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Long run and cyclical strong dependence in macroeconomic time series. Nelson and Plosser revisited

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ABSTRACT

This paper deals with the presence of long range dependence at the long run and the cyclical frequencies in macroeconomic time series. We use a procedure that allows us to test unit roots with fractional orders of integration in raw time series. The tests are applied to an extended version of Nelson and Plosser's (1982) dataset, and the results show that, though the classic unit root hypothesis cannot be rejected in most of the series, fractional degrees of integration at both the zero and the cyclical frequencies are plausible alternatives in some cases. Additionally, the root at the zero frequency seems to be more important than the cyclical one for all series, implying that shocks affecting the long run are more persistent than those affecting the cyclical part. The results are consistent with the empirical fact observed in many macroeconomic series that the long-term evolution is nonstationary, while the cyclical component is stationary.

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I. Introduction

Modelling macroeconomic time series is an area of research that has been widely investigated during the last twenty years. Many authors use a decomposition of the series into a seasonal movement, representing the persistent fluctuation of the series over the seasons, a trend movement, dealing with the long-run evolution, the business cycle movement, and an erratic component. With respect to the long run behaviour, unit root models have been widely employed. However, the unit root approach is merely one of the many models that can produce, via differencing, stationary series. In fact, it can be viewed as a particular case of a much more general class of models called I(d) (or fractionally integrated), where d can be any real number. These models assume that taking the d-difference of the data, the resulting series are I(0).[1] For empirical applications of I(d) models see, for example, the papers of Diebold and Rudebusch (1989), Baillie (1996) and Gil-Alana and Robinson (1997). The latter authors examined an extended version of Nelson and Plosser's (1982) dataset in terms of I(d) statistical models, and they came to the conclusion that fractional models may be plausible alternatives to the classical I(1) representations for these series.

The I(d) models can be interpreted as processes with a spectral density function that is unbounded at the origin but positive and finite at any other frequency. In other words, though they are useful to describe the time series dependence between the observations, they do not take into account the possibility of long memory at, for example, the cyclical frequencies.

The present paper extends earlier work by adopting a modelling approach which, instead of considering exclusively the component affecting the long-run frequency, also takes into account the cyclical structure. Using a large structure that involves simultaneously the zero and the cyclical frequencies, we can solve at least to some extent the problem of misspecification that may arise with respect to these two frequencies. We show that our proposed method represents an appealing alternative to the increasingly popular ARIMA (ARFIMA) specifications. It is also consistent with the widely adopted practice of modelling many economic series as two components, namely a secular or growth component and a cyclical one. The former, assumed in most cases to be nonstationary, is thought to be driven by growth factors, such as capital accumulation, population growth and technology improvements, whilst the latter, assumed to be covariance stationary, is generally associated with fundamental factors which are the primary cause of movements in the series.

The article is structured as follows: Section II describes the model of interest and its implications in terms of economic policy and planning. Section III briefly describes the procedure that allows us to test the model. In Section IV we include some Monte Carlo simulations, examining the size and the power properties of the tests in finite samples. In Section V we apply the tests to an extended version of Nelson and Plosser's (1982) dataset.

II. The statistical model

We assume that $\{x_t, t = 1, 2, \ldots, T\}$ is the observed time series, generated by the model:

$$(1 - L)^{d_1} \ (1 - 2\cos w\, L + L^2)^{d_2} \ x_t \ = \ u_t, \qquad t \ = \ 1, 2, .., \qquad (1)$$

where L is the lag operator ($Lx_t = x_{t-1}$ for all t), w is a given real number, u_t is I(0), and d_1 and d_2 can be real numbers. We first consider the case when $d_2 = 0$. Then, if $d_1 > 0$, the process is said to be long memory at the long run or zero frequency, also termed 'strong dependent', because of the strong association between observations widely separated in time. Note that the first polynomial in the left-hand-side of (1) can be expressed in terms of its Binomial expansion, such that for all real d_1,

$$(1-L)^{d_1} = \sum_{j=0}^{\infty} \binom{d_1}{j} (-1)^j L^j = 1 - d_1 L + \frac{d_1(d_1-1)}{2} L^2 - \dots . \qquad (2)$$

This type of process was introduced by Granger (1980, 1981) and Hosking (1981) and it was theoretically justified in terms of aggregation by Robinson (1978), Granger (1980) and more recently, in terms of the duration of shocks by Parke (1999). The differencing parameter d_1 plays a crucial role from both economic and statistical viewpoints. Thus, if $d_1 \in (0, 0.5)$, the series is covariance stationary and mean-reverting, having autocovariances which decay much more slowly than those of an ARMA process, in fact, so slowly as to be non-summable; if $d_1 \in [0.5, 1)$, the series is no longer stationary but it is still mean-reverting, with the effect of the shocks disappearing in the long run; while $d_1 \geq 1$ means nonstationarity and non-mean-reversion. It is therefore crucial to examine if d_1 is smaller than or equal to or larger than 1. We now consider the case of $d_1 = 0$ and $d_2 > 0$. The process is then said to be long memory at the cyclical part. It was examined by Gray et al. (1989, 1994), and they showed that the series is stationary if $|\cos w| < 1$ and $d < 0.50$ or if $|\cos w| = 1$ and $d < 0.25$. They also showed that the second polynomial in (1) can be expressed in terms of the Gegenbauer polynomial C_{j,d_2}, such that, calling $\mu = \cos w$,

$$(1 - 2\mu L + L^2)^{-d_2} = \sum_{j=0}^{\infty} C_{j,d_2}(\mu) L^j , \qquad (3)$$

for all $d_2 \neq 0$, where

$$C_{j,d_2}(\mu) = \sum_{k=0}^{[j/2]} \frac{(-1)^k (d_2)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}; \qquad (d_2)_j = \frac{\Gamma(d_2+j)}{\Gamma(d_2)},$$

where Γ(x) represents the Gamma function and a truncation will be required in (3) to make the polynomial operational. When $d_2 = 1$, we say that the process contains a unit root cycle, and its performance in the context of macroeconomic time series was examined by Bierens (2001).

