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Fractional integration and structural breaks at unknown periods of time

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ABSTRACT

This paper deals with the analysis of structural breaks in the context of fractionally integrated models. We assume that the break dates are unknown and that the different sub-samples possess different intercepts, slope coefficients and fractional orders of integration. The procedure is based on linear regression models using a grid of values for the fractional differencing parameters and least squares estimation. Several Monte Carlo experiments conducted across the paper show that the procedure performs well if the sample size is large enough. Two empirical applications are carried out at the end of the article.

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1. INTRODUCTION

In recent years, fractional integration has become a feasible alternative method of modelling many macroeconomic time series. The idea behind such specification is that the dependence between the observations, which are increasingly distant in time, can be adequately captured in terms of a hyperbolic rate of decay rather than the exponential rate associated to the autoregressive (AR) structure. Moreover, the nonstationary nature of many series that is usually solved by means of first differences might also be better described by using fractional integration.

There exist several sources that may produce fractional integration: the aggregation of heterogeneous AR processes (Robinson, 1978; Granger, 1980); error duration models (Parke, 1999), or regime-switching and structural break models (Diebold and Inoue, 2001). In fact, the existence of breaks may lead to spurious findings of long memory. Lobato and Savin (1998) argue that structural breaks may be responsible for the long memory in return volatility processes, and Engle and Smith (1999) investigated the relationship between structural breaks and long memory using a simple model where the data generating process consists of a mean process and a stationary error.

This paper contributes to the above-mentioned literature by proposing a simple procedure for determining fractional integration and structural breaks in a unified treatment. The procedure uses a grid of finite points for the fractional integration parameters. However, unlike other methods where the time of the break is known (Gil-Alana, 2003), we keep it unknown, and is implicitly determined in the model. A drawback of this approach is that since it uses a grid of finite values and given the real nature of the fractional differencing parameters, the resulting estimates for the break-fraction and the fractional differencing parameters will be inconsistent if the true values

of the differencing parameters are not included in the set of values chosen in the grid. Nevertheless, this is a limitation that faces all procedures based on this type of approach.

The structure of the paper is as follows. In Section 2 we define the concept of fractional integration and its relation with the existence of breaks. In Section 3 we present a procedure for fractional integration and structural breaks at unknown periods of time. Section 4 contains a small simulation study showing the performance of the procedure described in Section 3. Two empirical applications are carried out in Section 5, while Section 6 contains some concluding comments.

2. FRACTIONAL INTEGRATION AND STRUCTURAL BREAKS

For the purpose of the present paper, we define an I(0) process $\{u_t, t = 0, \pm 1, \ldots\}$ as a covariance stationary process with a spectral density that is positive and finite at the zero frequency. In this context, we say that a time series $\{x_t, t = 0, \pm 1, \ldots\}$ is I(d) if:

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, ..., \tag{1}$$

with $x_t = 0$, $t \leq 0$,[1] where L is the lag operator (i.e. $Lx_t = x_{t-1}$) and u_t is I(0). Clearly, if d = 0, $x_t = u_t$, and a 'weakly autocorrelated' x_t is allowed for. If d > 0, the process is said to be long memory, because of the strong association between observations widely separated in time. Note that the polynomial in the left hand side of (1) can be expressed in terms of its Binomial expansion, such that for all real d,

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - ...,$$

and (1) can be written as:

$$x_t = d\, x_{t-1} - \frac{d(d-1)}{2} x_{t-2} + ... + \varepsilon_t. \tag{2}$$

If d is an integer value, x_t will be a function of a finite number of past observations, while if d is real, x_t depends strongly upon values of the time series far away in the past. If $d \in (0, 0.5)$ in (1), x_t is covariance stationary and mean-reverting, with the effect of shocks disappearing in the long run; if $d \in [0.5, 1)$, the series is no longer covariance stationary but it is still mean-reverting, while $d \geq 1$ means nonstationarity and non-mean-reversion.

The implications of structural change on unit-root tests which take no account of this possibility attracted the attention of Perron (1989), who found that the 1929 crash and the 1973 oil price shock were a cause of non-rejection of the unit-root hypothesis, and that when these were taken into account, a deterministic trend model was preferable. This question was also pursued by other authors. Christiano (1992) argued that the date of the break should be treated as unknown, and suggested that tests for a structural break are themselves biased in favour of non-rejection. He proposed tests based on bootstrap critical values, reaching different conclusions from Perron (1989). Similarly, Zivot and Andrews (1992) allowed the structural break to be endogenous, finding less conclusive evidence against unit roots than did Perron (1989). Banerjee et al. (1992) also considered this problem, proposing sequential statistics based on the full sample, and a sequence of regressors indexed by a 'break' date. Using these techniques, they failed to reject the unit-root hypotheses in the real output in five industrialized countries (including the United States) but found evidence of stationarity around a shifted trend for Japan.

In the context of fractional processes, there are several works which show that neglecting occasional breaks may lead to spurious finding of long memory. Kuan and Hsu (1998) found that the least squares estimation of the change point may suggest a spurious change when data have long run dependence. Other studies have also investigated the effects of structural changes on persistence. Lobato and Savin (1998)

argue that structural breaks may be responsible for the long memory in return volatility processes. Engle and Smith (1999) investigate the relationship between structural breaks and long memory using a simple unit root process which occasionally changes over time. Beran and Terrin (1996) and Bos et al. (2001) proposed Lagrange Multiplier tests for fractional integration with breaks, while Diebold and Inoue (2001) relates long memory with regime-switching models.

3. THE STRUCTURAL CHANGE FRACTIONALLY INTEGRATED MODEL

To simplify matters, we consider the case of a single break, though the model can be easily extended for multiple breaks. We suppose that y_t is the observed time series, generated by the model

$$y_t = \alpha_1 + \beta_1 t + x_t; \quad (1-L)^{d_1} x_t = u_t, \qquad t = 1,...,T_b \tag{3}$$

$$y_t = \alpha_2 + \beta_2 t + x_t; \quad (1-L)^{d_2} x_t = u_t, \qquad t = T_b+1,...,T, \tag{4}$$

where the α's and the β's are the coefficients corresponding to the intercept and the linear trend; d_1 and d_2 may be real values, u_t is I(0) and T_b is the time of the break that is supposed to be unknown. Note that the model in (3) and (4) can also be written as:

$$(1-L)^{d_1} y_t = \alpha_1 \tilde{1}_t(d_1) + \beta_1 \tilde{t}_t(d_1) + u_t, \quad t = 1, ..., T_b, \tag{5}$$

$$(1-L)^{d_2} y_t = \alpha_2 \tilde{1}_t(d_2) + \beta_2 \tilde{t}_t(d_2) + u_t, \quad t = T_b+1, ..., T, \tag{6}$$

where $\tilde{1}_t(d_i) = (1-L)^{d_i} 1$, and $\tilde{t}_t(d_i) = (1-L)^{d_i} t$, $i = 1, 2$.

[Insert Figure 1 about here]

By way of illustration, we describe in Figure 1 the behaviour of the fractional processes $(1\text{-}L)^d 1_t$ and $(1\text{-}L)^d t_t$ with d = 0.25, 0.50, 0.75, 1 and 1.25. It is observed that if $d < 1$, the series $(1\text{-}L)^d 1_t$ decreases hyperbolically to zero, and becomes exactly 0 if $d = 1$.

For d > 1, the second observation in $\tilde{1}_t$ becomes negative, and the series decreases then hyperbolically to zero. With respect to the linear trend we see that $(1\text{-}L)^d t_t$ is explosive for d < 1, though it tends to a constant as d increases through 1; however, if d > 1 the values tend to zero.

The idea that is behind the model in (5) and (6) is based on the least square principle proposed by Bai and Perron (1998). First we choose a grid for the values of the fractionally differencing parameters d_1 and d_2, for example, d_{io} = 0, 0.01, 0.02, …, 1, i = 1, 2. Then, for a given partition $\{T_b\}$ and given initial d_1, d_2-values, $(d_{1o}^{(1)}, d_{2o}^{(1)})$, we estimate the α's and the β's by minimizing the sum of squared residuals,

$$\min_{\text{w.r.t.}\{\alpha_1,\alpha_2,\beta_1,\beta_2\}} \sum_{t=1}^{T_b}\left[(1-L)^{d_{1o}^{(1)}} y_t - \alpha_1 \tilde{1}_t(d_{1o}^{(1)}) - \beta_1 \tilde{t}_t(d_{1o}^{(1)})\right]^2 + \sum_{t=T_b+1}^{T}\left[(1-L)^{d_{2o}^{(1)}} y_t - \alpha_2 \tilde{1}_t(d_{2o}^{(1)}) - \beta_2 \tilde{t}_t(d_{2o}^{(1)})\right]^2 .$$

Let $\hat{\alpha}(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$ and $\hat{\beta}(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$ denote the resulting estimates for partition $\{T_b\}$ and initial values $d_{1o}^{(1)}$ and $d_{2o}^{(1)}$. Substituting these estimated values on the objective function, we have RSS(T_b; $d_{1o}^{(1)}$, $d_{2o}^{(1)}$), and minimizing this expression across all values of d_{1o} and d_{2o} in the grid we obtain $\text{RSS}(T_b) = \arg\min_{\{i,j\}} \text{RSS}(T_b; d_{1o}^{(i)}, d_{2o}^{(j)})$. Then, the estimated break date, $\hat{T}_k$, is such that $\hat{T}_k = \arg\min_{i=1,\ldots,m} \text{RSS}(T_i)$, where the minimization is taken over all partitions T_1, T_2, …, T_m, such that $T_i - T_{i-1} \geq |\varepsilon T|$. Then, the regression parameter estimates are the associated least-squares estimates of the estimated

k-partition, i.e., $\hat{\alpha}_i = \hat{\alpha}_i(\{\hat{T}_k\})$, $\hat{\beta}_i = \hat{\beta}_i(\{\hat{T}_k\})$, and their corresponding differencing parameters, $\hat{d}_i = \hat{d}_i(\{\hat{T}_k\})$, for i = 1 and 2.

