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Monopoly Extraction of an Exhaustible Resource with Two Markets

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Abstract

Although much has been written about the implications of monopoly power for the rate of extraction of natural resources, the specific case in which the resource can be sold in two markets with different elasticities of demand has escaped notice. We find that a monopolist facing two markets with differing iso-elastic demand schedules extracts more rapidly than the social planner, whether or not arbitrage prevents price discrimination between markets. This analysis is relevant in the case of many resources – such as natural gas used for power generation and household heating, or petroleum used for making plastics and as fuel.

Key Words: exhaustible resources, monopoly, markets, price discrimination.

JEL Classification Numbers: D42, Q3

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1   Introduction

A number of authors have written on the implications of monopoly for the extraction of an exhaustible resource[1]. However, based on these and our own review of the literature, there has been no previous analysis of a relatively simple but important case, namely, one in which the sole owner of a resource can sell to two markets at different prices. Although only a slight variation on the model in Stiglitz (1976), which features constant elasticity of demand with zero extraction costs, our basic model does not yield the result that the monopolist extracts at the same rate as the social planner. Nor does it confirm Hotelling's suggestion that the monopolist is the conservationist's best friend. Instead, a monopolist facing two markets with differing iso-elastic demand schedules extracts more rapidly than the social planner.[2]

When the monopolist can discriminate between the two markets, it sets prices so that they rise at the rate of interest, as does the social planner. However, unlike the planner, the monopolist finds it beneficial to set a different price in each market, and these prices are such that the quantity sold in the market with low demand elasticity is less than the socially optimal quantity, while the quantity sold in the market with relatively high demand elasticity is greater than the socially optimal quantity. The total quantity sold is higher than under the social planner. When arbitrage forces the monopolist to sell at the same price in both markets, the monopolist acts like a social planner in that prices are equal, but these prices differ from those of the social planner, rising in such a way that they encourage faster consumption of the resource.

These results are not just of academic interest, since many resources have multiple uses. In a number of cases, resources are sold in segmented markets where arbitrage is difficult. For example, petroleum is sold for both energy generation and plastics, and gold is mined for uses ranging from jewelry, coins and dentistry to coating electronic contacts and stereo speaker wires and toning silver images in photography. In such cases, the resource may have a different set of substitutes in each of the market in which it sold, which in turn influences its demand in that market. For instance, there may be many alternatives to gold for use in electronics, but few others for use in engagement

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[1] Reviews of this literature are included in Dasgupta and Heal (1979); Devarajan and Fisher (1981); and Krautkraemer (1998).

[2] A similar result in which the monopolist extracts too rapidly is provided by Lewis et al. (1979), who obtain this result both when there are fixed costs that do not vary with the extraction rate, as well as when demand elasticities vary with consumption, instead of time.

rings. With a different demand curve for gold in each market, the monopolist may, in addition to its ability set a higher price in each market, also be able to engage in price discrimination between markets. In this situation, we find that the monopolist extracts the resource at a rate that is greater than socially optimal in the market where demand is relatively elastic, and at a rate that is smaller than socially optimal in the market where demand is relatively inelastic.

2 Model

Consider a single exhaustible resource that is sold in two separate markets with two different elasticities of demand. Let us call these two markets A and B. Total stock of the available resource is K_0. We make the following assumptions:

1. There is no cost to extracting the resource other than the scarcity value.

2. The elasticity of demand in each market is constant *and greater than 1*.

3. There is no possibility for arbitrage between markets. Moreover, the monopolist cannot engage in price discrimination within each market.

The first two assumptions are identical to those made by Stiglitz (1976) and may be unduly restrictive. However, our choice of functional form is for essentially the same reason as the earlier paper, namely for analytical tractability and intuitive transparency. We find that the results of this paper are robust to other functional forms as well, although the analysis is far more tedious and far less intuitive. Since our purpose is, in part, to contrast the implications of monopoly for the rates of extraction and consumer welfare in the two markets, the choice of demand function is less critical than for comparing the overall outcome with that achieved by a social planner. The third assumption is added for our two-market problem in order to determine how a monopolist's behavior compares with that of a social planner, both with respect to the quantity sold in each market and to total extraction. Our final caveat relates to the structure of the model, which is set up with a eventual exhaustion constraint where the sum of quantities extracted over the planning horizon is set equal to the initial stock. This assumption is also made for ease of exposition, and an alternate approach to solving the problem using optimal control methods could be followed.

