University of Toronto
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Working Paper 437

Efficient Search on the Job and the Business Cycle

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June 19, 2011

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April 2011

Abstract

The paper develops a model of directed search on the job where transitions of workers between unemployment, employment and across employers are driven by heterogeneity in the quality of firm-worker matches. The equilibrium is such that the agents' value and policy functions are independent of the distribution of workers across employment states. Hence, the model can be solved outside of steady-state and used to measure the effect of cyclical productivity shocks on the labor market. Productivity shocks are found to generate large fluctuations in workers' transitions, unemployment and vacancies when matches are experience good, but not when matches are inspection goods.

1 Introduction

In the US labor market, workers move frequently between employment, unemployment and across different employers. On average, the rate at which unemployed workers move into employment (henceforth, the UE rate) is 42 percent a month, the rate at which employed workers move into unemployment (the EU rate) is 2.6 percent a month, and the rate at which workers move from one employer to the other (the EE rate) is 2.9 percent a month. These transition rates are not only large, but they are also very volatile at the business cycle frequency (relative to labor productivity), thus contributing to the large volatility of unemployment and vacancies. As documented in Table 1, the UE, EU and EE rates are five times as volatile as labor productivity, and the unemployment and vacancy rates are more

*We are grateful to the editor, Robert Shimer, and two referees for their enlightening comments. We have also benefited from the insights of Mike Elsby, Martin Gervais, Marcus Hagedorn, Bob Hall, Nir Jaimovich, Dale Mortensen, Giuseppe Moscarini, Toshihiko Mukoyama, Thijs van Rens, Ludo Visschers and Randy Wright and seminar participants at many conferences and universities. We thank Frank Diebold, Jason Faberman, Giuseppe Moscarini, Eva Nagypál, David Neumark and Daniel Polsky for generously sharing their data with us. Menzio gratefully acknowledges the financial support and the hospitality of the Hoover Institution. Shi gratefully acknowledges the financial support from the Social Sciences and Humanities Research Council of Canada and from the Bank of Canada Fellowship.

than ten times as volatile as labor productivity. Moreover, the cyclical fluctuations in the UE, EU and EE rates display a clear pattern of correlations with the cyclical fluctuations in unemployment and vacancies. As documented in Table 1, the UE and EE rates are strongly negatively correlated with unemployment and positively correlated with vacancies, and the EU rate is strongly positively correlated with unemployment and negatively correlated with vacancies.

This paper proposes a model of directed search on the job in which the workers' transitions between employment, unemployment and across different employers are driven by heterogeneity in the quality of different firm-worker matches. Like models of random search on the job (e.g. Burdett and Mortensen 1998, Postel-Vinay and Robin 2002), our model can account for the frequency and pattern of the transition of individual workers across employment states. Unlike models of random search on the job, our model can be easily solved in and out of steady-state and, hence, it can be used to study the behavior of workers' transitions, unemployment and vacancies over the business cycle.

In this paper, we use our model to measure the response of the labor market to cyclical fluctuations in aggregate productivity. We find that this response critically depends on whether the quality of a firm-worker match is observed before or after the match is created. If the quality is observed after the match is created (i.e. if matches are experience goods), aggregate productivity shocks generate large fluctuations in unemployment, vacancies and workers' transition rates. If the quality is observed before the match is created (i.e. if matches are inspection goods), the effect of aggregate productivity shocks on the labor market is negligible.

In our model, the search process is directed—as in Moen (1997) and Shimer (1996)—rather than random—as in Mortensen (1982) and Pissarides (1985). On one side of the market, firms choose how many and what type of vacancies to create. On the other side of the market, workers choose what type of vacancies to search. The type of a vacancy is defined by the conditions under which it hires a worker, and by the value of the employment contract that it offers to a new hire. Workers and vacancies searching for each other are brought into contact by a constant return to scale meeting function. Upon meeting, a worker and a firm observe a signal about the idiosyncratic productivity (i.e. quality) of their match. If the signal meets the conditions specified by the vacancy's type, the worker and the firm begin to produce and, eventually, observe the actual quality of their match. If the signal does not meet those conditions, the worker returns to his previous employment position. Depending on the informativeness of the signal, the model captures different views about the matching process. If the signal is completely uninformative, a match is an experience good. If the signal is perfectly informative, a match is an inspection good. If the signal contains some but not all information, a match is partly an inspection and partly an experience good.

In the theoretical part of the paper, we formulate the social planner's problem and characterize its solution. Then, we prove that there exists a unique equilibrium for the market economy. This equilibrium is block recursive, in the sense that the agents' value and policy functions depend on the aggregate state of the economy only through the realization of the aggregate shocks, and not through the entire distribution of workers across employment states (i.e. unemployment and employment in different matches). Because of this property, we can solve our model with heterogeneous agents and aggregate shocks as easily as one would solve a representative agent model. Moreover, we prove that the equilibrium is efficient, in the sense that it decentralizes the social planner's allocation. Because of this property, we can characterize the behavior of the economy using the first order conditions of the planner's problem.

The equilibrium is block recursive because the search process is directed. In fact, with directed search, workers in different employment states choose to search for different types of vacancies. Workers in low-value employment states (i.e. unemployment and employment in low quality matches) choose to search for vacancies that offer a low value but are easy to find (because the number of vacancies per applicant is high). Workers in high-value employment states (i.e. employment in high quality matches) choose to search for vacancies that offer a high value but are hard to find. As a result of this self-selection process, a firm that opens a particular type of vacancy knows that it will meet only one type of worker. Hence, the firm's expected value from meeting a worker does not depend on the distribution of workers across employment states and, because of firms' free entry, the probability that the firm meets an applicant must have the same property. In turn, the fact that the meeting probabilities are independent of the distribution of workers across employment states is sufficient to guarantee that the agents' value and policy functions will also be independent of the distribution.