III. The testing procedure

Robinson (1994) considers a model of the form:

$$y_t = \beta' z_t + x_t \qquad t = 1, 2, ..., \tag{4}$$

where y_t is a given raw time series; z_t is a (kx1) vector of exogenous variables; β is a (kx1) vector of unknown parameters; and the regression errors x_t are of form as in (1). The null hypothesis is

$$H_o: \quad d \equiv (d_1, d_2,)' = (d_{1o}, d_{2o})' \equiv d_o \tag{5}$$

where d_{1o} and d_{2o} may be real values and thus, equation (1) becomes:

$$(1 - L)^{d_{1o}} \; (1 - 2\cos wL + L^2)^{d_{2o}} \; x_t = u_t, \qquad t = 1, 2, \ldots . \tag{6}$$

Clearly, d_{1o} corresponds to the order of integration at the zero frequency, while d_{2o} refers to the degree of integration affecting the cyclical part. Additionally, we can take $w = w_r = 2\pi/r$, r = 2, … , T/2, where r means the number of periods required to complete the whole cycle.[2]

We next describe the test statistic. Based on H_o (5), the differenced series is given by:

$$\hat{u}_t = (1 - L)^{d_{1o}} (1 - 2\cos wL + L^2)^{d_{2o}} \; y_t - \hat{\beta}' s_t, \tag{7}$$

$$\hat{\beta} = \left(\sum_{t=1}^{T} s_t \, s_t' \right)^{-1} \sum_{t=1}^{T} s_t (1-L)^{d_{1o}} \, (1 - 2\cos w L \, + \, L^2)^{d_{2o}} \; y_t \; ,$$

with $s_t \; = \; (1-L)^{d_{1o}} (1 - 2\cos w L + L^2)^{d_{2o}} \; z_t$, and it is assumed to have spectral density:

$$f(\lambda;\tau) \; = \; \frac{\sigma^2}{2\pi} \; g(\lambda;\tau), \qquad -\pi < \lambda \; \leq \; \pi \, ,$$

where the scalar σ^2 is known and g is a function of known form, which depends on frequency λ and the unknown (qx1) parameter vector τ. Unless g is a completely known function (e.g., $g \equiv 1$, as when u_t is white noise), we have to estimate the nuisance parameter τ, for example by $\hat{\tau} \; = \; \arg\min_{\tau \in T^*} \; \sigma^2(\tau)$, where T^* is a suitable subset of the R^q Euclidean space, and

$$\sigma^2(\tau) = \frac{2\pi}{T} \sum_{s=1}^{T-1} g(\lambda_s;\tau)^{-1} \, I_{\hat{u}}(\lambda_s), \qquad \text{with}$$

$$I_{\hat{u}}(\lambda_s) = \left| (2\pi T)^{-1/2} \sum_{t=1}^{T} \hat{u}_t \, e^{i \lambda_s t} \right|^2 ; \;\; \lambda_s = \frac{2\pi s}{T} .$$

Note that the tests are purely parametric, requiring specific assumptions regarding the short memory specification of u_t. Thus, for example, if u_t is an AR process of form: $\phi(L)u_t = \varepsilon_t$, then, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ.

The test statistic, which is derived via Lagrange Multiplier principle, adopts the form:

$$\hat{R} \; = \; \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a} \, , \qquad (8)$$

where T is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_s^* \psi(\lambda_s)\, g(\lambda_s; \hat{\tau})^{-1}\, I(\lambda_s)\,; \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{s=1}^{T-1} g(\lambda_s; \hat{\tau})^{-1}\, I(\lambda_s)\,;$$

$$\hat{\varepsilon}(\lambda_s) = \frac{\partial}{\partial \tau} \log g(\lambda_s; \hat{\tau})$$

$$\hat{A} = \frac{2}{T} \left(\sum_s^* \psi(\lambda_s)\, \psi(\lambda_s)' - \sum_s^* \psi(\lambda_s)\, \hat{\varepsilon}(\lambda_s)' \left(\sum_s^* \hat{\varepsilon}(\lambda_s)\, \hat{\varepsilon}(\lambda_s)' \right)^{-1} \sum_s^* \hat{\varepsilon}(\lambda_s)\, \psi(\lambda_s)' \right)$$

$$\psi(\lambda_s)' = [\psi_1(\lambda_s), \psi_2(\lambda_s)]; \qquad \psi_1(\lambda_s) = \log \left| 2 \sin \frac{\lambda_s}{2} \right|;$$

$$\psi_2(\lambda_s) = \log \left| 2 \left(\cos \lambda_s - \cos w \right) \right|;$$

and the summation on * in the above expressions is over $\lambda \in M$ where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_k - \lambda_1, \rho_k + \lambda_1), k = 1, 2, \ldots, s\}$ such that ρ_k, $k = 1, 2, \ldots, s$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$. Based on H_o (5), Robinson (1994) established that, under certain regularity conditions:[3]

$$\hat{R} \to_d \chi_2^2, \qquad as \quad T \to \infty. \tag{9}$$

Thus, we are in a classical large-sample testing situation. A test of (5) will reject H_o against the alternative H_a: $d \neq d_o$ if $\hat{R} > \chi^2_{2,\alpha}$, where Prob $(\chi_2^2 > \chi^2_{2,\alpha} | H_o) = \alpha$. Moreover the test is efficient in the Pitman sense against local departures from the null, that is, if the test is directed against local departures of the form: H_a: $\theta = \delta T^{-1/2}$, for $\delta \neq 0$, the limit distribution is a $\chi_2^2(v)$, with a non-centrality parameter v, that is optimal under Gaussianity of u_t.

