A Theory of Vague Expected Utility

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Abstract

We propose a new theory of choice between lotteries, which combines an 'economic' view of decision making - based on a rational, though incomplete, ordering - with a 'psychological' view - based on heuristics. This theory can explain observed violations of EU theory, namely all cyclical patterns of choice as well as violations of independence.

Keywords: incomplete preference relation; cyclical preferences; expected utility.

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1 Introduction

In the face of the descriptive inadequacy of the Expected Utility (EU) model of decision under risk, many alternative theories have been proposed[1]. Most of these alternative theories address the violations of the Independence axiom, which were first highlighted by Allais [1] in his famous paradox. However far fewer are able to deal with the problem of *intransitivity*, which has also been extensively documented, starting with Tversky [24]. This type of violation is more fundamental, in that it seems to undermine not simply a specific theory of choice under risk, but the very principle of rationality (usually intended as maximisation of a transitive ordering). It is fair to say that there has been a tendency to sweep this problem under the carpet, and there are very few formal theories that can accommodate both types of violations outlined above. They are due to Bell [3], Fishburn [8] and Loomes and Sugden [15]. They are based on the psychological phenomenon of *regret* and can explain *specific* types of cycles in pairwise choices between three gambles.

In a different tradition, psychologists have tended to emphasize the role of *heuristics* and *rules of thumb* in human decision-making, at the expense of general formal properties, or axioms, which are rooted in economic theory.

In this paper we propose a theory that in a sense reconciles these two traditions, and which turns around the idea of *incompleteness* of rational preferences. We contend that of the usual rationality properties of preferences, it is that of completeness which is the critical one to explain the phenomena mentioned above. This may be surprising since completeness has been the least critically examined rationality property for preferences over gambles, at least until recently[2].

Our starting point is the recognition that choosing among lotteries is an unfamiliar and cognitively complex task. Moreover, some choices are intrinsically harder than others. For example, comparing two gambles one of which dominates the other is clearly simpler than comparing two gam-

[1] See e.g. Starmer [23] for a recent survey.

[2] See e.g. Dubra, Maccheroni and Ok[7], Manzini and Mariotti[17]. The pioneering contributions are by Aumann [2] and Bewley [4].

bles where such a relation of dominance does not exist. In general people are sometimes able to evaluate the trade-off between the probability and outcome dimensions, whereas sometimes they will find this operation just too difficult, and will rely on some secondary decision heuristics, or 'rule of thumb'. While economists in general have focused on the fact that people decide on the basis of some preference ordering, psychologists have tended to emphasise the *procedural* aspects of decision making, that is, the heuristics that people use. We suggest that a descriptively satisfactory theory should recognise that although people may not act on the basis of a *complete* rational preference ordering over gambles, still they possess a *partial* preference ordering (e.g. among gambles in a dominance relation). This partial preference ordering as distinct from a simple rule of thumb, should respect the usual axioms of rationality for preferences over lotteries.

In a general way, our approach is in line with Sen's [19] observation that:

"A chooser, who may have to balance conflicting considerations to arrive at a reflected judgement, may not, in many cases, be able to converge on a complete ordering when the point of decision comes. If there is no escape from choosing, a choice decision will have to be made even with incompleteness in ranking" (p. 746).

The 'reflected judgement' in this quotation is what we summarize in the 'rational' partial preference ordering. However, because a choice must be made, the cognitive 'holes' where the preference ordering fails must be dealt with in a way that transcends reflected judgement. It is here that we think that the heuristics that psychologists have emphasised may play a role.

Of course it would be easy to explain cycles by directly postulating a heuristic that fails to satisfy even transitivity, but this is not our approach. The potentially cyclical preferences we are able to explain are based on the combination of two *transitive* criteria, the partial preference ordering and the rule of thumb. One notable consequence of our theory is that it belies the standard economist view that cyclical preferences are diametrically opposed to the idea of rationality: we will show that, in a very precise sense, in order to explain intransitivities the individual will need to combine both rational judgements and heuristics. An individual guided solely by rational judgements or solely by heuristics will not reveal cyclical patterns of preferences.