In the quantitative part of the paper, we consider two versions of the model that, a priori, provide an equally plausible description of the labor market. Specifically, we consider a version of the model in which matches are experience goods, and a version in which matches are inspection goods. We calibrate the parameters of these two versions of the model using data on the frequency at which workers move between employment, unemployment and across different employers, as well as data on the relationship between tenure and the frequency at which workers leave their jobs.

Given the calibrated parameter values, we simulate the two versions of the model to measure the effect of aggregate productivity shocks on the labor market. When matches are experience goods, we find that the fluctuations in unemployment, vacancies and workers' transition rates generated by productivity shocks display the same pattern of comovement as in the data. Moreover, we find that the volatility of unemployment, vacancies and workers' transition rates generated by productivity shocks accounts for a large fraction of the empirical volatility of these variables. Specifically, we find that productivity shocks generate fluctuations in the UE, EU and EE rates that are (respectively) 3, 6 and 5 times larger than the fluctuations in the average productivity of labor, and fluctuations in unemployment and vacancies that are (respectively) 8 and 3 times larger than the fluctuations in average productivity. In contrast, when matches are inspection goods, we find that productivity shocks account only for a negligible fraction of the empirical volatility of the labor market. As we will discuss in section 6, the difference between the predictions of the two versions of the model is partly due to the fact that the informativeness of the signals affects the way in which the economy responds to the shocks (given the same parameter values), and partly due to the fact that the informativeness of the signal affects the calibrated parameter values.

The paper makes two contributions. On the theoretical side, the contribution of the paper is to develop a model of search on the job that is rich enough to match the pattern of workers' transitions between employment, unemployment and across employers, and tractable enough to study business cycles. The model is tractable because the equilibrium is block recursive. In earlier work, Shi (2009) proves the existence of a block recursive equilibrium for a stationary model of directed search on the job. In this paper and in a companion piece (Menzio and Shi, 2010a), we generalize proof of existence of a block recursive equilibrium to models of directed search on the job with aggregate shocks. These generalizations are not trivial as they require qualitatively different existence proofs than in a stationary environment. In the companion paper, where we consider a large class of employment contracts, we are only able to prove that the model admits a block recursive equilibrium. In this paper, where we restrict attention to bilaterally efficient contracts, we are able to prove that the only equilibrium is block recursive.

When the search process is random, models of search on the job are not block recursive, in the sense that the agents' value and policy functions depend on the entire distribution of workers across employment states. For this reason, models of random search on the job are difficult to solve outside of the steady state. To circumvent this difficulty, the existing literature has had to impose some strong restrictions on the environment. For example, in order to solve their models outside of the steady state, Moscarini and Postel-Vinay (2009) and Robin (2009) assume that rate at which workers and firms meet is exogenous. In contrast, in our model, this contact rate is endogenous and it is the key channel through which aggregate productivity shocks are transmitted to the workers' transition rates and unemployment. Similarly, in order to solve their models outside of the steady state, Mortensen (1994), Pissarides (1994, 2000) and Ramey (2008) assume that an employed worker moves into unemployment before bargaining the wage with his new employer. Hence, these models cannot capture the idea that on-the-job search affects the competitiveness of the labor market. Moreover, these models can only be solved outside of the steady state under the assumption that all matches are identical at the time they are created. Hence, these models cannot be used to study the cyclical behavior of the labor market when matches are inspection goods.

On the empirical side, the contribution of the paper is to measure the effect of aggregate productivity shocks on the labor market using a model that is calibrated to match the frequency and pattern on the workers' transitions between employment, unemployment and across employers. By calibrating the model, we discover that search on the job and match heterogeneity are both quantitatively important. By simulating the model, we discover that, if matches are experience goods, productivity shocks can account for the empirical pattern of comovement between unemployment, vacancies and workers' transition rates and for a large fraction of their empirical volatility. These findings are novel. In models that abstract from search on the job and match heterogeneity (e.g. Shimer 2005), productivity shocks generate very small movements in labor market variables. In models that allow for match heterogeneity but abstract from search on the job (e.g. Mortensen and Pissarides 1994, Merz 1995), productivity shocks generate a counterfactual comovement between vacancies, unemployment and workers' transition rates. We explain these differences at the end of section 5. Our findings are also different from Ramey (2008) who, using the model of random search on the job by Mortensen (1994), finds that productivity shocks generate implausibly small movements in the UE rate[1,2]. Moreover, we find that, if matches are inspection goods, productivity shocks account for a very small fraction of the empirical volatility of the labor market. This finding is novel since, as far as we know, there are no other papers that study the cyclical behavior of the labor market using a model in which matches are inspection goods.

The remainder of the paper is organized as follows. In section 2, we describe the physical environment of the economy, formulate the social planner's problem and characterize its

[1] There are two differences between our measurement exercise and Ramey's (2008) that might account for the differences in the results. First, we use a model of directed search on the job, while he uses a model of random search on the job by Mortensen (1994). As we have already discussed, there are important economic differences between these models. Second, while we calibrate all the parameters of the model to match the frequency and pattern of the transition of individual workers between employment states, Ramey chooses some parameters arbitrarily (e.g. the efficiency of search on the job and the scale and shape of the distribution of match-specific productivity).

[2] There are other, less related papers that study business cycle dynamics using models of search on the job. Nagypál (2007) studies a model of random search on the job in which workers have private information about the amenity value of their jobs. Using a calibrated version of the model, she finds that productivity shocks generate large fluctuations in unemployment and vacancies. Krause and Lubik (2007) reach similar conclusions using a model of segmented search on the job with two different types of vacancies. The amplification mechanism in these models is very different than in ours.

solution. In section 3, we describe the structure of the labor market and prove that its equilibrium is unique, efficient and block recursive. In section 4, we describe the strategy that we adopt to calibrate the parameters of the model. In sections 5 and 6, we measure the effect of aggregate productivity shocks on the labor market using, first, the version of the model in which matches are experience goods and, then, the version in which matches are inspection goods. Section 7 concludes. The proofs of all propositions and theorems are in the appendix.