The statistical properties of the resulting estimators are not derived though they should not differ much from those reported in Bai and Perron (1998) since we choose the values in a way such that they minimize the residuals sum squares and, under the appropriate specification, u_t must follow an I(0) process. In Appendix C we show that the model described by (3) and (4) can be expressed in a similar way as the one in Bai and Perron (1998) satisfying the same type of assumptions as in that paper. Several Monte Carlo experiments based on this model are provided in Section 4.

Clearly, the model can be extended to the case of multiple breaks. Thus, we can consider the model,

$$y_t = \alpha_j + \beta_j t + x_t;\ (1 - L)^{d_j} x_t = u_t,\quad t = T_{j-1}+1,...,T_j,$$

for j = 1, …, m+1, $T_0 = 0$ and $T_{m+1} = T$. Then, the parameter m is the number of changes. The break dates (T_1, …, T_m) are explicitly treated as unknown and for i = 1, …, m, we have $\lambda_i = T_i/T$, with $\lambda_1 < \ldots < \lambda_m < 1$. Following the same lines as in the previous case, for each j-partition, $\{T_1, \ldots T_j\}$, denoted $\{T_j\}$, the associated least-squares estimates of α_j, β_j and the d_j are obtained by minimizing the sum of squared residuals in the d_i-differenced models, i.e.,

$$\sum_{j=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_j} (1 - L)^{d_i} (y_t - \alpha_i - \beta_i t)^2,$$

where $\hat{\alpha}_i(T_j)$, $\hat{\beta}_i(T_j)$ and $\hat{d}(T_j)$ denote the resulting estimates. Substituting them in the new objective function and denoting the sum of squared residuals as $RSS_T(T_1, \ldots, T_m)$,

the estimated break dates $(\hat{T}_1, \hat{T}_2, ..., \hat{T}_m)$ are obtained by: $\min_{(T_1, T_2, ..., T_m)} RSS_T(T_1, ..., T_m)$ where the minimization is again obtained over all partition $(T_1, \ldots, T_m)$.

The above procedure requires the a priori determination of the number of breaks in the time series. Following standard procedures to select the number of breaks in the context of I(0) processes, Schwarz (1978) proposed the criterion: $SIC(m) = \ln[RSS_T(\hat{T}_1, ..., \hat{T}_m)/ \quad (T-m) + 2p^* \ln(T)/T$, where p^* is the number of unknown parameters.[2] The estimated number of break dates, $\hat{m}$, is then obtained by minimizing the above-mentioned criterion given M a fixed upper bound for m.

4. A MONTE CARLO SIMULATION STUDY

In this section we consider a data generating process given by:

$$y_r = 5 + 1t + x_t; \quad (1-L)^{d_1} x_t = u_t, \quad t = 1, ..., T_b \qquad (7)$$

$$y_r = 10 + 5t + x_t; \quad (1-L)^{d_2} x_t = u_t, \quad t = T_b + 1, ..., T, \qquad (8)$$

where $(d_1, d_2) = (0.2, 0.7)$; $(0.5, 0.5)$ and $(0.7, 0.2)$; $T_b = T/2$, $T/4$, $3T/4$, $T/10$ and $9T/10$, with sample sizes T = 200, 500, 700, 1000, 1500 and 2000 observations, and white noise u_t. We generate Gaussian series using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986).

[Insert Figure 2 about here]

Figure 2 contains plots of simple realizations of the model given by (7) and (8) with T = 300, $T_b = 150$, and $(d_1, d_2) = (0.2, 0.7)$, $(0.5, 0.5)$ and $(0.7, 0.2)$. We observe that if the deterministic components are included in the model (left-hand-side plots in the figure) the structural change becomes extremely clear, with a change in both the intercept and the slope coefficients. We also observe that the different orders of integration for

each sub-sample (upper and lower plots) are obscured because of the presence of the deterministic changes, and little thus can be said about them just from a simple visual inspection of the series. The plots in the right hand side correspond to the series without the deterministic terms. The upper plot refers to the case of $d_1 = 0.2$ and $d_2 = 0.7$, and a higher degree of dependence is observed between the observations in the second sub-sample. The lower plot refers to the opposite case, and the dependence is now higher in the first sub-sample.

In Tables 1 – 5 we report the probabilities of correctly determining the time break and the fractional differencing parameters in the model given by (7) and (8), using a grid of d_1, d_2 values = 0, 0.1, 0.2, …, 0.8, 0.9 and 1, and values for the break $T^* = (T/10, T/10 +1, …, (1), .., 9T/10 – 1, 9T/10)$[3]. We use 10,000 replications for each case.

The most noticeable thing observed from these tables is that the procedure accuracy determines the break date in all cases, and we find zero-probabilities for all values of d_1 and d_2 if T^* is different from the true time of the break. Thus, the probabilities corresponding to $T^* = T_b$ are presented exclusively in the tables. Note, however, that this might be a consequence of the deterministic pattern describing the equations in (7) and (8). At the end of this section we present the results for other deterministic models, where the optimal break is not so accuracy determined.

In Tables 1 - 3 we assume that the break takes place at T/2 and consider the three cases of stationarity for the first subsample ($d_1 = 0.2$) and nonstationarity for the second one ($d_2 = 0.7$) (in Table 1); nonstationarity in both subsamples, with d being in the boundary situation between stationarity and nonstationarity ($d_1 = d_2 = 0.5$) (Table 2); and nonstationarity in the first subsample and stationarity in the second subsample (Table 3).

[Insert Tables 1 – 3 about here]

The results are very similar in the three tables. Thus, if the sample size is small (e.g. T = 200) the probability of detecting the true break along with the true parameters for the orders of integration is very small (around 10% for the grid of values employed in the tables). However, increasing the sample size, the probabilities also increase; they are higher than 50% with T = 1000, and around 90% with T = 2000. Note here that these probabilities are based on the grid employed for the orders of integration and thus, the probabilities are smaller as we reduce the value for the increments in the ds. On the other hand, larger increments would produce higher probabilities of detecting the true values. Thus, for example, if we compute the procedure with (d_1, d_2)-values equal to 0, 0.2, 0.4, …, 0.8 and 1, the probabilities of correctly detecting the true parameters are higher than 75% with $T \geq 300$ and higher than 90% with $T \geq 700$.

[Insert Tables 4 and 5 about here]

Next, we perform the same experiment with the break dates taking place at T/10, T/4, 3T/4 and 9T/10, and for the same (d_1, d_2)-values as in the previous tables. The results were very similar. As an illustration, we reproduce only the results for the cases of $T_b = T/4$, $d_1 = 0.2$ and $d_2 = 0.7$ (in Table 4) and $T_b = 9T/10$, $d_1 = 0.7$ and $d_2 = 0.2$ (Table 5). We see that the probabilities are smaller. In fact, if T is very small, higher probabilities are obtained at other (d_1, d_2)-combinations. However, if T > 500, the highest probabilities are obtained at the true values. The same happens if the break occurs at $T_b = 9T/10$. Thus, for example, if T = 1000, the probability of correctly determining the true model is 51.5%, and if T = 2000, it becomes 88.7%.

As mentioned above, the accuracy in the estimation of the break date in the results presented so far might be a consequence of the coefficients used for the intercept and the

slope in the equations in (7) and (8). Thus, in Table 6, we examine the probability of correctly determining the break for different intercept and slope coefficients.