2.1   Social Planner's Problem

We frame the optimal resource allocation problem in terms of a social planner maximizing welfare (defined as total consumer surplus, since costs are absent) with respect to quantities sold in markets A and B, denoted by q_t^A and q_t^B, subject to the constraint that total extraction not exceed the available resource stock. The resulting equilibrium corresponds to the market allocation under perfect competition. The optimization problem is represented in the following Lagrangean:

$$U^S = \int_{t=0}^{\infty} e^{-rt} \left(\int_{s=0}^{q_t^A} P_A(s)ds + \int_{s=0}^{q_t^B} P_B(s)ds \right) dt - \lambda \left(\int_{t=0}^{\infty}(q_t^A + q_t^B)dt - K_0 \right), \quad (1)$$

where r denotes the discount rate, P_A and P_B denote the price of the resource in the two markets, and λ is the shadow price of the resource. The optimum is characterized by the first-order conditions with respect to resource sales in each market

$$q_t^A > 0, \qquad P_A(q_t^A) = e^{rt}\lambda; \tag{2}$$

$$q_t^B > 0, \qquad P_B(q_t^B) = e^{rt}\lambda; \tag{3}$$

and the stock constraint

$$\lambda > 0; \ \int_{t=0}^{\infty}(q_t^A + q_t^B)dt \leq K_0; \tag{4}$$

where a binding total extraction constraint leads to a positive shadow value of the resource (λ).[3]

The combination of (2) and (3) in any period t reveals that

$$P_i(q_t^{i,S}) = e^{rt}P_i(q_0^{i,S}); \quad i = A, B; \tag{5}$$

$$P_A(q_t^{A,S}) = P_B(q_t^{B,S}); \quad t = [0, \infty]. \tag{6}$$

First, prices in each market rise over time at the rate of interest. Second, in an equilibrium where the planner is selling in both markets, those prices must be equal, reflecting equivalent scarcity

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[3] The full complementary slackness conditions are

$$\lambda \geq 0; \quad \int_{t=0}^{\infty}(q_t^A + q_t^B)dt \leq K_0; \quad \lambda \left(\int_{t=0}^{\infty}(q_t^A + q_t^B)dt - K_0 \right) = 0$$

If some of the stock were left unexploited, then $\lambda = 0$, but this would imply from 2 and 3 that prices were zero and extraction positive in all periods, which would violate the constraint of a finite stock. Thus, $\lambda > 0$.

costs. Consider price functions with constant elasticity of demand, such as

$$P_i(q) = (q/\mu_i)^{-\frac{1}{\eta_i}},$$

where η is the elasticity (in absolute value terms), while μ represents a shifting parameter, indicating the relative size of the market. In our examples, we will designate market B as the one with the greater elasticity $(\eta_B > \eta_A)$. Since for these inverse demand functions, price goes to infinity as quantity goes to zero, we know that to satisfy the first-order conditions, some amount will be produced and sold in each market in every period, although those amounts will eventually become infinitessimally small. Solving for the extraction paths, we get

$$q_t^{A,S} = e^{-\eta_A rt} q_0^{A,S}; \quad q_t^{B,S} = e^{-\eta_B rt} q_0^{B,S}. \tag{7}$$

Since prices must be equal across the markets at all times including the start, we can solve for the initial quantity in market B in terms of that in A:

$$q_0^{B,S} = \mu_B \left(q_0^{A,S} / \mu_A \right)^{\frac{\eta_B}{\eta_A}}. \tag{8}$$

Although prices are the same, the quantities will differ according to the elasticities and the demand-shifting parameter. Integrating (7) over time, we can express cumulative extraction for each market as a function of initial resource sales, the interest rate, and its own market elasticity:

$$\int_{t=0}^{\infty} e^{-\eta_i rt} q_0^{i,S} dt = \frac{q_0^{i,S}}{r\eta_i}. \tag{9}$$

From (8), (9), and the stock constraint, we can solve for $q_0^{A,S}$:

$$\frac{q_0^{A,S}}{\eta_A r} + \frac{\mu_B}{\eta_B r} \left(\frac{q_0^{A,S}}{\mu_A} \right)^{\frac{\eta_B}{\eta_A}} = K_0. \tag{10}$$

With that solution, the entire optimal paths are determined.

