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Output-Based Allocation of Environmental Policy Revenues and Imperfect Competition

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Carolyn Fischer

Abstract

Environmental policies with output-based refunding of the revenues effectively combine a tax on emissions with a subsidy to output. Three similar forms exist: tradable performance standards, an emissions tax with rebates, and tradable permits with output-based allocation. Two arguments for including an output subsidy are imperfect competition, in which an environmental regulation alone could exacerbate output underprovision, and imperfect participation, in which imposing a regulation on a subset of polluters could cause output to shift to exempt firms. However, both these scenarios imply that output shares among program participants are likely to be significant. In this situation, output-allocated permits offer less of a subsidy than a fixed rebate, and they can lead to inefficient shifting of production among participants. Rebating the emission tax reduces the incentive to abate, nor will marginal abatement costs be equalized if costs differ. These results hold in a Cournot duopoly model whether emission rates are determined simultaneously or strategically in a two-stage model.

Key Words: emission tax, permit allocation, earmarking, tradable performance standards

JEL Classification Numbers: H21, H23, Q2

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1 Introduction

Combining a market-based environmental policy with an output subsidy for the participants can, in theory, be desirable in certain situations. For example, participation in the regulatory program may be incomplete, leading to concerns that imposing a cost on emissions will cause participants to lose competitiveness and emissions to leak outside the program to unregulated producers. The reasons for restricting regulatory scope may be jurisdictional—as with cross-border pollution—or technical—such as with point and non-point source pollution. For example, while it is feasible and cost-effective to monitor NO$_x$ emissions from large electric utilities, it may not be possible to monitor and regulate every vehicle tailpipe. Bernard, Fischer and Vielle (2001) explore whether output-based rebating is an appropriate response when one sector can be regulated while another cannot. Using a general equilibrium model, they find that a rebate to the regulated sector is only called for when the second-best response of taxing the output of the unregulated sector is also unavailable. Furthermore, the amount of the optimal rebate (i.e., the share of revenues or permits that should be allocated) depends on the substitutability between the goods and the extent to which the unregulated sector pollutes.

Imperfect competition provides another example. It is well known that an imperfectly competitive industry has an incentive to underprovide output; taxing emissions to reduce the pollution externality then tends to exacerbate this pre-existing distortion. Tying the emissions payments to an output subsidy can mitigate this effect. However, the solution is not likely to be simple, as imperfect competition can involve many complications (see Carraro, Katsoulacos, and Xepapadeas, 1996). Cost heterogeneity can make the emissions policy a tool for shifting production between low- and high-cost firms (Simpson, 1995). Optimal tax policy may be nonlinear (McKitrick, 1999), ad valorem (Shaffer, 1995), or otherwise related to cost

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variance (Carraro and Soubeyran, 1995), depending on how abatement costs, production costs, and environmental feedbacks are structured. The equivalence of tax, permit, and standards policies tends to break down in multi-stage games, due to first-mover advantages or investment decisions (Copeland, 1990), or due to knowledge of the policymaker's rules, which leads to issues of enforceability and time consistency (Requate, 1993; Petrakis and Xepapadeas, 2001). Emissions policy can influence entry and market structure (Katsoulacos and Xepapadeas, 1995; Lee, 1999), or it may facilitate strategic interactions (Carlsson, 2000; Long and Soubeyran, 2000).

Earmarking environmental revenues to subsidize the regulated producers is, by definition, a second-best policy, since the rebates are not optimized but tied explicitly to emissions rents. Meanwhile, in situtations where some output subsidy is desired, the optimal level of that subsidy is tied to many other factors. For example, with imperfect participation, the elasticities of substitution and pollution profiles of competing goods matter. With imperfect competition, the elasticity of demand is important, as is cost heterogeneity, innovation opportunities, game structure, etc.[1]

In this paper, we start from a point where a political decision has been made to rebate the revenues from an environmental policy.[2] Thus, we abstract from the question of whether to offer a rebate and ask how it might be done. The focus is on the specific effects of each rebate rule, rather than the general effect of an output subsidy under conditions of imperfect competition.

We compare the effects of three similar output-based rebating (OBR) regimes: tradable performance standards, emissions taxes with rebates according to output share, and output-allocated emissions permits. While these policies are not typically considered together, they are indeed similar forms of the same scheme: they each simultaneously impose a marginal cost to emissions and offer a subsidy to output. Furthermore, the marginal value of that subsidy is often not fixed but rather tied to the average value of inframarginal emissions to the affected industry.

In the perfect competition case examined by Fischer (2001), the marginal incentives of the three OBR schemes were similar from the point of view of the firm, although the equilibrium outcomes could differ.

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[1] While the focus here is on situations in which market shares are important, other grounds for some rebating may exist. Pre-existing tax distortions, upon which a large body of literature has been written, can make output-based rebating attractive in certain situations, relative to lump-sum allocations. See Fischer (2001).

[2] A distinct literature exists on the political economy of earmarking. See, for example, Wagner (1991) or Brett and Keen (2000). We remain agnostic here concerning the precise motivations for earmarking.

At Pigouvian emissions prices, the output subsidy causes the firm to produce more than a social planner would want. The dual to this problem is that, to achieve the same level of emissions as the optimal case, the marginal price of emissions must rise. Thus, for a given amount of emissions reduction, output-based rebating raises the marginal cost of emissions reduction relative to efficient policy. The combination of these points implies an equilibrium that has too much emissions rate reduction and/or too much production. Compared to the absence of any regulation, of course, output will still be lower. Thus, output-based rebating mitigates the rise in the equilibrium output price due to regulation.

In a second-best world with imperfect competition or imperfect participation, such a diminishing of the price impact may offer some welfare improvements. However, in these situations market shares among participants are likely to be significant. Under these conditions, the marginal incentives of the three OBR policies are no longer equivalent.

Compared to an emissions tax with a fixed subsidy, allocating emission permits based on market share provides less of an output incentive for firms with larger market shares. To the extent that firms have smaller market shares due to higher production costs, output-based allocation can inefficiently shift production toward higher-cost firms. In the case of the tax-rebate scheme, larger firms have diminished incentives to reduce emission rates, and larger marginal subsidies are given to firms with higher emission rates and greater market shares.

This paper investigates the impact of imperfect competition and imperfect participation on the incentives created by output-based rebating. The next section develops a model to compare the marginal incentives of a firm facing the different output-rebating schemes. The different rebating methods are compared to an equivalent fixed subsidy, focusing on the impact of significant market shares among program participants. The third section considers the case in which firms also have significant market shares in the overall output market. A duopoly model is employed to investigate how the schemes compare, both when emission rates and output are chosen simultaneously, and when emission rates are established before competition in the product market. The final section offers conclusions.

2 Model

In this section, we focus on the effects of rebate design, as opposed to the net effect of adopting a regulation. Thus, the fixed-subsidy case will be our baseline of comparison for the other rebate mechanisms, rather than a no-policy or optimal policy scenario. In keeping with this emphasis, for the purposes of this section, we will initially abstract from issues of competition in the output market (price effects) and concentrate on the effects of a participant having significant market share among the regulated firms (rebate effects). In many situations competition in the "rebate market" may be unrelated to competition in the product market; for example, output prices could be set on broader international markets, while only the local industry is regulated. This distinction allows us to focus on the unique effects of each rebate scheme. In the next section of the paper, we consider the joint effects of market power in both the rebate and product markets.

To represent these policies more generally, let us define the following variables:

q_i	Output of firm i
μ_i	Emissions rate of firm i
t	Price of emissions
$s(q_i, \mu_i)$	Subsidy (rebate) per unit of output
$R(q_i)$	Revenue function
$C_i(q_i, \mu_i)$	Production costs of firm i
$\pi_i(q_i, \mu_i)$	Profits of firm i

We assume that production costs are convex in output, and declining and convex in the emissions rate: $\partial C_i/\partial q_i > 0$, $\partial^2 C_i/(\partial q_i)^2 > 0$, $\partial C_i/\partial \mu_i \leq 0$, $\partial^2 C_i/(\partial \mu_i)^2 > 0$, and $\partial^2 C_i/(\partial q_i \partial \mu_i) \leq 0$, where $\partial^2 C_i(q_i, \mu_i^0)/(\partial q_i \partial \mu_i) = 0$ for some finite μ_i^0 (else emissions would be infinite in the absense of policy). Furthermore, we assume that the price of emissions is always fixed from the firm's viewpoint.[3]

The rebate mechanism generates an implicit per-unit output subsidy that may itself be a function of the firm's output; therefore, the marginal rebate may differ from the average.

