies the result. $\square$

Since as is well-known points in E define the j.s.s.p.e. of $\Gamma\left(\mathcal{P}_{SAFE}^{UNSIM}\right)$ which are stationary, we have thus established the existence of an equilibrium. An example of this type of equilibrium is depicted in figure 3.

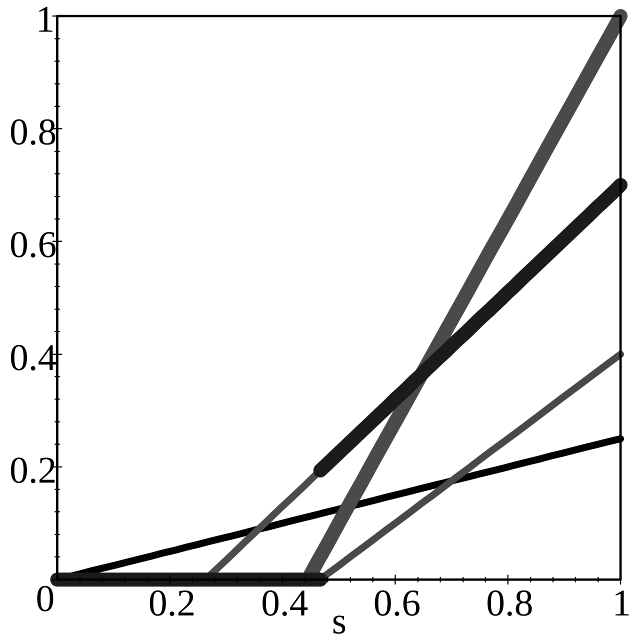

Figure 3: Equilibria in the $\Gamma\left(\mathcal{P}_{SAFE}^{UNSIM}\right)$ game $(A = \{2,3,4\})$

To characterise the possible equilibrium configurations, consider the following four exhaustive cases:

1. $d_G(\underline{s}_A) = 0 = d_G(\overline{s}_A)$. Then system 13-16 yields $\underline{s}_1 = \overline{s}_1 = 0$ and $\underline{s}_A = \overline{s}_A = d_1(0)$. So we have $0 = d_G(d_1(0))$, and therefore the pair $(d_1(0), 0)$ is also a fixed point of system 17-18. We can conclude that, $d_1(0)$ is the unique j.s.s.p.e. outcome of $\Gamma\left(\mathcal{P}_{SAFE}^{UNSIM}\right)$, and the j.s.s.p.e. is stationary.

2. $d_G(\underline{s}_A) = \max\limits_{i \in A} d_i(\underline{s}_A) > 0$, $d_G(\overline{s}_A) = \max\limits_{i \in A} d_i(\overline{s}_A) > 0$. System 13-16 reduces to

system 5-8. From the proof of Theorem 1 we know that the function $d_1\left(\max_{i\in A} d_i\right)$ has a unique fixed point, so that $\underline{s}_A = \overline{s}_A = s_A^* \in (0,1]$ and $\underline{s}_1 = \overline{s}_1 = s_1^* \in (0,1)$. Therefore s_A^* is the unique j.s.s.p.e. outcome of $\Gamma\left(\mathcal{P}_{SAFE}^{UNSIM}\right)$; the j.s.s.p.e. is stationary and coincides with the j.s.s.p.e. of. $\Gamma\left(\mathcal{P}^{UNSIM}\right)$.

3. $d_G(\underline{s}_A) = \max_{i\in A} d_i(\underline{s}_A) > 0$, $d_G(\overline{s}_A) = 0$. This is not possible, since by the fact that d_G is non decreasing we have $d_G(\overline{s}_A) \geq d_G(\underline{s}_A)$.

