Backward unraveling over time: the evolution of strategic behavior in the entry-level British medical labor markets

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Abstract

This paper studies an adaptive artificial agent model using a genetic algorithm to analyze how a population of decision-makers learns to coordinate on the selection of an equilibrium or a social convention in a two-sided matching game. In the contexts of centralized and decentralized entry-level labor markets, evolution and adjustment paths of unraveling are explored using this model in an environment inspired by the Kagel and Roth (2000) experimental study. As an interesting result, it is demonstrated that stability need not be required for the success of a matching mechanism under incomplete information in the long run.

Keywords: Genetic algorithms, linear programming matching, stability, two-sided matching, unraveling

Journal of Economic Literature Classification Numbers: C63 computational techniques; C78 bargaining theory, matching theory; D83 search, learning and information

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1 Introduction

In the labor markets where agents seek employment, the contract dates sometimes shift far earlier than the start of employment. This dynamic phenomenon, known as unraveling, causes ex-post inefficiencies in market matching even though it may lead to an equilibrium. Contracts are made before most of the needed information becomes available in markets in which unraveling is observed.

There are many real life examples of markets, where better qualified agents choose to contract earlier than less qualified agents who wait to make contracts until the start of employment. Elite colleges have both early and regular admission programs. Another example can be seen in pro-sports draft selections in the USA.[1] Post-season college football bowl selections and entry-level labor markets for judges and for medical interns are examples of natural experiments involving unraveling.

This study will focus on entry-level medical intern-hospital labor markets in Britain. It will explore the nature of strategic behavior in these markets. Centralized institutions were established to control the dates of contracts in these markets. The entry-level medical intern labor markets in Britain are regional. We assume that almost every region is similar to the others in terms of the preferences of agents and information structure. Therefore, the only difference between these markets appears in the manner through which agents are appointed. British markets employ different mechanisms to match interns and hospital consultants to each other. These markets are not competitive. Wage negotiations do not exist. The markets are organized mostly in an oligopolistic structure that gives all the power to the hospital consultants in the design of these matching mechanisms. The jobs last only six months, so initial appointments are binding.

Kagel and Roth (KR) (2000) consider a study similar to the environment here. In that study, they conduct a laboratory experiment on the mechanisms used in Britain. Unver (2000b) works on an extended experiment. This study will be a complement to the KR (2000) and to the Unver (2000b) studies, and will answer the questions regarding the success of matching mechanisms using computational methods.

The basic properties of these labor markets are given in section 2. After proposing matching games to explore unraveling in these markets in section 3, we will study game theoretic properties of these matching games in section 4. Then in section 5, we will propose the use of an adaptive artificial agent model using a genetic algorithm (GA), to analyze the strategic behavior that employees and employers follow to adjust to an equilibrium from different initial conditions. This model will be used to analyze the genetic evolutionary strategic behavior of artificial agents who can be viewed as learning over time which strategies are best to adopt. In section 6, evolution experiments are simulated to examine different properties of institutions that are used to control the dates of

[1] Roth and Xing (1994), Li and Rosen (1998) present several other examples.

contracts. Robust features of evolution will be examined.

This essay allows us to explore an adaptive artificial agent model, that can be used in combination with laboratory experimentation and even with field applications. This model can be used in equilibrium selection of a wide variety of equilibria of the matching game.

One interesting result will be the demonstration of how some theoretically unstable mechanisms, namely linear programming (LP) mechanisms, may not lead to early contracts under the assumption that agents learn in an evolutionary manner and initially randomize their rank-order lists. This may help explain the field success of LP mechanisms used in Britain.

1.1 A Brief Literature Survey on Two-Sided Matching

An experimental study by KR (2000) works on a similar environment to the one handled in this study to analyze timing of transactions. They consider two mechanisms. The main result of the experiment is that lower levels of early contracts are observed in a stable deferred acceptance (DA) matching market than in an unstable priority matching market. More qualified agents make early contracts; less qualified do not arrange early under the unstable priority mechanism. Both type of agents do not make early contracts under the stable mechanism.

The Unver (2000b) study, an extension to the KR study, shows that a LP mechanism will not do as well as a stable DA mechanism in terms of preventing harmful early contracts in the laboratory. Agents do not game their preferences in an optimal way and the finitely repeated play of the games in the experiment is not as long as the computational markets considered in this study. But, it decreases the unraveling occurred in the decentralized markets.

The timing of transactions in an entry-level labor market framework has been studied by Roth and Xing (1997) to investigate the turnaround time and bottlenecks in market clearing for clinical psychologists.

Game theoretical properties of the two-sided matching mechanisms employed in Britain are studied by Roth (1991b). Roth and Xing (1994), Li and Rosen (1998) and Sonmez (1999) study the unraveling issue in a theoretical framework. Roth and Rothblum (1999) search advice to participants in two-sided matching markets.

Entry-level labor markets have been studied in the two-sided matching framework (see Roth and Sotomayor, 1990 for a theoretical background and motivation) through the marriage and assignment models. The links between the theoretical framework of Gale and Shapley's (1962) marriage model and the applied frameworks observed in the field (such as the entry-level labor markets for American physicians, osteopaths, dentists, lawyers, sororities, judges, etc.) are established by Roth (1984), Roth and Sotomayor (1990), Mongell and Roth (1991) and in the above mentioned studies. Roth (1990) gives a survey about these markets.

1.2 Coevolution in Games with Multiple Populations

However, all of these listed studies except KR's (forthcoming) and Unver's (2000b) investigate the problem in a mainly static and cooperative framework. These do not exactly describe the reasons why some unstable mechanisms (such as the London and Cambridge hospitals' selection process for medical interns) are still in use.

Some other recent studies involving market transition behavior relax the "best response play" hypothesis of game theory. In place of best response, these studies have adopted an evolutionary dynamic to update the strategy choices. This dynamic is partly adapted from biological research. Each agent selects strategies in a game theoretic setting in proportion to their relative ratio in the population. Evolution continues over time in each repeated play of a game.

Admission markets are perfect settings for application of this hypothesis. Workers change every year, but they have strong incentives to adapt the experience of the previous generation. Firms are usually the permanent players. They learn from the previous year's experience. Equilibrium selection and the nature of the adjustment process can be examined thoroughly with different evolution dynamics. We will use a GA for analysis in a computational framework.

Entry-level labor market games possess multiple equilibria which reflect different modes of coordination of two different sides of agents (workers and firms). In matching games, enumeration of the huge strategy space[2] is quite difficult. Any analytical evolutionary concept with a fixed set of strategies seems limiting the complex nature of the games. As proposed by Fudenberg and Levine (1998), GAs seem the best hope in exploration of the strategy spaces using artificial agents. Therefore, we will focus on GAs.

1.2.1 Description of a Genetic Algorithm

Basically, a GA is a computational tool that permits adaptive optimization or evolution over time, using genetics-based operators and assumptions of biological evolution. In a game theoretic setting, a GA is a technique that helps in the analysis of evolution dynamics and determines paths to a social equilibrium concept or a social convention. GAs search strategy spaces on the trade-off between exploration and exploitation of results achieved by difference equation and calculus-based procedures.

By mutating, crossing over, and reproducing strategies, a "population" of existing strategies in a game can be replaced to form new strategies as the offspring. An existing set of strategies of agents is called a generation. The fitness of the strategy measures the success of a strategy in a generation after a tournament against all strategies in that generation. Using this measure, parent strategies are selected to produce offspring, which makes up the next generation.

[2]For example, in a "mixed game" considered in this study there are $7 \times 7 \times 2^7 = 6272$ meaningful firm strategies, as programmed in the GA..

GAs are used in the economics literature generally to find evolution dynamics in macroeconomic models, where social learning is a major concern. Overlapping generation models and general equilibrium frameworks are two of the applications of evolution programming in economics. In game theory, GAs can be applied to complicated games where the strategy sets are too large for learning models such as the Roth and Erev (1995) model, the Camerer and Ho (1999) model or stochastic fictitious play. They can support laboratory experimentation via similarities between evolution strategies and actual human strategies. As an example, Miller (1996) investigates the evolved automata of a repeated prisoners' dilemma using a GA. As another example, Andreoni and Miller (1995) use a genetic algorithm in auction market simulations to find evidence to support the behavior observed in laboratory experiments. Technical and some theoretical issues regarding the application of a GA are addressed by Goldberg(1989), Holland (1992), Judd (1998) and Michalewicz (1994).

The reproduction and crossover process of the GA can be interpreted as the transmission of experience between generations of agents. Since the KR (2000) experiment treats also workers as permanent players across generations and since we base our matching game design on this experiment, we use the same genetics-based learning for both firms and workers. Agents of the same type will use the same reproductive selection. We will also consider only symmetric strategies.

2 The Two-Sided Matching Markets in Britain

The reader can follow Roth (1991b) for a detailed discussion of the matching markets in Britain. In summary, the matches for consultant and medical intern pairs were realized in a decentralized manner at the beginning of the century in Britain. However, unraveling appeared in these markets. Almost simultaneously, many regions adopted different centralized matching mechanisms to prevent unraveling. A mechanism is a function from the set of preferences to the set of matchings. In practice, computers process rank-order lists submitted by agents to produce a matching according to the mechanism. .

In several of the areas, including Newcastle, Birmingham and Edinburgh (1967), the unraveling problem could not be resolved although centralized but unstable "priority" matching mechanisms were introduced. They were abandoned after field trial. A mechanism is unstable if a preference profile exists for which the produced matching is unstable. A matching is unstable if a firm and a worker exist who prefer each other rather than their matches or an agent exists who prefers staying unmatched rather than being matched to the partner assigned.