The firm wants to maximize profits—total revenues from the sale of output less the costs of production, less emissions costs, plus its rebated subsidy. We assume that competition is Cournot and firms choose

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[3] This assumption may not be innocuous in the case of tradable emissions permits. However, it is a useful simplification, as emissions markets raise complicated questions about the incentive and ability to exercise market power. Incentive is a function of the net permit liability, and ability must balance market power on both the supply and demand sides.

emission rates and output quantities to maximize profits:

$$\pi_i = R(q_i) - C_i(q_i, \mu_i) - t\mu_i q_i + s(q_i, \mu_i)q_i. \tag{1}$$

The firm lowers its emissions rate until the marginal cost per unit of output equals the marginal price of emissions minus any impact on the firm's subsidy:

$$-\frac{\partial C_i(q_i, \mu_i)/\partial \mu}{q_i} = t - \frac{\partial s(q_i, \mu_i)}{\partial \mu_i}. \tag{2}$$

To put this in a traditional context, suppose t equals the marginal damage imposed by emissions on society. Then, to the extent that $\partial s/\partial \mu_i$ differs from zero, the firm's first-order condition for the emissions rate (2) will not equate marginal abatement costs with marginal damages.

Meanwhile, the firm will increase its output until the marginal revenue, net of production costs, just offsets the marginal cost of emissions, net of the marginal subsidy:

$$R'(q_i) - \frac{\partial C_i(q_i, \mu_i)}{\partial q} - t\mu_i + s(q_i, \mu_i) + \frac{\partial s(q_i, \mu_i)}{\partial q_i} q_i = 0. \tag{3}$$

In a case where output prices are determined exogenously to the firm (as with perfectly competitive markets), marginal revenue would equal the market price. A nonzero marginal subsidy ($s + \partial s/\partial q_i$) then creates a wedge between the price and the social marginal cost. In other words, in the absence of other distortions (including imperfect competition in output), a rebate policy other than a lump-sum transfer would create inefficient distortions.

In equilibrium for each self-contained policy x, we impose a revenue neutrality requirement that total subsidies equal total emissions payments:

$$\sum_{j=1}^{n} t^x \mu_j q_j = \sum_{j=1}^{n} s^x(q_j, \mu_j) q_j. \tag{4}$$

In one case—that of a broad-based permit system not limited to the firms receiving the rebates—the requirement is that the total subsidy equal the total value of the emissions permits allocated.

We next consider each of the different OBR schemes, focusing on the incentive effect of the marginal subsidy. The fixed tax and subsidy policy, represented with tradable performance standards, will be the benchmark. Policies will then be compared to the fixed tax-subsidy given either the same emissions price or the same level of emissions (the dual to the price problem).

2.1 Fixed Rebate Rate

The basic output-based rebating format is the fixed rebate rate, the revenues from which equal, in equilibrium, those generated by the emissions tax. In addition to presenting a useful baseline for analysis, this case also can represent a tradable performance standard, with the caveat that participating firms must be price takers in the permit market. With tradable performance standards, the average emissions rate is fixed by policy. To the extent a firm produces with emissions rates below the standard, it creates permits that it can sell; to the extent it produces with above-average emissions, it must purchase permits to cover the gap. Thus, for each unit of output, each firm gets a rebate equal in value to the emissions standard multiplied by the permit price. The subsequent equilibrium determines the price of emissions and total amount of emissions, such that the industry emission rate average equals the performance standard.

An explicit program of tradable performance standards was implemented in the program that phased lead out of gasoline in the United States. In 1982, the Environmental Protection Agency (EPA) set an inter-refinery average for lead usage among importers and refineries producing leaded gasoline.[4] Refineries using less lead than the standard could sell these credits to others using more than average. Rate-based mechanisms also are surfacing in climate policy: The Netherlands has decided to use tradable performance standards for its energy-intensive sectors that are sensitive to trade, while the other sectors remain subject to a cap-and-trade program. Although not formalized as tradable performance standards, the Bush climate plan for the United States also focuses on an emissions intensity goal.

In our formal presentation of the tradable performance standard as a fixed rebate rate, we assume that firms are price takers in emissions markets, as previously noted. As a result, the average subsidy will then also be taken as given. Thus, we represent the tradable performance standard as a fixed tax- and subsidy-rate

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[4] This standard was 1.10 gplg. In 1985, banking of permits was introduced as the the standard was reduced to 0.50 gplg and ultimately 0.10 gplg in 1986. The trading program ended in 1988. (EPA, 1997).

policy. We denote this policy with superscript S.

Let t represent the price of emissions and $\bar{\mu}$ the standard for average emissions. Under a tradable performance standard, the firm must buy permits to the extent it emits more than average: $t(\mu_i - \bar{\mu})q_i$. In other words, for each unit of output the firm pays an emissions tax of $t\mu_i$ and receives a subsidy equal to the average value of emissions per unit:

$$s^S(q_i, \mu_i) = t\bar{\mu}$$

Let us assume that firms are price takers in the market for emissions permits. In this case, the firm's choice of emissions rate has no impact on its implicit subsidy:

$$\frac{\partial s^S(q_i, \mu_i)}{\partial \mu_i} = 0. \tag{5}$$

Furthermore, with the performance standard fixed by policy and permit prices fixed by the market, the implicit average subsidy is then fixed in the eyes of the firm:

$$\frac{\partial s^S(q_i, \mu_i)}{\partial q_i} = 0. \tag{6}$$

Thus, market shares do not change the marginal subsidy in a tradable performance standard program.

2.2 Tax and Rebate

One tax-rebate scheme has received some attention in policy circles recently. In 1990, the Swedish government announced the implementation of an environmental charge on nitrogen oxide (NO_x) emissions beginning in 1992, its first to be based on actual emissions. The revenue is rebated to the affected plants in proportion to the amount of energy produced. The intent of the tax is to promote emissions reduction, while the rebate is intended to ameliorate the distributional impact of the tax, since only large producers are affected.[5]

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[5] The rate of SEK 40 per kilo (about $2.80/lb) was set to approximate the cost of reducing (and asserted to be the marginal damage of) NO_x emissions. The charge applies only to large combustion plants, since the measurement equipment is costly. Initially, the program applied to heat and power producers with a capacity of more than 10 MW and production exceeding 50 GWh. The latter threshold was to be lowered to 40 GWh in 1995 and 25 GWh in 1997. (*The Swedish Experience*, pp. 45-6.) Only final energy producers are included, not industrial process burning. The original participating installations were responsible for about 6.5% of total Swedish NO_x emissions. (*Managing the Environment*, p. 59.)

Consider a basic tax-rebate scheme similar to the Swedish NO$_x$ program. Producers face tax t on their emissions $\mu_i q_i$. Total revenue from the program is represented by the tax rate t multiplied by total emissions $E^T = \sum_{j=1}^n \mu_j q_j$. This revenue is then rebated to the firms according to their shares of total output among participants: $1/Q^T$ per unit of output, where $Q^T = \sum_{j=1}^n q_j$.

Written explicitly, the per-unit output subsidy to the firm is

$$s^T(q_i, \mu_i) = \frac{\sum_{j=1}^n t\mu_j q_j}{\sum_{j=1}^n q_j}$$

Let $\mu^T = \sum_{j=1}^n \mu_j q_j / \sum_{j=1}^n q_j$. The average subsidy can be rewritten as a per-unit subsidy of the tax rate multiplied by the average emissions rate:

$$s^T(q_i, \mu_i) = t\mu^T.$$

In this formulation, the rebate appears to be the same as the performance standard subsidy. However, the key difference is that the average emissions rate—thereby the average subsidy—is endogenous to the decisions of the firm.

First of all, a higher emissions rate for firm i raises the average emissions rate in proportion to that firm's market share, and thus raises the firm's average subsidy:

$$\frac{\partial s^T(q_i, \mu_i)}{\partial \mu_i} = t\frac{q_i}{Q^T}. \tag{7}$$

The result is that, to the extent the firm's market share among the program participants is non-negligible, the rebate diminishes the effect of the tax on the emissions rate choice. Essentially, the firm knows that if it raises its emissions, it will get back some of the additional tax payments in its share of the rebated revenues. Or, if it reduces emissions, it does not get the full benefit of reduced tax payments because its rebate also falls.

Second, by expanding its output, firm i not only gets another unit of the rebate, but it also changes the average amount of that subsidy for all its inframarginal output:

$$\frac{\partial s^T(q_i, \mu_i)}{\partial q_i} q_i = t(\mu_i - \mu^T)\frac{q_i}{Q^T}. \tag{8}$$

The net effect on the average emissions rate depends on whether the firm is an above-average or below-average emitter. In fact, the marginal subsidy is increasing in the emissions rate. Thus, the tax-rebate formulation can actually encourage more production by higher-emitting firms.