There exist other procedures for estimating and testing the fractionally differenced parameters. Ooms (1997) proposed tests based on seasonal fractional models. Also, Hosoya (1997) established the limit theory for long memory processes with the singularities not restricted at the zero frequency, and proposed a set of quasi log-likelihood statistics to be applied to raw time series. As in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives will have the same null and local limit theory as the LM tests of Robinson (1994).

IV. A Monte Carlo simulation study

The first thing we do is to compute finite-sample critical values of the version of the tests of Robinson (1994) described in Section III. We generate Gaussian series obtained by the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986), with 10,000 replications of each case, and compute the empirical distribution of the tests for sample sizes T = 100, 200, 300, 500 and 1000 and nominal sizes of 10%, 5% and 1%. Note that the empirical distribution is numerically invariant to the orders of integration, since the test statistic is computed based on the null differenced model, which is supposed to be I(0). However, it will take different sample values for each w_r, since this parameter appears in the specification of the test statistic, via $\psi_2(\lambda)$. We computed the test statistic given by $\hat{R}$ in (8), testing H_o (5) in a model given by (1) with white noise u_t, $w = w_r$, and values of r = 3, 4, 5, 6, 7 and 8.

(Insert Table 1 about here)

We observe that the finite-sample critical values slightly vary across r. If T = 100, they are greater than those corresponding to the χ_2^2 distribution, however, increasing the sample size, they approximate (non-monotonically) to the standard values of the χ_2^2 distribution, and, if T = 1000 the values are very close to the χ_2^2-distribution in all cases. Table 2 examines the size and the power properties of the tests. We assume that the true model is given by (1) with $d_1 = d_2 = 1$, white noise u_t, $w = w_r$, and r = 6. The choice of r is arbitrary. We tried other values of r and the results were very similar to those reported in the table. The alternatives are in all cases fractional of form as in (6), with d_{1o}, d_{2o} = 0.50 (0.25), 1.50, with r still equal to 6. That is, we try all possible combinations from (0.50, 0.50) to (1.50, 1.50) with 0.25 increments. The rejection frequencies corresponding to d_{1o}, = d_{2o} = 1 will then indicate the sizes of the tests. The nominal size is 5%, 10,000 replications were used in each case, and we compute the rejection probabilities based on both the asymptotic and the finite sample critical values.

(Insert Tables 2 and 3 about here)

We see that the sizes of the asymptotic tests are in all cases too large though they approximate to the nominal value of 5% with T. The larger size of the asymptotic tests is also associated with some superior rejection frequencies relative to the finite sample tests. However, we observe that even if the sample size is 100, the rejection probabilities are relatively high for both tests, exceeding 0.500 in practically all cases. Increasing the sample size, the rejection frequencies become higher, and if T = 300, they are close to 1 for all types of alternatives.

In the following table, we examine if the tests are sensitive to the choice of r. Table 3 reports the rejection frequencies of $\hat{R}$ in (8), testing the null of two unit roots (i.e., d_{1o}, = d_{2o} = 1) for values r = 3, 4, 5, 7, 8 and 9, in a true model where $d_{1o} = d_{2o} = 1$ and r = 6. We see that if T = 100, the rejection probabilities are low with r = 4 and 5. If T = 200, the values are around 0.750 with r = 5, and if T = 300, they are practically 1 for all r. Thus, we can conclude by saying that, though there is some bias toward small r in small samples, the tests have enough power to detect the correct choice of r, especially if T is large.

V. The analysis of Nelson and Plosser's (1982) dataset

The extended version of the annual data set of fourteen US macroeconomic variables analysed by Nelson and Plosser (1982) ends in 1988. The starting date is 1860 for consumer price index and industrial production; 1869 for velocity; 1871 for stock prices; 1889 for GNP deflator and money stock; 1890 for employment and unemployment rate; 1900 for bond yield, real wages and wages; and 1909 for nominal and real GNP and GNP per capita. As Nelson and Plosser (1982), all series except the bond yield (interest rate) are transformed to natural logarithms.

Gil-Alana and Robinson (1997) examined exactly the same dataset. However, they exclusively concentrated on the zero frequency and did not pay any attention to the possible cyclical structure underlying the series. Across Tables 1 and 2 in that paper the authors displayed the first fourteen sample autocorrelations of the original series and their first differences. In the latter table, they obtained significant values, especially at lag 1, but also values with some slow decay and/or cyclical oscillation in some cases, which could be indicative not only of fractional integration but also of some cyclical dependence.

Denoting each of the series by y_t, we employ throughout the model given by (4) and (1) with $z_t = (1,t)'$, $t \geq 1$, $(0, 0)'$ otherwise. Thus, under H_o (5), the model becomes:

$$y_t \;=\; \beta_0 \;+\; \beta_1 t \;+\; x_t, \qquad t \;=\; 1, 2, \ldots \tag{10}$$

$$(1 - L)^{d_{1o}} \;(1 - 2\cos w\, L + L^2)^{d_{2o}}\; x_t \;=\; u_t, \quad t \;=\; 1, 2, \ldots., \tag{11}$$

and if $d_{2o} = 0$, the model reduces to the case of long memory exclusively at the long run or zero frequency. We consider separately the cases of $\beta_0 = \beta_1 = 0$ a priori, (i.e., including no regressors in the undifferenced model (10)); β_0 unknown and $\beta_1 = 0$ a priori, (i.e., with an intercept), and β_0 and β_1 unknown (with an intercept and with a linear time trend), and assume that $w = w_r = 2\pi/r$, r indicating the number of time periods per cycle.