Several regions were successful in preventing the problem by introducing centralized mechanisms. Roth (1991b) argues that stability determined the winners. Indeed, the successful Edinburgh'69 and Cardiff mechanisms were adaptations of stable mechanisms.

But there are two unstable mechanisms still in use in the London and Cambridge regional markets of Britain. These, called LP mechanisms, have not been subject to the unraveling problem. Therefore, they deserve attention, because stability may not be a necessary condition for survival of a centralized mechanism in the field.

2.1 Description of Centralized Matching Mechanisms

2.1.1 Priority Matching Mechanisms

The mechanisms introduced in Newcastle, Birmingham, and Edinburgh (1967) give a priority rank to each match, according to the preferences of agents, and matches are realized from the highest to the lowest priority. Details of the algorithms are given by Roth (1991b).

The Birmingham and Newcastle mechanisms give a priority to a consultant-intern pair as follows: First note that in a (k, l) match, the consultant lists the student l'th, the student lists the consultant k'th in the rank-order lists. The number of such a match is $k \times l$. Smaller numbered matches have higher priorities.

Priorities are "lexicographic" in consultants' preferences for the Edinburgh (1967) mechanism. If $k < l$ then a (x, k) match is favored before a (y, l) match for any value of x, y. If $k = l$ and $x < y$ then a (x, l) match has a higher priority than a (y, k) match.

These mechanisms are unstable and have unraveled, so they were abandoned (see example 1 in appendix A). $(1, 1)$ matches are guaranteed to be realized by these priority mechanisms.

2.1.2 Linear Programming Matching Mechanisms

The mechanisms introduced in London and Cambridge solve an assignment problem to find a matching. Shah and Farrow (1976) and Roth (1991b) give details of the algorithms. After weights are assigned to each possible pair with respect to the submitted rank-order lists, they are summed up for potential pairs in each matching. The resulting weights are used in an integer programming problem to find a matching that maximizes these weights. For our purposes, the problem reduces to the linear programming problem

$$\begin{aligned}
&\max_{x_{f,w}} \sum_{f,w} \alpha_{f,w} x_{f,w} \\
&\text{s.t.} \\
&(i) \sum_{f} x_{f,w} \leq 1 \\
&(ii) \sum_{w} x_{f,w} \leq 1 \\
&(iii) 0 \leq x_{f,w} \leq 1 \ \forall\ f \text{ and } w
\end{aligned} \tag{1}$$

where $\alpha_{f,w}$ is a weight. $x_{f,w} = 1$ denotes a match between f and w, $x_{f,w} = 0$ denotes no match in the solution.

Different regions use different methods to determine these weights. In the London region, weights are determined by summing up the firm's weight for the worker and the worker's weight for the firm in a pair. These individual weights are decreasing in rank of agent in the rank-order list. In the Cambridge region, the weight for a pair is "lexicographic" in consultant's preferences.[3] Although they are unstable, these algorithms have survived and are still in use. One hypothesis to account for their survival is that it is easy for the agents to adjust to the system by manipulating their rank-order lists (see Shah and Farrow, 1976). $(1,1)$ matches may not be realized (see examples 1 and 2 in appendix A).

2.1.3 Deferred Acceptance Matching Mechanisms

The mechanisms introduced in the Edinburgh (1969) and Cardiff regions are adaptations of Gale and Shapley's (1962) stable mechanisms.

In modeling Edinburgh (1969 and later) and Cardiff markets, it will be assumed that these markets use one-to-one DA mechanisms. The Edinburgh (1969) mechanism will be approximated by the consultant proposing DA scheme, and the Cardiff mechanism will be approximated by the intern proposing DA algorithm. The consultant proposing DA algorithm can be stated as follows for future reference. Suppose Q is the rank-order list profile submitted, that is the vector of rank-order lists submitted by each consultant and student.

At the first step, each consultant, f, proposes to his most favored intern with respect to $Q(f)$. Each intern, w, holds only the best consultant's proposal with respect to $Q(w)$ if that is acceptable to her; she refuses all others.

At any other step, each consultant, f, who does not have an offer held by an intern proposes to the remaining most favored intern with respect to $Q(f)$. Each intern, w, holds only the best offer with respect to $Q(w)$ among all the new proposals at this step and the offer held from the previous step.

When no offers are rejected at a step, the algorithm terminates and tentative agreements are realized as matches.

These are stable mechanisms, in the sense that the matching produced is stable with respect to the submitted rank-order list profile. They have not unraveled and are still in use.

[3] For the London mechanism, we use the weights $36, 28, 21, 15, 10, 6$ for choices $1, 2, 3, 4, 5, 6$ and a negative weight for an unlisted choice (as given in Shah and Farrow, 1976). A $(1,1)$ match has the weight 72. $(1,2)$ and $(2,1)$ matches have the weight 64, and so forth. For the Cambridge mechanism, we use the weights 9 for (A,A), 8 for (A,B), 7 for (A,C), 6 for (B,A), 5 for (B,B), 4 for (B,C), 3 for (C,A), 2 for (C,B) and 1 for (C,C) matches. We assign, a negative weight for unacceptable matches ((U,x) or (x,U) where x can be A,B,C or U for unacceptable). For the Cambridge mechanism, an artificial agent's rank-order list translates as first listed choice A, second and third B, fourth, fifth and sixth C. We assume that agents can make one A, two B, three C choices.

3 An Incomplete Information Model for Unraveling

We begin with an entry-level labor market model of n firms (set F) and m workers (set W). In order to motivate the British experience, this study considers a market that clears at three consecutive rounds. We will refer to consultants as firms and to interns as workers in our model. (Some preliminaries about the cooperative two-sided matching theory are given in appendix A.) The games considered are partially adopted from the KR (2000) laboratory experiment, because this is the only work that studies human dynamics in unraveling markets. So, we can compare our results with those of that study. We also adopt primary computational values from these experimental games. First, we consider a decentralized matching game.

3.1 The "Decentralized" Matching Game

This game has three periods: In round -2, a firm has the option to make an offer to one worker and a worker has the option to accept one offer. An accepted offer binds parties for an early match. Round -1 is a replay of round -2 among the players who have not made a contract in round -2. Similarly, round 0 consists of replay of round -1 among the players who have not made early contracts yet. The rounds differ from each other in terms of the costs of contract. A match in round -2 has the cost \$2 , round -1 matches cost \$1 and round 0 matches do not incur any costs. Application of these costs is motivated by the decrease in planning flexibility. Next, we consider mixed games.

3.2 The "Mixed" Matching Games with Decentralized Early Offers

These games have also three periods: in each of the rounds -2 and -1 a firm has the option to make an offer to one worker and a worker has the option to accept one offer. A contract is costly. Round 0 is replaced by a centralized market. Agents submit rank-order lists, these are processed by a centralized mechanism. We study two different early contract technologies. Through the first contract technology, we assume that early contracts are tentative. Agents only commit to list each other in first place in round 0 rank-order lists. They freely fill the rest of rank-order lists. The second contract technology studies the case in which these contracts are binding. Only agents who do not make early contracts participate in the mechanism matching and they do not pay any costs. The costs of early contracts are set to \$2 in round -2, \$1 in round -1. A cost is charged regardless of whether the contract was successful or not.[4] The first two rounds are exactly the same in the binding contract mixed games and in the decentralized game. In all the games, agents are not informed about the early matches and about the early offers and acceptances/rejections.

[4] "Mixed matching games with early decentralized offers" will be referred as "mixed matching games" from now on.

These contract technologies are motivated by the British experience where agents have to participate in the mechanism match. Since priority and DA mechanisms turn $(1,1)$ lists into a match these two technologies coincide strategically.

In summary, we consider 10 different Bayesian games: a decentralized game, mixed games under 3 priority matching mechanisms, 2 DA markets, and tentative and binding contract treatments of 2 LP markets.

3.3 The Partially Correlated Preferences

The preferences of agents are determined by partially correlated rank-order lists. There are two disjoint set of players, workers and firms. Each firm and worker is of one of the two types, "high productivity" or "low productivity". $n/2$ firms are high, $n/2$ firms are low productive types. Similarly, $m/2$ workers are high and $m/2$ workers are low productive types. The types of agents are common knowledge. We will only consider $n = 6$ and $m = 6$ in our learning simulations.

The utility of a firm f from a worker w can be given by

$$u_f(w) = t_w + \theta_{f,w} \quad (2)$$

with $\theta_{f,w} \sim g$, a density function.

Similarly for a worker w, the payoff from a firm f can be given by

$$u_w(f) = t_f + \theta_{w,f} \quad (3)$$

with $\theta_{w,f} \sim g$ where the primary computational values are

$$t_v = \begin{cases} 5 & \text{if } v \text{ is a low type agent} \\ 15 & \text{if } v \text{ is a high type agent} \end{cases} \quad \text{for agent } v \quad (4)$$

and $g = U(-1, 1)$, the uniform density with the support $[-1, 1]$. While t_f and t_w are common knowledge, $\theta_{w,f}$ is private to w and $\theta_{f,w}$ is private to f. The net payoff of an agent in the market is the utility she gets from being matched minus the early contract cost. The net payoff of being unmatched is zero for an agent v, even if she made an unsuccessful early contract.