[Insert Table 6 about here]

We now assume that the break date takes place at T/2, with $d_1 = d_2 = 0.5$, and look at the probability of detecting the true break date for a grid of values (T/5, T/5+1, …, 4T/5 – 1, 4T/5), using the following coefficients for the deterministic trends (α_1, β_1, α_2, β_2) = (5.0, 1.0, 10.0, 5.0); (0.5, 0.1, 1.0, 0.5); (0.5, -0.1, -1.0, 0.5) and (-0.5, 0.1, 1.0, -0.5). We observe that using the coefficients in (7) and (8) the procedure correctly determines the break at the 100% of the cases even for a sample size of T = 100. However, reducing the magnitude of these coefficients the probabilities are very small for small sample sizes, though, if T = 500, it reaches 100% in all cases. Note that in this simulation we have only considered for the possible breaks 60% of the sample period. Increasing the set of break dates the probabilities considerably reduce in some cases, implying that, in small samples, it is important to have some a priori knowledge about the period of the break.

5. THE EMPIRICAL WORK

Two different datasets are analysed in this section. The first one is a monthly series of US money stock, while the second refers to the US monthly inflation rate. We choose these series because they seem to have a single break across the sample. In fact, we performed the procedure described in Section 3 and the evidence was in favour of a single break.

5.a The US H-6 money stock

The data analyzed here is the U.S. Total Large Time Deposits (H-6 Money Stock), monthly, seasonally adjusted, for the time period January, 1959 to August, 2004, obtained from the Board of Governors of the Federal Reserve System. Further information and definitions are available at: http://research.stlouisfed.org/publications/mt/.

[Insert Figure 3 about here]

Figure 3 displays plots of the original series and its first differences, along with their corresponding correlograms and periodograms. It is observed that the values of the original data increase across the sample implying that the series is nonstationary. This is substantiated by the correlogram, with values decreasing very slowly, and the periodogram, with a large peak at the smallest frequency. If we take first differences, the plot in the up-right side in Figure 3 shows an increasing variance with T, and we also observe significant values in the correlogram even at some lags far away from zero. The periodogram of the first differences still shows its highest value at the zero frequency, which may suggest that long memory is still present in the differenced data, though this latter result might be a consequence of the existence of a structural break in the data.

The first thing we do is to estimate the fractional differencing parameter assuming that there are no breaks in the data. For this purpose we employ both parametric and semiparametric methods. First we use a parametric testing approach suggested by Robinson (1994) that is described in Appendix A. In this approach we test:

$$H_o : d = d_o, \qquad (9)$$

in a model given by:

$$y_t = \alpha + \beta t + x_t, \quad t = 1, 2, ... \qquad (10)$$

with x_t given by (1). We take d_o-values equal to 0, (0.02), 2, and assume first that $\alpha = \beta = 0$ (i.e., there are no deterministic terms), and then with α and β unknown.

The results are given in Table 7. We report the confidence intervals of those values of d_o where the null hypothesis cannot be rejected at the 5% level.[4] We present the results for the two cases of no regressors and a linear trend, with white noise, AR and Bloomfield (1973) disturbances.[5] It is observed that the values are very similar independently of the inclusion or not of deterministic trends, which might suggest that a linear time trend is not required when modelling this series. The non-rejection values of d are in all cases higher than 1, ranging between 1.21 and 1.58. We also display in the table the value of d that produces the lowest statistic for each type of disturbances. This value should be an approximation to the maximum likelihood estimate.[6] We note that the values are higher than 1.30 in all cases.

[Insert Table 7 and Figure 4 about here]

Next we perform a semiparametric method (Robinson, 1995) that is described in Appendix B. Figure 4 displays the estimates of d across the whole range of values for the bandwidth number m, along with the 95% interval corresponding to the I(1) hypothesis. We see that the estimated values of d are in all cases above the I(1) interval, implying that d is higher than 1, which is consistent with the results based on the parametric approach.

However, the large values of d obtained from the previous results might be in large part due to the fact that no structural breaks are taken into account. Though not reported in the paper, we computed the residuals from the d-differenced series, for values of d from 1.30 to 1.40 (with 0.01 increments) and in all cases, the residuals showed evidence of a structural break, which might be producing a bias in favour of higher orders of

integration. To illustrate this point, we include in this section a simple Monte Carlo experiment. We consider again the model given by (7) and (8) with T = 300, T_b = 150, d_1 = 0.2 and d_2 = 0.7, and perform both the parametric and the semiparametric methods described in the appendices, using 10,000 replications.

[Insert Table 8 and Figure 5 about here]

We employ first Robinson's (1994) parametric tests for d_o-values = 0, (0.02), 2, and the null hypothesis was rejected in the 100% of the cases for $d_o < 1.02$ and $d_o > 1.14$. However, for values of d_o constrained between these two numbers, the rejection probabilities were smaller than 1, and, if d_o = 1.04, 1.06, 1.08 and 1.10, the null hypothesis was never rejected (see Table 8). This happens for the two cases of no regressors and when both the intercept and the slope are included in the model. Thus, using the tests of Robinson (1994) when the true data generating process contains a structural break leads to spurious conclusions about the order of integration of the series. Figure 5 displays the averaged estimates of d for each bandwidth number using the semiparametric method. We observe that this procedure also leads to spurious conclusions about d, finding orders of integration higher than 1 for practically all the values of the bandwidth number m.

Next we perform the procedure described in Section 3. Initially we consider the case of a linear time trend in both subsamples, with white noise disturbances, and the selected model is:

$$y_t = \underset{(1.658)}{1.449} + \underset{(1.611)}{0.156}t + x_t; \quad (1-L)^{1.64}x_t = \varepsilon_t, \quad t = 1, 2, \ldots, T_b,$$

$$y_t = \underset{(785.971)}{-427.583} + \underset{(2.724)}{2.537}t + x_t; \quad (1-L)^{1.37}x_t = \varepsilon_t, \quad t = T_b + 1, \ldots, T$$

with T_b = 289, which corresponds to January 1983. Standard errors are displayed in parenthesis. We observe that both the intercept and the slope coefficients are different for each sub-sample. Also, the fractional differencing parameters are different in both cases, d_1 = 1.64 and d_2 = 1.37. If we allow for short run dynamics and model u_t in terms of an AR(1) process, the selected model is now:

$$y_t = \underset{(2.305)}{2.055} + \underset{(0.676)}{1.050}t + x_t; \quad (1-L)^{1.24}x_t = u_t; \quad u_t = 0.483u_{t-1} + \varepsilon_t,$$

for t = 1, 2, …, T_b = 289 (January, 1983), and

$$y_t = \underset{(677.011)}{-973.432} + \underset{(2.369)}{4.545}t + x_t; \quad (1-L)^{1.32}x_t = u_t; \quad u_t = 0.075u_{t-1} + \varepsilon_t,$$

for t = T_b + 1, …, T. Thus, the values for the orders of integration are slightly smaller than in the uncorrelated case though still above 1. Another remarkable thing observed in this table is that the coefficients associated with the intercept and the linear trend are both insignificantly different from zero. Thus, we perform the same procedure but assuming now that there are no deterministic components. In this case, the selected models are

$$(1-L)^{1.59}y_t = \varepsilon_t, \quad t = 1,2,...,T_b, \quad \text{and} \quad (1-L)^{1.09}y_t = \varepsilon_t, \quad t = T_b+1,...,T,$$

with uncorrelated disturbances, and

$$(1-L)^{1.36}y_t = u_t; \quad u_t = 0.322u_{t-1} + \varepsilon_t, \quad t = 1,2,...T_b,$$

$$(1-L)^{0.34}y_t = u_t; \quad u_t = 0.862u_{t-1} + \varepsilon_t, \quad t = T_b+1, ... T,$$

with AR(1) u_t. In these cases the break date takes place at T_b = 423, which is March, 1994. This period for the break seems to be more realistic if we look back at the plot of the original series in Figure 3. The fractional differencing parameters substantially change depending on how we model the I(0) disturbances. Thus, if u_t is white noise, d_1 = 1.59 and d_2 = 1.09. However, if we model u_t in terms of an AR(1) process, d_1 = 1.36 and

d_2 = 0.34. Note that in this case the order of integration for the second subsample is smaller than 1 (d_2 = 0.34). Therefore, the dependence between the observations is captured by both the fractional differencing parameter and the AR coefficient, which is substantially large (α = 0.862). Moreover, the results are very sensitive to the specification of the serial correlation in the disturbance term. In order to check if they are correlated or not, we perform a test for autocorrelation (Ljung-Box statistic at different lags) in both residuals and the results support the existence of an AR(1) structure for the disturbance term.

5.b The US inflation rate

Here we examine the US inflation rate by looking at the log of the first differences in the Consumer Price Index for All Urban Consumers, monthly from January 1947 to December 2004. The data were collected from the Federal Reserve Bank database of St. Louis. This is a very popular time series in applications with long memory (Hassler and Wolters, 1995; Bos et al., 2001; etc.).