We computed the statistic $\hat{R}$ given by (8) for values d_{1o} and d_{2o} = 0, (0.01), 2, and r = 2, …, T/2, assuming that u_t is white noise. In other words, for each r, we compute the test statistic for all possible combinations of d_1 and d_2, with 0.01 increments. We do not report, however, the results for all statistics, though it was obtained that the null hypothesis (5) was rejected for all values of d_{1o} and d_{2o} if r was smaller than 4 or higher than 7, implying that if a cyclical component is present, its periodicity is constrained between these two years. This is consistent with the empirical findings in Canova (1998), Burnside (1998), King and Rebelo (1999) and others that cycles occur between 3 and 8 years.

(Insert Figure 1 about here)

Figure 1 displays the (d_{1o}, d_{2o}) combinations where H_o (5) cannot be rejected at the 5% significance level, with r = 6 and $\beta_0 = \beta_1 = 0$. We see in this figure that the results substantially vary across the series. Thus, for example, starting with consumer prices, we see that the non-rejection values are constrained between 0.9 and 1.3 for d_{1o} and between 0 and 0.40 for d_{2o}. For GNP deflator, industrial production, S&P500 and money stock, d_{1o} still ranges between 0.75 and 1.5 while d_{2o} is now between 0 and 0.70. There are three series (interest rates, unemployment and velocity), with d_{1o} around the unit root, and with d_{2o} widely varying from 0 to 1.2 (interest rate); from 0 to 1.5 (unemployment), and from 0 to 2 in case of velocity.

(Insert Figure 2 about here)

For the remaining six series, (employment, wages, real wages, and nominal, real and real per capita GNP), the results (not reported) were less conclusive, obtaining two unconnected sets of non-rejection values for each series. This may be an indication of model misspecification. Thus, we also performed the tests, including an intercept, and with an intercept and a linear time trend. The results for these series were similar in both cases, and we display in Figure 2 those corresponding to the case of an intercept.[4] The results were here much more conclusive, with the non-rejection values of d_{1o} and d_{2o} forming a single compact set for each series. For wages, nominal, real and real per capita GNP, d_{1o} moves between 0.80 and 1.75, with d_{2o} ranging between 0 and 0.50. For real wages and employment, the unit root at the zero frequency is excluded in favour of higher orders of integration, with d_{1o} ranging between 1.05 and 1.75 and d_{2o} between 0 and 0.5. However, for these two series the intercept was found to be statistically insignificant across all non-rejected models, and thus, in what follows, we rely for employment and real wages on the results based on β_0 and $\beta_1 = 0$. Also, it is important to note here that, with respect to the cyclical frequency, the null hypothesis cannot be rejected in any series with $d_{2o} = 0$. However, this case is in many cases "less clearly non-rejected" (in the sense that they display lower p-values) than with positive values of d_{2o}.

Table 4 shows, for each series, the values of d_{1o} and d_{2o} that produce the lowest statistics across the d's, for a given r = 6 and β_0 and $\beta_1 = 0$ a priori in case of the eight series presented in Figure 1, plus employment and real wages, and β_0 unknown and $\beta_1 = 0$ for the remaining four series in Figure 2. These values should approximate to the maximum likelihood estimates.[5] We observe that, only for CPI and money stock, d_{1o} is higher than 1. It is exactly 1 for stock prices, and it is strictly below 1 (and thus showing mean reversion) for the remaining series. With respect to the cyclical component, the values are exactly 0 for half of the series (nominal, real, real per capita GNP, GNP deflator, CPI, stock prices and employment), and the highest values are obtained in the cases of interest rate (d_{2o} = 0.07), industrial production index (0.08) and unemployment (0.11). Of particular interest is the case of the unemployment rate: it presents the lowest degree of integration at the zero frequency (d_{1o} = 0.84) and the highest one at the cyclical frequency (d_{2o} = 0.11). The results presented across this section show little evidence of fractional integration in the Nelson and Plosser's (1982) dataset. Thus, the null hypothesis of a unit root (i.e., d_1 = 1, d_2 = 0) is practically never rejected, though fractional degrees of integration at both the zero and the cyclical frequencies seem to be plausible alternatives in some of the series.

(Insert Tables 4 and 5 about here)

Table 5 reports the first 20 impulse responses for each of the selected models in Table 4. These values were obtained through the lag polynomials in (2) and (3), noting that x_t in (1) can be expressed as

$$x_t \;=\; (1-L)^{-d_1}\,(1-2\cos w\,L+L^2)^{-d_2}\,u_t, \qquad t \;=\; 1,2,..,$$

and thus,

$$x_t \;=\; \left(\sum_{j=0}^{\infty} a_j L^j\right)\left(\sum_{j=0}^{\infty} b_j L^j\right)u_t \;=\; \sum_{j=0}^{\infty} c_j u_{t-j}, \tag{12}$$

where the c_j are obtained using all the linear combinations in the two polynomials above. Note, however, that since we have a single innovation term, we cannot use the interpretation based on a zero-cyclical frequency decomposition with different shocks, but it allows us to examine the effect that a shock has on the system. As expected, all series are highly persistent, though all except three of them (CPI, stock prices and money stock) present mean reverting behaviour, with shocks disappearing in the very long run.

We observe that even 20 periods after the initial shock, 90% of the effect remains in most of the series; 75% in case of velocity and real wages, and around 50% for unemployment and industrial production index.

(Insert Figure 3 about here)

In spite of the fact that we cannot separate the effects from zero and the cyclical frequencies in terms of the impulse responses since we use a unique innovation term, we can still consider, ceteris paribus, each of the effects separately. Figure 3 displays the plots of the impulse responses for the joint effect (c_j) and for each component (a_j and b_j) separately, for a 50-period horizon, in the four series with a potential fractional degree of cyclical behaviour: unemployment rate, industrial production index, interest rate and velocity). It is observed that the effect of the stationary cyclical frequency is very small compared with the long-term evolution, and it becomes negligible 10 periods after the shock.