2 Planner's Problem

2.1 Preferences and technologies

The economy is populated by a continuum of workers with measure 1 and a continuum of firms with positive measure. Each worker is endowed with an indivisible unit of labor and maximizes the expected sum of periodical consumption discounted at the factor $\beta \in (0,1)$. Each firm operates a constant return to scale technology that turns one unit of labor into $y+z$ units of output. The first component of productivity, y, is common to all firms and its value lies in the set $Y = \{y_1, y_2, ..., y_{N(y)}\}$, where $y_1 < y_2 < ... < y_{N(y)}$ and $N(y) \geq 2$ is an integer. The second component of productivity, z, is specific to a firm-worker pair, and its value lies in the set $Z = \{z_1, z_2, ..., z_{N(z)}\}$, where $z_1 < z_2 < ... < z_{N(z)}$ and $N(z) \geq 2$ is an integer.[3] Each firm maximizes the expected sum of profits discounted at the factor β.

Time is discrete and continues forever. At the beginning of each period, the state of the economy can be summarized by the triple $\psi = (y, u, g)$. The first element of ψ denotes aggregate productivity, $y \in Y$. The second element denotes the measure of workers who are unemployed, $u \in [0,1]$. The third element is a function $g : Z \to [0,1]$, with $g(z)$ denoting the measure of workers who are employed in matches with the idiosyncratic productivity z. Let Ψ denote the set in which ψ belongs.

Each period is divided into four stages: separation, search, matching and production. At the separation stage, the planner chooses the probability $d \in [\delta, 1]$ with which a match between a firm and a worker is destroyed. The lower bound on d denotes the probability that a match is destroyed for exogenous reasons, $\delta \in (0,1)$.

At the search stage, the planner sends workers and firms searching for new matches across different locations. Specifically, the planner chooses how many vacancies a firm should open

[3] The assumption that y and z are discrete random variables simplifies the notation but plays no role in the derivation of our theoretical results. In fact, it is straightforward to generalize the proof of the linearity of the planner's problem (Theorem 1) and the proof of the existence, uniqueness and block recursivity of the equilibrium (Theorem 4) to the case in which y and z are continuous random variables. Moreover, the assumption plays no role in the derivation of our quantitative results, because continuous random variables would eventually have to be discretized in order to simulate the model.

in each different location, and which location a worker should visit if he has the opportunity to search. The cost of maintaining a vacancy for one period is $k > 0$. The worker has the opportunity to search with a probability that depends on his employment status. If the worker was unemployed at the beginning of the period, he can search with probability $\lambda_u \in [0, 1]$. If the worker was employed at the beginning of the period and did not lose his job during the separation stage, he can search with probability $\lambda_e \in [0, 1]$. Finally, if the worker lost his job during the separation stage, he cannot search. As is standard in models of directed search (e.g. Acemoglu and Shimer 1999, Burdett et al. 2001, and Shi 2001), the planner finds it optimal to send workers in different employment states (i.e. unemployment and employment in a match of type z) to search in different locations, but has no incentive to send workers in the same employment state to different locations. Thus, there is no loss in generality in assuming that there are exactly $N(z) + 1$ locations.

At the matching stage, the workers and the vacancies who are searching in the same location are brought into contact by a meeting technology with constant returns to scale that can be described in terms of the vacancy-to-worker ratio θ (i.e., the tightness). Specifically, the probability that a worker meets a vacancy is $p(\theta)$, where $p : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable, strictly increasing, and strictly concave function which satisfies the boundary conditions $p(0) = 0$ and $p(\infty) = 1$. Similarly, the probability that a vacancy meets a worker is $q(\theta)$, where $q : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable and strictly decreasing function such that $q(\theta) = p(\theta)/\theta$, $q(0) = 1$ and $q(\infty) = 0$.

When a firm and a worker meet, Nature draws the idiosyncratic productivity of their match, z, from the probability distribution $f(z)$, $f : Z \to [0, 1]$. Nature also draws a signal about the idiosyncratic productivity of their match, s. With probability $\alpha \in [0, 1]$, the signal is equal to z; with probability $1-\alpha$, the signal is drawn from the distribution f independently of z. After observing s but not z, the planner chooses whether to create the match or not. If the planner chooses to create the match, the worker's previous match is destroyed (if the worker was employed). If the planner chooses not to create the match, the worker returns to his previous status (unemployment or employment in the previous match).

Notice that the information structure above encompasses a number of interesting special cases. If $\alpha = 0$, the planner has no information about the quality of a match when choosing whether to create it or not, in which case a match is a pure *experience good.* If $\alpha = 1$, the planner has perfect information about the quality of a match before choosing whether to create it or not, in which case a match is a pure *inspection good.* If $\alpha \in (0, 1)$, a match is partly an experience good and partly an inspection good.

At the production stage, an unemployed worker produces $b > 0$ units of output. A worker employed in a match with idiosyncratic productivity z produces $y+z$ units of output[4], and z is observed. At the end of this stage, Nature draws next period's aggregate component of productivity, $\hat{y}$, from the probability distribution $\phi(\hat{y}|y)$, $\phi : Y \times Y \to [0,1]$. Throughout the paper, the caret indicates variables or functions in the next period.