Our theory is simple and parsimonious, and we delimit it in a clear way by focussing our attention on elementary monetary gambles[3]. We propose that the 'rational' part of the individual's judgements is modeled just as in EU theory. The incompleteness of rational preferences is modeled by means of a 'vagueness' function, which expresses the cognitive difficulty the individual faces in applying rationality. This (incomplete) version of EU theory is combined with a well-known lexicographic heuristics for multi-attribute decision making (studied for example by Slovic [21]).

This heuristics is a simple lexicographic (transitive) criterion[4]: the individual will either first scan the probability dimension and if this is not conclusive the outcome dimension; or he will first privilege the outcome dimension and then follow with the probability dimension. We show that in this way violations of EU theory - such as Allais-type phenomena and cycles - can be reconciled even with a theory that is extremely close to it - our σEU model.

Although in principle our approach can be easily extended to non elementary gambles, we prefer to narrow our applications to this domain because we believe that additional cognitive issues might affect choices over non elementary gambles. In the domain of elementary gambles there is a clear cut trade-off to be made between probabilities and outcomes; there is limited possibility for framing effects to have a bite; and there is no issue of what the attributes are. With general gambles the situation is not so clear cut, and there is likely to be more discussion about the proper heuristics to be used. However, we emphasize that there is no intrinsic impossibility in principle to extend our theory to general gambles. We are just not expert enough in psychology to be ready to do so.

To the best of our knowledge our approach is new[5], though it is con-

[3] By elementary monetary gambles we intend lotteries that attach a given probability to an amount of money in a given set, and the complementary probability to getting nothing.

[4] The use of heuristics which treat alternatives as sets of characteristics, and lexicographically consider those characteristics, is well documented in the psychological literature (from e.g. Tversky [25] to the 'Take the Best and Leave the Rest' heuristic introduced in Gigerenzer and Goldstein [9]).

[5] In Manzini and Mariotti [16] we have proposed to use an analogous approach to the

ceptually related to Rubinstein [18], who pioneered a theory of choice over elementary gambles based on similarity relations. A similarity is formally analogous to our notion of vagueness. However, the main departure from Rubinstein is that in our theory the crucial feature is vagueness *between* gambles rather than in each dimension (probability and outcome). The combination of this characteristic with the secondary heuristics is able to generate intransitivities, whereas Rubinstein's theory implies transitive choices.

2 σEU and its applications

Let X be a set of monetary consequences, and for simplicity identify X with a closed real interval with 0 as its lower extreme. We consider individual preferences on the set G of elementary monetary gambles on X. These are the lotteries of the type $(x, p; 0, 1-p)$ to mean that the monetary prize $x \in X$ is won with probability p, and zero with the complementary probability (see figure 1). To shorten notation we will denote such a gamble simply by (x, p).

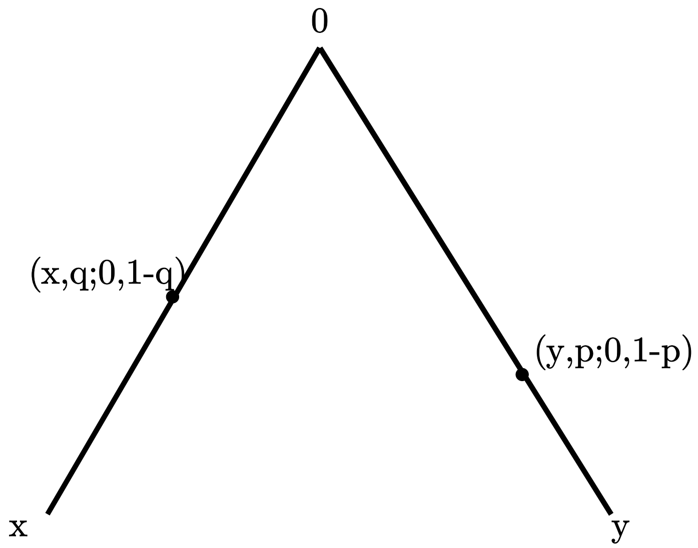

Figure 1: Restriction to elementary gambles

study of the theory of choice over time. The axiomatics of that paper is, however, entirely separate from the theory developed in the present paper.