4. $d_G(\underline{s}_A) = 0$, $d_G(\overline{s}_A) = \max_{i\in A} d_i(\overline{s}_A) > 0$. System 13-16 yields $d_1\left(\max_{i\in A} d_i(\overline{s}_A)\right) \geq \overline{s}_A \geq \underline{s}_A \geq 0$. Clearly if it was $d_1\left(\max_{i\in A} d_i(\overline{s}_A)\right) > \overline{s}_A$ there would exist $s' > \overline{s}_A$ such that $d_1\left(\max_{i\in A} d_i(s')\right) = s'$, so that s' in round 0 would be a (stationary) j.s.s.p.e. outcome, in contradiction with the definition of $\overline{s}_A$. So it must be $d_1\left(\max_{i\in A} d_i(\overline{s}_A)\right) = \overline{s}_A$. Thus from the proof of Theorem 1 we can deduce that in $\Gamma\left(\mathcal{P}_{SAFE}^{UNSIM}\right)$ $\overline{s}_A > 0$ in round 0 coincides with the unique j.s.s.p.e. outcome of $\Gamma\left(\mathcal{P}^{UNSIM}\right)$ and is attained in a stationary j.s.s.p.e. Consider now $\underline{s}_A$. From $d_G(\underline{s}_A) = 0$ and $d_G(\overline{s}_A) > 0$ it follows that $\underline{s}_A < \overline{s}_A$. Moreover $\max_{i\in A} d_i(\underline{s}_A) = 0$, which implies $\underline{s}_1 = 0$. It is then trivial to show that there exists a stationary j.s.s.p.e. in which A always proposes $\underline{s}_A = d_1(0)$ and 1 always proposes 0.

Similar arguments can be used for $\Gamma\left(\mathcal{P}_{SAFE}^{MAJSIM}\right)$. We can then summarise the analysis as follows:

Theorem 6 *Let $\mathcal{P}_{SAFE}^X$ be a procedure, with $X \in \{MAJSIM, UNSIM\}$. In negotiations over a parameter there exists a j.s.s.p.e. of the game $\Gamma\left(\mathcal{P}_{SAFE}^X\right)$. The equilibrium set can be of three types: either (1) a unique and stationary j.s.s.p.e. coinciding with the j.s.s.p.e. of $\Gamma\left(\mathcal{P}^X\right)$; or (2) a unique and stationary equilibrium in which A always proposes $d_1(0)$ and 1 always proposes 0; or (3) a set of j.s.s.p.e. whose outcomes are bounded by the equilibrium outcomes of cases (1) and (2). In particular, any j.s.s.p.e. outcome of $\Gamma\left(\mathcal{P}_{SAFE}^X\right)$ is not greater, and can be strictly lower, than the j.s.s.p.e. outcome of $\Gamma\left(\mathcal{P}^X\right)$.*

We see then that, whatever the procedure, the introduction of safeguards for agents $i \in A$ can have at best no effect and at worst a negative effect on the outcome of negotiations.

Consider for instance the initial situation where negotiations between agent 1 and A proceed according to $\Gamma\left(\mathcal{P}^{UNSIM}\right)$, where preferences of the members of the alliance are such that they can be represented by the d_i functions in the left panel of figure 4, so that in equilibrium an agreement is reached immediately on some $(s_A^*, s_1^*) \in (0,1)^2$.