2.2 Formulation of the planner's problem

At the beginning of a period, the social planner observes the aggregate state of the economy $\psi = (y, u, g)$. At the separation stage, the planner chooses the probability $d(z)$ of destroying a match of quality z, $d : Z \to [\delta, 1]$. At the search stage, the planner chooses θ_u, the ratio of vacancies to workers at the location where unemployed workers look for matches, and $\theta_e(z)$, the ratio of vacancies to workers at the location where workers employed in matches of quality z look for new matches, $\theta_u \in \mathbb{R}_+$, $\theta_e : Z \to \mathbb{R}_+$. At the matching stage, the planner chooses the probability $c_u(s)$ with which a meeting between an unemployed worker and a firm is turned into a match given the signal s, $c_u : Z \to [0,1]$. Also, the planner chooses the probability $c_e(s,z)$ with which a meeting between an employed worker and a firm is turned into a match given the signal s, $c_e : Z \times Z \to [0,1]$. Given the choices $(d, \theta_u, \theta_e, c_u, c_e)$, aggregate consumption is given by

$$F(d, \theta_u, \theta_e, c_u, c_e|\psi) = -k\left\{\lambda_u \theta_u u + \textstyle\sum_z \left[(1-d(z))\lambda_e \theta_e(z) g(z)\right]\right\} + b\hat{u} + \textstyle\sum_z \left[(y+z)\hat{g}(z)\right], \quad (1)$$

where $(\hat{u}, \hat{g})$ denotes the distribution of workers across employment states at the production stage and, hence, at the beginning of next period.

To compute $\hat{u}$ and $\hat{g}$, it is useful to derive the transition probabilities for an individual worker. First, consider a worker who enters the period unemployed. With probability $1 - \lambda_u p(\theta_u)$, the worker does not meet any firm at the matching stage. In this case, the worker remains unemployed. With probability $\lambda_u p(\theta_u)$, the worker meets a firm during the matching stage. In this case, the worker and the firm receive a signal s about the quality of their match. With probability $1 - c_u(s)$, the match is not created and the worker remains unemployed. With probability $c_u(s)\left[\alpha + (1-\alpha)f(s)\right]$, the match is created and its

[4] The assumption that y and z enter additively in the production function plays no role in the derivation of our theoretical results. Indeed, we can prove Theorems 1 and 4 using a generic production function. Moreover, the assumption does not appear to have a large effect on our empirical findings. Indeed, we find that the predictions of the model regarding the effect of aggregate productivity shocks on the labor market are similar if we assume that y and z enter additively or multiplicatively in the production function. Specifically, for the version of the model in which matches are experience goods, the volatility of the UE, EU and EE rates generated by aggregate productivity shocks is, respectively, 2.5, 6.2 and 5.5 times larger than the volatility of labor productivity if y and z are additive, and 2.3, 5.4 and 4.8 times larger if y and z are multiplicative. The volatility of unemployment and vacancies is, respectively, 7.8 and 2.5 times larger than the volatility of labor productivity if y and z are additive, and 7 and 2.3 times larger if y and z are multiplicative.

idiosyncratic productivity is $z' = s$. With probability $c_u(s)(1-\alpha)f(z')$, the match is created and its idiosyncratic productivity is $z' \neq s$. Overall, at the production stage, the worker is unemployed with probability $1-\lambda_u p(\theta_u)m_u$, where $m_u = \sum_s [c_u(s)f(s)]$, and he is employed in a match of type z' with probability $\lambda_u p(\theta_u)[\alpha c_u(z') + (1-\alpha)m_u] f(z')$. Next, consider a worker who enters the period in a match of type z. It is easy to verify that, at the production stage, this worker is unemployed with probability $d(z)$, he is employed in the same match as at the beginning of the period with probability $(1-d(z))\ (1-\lambda_e p(\theta_e(z))m_e(z))$, where $m_e(z) = \sum_s [c_e(s,z)f(s)]$, and he is employed in a new match of type z' with probability $(1-d(z))\lambda_e p(\theta_e(z))\ [\alpha c_e(z',z) + (1-\alpha)m_e(z)]\ f(z')$.

After aggregating the transition probabilities of individual workers, we find that the measure of workers who are unemployed at the production stage is given by

$$\hat{u} = u\left[1-\lambda_u p(\theta_u)m_u\right] + \textstyle\sum_z \left[d(z)g(z)\right]. \tag{2}$$

Similarly, the measure of workers who are employed in matches of type z' is given by

$$\begin{aligned} \hat{g}(z') = \quad & u\lambda_u p(\theta_u)\left[\alpha c_u(z') + (1-\alpha)m_u\right] f(z') \\ & + g(z')\left[1-d(z')\right]\left[1-\lambda_e p(\theta_e(z'))m_e(z')\right] \\ & + \textstyle\sum_z g(z)\left\{\left[1-d(z)\right]\left[\lambda_e p(\theta_e(z))\right]\left[\alpha c_e(z',z) + (1-\alpha)m_e(z)\right] f(z')\right\}. \end{aligned} \tag{3}$$

The planner maximizes the sum of present and future consumption discounted at the factor β. Hence, the planner's value function, $W(\psi)$, solves the following Bellman equation

$$\begin{aligned} W(\psi) = \quad & \max_{(d,\theta_u,\theta_e,c_u,c_e)} F(d,\theta_u,\theta_e,c_u,c_e|\psi) + \beta\mathbb{E}W(\hat{\psi}) \\ \text{s.t.} \quad & (2) \text{ and } (3), \qquad d: Z \to [\delta,1], \quad \theta_u \in \mathbb{R}_+, \\ & \theta_e: Z \to \mathbb{R}_+, \quad c_u: Z \to [0,1], \quad c_e: Z\times Z \to [0,1]. \end{aligned} \tag{4}$$

Throughout this paper, the expectation operator is taken over the future state of the aggregate economy, $\hat{\psi}$, unless it is specified otherwise.

The planner's problem depends on the aggregate productivity, y, the measure of workers who are unemployed, u, and the measure of workers who are employed in the $N(z)$ different types of matches, g. If $N(z)$ is large—as it is needed to properly calibrate and simulate the model—solving the planner's problem might be difficult as it involves solving a functional equation in which the unknown function has many dimensions. Theorem 1 below shows that this potential difficulty does not arise in our model because the planner's problem breaks down into $N(z)+1$ problems that only depend on the aggregate productivity y.