The individual decides in the first instance on the basis of a *primary* criterion $\succ$ which partially orders G. Therefore $g_1 \succ g_2$ means that gamble g_1 is rationally preferred to gamble g_2. When g_1 and g_2 cannot be rationally compared, we say that individual is 'vague', and we write $g_1 \sim g_2$. In order to 'break' vagueness and formulate a choice, the individual will have to resort to a *secondary* criterion. Following a considerable body of work in psychology literature (see e.g. Slovic [21], Tversky, Sattath and Slovic [26] and Shafir, Simonson and Tversky [20]) we contend that when faced with multi attribute alternatives, decision makers focus *lexicographically* on each individual attribute. In the case of elementary gambles the relevant attributes or dimensions are clear (and as explained this is our reason for focussing on them): they are probability and outcome. So, when vague, the individual will either choose the alternative which offers the higher outcome or the one which offers the higher probability of a positive outcome.

Individual choice is finally determined by a (complete) ordering $\succcurlyeq^*$ constructed by combining the primary and secondary criteria.

Under standard conditions the partial order $\succ$ which describes the primary criterion can be represented by a utility function $u : G \to \mathcal{R}$ and by a symmetric 'vagueness' function $\sigma : G \times G \to \mathcal{R}$, so that $g_1 \succ g_2$ if and only $u(g_1) > u(g_2) + \sigma(g_1, g_2)$.

Now we proceed to explain how $\succcurlyeq^*$ can be constructed in two different ways, according to which attribute has prominence.

Let $g_i = (x_i, p_i) \in G$. Then:

Outcome Prominence:

1. $a \succ^* g_2 \Leftrightarrow$
   (a) $u(g_1) > u(g_2) + \sigma(g_1, g_2)$ (primary criterion), or
   (b) $u(g_i) \leq u(g_j) + \sigma(g_i, g_j)$ $(i = 1, 2)$ and either $x_1 > x_2$ or $x_1 = x_2$ and $p_1 > p_2$ (secondary criterion)

2. $g_1 \sim^* g_2 \Leftrightarrow u(g_i) \leq u(g_j) + \sigma(g_i, g_j)$ $(i = 1, 2)$ and $x_1 = x_2$ and $p_1 = p_2$.

Probability Prominence:

1. $g_1 \succ^* g_2 \Leftrightarrow$

   (a) $u(g_1) > u(g_2) + \sigma(g_1, g_2)$ (primary criterion), or

   (b) $u(g_i) \leq u(g_j) + \sigma(g_i, g_j)$ $(i = 1, 2)$ and either $p_1 > p_2$ or $p_1 = p_2$ and $x_1 > x_2$ (secondary criterion)

2. $g_1 \sim^* g_2 \Leftrightarrow u(g_i) \leq u(g_j) + \sigma(g_i, g_j)$ $(i = 1, 2)$ and $x_1 = x_2$ and $p_1 = p_2$.

Notice that although both the primary partial order $\succ$ and the lexicographic procedure assumed above are transitive, this does not imply that the whole procedure (i.e. the combination $\succcurlyeq^*$ of the primary and secondary criteria) is also transitive, independently of which secondary criterion applies.

In what follows we concentrate on a model which is - so to say - the smallest possible departure from standard EU theory, and still is able to account for a number of profound violations of that theory.

First, we assume the linearity of u in probability and therefore we impose on u the expected utility property, that is $u(x, p) = pu(x) + (1 - p)u(0)$ for all $x \in X$, $p \in [0, 1]$, where with abuse of notation we denote $u(x)$ as the utility of the degenerate gamble $(x, 1)$. We further assume that u is concave and normalise $u(0)$ to zero. Finally we let $\sigma(g_i, g_j) = \sigma$ for all $(g_i, g_j) \in G$. This last simplification makes vagueness independent of the specific gambles being compared. However, note that *relative* vagueness does depend on them, and in particular - ceteris paribus - it increases as the probabilities of the positive prize decrease.

In this simple version of our model, which we will refer to as σEU, the decision maker chooses $g_1 = (x, p)$ over $g_2 = (y, q)$ by the primary criterion if and only if $pu(x) > qu(y) + \sigma$.