2.2 Monopolist's Problem

The monopolist's problem is to maximize profits (revenues) subject to the total resource stock constraint:

$$U^M = \int_{t=0}^{\infty} e^{-rt} \left(P_A(q_t^A)q_t^A + P_B(q_t^B)q_t^B \right) dt - \lambda \left(\int_{t=0}^{\infty} (q_t^A + q_t^B) dt - K_0 \right). \tag{11}$$

Along the profit-maximizing path of resource sales in each market, the following first-order conditions must hold:

$$q_t^A > 0, \qquad MR_A(q_t^A) = e^{rt}\lambda; \tag{12}$$

$$q_t^B > 0, \qquad MR_B(q_t^B) = e^{rt}\lambda; \tag{13}$$

where MR_i denotes marginal revenue in market i. The final condition is the same stock constraint as (4), although the actual shadow value may differ. Thus, for the monopolist to sell in both markets, their marginal revenues must be equal in each period, reflecting equivalent scarcity costs. Marginal revenues in each market must rise at the rate of interest. With constant elasticity demand, marginal revenue is a constant times the price:

$$MR_i(q) = \frac{\eta_i - 1}{\eta_i} \left(\frac{q}{\mu_i} \right)^{-\frac{1}{\eta_i}} = \frac{\eta_i - 1}{\eta_i} P_i(q)$$

This latter result implies that prices will also be rising at the rate of interest in each market:

$$P_i(q_t^{i,M}) = e^{rt} P_i(q_t^{i,M}); \quad i = A, B. \tag{14}$$

Solving for $q_t^{i,M}$, we then get the same result as (7): $q_t^{A,M} = e^{-\eta_A rt} q_0^{A,M}$ and $q_t^{B,M} = e^{-\eta_B rt} q_0^{B,M}$. However, although this Hotelling rule seems the same for the planner and the monopolist, the relative prices are different, meaning that the paths do diverge. Setting marginal revenues equal to each other, we see the monopolist does not want prices to be equal in the two markets if the elasticities of demand are different:

$$P_B(q_t^{B,M}) = \frac{\eta_B}{(\eta_B - 1)} \frac{(\eta_A - 1)}{\eta_A} P_A(q_t^{A,M}). \tag{15}$$

From this result, we can prove three statements comparing monopolist resource use with the planner's with respect to prices, quantities, and welfare:

Theorem 1 *Proposition 2* *When assumptions 1 to 3 hold, the monopolist raises the price in the market with relatively inelastic demand and lowers it in the market with relatively elastic demand compared with the optimal (planner's) price.*

Proof. Let $\eta_B > \eta_A > 1$. From (15), we find that $P_t^{B,M} < P_t^{A,M}$; the monopolist wants a lower price in the market with more elastic demand for all t. From (12) and (13), monopoly prices in each market must rise at the rate of interest; from (2) and (3), optimal prices are the same in each market and rise at the rate of interest. Thus, if $P_t^S < P_t^{A,M}$ for any t, it must hold for all t. Suppose now that the price path in market A were the same as the planner's, while that in B were lower, implying greater sales in that market in every period. Over the same horizon, then, total extraction would be greater for the monopolist, thereby violating the stock constraint. (By the same logic, if $P_t^{B,M} = P_t^S$ and $P_t^{A,M} > P_t^S$ for all t, then the monopolist would extract less over the same horizon, implying a slack stock constraint, which violates the monopolist's conditions for optimality.) Thus, the monopolist's equilibrium must have higher prices than optimal in market A in all periods, and prices in market B must be lower than optimal. ∎

Corollary 3 *Consumers in the market with relatively inelastic demand obtain less of the resource in each period and overall with monopoly provision, while those in the market with relatively elastic demand obtain more of the resource in each period and overall.*