[Insert Figure 6, Table 9 and Figure 7 about here]

Figure 6 displays the time series corresponding to the log of the US CPI. Table 9 reports the 95% intervals of the values of d_o where the null hypothesis cannot be rejected using Robinson's (1994) tests. If a linear trend is included in the model, the values are very similar across all type of disturbances, with the values of d ranging between 1.32 and 1.52. However, if these components are not taken into account, the non-rejection values of d are smaller than one in some cases, ranging between 0.89 and 1.62. Note, however, that the lowest statistics take place at d equal to or higher than 1 and, in the context of a linear time trend, the lowest statistics are obtained at d around 1.40. Using

the semiparametric method (Figure 7) the results are in line with the parametric ones, and the estimated values of d are slightly below 1.5 for practically all the values of the bandwidth number m. These results are in line with other empirical works on the US inflation rate when no breaks are taken into account. Hassler and Wolters (1995) find estimates of d of about 0.40 for the inflation rates, which correspond to values around 1.40 in the log prices. However, these results do not consider the possibility of a structural break.[7]

If a break is taken into account the results are as follows: first we permit the existence of deterministic trends and the break date is found in September 1982. If u_t is white noise the selected model is:

$$\underset{\quad (4.612)\quad (0.235)}{y_t = 2.842 + 0.237t + x_t;} \quad (1-L)^{1.01}x_t = \varepsilon_t, \qquad t = 1,2,..., T_b = 429\ (Sept.1982),$$

$$\underset{\quad (64.439)\quad (0.139)}{y_t = -93.103 + 0.216t + x_t;} \quad (1-L)^{0.63}x_t = \varepsilon_t, \qquad t = T_b + 1, ..., T.$$

However, if u_t follows an AR(1) process, the orders of integration are smaller, and the selected model is then

$$\underset{\quad (15.851)\quad (0.058)}{y_t = -4.176 + 0.083t + x_t;} \quad (1-L)^{0.67}x_t = u_t; \qquad u_t = 0.318u_{t-1} + \varepsilon_t.$$

for t = 1, …, T_b = 429 (September, 1982), and

$$\underset{\quad (7.093)\quad (0.022)}{y_t = 18.812 - 0.028t + x_t;} \quad (1-L)^{0.03}x_t = u_t; \qquad u_t = 0.473u_{t-1} + \varepsilon_t$$

for t = T_b + 1, ..., T. Moreover, the coefficients associated to the deterministic terms are also different in both subsamples, though again they are not significant at conventional levels. Thus, we performed the procedure without the deterministic terms. The break date

takes place at the same period as in the previous case (September, 1982). If u_t is white noise, $d_1 = 1.00$, while $d_2 = 0.43$, and allowing for autocorrelated disturbances, the selected model is

$$(1-L)^{0.67} y_t = u_t; \quad u_t = 0.320 u_{t-1} + \varepsilon_t, \quad t = 1, 2, ..., T_b = 429,$$

$$(1-L)^{0.03} y_t = u_t; \quad u_t = 0.499 u_{t-1} + \varepsilon_t, \quad t = T_b + 1, ..., T.$$

Note that, similarly to the previous application, if we allow for autoregressions, the orders of integration reduce in both subsamples. This may be due to the competition between the fractional differencing parameters and the AR coefficients in describing the dependence between the observations. We also performed here a Ljung-Box test for autocorrelation on the residuals of the estimated models. The results were a bit ambiguous, finding evidence of autocorrelation at the 10% significance level but not at the 5% level.

6. CONCLUSIONS

In this paper we have proposed a procedure for determining the time of structural breaks along with the parameters associated to the models at each sub-sample. In particular, we allow different orders of integration and different coefficients for the time trends. The procedure is similar to the one proposed by Bai and Perron (1998) for the case of I(0) disturbances and is based on least squares estimation of the coefficients for a grid of finite points for the orders of integration at different periods of time. The break date is then determined as the value that produces the lowest squared residuals. Several Monte Carlo experiments were conducted across the paper and the results showed that the procedure performs well if the sample size is large enough (e.g. $T \geq 300$). A drawback of

the present approach is that given the fractional nature of the fractional differencing parameters, if the true values are not included in the grid, the resulting estimates of the d's and the break fraction are likely to be inconsistent. This can be sorted out by using a shorter (finer) grid, e.g. with 0.001 increments. Two empirical applications were also performed at the end of the article. In particular we examined two monthly series corresponding to the US money stock and inflation. In both series we observed a single break, in March 1994 for money stock and in September 1982 for inflation. With respect to the coefficients associated to the linear trends, they were found to be insignificantly different from zero in all cases. This is not surprising since the orders of integration are then capturing most of the stochastic trends of the series. For the money stock, the orders of integration are 1.36 for the first subsample and 0.34 for the second one, and for inflation these values are 0.67 and 0.03, implying thus in both cases nonstationarity for the first subsamples and stationarity for the second parts of the samples.

APPENDIX A

The LM test of Robinson (1994) for testing H_o (9) in (1) and (10) is $\hat{r} = \frac{T^{1/2}}{\hat{\sigma}^2}\hat{A}^{-1/2}\hat{a}$, where T is the sample size and:

$$\hat{a} = \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\,g(\lambda_j;\hat{\tau})^{-1}I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1}g(\lambda_j;\hat{\tau})^{-1}I(\lambda_j);$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 - \sum_{j=1}^{T-1}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)'\times\left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1}\times\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}); \qquad \lambda_j = \frac{2\pi\, j}{T}; \qquad \hat{\tau} = \arg\min\sigma^2(\tau).$$

$\hat{a}$ and $\hat{A}$ in the above expressions are obtained through the first and second derivatives of the log-likelihood function with respect to d (see Robinson, 1994, page 1422, for further details). $I(\lambda_j)$ is the periodogram of u_t evaluated under the null, i.e.:

$$\hat{u}_t = (1 - L)^{d_o} y_t - \hat{\beta}' w_t; \quad \hat{\beta} = \left(\sum_{t=1}^{T} w_t w_t' \right)^{-1} \sum_{t=1}^{T} w_t (1 - L)^{d_o} y_t; \quad w_t = (1 - L)^{d_o} z_t,$$

and g is a known function related to the spectral density of u_t: $f(\lambda;\sigma^2;\tau) = \frac{\sigma^2}{2\pi} g(\lambda;\tau), \ -\pi < \lambda \leq \pi.$

APPENDIX B

The Whittle estimate of Robinson (1995) is defined by: $\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2d \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right)$, for

$d \in (-1/2, 1/2)$; $\overline{C(d)} = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j) \lambda_j^{2d}$, $\lambda_j = \frac{2\pi j}{T}$, $\frac{m}{T} \to 0$, where m is a bandwidth number.

APPENDIX C

The starting point is our model in (3) and (4), which can be written as:

$$(1-L)^{d_1} y_t = \alpha_1 (1 - L)^{d_1} 1_t + \beta_1 (1 - L)^{d_1} t_t + u_t, \quad t = 1, ..., T_b,$$

$$(1-L)^{d_2} y_t = \alpha_2 (1 - L)^{d_2} 1_t + \beta_2 (1 - L)^{d_2} t_t + u_t, \quad t = T_b + 1, ..., T.$$

We call

$$y_t^* = \begin{pmatrix} (1 - L)^{d_1} y_t & t = 1, ..., T_b \\ (1 - L)^{d_2} y_t & t = T_b + 1, ..., T \end{pmatrix},$$

$$\delta_1 = (\alpha_1, \beta_1)^T; \quad \delta_2 = (\alpha_2, \beta_2)^T; \quad z_t^* = (1_t^*, t_t^*)^T,$$

$$1_t^* = \begin{pmatrix} (1 - L)^{d_1} 1_t & t = 1, ..., T_b \\ (1 - L)^{d_2} 1_t & t = T_b + 1, ..., T \end{pmatrix}, \quad t_t^* = \begin{pmatrix} (1 - L)^{d_1} t_t & t = 1, ..., T_b \\ (1 - L)^{d_2} t_t & t = T_b + 1, ..., T \end{pmatrix},$$

Thus, the model can be expressed as

$y_t^* = z_t^{*T} \delta_j + u_t, \quad t = T_{j-1} + 1, ..., T_j,$ j = 1,2, with $T_0 = 0$, $T_1 = T_b$ and $T_2 = T$. That is,

$$y_t^* = z_t^{*T} \delta_1 + u_t, \quad t = 1, ..., T_b, \quad y_t^* = z_t^{*T} \delta_2 + u_t, \quad t = T_b + 1, ..., T,$$

which is precisely the same model as in Bai and Perron (1998) for the case of a single break.

ENDNOTES

1. "This convention" applies to all formulae like (1) and is usually employed in applied work. In fact, it is a standard assumption in the empirical work on fractional integration (see Gil-Alana and Robinson, 1997) and is made so that even within the "stationary" region ($d < 0.5$) x_t is actually not covariance stationary, though it may be thought of as "asymptotically stationary" for such d. In general, this truncation is introduced to cater for "nonstationary" values, ($d \geq 0.5$), where x_t would otherwise blow up.