It is worth noting that σEU satisfies the following form of independence:

$$g_1 \sim g_2 \Rightarrow \alpha g_1 \sim \alpha g_2 \text{ for all } \alpha \in [0, 1]$$

where αg_i, with $g_i = (x_i, p_i)$, denotes the elementary gamble $(x_i, \alpha p_i)$. In fact:

$$g_1 \sim g_2 \Leftrightarrow p_i u(x_i) \leq p_j u(x_j) + \sigma,\ i,j = 1,2 \text{ and } i \neq j \Leftrightarrow$$
$$\alpha p_i u(x_i) \leq \alpha p_j u(x_j) + \alpha\sigma \Rightarrow \alpha p_i u(x_i) \leq \alpha p_j u(x_j) + \sigma \Leftrightarrow \alpha g_1 \sim \alpha g_2$$

However, it is not necessarily true that $g_1 \succ g_2 \Rightarrow \alpha g_1 \succ \alpha g_2$, as it is easily verified. Only the implication $g_1 \succ g_2 \Rightarrow \alpha g_1 \not\prec \alpha g_2$ holds.

2.1 Ratio effect

Kahneman and Tversky [10] highlighted a common violation of the independence axiom using elementary gambles in a series of experiments, based on the original Allais [1] paper. These examples exhibit the so called 'probability ratio' effect, which is a simpler demonstration of the violation of the independence axiom discovered by Allais. Two pairwise comparisons of gambles are made. Denoting the first pair of gambles to be compared (x, p) and (y, q), the second pair has the form $(x, \alpha p)$ and $(y, \alpha q)$ where $0 < \alpha < 1$. The independence axiom implies that (x, p) is preferred to (y, q) if and only if $(x, \alpha p)$ is preferred to $(y, \alpha q)$. However, Kahneman and Tversky showed that for some choices of outcomes and probabilities this predicted pattern of choice was contradicted in actual fact. This phenomenon has been replicated in several subsequent studies (refer to Starmer [23]). We will show that the σEU model, despite constituting a minimal departure from the standard EU model, can easily account for the 'paradoxical' choice patterns.

To see this, consider the gambles[6] $g_1 = (4000, 0.8)$, $g_2 = (3000, 1)$, $g_3 = (4000, 0.2)$ and $g_4 = (3000, 0.25)$. Note that g_3 and g_4 are the same as g_1 and g_2, respectively, with the probability of the positive prize reduced by a factor $\alpha = 0.25$. In experiments it is normally found that a significant majority of choosers picks g_2 over g_1 and g_3 over g_4, violating independence and EU. These choices, however, are consistent with σEU. In fact, for this pattern of choice it is simply required that

$$\begin{aligned}
&u(3000) > (0.8)\, u(4000) + \sigma \\
&(0.25)\, u(3000) \leq (0.2)\, u(4000) + \sigma \\
&(0.2)\, u(4000) \leq (0.25)\, u(3000) + \sigma
\end{aligned}$$

[6] Taken from Kahneman and Tversky [10].

The first line ensures that g_2 is chosen over g_1 by the primary criterion. The last two inequalities assert that the comparison between g_3 and g_4 is 'vague'. Then, based on the secondary criterion of Outcome Prominence, the individual selects g_3. The inequalities are compatible because there exists a positive constant σ and a concave u such that $\sigma < u(3000) - (0.8)\,u(4000)$ and $\sigma \geq |(0.2)\,u(4000) - (0.25)\,u(3000)|$. For instance, take $u(x) = \ln(x+1)$ and $\sigma \in [0.343, 1.371)$. In this calibration the value of σ is realistically small compared to the size of the prizes, as the impact of a σ close to the lower end of its admissible range is equivalent to a monetary prize of about 0.4 (in the sense that a change by that amount in the prize impacts on utility as much as σ does).

Whether Outcome Prominence or Probability Prominence occurs is just a feature of the preferences of the decision maker, much as the degree of risk aversion is. The observed pattern of choice in the class of gambles outlined above seems to imply, if our explanation is correct, that for those gambles Outcome Prominence is what drives choice, rather than Probability Prominence. Is this reasonable? We think it is. There is experimental evidence that in risky choices where either the outcome is affectively significant (as a large monetary win certainly is) and/or probabilities are low, the probability component of the gamble tends to be neglected.[7] Sometimes this tendency even generates violations of the more fundamental principle of dominance. For example in an experiment by Denes-Raj and Epstein [6] people were asked to choose which of two urns was to be used to determine whether or not they win a monetary prize. A sizable number of subjects chose an urn with a smaller proportion of winning chips (jelly beans) favouring the higher absolute number of winning chips (e.g. 7 in 100) of this urn over the higher probability of winning of the other urn (e.g. 1 winning chip in 10)!