4 Strategic Behavior in the Stage Games

Suppose $F_H = \{f_1, \ldots, f_{n/2}\}$ is the set of high firms and let $F_L = \{f_{n/2+1}, \ldots, f_n\}$be the set of low firms. Similarly define W_H and W_L. Let the set of high agents be $H = F_H \cup W_H$ and the set of low agents be $L = F_L \cup W_L$. Let P be the true preference profile of agents, the realization of the partially correlated preference profile $\widetilde{P}$. Let $\mathcal{P}$ be the set of those profiles where the preferences are rational and admissible by the partially correlated preference distribution. $P(v)$ is only known by agent v, while the distribution of $\widetilde{P}$ is common knowledge. A preference profile is rational if it is transitive and complete. Admissible preferences always rank high agents strictly preferred to low agents who are preferred to being unmatched. Define $U, U(v)$ and $\widetilde{U}$ for the utility values and the set $\mathcal{U}$ similarly. Define the pure strategy of agents in the Bayesian games for each realization of utilities by a function $s : \mathcal{U} \to \mathcal{S}_c$ onto the set of complete information strategies from the set of utility profiles. Let productivity types be defined as $\mathcal{T} = \{F_H, W_H, W_L, F_L\}$. Let $\mathcal{S}$ be the set of pure strategies. Let strategies be defined in terms of rank-orders instead of identities of agents. For example, a strategy might specify that a firm makes its first offer to its highest ranked worker, at period -1. A strategy $s \in \mathcal{S}$ is symmetric if it is employed by every agent in a type that is $s_v(U(v), \widetilde{U}_{-v}) = s_{v'}(U(v'), \widetilde{U}_{-v'}) \ \forall \ v, v' \in T \quad \forall \ T \in \mathcal{T}$. We will say a worker $w \in W$ and a firm $f \in F$ unravel if they arrange in rounds -2 or -1 in any of the games.

In examining stability under incomplete information, we will use ex-post stability and instability (i.e., stability and instability of the outcome matching for all possible realizations of the preference profiles).

The next section considers some of the equilibrium properties of the 10 games considered in this study.

4.1 The Equilibria

Here we will concentrate on symmetry, thus the outcomes will be simple lotteries over matchings for symmetric strategies.

Lemma 1 *Under the binding contract mixed Bayesian games with the priority and DA mechanisms, symmetric strict equilibria exist, whose outcomes match all agents, involve no mismatches between high and low types of agents, and involve no early contracts. (See appendix B for a sketch of proof.)*

There may or may not exist such equilibria for the LP mechanisms depending on the weighing schemes. For the weights considered here, such equilibria do not exist:

Lemma 2 *There does not exist any pure strategy symmetric equilibrium whose outcome matches all agents, involves no mismatches between high and low agents, and involves no early contracts for the mixed Bayesian game with the LP mechanisms under the binding contracts. (See appendix B for sketch of proof.)*

4.2 Incentive Compatibility of Mechanisms under Incomplete Information

It can be shown that the DA mechanisms are incentive-compatible for this information structure and utility parameters (i.e., each agent has a truthful revelation in response to everybody else's truthful revelation of preferences.)[5] However, the priority and LP mechanisms are not.

[5] Under complete information, no incentive compatible mechanism exists. See Roth and Sotomayor (1990).

Lemma 3 *The DA mechanisms are incentive-compatible under the proposed model. That is the strategy s such that $s(P) = P$ for any P is an equilibrium in the direct revelation game. (See appendix B for sketch of a proof.)*

5 Genetic Evolution

In order to model a behavior that determines the adjustment paths of artificial agents to an equilibrium or a social convention, a genetic algorithm (GA) is used. This study will be interested in the evolution of strategies and the dynamics of markets. The evolution and the adjustment process will be the primary concern, not the steady-state evolution stage. The evolution of market behavior will be examined after the adoption of new market organizations. For example, we examine the change in behavior after decentralized markets are replaced by mixed markets.

The basic GA,[6] which is run independently SN number of times to determine the average behavior using Monte Carlo experimentation, is stated as pseudo-code in figure 1.

Figure 2 shows the representation of the strategies in the GA. Under this representation, each string represents a valid strategy, although some parts of the string can be redundant.

The average simulated behavior is determined after $SN = 30$ runs of the basic algorithm. The number of agents in each side are set as $m = n = 6$. The values of parameters and genetic operators can be chosen freely, and do not affect the ordering of the results to be presented as will be described later. The speed and magnitude of adjustment dynamics can be calibrated by these parameters. For the simulations presented here, the following parameter values are chosen: $p = 0.90$, the crossover probability, $q = 0.05$, the mutation probability. The number of generations is determined as $DG = 40$ decentralized market game generations followed by 120 mixed game generations, a total of $G = 160$ generations. The population of strategies is determined to be $4s_t = 28$ ($s_t = 7$ for each type).[7] The number of reproduced best strategies is set to be $h = 1$.[8]

6 Evolution Experiments for Random Initial Strategies

First, we present some of the terminology used to define the properties of the experiments in the graphs in appendix 3. The average total cost of unraveling is the total cost incurred by all players

[6] The genetic algorithm, all the matching mechanisms, and the games are coded in PASCAL and run on a IBM-PC compatible machine. PASCAL implementations of "Numerical Recipes" routines by Press et. al (1996) are used whenever needed.

[7] Increasing the population size is very expensive in terms of CPU run-time. For example introducing a new strategy to each type (i.e. increasing population size from 28 to 32) increases the CPU-time by 1.71 times (slightly less then doubling the CPU run-time).

[8] The number of plays of a single strategy in the tournament of the genetic algorithm is therefore $7^3 = 343$. The fitness of the strategy is the sum of the average payoffs the strategy earns for the three agents of a type in these 343 plays.

Figure 1: The Genetic Algorithm

For $i = 1$ to SN,simulation number

1 Randomly generate the initial population s_t of strategies for each type T =High firms, high workers, low firms, low workers.

2 For $g = 1$ to G, number of generations

2.1 Generate a preference profile using the partially correlated preference distribution.

2.2 Conduct a tournament among each strategy of each type, treating them as symmetric strategies.

2.2.1 If $g \leq DG$, decentralized game generation, then use decentralized game in the tournament.

2.2.2 Otherwise use one of the centralized games in the tournament

2.2.3 Fitness of a strategy is sum of payoffs strategy achieves in each play

2.3 For $k = 1$ to h, highest fitness strategy number, reproduce the highest fitness strategies for each type.

2.4 For $k = 1$ to $s_t - h$, crossover the parents linearly for each type.

2.4.1 Randomly choose four parent candidates C_1, C_2, C_3, C_4 for each type among the current generation.

2.4.2 The higher fitness strategies of C_1, C_2 and C_3, C_4 become the two parents P_1, P_2 for each type.

2.4.3 With probability p crossover the parents P_1, P_2, with probability $1 - p$ directly clone the parents as the offspring (Bernoulli density).

2.4.3.1 Randomly draw a crossover “joint” digit in the strategy string of the size “length.”

2.4.3.2 Copy the digits 1,..,“joint” of P_1 and “joint” + 1 ,$\ldots$, “length” digits of P_2 to form the child O_1.

2.4.3.3 Copy the digits 1,..,“joint” of P_2 and “joint”+1,$\ldots$,“length” digits of P_2 to form the child O_2.

2.5 Mutate each digit in the offspring strategies of each type with probability q (Bernoulli density).

2.5.1 For mutation of offer/acceptances draw a digit from $\{1, 2\}$.

2.5.2 For mutation of rank-orders draw a digit from $\{1, 2, 3, \ldots, m\}$ for a firm strategy ($\{1, 2, 3, \ldots, n\}$ for a worker strategy).

Figure 2: Representation of strategies as "integer" strings in the genetic algorithm for games with two early offer rounds

THE DECENTRALIZED GAME "FIRM" STRATEGY STRING

$o_{-2}r_{-2} - o_{-1}r_{-1} - o_0r_0$

$o_t \in \{1, 2\}$: 1 for offer, 2 for no offer in round $t \in \{-2, -1, 0\}$

$r_t \in \{1, \ldots, m\}$:the rank-order of the worker to make an offer in round $t \in \{-2, -1, 0\}$

THE DECENTRALIZED GAME "WORKER" STRATEGY STRING

$a_{-2}r_{-2} - a_{-1}r_{-1} - a_0r_0$

$a_t \in \{1, 2\}$: 1 for accept, 2 for reject the best offer in round $t \in \{-2, -1, 0\}$

$r_t \in \{1, \ldots, n\}$: the threshold rank-order of the best firm whose offer is accepted in round $t \in \{-2, -1, 0\}$

ANY CENTRALIZED GAME "FIRM" STRATEGY STRING

$o_{-2}r_{-2} - o_{-1}r_{-1} - r_{0,1}r_{0,2} \ldots r_{0,m}$

$o_t \in \{1, 2\}$: 1 for offer, 2 for no offer in round $t \in \{-2, -1\}$

$r_t \in \{1, \ldots, m\}$: the rank-order of the worker the firm makes an offer to in round $t \in \{-2, -1\}$

$r_{0,1}r_{0,2} \ldots r_{0,m}$: length m rank-order list of the firm for round 0 ($r_{0,k} \in \{1, \ldots, m\}$)

ANY CENTRALIZED GAME "WORKER" STRATEGY STRING

$a_{-2}r_{-2} - a_{-1}r_{-1} - r_{0,1}r_{0,2} \ldots r_{0,n}$

$a_t \in \{1, 2\}$: 1 for accept, 2 for reject the best offer in round $t \in \{-2, -1\}$

$r_t \in \{1, \ldots, n\}$: the threshold rank-order of the best firm whose offer is accepted in round $t \in \{-2, -1\}$

$r_{0,1}r_{0,2} \ldots r_{0,n}$: length n rank-order list of the worker for round 0 ($r_{0,k} \in \{1, \ldots, n\}$)

The space of rank-order lists consider all possible misrepresentations and truncations of the preferences.