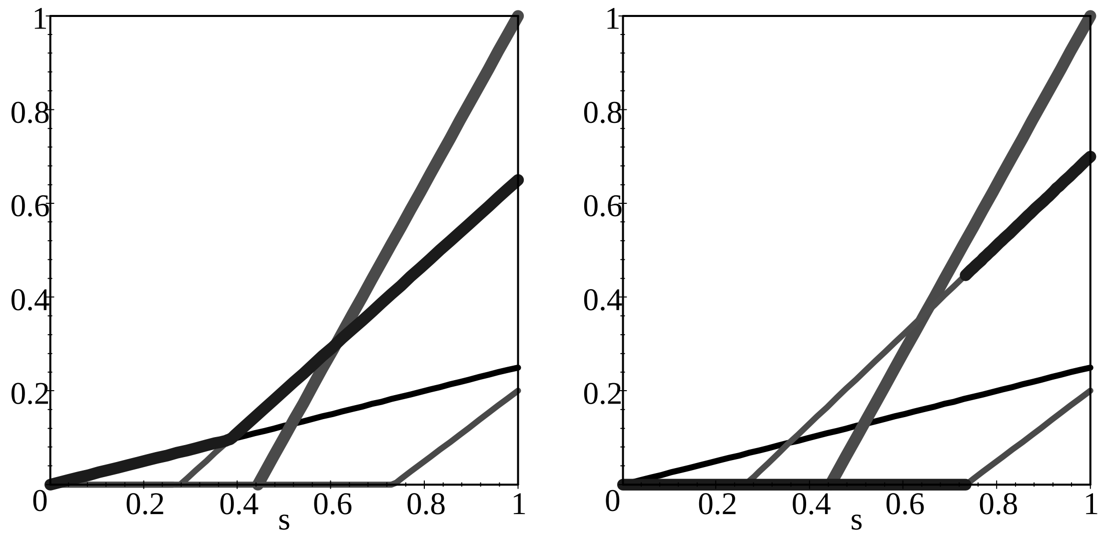

Figure 4: The unique equilibrium in $\Gamma\left(\mathcal{P}^{UNSIM}\right)$ differs from the one in $\Gamma\left(\mathcal{P}^{UNSIM}_{SAFE}\right)$.

Consider instead the case where members of the alliance negotiate with agent 1 according to $\Gamma\left(\mathcal{P}^{UNSIM}_{SAFE}\right)$. Now the equilibrium outcome is with immediate agreement on $s_A^* = d_1(0)$; furthermore, no other equilibrium outcome can be supported (see right panel in figure 4). Under unanimity, a proposal of 0 by agent 1 is deterred in equilibrium by the threat that the most aggressive agent will prefer to reject, take the risk of breakdown and move on to the next round. This threat is credible since every member of the alliance enjoys the power to veto an agreement. On the other hand, when safeguards are introduced, the veto power of each member of the alliance is limited by the requirement to take into account the wishes of the less aggressive members (i.e. those with a zero certainty equivalent). Then, the more aggressive members' threat to reject a proposal of 0 by agent 1 may no longer be credible.

Depending on preferences, however, multiple equilibria can result, as for instance in the example of figure 3. As long as a strictly positive certainty equivalent of $s_1^* \in (0,1)$ is defined for all members of the alliance, the safeguard requirement does not hold, so that the threat to use one's veto power is restored to its credibility.

Remark 2 *It is easy to show that in case (3) of Theorem 6 there is in fact a continuum of equilibrium outcomes, supportable in the standard way by using as 'punishments' the extreme stationary equilibria. In other words, all agreements* $s^* \in [d_1(0), s_A^*]$ *can be supported at an equilibrium with immediate agreement where, along the equilibrium path agents in* A *propose* s^**, which is accepted. Deviations from the equilibrium are punished by reverting to the (equilibrium) strategy profile which supports the worst equilibrium outcome for the deviator.*

4 Negotiating over a general set of alternatives

In this section we show that the results of the previous sections to a large extent do not depend on the choice of a real interval as the set of alternatives. Consider a generic set of alternatives S. Each pairwise bargaining game between agent 1 and each agent i generates a feasible set of utility pairs, $B_i \subset [\underline{u}_1, \overline{u}_1] \times [\underline{u}_i, \overline{u}_i]$, where $\overline{u}_j$ (resp. $\underline{u}_i$) $j \in \{1\} \cup A$ is the maximum (resp. minimum) feasible utility for player j. We assume as in standard models that B_i can be described as $B_i = \{u = (u_1, u_i) \,|u \geq (u_1(b), u_i(b)), u_i \leq \phi_i(u_1)\}$, where for all $i \in A$ $\phi_i : [\underline{u}_1, \overline{u}_1] \rightarrow [\underline{u}_i, \overline{u}_i]$ is a continuous strictly decreasing and concave function. However, as explained in section 2 we depart from those models by not assuming that $\underline{u}_i \leq u_i(b)$ or $\underline{u}_i = u_i(b)$.

We consider for the sake of brevity only the procedure $\mathcal{P}^{UNSIM}$. It is straightforward to extend all the other results of the previous sections to the case of a general set S.