Arguably, what constitutes a 'low' probability varies across individuals and is a fundamental component of their own preferences. At any rate, one would expect a stronger probability ratio effect the smaller α is. Indeed, Kahneman and Tversky [10] find precisely this effect with gambles $g_1 = (6000, 0.45)$, $g_2 = (3000, 0.90)$, $g_3 = (6000, 0.001)$ and $g_4 = (3000, 0.0.002)$.

[7] See for example Kunreuther, Novemsky and Kahneman [11] as well as Slovic [22] and the bibliography therein.

In this case, most people would regard the probabilities of winning in gambles g_3 and g_4 as negligible, and we would feel confident that Outcome Prominence occurs. Kahneman and Tversky [10] themselves observe that, for gambles g_1 and g_2 "the probabilities of winning are substantial", whereas for the other gambles there is "a *possibility* of winning, although the probabilities of winning are minuscule... In this situation, where winning is possible but not probable, most people choose the prospect that offers the larger gain" (p. 267, italics in the original).

2.2 Cycles

A second class of experimental violations of the EU model concerns intransitivities which generate cyclical patterns of choice. We want to show that the combination of the two transitive criteria in our σEU model can account for this apparent irrationality.

To illustrate this point we use elementary gambles of the type employed for example in Loomes, Starmer and Sugden [14]. Let $g_1 = (x, p)$, $g_2 = (y, q)$ and $g_3 = (z, r)$, where $x > y > z$ and $p < q < r \leq 1$. One possible cycle which has been observed experimentally is generated when in pairwise choices g_1 is chosen over g_2, g_2 is chosen over g_3 and g_3 is chosen over g_1. This is consistent with σEU generated preferences under Outcome Prominence - hence we will refer to them as OP cycles. They can be generated as follows

$$\begin{aligned} ru(z) &> pu(x) + \sigma \\ pu(x) + \sigma &\geq qu(y) \\ qu(y) + \sigma &\geq ru(z) \end{aligned}$$

The first inequality establishes that g_3 is chosen over g_1 by the primary criterion. The second inequality instead provides conditions for g_1 to be chosen over g_2. To see this, observe that there are two cases. Either *(i)* $pu(x) > qu(y) + \sigma$, or *(ii)* the comparison between g_1 and g_2 is vague. In case *(i)* g_1 is chosen by the primary criterion. In case *(ii)* g_1 'wins' by the secondary criterion. Similarly, the last inequality in the display provides conditions for g_2 to be chosen over g_3.

The other observed cycle is generated when g_1 is chosen over g_3, g_3 is chosen over g_2 and g_2 is chosen over g_1. This is consistent with σEU

preferences, this time under Probability Prominence (PP cycles), as follows:

$$\begin{aligned} pu(x) &> ru(z) + \sigma \\ pu(x) &\leq qu(y) + \sigma \\ qu(y) &\leq ru(z) + \sigma \end{aligned}$$

The first inequality establishes that g_1 is chosen over g_3 by the primary criterion. The second (resp. third) inequality instead provides conditions for g_2 (resp. g_3) to be chosen over g_1 (resp. g_2) by either the primary criterion or the secondary criterion.

As we emphasised above, being in a state of vagueness and favouring either OP or PP is a structural feature of individual preferences. However, keeping the prizes fixed, on average we would expect the OP criterion to be relied upon more in correspondence with lower probabilities. Indeed, the data in Loomes, Starmer and Sugden [14] appear to support this conclusion. For example, when the gambles where $(16, 0.4)$, $(9, 0.6)$ and $(4, 1)$, there were 2 OP cycles against 15 PP cycles (out of 100 choices made by different subjects). However once the probabilities of the uncertain outcomes where reduced to generate the gambles $(16, 0.2)$, $(9, 0.3)$ and $(4, 0.5)$ the number of OP cycles shot up to 9 against a reduction of the PP cycles to 13. In the four comparisons of this type which can be made based on their data a reduction in probabilities keeping prizes constant never reduces the number of OP cycles and in three cases it increases them. Also, in three cases the number of PP cycles is reduced.