Proof. This result follows from the previous theorem: since $P_t^{A,M} > P_t^S$, then $q_t^{A,M} < q_t^{A,S}$ for all t. Since $P_t^{B,M} < P_t^S$, then $q_t^{B,M} > q_t^{B,S}$ for all t. Thus, $\int_{t=0}^{\infty} q_t^{A,M} dt < \int_{t=0}^{\infty} q_t^{A,S} dt$ and $\int_{t=0}^{\infty} q_t^{B,M} dt > \int_{t=0}^{\infty} q_t^{B,S} dt$. ∎

Corollary 4 *Compared with the social optimum, consumers in the market with relatively elastic demand are better off under monopoly provision, as is the monopolist, while consumers in the market with relatively inelastic demand are worse off, as is society overall.*

Proof. This result follows from the previous corollary. Consumer surplus is higher with more consumption; since $q_t^{B,M} > q_t^{B,S}$ and $q_t^{A,M} < q_t^{A,S}$ for all t, consumers in B are unambiguously

better off with monopoly provision while those in A are unambiguously worse off. The monopolist is better off by definition of profit maximization, while society overall is worse off by definition of welfare maximization, since the two paths differ. ∎

We can show that the price path for relatively inelastic market A gets shifted up, while that of B gets shifted down, compared with the planner's path. The dual to this result is that the extraction path for market B gets shifted up, while that of A gets shifted down, compared with the planner's. However, either market could still be the larger than the other, depending again on the relative demand parameters.

To solve for the complete system of extraction, we must combine the first-order conditions with the stock constraint. Cumulative extraction for each market is the same function of initial resource sales, the interest rate, and its own market elasticity as we saw for the planner's problem:

$$\int_{t=0}^{\infty} e^{-\eta_i r t} q_0^{i,M} dt = \frac{q_0^{i,M}}{r \eta_i}. \tag{16}$$

However, the initial extraction terms will differ for the monopolist. From (15) we solve for initial extraction in B in terms of that in A:

$$q_0^{B,M} = \mu_B \left(\frac{\eta_B}{(\eta_B - 1)} \frac{(\eta_A - 1)}{\eta_A} \right)^{-\eta_B} \left(\frac{q_0^{A,M}}{\mu_A} \right)^{\frac{\eta_B}{\eta_A}}. \tag{17}$$

Then, substituting this result with (16) into the stock constraint (4), we arrive at the equation determining the monopolist's optimal $q_0^{A,M}$:

$$\frac{q_0^{A,M}}{\eta_A r} + \frac{\mu_B}{\eta_B r} \left(\frac{\eta_B}{(\eta_B - 1)} \frac{(\eta_A - 1)}{\eta_A} \right)^{-\eta_B} \left(\frac{q_0^{A,M}}{\mu_A} \right)^{\frac{\eta_B}{\eta_A}} = K_0. \tag{18}$$

Recall that μ represents the relative size of the market. From (10) and (18), we can observe that as $\mu^B \to 0$ and one market shrinks to nothing, $q_0^{A,M} \to q_0^{A,S}$, and we are back in the Stiglitz world, where the monopolist behaves like the planner in providing to a single market.

2.3 Total Extraction with Two Markets

We have shown that the extraction paths of the planner and the monopolist diverge when facing two markets with different elasticities. But what does this imply for the path of total extraction

and thereby the rate of depletion of the single resource?

Theorem 5 *Proposition 6 When assumptions 1 to 3 hold, total extraction by the monopolist is greater than that by the planner in the initial period.*

Proof. From (9) and (16) we can rewrite the stock constraint (4) as

$$\frac{q_0^{A,j}}{r\eta_A} + \frac{q_0^{B,j}}{r\eta_B} = K_0 \tag{19}$$

for $j = S, M$. Solving for $q_0^{B,j}$, we get $q_0^{B,j} = \eta_B r K_0 - \frac{\eta_B}{\eta_A} q_0^{A,j}$, which lets us write the total initial extraction Q_0 for each actor, j, as $Q_0^j = \eta_B r K_0 + \left(1 - \frac{\eta_B}{\eta_A}\right) q_0^{A,j}$. Subtracting the planner's initial extraction from the monopolist's, we see that the difference is positive:

$$Q_0^M - Q_0^S = \left(1 - \frac{\eta_B}{\eta_A}\right)(q_0^{A,M} - q_0^{A,S}), \tag{20}$$

since $\eta_B > \eta_A$ implies that $q_0^{A,M} < q_0^{A,S}$. ∎

Although the monopolist contracts supply in the market with less elastic demand, it expands it in the market with more elastic demand, with the net effect being an initial increase in total supply.