The fact that the firm's share of output among program participants is in the firm's first-order conditions also means that firms of different sizes face different effective tax and subsidy rates. Consequently, another one of the advantages of market-based environmental policy—marginal cost equalization—is compromised.

In the case of this tax-rebate policy, output share has two effects:

1. it tempers the effect of the emissions tax; and
2. it also tempers the effect of the average output subsidy.

The first effect applies to both the output and the emissions rate reduction decision. Substituting (7) into (2) and solving, we see that the effective marginal tax on emissions is

$$-\frac{\partial C_i(q_i, \mu_i)/\partial \mu}{q_i} = t\left(1 - \frac{q_i}{Q^T}\right). \tag{9}$$

Larger firms then have less incentive to reduce their emissions rate than smaller firms, since they have a lower effective marginal emissions tax rate.

With respect to output, by the first effect, market share decreases the marginal tax on emissions. By the second effect, it decreases the effect of the rebate rate. By increasing production, the firm not only raises the share of the emissions permit or revenue "pie" it is allocated, but it also lowers the size of each "slice" by raising industry output. Thus, by this second effect, larger firms have less incentive to expand output, since it dilutes the value of existing shares of the rebate. However, by the first effect, they also have less incentive to contract output because of the lower effective tax. Substituting (8) into the right-hand side of (3), we see that the net marginal tax on output depends on whether the firm is an above-average or below-average emitter:

$$R'(q_i) - \frac{\partial C_i(q_i, \mu_i)}{\partial q} = t(\mu_i - \mu^T) \left(1 - \frac{q_i}{Q^T}\right). \tag{10}$$

Importantly, these effects can interact. Referring back to the marginal subsidy (8), we see that above-average emitters face larger-than-average output subsidies on the margin. From the first-order condition with respect to μ_i (9), we also see that larger firms, all else being equal, will have higher emissions rates, due to the first effect of output share. This effect then reinforces that of the marginal output subsidy, making the marginal subsidy increasing in market share. However, the increase in their emission rate also increases the larger firm's net tax. In other words, larger firms will emit more, and higher emitters will become larger.

Only as market shares become negligible, from the firm's point of view, does the rebated tax scheme converge to the previous case of the tradable performance standard. As $q_i/Q^T \to 0$, Equation (7) converges to (5) and Equation (8) converges to (6), for equivalent tax rates. Essentially, if the firm is perfectly competitive in the rebate program, it becomes a "rebate taker." For each unit of output it gets the average revenue ($t\mu^T$), but it does not believe it is large enough to affect that average; in essence, it faces a performance standard.[6]

The combination of the previous results is that, when market shares are significant, an emissions tax with OBR tends to induce an equilibrium with higher total emissions than does an equivalent emissions tax with a fixed output subsidy. Given any emissions tax rate, with OBR all firms have less incentive to reduce emissions rates, and to the extent that firms differ, above-average emitters are subsidized more.

In an equilibrium with Cournot competition, not only does the OBR tax raise average emissions compared to the fixed rebate rate, but it also raises total output. The next section uses a Cournot duopoly to illustrate these results as well as the production-shifting effects.

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[6] Tradable performance standards avoid the problem of differing effective tax rates, as long as the standard is perceived as fixed. If the regulator responds to current deviations from the total emissions target by adjusting future performance, the policy may no longer be fixed in the eyes of the firm. If the firm producing today believes it can affect future performance standards, then the effect on output is much like that in the permit system, and different marginal subsidies are still a potential problem. The difference is that the subsidy would be the discounted permit price multiplied by the marginal impact on the performance standards in all future periods.

2.3 Permits and Output-Based Allocation

Output-based allocation has surfaced recently as a proposed rule for distributing emissions permits in a variety of cap-and-trade systems. It has been proposed for allocating NO$_x$ emission allowances in some states under the SIP Call and in EPA's Federal Implementation Plan. Output-based allocations are likely to be part of the greenhouse gas trading program designs for countries in the European Union. Canada is considering the use of output-based allocation as a means of limiting the adverse impacts on industries that compete in export markets with firms in countries that will not have a national greenhouse gas emissions limitation commitment. In a broad-based cap-and-trade system, the policy envisioned here would combine inter-industry trading and intra-industry allocation: each sector would be granted a fixed number of permits and, within that sector, firms would receive permits proportional to their share of their industry's output. We also consider a restricted cap-and-trade program, where trading only occurs within the one sector.

An output-allocated permit trading program presents yet again different incentives when output shares are nontrivial. Unlike the tax-rebate scheme, in this market, the total value of the rebated "pie" is fixed from the firm's point of view. But, as with the previous case, the number and size of the slices adjust.

Let us continue to assume that no individual firm believes it can affect the equilibrium permit price. However, among the regulated producers in its sector, the firm may still have a significant market share. These conditions can coincide, for example, if prices are determined in broader markets: permits may be traded across sectors but, in competing for its share of the permit allocations, the firm faces only rivals in its own sector. Even in a self-contained program, the existence of market power does not necessarily imply an incentive or ability to move emissions prices.[7]

To illustrate this case with the model, let γ represent the market price of permits (exogenous to the firm), and $\bar{E}$ the capped level of total emissions for the sector. The value of firm i's permit allocation per unit of output equals

$$s^P(q_i, \mu_i) = \gamma \frac{\bar{E}}{\sum_{j=1}^{n} q_j}.$$

As with tradable permits, the implicit subsidy is unaffected by the firm's choice of emissions rate. Thus,

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[7] The distinction between competition in the product market and competition in the "permit market" can be important. If program participation is restricted to a subset of the relevant industry, the regulated firms could be price takers in the product market but still be able to influence the equilibrium permit price. On the other hand, the reverse situation can occur if an imperfectly competitive industry must participate in a broader inter-industry permit trading system, such as is foreseen for CO$_2$.

$\partial s^P(q_i, \mu_i)/\partial \mu_i = 0$, and the firm equalizes marginal abatement costs with the marginal cost of emissions.

Like the tax scheme, to the extent they have significant market share, firms can influence the average subsidy. However, they can do so only by affecting share sizes; in contrast to the tax case, individual firms cannot affect the total value of their sector's permits, due to the assumption of exogenous permit prices.[8] As a result, the effect on the marginal subsidy is to reduce it as market share increases:

$$\frac{\partial s^P(q_i, \mu_i)}{\partial q_i} q_i = \gamma \frac{\bar{E}}{Q^P} \left(-\frac{q_i}{Q^P} \right) \tag{11}$$

where the superscript P denotes equilibrium values of variables in the case of output-allocated permits, and Q^P is the sum of all the participants' outputs.

In other words, while expanding output gets the firm an additional allocation of permits s^P, it also tends to drive down the average allocation as the subsidy gets spread more thinly. Consequently, smaller firms have more incentive to expand output than larger firms. This effect becomes evident by looking at the net marginal tax on output:

$$R'(q_i) - \frac{\partial C_i(q_i, \mu_i)}{\partial q} = \gamma \mu_i - \gamma \frac{\bar{E}}{Q^P} \left(1 - \frac{q_i}{Q^P} \right). \tag{12}$$

This issue is particularly problematic if firms are not perfectly competitive in output, as this policy can shift production from low- to high-cost producers. If market share is decreasing with production costs (i.e., small firms are small because they have higher marginal costs), output-based rebating implies a larger subsidy to high-cost firms. If anything, society would want to do the opposite: lower overall production costs by shifting output toward low-cost firms.[9]

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[8] If participating firms could influence the market price of permits, the result would be similar to the tax-rebate policy. Increasing output would drive up the permit price and increase the total value of the permit "pie"; meanwhile, lowering emission rates would tend to lower permit prices and have the effect of decreasing total revenues.

[9] For example, Simpson (1995) considers an industry with variation in the costs of emissions rate reduction (as opposed to marginal production costs). He shows that in this type of Cournot duopoly, the optimal tax may exceed the Pigouvian rate, since the tax can facilitate shifting more production to the firm with lower production costs.

2.4 Comparison

The following tables summarize these results, comparing the effects of significant market shares on the marginal incentives for the firms under the different schemes. Note that strategic effects with respect to output prices are not included—they will be considered in the next section.