Theorem 7 *In negotiations over a general set of alternatives, the game* $\Gamma\left(\mathcal{P}^{UNSIM}\right)$ *has a unique j.s.s.p.e., which is stationary and with immediate agreement, as described in table 1.*

Proof. Part of the statement of Theorem 1 in Manzini and Mariotti (2000) establishes:

Lemma 8 *The bargaining problem* B_i *has a unique s.p.e. equilibrium.*

For each such bargaining problem we can define the unique equilibrium payoff pair as the solution, for some $s_i^1, s_1^i \in S$, to the following system:

$$u_i\left(s_i^1\right) = \max\left\{pu_i\left(s_1^i\right) + (1-p)\,u_i(b)\,, \underline{u}_i\right\}$$
$$u_1\left(s_1^i\right) = \max\left\{pu_1\left(s_i^1\right) + (1-p)\,u_1(b)\,, \underline{u}_i\right\}$$

Indeed, there may exist more than one alternatives $s \in S$ such that $u_i(s) = u_i\left(s_i^1\right)$. Since however all agents $i \in A$ have the same preference ordering over alternatives, we abuse notation and let s_i^1 be any alternative from the indifference set $I_i\left(s_i^1\right) = \left\{s \in S | u_i(s) = u_i\left(s_i^1\right)\right\}$. Similarly, for agent 1 we denote by s_1^i any alternative from the indifference set $I_1\left(s_1^i\right) = \left\{s \in S | u_1(s) = u_1\left(s_1^i\right)\right\}$.

For simplicity, without loss of generality, index equilibrium alternatives in increasing order of preference for agents in A, so that $s_{j+1}^1 \geq s_j^1$ for all $j = 2...N$. Similarly, $s_1^{j+1} \geq s_1^j$ for all $j = 2, ..., N$. Then we can use the equilibrium payoffs of the N pairwise bargains between agent 1 and each agent $i \in A$ to construct a strategy profile which constitutes a j.s.s.p.e. for the $\mathcal{P}^{UNSIM}$ game over a general set of alternatives. This strategy profile, which is stationary, is described in Table 1.

Agent 1:
proposes alternative s_{N+1}^1
accepts any alternative $s \leq s_1^{N+1}$ and rejects otherwise
Agent $i \in A$:
proposes alternative s_1^{N+1},
accepts any alternative $s \geq s_1^{N+1}$ and rejects otherwise
Note: s_{N+1}^1 and s_1^{N+1} are the equilibrium proposals by agent 1 and agent $N+1$, in a pairwise bargaining respectively, in a pairwise bargain between them alone.

Table 1: Equilibrium strategies for Theorem 7

It easily checked that the strategies in Table 1 constitute an equilibrium. Just notice that agent $i \in A$ has no incentive to accept $s_i^1 < s_{N+1}^1$ since at least agent $N+1$ would reject such an alternative, which under $\mathcal{P}^{UNSIM}$ ensures that the alternative is rejected by A.

We now show that the j.s.s.p.e. equilibrium just described is unique. To this end, we first show that no other alternative can be supported in an equilibrium with immediate agreement[12].

Lemma 9 *There can be no subgame where agent* 1 *proposes an alternative* $s < s^1_{N+1}$ *which is accepted.*

Proof. Suppose not, so that the supremum equilibrium payoff for agent 1 in subgames G^1 is $M_1 > u_1\left(s^1_{N+1}\right)$, and the corresponding infima for $i \in A$ are m_i. Then we show that members of the alliance would have a profitable deviation, that is, they could reject and make a profitable counteroffer which would be accepted.

Since the equilibrium outcome considered is precisely the (unique) one that would be established in a two-person bargain between agent 1 and agent $N+1$, by standard arguments it must be that

$$pu_{N+1}\left(u_1^-\left(pM_1\right)\right) > m_{N+1}$$

where $u_1^- : R \rightarrow S$ is a function that selects one alternative from $I_1(s) \subset S$ such that $u_1\left(u^{-1}\left(pM_1\right)\right) = pM_1$.