2. Other well-known criteria are the Bayesian criterion: $BIC(m) = \ln [RSS_T(T_1, \ldots, T_m)/T] + p^* \ln(T)/T$, and the $YIC(m) = \ln [RSS_T(T_1, \ldots, T_m)/T] + mC_T/T$, where C_T is any sequence satisfying $C_T T^{-2d/k} \rightarrow \infty$ as $T \rightarrow \infty$ for some positive integer k.

3. In case of $T_b = T/10$ and $9T/10$, we use $T^* = T/10 - 10$, ..., (1), ..., $9T/10 + 10$.

4. These intervals were constructed as follows: First, we choose a value of d from a grid. Then, we form the test statistic testing the null for this value. If the null is rejected at the 5% level, we discard this value of d. Otherwise, we keep it. An interval is then obtained after considering all the values of d in the grid.

5. The Bloomfield (1973) model is a non-parametric approach of modeling the I(0) disturbances that produces autocorrelations decaying exponentially as in the AR(MA) case.

6. Note that Robinson's (1994) method is based on the LM principle and uses the Whittle function, which is an approximation to the likelihood function

7. Similarly to the first application, the plot of the residuals of the 1.40-differenced series showed some evidence of structural breaks.

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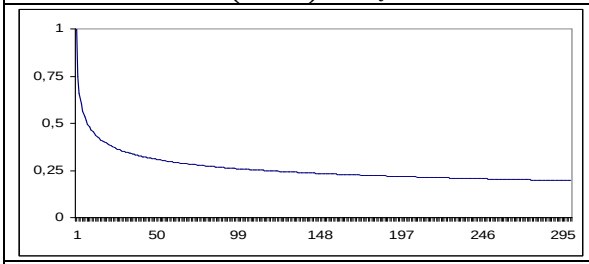
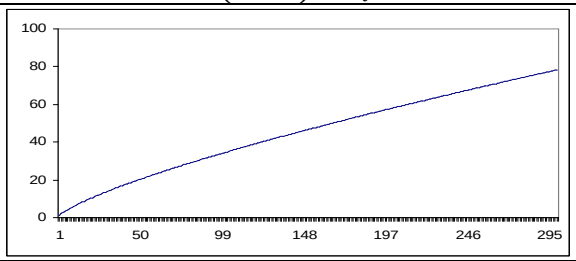
FIGURE 1

Illustrations of fractional processes for a constant and a linear trend

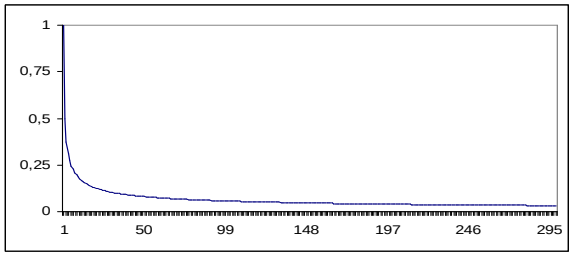
$(1 - L)^{0.25}\, 1_t$



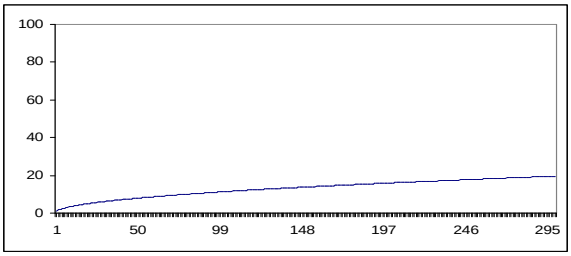
$(1 - L)^{0.25}\, t_t$



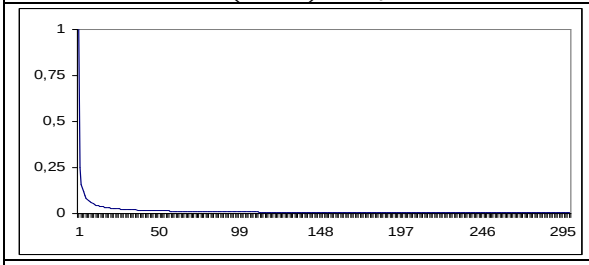
$(1 - L)^{0.50}\, 1_t$



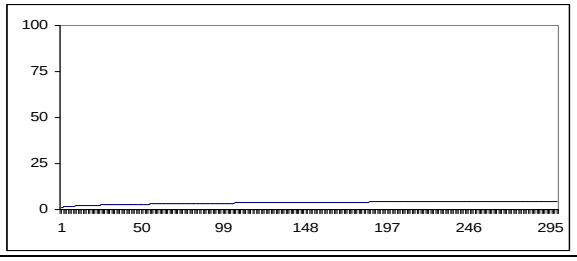
$(1 - L)^{0.50}\, t_t$



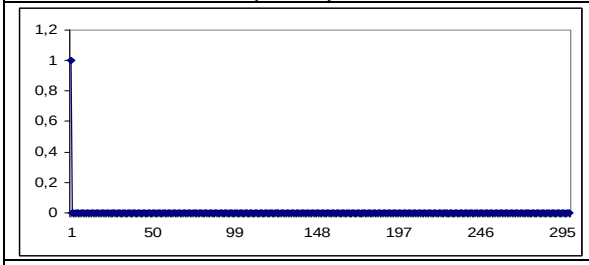
$(1 - L)^{0.75}\, 1_t$



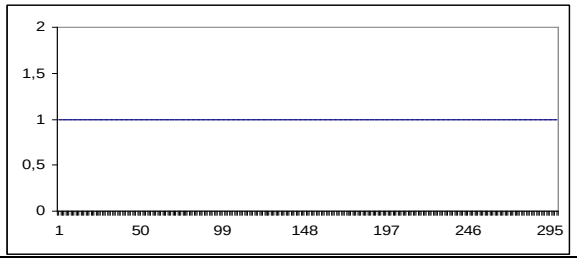
$(1 - L)^{0.75}\, t_t$



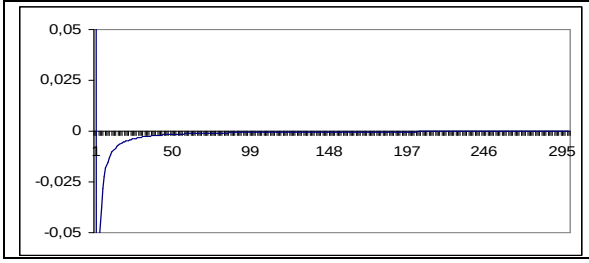
$(1 - L)\, 1_t$



$(1 - L)\, t_t$



$(1 - L)^{1.25}\, 1_t$



$(1 - L)^{1.25}\, t_t$

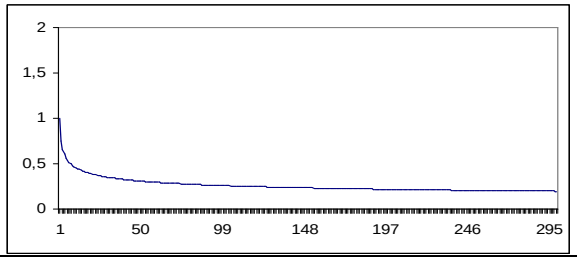


FIGURE 2

Examples of simple realizations with deterministic trends, fractional integration and structural breaks