Table 1: Marginal Incentives of OBR Policies

	Fixed Rebate Rate (TPS)	Tax-Rebate	OBR Permits
Total Subsidy	$t\bar{\mu}q_i$	$\left(\sum_{j=1}^{n} t\mu_j q_j\right) \frac{q_i}{\sum_{j=1}^{n} q_j}$	$\gamma \bar{E} \frac{q_i}{\sum_{j=1}^{n} q_j}$
Change in Subsidy w.r.t. μ	0	$t\frac{q_i}{Q^T}$	0
Effective Emissions Tax	t	$t\left(1 - \frac{q_i}{Q^T}\right)$	γ
Marginal Subsidy	$t\bar{\mu}$	$t\bar{\mu}^T + t\left(\mu_i - \mu^T\right)\frac{q_i}{Q^T}$	$\gamma\frac{\bar{E}}{Q^P}\left(1 - \frac{q_i}{Q^P}\right)$
Marginal Net Tax on Output	$t(\mu_i - \bar{\mu})$	$t(\mu_i - \mu^T)\left(1 - \frac{q_i}{Q^T}\right)$	$\gamma\left(\mu_i - \frac{\bar{E}}{Q^P}\left(1 - \frac{q_i}{Q^P}\right)\right)$

With heterogeneous firms, a larger market share implies a larger net tax on output with OBR permits, no change with tradable performance standards, and a dampened net tax or subsidy with the OBR tax. Those with larger market shares have less incentive to reduce the emissions rate under the tax-rebate scheme, but identical incentives under the other policies.[10] A higher emissions rate also implies a higher marginal subsidy in the case of the tax-rebate policy.

Using a simplification of symmetry, we can consider the equilibrium effects of the different policies when market shares among participants are nontrivial. Let there be n identical firms. For now, we remain agnostic about competition in the goods market. The firms participating in the emissions program might be price takers in the output market (for example, due to international or out-of-state competition), or they could influence prices through their output decisions. The conclusions in Table 2.4 focus on the output and emissions effects for the participating firms alone, and require only that their total supply monotonically decrease with increases in their total marginal costs (including production costs and net output taxes).

Compared to a fixed-rebate policy (represented by TPS), for a given tax price, emission rates are higher under the OBR tax, due to the market-share effect on the marginal emissions tax. With the OBR tax, output

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[10] Identical in the sense of the same effective tax rate. Firms with different output levels but identical abatement cost functions will then choose the same emissions rate if market share does not affect the cost of emissions rate reduction per unit of output: i.e., $\partial(\partial C/\partial\mu/q)\partial q = 0$.

is higher than with the standard since higher emissions rates implies lower marginal production costs; as a result, total emissions are higher as well.

For evaluating the equilibrium effects of output-based permits, two bases of comparison are available: equivalent prices and equivalent emissions. For the same total allocation, the average allocation will differ in the two scenarios. Let $\bar{e} = \bar{E}/Q^P$ be the average allocation, so $s^P = \gamma\bar{e}$. Let us also assume for comparison that the total allocation of permits is the same as emissions under the fixed rebate plan: $\bar{E} \equiv \bar{\mu}Q^S$. Thus, $\bar{e} = \bar{\mu}Q^S/Q^P$. The marginal subsidy (the latter term in (12)) is then $\gamma\bar{\mu}(\frac{n-1}{n})Q^S/Q^P$.

Suppose first that $\gamma = t$, as in the case where permits are being traded outside the sector in a broad cap-and-trade program (denoted with superscript B). Correspondingly, the incentives for the emission rate are the same as in the fixed rate case. If we evaluate (12) at $Q^B = Q^S$, we see that the market share effect implies that the marginal subsidy is smaller than under the tradable performance standard. Thus, in equilibrium, total output also must be lower than it would be under the performance standard ($Q^B \leq Q^S$). Consequently, total emissions would be lower as well.

If the program is self-contained and the allocation cap also is the emissions cap, then the average allocation also must equal average emissions: $\bar{e} = \mu^P$. Then lower output in the output-allocated permit equilibrium (for any permit price) implies that, for equilibrium in the permit market, the emissions rate must be higher and permit prices lower: $Q^P < Q^S$, $\mu^P > \bar{\mu}$ and $\gamma < t$.[11]

Table 2: Symmetric Equilibria with OBR Policies

	Fixed Rebate Rate (TPS)	Tax-Rebate	OBR Permits (Restricted)	OBR Permits (Broad)
Effective Emissions Tax	t	$t(\frac{n-1}{n})$	$\gamma < t$	$\gamma \equiv t$
Marginal Net Tax on Output	0	0	γ/n	$t\bar{\mu}\left(1-(\frac{n-1}{n})Q^S/Q^P\right)$
Emissions Rate	$\bar{\mu}$	$\mu^T > \bar{\mu}$	$\mu^P > \bar{\mu}$	$\bar{\mu}$
Total Output	Q^S	$Q^T > Q^S$	$Q^P < Q^S$	$Q^B < Q^S$
Total Emissions	E^S	$E^T > E^S$	$\bar{E} \equiv E^S$	$E^B < E^S$

The symmetric case also is useful for thinking about interactions with competition in the goods market. It demonstrates that, for an imperfectly competitive industry which already underprovides output, incor-

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[11] Compared to a Pigouvian policy, however, because the subsidy still exists, permit prices are higher. Of course, these comparisons assume emissions allocations equivalent to those that would occur under the fixed-subsidy scheme. On the other hand, if we assume allocations equal to the Pigouvian level of emissions ($\bar{E} = \bar{\mu}Q^*$), output will fall further (though not so far as Q^*) and, in the self-contained case, emissions rates will fall as well (and be lower than optimal).

porating an output subsidy into the environmental policy could, in theory, raise welfare. In essence, two problems exist—insufficient output due to imperfect competition and overproduction of emissions due to the externality. Thus, two policy tools are needed to address them both, one to internalize the externality and one to encourage output. In theory, the social optimum can actually be achieved in the symmetric case. The optimal policy is composed of a Pigouvian emissions tax and an output subsidy that depends inversely on the price elasticity of demand. The implicit subsidy counteracts the increase in price due to a firm's incentive to raise revenues by withholding some output. For the net tax on output to be zero (as with tradable performance standards) at the optimum, the lost consumer surplus from a unit withheld would have to exactly equal the marginal environmental cost of increasing one firm's production by a unit.[12] In other words, if the deadweight loss due to imperfect competition is relatively large compared to the cost of the externality, the optimal output support will be greater than the average value of embodied emissions—the planner will want to provide a net subsidy. On the other hand, if the pollution cost is relatively more important, the planner will want to impose a net tax.

Similar results can be obtained for the tax-rebate and output-allocated permit systems, although the impact of market share on the implicit marginal subsidy must be accounted for.[13] However, if firms are allowed to be heterogeneous, optimality cannot be achieved. As previously shown and illustrated next, heterogeneity in market shares compromises marginal cost equalization and causes inefficient production and emissions shifting.

3 Duopoly Competition

3.1 Simultaneous Cournot Game

In the preceding section, we considered equilibrium output merely to be declining with marginal costs. In this section, we explicitly model product markets and investigate the interactions between imperfect

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[12] The optimal net tax on output is $\delta\mu + P'(Q)Q/n$, where δ is the marginal damage from emissions. As $n \to \infty$, $\sigma \to \delta\mu$; in other words, as the industry approaches perfect competition, the tax approaches the marginal damages from the emissions embodied in each unit of output.

[13] For example, in the tax-rebate case, because of the market share distortion to the emissions rate decision, the planner must set the tax rate as a function of the rebate rate α to get the firms emissions rate incentives to correspond to society's: $t = \delta/(1 - \alpha/n)$. Logically restricting α to the interval $[0, n)$, we see that the optimal tax rate will be greater than or equal to the Pigouvian tax rate, δ.

competition, output-based rebating, and cost heterogeneity. To this end, let us employ a simple Cournot duopoly with linear demand and cost functions. Given that output-based rebating, by definition, must be implemented across identical or sufficiently similar products, Cournot competition seems like a reasonable choice to characterize the likely form of imperfect competition. For example, electricity generators provide an identical good and make quantity decisions regarding their production capacity. Of course, one could envision OBR applied to differentiated products—tradable fuel economy standards for cars, for example—for which Bertrand competition might be more appropriate. However, for now, we focus on output decisions and Cournot competition.