Consequently, agent $N+1$ will always reject a proposal yielding agent 1 more than $u_1\left(s^1_{N+1}\right)$ unless in the internal procedure among agents in A a proposal is made yielding agent 1 more than M_1. But this is ruled out by the requirement that the equilibrium be jointly stable. □

A similar argument can be used to establish:

Lemma 10 *There can be no subgame where agents in A propose an alternative* $s > s^{N+1}_1$ *which is accepted.*

To conclude the proof, it is necessary to show that there can be no delayed agreement equilibria. Such a proof is standard[13] and thus omitted. ■

[12]Clearly, there will be other outcome-equivalent equilibria in which agents $i < N+1$ in A accept $s \prec_i s^1_{N+1}$ while $N+1$ still rejects.

[13]See for instance Muthoo (1999).

5 Concluding Remarks

Although for simplicity we have limited ourselves to analysing negotiations between an alliance and a single opponent, it is possible, by using the same techniques, to extend all results to the case of negotiations between two alliances. Let $A = \{1, ..., N\}$ and $B = \{N+1, ..., N+M\}$ be two alliances with opposite preferences over alternatives in S. Denote by $\Gamma(\mathcal{P}_A, \mathcal{P}_B)$ the game between two alliances in which A uses the internal procedure $\mathcal{P}_A$ and B uses the internal procedure $\mathcal{P}_B$, and where A is the first proposer. Then a typical result would read as follows:

Theorem 11 *In games* $\Gamma\left(\mathcal{P}_A^{UNSIM}, \mathcal{P}_B^{UNSIM}\right)$ *and* $\Gamma\left(\mathcal{P}_A^{MAJSIM}, \mathcal{P}_B^{MAJSIM}\right)$ *let* s^*_{iX}*, with* $X = A, B$ *and* $i \in \{A \cup B\} \backslash X$*, be the unique*[14] *equilibrium alternative which would be agreed upon in a game between agent* i *alone and alliance* X*. Then: (1) The game* $\Gamma\left(\mathcal{P}_A^{UNSIM}, \mathcal{P}_B^{UNSIM}\right)$ *has a unique j.s.s.p.e., which is stationary and with immediate agreement on an alternative* s^*_{AB} *such that* s^*_{AB} *is preferred by all agents in* $\{A \cup B\} \backslash X$ *to all* s^*_{iX}*. (2) The game* $\Gamma\left(\mathcal{P}_A^{MAJSIM}, \mathcal{P}_B^{MAJSIM}\right)$ *has a unique j.s.s.p.e., which is stationary and with immediate agreement on the alternative* s^*_{AB} *which is the median of the set* $\{s^*_{iX}\}_{i \in A}$*.*

Our model has highlighted several interesting features of 'bilateral' negotiations when at least one of the sides is an alliance representing the non identical interests of a number of individuals. Our main findings can be summarised as follows:

1. Unanimity rules are 'better' for an alliance than majority rules, in the sense that the equilibrium outcome for an alliance under unanimity dominates the one under majority, whatever the bargaining behaviour of the opponent. More specifically, unanimity rules, lead the alliance to behave as if the 'most agreesive' member had been delegated to bargain on its behalf.

2. In sequential procedures the protocol does not matter, in the sense that the optimal actions arrived at are the same regardless of the protocol followed. Furthermore,

[14]From Theorems 1 and 3.

the s.p.e. outcome of sequential procedure coincides with the jointly stable s.p.e. of simultaneous procedures.

3. ‘Weaker’ members benefit from the presence of ‘stronger members’ in an alliance, in the sense that, for any procedure, acquiring a more aggressive member (one which would obtain a better result in individual bargaining) does not worsen, and possibly strictly improves, the equilibrium outcome for an alliance. However, weaker members may be forced, in virtue of their being bound by the internal procedural rules, to take actions they regard as suboptimal at out of equilibrium nodes.

4. Surprisingly, introducing safeguards for ‘weaker members’ in order to avoid the feature in 3 harms everybody (including the very weaker members to be protected) and may ‘soften’ the negotiating tactics of the alliance to an extreme degree. The bargaining strength of an alliance must be based on internal procedures being sufficiently rigid to make credible threats of actions on the part of the alliance which would harm some of its own members.

Clearly, the choice of a procedure for an alliance has many other facets which we have not considered in the paper. A major example of these, often debated in practice[15], is the extent to which the majority rule would yield more *timely* decisions. Analysing such features is left for future research.

[15]For instance within the European Union.

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