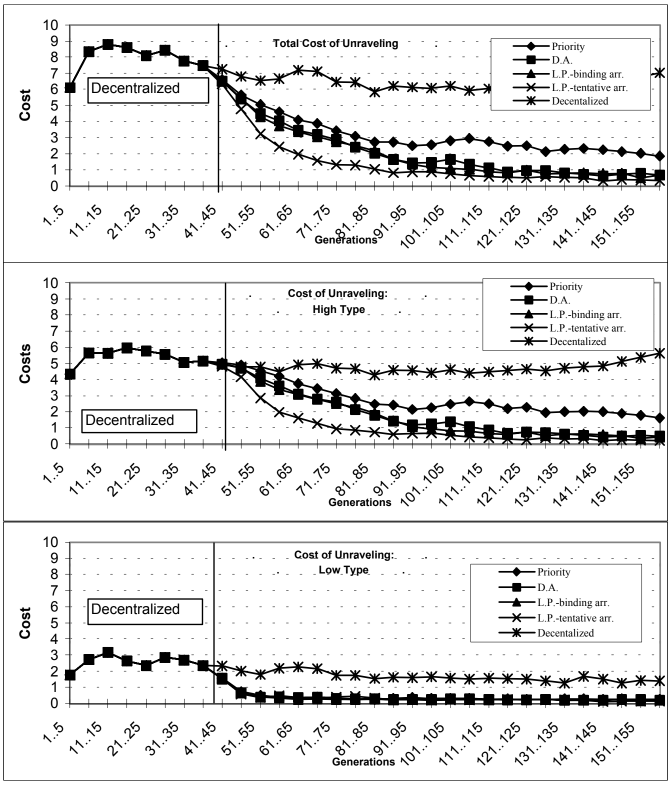

Figure 3: Evolution with Random Initial Strategies and with Rank-Order List Updating

by making early contracts in rounds -2 and -1 in one generation of a simulation. Firm offer rates in a round are determined for both types. In that round, the ratio of firms making an offer for a contract to currently unmatched firms of the type is considered. The low worker acceptance rate is the ratio of low workers accepting a best offer to the current number of unmatched low workers who receive at least one offer in that round. The high worker acceptance rate is differentiated from the low worker acceptance rate, since low firms almost always offer early contracts to high workers and almost always workers reject the offers. Therefore, the high worker acceptance rate is defined as the ratio of high workers who accept an offer from a high firm to the high unmatched workers who receive at least one offer from a high firm. The average number of early contracts is the total number of contracts in the society done in rounds -2 and -1 respectively. Basically, costs, the number of early contracts, and offer/acceptance rates will be indicators of the evolution dynamics.

6.1 Results

The basic model is considered with 30 independent runs of the basic GA, each of 160 generations (40 decentralized +120 decentralized/mixed market games).[9] The basic payoffs are employed as \$15 for (average) high, \$5 for (average) low, \$0 for unmatched. The costs for early contracts are imposed as \$2 for round -2, \$1 for round -1, \$0 for round 0 contracts. As described before, these values are adopted from the KR (2000) laboratory experiment. We study the experiments involving decentralized markets, mixed games with the priority, DA and LP matching mechanisms under the tentative and binding contracts. We study these experiments from random initial strategies. In the first mixed game generation, the rank-order lists are generated randomly and then are updated in the following generations. We permit a random rank-order list to start from an arbitrary firm (or work) rank-order, and to be terminated by any truncation. The evolution paths look qualitatively like the KR (2000) experiment for the priority and DA markets. The dynamic comparisons between the acceptance and offer rates, and costs qualitatively hold for both GA and laboratory evolution. Figures 3-6 summarize the results outlined in this section.

6.1.1 The Decentralized Matching Game

In the decentralized treatment, it is observed that the total cost of unraveling increases after initial generations (as seen in figure 3a). As seen in figures 3b and c, high agents are observed to make more early contracts than low agents after experience accumulation. This is because high agents have higher opportunity costs to avoid mismatches. Round -1 matches are higher in number than round -2 matches as seen in figures 4b and c.

[9] After the end of decentralized markets, the schemata that represent actions in rounds -2 and -1 are communicated to the first generation of centralized game as schemata for actions in rounds -2 and -1. In round 0, the initial rank-order lists are randomly determined by discrete uniform density.

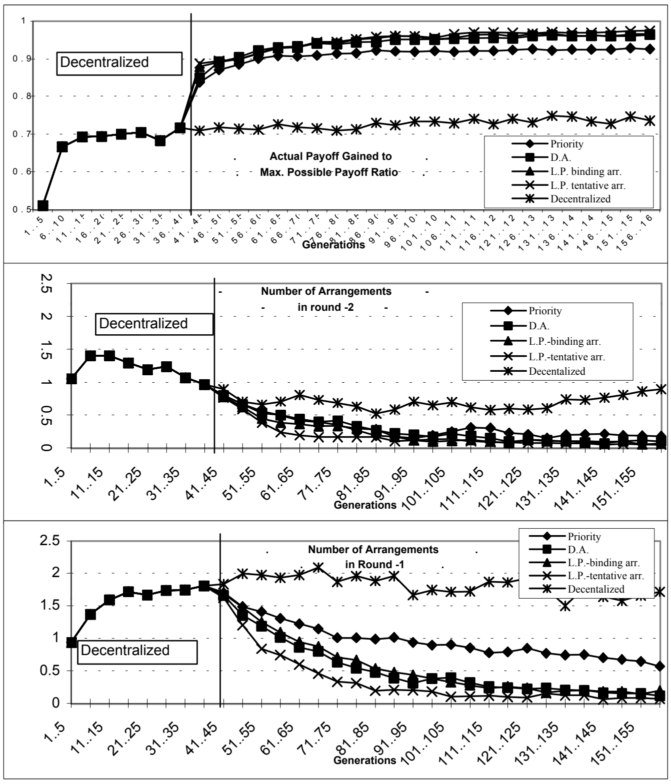

Figure 4: Evolution with Random Initial Strategies and with Rank-Order List Updating

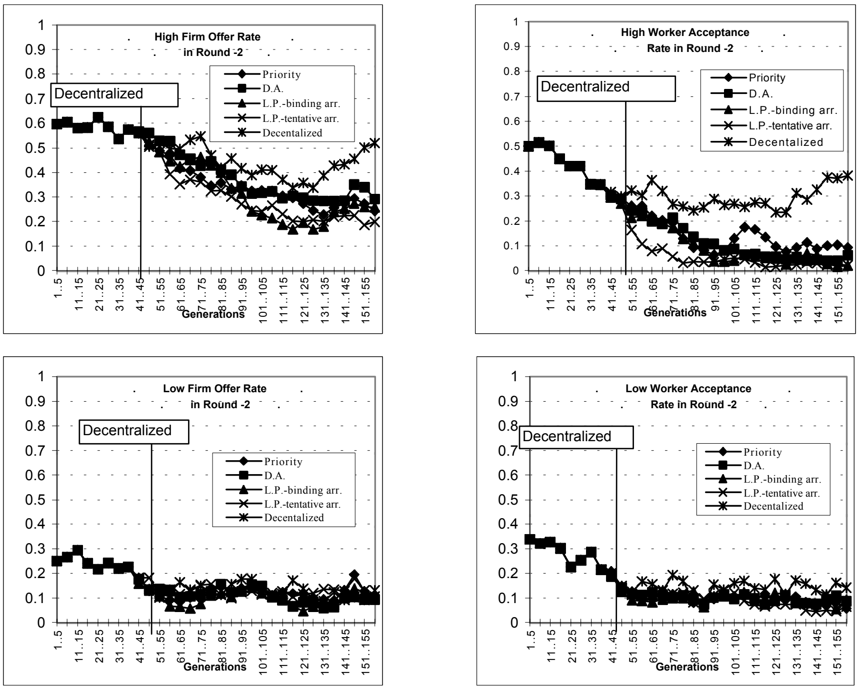

Figure 5: Evolution with Random Initial Strategies and with Rank-Order List Updating

6.1.2 The Mixed Matching Games

Introduction of the centralized mechanisms makes unraveling costs fall dramatically as seen in figure 3a. This is caused by low agents who literally stop making early matches (see figure 3c).

Under the stable DA mechanisms and the LP markets, unraveling costs approach zero in the long run (see figure 3a). With random initial conditions, the priority markets perform most poorly under evolution in terms of costs. The major difference between the DA, LP, and priority markets appears in the acceptance ratio of high workers in round -1 (see figure 6b). In the DA and LP markets, almost all high workers reject high firm offers in round -1. However the priority market high workers continue accepting a substantial ratio of high firm offers in round -1.

The LP markets are also differentiated among themselves. The Cambridge mechanism leads to more early contracts than the London mechanism. However in general, when the contracts are binding, agents adapt to the mechanism and do not circumvent it. The costs and acceptance/offer rates fall further, when we consider the tentative contract technology. The binding contract treatment produces a comparable number of ex-post market matching blocking pairs with the DA mechanisms. However priority matching markets produce a much higher number of ex-post blocking pairs. The tentative contract treatment of the LP markets produces the lowest numbers (see table

1). The tentative contract technology, by itself, presents an evolutionary evidence for the reasons unstable LP mechanisms are still in use and stopped unraveling.

6.1.3 Strategy Evolution

Each treatment's strategy evolution converges to different sets. The fitness of a strategy shows the rate of increase of the population of a specific schema. A "schema" is a specific portion of the strategy code. For example, first and third digits in the firm strategy representation are the early offer schema. In the experiments, the offer/acceptance schemata in the strategies are different. A priority market high workers still tend to accept offers from the high firms in round -1. (In Newcastle, the percentage of such schema is 27.85% in generations $121-160$.) In the DA and LP markets, schema involving no acceptances in rounds -2 and -1 are in the highest ratio. (For example the Edinburgh'69 market involves such schema 59.77% of the population in generations $121-160$, for the London market under tentative contracts 56.23%.) We do not observe a clear convergence in the rank-order submission schemata of the mechanisms. However the length of the rank-order lists are shortest under the LP markets and longest under the DA market for the high agents.