The PP cycle (and therefore regret theory) explains one other major 'anomaly' first noted by Lichtenstein and Slovic [12] and Lindman [13], termed *preference reversal*. They noticed that while some people preferred a lottery with higher prize obtained with lower probability ($-bet), they were prepared to pay less for it than for the competing lower prize-higher probability elementary gamble (P-bet). These preferences can be interpreted as a PP cycle with g_1 representing $-bet, g_2 representing the P-bet and where $g_3 = (z, 1)$ is a sure bet.

The preference reversal phenomenon was detected by engaging experimental subjects in both a 'choice task' (i.e. selecting either the $-bet or the

P-bet) and a ‘matching task’ (i.e. ‘match’ a price to a gamble), where the certainty equivalent of both the \$-bet and the P-bet is elicited. In a recent paper Cubitt, Munro and Starmer [5] carry out tests for preference reversal where the choice task can be paired with either the standard matching task, thereby providing a *monetary valuation* of each gamble; or a non standard matching task (*probability valuation*) where what is elicited is the probability p that makes the individual indifferent between either the \$-bet or the P-bet, and another gamble where some monetary amount X (determined by the experimenter) is obtained with the elicited probability p. They find that the pairing of the choice task and the monetary valuation task generates a much higher frequency of PP cycles[8]. However with pairings of the choice task and the probability valuation task, the frequency of OP cycles increases considerably, and for some set of parameters exceeds that of PP cycles. They discuss possible explanations for the experimental results, and highlight how traditional economic theory[9], which require agents' choice to be free from framing effects, cannot be reconciled with the experimental evidence[10]. On the other hand our σEU model might help explain the evidence.

An interesting aspect of our theory is that it is compatible with both types of possible cycles, offering a psychological support for each of them. In our framework we do not postulate which secondary criterion a subject might employ, and we cannot rule out the possibility that the same individual might employ a different secondary criterion *depending on the context*. For instance, once the probability dimension is highlighted (as in the probability valuation) it is not unreasonable to expect that the probability dimension becomes the most prominent, thus favouring PP cycles.

A second remarkable aspect is that cycles can only occur when vagueness is not too great. When for example $\sigma$is so large as to prevent any application of expected utility, no cycle can occur. Similarly, when σ is so small as to prevent any application of the heuristics, equally no cycle can occur. In

[8]Our PP cycles correspond to the ‘standard reversals’ in Cubitt, Munro and Starmer [5], that is the P-bet is chosen over \$-bet, but the latter has higher monetary value. On the other hand our OP cycles correspond to their ‘counter reversals’, where the \$-bet is chosen over P-bet, which now has higher monetary value.

[9]Including regret theory. See Loomes, Starmer and Sugden [14].

[10]See also Tversky, Slovic and Kahneman [27].

order to produce cycles one needs intermediate levels of vagueness.

3 Concluding remarks

In this paper we have argued that some 'EU paradoxes' of decision making under risk, including cyclical choices, can be explained by the way decision makers tackle cognitive difficulties in assessing probabilistic outcomes. Our core argument is that the decision maker is able to make at least *some* rational, reflected judgements. Such judgements are embodied in a partial ordering which satisfies usual rationality properties. The situation of 'vagueness', where such judgement fails, automatically calls for heuristics that enable the decision maker to express a preference in order to arrive at making a choice. We have thus combined an economic view of decision making - focused on rationality - and a psychological view - based on heuristics.

The strength of our σEU model, which is extremely close to the EU model, is that it can explain not only 'Allais type' violations of independence, but also cyclical choice patterns, regardless of whether or not they are consistent with alternative theories (e.g. regret theory). Indeed, although the heuristic we postulate in the σEU model is not itself cyclical, the whole choice procedure may fail to be transitive.

References

[1] Allais, M., 1953, "Le Comportement de l'Homme Rationelle devant le Risque: Critique des Postulats et Axiomes de l'Ecole Americaine", *Econometrica*, 21: 503-546.