Theorem 1 *(Separability of the planner's problem): (i) The planner's value function, $W(\psi)$, is the unique solution to (4). (ii) $W(\psi)$ is linear in u and g. That is, $W(\psi) = W_u(y)u + \sum_z [W_e(z,y)g(z)]$, where $W_u(y)$ and $W_e(z,y)$ are called the component value functions. The* *component value function $W_u(y)$ is given by*

$$W_u(y) = \max_{(\theta_u, c_u)} \{-k\lambda_u\theta_u + [1-\lambda_u p(\theta_u)m_u]\,[b + \beta\mathbb{E}W_u(\hat{y})] \\ +\lambda_u p(\theta_u)\textstyle\sum_{z'} \{[\alpha c_u(z') + (1-\alpha)m_u]\,[y + z' + \beta\mathbb{E}W_e(z',\hat{y})]\, f(z')\}\} \\ s.t.\ \theta_u \in \mathbb{R}_+, \quad c_u : Z \to [0,1]. \tag{5}$$

The component value function $W_e(z, y)$ is given by

$$W_e(z,y) = \max_{(d,\theta_e,c_e)} \{d\,[b + \beta\mathbb{E}W_u(\hat{y})] - (1-d)\,k\lambda_e\theta_e \\ +(1-d)\,[1-\lambda_e p(\theta_e)m_e]\,[y + z + \beta\mathbb{E}W_e(z,\hat{y})] \\ +(1-d)\lambda_e p(\theta_e)\textstyle\sum_{z'} \{[\alpha c_e(z') + (1-\alpha)m_e]\,[y + z' + \beta\mathbb{E}W_e(z',\hat{y})] f(z')\}\} \\ s.t.\ d \in [\delta, 1], \quad \theta_e \in \mathbb{R}_+, \quad c_e : Z \to [0,1]. \tag{6}$$

(iii) $W_e(z, y)$ is strictly increasing in z. (iv) The policy correspondences $(d^*, \theta_u^*, \theta_e^*, c_u^*, c_e^*)$ associated with (4) depend on ψ only through y and not through (u, g).

Each of the $N(z)+1$ planner's problems is associated with a worker in a different employment state (unemployment and employment in a match of different quality). In the problem associated with an unemployed worker, (5), the planner chooses θ_u and $c_u(s)$ to maximize the present value of the output generated by this worker, net of the cost of the vacancies assigned to him. Similarly, in the problem associated with a worker employed in a match of type z, (6), the planner chooses $d(z)$, $\theta_e(z)$ and $c_e(s, z)$ to maximize the present value of the output generated by this worker, net of the cost of the vacancies assigned to him. Since each of these worker-specific problems only depends on the aggregate productivity, y, solving the planner's problem in our model is just as easy as solving the planner's problem in a representative agent model.

The planner's problem can be decomposed into worker-specific problems that only depend on the aggregate productivity, y, because the search process is directed rather than random. Under random search, the planner has to choose the same tightness for workers in different employment states, because all workers search in the same location. For this reason, the planner's problem cannot be decomposed into worker-specific problems and its solution will depend not only on the aggregate productivity, y, but also on the distribution of workers across employment states, (u, g). In contrast, under directed search, the planner can choose a different tightness for each different worker, because different workers search in different locations. This property, together with the linearity of the production function, is sufficient to guarantee that the planner's problem can be decomposed into $N(z) + 1$ worker-specific problems that depend on the aggregate productivity, y, but not on the distribution of workers,

(u, g).

2.3 Solution to the planner's problem

The efficient choice for the probability of turning a meeting between a firm and an unemployed worker into a match is $c_u^*(s, y) = 1$ if

$$b + \beta \mathbb{E} W_u(\hat{y}) \leq \alpha[y + s + \beta \mathbb{E} W_e(s, \hat{y})] + (1 - \alpha)\mathbb{E}_{z'}[y + z' + \beta \mathbb{E} W_e(z', \hat{y})], \tag{7}$$

and $c_u^*(s, y) = 0$ otherwise, where s is the signal about the quality of the match. Similarly, the efficient choice for the probability of turning a meeting between a firm and an employed worker into a match is $c_e^*(s, z, y) = 1$ if

$$y + z + \beta \mathbb{E} W_e(z, \hat{y}) \leq \alpha[y + s + \beta \mathbb{E} W_e(s, \hat{y})] + (1 - \alpha)\mathbb{E}_{z'}[y + z' + \beta \mathbb{E} W_e(z', \hat{y})], \tag{8}$$

and $c_e^*(s, z, y) = 0$ otherwise, where z is the quality of the worker's current match and s is the signal about the quality of the new match. These conditions are intuitive. The left hand side in (7) and (8) is the value of keeping the worker in his current employment position (unemployment and employment in a match of type z). The right hand side of (7) and (8) is the value of moving the worker to the new match. This is equal to the value of a worker employed in a match with idiosyncratic productivity z', where z' is equal to s with probability α and to a value drawn randomly from the distribution f with probability $1-\alpha$. The planner finds it optimal to create the match if and only if the left hand side is smaller than the right hand side. Notice that the left hand side of (7) is independent of s, while the right hand side is strictly increasing in s. Hence, the creation probability $c_u^*(s, y)$ is an increasing function of s, and can be represented by a reservation signal $r_u^*(y)$ such that $c_u^*(s, y) = 0$ if $s < r_u^*(y)$ and $c_u^*(s, y) = 1$ if $s \geq r_u^*(y)$. For the same reason, the creation probability $c_e^*(s, z, y)$ can be represented by a reservation signal $r_e^*(z, y)$ such that $c_e^*(s, z, y) = 0$ if $s < r_e^*(z, y)$ and $c_e^*(s, z, y) = 1$ if $s \geq r_e^*(z, y)$. Moreover, since the right hand side of (8) is strictly increasing in z, $r_e^*(z, y)$ is increasing in z.

