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Are Absolute Emissions Better for Modeling?

It's All Relative

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Abstract

Some environmental policies focus on emissions intensity rather than total emissions, or they try to mitigate the regulatory impact on the final product market. To analyze the effects of these policies, or to evaluate the distributional effects of any regulation on consumers and producers, output must be incorporated explicitly into an economic model of abatement, separately from the emissions variable. This provides two options. Traditionally, total emissions and output are the independently controlled variables, leaving emissions intensity as endogenously determined. Alternatively, one can make emissions intensity and output the control variables, leaving total emissions as the endogenously determined variable. One is the dual of the other and the problems are equivalent, but the latter method offers more transparency for examining intensity-based policies. This note shows how the intensity-based model fits into the traditional context.

Key Words: emissions intensity, emissions standards, environmental tax, pollution, tradable emissions permits

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Introduction

In typical models of emissions taxes or tradable permit policies, economists focus on the emissions side of the market—the costs and benefits of abatement—while the final product market remains in the background. On the other hand, for some emissions control policies, one of the regulatory goals is explicitly to mitigate price impacts in the product market—not just to affect the emissions market. Primary examples are intensity-based policies, like tradable performance standards or emissions price policies that incorporate output-based refunding. These range from U.S. EPA's lead phasedown program, to the Corporate Average Fuel Economy standards, to the Swedish NO_x tax-rebate plan, to proposals to allocate tradable emissions permits according to output shares within an industry.[1]

To analyze the effects of these kinds of policies, or to evaluate the distributional effects of any regulation on consumers and producers, one must incorporate output explicitly into the economic model, separately from the emissions variable. Then, two options are available. One option is to choose total emissions and output as independently controlled variables, leaving emissions intensity as endogenously determined.[2] Alternatively, one can make emissions intensity and output the control variables, leaving total emissions as the endogenously determined variable.[3] One is the dual of the other and the problems are equivalent. I often choose the latter method in my work as a bit more transparent method for examining intensity-based policies. However, since this choice of focusing on emissions intensity is less common, I present this note to show how it fits into the traditional context.

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[1] These kinds of rate-based allocations more generally are part of the U.S. NO_x trading program under development for the Northeastern states and a likely part of national allocation plans for CO_2 in the European Union.

[2] For examples in the rebating literature, see Sterner and Isaakson (forthcoming), Gersbach and Requate (2004) or Bernard et al. (2005).

[3] For example, see Fischer (2003), as well as Quirion (2003). Millock and Nauges (2006) use input efficiency as a choice parameter.

Modeling Choices

Let us consider the problem of a representative competitive firm facing an emissions price (or tax) and receiving a permit allocation (or tax refund) according to its output. Let P be the output price, q be the output quantity, e be emissions, t be the price of emissions, and a be the per-unit allocation, which is also valued at t. Emissions intensity is then the ratio of total emissions per unit of output: $\mu = e / q$. The firm's production costs are some function $C(\mu, q)$ that is increasing and weakly convex in output and declining and convex in emissions intensity. Furthermore, targeting a higher emissions rate lowers the cost of additional output.

Emissions Framework

Let us examine the problem for firm *i* in the framework with total emissions and output as the control variables, with intensity endogenous. In this case, we write the profit function as

$$\pi_i = Pq_i - C(e_i, q_i) - te_i + taq_i \tag{1}$$

Then, the first-order condition with respect to total emissions is

$$-\frac{\partial C(e_i, q_i)}{\partial e_i} = t \tag{2}$$

or that the marginal cost of reducing emissions must just balance the emissions price.

Maximizing with respect to output, we get

$$P = \frac{\partial C(e_i, q_i)}{\partial q_i} - ta \tag{3}$$

In other words, the equilibrium price will equal the marginal cost of production, given the level of emissions, plus the value of the emissions allocation. Note that with emissions as an independently controlled variable, output only affects the refund on the margin, not the emissions payment.