[2] Aumann, R.J., 1962, "Utility Theory without the Completeness Axiom", *Econometrica,* 30: 445-462

[3] Bell, D.,1982, "Regret in Decision Making under Uncertainty", *Operations research*, 20: 961-981..

[4] Bewley, T., 1986, "Knightian Uncertainty Theory: Part I", Cowles Foundation Discussion Paper 807.

[5] Cubitt, R., A. Munro and C. Starmer, 2001, "Preference Reversal: an Experimental Investigation of Economic and Psychological Hypotheses", Cedex Working Paper, University of Nottingham.

[6] Denes-Raj, V., and S. Epstein, 1995, "Conflict Between Intuitive and Rational Processing: When People Behave Against Their Better Judgment", *Journal of Personality and Social Psychology*, 66: 819-829.

[7] Dubra, J, F. Maccheroni and E. A. Ok, "Expected Utility Theory without the Completeness Axiom", *Journal of Economic Theory*, forthcoming.

[8] Fishburn, P., 1982, "Nontransitive Measurable Utility", *Journal of Mathematical Psychology*, 26: 31-67.

[9] Gigerenzer, G. and D. G. Goldstein, 1996, "Reasoning the Fast and Frugal Way: Models of Bounded Rationality", *Psychological Review*, 103: 650–669.

[10] Kahneman, D. and A. Tversky, 1979, "Prospect Theory: an Analysis of Decision Under Risk", *Econometrica*, 47: 263-291.

[11] Kunreuther, H., N. Novemsky and D. Kahneman, 2001, "Making Low Probabilities Useful", *Journal of Risk and Uncertainty*, 23: 103-120.

[12] Lichtenstein, S. and P. Slovic, 1971, "Reversals of Preferences between Bid and Choices in Gambling Decisions", *Journal of Experimental Psychology*, 89: 46-55.

[13] Lindman, H. R., 1971, "Inconsistent Preferences among Gambles", *Journal of Experimental Psychology*, 89: 390-397

[14] Loomes, G., C. Starmer and R. Sugden, 1991, "Observing Violations of Transitivity by the Experimental Method", *Econometrica*, 59: 435-439.

[15] Loomes, G., and R. Sugden, 1982, "Regret Theory: an Alternative Theory of Rational Choice under Uncertainty", *Economic Journal*, 92: 805-824

[16] Manzini, P. and M. Mariotti, 2002, "A Vague Theory of Choice over Time", Economics WPA n.0203004 (Game Theory and Information series).

[17] Manzini, P. and M. Mariotti, 2003, "On the Representation of Incomplete Preferences over Risky Alternatives", mimeo.

[18] Rubinstein, A., 1988, "Similarity and Decision Making Under Risk (Is there a utility theory resolution to the Allais Paradox?)", *Journal of Economic Theory*, 46: 145-153.

[19] Sen, A., 1997, "Maximization and the Act of Choice", *Econometrica* 65: 745-779.

[20] Shafir, E., I. Simonson and A. Tversky, 1993, "Reason-based choice", *Cognition*, 49:11-36.

[21] Slovic, P., 1975, "Choice between equally valued alternatives", *Journal of Experimental Psychology: Human Perception and Performance*, 1: 280-287.

[22] Slovic, P., 2002, "Rational Actors or Rational Fools: Implications of the Affect Heuristic for Behavioural Economics", mimeo, Decision Research, Eugene, Oregon.

[23] Starmer, C., 2000, "Testing Alternative Explanations of Cyclical Choice", *Journal of Economic Literature*, 38: 332-382.

[24] Tversky, A., 1969, "Intransitivity of Preferences", *Psychological Review*, 76: 31-48.

[25] Tversky, A., 1972, "Elimination by Aspects: a Theory of Choice", *Psychological Review*, 79: 281-299.

[26] Tversky, A., S. Sattath and P. Slovic, 2000, "Contingent weighting in judgement and choice", in D. Kahneman and A. Tversky (eds.), *Choices, Values and Frames*, Cambridge University Press, Cambridge, UK.

[27] Tversky, A., P. Slovic and D. Kahneman, 1990, "The Causes of Preference Reversal", *American Economic Review*, 80: 204-217.