Corollary 7 *The monopolist follows a faster extraction path than the planner.*

Proof. The previous theorem states that total extraction is higher for the monopolist initially, but it cannot be so indefinitely, else the stock constraint would be violated. Thus, at some point the extraction paths must cross, with the monopolist's total extraction being less than the planner's in later periods. Therefore, the monopolist's extraction path must be steeper. ∎

In other words, the monopolist will tend to be less "conservationist" than the planner. Furthermore, we see from (20) that this tilting of the total extraction path must be greater when the market elasticities diverge to a greater extent.

2.4   Monopolist Extraction with Arbitrage

Does arbitrage between the two markets eliminate the monopolist's incentive to behave differently than the planner? If the markets can resell to each other, a single equilibrium price will result.

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Let $P_t^1 = P_A(q_t^A) = P_B(q_t^B)$. Whereas in the segregated markets case we worked with inverse demand, in this case we will simplify the expressions as a function of the common price P_t^1. With our constant-elasticity demand, we can write $q_t^i = \mu_i(P_t^1)^{-\eta_i}$. In other words, at any price the monopolist faces a single, unified demand schedule of $Q(P_t^1) = \mu_A(P_t^1)^{-\eta_A} + \mu_B(P_t^1)^{-\eta_B}$, which no longer displays constant elasticity. The point elasticity is now the average of the individual market elasticities, weighted by output:

$$\bar{\eta}_t = -\frac{dQ/Q}{dP/P} = \frac{\eta_A\mu_A(P_t^1)^{-\eta_A} + \eta_B\mu_B(P_t^1)^{-\eta_B}}{\mu_A(P_t)^{-\eta_A} + \mu_B(P_t^1)^{-\eta_B}},$$

which collapses to a constant as $\mu_i \to 0$ or $\eta_A \to \eta_B$.

On the optimal path, the monopolist always wants marginal revenue—this time with respect to total output—to rise at the rate of interest, or $P_t^1\left(1 - 1/\bar{\eta}_t\right) = e^{rt}\lambda$. The rate of change in the price path is then

$$\frac{\dot{P}_t^1}{P_t^1} = r - \frac{\dot{\bar{\eta}}}{\bar{\eta}_t}\left(\frac{1}{\bar{\eta}_t - 1}\right).$$

Since we assume $\eta_B > \eta_A > 1$, then $\bar{\eta}_t > 1$. Thus, whether the price rises faster or slower than the interest rate depends on whether the total elasticity is rising or falling, and we show it to be falling:

$$\dot{\bar{\eta}} = -\frac{\dot{P}_t^1}{P_t^1}\left(\frac{(\eta_B - \eta_A)^2\mu_A\mu_B(P_t^1)^{-(\eta_A+\eta_B)}}{(\mu_A(P_t^1)^{-\eta_A} + \mu_B(P_t^1)^{-\eta_B})^2}\right) < 0.$$

Since the effective elasticity falls as the price rises, in equilibrium $P_t^1 > e^{rt}P_0^1$. In other words, with an arbitrage constraint, the monopolist will choose a steeper price path than the planner: the monopolist prefers to lower prices in relatively elastic periods and raise prices in relatively inelastic ones, resulting in lower initial prices and higher ones later on. A steeper price path over the same time horizon with the same stock constraint implies that the monopolist's initial price must be lower than the planner's: $P_0^1 < P_0^S$. This in turn implies that starting consumption is greater in both markets: not only $q_0^{1B} > q_0^{B,S}$, but also $q_0^{1A} > q_0^{A,S}$. Thus, total extraction is greater at the beginning, and the monopolist is again less conservationist than the planner. This result is similar to that of Stiglitz (1976) and others who note that if the elasticity of demand increases over time, the monopolist picks a more conservationist path relative to the social planner. However, these

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increases are modeled as exogenous changes to an elasticity that is otherwise constant with respect to price within any period. Here, by presenting overall demand as the sum of distinct markets each with constant elasticities of demand, we arrive at an effective elasticity that is decreasing in price, and thus decreasing over time. Thus, our example corresponds to the extension by Lewis (1976) and Lewis, Matthews, and Burness (1979), who consider a stationary demand schedule with elasticity increasing in consumption.