The tests were also performed allowing autocorrelated disturbances. In particular, we use AR(1) and AR(2) processes, and the results were practically the same as those reported here for the case of white noise u_t. Attempting to summarize the conclusions, we are left with the impression that the I(1) hypothesis advocated by Nelson and Plosser (1982) cannot statistically be rejected in most of the series, though for some of them the zero and the cyclical frequencies have a component of long memory behaviour. Also, the order of integration seems to be higher at the long run or zero frequency than at the cyclical one, implying that shocks affecting the former component are more persistent than those affecting the cyclical part. For the zero frequency, these values fluctuate around the unit root in all series, being possibly higher than 1 for CPI and money stock. For the cyclical part, d_2 ranges between 0 and 0.5 for most of the series, implying that cycles are stationary. These values are in some cases 0 or close to 0, and the highest values are obtained for unemployment, industrial production index, interest rate and velocity.

VI. Concluding comments

In this paper we have presented a procedure for simultaneously consider roots with integer and fractional orders of integration at the long run and the cyclical frequencies. The tests are very general and allow us to consider as particular cases the situations of unit roots either at zero or at the cyclical components. However, unlike other procedures, they have standard null and local limit distributions. A simulation study was conducted to examine the size and the power properties of the tests in finite samples, the results showing that they behave relatively well even with small sample sizes.

The tests were applied to an extended version of Nelson and Plosser's (1982) dataset. These series were also examined by Crato and Rothman (1994) and Gil-Alana and Robinson (1997). However, in these two papers the authors exclusively concentrate on the long run or zero frequency and do not pay any attention to the cyclical structure underlying the series. Using our approach, the results substantially vary across the series. However, a common pattern is obtained for all of them, with values of d_{1o} ranging around 1 and d_{2o} constrained to be 0 or slightly above. Thus, we obtain evidence of long memory with respect to the long run frequency and also in some cases with respect to the cyclical frequency, though the root at the zero frequency seems to present a higher degree of integration, with shocks persisting forever.

It should also be important to stress that the existence of unit roots in most of the series implies a stochastic trend and thus, the model can be alternatively written in the form of an orthogonal zero-cyclical frequency decomposition with an ARMA cycle, which does not exhibit long memory, especially in those series with d_{2o} equal to or close to 0. This specification is not nested in the model presented here, but it might be an alternative way of modelling its behaviour. Finally, the issue of data mining is another worry for economists when looking at time series models. There are so many possible models that may be relevant and so many modelling choices that econometricians are almost sure to find something purely by data mining. For this reason, sequential testing and other procedures based on information criteria are widely distrusted, and model averaging methods have become very popular. Thus, it might also be worthwhile to broaden the class of models under consideration and address the data mining problem, along with other issues (e.g., structural breaks) using averaging approaches. Work in these directions is now under progress.

Notes

1. We define an I(0) process as a covariance stationary process with spectral density function that is positive and finite at any frequency on the spectrum.

2. Note that if r = 1, the cyclical part reduces to an I(d) process, with the singularity occurring exclusively at the long run or zero frequency.

3. These conditions are very mild, and impose a martingale difference assumption on u_t, which is substantially weaker than Gaussianity.

4. In fact, the inclusion of a linear time trend was found to be insignificant in practically all cases. Note that the tests are evaluated under the null, which is I(0) and thus, standard t-tests apply.

5. Note that Robinson's (1994) procedure is based on the Whittle function, which is an approximation to the likelihood function.

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TABLE 1							
Finite sample critical values							
α%	T / r	3	4	5	6	7	8
10%	100	5.47	4.04	4.36	4.60	4.26	4.24
	200	4.87	4.03	4.16	4.54	4.23	4.10
	300	4.05	4.19	4.22	4.27	4.40	4.46
	500	4.79	4.39	4.24	4.56	4.54	4.65
	1000	4.63	4.61	4.65	4.60	4.68	4.61
5%	100	6.78	5.70	5.83	6.22	5.84	5.63
	200	6.17	5.48	5.09	5.60	5.61	5.50
	300	5.16	5.27	5.35	5.34	5.36	5.45
	500	6.04	5.44	5.46	5.69	5.65	5.75
	1000	5.93	5.96	5.97	5.94	5.90	6.00
1%	100	9.35	9.19	9.22	11.16	8.71	9.09
	200	8.88	8.95	7.93	9.22	9.84	9.43
	300	7.62	9.64	7.86	7.34	8.19	8.45
	500	10.32	9.10	8.76	9.66	8.95	10.27
	1000	9.24	9.32	9.19	9.19	9.17	9.22

10,000 replications were used in each case. The critical values corresponding to the χ_2^2 – distribution are 4.605, 5.991 and 9.210 at the 10%, 5% and 1% significance levels respectively.