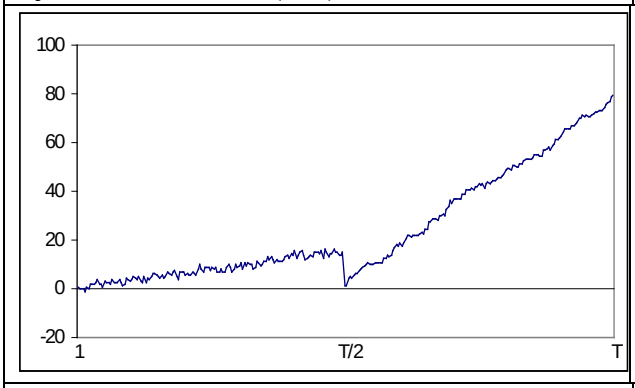
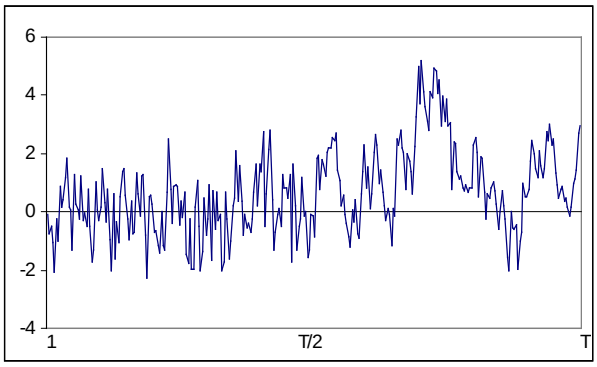
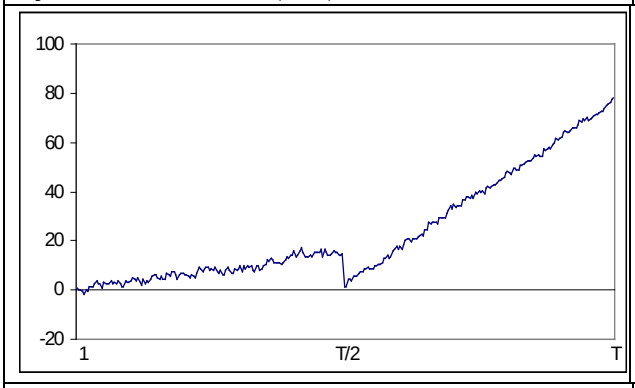
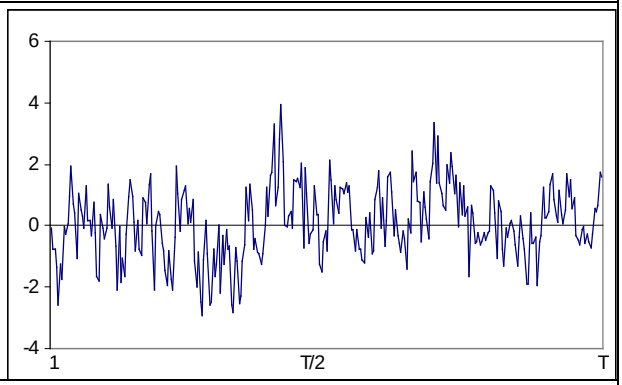
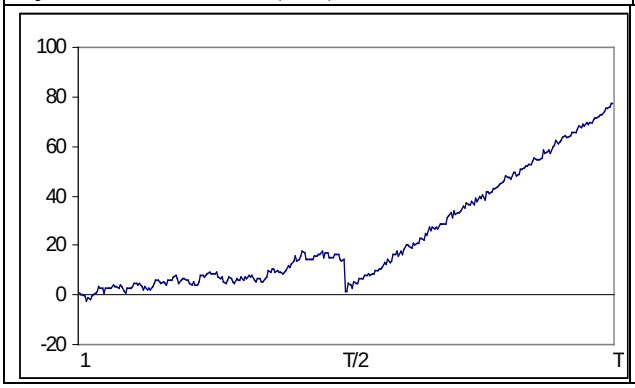
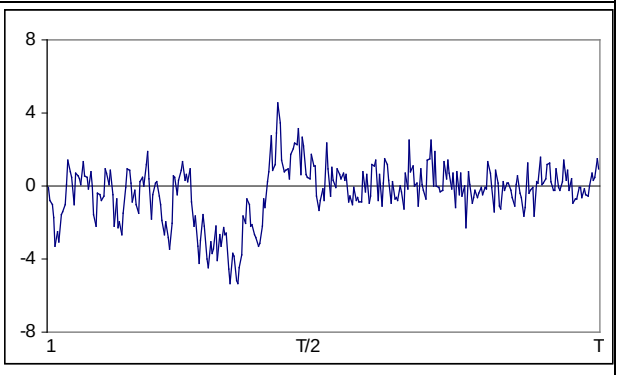
$y_t = 5 + t + x_t$; $(1-L)^{0.2} x_t = \varepsilon_t$ $t = 1, \ldots, 150$ $y_t = 10 + 5t + x_t$; $(1-L)^{0.7} x_t = \varepsilon_t$ $t = 151, \ldots$	$(1-L)^{0.2} x_t = \varepsilon_t$ $t = 1, \ldots, 150$ $(1-L)^{0.7} x_t = \varepsilon_t$ $t = 151, \ldots 300$
	
$y_t = 5 + t + x_t$; $(1-L)^{0.5} x_t = \varepsilon_t$ $t = 1, \ldots, 150$ $y_t = 10 + 5t + x_t$; $(1-L)^{0.5} x_t = \varepsilon_t$ $t = 151, \ldots$	$(1-L)^{0.5} x_t = \varepsilon_t$ $t = 1, \ldots, 150$ $(1-L)^{0.5} x_t = \varepsilon_t$ $t = 151, \ldots 300$
	
$y_t = 5 + t + x_t$; $(1-L)^{0.7} x_t = \varepsilon_t$ $t = 1, \ldots, 150$ $y_t = 10 + 5t + x_t$; $(1-L)^{0.2} x_t = \varepsilon_t$ $t = 151, \ldots$	$(1-L)^{0.7} x_t = \varepsilon_t$ $t = 1, \ldots, 150$ $(1-L)^{0.2} x_t = \varepsilon_t$ $t = 151, \ldots 300$
	

TABLE 1							
Probabilities of detecting the true model with a break at T/2 and $d_1 = 0.50$ and $d_2 = 0.50$							
d_1	d_2	T = 200	500	700	1000	1500	2000
0.1	0.3	0.001	--	--	--	--	--
0.1	0.4	0.002	--	--	--	--	--
0.1	0.5	0.002	--	--	--	--	--
0.2	0.1	0.001	--	--	--	--	--
0.2	0.2	0.001	--	--	--	--	--
0.2	0.3	0.006	--	--	--	--	--
0.2	0.4	0.008	--	--	--	--	--
0.2	0.5	0.002	--	--	--	--	--
0.2	0.6	0.005	--	--	--	--	--
0.3	0.2	0.004	--	--	--	--	--
0.3	0.3	0.020	0.001	--.	--	--	--
0.3	0.4	0.052	0.008	0.001	--	--	--
0.3	0.5	0.054	0.012	0.002	--	--	--
0.3	0.6	0.017	0.001	--	--	--	--
0.3	0.7	0.002	--	--	--	--	--
0.4	0.1	0.001	--	--	--	--	--
0.4	0.2	0.011	--	--	--	--	--
0.4	0.3	0.040	0.005	--	--	--	--
0.4	0.4	0.113	0.094	0.057	0.020	0.003	0.001
0.4	0.5	0.120	0.173	0.173	0.130	0.091	0.044
0.4	0.6	0.037	0.032	0.017	0.006	0.001	0.001
0.4	0.7	0.003	0.001	--	--	--	--
0.5	0.1	0.001	--	--	--	--	--
0.5	0.2	0.017	--	--	--	--	--
0.5	0.3	0.055	0.012	0.002	--	--	--
0.5	0.4	0.102	0.164	0.152	0.126	0.073	0.046
0.5	**0.5**	**0.114**	**0.359**	**0.470**	**0.637**	**0.788**	**0.882**
0.5	0.6	0.043	0.051	0.046	0.029	0.015	0.017
0.5	0.7	0.005	0.001	--	--	--	--
0.6	0.2	0.004	--	--	--	--	--
0.6	0.3	0.017	0.003	--	--	--	--
0.6	0.4	0.040	0.026	0.018	0.006	0.002	0.0-01
0.6	0.5	0.039	0.048	0.058	0.044	0.027	--
0.6	0.6	0.018	0.008	0.004	0.001	--	--
0.7	0.2	0.001	--	--	--	--	--
0.7	0.3	0.001	--	--	--	--	--
0.7	0.4	0.003	0.001	--	--	--	--
0.7	0.5	0.004	--	--	--	--	--
0.7	0.6	0.002	--	--	--	--	--
0.7	0.7	0.001	--	--	--	--	--
0.8	0.6	0.001	--	--	--	--	--

-- means that the probability of choosing the model is 0.

TABLE 2							
Probabilities of detecting the true model with a break at T/2 and d_1 = 0.20 and d_2 = 0.70							
d_1	d_2	T = 200	500	700	1000	1500	2000
0.1	0.3	0.004	--	--	--	--	--
0.1	0.4	0.012	--	--	--	--	--
0.1	0.5	0.066	0.006		--	--	--
0.1	0.6	0.188	0.103	0.057	0.020	0.003	--
0.1	0.7	0.225	0.205	0.179	0.147	0.099	0.048
0.1	0.8	0.078	0.038	0.022	0.006	0.001	--
0.1	0.9	0.006	0.001	--	--	--	--
0.2	0.4	0.015	--	--	--	--	--
0.2	0.5	0.048	0.007	--	--	--	--
0.2	0.6	0.097	0.153	0.140	0.113	0.070	0.035
0.2	**0.7**	**0.097**	**0.355**	**0.484**	**0.632**	**0.781**	**0.917**
0.2	0.8	0.048	0.057	0.048	0.036	0.018	--
0.2	0.9	0.004	0.001	--	--	--	--
0.3	0.4	0.001	--	--	--	--	--
0.3	0.5	0.012	0.001	--	--	--	--
0.3	0.6	0.032	0.023	0.010	0.007	0.004	--
0.3	0.7	0.040	0.043	0.053	0.038	0.024	--
0.3	0.8	0.015	0.006	0.005	0.001	--	--
0.3	0.9	0.002	--	--	--	--	--
0.4	0.4	0.001	--	--	--	--	--
0.4	0.5	0.001	--	--	--	--	--
0.4	0.6	0.002	0.001	--	--	--	--
0.4	0.7	0.004	--	--	--	--	--
0.4	0.8	0.001	--	--	--	--	--
0.5	0.7	0.001	--	--	--	--	--