6.1.4 Efficiency of Mechanisms

We consider the total payoff to maximum payoff ratio for a measure of over all efficiency. According to this measure, the LP market agents reach 97.4% of total payoff under the tentative contract treatment, 96.5% under the binding contract treatment. The DA market agents reach 96.3%, the priority market agents reach 92.5%, in the decentralized markets agents obtain 73.5%.

6.2 Summary for Randomly Chosen Initial Strategies

We use a regression model to find the differences between evolution processes of the "average total cost of unraveling" under 10 different markets/institutions with our random initial condition experiments. One of the models is presented below. Note that we find a strong autoregressive process for the error terms. Therefore, we make a 2-step feasible GLS estimation by first estimating ϕ and then estimating regression coefficients. We present the average total cost of unraveling as:

$c_{i,g} = \beta_i + \frac{\alpha_i}{g-g_0} + \gamma \overline{u_v}(v_H) + \varepsilon_{i,g}$

$\varepsilon_{i,g} = \phi \varepsilon_{i,g-1} + \eta_{i,g}$

for i denoting one of the 10 market treatments. $\eta_{i,g} \overset{i.i.d.}{\sim} N(0, \sigma^2)$ and g, generations, change from 41 to 160 (i.e., the era after the artificial agents gain experience in the decentralized market)

Table 1: Statistics from Experiments with Rank-Order List Updating and with Random Initial Conditions

MARKET GAME	Pairs in the Market Matching That Block The True Revelation Matching (of 6 maximum)	Pairs That Block The Market Matching (of 36)	Early Contracts	Percentage of Successful Early Contracts
Generation: 40				
Decentralized	-	8.0610 (3.1913)	2.7837 (1.0036)	100 %
Generation: 80				
Newcastle P	0.2178 (0.3527)	5.4081 (1.8168)	1.1692 (0.8672)	100 %
Birmingham P	0.2095 (0.3351)	4.9544 (1.8795)	1.0256 (0.9102)	100 %
Edinburgh'67 P	0.2093 (0.3025)	4.9539 (1.5916)	1.2249 (0.8503)	100 %
Edinburgh'69 DA	0 (0)	4.3600 (1.3421)	0.8673 (0.8615)	100 %
Cardiff DA	0 (0)	4.5583 (1.2682)	0.9164 (0.7449)	100 %
London LP (tent.)	1.0807 (0.6979)	3.5488 (1.9867)	0.3194 (0.5583)	66.1958 %
CambridgeLP(tent.)	1.7471 (0.8128)	4.3245 (2.3195)	0.5033 (0.7578)	88.1968 %
London LP (bind.)	1.0208 (0.7196)	4.0567 (1.6164)	0.7825 (0.9474)	100 %
CambridgeLP(bind.)	1.6111 (0.6950)	4.5576 (1.9314)	0.9970 (0.9084)	100 %
Decentralized	-	9.4629 (3.6740)	2.5701 (0.9153)	100 %
Generation:120				
Newcastle P	0.2616 (0.4215)	4.5145 (1.5217)	1.2321 (1.0514)	100 %
Birmingham P	0.2273 (0.3426)	5.1746 (2.0009)	0.6598 (0.8558)	100 %
Edinburgh'67 P	0.4586 (0.3953)	5.6615 (1.9637)	1.0139 (0.7481)	100 %
Edinburgh'69 DA	0 (0)	3.9886 (1.6239)	0.3152 (0.5932)	100 %
Cardiff DA	0 (0)	3.6978 (1.4764)	(0.3663 0.5912)	100 %
London LP (tent.)	1.3150 (0.8121)	2.4404 (1.5322)	0.1274 (0.1763)	59.2133 %
CambridgeLP(tent.)	1.7794 (0.5869)	3.6264 (1.9608)	0.3220 (0.4980)	89.4701 %
London LP (bind.)	1.3370 (0.6228)	3.0680 (2.0191)	0.2140 (0.4239)	100 %
CambridgeLP(bind.)	1.6839 (0.6329)	4.3344 (2.1775)	0.3858 (0.5514)	100 %
Decentralized	-	8.7771 (4.3224)	2.4868 (0.7253)	100 %
Generation:160				
Newcastle P	0.1871 (0.3196)	4.6220 (1.3655)	0.7313 (0.8245)	100 %
Birmingham P	0.2147 (0.3667)	5.0770 (2.1852)	0.6737 (0.9153)	100 %
Edinburgh'67 P	0.4294 (0.4066)	4.7671 (2.1735)	0.7384 (0.8005)	100 %
Edinburgh'69 DA	0 (0)	3.7240 (1.8856)	0.1628 (0.1977)	100 %
Cardiff DA	0 (0)	3.6538 (1.5186)	0.2796 (0.4586)	100 %
London LP (tent.)	0.9041 (0.7526)	2.2565 (1.5883)	0.1351 (0.2236)	73.2991 %

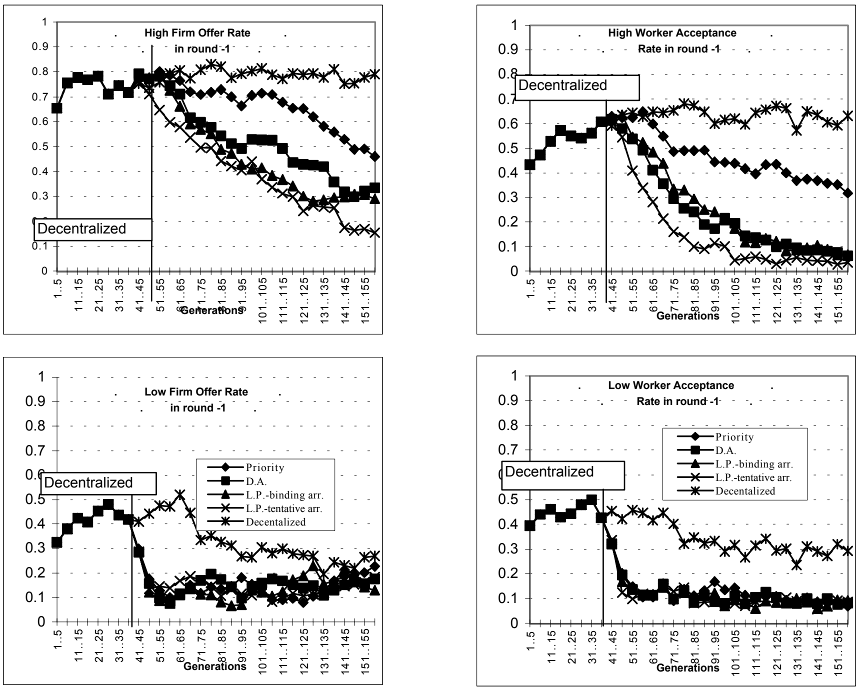

Figure 6: Evolution with Random Initial Strategies and with Rank-Order List Updating

so that $g_0 = 40$, $\overline{u_v}(v_H)$ is the average payoff from a high agent (\$15 in original, \$10 in subsequent experiments), α_i the speed parameter for decrease in cost, β_i the constant dummy coefficient denote the market specific coefficients for 10 of the markets/institutions with their relevant subscripts i.[10] Table 2 presents to feasible GLS estimates of coefficients. Table 3 presents some simple hypotheses testing results. We use market specific variables to capture the individual effects of each of the mechanisms after pooling all data to estimate γ. We change the average payoff from the high type agents in some of the simulations. This captures the payoff effects. This is particularly important because the KR (2000) study includes different sessions when high type agent payoffs are on average \$10 and \$15. On average their findings conjecture a direct relationship between the cost of unraveling and average payoff from the high types. We try to characterize those effects in our regression equation and in our GA model. To capture the indirect relationship between cost and generations after $g_0 = 40$, we use $\frac{1}{g-g_0}$ as one of the regressors.[11]

The payoff coefficient (γ) is found positive and significant. Therefore, we find the same payoff

[10] The subscripts are NE for the Newcastle priority, BI for the Birmingham priority, $ED'67$ for the Edinburgh'67 priority, $ED'69$ for the Edinburgh'69 DA, CR for the Cardiff DA, LO for the London LP and CM for the Cambridge LP mechanisms. We use DC to denote the decentralized markets.

[11] We also run different linear regressions using different regressors such as log generation. Those usually give similar results with those found below in presence of $AR(1)$ process.