TABLE 2							
Rejection frequencies against fractionally integrated alternatives							
d_1	d_2	T = 100		T = 200		T = 300	
		FSCV	ASCV	FSCV	ASCV	FSCV	ASCV
0.50	0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.50	0.75	0.992	0.999	1.000	1.000	1.000	1.000
0.50	1.00	0.987	0.998	1.000	1.000	1.000	1.000
0.50	1.25	0.992	0.999	1.000	1.000	1.000	1.000
0.50	1.50	0.991	0.999	1.000	1.000	1.000	1.000
0.75	0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.75	0.75	0.776	0.790	0.998	0.999	1.000	1.000
0.75	1.00	0.564	0.534	0.887	0.876	0.999	1.000
0.75	1.25	0.580	0.560	0.973	0.973	0.997	1.000
0.75	1.50	0.589	0.623	1.000	1.000	0.943	0.956
1.00	0.50	1.000	1.000	1.000	1.000	1.000	1.000
1.00	0.75	0.690	0.678	0.954	0.966	0.997	0.999
1.00	**1.00**	**0.050**	**0.060**	**0.050**	**0.056**	**0.050**	**0.052**
1.00	1.25	0.311	0.345	0.380	0.460	0.444	0.490
1.00	1.50	0.576	0.657	0.999	0.999	0.999	1.000
1.25	0.50	1.000	1.000	1.000	1.000	1.000	1.000
1.25	0.75	0.834	0.876	1.000	1.000	1.000	1.000
1.25	1.00	0.354	0.355	0.798	0.808	1.000	1.000
1.25	1.25	0.674	0.786	0.999	1.000	1.000	1.000
1.25	1.50	0.994	0.999	1.000	1.000	1.000	1.000
1.50	0.50	1.000	1.000	1.000	1.000	1.000	1.000
1.50	0.75	0.970	0.989	1.000	1.000	1.000	1.000
1.50	1.00	0.889	0.923	1.000	1.000	1.000	1.000
1.50	1.25	0.997	0.999	1.000	1.000	1.000	1.000
1.50	1.50	1.000	1.000	1.000	1.000	1.000	1.000

FSCV means finite-sample critical values and ASCV refers to the asymptotic values. In bold, the size of the test.

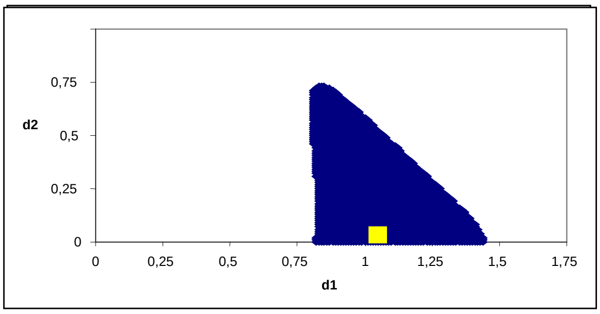
TABLE 3						
Rejection frequencies against misspecification in r						
r	T = 100		T = 200		T = 300	
	FSCV	ASCV	FSCV	ASCV	FSCV	ASCV
3	1.000	1.000	1.000	1.000	1.000	1.000
4	0.050	0.049	0.034	0.030	0.110	0.100
5	0.019	0.016	0.780	0.750	0.990	0.978
7	1.000	0.979	1.000	1.000	1.000	1.000
8	1.000	1.000	1.000	1.000	1.000	1.000
9	1.000	1.000	1.000	1.000	1.000	1.000

FSCV means finite-sample critical values, while ASCV refers to the asymptotic values.

FIGURE 1

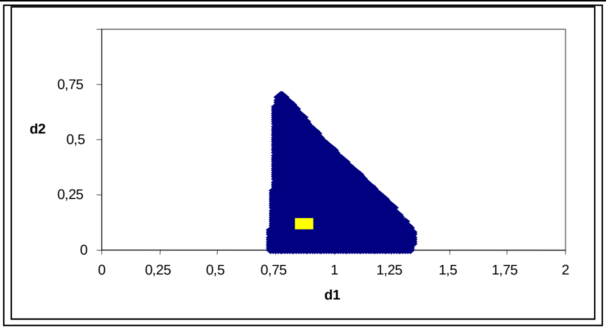
Non-rejection values of d_1 and d_2 for an extended version of Nelson and Plosser's (1982) dataset

CONSUMER PRICE INDEX



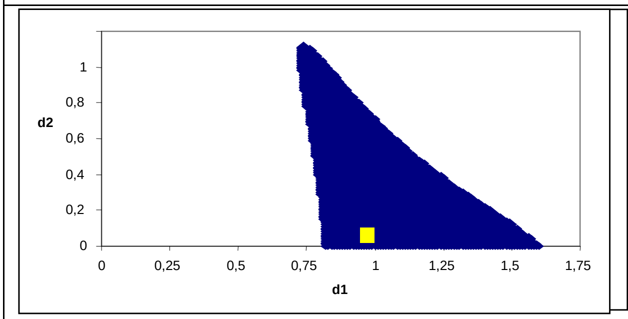
STOCK PRICES S&P 500

GNP DEFLATOR

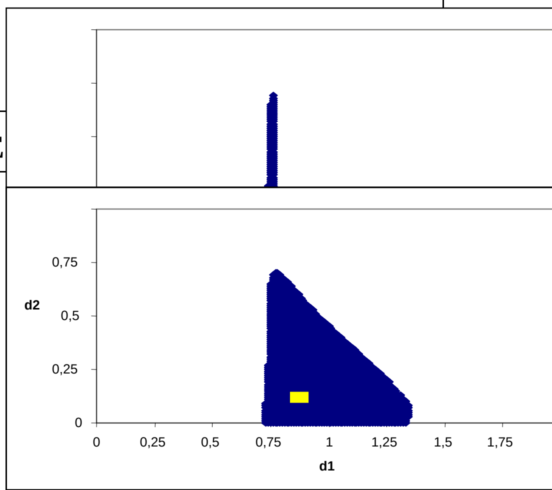


INDUSTRIAL PRODUCTION

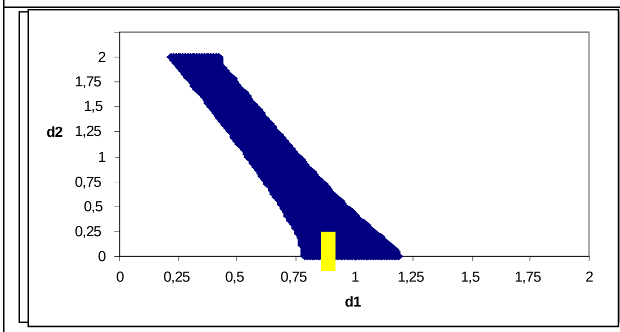
MONEY STOCK



INTERE



UNEMPLOYMENT RATE



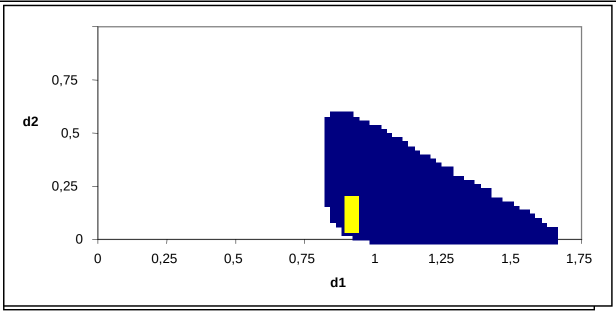
VELOCITY

d_1 refers to the order of integration at the zero frequency, while d_2 corresponds to the cyclical component.