Now, let us use emissions intensity and output as the control variables, with total emissions being endogenous. Then, the problem of our representative firm is

$$\pi_i = Pq_i - C(\mu_i q_i, q_i) - t\mu_i q_i + taq_i \tag{4}$$

Maximizing profits with respect to emissions intensity, we have

$$\frac{\partial \pi_i}{\partial \mu_i} = \left(-\frac{\partial C(\mu_i q_i, q_i)}{\partial e_i} - t \right) q_i = 0 \tag{5}$$

which yields the same result as (2). Maximizing now with respect to output, we get

$$\frac{\partial \pi_i}{\partial q_i} = P - \frac{\partial C(e_i, q_i)}{\partial q_i} - \frac{\partial C(e_i, q_i)}{\partial e_i} \mu_i - t\mu_i + ta = 0 \tag{6}$$

However, using (5), this condition simplifies to the same result as (3).

Implicit, but not explicit, in this analysis is that increasing output raises emissions. Totally differentiating (2), we see that $\frac{de_i}{dq_i} = -\frac{\partial^2 C}{\partial e_i \partial q_i} / \frac{\partial^2 C}{\partial e_i^2} > 0$. Furthermore, the tax costs of those additional emissions are implicitly in the production costs. However, recognizing that one way to reduce total emissions is to conserve output is often easier to see in a model of emissions intensity, in which this tradeoff is explicit.

Intensity Framework

In this explicit intensity framework, we designate production costs as a function of emissions intensity and output, although these costs could still represent the same underlying function as before. With $C(\mu_i, q_i)$, we similarly assume that costs are increasing and weakly convex in output and declining and convex in emissions intensity, and that targeting a higher emissions rate lowers the cost of additional output.

Now, the problem becomes

$$\pi_i = Pq_i - C(\mu_i, q_i) - t\mu_i + taq_i \tag{7}$$

The first-order condition with respect to emissions intensity reduces to

$$-\frac{\partial C(\mu_i, q_i)}{\partial \mu_i} / q_i = t \tag{8}$$

or that the per-unit cost of reducing emissions intensity must equal the emissions price. We write the condition in this manner to show the result is the same as before, since $\frac{\partial C(e_i / q_i, q_i)}{\partial e_i} = \frac{\partial C(\mu_i, q_i)}{\partial \mu_i} / q_i$.

Maximizing with respect to output, we now get

$$P = \frac{\partial C(\mu_i, q_i)}{\partial q_i} + t(\mu_i - a) \tag{9}$$

In other words, the firm will produce until the market price reflects the sum of the marginal cost of output and the cost of the emissions embodied in that output, net of the value of the additional allocation. Again, in the same manner as before, if we set up the problem as $\pi_i = Pq_i - C(e_i/q_i, q_i) - te_i + taq_i$ and maximized with respect to total emissions and output, we would end up with the same first-order conditions as with emissions intensity.

The key difference between (9) and (3) is that the cost function is holding emissions intensity, rather than total emissions, constant when we evaluate the cost of an additional unit of output. In other words, the answers differ because we are asking different questions. In (3) we ask, what does the next unit of output cost if we adjust our behavior to keep our *emissions* constant? In (9) we ask, what does the next unit of output cost if our behavior keeps our *emissions rate* constant?

This is a subtle but important difference. In terms of the optimal allocation of emissions and output, the choice of framework doesn't matter. However, the latter formulation seems more intuitive for evaluating the interaction between emissions markets and output markets. When we think about how firms approach emissions reductions, we focus on technologies and changes to production practices that reduce emissions, given any level of output—that is, that reduce emissions intensity. We also recognize that if we continue with the current business as usual, increasing output tends to increase emissions. The flip side of that coin is that another option for reducing emissions is conservation—that is, producing less. These tradeoffs are all readily apparent when the modeling framework focuses on emissions intensity.

Abatement Cost Framework

Policymakers have a penchant for tracking emissions intensity, since they are focused on product markets. An oft-stated goal is restraining emissions without sacrificing growth. Most traditional command-and-control regulations limit emissions rates, not total emissions. Now, market-based programs are also being given a rate-based focus. An important part of the reasoning behind programs that allocate emissions permits or refund emissions tax revenues according to output is to mitigate the impact of a regulation on prices and production.

Environmental economists, on the other hand, tend to focus on emissions markets. Output consequences stay in the background, and lost surplus from higher costs and lower output merely forms part of the abatement costs. Fortunately, we can easily map this output-oriented framework back into the traditional framework of abatement costs.