The efficient choice for the vacancy-to-worker ratio at the location visited by unemployed workers is $\theta_u^*(y)$ such that

$$k \geq p'(\theta_u^*(y)) \sum_{s \geq r_u^*(y)} \left\{ \begin{array}{l} \alpha \left[y + s - b + \beta \mathbb{E} \left(W_e(s, \hat{y}) - W_u(\hat{y})\right)\right] \\ +(1 - \alpha)\mathbb{E}_{z'} \left[y + z' - b + \beta \mathbb{E} \left(W_e(z', \hat{y}) - W_u(\hat{y})\right)\right] \end{array} \right\} f(s), \tag{9}$$

and $\theta_u^*(y) \geq 0$, with complementary slackness. Similarly, the efficient choice for the vacancy-to-worker ratio in the location visited by workers employed in matches of quality z is $\theta_e^*(z, y)$

such that

$$k \geq p'(\theta_e^*(z,y)) \sum_{s \geq r_e^*(z,y)} \left\{ \begin{array}{l} \alpha\left[s - z + \beta \mathbb{E}\left(W_e(s,\hat{y}) - W_e(z,\hat{y})\right)\right] \\ +(1-\alpha)\mathbb{E}_{z'}\left[z' - z + \beta\mathbb{E}\left(W_e(z',\hat{y}) - W_e(z,\hat{y})\right)\right] \end{array} \right\} f(s), \tag{10}$$

and $\theta_e^*(z,y) \geq 0$, with complementary slackness. We only discuss (10) as the two conditions above are similar. The left-hand side is the marginal cost of increasing the vacancy-to-worker ratio at the location visited by workers employed in matches of quality z. The right-hand side is the marginal benefit of increasing this vacancy-to-worker ratio, which is given by the product of two terms. The first term is the marginal increase in the probability with which a worker employed in a match of quality z meets a firm. The second term is the value of a meeting between a worker employed in a match of quality z and a firm. If $\theta_e^*(z,y)$ is positive, the marginal cost and the marginal benefit of increasing the vacancy-to-worker ratio must be equal. Otherwise, the marginal cost must be greater than the marginal benefit. Notice that the left-hand side does not depend on z, while the right-hand side strictly decreases with z. Hence, as long as $\theta_e^*(z,y) > 0$, the vacancy-to-worker ratio $\theta_e^*(z,y)$ is a strictly decreasing function of z.

Finally, the efficient choice for the probability of destroying a match is $d^*(z,y) = 1$ if

$$\begin{aligned} b + \beta\mathbb{E}W_u(\hat{y}) > & -k\lambda_e\theta_e^*(z,y) + (1 - \lambda_e p(\theta_e^*(z,y))m_e^*(z,y))\left[y + z + \beta\mathbb{E}W_e(z,\hat{y})\right] \\ & +\lambda_e p(\theta_e^*(z,y))\mathbb{E}_{z'}\left\{\left[\alpha c_e^*(z',z,y) + (1-\alpha)m_e^*(z,y)\right]\left[y + z' + \beta\mathbb{E}W_e(z',\hat{y})\right]\right\}, \end{aligned} \tag{11}$$

and $d^*(z,y) = \delta$ otherwise, where z is the idiosyncratic productivity of the match. The left-hand side of (11) is the value of a worker who is unemployed and does not have the opportunity to search for a new match in the current period. This is the value of destroying the match. The right-hand side is the value of a worker who is employed in a match of type z and has the opportunity to search for a new match with probability λ_e. This is the value of keeping the match alive. When the left-hand side is greater than the right-hand side, the planner destroys the match with probability 1. Otherwise, Nature destroys the match with probability δ. Notice that the left-hand side does not depend on z, while the right-hand side is strictly increasing in z. Hence, the destruction probability $d^*(z,y)$ is a decreasing function of z, and can be represented by a reservation productivity $r_d^*(y)$ such that $d^*(z,y) = 1$ if $z < r_d^*(y)$ and $d^*(z,y) = \delta$ if $z \geq r_d^*(y)$.

We summarize the properties of the efficient choices in the proposition below.

Proposition 2 *(Planner's policy functions): (i) The policy correspondences $(d^*, \theta_u^*, \theta_e^*, c_u^*, c_e^*)$ are single valued. (ii) There is $r_d^*(y)$ such that $d^*(z,y) = 1$ if $z < r_d^*(y)$ and $d^*(z,y) = \delta$ else. (iii) There is $r_u^*(y)$ such that $c_u^*(s,y) = 0$ if $s < r_u^*(y)$ and $c_u^*(s,y) = 1$ else. Similarly, there is $r_e^*(z,y)$ such that $c_e^*(s,z,y) = 0$ if $s < r_e^*(z,y)$ and $c_e^*(s,z,y) = 1$ else. Moreover, $r_e^*(z,y)$*

is increasing in z. (iv) $\theta_e^*(z, y)$ is decreasing in z.

With respect to a standard search model (e.g. Pissarides 2000, Chapter 1), our model identifies a number of additional channels through which an aggregate productivity shock may affect the transitions of workers across employment states. First, by affecting not only θ_u^* and θ_e^* but also r_u^* and r_e^*, an aggregate productivity shock may affect not only the probability that a worker meets a firm but also the probability that a meeting between a firm and a worker turns into a match. Clearly, both channels may contribute to the response of the UE and EE rates to an aggregate productivity shock. Second, by affecting r_d^*, an aggregate productivity shock may affect the probability that the match between a firm and a worker is destroyed and, hence, it may affect the EU rate. As we shall see in sections 5 and 6, the quantitative importance of these additional channels depends on the informativeness of the signals, and on the shape of the distribution of match-specific productivity.