FIGURE 2

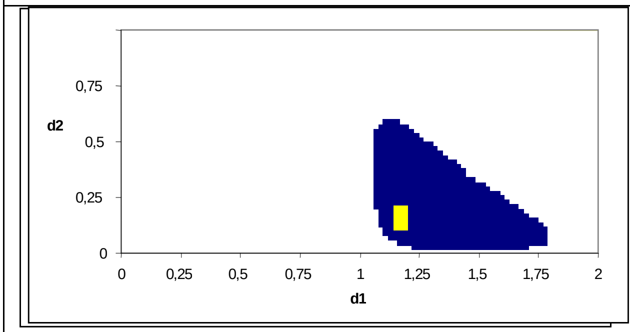
Non-rejection values of d_1 and d_2 for an extended version of Nelson and Plosser's (1982) dataset

EMPLOYMENT



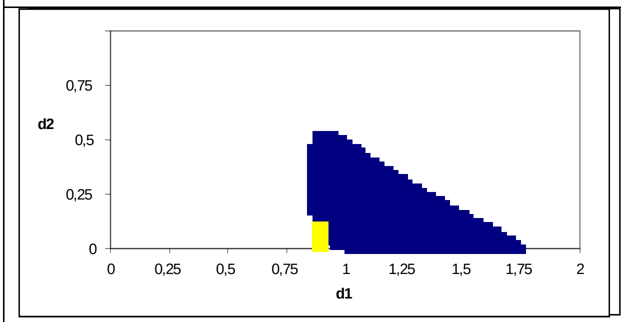
NOMINAL GNP

WAGES



REAL WAGES

REAL GNP



REAL PER CAPITA GNP

d_1 refers to the order of integration at the zero frequency, while d_2 corresponds to the cyclical component.

TABLE 4			
Values of d_{1o} and d_{2o} that produce the lowest statistics across d_1 and d_2			
Series	d_{1o} (0-frequency)	d_{2o} (Cyclical frequency)	Test statistic
CONSUMER PRICE INDEX	1.06	0.00	0.31115
STOCK PRICES S&P	1.00	0.00	0.00952
GNP DEFLATOR	0.96	0.00	0.00097
INDUSTRIAL PRODUCTION	0.85	0.08	0.00277
MONEY STOCK	1.07	0.01	0.00039
INTEREST RATE (BOND YIELD)	0.99	0.07	0.00100
UNEMPLOYMENT RATE	0.84	0.11	0.00195
VELOCITY	0.92	0.06	0.00289
EMPLOYMENT	0.97	0.00	0.00835
NOMINAL GNP	0.97	0.00	0.02335
WAGES	0.95	0.01	0.00039
REAL WAGES	0.93	0.02	0.00279
REAL GNP	0.97	0.00	0.01968
REAL PER CAPITA GNP	0.97	0.00	0.02220