Production costs are assumed to take the form $C(q_i, \mu_i) = (c_i + a(\mu_i))q_i$, which exhibits constant marginal production costs that vary with the emissions rate. Let $a(\mu_i)$ be a decreasing function of the emissions rate, with $a'(\mu_i) < 0$, $a''(\mu_i) > 0$, and $a'(\mu_i) = 0$ for some finite μ^0 where $a(\mu^0) = 0$. Thus,

$$\frac{\partial C_i(q_i, \mu_i)/\partial \mu}{q_i} = a'(\mu_i). \tag{13}$$

Revenues for the duopolist are now defined as $R(q_i) = P(q_i + q_j)q_i$, where according to Cournot competition, firm i takes its rival's production as given. Let h_i represent firm i's market share: $h_i = q_i/(q_i + q_j)$; consequently, $\partial h_i/\partial q_i = (1 - h_i)/(q_i + q_j)$. We will further assume that the demand function take the following form: $P(Q) = y - bQ$, where $Q = q_i + q_j$. Thus,

$$\begin{aligned} R'(q_i) &= P(Q) + P'(Q)q_i \\ &= y - bQ(1 + h_i). \end{aligned} \tag{14}$$

The analysis proceeds with the profit maximization expressions derived in Section 2. By imposing on them these functional forms in a duopoly model, we can consider the equilibrium effects of the different OBR policies. In examples with cost heterogeneity, we will generally consider firm 1 to be the relatively high-cost firm.

Cournot competition with linear demand offers the straightforward result that market share is a function of the relative marginal costs. Let $\psi_i^x = c_i + a(\mu_i) + t\mu_i - s^x - \partial s^x/\partial q_i$ represent the full marginal cost to the firm (inclusive of the relevant emissions price and subsidy) of an additional unit of output under regime

x. Substituting our expressions into the first-order conditions for output (3), we get the standard result that the firm's profits are maximized when marginal revenues equal these full marginal costs:

$$y - bQ^x - bq_i^x = \psi_i^x.$$

Adding the conditions for each firm, we can solve for Q^x:

$$Q^x = \frac{1}{3b}(2y - \psi_1^x - \psi_2^x). \tag{15}$$

Substituting back into each firm's first-order condition and solving for output, we get

$$q_i^x = \frac{1}{3b}(y - 2\psi_1^x + \psi_2^x). \tag{16}$$

Taking the ratio for the market share of firm 1 in regime x, we see that it declines with the difference in marginal costs compared to the rival:

$$\begin{aligned} h_1^x &= \frac{q_i^x}{Q^x} = \frac{y - 2\psi_1^x + \psi_2^x}{2y - \psi_1^x - \psi_2^x} \qquad (17) \\ &= \frac{1}{2} - \frac{3(\psi_1^x - \psi_2^x)}{2y - \psi_1^x - \psi_2^x}. \qquad (18) \end{aligned}$$

Using this model structure, we now compare the different policy regimes.

3.1.1 Fixed Rebate Rate

The fixed tax-subsidy case is again our reference baseline. Substituting the defined marginal abatement costs (13) into the first-order condition for the emission rate (2), we see that each firm equalizes marginal abatement costs to the tax rate: $-a'(\mu_i) = t$.

With the fixed rebate rate, the requirement of revenue neutrality is equivalent to actual average emissions being equal to the standard. In the duopoly equilibrium,

$$\bar{\mu} = h_1\mu_1 + h_2\mu_2. \tag{19}$$

From the previous analysis, we know that both firms get the same marginal output subsidy. Full marginal costs for firm i then, in equilibrium, include a net tax that depends on market shares, as well as emission rates:

$$\begin{aligned} \psi_i^S &= c_i + a(\mu_i) + t\mu_i - t\bar{\mu} \\ &= c_i + a(\mu_i) + t(\mu_i - \mu_j)(1 - h_i) \end{aligned} \tag{20}$$

With Cournot competition, industry output is a function of the sum of the marginal costs. With identical abatement-related costs, given the same tax, each firm will choose the same emissions rate, or $\mu_i = \bar{\mu}$. Substituting into (15) gives us the equilibrium Q^S:

$$\begin{aligned} Q^S &= \frac{2y - c_1 - c_2 - a(\mu_1) - a(\mu_2) + t(\mu_1 - \mu_2)(h_1 - h_2)}{3b} \\ &= \frac{2y - c_1 - c_2 - 2a(\bar{\mu})}{3b} \end{aligned} \tag{21}$$

Substituting into (18), we see that that firm 1 gets less (more) than half of the market if its marginal costs are more (less) than firm 2's:

$$h_1^S = \frac{1}{2} - \frac{3(c_1 - c_2)}{2Q^S}.$$

Since both firms get the same rebate, it does not affect market share.

3.1.2 Tax-Rebate Policy

Section 2.2 showed that when firms are heterogeneous, the tax-rebate scheme discourages large firms from abating emissions and subsidizes high emitters to a greater extent. In a system of Cournot competition, this scheme leads to production shifting toward the higher-emitting firm. The result is an equilibrium with higher average and total emissions and greater output compared to a fixed rebate rate.

Simplifying (9) with the duopoly example, we see that the bigger i's market share, the higher its emissions rate:

$$-a'(\mu_i) = t(1 - h_i). \tag{22}$$

Thus, both firms emit more than with the fixed rebate, and a lower-cost firm with higher market share will emit at a higher rate.

Meanwhile, when the firm simultaneously chooses its output quantity, as in (10), its full marginal costs are

$$\psi_i^T = c_i + a(\mu_i) + t(\mu_i - \mu_j)(1 - h_i)^2 \tag{23}$$

Compared to the fixed-subsidy policy, marginal costs are lower here by $th_i(1 - h_i)(\mu_i - \mu_j)$, meaning the higher emitter faces a lower marginal net tax—given any emissions rate. However, since emission rates diverge, the higher-emitting firm now pays a positive net tax, while the lower-emitting firm receives a net subsidy.

Substituting (23) into (15) we get an expression for industry output Q^T:

$$Q^T = \frac{2y - c_1 - c_2 - a(\mu_1) - a(\mu_2) + t(\mu_1 - \mu_2)(h_1 - h_2)}{3b}. \tag{24}$$

Initially, this expression looks identical to that from the fixed-subsidy world; however, the underlying variables are different, since emission rates are higher and diverge. As a result, we can formulate two propositions.

Proposition 1 *The marginal cost for each firm is strictly lower when a tax is combined with an output-based rebate than with a fixed, revenue-neutral subsidy ($\psi_i^T < \psi_i^S$).*

Proof. See Appendix. ■

Proposition 2 *With Cournot competition, an output-rebated tax raises output and emissions compared to a comparable fixed rebate rate.*

Proof. Proof. From (15) we see that Q increases when $(\psi_1 + \psi_2)$ decreases. Thus, from Proposition 1, $\psi_1^S + \psi_2^S > \psi_1^T + \psi_2^T$, implying $Q^T > Q^S$. Since $\mu_i^T > \mu_i^S = \bar{\mu}$ from (22), $\mu^T = h_1^T \mu_1^T + h_2^T \mu_2^T > \bar{\mu}$. Thus, $\mu^T Q^T > \bar{\mu} Q^S$. ■

From the first-order conditions for emissions, we see that both individual and average emissions are higher compared to the fixed rebate rate, due to the weaker incentive to abate with the endogenous rebate.

Higher emissions rates mean higher tax payments, but also higher subsidies and lower abatement-related costs. The overall effect is thus to raise total output compared to the fixed rebate rate. Since both output and emission rates are higher, total emissions must then be higher. In other words, the higher emission rate lowers i's costs more than it affects the net marginal subsidies, regardless of which firm has higher market share. As a result, with the tax-rebate policy, industry output is higher, and total emissions are higher still, compared to the fixed-subsidy policy.

Meanwhile, cost heterogeneity has mixed effects in this case. If $c_1 > c_2$, then the high-cost firm has lower market share ($h_1^T < 1/2$), and thereby from (22) a lower emission rate $\mu_1^T < \mu_2^T$. The low-cost firm then has even lower abatement-related costs, along with higher subsidies, although they may be counterbalanced by the higher tax payments. The net effect on the market share of the high-cost, lower-emissions firm is uncertain:

$$h_1^T = \frac{1}{2} - \frac{3(c_1 - c_2 + A)}{Q^T}$$

where $A = a(\mu_1^T) - a(\mu_2^T) - t(\mu_2^T - \mu_1^T)(1 - 2h_1^T + 2(h_1^T)^2)$. This additional cost disparity may or may not be sufficient to outweigh the effects of greater output in diminishing the market share differential.

3.1.3 Output-Allocated Permits

Allocating permits based on output has the effect of subsidizing higher-cost firms to a greater extent when market shares are important. Market share effects also reduce the total impact of the subsidy, leading to lower output—and possibly higher emissions rates—compared to a fixed rebate rate.