TABLE 3							
Probabilities of detecting the true model with a break at T/2 and $d_1 = 0.70$ and $d_2 = 0.20$							
d_1	d_2	T = 200	500	700	1000	1500	2000
0.2	0.1	0.001	--	--	--	--	--
0.3	0.1	0.002	--	--	--	--	--
0.3	0.2	0.003	--	--	--	--	--
0.4	0.1	0.030	--	--	--	--	--
0.4	0.2	0.007	--	--	--	--	--
0.4	0.3	0.005	--	--	--	--	--
0.5	0.1	0.075	--	--	--	--	--
0.5	0.2	0.038	0.007	0.001	--	--	--
0.5	0.3	0.015	0.008	--	--	--	--
0.5	042	0.003	0.001	--	--	--	--
0.6	0.1	0.172	0.096	0.049	0.023	0.004	--
0.6	0.2	0.121	0.167	0.159	0.107	0.083	--
0.6	0.3	0.034	0.023	0.015	0.002	0.001	--
0.6	0.4	0.001	0.001	--	--	--	--
0.7	0.1	0.194	0.207	0.182	0.141	0.083	0.055
0.7	**0.2**	**0.119**	**0.336**	**0.457**	**0.641**	**0.781**	**0.907**
0.7	0.3	0.036	0.049	0.044	0.028	0.014	0.011
0.7	0.4	0.005	0.001	--	--	--	--
0.8	0.1	0.069	0.037	0.021	0.010	0.002	--
0.8	0.2	0.040	0.060	0.069	0.047	0.032	0.017
0.8	0.3	0.013	0.006	0.003	0.001	--	--
0.8	0.4	0.005	--	--	--	--	--
0.9	0.1	0.006	0.001	--	--	--	--
0.9	0.2	0.008	--	--	--	--	--
0.9	0.4	0.001	--	--	--	--	--
1.0	0.2	0.001	--	--	--	--	--

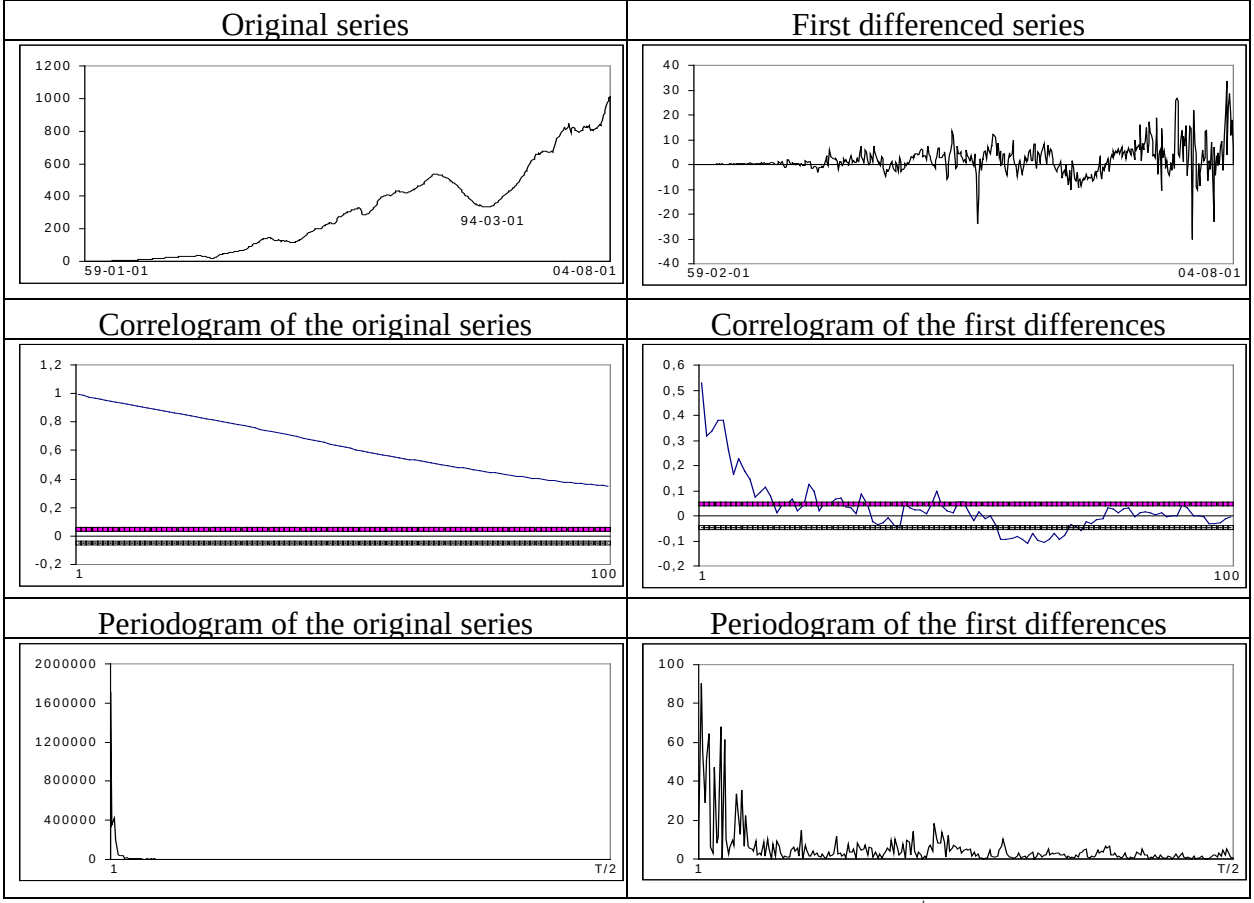
TABLE 4							
Probabilities of detecting the true model with a break at T/4 and $d_1 = 0.20$ and $d_2 = 0.70$							
d_1	d_2	T = 200	500	700	1000	1500	2000
0.1	0.1	0.012	--	--	--	--	--
0.1	0.2	0.012	--	--	--	--	--
0.1	0.3	0.048	--	--	--	--	--
0.1	0.4	0.089	0.012	0.003	--	--	--
0.1	0.5	0.118	0.038	0.016	0.003	--	--
0.1	0.6	0.182	0.176	0.148	0.091	0.054	0.022
0.1	0.7	0.138	0.232	0.212	0.212	0.174	0.160
0.1	0.8	0.102	0.074	0.065	0.040	0.015	--
0.1	0.9	0.030	0.008	0.002	--	--	--
0.1	1.0	0.004	--	--	--	--	--
0.2	0.1	0.001	--	--	--	--	--
0.2	0.2	0.005	--	--	--	--	--
0.2	0.3	0.008	--	--	--	--	--
0.2	0.4	0.019	0.004	0.001	--	--	--
0.2	0.5	0.030	0.036	0.022	0.011	--	--
0.2	0.6	0.040	0.120	0.137	0.151	0.118	0.022
0.2	**0.7**	**0.034**	**0.136**	**0.250**	**0.354**	**0.530**	**0.896**
0.2	0.8	0.016	0.047	0.052	0.064	0.047	--
0.2	0.9	0.006	0.003	0.004	--	--	--
0.2	1.0	0.001	--	--	--	--	--
0.3	0.2	0.001	--	--	--	--	--
0.3	0.3	0.007	--	--	--	--	--
0.3	0.4	0.004	0.001	0.001			--
0.3	0.5	0.020	0.009	0.001	0.001		--
0.3	0.6	0.020	0.044	0.027	0.021	0.014	--
0.3	0.7	0.014	0.039	0.047	0.043	0.044	--
0.3	0.8	0.009	0.014	0.010	0.008	0.003	--
0.4	0.2	0.001	0.001	--	--	--	--
0.4	0.3	0.004	--	--	--	--	--
0.4	0.5	0.003	--	--	--	--	--
0.4	0.6	0.005	--	--	--	--	--
0.4	0.7	0.006	0.003	0.001	0.001	--	--
0.4	0.8	0.004	0.003	0.001	--	--	--
0.4	0.9	0.002	--	--	--	--	--
0.4	1.0	0.001	--	--	--	--	--
0.5	0.6	0.003	--	--	--	--	--
0.5	0.7	0.001	--	--	--	--	--