3 Decentralization

In this section, we describe a market economy that decentralizes the efficient allocation. We first describe the structure of the labor market and the nature of the employment contracts. We then derive the conditions on the individual agents' value and policy functions that need to be satisfied in the market equilibrium. Finally, we establish that there exists a unique equilibrium for the market economy and that this equilibrium is efficient, in the sense that it decentralizes the solution to the planner's problem, and block recursive, in the sense that the agents' value and policy functions depends on the aggregate state of the economy, ψ, only through the aggregate productivity, y, and not through the entire distribution of workers across employment states, (u, g). The equilibrium is block recursive because, with directed search, workers *choose* to search in different submarkets.

3.1 Market economy

For the planner's problem in section 2, we only needed to describe the physical environment of the economy. For the analysis of equilibrium here, we also have to describe the structure of the labor market and the nature of the employment contracts. We assume that the labor market is organized in a continuum of submarkets indexed by (x, r), $(x, r) \in \mathbb{R} \times Z$, where x is the value offered by a firm to a worker and r is a selection criterion based on the signal s. Specifically, when a firm meets a worker in submarket (x, r), it hires the worker if and only if the signal s about the quality of their match is greater than or equal to r. If the firm hires the worker, it offers him an employment contract worth x in lifetime utility. The vacancy-to-worker ratio of submarket (x, r) is denoted as $\theta(x, r, \psi)$. In equilibrium, $\theta(x, r, \psi)$ will be consistent with the firms' and workers' search decisions.

At the separation stage, an employed worker moves into unemployment with probability $d \in [\delta, 1]$. At the search stage, each firm chooses how many vacancies to create and in which submarkets to locate them. On the other side of the market, each worker who has the opportunity to search chooses which submarket to visit. At the matching stage, each worker searching in submarket (x, r) meets a vacancy with probability $p(\theta(x, r, \psi))$. Similarly, each vacancy located in submarket (x, r) meets a worker with probability $q(\theta(x, r, \psi))$. When a worker and a vacancy meet in submarket (x, r), the hiring process follows the rule specified for that submarket; i.e., the worker is hired if and only if the signal is higher than r and, conditional on being hired, he receives the lifetime utility x. At the production stage, an unemployed worker produces b units of output, and a worker employed in a match of type z produces $y + z$ units of output.

We assume that the contracts offered by firms to workers are bilaterally efficient, in the sense that they maximize the joint value of the match, i.e., the sum of the worker's lifetime utility and the firm's lifetime profits. We make this assumption because there are a variety of specifications of the contract space under which the contracts that maximize the profits of the firm are, in fact, bilaterally efficient. In a previous version of this paper (Menzio and Shi 2009), we prove that the profit maximizing contracts are bilaterally efficient if the contract space is complete, in the sense that a contract can specify the wage, w, the separation probability, d, and the submarket where the worker searches while on the job, (x_e, r_e), as functions of the history of the aggregate state of the economy, ψ, and the quality of the match, z. This result is intuitive. The firm maximizes its profits by choosing the contingencies for d, x_e and r_e so as to maximize the joint value of the match and by choosing the contingencies for w so as to deliver the promised value x. Moreover, we can prove that the profit maximizing contracts are bilaterally efficient even if they can only specify the wage as a function of tenure and productivity (while the separation and search decisions are made by the worker). This result is also intuitive. The firm maximizes its profits by choosing the wage in the first period of the employment relationship so as to deliver the promised value x, and by choosing the wage in the subsequent periods so as to induce the worker to maximize the joint value of the match (this is accomplished by setting the wage equal to the product of the match). Alternatively, profit maximizing contracts are bilaterally efficient if they can specify severance transfers that induce the worker to internalize the effect of his separation and search decisions on the profits of the firm.

3.2 The problem of the worker and the firm

First, consider an unemployed worker at the beginning of the production stage, and let $V_u(\psi)$ denote his lifetime utility. In the current period, the worker produces and consumes b units of output. In next period, the worker matches with a vacancy with probability $\lambda_u p(\theta(x,r,\psi))m(r)$, where (x,r) is the submarket where the worker searches and $m(r) = \sum_{s\geq r} f(s)$ is the probability that the signal about the quality of the match is above the selection cutoff r. If the worker matches with a vacancy, his continuation utility is x. If the worker does not match with a vacancy, his continuation utility is $V_u(\hat{\psi})$. Thus,

$$V_u(\psi) = b + \beta \mathbb{E} \max_{(x,r)} \left\{ V_u(\hat{\psi}) + \lambda_u D(x,r,V_u(\hat{\psi}),\hat{\psi}) \right\}, \tag{12}$$

where D is defined as

$$D(x,r,V,\psi) = p(\theta(x,r,\psi))m(r)(x - V), \tag{13}$$

We denote as $(x_u(\hat{\psi}), r_u(\hat{\psi}))$ the policy functions for the optimal choices in (12).

Second, consider a worker and a firm who are matched at the beginning of the production stage. Let $V_e(z,\psi)$ denote the sum of the worker's lifetime utility and the firm's lifetime profits. In the current period, the sum of the worker's utility and the firm's profit is equal to the output of the match, $y + z$. In the next period, the worker and the firm separate at the matching stage with probability d, in which case the worker's continuation utility is $V_u(\hat{\psi})$ and the firm's continuation profit is zero. The worker and the firm separate at the next matching stage with probability $(1-d)\left[\lambda_e p(\theta(x,r,\psi))m(r)\right]$, where (x,r) is the submarket where the worker searches for a new match. In this case, the continuation utility of the worker is x and the firm's continuation profit is zero. Finally, the worker and the firm remain together until the next production stage with probability $(1-d)[1-\lambda_e p(\theta(x,r,\psi))m(r)]$, in which case the sum of the worker's continuation utility and the firm's continuation profit is $V_e(z,\hat{\psi})$. Thus,

$$V_e(z,\psi) = y + z + \beta \mathbb{E} \max_{(d,x,r)} \left\{ d\ V_u(\hat{\psi}) + (1-d) \left[V_e(z,\hat{\psi}) + \lambda_e D(x,r,V_e(z,\hat{\psi}),\hat{\psi}) \right] \right\} \tag{14}$$

where D is the function defined in (13). We denote as $d(z,\hat{\psi})$ and $(x_e(z,\hat{\psi}), r_e(z,\hat{\psi}))$ the policy functions for the optimal choices in (14).