In the duopoly case, the value of each firm's output-based allocation is $s^P(q_i, \mu_i) = \gamma\bar{e}$, where $\bar{e} = \bar{E}/(q_1 + q_2)$. We continue to assume that the two firms remain price takers in the permit market.[14] Since $\partial s^P(q_i, \mu_i)/\partial\mu_i = 0$, we know from (2) that both firms equalize marginal abatement costs:

$$-a'(\mu_i) = \gamma. \tag{25}$$

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[14] Market power in the permit market is hard to model for a duopoly, since both the seller and buyer would have market power. Hahn (1984) shows that incentives to exert market power in permit markets exist only to the extent a firm is a net buyer or seller. Thus, identical firms would not create a problem, and for heterogeneous costs it is unclear whether the monopolist or monopsonist would dominate. Thus, for our purposes here, we abstract from permit price effects in the simultaneous game to focus on the allocation effects.

Let us define $\mu^P = h_1\mu_1 + h_2\mu_2$ as average emissions in this output-allocation case. With identical abatement-related costs, facing the same permit price, each firm will choose the same emission rate, $\mu_i = \mu^P$. However, their marginal costs differ according to their market shares:

$$\psi_i^P = c_i + a(\mu_i) + \gamma\mu_i - \gamma\bar{e}(1 - h_i). \tag{26}$$

Solving for Q^P with our linear demand function, we get

$$Q^P = \frac{2y - c_1 - c_2 - 2a(\mu^P) - \gamma(2\mu^P - \bar{e})}{3b}. \tag{27}$$

Since a "comparable" fixed rebate rate can be defined either from an emissions price or quantity perspective, we consider these two scenarios. In both cases, the total allocation is assumed to be equal to total emissions in the fixed rebate case: $\bar{E} = \bar{\mu}Q^S$, therefore the average allocation is $\bar{e} = \bar{\mu}Q^S/Q^P$. In the first scenario, we consider an equilibrium with an equivalent prevailing emissions price: $\gamma = t$. This could be the case of a broad-based permit market with many sectors, and this one has its permits allocated based on output shares. If permits are traded on a broader market, average emissions need not equal average allocations. In the second scenario, the permit market may be restricted to the duopoly with equivalent total emissions to the fixed rebate rate case; then average emissions also must equal average allocations: $\mu^P = \bar{e}$.

Proposition 3 *Output-allocating permits lowers output relative to a comparable fixed rebate rate.*

Proof. First, consider an equilibrium with an equivalent prevailing emissions price: $\gamma = t$. From (25), this implies $\mu_i^P = \mu_i^S = \bar{\mu}$. Subtracting (21) from (27), we get $Q^P - Q^S = t(\bar{e} - 2\bar{\mu})/(3b) = t\bar{\mu}(Q^S/Q^P - 2)/(3b) < 0$. While a comparable fixed rebate rate features no net tax on output, here the market share effect implies a positive net tax; thus, total output must be smaller. ■

Proof. Second, consider a cap and an allocation of permits equal to emissions under the fixed rebate rate scheme, but the permit price may vary. In this case, $\mu^P = \bar{e} = \bar{\mu}Q^S/Q^P$. Suppose instead that $Q^P = Q^S$. Then $\bar{e} = \bar{\mu}$; with identical abatement costs, this implies from (25) that $\gamma = t$. We have just shown that this implies $Q^P < Q^S$. ■

With broad-based permit trading, since emission rates are the same, lower output then also implies lower emissions compared to the fixed rebate. With restricted trading, lower ouput implies that emission rates and prices must equilibrate to achieve the same cap. Since $\bar{\mu}Q^P < \bar{E}$, in order to have equilibrium in the permit market, it must be that the emissions price is lower ($\gamma < t$), which means that emission rates for both firms are higher and $\bar{e} = \mu^P > \bar{\mu}$. Since abatement costs are then also lower, total output will be higher compared to the broad-based trading example but still lower than in the fixed rebate rate example.[15]

Proposition 4 *Output-allocating permits shifts production toward the higher-cost producer, relative to a comparable fixed rebate rate.*

Proof. See Appendix.

Consider the case in which firm 1 has higher marginal production costs than 2: $c_1 > c_2$. Then, all else equal, $h_2 > h_1$. It follows from (25) that firm 1 has a higher marginal subsidy than firm 2.

$$\begin{aligned}\psi_1^P - \psi_2^P &= c_1 - c_2 - \gamma\bar{e}(1 - 2h_1^P) \\ &< c_1 - c_2 = \psi_1^S - \psi_2^S.\end{aligned}$$

While both marginal costs are higher than with the fixed rebate rate, the disparity drives the change in market share. Normally, with Cournot competition, cost disparities cause production to shift toward the lower-cost producer. However, with output-based allocation, the marginal subsidy is decreasing in market share, halting some of this shifting and raising average production costs, given any emissions rate.

3.2 Two-Stage Game

Suppose the emission rate and output decisions are not made simultaneously, but rather the emission rate is chosen before output. In this case, each firm knows its emission rate choice will help determine the

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[15] To see that total output is higher in the restricted trading scenario than the broad-based one, suppose first that output were the same in both scenarios; then $\bar{e}$ is also the same. From the first-order conditions, $a(\bar{e}) + \gamma\bar{e} < a(\mu) + \gamma\mu$, for any $\mu \neq \bar{e}$. Thus, we can show that total costs are lower in the restricted permit case than in the broad-based trading case:

$$\begin{aligned}2a(\bar{e}) + \gamma\bar{e} &< 2a(\bar{\mu}) + 2\gamma\bar{\mu} - \gamma\bar{e} \\ &< 2a(\bar{\mu}) + t(2\bar{\mu} - \bar{e})\end{aligned}$$

subsequent output and market-share equilibrium. Do the different incentives with respect to output then change the emission rate decision?

In general, with two-stage Cournot competition, players attempt to raise their rival's marginal cost and lower their own, in order to have a market-share advantage. Shaffer and Salant (1998) show that, in a game of marginal-cost reducing investments among Cournot players, the symmetric noncooperative equilibrium represents a local minimum. The reason is that each overinvests in order to achieve a marginal cost (and thereby market share) advantage in the subsequent output game. This result would hold for emissions abatement if high fixed cost investments were chosen consistently over high marginal cost abatement techniques, when a mix would be preferred. Here, however, total marginal costs depend not only on abatement costs, but also the tax and subsidy, which may be endogenous in revenue-neutral policies.

Consider a Cournot duopoly playing a two-stage game in which firms simultaneously choose emission rates and then simultaneously choose output. We consider the problem from the point of view of firm 1, recognizing that 2's problem is a mirror image. The profit function of firm 1 is now

$$\pi_1^{2S} = P(Q)q_1 - (c_1 + a(\mu_1))q_1 - t\mu_1 q_1 + s(q_1, q_2, \mu_1, \mu_2)q_1 \tag{28}$$

The distinction in this game is that the implicit subsidy is now not only a function of the firm's own emission rate and output, but the competitor's equilibrium output, given the competitor's chosen emission rate.

In the second stage, firms choose output quantities given their emission rates and their competitor's output, with the first-order conditions being the same as in the previous section for the simultaneous game. In the first stage, however, each firm chooses its emission rates, given its competitor's rate, knowing how that will affect the subsequent output equilibrium:

$$\frac{\partial \pi_1^{2S}}{\partial \mu_1} = -a'(\mu_1)q_1 - tq_1 + \frac{\partial s_1}{\partial \mu_1}q_1 + \frac{\partial \pi_1}{\partial q_1}\frac{dq_1}{d\mu_1} + \frac{\partial \pi_1}{\partial q_2}\frac{dq_2}{d\mu_1} \tag{29}$$

By the second-stage first-order conditions, $\partial \pi_1/\partial q_1 = 0$, therefore, the impact of small changes in the emission rate on the firm's own output does not affect profits. However, changes in the competitor's output do. We can rewrite this equation as

$$-a'(\mu_1) = t - \frac{\partial s_1}{\partial \mu_1} - \frac{\partial \pi_1}{\partial q_2}\frac{dq_2}{d\mu_1}\frac{1}{q_1} \tag{30}$$

Profit is decreasing in the competitor's output ($\partial \pi_1/\partial q_2 < 0$) by reducing prices and average subsidies; the competitor's output is decreasing in the firm 1's emissions rate to the extent it increases firm 1's output or decreases firm 2's subsidy.

3.2.1 Fixed Rebate Rate

With the fixed rebate rate or tradable performance standard, we continue to assume that the firm takes the marginal subsidy as invariant to its own behavior. Thus, no permit price change is expected, nor a policymaker response.