Let $F(A)$ be the abatement cost function, where $A = \mu_0 q_0 - \mu q$. First, let us identify baseline emissions and output as the values of μ_0 and q_0 that jointly solve $-C_\mu(\mu_0, q_0) = 0$ and $P(q_0) = C_q(\mu_0, q_0)$, since our representative firm represents total industry output, which must in equilibrium equal demand.

Next, for this partial equilibrium model, total surplus is the area under the demand curve net of production costs:

$$S(\mu, q) = \int_0^q P(s)ds - C(\mu, q) \tag{10}$$

The abatement cost curve then is the surplus foregone—the increase in production costs and the lost surplus of reduced output:

$$\begin{aligned} F(\mu_0 q_0 - \mu q) &= S(\mu_0, q_0) - S(\mu, q) \\ &= \int_q^{q_0} P(s)ds + C(\mu, q) - C(\mu_0, q_0) \end{aligned} \tag{11}$$

These costs depend on the choice of μ and q, not on redistributive measures like the tax revenues or payments by consumers to producers. However, the combination of μ and q chosen by the firm does depend on the combination of tax and allocation rates. Thus, if we are evaluating policies that do more than set an emissions price or abatement target, we cannot think of a single abatement supply curve, since it will shift with the output allocation incentives.

$$F'(A) = (C_q(\mu, q) - P(q))\left(\frac{dq}{dA}\right) + C_\mu(\mu, q)\left(\frac{d\mu}{dA}\right) \tag{12}$$

In a decentralized framework, μ and q must jointly satisfy $-C_\mu(\mu, q) = t$ and $P(q) = C_q(\mu, q) + t(\mu - a)$. By definition, $dA = -qd\mu - \mu dq$, so $d\mu/dA = -(1 + \mu dq/dA)/q$. Substituting and simplifying, we can express total marginal abatement costs as a function of the marginal unit costs of reducing emissions intensity and the effect of the industry's output response on its allocation:

$$F'(A) = -\frac{C_\mu(\mu, q)}{q} + ta\left(\frac{dq}{dA}\right) \qquad (13)$$

Thus, in the absence of any output-based allocation ($a = 0$), we see that total marginal abatement costs equal the marginal unit costs of reducing emissions intensity, which in turn equals the emissions price. Positive output incentives, however, increase the overall marginal abatement costs—since less conservation is being made, more effort has to be put into reducing emissions intensity. A caveat for this result is the partial equilibrium framework, as the welfare costs in a general equilibrium framework are more complex than the partial costs if there are other market imperfections present.

Conclusion

Ultimately, choosing whether to model environmental policy effects using emissions or intensity is a matter of taste and convenience. This choice does not affect the absolute results with respect to the economic tradeoffs. It can, however, affect the relative emphasis in the interpretation of those results. In particular, the tradeoffs between conservation and changing production techniques can be more transparent with intensity choice models.

The key difference is in the modeling of costs. In the first method, we evaluate the cost of the next unit of output, assuming we adjust our behavior to keep our emissions constant. In the second method, we evaluate the cost of the next unit of output assuming our behavior keeps our emissions rate constant.

When simplified forms of production costs are useful, intensity is often a more logical choice. For example, computable general equilibrium (CGE) models of the economy often assume constant returns to scale in production. Emissions reductions then come from the production factor mix (effectively determining intensity) and the contraction of the industry.[4] One can then assume that the chosen emissions intensity determines the level of the constant marginal production costs. These kinds of simplifications are suitable for pollution reductions made through changes to input uses (like fuel switching), production methods, or end-of-pipe technologies that scrub a certain percentage of emissions. However, they are less able to take into account technologies that remove absolute quantities of emissions. Therefore, the available

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[4] See Dissou (2005).

abatement options need to factor into the choice of simplified forms of production costs that are inclusive of abatement.

Although this note demonstrates examples in a partial equilibrium model, the duality concept applies equally to general equilibrium models with multiple sectors.[5] This lesson is simply that if the underlying functions are the same, and the regulatory policies are the same, the markets (and the economists) will arrive at the same point, whether they get there focusing on reducing emissions intensity or total emissions.

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[5] In CGE models comparing emissions permit allocation systems, Dissou (2005) uses emissions factors for production inputs while Boehringer and Lange (2005) use total emissions as the control variable.

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