In summary, without arbitrage, the monopolist will follow the optimal (Hotelling) rule for the price paths but will differ from the social planner with respect to the relative prices between markets. With arbitrage, the monopolist's relative prices between markets coincide with the planner's, but as a consequence the monopolist's price path is distorted. In both cases, the existence of two markets with different elasticities causes the monopolist to behave differently from the planner and, overall, to follow a steeper path of depletion. A caveat to this conclusion comes from Stiglitz (1976), who notes that when the price of a resource is projected to rise faster than the interest rate, speculators will seize the opportunity and buy the resource, causing its current price to increase until, with no storage costs, the price is projected to rise no faster than the interest rate. Thus, when there is arbitrage both between time periods and among markets, the monopolist is further restricted in its ability to control prices, as speculation may limit increases to the rate of interest. One then concludes that the monopolist's equilibrium price path will rise at least as steeply as that of the social planner.

3 Conclusion

This paper illustrates an important extension to the literature on the extraction of a depletable resource by a monopoly. We find that the ability to extract at zero marginal cost and sell in two constant-elasticity markets, with or without arbitrage between them, leads a monopolist to extract a resource at a faster-than-optimal rate. This result suggests that when a monopolist is able to price-discriminate between two sets of consumers, then the group with relatively elastic demand is better off, and the group with relatively inelastic demand is worse off with monopoly provision than under the social planner. Overall, there may be significant welfare losses associated with unregulated monopoly provision.

Needless to say, our assumption of iso-elastic demand, though a "natural first approximation" (Stiglitz 1976), is restrictive and the assumption of exogeneity of the resource stock needs to be tested. Indeed, incorporating the ability of resource owners to expand the supply of resource through exploration and innovation may prove a particularly fruitful avenue of research, as models that incorporate an endogenous stock of resource, without the potential for two markets (Gaudet and Lasserre 1988), indicate that the monopolist extracts too slowly. This suggests that a model incorporating the potential for both discovering new stock of the resource and selling it in multiple markets may be worthwhile to pursue.

References

Dasgupta, P. and G. Heal (1979). *Economic Theory and Exhaustible Resources*. Cambridge University Press.

Devarajan, S. and A. C. Fisher (1981). Hotelling's "economics of exhaustible resources": Fifty years later. *Journal of Economic Literature 19*(1), 65–73.

Eswaran, M. and T. R. Lewis (1984). Ultimate recovery of an exhaustible resource under different market structures. *Journal of Environmental Economics and Management 11*(1), 55–69.

Gaudet, G. and P. Lasserre (1988). On comparing monopoly and competition in exhaustible resource exploitation. *Journal of Environmental Economics and Management 15*(4), 412–418.

Krautkraemer, J. A. (1998). Nonrenewable resource scarcity. *Journal of Economic Literature 36*(4), 2065–2107.

Lewis, T. R. (1976). Monopoly exploitation of an exhaustible resource. *Journal of Environmental Economics*

and Management   3(3), 198–204.

Lewis, T. R., S. A. Matthews, and H. S. Burness (1979). Monopoly and the rate of extraction of exhaustible resources: Note. *American Economic Review 69*(1), 227–230.

Pindyk, R. S. (1987). On monopoly power in extractive resource markets. *Journal of Environmental Economics and Management   14*(2), 128–142.

Solow, R. M. (1974). The economics of resources or the resources of economics. *American Economic Review 64*(2), 1–14.

Stiglitz, J. E. (1976). Monopoly and rate of extraction of exhaustible resources. *American Economic Review 66*(4), 655–661.