TABLE 5							
Probabilities of detecting the true model with a break at 9T/10 and $d_1 = 0.70$ and $d_2 = 0.20$							
d_1	d_2	T = 200	500	700	1000	1500	2000
0.5	0.1	0.034	--	--	--	--	--
0.5	0.2	0.004	--	--	--	--	--
0.5	0.3	0.002	--	--	--	--	--
0.5	0.4	0.001	--	--	--	--	--
0.5	0.6	0.001	--	--	--	--	--
0.6	0.1	0.300	0.108	0.064	0.024	0.012	--
0.6	0.2	0.012	0.034	0.060	0.019	0.008	--
0.6	0.3	0.007	0.012	0.10	0.003	0.002	--
0.6	0.4	0.010	0.005	0.001	--	--	--
0.6	0.5	0.004	0.001	--	--	--	--
0.6	0.6	0.001	0.001	--	--	--	--
0.7	0.1	0.410	0.549	0.388	0.324	0.280	0.102
0.7	**0.2**	**0.034**	**0.140**	**0408**	**0.515**	**0.545**	**0.887**
0.7	0.3	0.021	0.059	0.063	0.084	0.148	0.011
0.7	0.4	0.009	0.028	0.004	0.012	--	--
0.7	0.5	0.007	0.003	--	--	--	--
0.7	0.6	0.004	--	--	--	--	--
0.7	0.7	0.002	--	--	--	--	--
0.7	0.8	0.001	--	--	--	--	--
0.8	0.1	0.118	0.046	0.026	0.010	0.003	--
0.8	0.2	0.005	0.009	0.024	0.008	0.001	--
0.8	0.3	0.001	0.002	0.003	0.001	0.001	--
0.8	0.4	0.002	0.003	--	--	--	--
0.8	0.6	0.003	--	--	--	--	--
0.8	0.8	0.001	--	--	--	--	--
0.9	0.1	0.002	--	--	--	--	--
0.9	0.4	0.002	--	--	--	--	--

TABLE 6						
Probabilities of detecting the true break fraction for different deterministic patterns						
(α_1, β_1, α_2, β_2)	T = 100	T = 150	T = 200	T = 250	T = 300	T = 500
(5.0, 1.0, 10.0,	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
(0.5, 0.1, 1.0,	5.51%	19.98%	47.12%	72.63%	95.76%	100.00%
(0.5, -1.0, -1.0,	10.92%	38.37%	62.16%	78.97%	98.21%	100.00%
(-0.5, 1.0, 1.0, -	10.99%	39.25%	63.80%	79.06%	98.36%	100.00%

FIGURE 3

Original time series and first differences, with the correlograms and periodograms

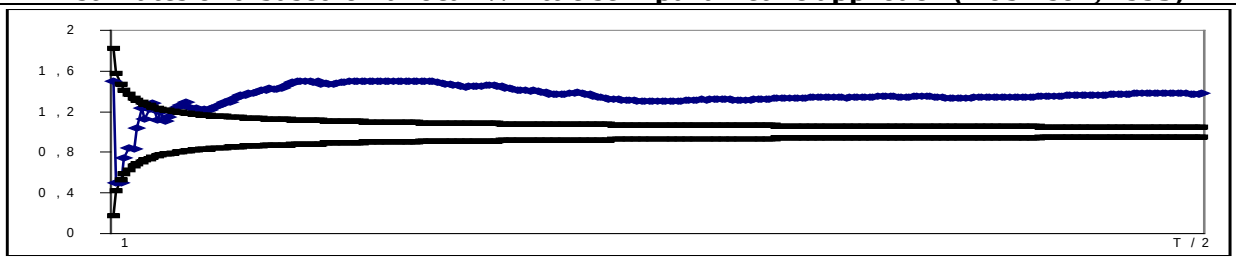


The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly ±0.046.

TABLE 7		
95%-confidence intervals of the non-rejection values of d using Robinson's (1994) procedure		
Type of disturbances	With no regressors	With a time trend
White noise	[1.34 (1.39) 1.45]	[1.34 (1.39) 1.45]
AR(1)	[1.24 (1.31) 1.39]	[1.24 (1.31) 1.39]
AR(2)	[1.35 (1.47) 1.58]	[1.37 (1.47) 1.58]
Bloomfield (1)	[1.21 (1.31) 1.37]	[1.22 (1.32) 1.38]
Bloomfield (2)	[1.32 (1.41) 1.54]	[1.31 (1.40) 1.54]

FIGURE 4

Estimates of d based on a local Whittle semiparametric approach (Robinson, 1995)

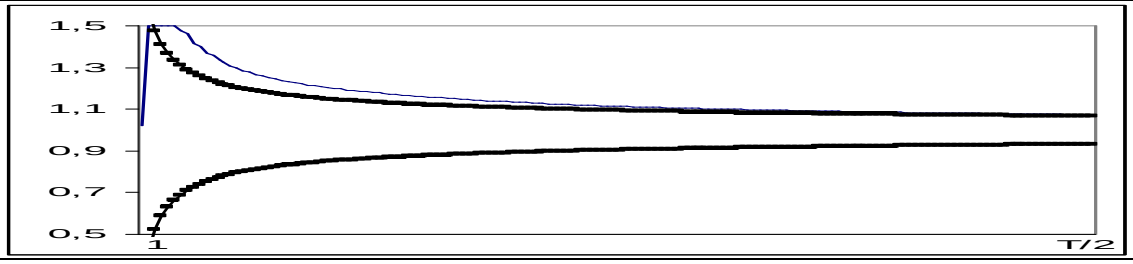


The horizontal axes corresponds to the bandwidth parameter number m, while the vertical one refers to the order of integration

TABLE 8											
Rejection probabilities of Robinson's (1994) parametric procedure with a single											
u_t / d_o	1.00	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20
No regressors	1.000	0.504	0.000	0.000	0.000	0.000	0.043	0.961	1.000	1.000	1.000
A linear trend	1.000	0.637	0.000	0.000	0.000	0.000	0.004	0.694	1.000	1.000	1.000

FIGURE 5

Estimates of d based on a Whittle semiparametric approach (Robinson, 1995) with



The horizontal axes corresponds to the bandwidth parameter number m, while the vertical one refers to the order of integration

FIGURE 6

Log of the U.S. Consumer Price Index

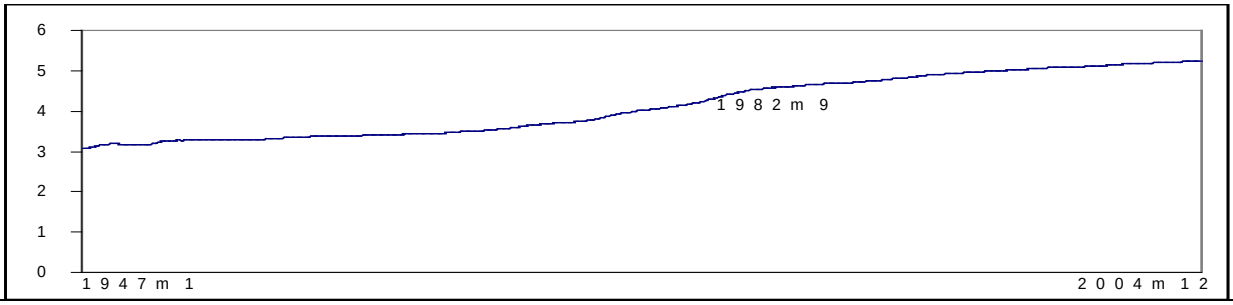


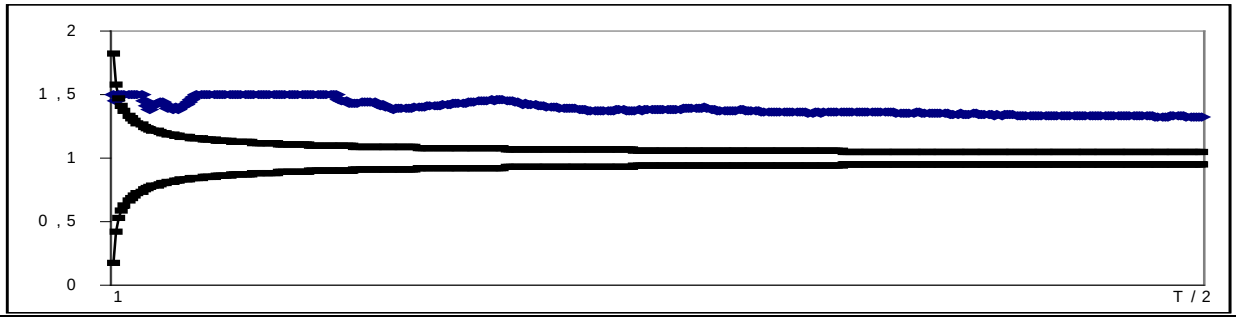
TABLE 9

95%-confidence intervals of the non-rejection values of d using Robinson's (1994) parametric procedure

Type of disturbances	With no regressors	With a linear trend
White noise	[0.95 (1.00) 1.05]	[1.32 (1.35) 1.39]
AR(1)	[1.31 (1.39) 1.49]	[1.37 (1.42) 1.48]
AR(2)	[0.44 (1.57) 1.62]	[1.35 (1.43) 1.52]
Bloomfield (1)	[0.92 (0.99) 1.08]	[1.37 (1.42) 1.50]
Bloomfield (2)	[0.89 (1.02) 1.11]	[1.37 (1.41) 1.52]

FIGURE 7

Estimates of d based on a local Whittle semiparametric approach (Robinson, 1995)



The horizontal axes corresponds to the bandwidth parameter number m, while the vertical one refers to the order of integration.