Table 2: 2-Step Feasible GLS Coefficients

GLS Coefficients	Estimate	Standard Error	t statistic	p value
$\hat{\gamma}$	0.0553	0.0119	4.6550	0.0000
Speed Coefficient Estimates				
a_{NE} priority	4.5417	0.2733	16.6190	0.0000
a_{BI} priority	5.0818	0.2360	21.5284	0.0000
$a_{ED'67}$ priority	4.9885	0.2359	21.1465	0.0000
$a_{ED'69}$ DA	5.7941	0.2359	24.5613	0.0000
a_{CR} DA	5.7859	0.2359	24.5268	0.0000
$a_{LO-tent.}$ LP	5.6910	0.2359	24.1243	0.0000
$a_{CM-tent.}$ LP	6.2455	0.2359	26.4747	0.0000
$a_{LO-bind.}$ LP	5.8518	0.2359	24.8063	0.0000
$a_{CM-bind.}$ LP	5.6796	0.2363	24.0373	0.0000
a_{DC} decentralized	1.8476	0.2359	7.8321	0.0000
Constant Dummy Coefficient Estimates				
b_{NE} priority	2.0030	0.1990	10.0676	0.0000
b_{BI} priority	1.8949	0.1942	9.7557	0.0000
$b_{ED'67}$ priority	2.0997	0.1940	10.8228	0.0000
$b_{ED'69}$ DA	1.0146	0.1940	5.2295	0.0000
b_{CR} DA	1.1203	0.1940	5.7745	0.0000
$b_{LO-tent.}$ LP	-0.0206	0.1940	-0.1064	0.9152
$b_{CM-tent.}$ LP	0.4336	0.1937	2.2385	0.0253
$b_{LO-bind.}$ LP	0.7292	0.1940	3.7587	0.0002
$b_{CM-bind.}$ LP	0.9752	0.1911	5.1034	0.0000
b_{DC} decentralized	5.5505	0.1940	28.6092	0.0000

coefficient of determination: 0.7389

$\hat{\phi} = 0.8366$ Wald stat$= 5548.5$ $p = 0.0000$

Table 3: Hypotheses Testing for Feasible GLS Estimation

H_0	statistics	degrees of freedom	decision for 5%
Coef. of payoff from high agents >0:	$t = 4.6550$ $p = 0.0000$	2386	not rejected
Speed of priority mechanisms equal to each other:	$f = 1.2064$ $p = 0.2095$	$2, 2386$	not rejected
Speed of d.a. mechanisms equal to each other:	f_stat=0.0006 $p = 0.9809$	$1, 2386$	not rejected
Speed of l.p. mechanism tent. tech. dummies equal to each other:	$f = 2.7552$ $p = 0.0971$	$1, 2386$	not rejected
Speed of l.p. mechanism bind. tech. dummies equal to each other:	$f = 0.2654$ $p = 0.6065$	$1, 2386$	not rejected
Speed of dummies of l.p. tent. tech. and bind. tech. are equal:	$f = 1.8241$ $p = 0.1406$	$2, 2386$	not rejected
Speed of l.p. bind. tech. and d.a. mech. same:	$f = 0.0922$ $p = 0.9644$	$3, 2386$	rejected
Speed of priority and d.a. mech. same:	$f = 4.7398$ $p = 0.0008$	$4, 2386$	rejected
Speed of priority and l.p. bind. tech. dummies same:	$f = 4.5921$ $p = 0.0011$	$4, 2386$	rejected
Speed of all centralized mechanism dummies equal under bind. tech.:	$f = 4.1905$ $p = 0.0003$	$6, 2386$	rejected

Table 4: Hypotheses Testing for Aysmptotic Values of Cost

H_0	statistics	degrees of freedom	decision for 5%
Asymp. cost of priority mechanisms equal to each other:	$f = 0.6762$ $p = 0.5087$	$2, 2386$	not rejected
Asymp. cost of d.a. mechanisms equal to each other:	$f = 0.3846$ $p = 0.5352$	$1, 2386$	not rejected
Cost of l.p. mechanism tent. games equal to each other:	$f = 6.8482$ $p = 0.0089$	$1, 2386$	rejected
Cost of l.p. mechanism bind. games equal to each other:	$f = 1.8941$ $p = 0.1689$	$1, 2386$	not rejected
Cost of l.p. tent. tech. and bind. tech. are equal:	$f = 18.2170$ $p = 0.0000$	$2, 2386$	rejected
Cost of l.p. bind. tech. and d.a. mech. same:	$f = 1.7488$ $p = 0.1549$	$3, 2386$	not rejected
Cost of priority and d.a. mech. same:	$f = 17.0391$ $p = 0.0000$	$4, 2386$	rejected
Cost of priority and l.p. bind. tech. same:	$f = 25.3097$ $p = 0.0000$	$4, 2386$	rejected
Cost of all centralized mechanisms equal under bind. tech.:	$f = 20.6003$ $p = 0.0000$	$6, 2386$	rejected

effect that KR found in their laboratory experiments. The average cost of unraveling is affected by the payoff from high types directly. Following table 3, the null hypotheses involving the equality of speed of decrease in average cost within the priority mechanisms, within the LP mechanisms under the tentative transactions, within LP mechanisms under the binding contract technology, and within the DA mechanisms are not rejected. The slowest decrease is seen under the decentralized market, followed by the priority matching markets. DA mechanism and LP mixed games with binding transactions lead to comparable speeds of convergence, although DA mixed market speeds seem faster. The fastest decrease is observed under the LP mixed markets with tentative transaction game. The next test asks whether the speed of decrease is equal under two different institutional restrictions in the LP markets: hypothesis is not rejected. Then, we determine whether the DA and LP mechanisms under binding technology generate similar processes: the speed estimates are not significantly different. We test whether speed coefficients of priority and DA markets are the same: the null hypothesis is rejected. It is next tested whether speed coefficients of the priority and the LP markets under the binding technology are the same: the null hypothesis is rejected. Then, we test whether all the centralized mechanisms are have the same convergence speed under the binding contract technology: the coefficients are not equal to each other. The next set of tests are about the asymptotic values of costs under the mixed mechanisms. This asymptotic estimate is determined to be $\widehat{c}_{i,g}$ for $g = 160$. So we test whether $\frac{a_i}{120} + b_i = \frac{a_j}{120} + b_j$ for different markets i and j. Following table 4, the binding early contracts produce similar results within the LP and within the DA markets. The order of the asymptotic costs can be given as: decentralized markets > priority mixed markets > LP mixed markets with binding transactions and DA mixed markets > LP mixed markets with tentative transactions. One can follow table 3 and 4 for the statistic values of the tests mentioned above.

We also make a sensitivity analysis for the GA evolution.[12]

7 Conclusions

This study considers the adjustment dynamics of agents to three types of two-sided matching mechanisms that came into use in the field in Britain after a decentralized matching era. Unraveling is studied with specific matching games with several rounds of clearance. We use a GA in the evolutionary programming framework. Static game theoretic analyses fail to explain the field success of a class of theoretically unstable mechanisms. Analytical evolutionary analyses seem

[12] Sensitivity Analysis: Same experiments are run for different parameter values under the ceteris paribus assumption:

Genetic Operator Changes:

Increase in the mutation rate: The uniform mutation rate for each digit of the strategy strings is increased from

Decrease in the crossover rate: The uniform crossover rate is decreased from

Change in the reproduction operator: We adopt biased-roulette-wheel selection instead of first-past-the-post. According to selected bias, we observe slower convergence and higher costs.

Change in the crossover operator: Instead of linear crossover, we consider circular version. The convergence results occur slightly faster; we observe lower costs.

Change in the initial conditions: Suppose that instead of random rank-order list generation in round 0, we consider truthful revelation (straightforward) of rank-order lists in the centralized games. In the short run, the LP markets unravel "most" under both communication technologies. The priority markets unravel earlier than the DA markets do but not as much as the LP markets do. In the long run we return to our prior findings: the LP markets under the tentative arrangement technology unravels least, which are followed by the DA and LP markets under the binding arrangement technology. Finally, priority markets unravel most in the long run. (See figure 7.)

Change in Strategy Form: When we assume agents only decide for offer and acceptances but do not make decisions on rank-order lists instead always submit full truthful lists, we observe that DA mechanism is most successful algorithm. Priority and LP tentative, LP binding treatments have less success in preventing early contracts. Next section's simulations consider these kinds of strategies.

Changes in Model Parameters:

We consider two changes in the model parameters.

The first one is increase in the range of the random parameter in the payoff: The random parameter $\theta_{f,w}$ for firm f for each w and $\theta_{w,f}$ for the worker w for each f (described in section 3) are drawn from $U(-2.5, 2.5)$ instead of $U(-1, 1)$ independently and identically. The average cost of unraveling is observed to increase for the unstable markets slightly and decrease for the stable markets slightly. Under the binding arrangement technology, deferred acceptance markets lead to comparable levels of unraveling with the linear programming markets. Both lead to lower number of early arrangements than the priority markets.

The second is a change in the average payoff from the high type agents: Average payoff from the high, t_v when v is a high agent is then set to \$10 and \$100 respectively instead of \$15. When it was set to \$10, the total cost of unraveling on average fell for all the markets, and when it was set to \$100, the total cost of unraveling on average increased for all the markets over $t_v = 15$ (for v is high) levels. The major difference was the rate of acceptances done by the high workers in round -1. The rates were highest for $t_v = 100$ (for v is high) and lowest for $t_v = 10$ (for v is high). Also high firms make more offers in round -1 for $t_v = 100$ (for v is high).

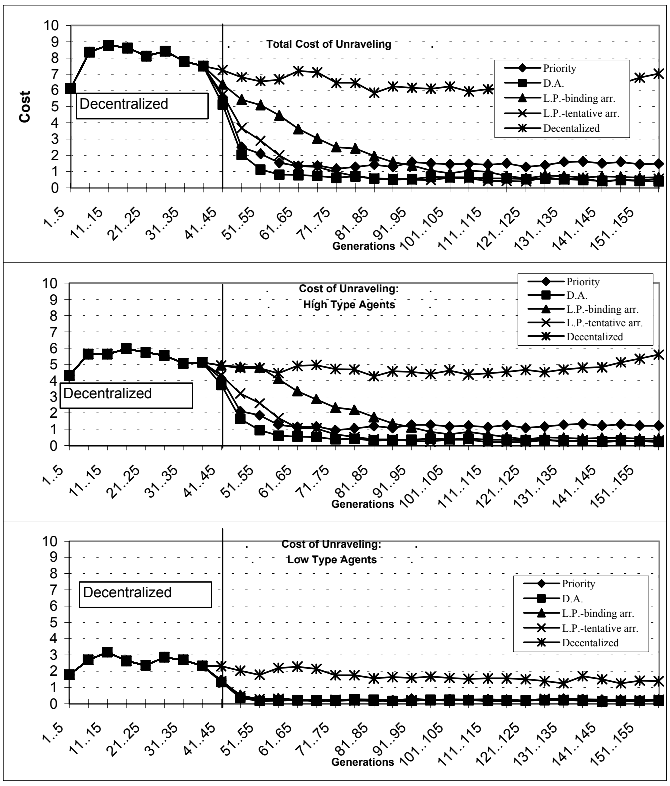

Figure 7: Evolution with Random Initial Strategies for the Decentralized Game, Truthful Initial Rank-Order Lists for the Mixed Games, and with Rank-Order List Updating

limited to examine the complex matching games for equilibrium selection. The main contribution of this paper is a dynamic analysis of unraveling using GAs.