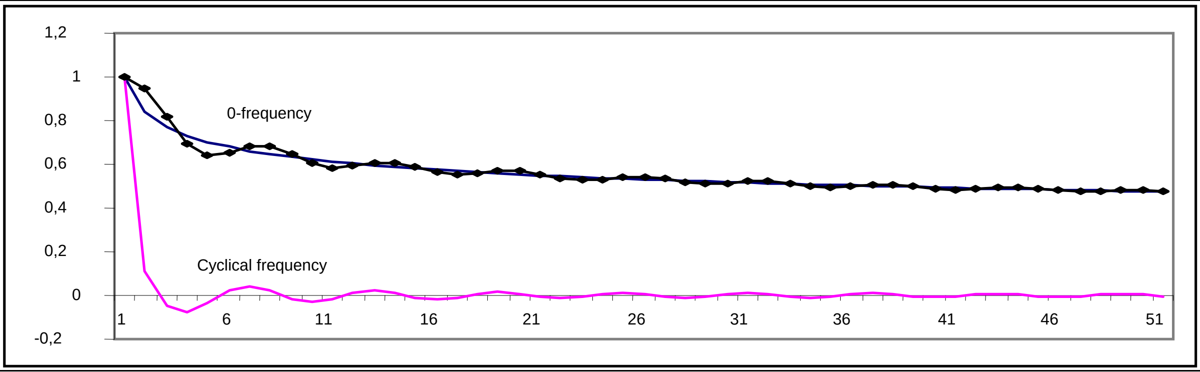
TABLE 5														
Impulse response functions for each of the Nelson and Plosser's (1982) dataset														
	CPI	SP	DEF	IPI	MON	BY	UNE	VEL	EMP	NOM	WAG	RWG	REA	CAP
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	1.059	1.000	0.959	0.930	1.080	1.060	0..949	0.980	0.970	0.970	0.959	0.950	0.970	0.970
2	1.091	1.000	0.940	0.817	1.113	1.021	0.816	0.910	0.955	0.955	0.930	0.906	0.955	0.955
3	1.113	1.000	0.928	0.722	1.132	0.969	0.696	0.844	0.945	0.945	0.908	0.871	0.945	0.945
4	1.130	1.000	0.918	0.678	1.149	0.947	0.644	0.813	0.938	0.938	0.895	0.852	0.938	0.938
5	1.143	1.000	0.911	0.679	1.167	0.959	0.653	0.815	0.933	0.933	0.888	0.844	0.933	0.933
6	1.155	1.000	0.905	0.692	1.183	0.984	0.680	0.826	0.928	0.928	0.884	0.841	0.928	0.928
7	1.165	1.000	0.900	0.688	1.197	0.994	0.680	0.825	0.924	0.924	0.879	0.836	0.924	0.924
8	1.173	1.000	0.895	0.662	1.206	0.984	0.647	0.808	0.921	0.921	0.872	0.825	0.921	0.921
9	1.181	1.000	0.891	0.630	1.213	0.965	0.605	0.786	0.917	0.917	0.865	0.814	0.917	0.917
10	1.188	1.000	0.888	0.613	1.221	0.955	0.584	0.773	0.915	0.915	0.860	0.807	0.915	0.915
11	1.195	1.000	0.885	0.615	1.229	0.961	0.591	0.774	0.912	0.912	0.857	0.804	0.912	0.912
12	1.201	1.000	0.882	0.623	1.238	0.974	0.607	0.781	0.910	0.910	0.855	0.803	0.910	0.910
13	1.206	1.000	0.879	0.622	1.246	0.980	0.608	0.781	0.908	0.908	0.852	0.800	0.908	0.908
14	1.212	1.000	0.876	0.607	1.251	0.974	0.589	0.771	0.906	0.906	0.848	0.794	0.906	0.906
15	1.216	1.000	0.874	0.588	1.256	0.962	0.563	0.758	0.904	0.904	0.844	0.788	0.904	0.904
16	1.221	1.000	0.872	0.578	1.261	0.956	0.550	0.750	0.902	0.902	0.841	0.783	0.902	0.902
17	1.225	1.000	0.870	0.580	1.266	0.960	0.556	0.751	0.901	0.901	0.839	0.781	0.901	0.901
18	1.229	1.000	0.868	0.586	1.272	0.969	0.567	0.755	0.899	0.899	0.838	0.780	0.899	0.899
19	1.233	1.000	0.866	0.585	1.278	0.973	0.569	0.756	0.898	0.898	0.836	0.779	0.898	0.898
20	1.237	1.000	0.864	0.575	1.282	0.969	0.555	0.749	0.897	0.897	0.834	0.775	0.897	0.897

CPI: Consumer price index; SP: Stock prices; DEF: GNP deflator; IPI: Industrial production index; MON: Money stock; BY: Bond Yield; UNE: Unemployment rate; VEL: Velocity; EMP: Employment; NOM: Nominal GNP; WAG: Wages; REW: Real wages; REA: Real GNP; CAP: Real GNP per capita.

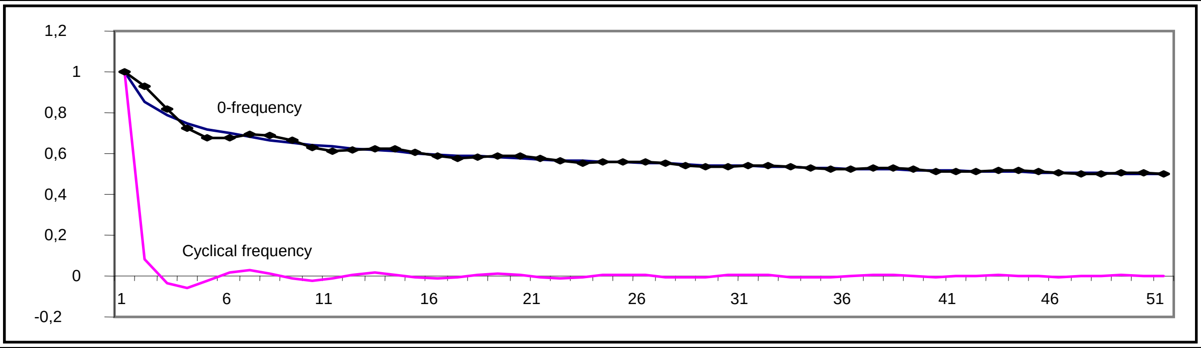
FIGURE 3

Impulse response functions

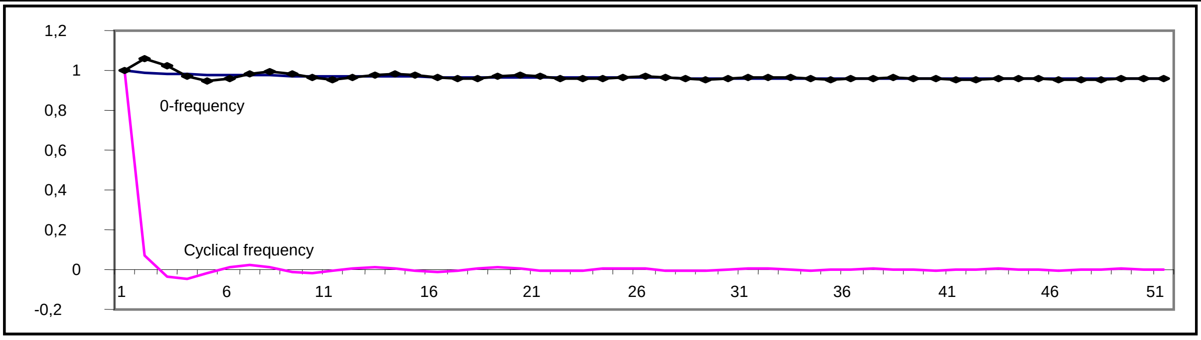
Unemployment rate



Industrial Production Index



Interest rate (Bond yield)



Velocity