Using our linear demand example, solving from (20) and (21) for each firm's output in the second stage, we get

$$\begin{aligned} q_1 &= \frac{1}{3b}\left(y - 2c_1 - 2a_1(\mu_1) + c_2 + a(\mu_2) - t(2\mu_1 - \mu_2 - \bar{\mu})\right) && (31) \\ q_2 &= \frac{1}{3b}\left(y - 2c_2 - 2a_2(\mu_2) + c_1 + a(\mu_1) - t(2\mu_2 - \mu_1 - \bar{\mu})\right) && (32) \end{aligned}$$

From (32) and (28) we get

$$\begin{aligned} \frac{dq_2}{d\mu_1} &= \frac{a'(\mu_1) + t}{3b} && (33) \\ \frac{\partial \pi_1}{\partial q_2} &= -bq_1 && (34) \end{aligned}$$

Substituting into (29), we get

$$-a'(\mu_1) = t + \frac{a'(\mu_1) + t}{3} \tag{35}$$

which can only hold when $-a'(\mu_1^S) = t$ in the two-stage game. Thus, if the subsidy is fixed (or the firms are price-takers in the tradable performance standards market and the average emission rate constraint binds in equilibrium), the two-stage game produces the same results as the simultaneous game.

3.2.2 Tax and Rebate

With the tax-rebate program, the impact of market shares on marginal subsidies and marginal abatement incentives leads to somewhat different (and more complicated) outcomes in the two-stage framework. Solving from (16) and (23) for each firm's output in the second stage, we get

$$\begin{aligned} q_1 &= \frac{1}{3b}\left(y - 2c_1 - 2a(\mu_1) + c_2 + a(\mu_2) - t(\mu_1 - \mu_2)(2 - 4h_1 + 3h_1^2)\right) && (36) \\ q_2 &= \frac{1}{3b}\left(y - 2c_2 - 2a(\mu_2) + c_1 + a(\mu_1) + t(\mu_1 - \mu_2)(1 - 2h_1 + 3h_1^2)\right) && (37) \end{aligned}$$

In the first stage, firm 1 chooses its emission rate, recognizing that $(\partial s_i/\partial \mu_i)q_i = th_i$. From (29) we get

$$-a'(\mu_1) = t(1 - h_1) - \frac{\partial \pi_1}{\partial q_2}\frac{dq_2}{d\mu_1}\frac{1}{q_1} \qquad (38)$$

Deriving the components of the latter term,

$$\begin{aligned} \frac{dq_2}{d\mu_1} &= \frac{1}{3b}\left(a'(\mu_1) + t(1 - 2h_1 + 3h_1^2) + t(\mu_1 - \mu_2)(6h_1 - 2)\frac{dh_1}{d\mu_1}\right) && (39) \\ \frac{\partial \pi_1}{\partial q_2} &= -bq_1 - th_1^2(\mu_1 - \mu_2) && (40) \end{aligned}$$

Here it is evident that emission rate choice has complicated strategic effects, due to the effect on tax payments and shares of the revenues. Since market share affects the expected subsidy and the rival's output in a way that is not proportional to the direct effect on costs, the strategy for choosing the emission rate is different in the two-stage game.

To simplify this problem, let us restrict the analysis to identical firms, where $c_1 = c_2 = 0$. In the symmetric equilibrium, $\mu_1 = \mu_2$, $h_1 = 1/2$. Substituting into (38) and solving for the marginal cost of abatement per unit:

$$-a'(\mu^{T_{2S}}) = \frac{9}{16}t > \frac{1}{2}t.$$

As a result, we see that marginal abatement costs are somewhat higher than in the simultaneous Cournot game, in which $dq_2/d\mu_1 = 0$ and $-a'(\mu^T) = t/2$. Emission rates will then be lower, since each firm competes for a higher net subsidy from the program to maintain and enhance its market share. This strategic effect mitigates, to some extent, the effect of the tax-rebate program to raise emission rates compared to a tradable performance standard with an equivalent price for emissions.

We also see from (39) and (40) that cost heterogeneity may have important effects. When costs are not symmetric, the analysis becomes complicated, and the results are likely to be ambiguous. Whether or not the firm increases or decreases abatement relative to the simultaneous game depends on the sign of the marginal profits from influencing the rival. This has the effect of creating a wedge between $-a'(\mu_1)$ and $t(1-h_1)$. Since profits always fall as the rival's production increases ($\partial\pi_1/\partial q_2 < 0$), the sign of this wedge depends on $dq_2/d\mu_1$. Let $\Delta = -a'(\mu_1) - t(1-h_1)$. Substituting into (38) and solving for Δ, we see the importance of market share in determining the sign:

$$\Delta = \frac{-\partial\pi_1/\partial q_2}{-\partial\pi_1/\partial q_2 + 3bq_1}\left[t(3h_1-1)\left(h_1 + 2t(\mu_1-\mu_2)\frac{dh_1}{d\mu_1}\right)\right] \tag{41}$$

Holding emission rates identical, the bracketed term reduces to $th_1(3h_1-1)$. Starting at a point of equal market share, reducing 1's market share shrinks this term, reducing it to zero when firm 1's share falls to one third, after which it turns negative. This means that, all else equal, a very small player would have less incentive to abate in the two-stage game, while a larger player would abate more than in the simultaneous game and more than if its rival were more equal. Of course, the direct, first-order effect of falling market share is to increase abatement incentives, so a smaller firm 1 would have a lower emission rate than larger firm 2, other things equal. The sign of $dh_1/d\mu_1$ is generally negative (when evaluated at the profit-maximizing rate), although it may in theory be ambiguous.[16]

As long as the impact of changing market share with the emissions rate is a second-order effect, increasing cost heterogeneity means that in the two-stage game, larger firms (with market share $>$1/3) will have a strategic incentive to choose a lower emission rate than in the simultaneous game, while smaller firms (with market share $<$1/3) will have a strategic incentive to select a higher emission rate than they would otherwise.

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[16] The effect of the emission rate on market share follows from its direct effect on output, and

$$\frac{\partial q_1}{\partial \mu_1} = \frac{1}{3b}\left(2\Delta - 3th_1^2)\right). \tag{42}$$

For example, at $h_1 = 1/2$, $\partial q_1/\partial\mu_1 = -5t/(3b)$, and at $h_1 = 1/3$, $\partial q_1/\partial\mu_1 = -t/(3b)$. If Δ turns negative, $\partial q_1/\partial\mu_1$ remains unambiguously negative (though small as market share gets small). The question would be whether at larger market shares it could become positive. Evaluating at $h_1 = 1$, $\Delta > 3t/2$ if $dh_1/d\mu_1$ is positive since $\frac{-\partial\pi_1/\partial q_2}{-\partial\pi_1/\partial q_2 + 3bq_1} = \frac{bq_1 + th_1^2(\mu_1-\mu_2)}{2bq_1 + th_1^2(\mu_1-\mu_2)} > 1/2$, and $3t(1+2(\mu_1-\mu_2)dh_1/d\mu_1) > 3t$ (assuming abatement costs are similar), confirming that $\partial q_1/\partial\mu_1 > 0$.

Thus, for large firms the dampening effect of market share is somewhat mitigated, while for smaller firms it is exacerbated.

3.2.3 Output-Allocated Permits

With output-allocated permits, market share affects the marginal subsidy but not marginal abatement incentives in the simultaneous game. This result is shown to carry over into the two-stage framework as well. It should be noted, however, that the assumption of being price takers in the permit market may be more strained in the two-stage game.

From (16) and (26), we solve for each firm's output in the second stage:

$$\begin{aligned} q_1 &= \frac{1}{3b}\left(y - 2c_1 - 2a(\mu_1) + c_2 + a(\mu_2) - \gamma(2\mu_1 - \mu_2 + (3h_1 - 2)\bar{e})\right) && (43) \\ q_2 &= \frac{1}{3b}\left(y - 2c_2 - 2a(\mu_2) + c_1 + a(\mu_1) + \gamma(\mu_1 - 2\mu_2 + (3h_1 - 1)\bar{e})\right) && (44) \end{aligned}$$

In the first stage,

$$\frac{\partial \pi_1^{P,2S}}{\partial \mu_1} = -a'(\mu_1) - \gamma + \frac{\partial \pi_1}{\partial q_2}\frac{dq_2}{d\mu_1}\frac{1}{q_1} = 0, \tag{45}$$

with

$$\begin{aligned} \frac{dq_2}{d\mu_1} &= \frac{1}{3b}\left(a'(\mu_1) + \gamma - 3\gamma\bar{e}\frac{dh_1}{d\mu_1} + \gamma(3h_1 - 1)\frac{d\bar{e}}{d\mu_1}\right) && (46) \\ \frac{\partial \pi_1}{\partial q_2} &= -bq_1 - \gamma\frac{q_1}{Q^2}\bar{E} && (47) \end{aligned}$$

The Appendix shows that each part of our $dq_2/d\mu_1$ is a ratio of $-a'(\mu_1) - \gamma$. Substituting, we can rewrite the first-order condition with respect to the emission rate as

$$(-a_1'(\mu_1) - \gamma)\chi = 0, \tag{48}$$

where χ is the sum of the ratios in each part. As in the fixed rebate rate case, this can only hold if $-a'(\mu_1) = \gamma$. Therefore, the solution to the two-stage game will be identical to the simultaneous game. This result extends to the case of heterogeneous costs as well, since the impact on the opponent's output is from each firm's own costs, leaving the essential part of (48) unchanged while only the bracketed term will vary with unequal market shares.