The GA serves as an evolution environment to study learning under incomplete private information. The learning occurs through communication and transmission of experience. Following KR's (forthcoming) experimental design, we assume that both firms and workers are permanent players.

In summary, decentralized market evolution leads to unraveling for both better and less qualified agents. Less qualified agents do not unravel once mixed mechanisms are introduced. We find asymptotic evolution paths with low levels of unraveling for highly unstable LP markets. These are also successful in the field in Britain. We find also qualitative evidence for plausibility of an evolution process in KR's (forthcoming) experiment. The unstable priority matching mechanisms that failed in the field and laboratory also perform most poorly in the evolution environment in terms of costs.

The difference between the priority and LP mechanisms arises from the intrinsic optimization procedure used in the LP mechanisms. For agents, manipulation of rank order lists occur under the mixed games.[13] The difference in the evolution stage indicates that early offers and acceptances are in higher percentages under the priority mechanisms than they are under the LP mechanisms.

When we consider different initial conditions, the stability results hold only asymptotically. For example, if inexperienced agents submit truthful rank-order lists as they would in the laboratory, "LP markets unravel the most" in the short run.

In the literature, the unraveling issue has mostly been considered in a static framework. However field results show that, unstable mechanisms that are not susceptible to unraveling do exist. It seems necessary to consider models of adaptive behavior to illuminate the dynamics observed in the field when new market institutions are introduced. Computational tools of modern game theory present such opportunities.

Appendix A. Preliminaries about the Marriage Model (Section 3)

Marriage model is used to analyze the unraveling problem in entry-level labor markets. The models in the literature are based mostly on works of Roth, Sotomayor, Gale and Shapley. Here, this general framework will be presented briefly. The set of players $N = F \cup W$ (such that $F \cap W = \phi$) consists of the set of firms $F = \{f_1, f_2, ..., f_n\}$ and the set of workers $W = \{w_1, w_2, ..., w_m\}$ where each firm can hire one worker, while one worker can work for one firm. The preferences of each worker $w \in W$ over firms and herself are denoted by $P(w)$, an ordered list of elements in $\{w\} \cup F$.

[13] We ran several artificial agent simulations for the Unver (2000a and b) studies in which agents always submit truthful lists. We do not permit gaming of preference lists in these simulations. Under this implementation, the LP mechanisms are not as successful as the DA mechanisms are. They perform poorest with the priority matching mechanisms. Therefore manipulation of rank-order lists seems as an important factor in success of LP mechanisms.

Similarly the preferences of each firm $f \in F$ over workers and itself, $P(f)$ is an ordered list of elements of $\{f\} \cup W$. All preferences are assumed to be rational (i.e., complete and transitive). Let "$\geq_{P(v)}$" denote "weak preference" relation and "$>_{P(v)}$" denote "strict preference" relation with respect to the preference list $P(v)$, the component of the preference profile P for the agent v. A worker w is acceptable given $P(f)$ to firm f if $w \geq_{P(f)} f$. Similarly a firm f is acceptable given $P(w)$ to worker w if $f \geq_{P(w)} w$. Let $P = (P(f_1), \ldots, P(f_n), P(w_1), \ldots, P(w_m))$. Now suppose $\mathcal{P}$ is the set of rational and admissible preference profiles on N so that $P \in \mathcal{P}$.

We can now define a one-to-one matching.

Definition 4 *A one-to-one matching μ in the market is a function defined on the set of players. Formally $\mu : N \to N$. μ satisfies $\forall w \in W$ and $\forall f \in F$:*
(i) $\mu(w) = f$ if and only if $w = \mu(f)$,
(ii) $\mu(w) = w$, if $\mu(w) \notin F$, and
(iii) $\mu(f) = f$, if $\mu(f) \notin W$.

Workers (firms) have preferences over matchings identical to their preferences over firms (workers) and themselves. A matching μ is said to be individually rational given $P(v)$ if $\forall\ v \in N$ $\mu(v) \geq_{P(v)} v$. A matching μ is stable given P, if it is individually rational given P and if there exist no $f \in F$ and $w \in W$ such that $\mu(w) \neq f$, $w >_{P(f)} \mu(f)$ and $f >_{P(w)} \mu(w)$.Let $\mathcal{M}$ be the set of matchings defined on N.

There exists at least one stable matching for any set of players and rational strict preference profiles.[14] A matching mechanism , is defined by a function $\pi : \mathcal{P} \to \mathcal{M}$. A mechanism is stable within $\mathcal{P}$ if there exists no $P \in \mathcal{P}$ such that $\pi[P]$ is unstable. A mechanism is individually rational within $\mathcal{P}$ if $\pi[P]$ is individually rational for all $P \in \mathcal{P}$. Unless otherwise denoted, let π denote also the matching $\pi[P]$ under the true preferences P. A matching problem is denoted by a set of firms, a set of workers, a preference profile and a mechanism such as (F, W, P, π).

Suppose that preferences of agents over individuals are strict. Therefore, suppose that $\mathcal{P}$ is further restricted in this sense. There exists a unique stable matching μ_W which is weakly preferred to any other stable matching by all workers in W, at least one of the weak preferences is strict. Moreover there exists a unique stable matching μ_F which is weakly preferred to any other stable matching by all firms in F, at least one of the weak preferences is strict. μ_F is known to be the firm optimal stable matching. μ_W is known to be the worker optimal stable matching.[15] Firm proposing DA algorithm determines μ_F given the rank-order lists Q, worker proposing DA algorithm determines μ_W given the rank-order lists Q. The other mechanisms are described in section 2. Let $P(v, k)$ denote the kth ranked agent in the preference of $v \in N, P(v)$. A match between firm f, and worker w is a (k, l) match given the rank-order list profile Q if $Q(f, l) = w$ and $Q(w, k) = f$.

[14] Theorem proven by Gale and Shapley (1962)
[15] A result by Gale and Shapley (1962)

One simple example can show that the Birmingham priority, μ_{BI}, the London LP, μ_{LO}, mechanisms are unstable. Reader can verify for other priority and LP mechanisms.

Example 5 *If* $P(f_1) = (w_1, w_2, w_3)$*,* $P(f_2) = (w_2, w_1, w_3)$*,* $P(f_3) = (w_2)$*,* $P(w_1) = (f_3, f_2, f_1)$*,* $P(w_2) = (f_3)$*,* $P(w_3) = (f_2, f_1)$ *then* $\mu_{BI}(f_1) = \mu_{LO}(f_1) = w_1$*,* $\mu_{BI}(f_2) = \mu_{LO}(f_2) = w_3$*,* $\mu_{BI}(f_3) = \mu_{LO}(f_3) = w_2$*. For the London LP mechanism, this creates the weight* 57 *for* $(1,3)+57$ *for* $(3,1)+72$ *for* $(1,1) = 186$ *as maximum. The only stable matching is* $\mu(f_1) = w_3$*,* $\mu(f_2) = w_1$*,* $\mu(f_3) = w_2$*. It creates the weight* 49 *for* $(3,2) + 56$ *for* $(2,2) + 72$ *for* $(1,1) = 177$*. Note that* $w_1 >_{P(f_2)} w_3$ *and* $f_2 >_{P(w_1)} f_1$ ■

Also $(1,1)$ matches might not be realized under the London LP mechanism. It can also be verified for the Cambridge LP mechanism.

Example 6 *If* $P(f_1) = (w_3, w_2, w_1)$*,* $P(f_2) = (w_2, w_1, w_3)$*,* $P(f_3) = (w_2, w_1, w_3)$*,* $P(w_1) = (f_3, f_2, f_1)$*,* $P(w_2) = (f_3, f_2, f_1)$*,* $P(w_3) = (f_2)$*.Then the highest weighted matching* μ *that realizes* f_3, w_2 $(1,1)$ *match is* $\mu(f_1) = w_1, \mu(f_2) = w_3$ *and* $\mu(f_3) = w_2$ *brings the weight* 42 *for* $(3,3), 57$ *for* $(3,1), 72$ *for* $(1,1)$ *a total of* 171*. However the following matching* ν *brings the weight* 177 *as maximum* $\nu(f_1) = w_2$*,* $\nu(f_2) = w_3$*,* $\nu(f_3) = w_1$ *with* 56 *for* $(2,2), 57$ *for* $(1,3), 64$ *for* $(2,1)$ *matches.* ■

By the revelation principle, we can focus on the matching mechanisms under rank-order list submission games. Stability of a mechanism appears as a cooperative, complete information issue. A matching is stable if and only if it is in the core of the mechanism rank-order list submission game. Theory also implies that there exist no incentive-compatible mechanisms under complete information.

Appendix B. Proofs (Section 4)

Sketch of Proof of Lemma 1: Consider the strategy s which tells firms, not to make any offers in rounds -2 and -1; which tells workers, not to accept any offers in rounds -2 and -1; and which tells a high type firm (worker) to rank only 3 high type workers (firms) in its (her) rank-order list, a low type firm (worker) to list all 6 workers (firms) in (her) rank-order list in accordance with its (her) true preferences in round 0. Denote this profile with Q. Now s leaves no one unmatched, causes no mismatches and does not involve any early contracts.