4 Conclusion

The intent of rebating environmental policy revenues is to mitigate the cost burden on participants. The reasons may be to maintain equity, to prevent production from shifting to unregulated sectors, or plainly to garner political support of regulation. Output-based rebating is attracting attention because it provides a seemingly fair rule of distribution of the policy rents and because it allows the allocations to respond to changes in market conditions over time. Furthermore, the subsidy to output may help counteract the effects of imperfectly competitive markets or counteract leakage due to imperfect participation.

However, output-based rebating can cause some problems. First, the effective subsidy from an earmarking program is unlikely to be the optimal one. The marginal rebate is tied to the value of emissions in the program, rather than the degree of output underprovision or leakage. It can be greater or less than optimal, depending on the relative costs of emissions and demand elasticities. In particular if it is greater, the wrong subsidy can be worse than no subsidy. Thus, a tailored fixed subsidy implemented with the environmental policy may be prefered to full earmarking of the implicit (or explicit) revenues.

Second, certain forms of output-based rebating—notably the tax-rebate scheme and output-allocated permits—can lead to different effective tax and/or subsidy rates when market shares among program participants are significant. These additional distortions occur because firms then know that part of any emissions rents they create will be returned to them in their allocation share. The result is the potential for inefficient production shifting and suboptimal abatement incentives. For this reason, a fixed subsidy is again prefered to endogenous refunding when output support is desired.

With output-allocated permits, market share diminishes the effect of the output subsidy, but not the emissions price. We show that with heterogeneous firms engaged in Cournot competition, while one would typically prefer to shift production toward the lower cost firm, the output-allocated permit scheme tends to encourage the opposite. The tax-rebate scheme also suffers when market shares are significant. Since changing emissions changes the tax revenues that will be rebated, the expectation of a large rebate share reduces the incentive to reduce emissions. Thus, the endogenous refund has the effect of increasing both emissions and output, relative to an equivalent tax rate with a fixed rebate. Furthermore, when market shares differ, marginal abatement cost equalization is also sacrificed. This result still holds when emission rates are chosen before output, although strategic effects may then induce more abatement than in a simultaneous game.

Conclusions about the impact on overall welfare are harder to draw, since one is by definition starting from a second-best point. Therefore, the tradeoff between more or less output and more or less emissions depends on whether, in the fixed rebate rate case, one has too much output or emissions from a societal perspective. With the tax-rebate, emission rates are higher; so the question is whether the lower production costs and extra output outweigh the extra pollution damages. With output-allocated permits, since output is smaller due to the lower average marginal subsidy, consumer surplus is then necessarily lower. Since emissions are capped, the remaining welfare effects depend on producer costs within the sector. When permit trading is restricted to the duopoly, emission rates are higher; the net impact on average production costs depends on whether the cost-shifting effect dominates any cost-reducing effect of looser emissions standards.[17]

Obviously, efficient, revenue-raising policies and independent tools for correcting market distortions are preferred. However, political realities must be taken into account, and market-based environmental policies with output-based rebating may still dominate command-and-control policies and no policy or no subsidy. Given the potential for quite different outcomes, more research is required to assess the relative size of the efficiency losses from using different output-based rebating instruments to address environmental externalities in highly concentrated industries.

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[17] I.e., whether $(h_1^P - h_1^S)(c_1 - c_2) >< a(\mu^S) - a(\mu^P)$.

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5 Appendix

Proof of Proposition 1:

Since abatement-related cost functions are identical, $\mu_1^S = \mu_2^S$. From (22) we know that, given any t, $\mu_i^T > \mu_i^S$, which implies $a(\mu_i^S) > a(\mu_i^T)$. Furthermore, from the combination of these conditions, if $h_1^T < h_2^T$ (i.e., if $h_1^T < 1/2$), then $\mu_1^T > \mu_2^T$. Assuming $c_1 > c_2$, then $h_1^T < 1/2$. Thus,

$$\begin{aligned} \psi_1^S &= c_1 + a(\mu_1^S) \\ &> c_1 + a(\mu_1^T) + t(\mu_1^T - \mu_2^T)(1 - h_1^T)^2 = \psi_1^T, \end{aligned}$$

since $\mu_2^T > \mu_2^S$ and $\mu_1^T < \mu_2^T$, and

$$\begin{aligned} \psi_2^S &= c_2 + a(\mu_2^S) \\ &> c_2 + a(\mu_2^T) + t(\mu_2^T - \mu_2^S)(h_1^T) \\ &> c_2 + a(\mu_2^T) + t(\mu_2^T - \mu_1^T)(h_1^T)^2 = \psi_2^T; \end{aligned}$$

since $\mu_1^T > \mu_1^S = \mu_2^S$ and $\mu_1^T < \mu_2^T$ and $h_1^T < 1$.

Proof of Proposition 4:

Consider the case in which firm 1 has higher marginal production costs than 2: $c_1 > c_2$. Then, all else equal, $h_2 > h_1$. It follows from (25) that firm 1 has a higher marginal subsidy than firm 2.

$$\begin{aligned} \psi_1^P - \psi_2^P &= c_1 - c_2 - \gamma\bar{e}(1 - 2h_1^P) \\ &< c_1 - c_2 = \psi_1^S - \psi_2^S. \end{aligned}$$

$$h_1^P = \frac{1}{2} - \frac{3(c_1 - c_2 - \gamma\bar{e}(1 - 2h_1^P))}{2y - 2a(\mu^P) - c_1 - c_2 - 2\gamma\mu^P + \gamma\bar{e}}.$$

Solving for h_1^P,

$$\begin{aligned} h_1^P &= \frac{1}{2} - \frac{3(c_1 - c_2)}{2y - 2a(\mu^P) - c_1 - c_2 - 2\gamma\mu^P + 7\gamma\bar{e}} \\ &> \frac{1}{2} - \frac{3(c_1 - c_2)}{2y - 2a(\bar{\mu}) - c_1 - c_2} = h_1^S \end{aligned}$$

If $\gamma = t$, then $\mu_i^P = \mu_i^S = \bar{\mu}$, and $\bar{\mu} < \bar{e} < 2\bar{\mu}$. Since $\bar{\mu} < \bar{e}$, the denominator is larger than with the fixed rebate; therefore, firm 1's market share is also larger. If, instead, total emissions are equal and the permit market is restricted to the duopoly ($\mu^P = \bar{e}$, and $a(\mu^P) < a(\bar{\mu})$), firm 1's relative market share would still be higher with the permit system, since $-2a(\bar{\mu}) < -2a(\mu^P) + 5\gamma\bar{e}$ ensures the denominator of the second term in h_1^P remains larger.

Demonstration of change in rival's output under OBA permits in the two-stage game:

The change in the average allocation comes from the change in total output:

$$\frac{d\bar{e}}{d\mu_1} = -\frac{\bar{E}}{(Q^P)^2}\frac{dQ^P}{d\mu_1}$$

Recognizing $\bar{e} = \bar{E}/Q^P$ and totally differentiating (27), we get

$$\frac{dQ^P}{d\mu_1} = \frac{-a'(\mu_1) - \gamma}{3b + \gamma\bar{E}/(Q^P)^2}$$

Meanwhile, the change in market share can be derived from (17) (using the simplified cost functions), which also includes $\bar{e}$. From the Chain Rule,

$$\frac{dh_1}{d\mu_1} = \frac{\partial h_1}{\partial \mu_1} + \frac{\partial h_1}{\partial \bar{e}}\frac{d\bar{e}}{d\mu_1}$$

where

$$\frac{\partial h_1}{\partial \mu_1} = (-a'(\mu_1) - \gamma)\frac{3(y - a(\mu_2) - \gamma\mu_2 + 2\gamma\bar{e})}{2y - (a(\mu_1) + \gamma\mu_1) - (a(\mu_2) + \gamma\mu_2) + 4\gamma\bar{e}}$$

Thus, each part of our $dq_2/d\mu_1$ is a ratio of $-a'(\mu_1) - \gamma$.