Now, we need to show that s is an equilibrium.

(i) This strategy gives the same outcome with the truthful revelation equilibrium of the direct revelation game. A similar proof to the proof of lemma 3 can show that Q is an equilibrium of the round 0 subgame. Since nobody else makes early contracts, one's deviation in round -2 or -1 will not change her payoff. So, s is an equilibrium for the DA mixed game.

(ii) We will give the proof only for the Newcastle priority mixed game. Consider the following most profitable deviation from s: an agent v omits the last ranked agent in $Q(v)$ from her rank-order list, and all other nodes of this strategy coincide with those of $s(v)$. Note that a deviation to early offers or acceptances do not change the payoff of an agent given that no one else is making early offers or accepting early offers.

Suppose a high worker w omits her third choice i.e. plays $Q'(w) = (f_1, f_2)$ while $Q(w) = (f_1, f_2, f_3)$.[16] This will result no change for profiles where she was matched to her top choice. It can be shown that her expected payoff[17] at s is

$u = \frac{637}{972} u_w(f_1) + \frac{791}{3888} u_w(f_2) + \frac{61}{432} u_w(f_3)$

and her expected payoff at the strategy which only has a deviation from s in round 0 using $Q'(w)$ is

$u' = \frac{637}{972} u_w(f_1) + \frac{815}{3888} u_w(f_2) + \frac{175}{432} u_w(w)$

$u > u'$ if and only if $183 u_w(f_3) > 8 u_w(f_2)$ since $u_w(w) = 0$.

$u_w(f_2) - u_w(f_3) < 2$ and $u_w(f_3) > 0$ imply that the inequality holds in our model. Early offers do not profit w, while everybody else is playing s_{-w}.

The Newcastle mechanism does not treat firms and workers symmetrically. A (k, l) match is favored before an (a, b) match if $k \times l = a \times b$ and $k < a$. That is, workers' preferences are favored over firms' in cases of ties in the product numbers between two matches.

Suppose a high firm f omits its third choice i.e. it plays $Q'(f) = (w_1, w_2)$ when $Q(f) = (w_1, w_2, w_3)$. This will result with also some changes for the profiles where it was matched to its top choice. It can be shown that its expected payoff at s is

$u = \frac{641}{1296} u_f(w_1) + \frac{1247}{3888} u_f(w_2) + \frac{359}{1944} u_f(w_3)$

and her expected payoff at the strategy which deviates from s only in round 0 using $Q'(f)$ is

$u' = \frac{245}{486} u_f(w_1) + \frac{1303}{3888} u_f(w_2) + \frac{625}{3888} u_f(f)$

$u > u'$ if and only if $718 u_f(w_3) > 37 u_f(w_1) + 56 u_f(w_2)$ since $u_f(f) = 0$.

$u_f(w_1) - u_f(w_3) < 2$, $u_f(w_2) - u_f(w_3) < 2$ and $u_f(w_3) > 0$ imply that the inequality holds in our model. Early offers do not profit f, while everybody else is playing s_{-f}.

Similar statements can be stated for a low type worker and a low type firm deviations. We have shown that s is an equilibrium of the Newcastle priority mixed game. Since the Birmingham mechanism is dual to Newcastle's, a proof changing roles of firms and workers will work. Reader can verify the lemma for the Edinburgh'67 mechanism. ■

Sketch of Proof of Lemma 2: Consider any symmetric strategy profile s with non-empty rank-order lists. In s in round 0, high firms (workers) should list all high workers (firms), and low firms (workers) should list all low workers (firms) so that there are no mismatches and unmatched

[16] Let f_i denote the i'th ranked firm in $P(w)$ that is $P(w, i)$

[17] One can check all possible preference configurations which occur with equal probability and find the match of w in each case to determine the expected payoff.

agents. Moreover, high agents do not list low agents in round 0, since otherwise mismatches can occur with positive probability or a low agent can have a more profitable deviation from s. Now in round -2 or -1, no contracts can occur. Without loss of generality, consider the rank-order lists of round 0 that consist of high (or low) firms ranking only high (or low) workers in order with their true preferences, and high (or low) workers ranking only high (or low) firms in order with their true preferences. This random preference profile $\widetilde{P}'$can be represented for high firms by $\widetilde{P}'(f) = (\widetilde{P}(f,1), \widetilde{P}(f,2), \widetilde{P}(f,3))$, high workers $\widetilde{P}'(w) = (\widetilde{P}(w,1), \widetilde{P}(w,2), \widetilde{P}(w,3))$, low firms by $\widetilde{P}'(f) = (\widetilde{P}(f,4), \widetilde{P}(f,5), \widetilde{P}(f,6))$, low workers by $\widetilde{P}'(w) = (\widetilde{P}(w,4), \widetilde{P}(w,5), \widetilde{P}(w,6))$. Let P' be the realization of this profile.

Let f be a high firm with $P(f) = (w_1, w_2, w_3, w_4, w_5, w_6)$ only ranking his first choice by deviating from these profiles where $P'(f) = (w_1, w_2, w_3)$. Let this deviation be $P''(f) = (w_1)$.

The weights are (actual weights used in London) $36, 28, 21, 15, 10, 6$ for choices ranked 1 to 6. Let P be a realization of preferences. Now the two matching problems are $A = (F, W, P', \mu_{LO})$ versus $B = (F, W, (P''(f), P'_{-f}), \mu_{LO})$ using the London matching mechanism.

Consider any matching μ that leaves f unmatched i.e. $\mu(f) = f$ and brings total weight "a" in problem B. Let $\mu(f')$=w_1. Suppose $\mu(w) = w$ for the high worker w.

Consider the following matching ν with $\nu(f) = w_1$ this will bring at least $(1,3)$ match weight $36+21 = 57$, and $\nu(f') = w$ this will bring at least $(3,3)$ match weight $21+21 = 42$. But we might have broken at best a $(1,1)$ match between w_1 and f' with a loss of weight $36 + 36 = 72$. Let ν and μ match the same agents for the remaining of the market.

ν brings at least 27 points more weight than μ in problem B.

Therefore, in the outcome matching of problem B, f will never be unmatched and will be matched to w_1.

In problem A:

$Pr\{\mu_{LO}[P'(f), \widetilde{P}'_{H-f}](f) <_{P(f)} w_1\} > 0$.

i.e. with positive probability f can be matched to its 2'nd or 3'rd choices.

Thus, outcome of B is preferable to A for f. Therefore, f has an incentive to truncate its rank-order list under the symmetric strategy profile mentioned above.

Similarly, it can be stated that a high (or low) agent has incentive to deviate for any symmetric strategy profile with no mismatches, no early contracts and no unmatched agents.

A symmetric strategy profile cannot constitute an equilibrium with no-early contracts no mismatches and non-empty rank-order list submission.

A similar proof can be stated for the Cambridge mechanism. ■

Sketch of Proof of Lemma 3: Consider the firm proposing DA mechanism, μ_F. The lemma can be proven in 3 steps. Let P be the preference profile of agents.

(i) By theorem 4.7 of Roth and Sotomayor (1990), it is dominant for firms to reveal their preferences truthfully.

(ii) Consider workers. By corollary 5 by Roth and Rothblum (1999), it is stochastically dominant for worker w to reveal $P(w)$ or truncations of $P(w)$ in response to P_{-w}. First suppose that w is a high type worker. Let $P(w) = (f_1, f_2, f_3, f_4, f_5, f_6)$. Now the potentially most profitable deviation is a truncation as $P'(w) = (f_1, f_2)$

The expected payoff of w under the strategy profile P can be given as

$u = \frac{35}{72}u_w(f_1) + \frac{67}{216}u_w(f_2) + \frac{11}{54}u_w(f_3)$.

The expected payoff of w under the strategy profile $(P'(w), P_{-w})$ is

$u' = \frac{233}{432}u_w(f_1) + \frac{157}{432}u_w(f_2) + \frac{7}{72}u_w(w)$.

$u > u'$ if and only if $88u_w(f_3) > 23u_w(f_1) + 23u_w(f_2)$ since $u_w(w) = 0$.

Now $u_w(f_1) - u_w(f_3) < 2$, $u_w(f_2) - u_w(f_3) < 2$, and $u_w(f_3) > 0$ imply that above inequality holds. So w has the best response $P(w)$ to P_{-w}.

(iii) For a low type worker w, the most profitable deviation from $P(w) = (f_1, f_2, f_3, f_4, f_5, f_6)$[18] is $P'(w) = (f_1, f_2, f_3, f_4, f_5)$ in response to P_{-w}. The expected payoff of w under strategy profile P is

$u = \frac{35}{72}u_w(f_4) + \frac{67}{216}u_w(f_5) + \frac{11}{54}u_w(f_6)$.

The expected payoff of w under the strategy profile $(P'(w), P_{-w})$ is

$u' = \frac{233}{432}u_w(f_4) + \frac{157}{432}u_w(f_5) + \frac{7}{72}u_w(w)$.

$u > u'$ if and only if $88u_w(f_6) > 23u_w(f_4) + 23u_w(f_5)$ since $u_w(w) = 0$.

Now $u_w(f_4) - u_w(f_6) < 2$, $u_w(f_5) - u_w(f_6) < 2$ and $u_w(f_6) > 0$ imply that above inequality holds. So w has the best response $P(w)$ to P_{-w}.

We have shown that P is a strict equilibrium of the direct revelation game of μ_F.

Proof can be modified by changing roles of firms and workers for the worker proposing DA mechanism μ_W. ■

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