At the search stage, a firm chooses how many vacancies to create and where to locate them. The firm's cost of creating a vacancy in submarket (x,r) is k. The firm's benefit from creating a vacancy in submarket (x,r) is

$$q(\theta(x,r,\psi)) \textstyle\sum_{s\geq r} \left\{ \left[\alpha V_e(s,\psi) + (1-\alpha)\mathbb{E}_z V_e(z,\psi) - x \right] f(s) \right\}, \tag{15}$$

where $q(\theta(x,r,\psi))$ is the probability of meeting a worker, $V_e(s,\psi)$ is the joint value of the match if the signal is correct, $\mathbb{E}_z V_e(z,\psi)$ is the joint value of the match if the signal is not correct, and x is the part of the joint value of the match that the firm delivers to the worker. When the cost is strictly greater than the benefit, the firm does not create any vacancy in submarket (x,r). When the cost is strictly smaller than the benefit, the firm creates infinitely many vacancies in submarket (x,r). And when the cost and the benefit are equal, the firm's profit is independent of the number of vacancies it creates in submarket (x,r).

In any submarket visited by a positive number of workers, the tightness $\theta(x,r,\psi)$ is consistent with the firm's incentives to create vacancies if and only if

$$k \geq q(\theta(x,r,\psi)) \sum_{s \geq r} \left\{ \left[\alpha V_e(s,\psi) + (1-\alpha)\mathbb{E}_z V_e(z,\psi) - x \right] f(s) \right\}, \tag{16}$$

and $\theta(x,r,\psi) \geq 0$ with complementary slackness. In any submarket that workers do not visit, the tightness $\theta(x,r,\psi)$ is consistent with the firm's incentives to create vacancies if and only if k is greater or equal than (15). However, following the literature on directed search on the job with heterogeneous workers (i.e. Shi 2009, Menzio and Shi 2010 *a*, 2010 *b*, and Gonzalez and Shi, 2010), we restrict attention to equilibria in which $\theta(x,r,\psi)$ satisfies the above complementary slackness condition in every submarket.[5]

3.3 Equilibrium, block recursivity and efficiency

Definition 3 *A Block Recursive Equilibrium (BRE) consists of a market tightness function* $\theta : \mathbb{R} \times Z \times Y \to \mathbb{R}_+$*, a value function for the unemployed worker* $V_u : Y \to \mathbb{R}$*, a policy function for the unemployed worker* $(x_u, r_u) : Y \to \mathbb{R} \times Z$*, a joint value function for the firm-worker match* $V_e : Z \times Y \to \mathbb{R}$*, and policy functions for the firm-worker match* $d : Z \times Y \to [\delta, 1]$ *and* $(x_e, r_e) : Z \times Y \to \mathbb{R} \times Z$*. These functions satisfy the following conditions:*
(i) $\theta(x,r,y)$ *satisfies (16) for all* $(x,r,\psi) \in \mathbb{R} \times Z \times \Psi$*;*
(ii) $V_u(y)$ *satisfies (12) for all* $\psi \in \Psi$*, and* $(x_u(y), r_u(y))$ *are the associated policy functions;*
(iii) $V_e(z,y)$ *satisfies (14) for all* $(z,y) \in Z \times \Psi$*, and* $d(z,y)$ *and* $(x_e(z,y), r_e(z,y))$ *are the associated policy functions.*

Condition (i) guarantees that the search strategy of an unemployed worker maximizes his lifetime utility, given the market tightness function θ. Condition (ii) guarantees that

[5] This assumption pins down the tightness of an inactive submarket by a firm's indifference condition. That is, the tightness is such that a firm's expected profit from visiting any inactive submarket is equal to the firm's expected profit from visiting one of the active submarkets. A justification for this assumption comes from the following thought experiment. Imagine a sequential game in which unemployed workers choose (with a tremble) where to look for vacancies and, then, firms choose where to create their vacancies. Because of the tremble, the tightness is well defined everywhere. As the probability of the tremble goes to zero, the tightness of every submarket remains well defined and converges to the one given by (16).

the employment contract maximizes the sum of the worker's lifetime utility and the firm's lifetime profits, given the market tightness function θ. Condition (iii) guarantees that the market tightness function θ is consistent with the firm's incentives to create vacancies. Taken together, conditions (i)-(iii) insure that in a BRE, just like in a recursive equilibrium, the strategies of each agent are optimal given the strategies of the other agents. However, unlike in a recursive equilibrium, in a BRE, the agent's value and policy functions depend on the aggregate state of the economy, ψ, only through the aggregate productivity, y, and not through the distribution of workers across different employment states, (u, g). For this reason, a BRE is much easier to solve than a recursive equilibrium. But does a BRE exist? And why should we focus on a BRE rather than on a recursive equilibrium?

The following theorem answers these questions. Specifically, the theorem establishes that a BRE exists, that a BRE is unique and that it decentralizes the solution to the social planner's problem. Moreover, the theorem establishes that there is no loss in generality in focusing on the BRE because all equilibria are block recursive.

Theorem 4 *(Block recursivity, uniqueness and efficiency of equilibrium):(i) All equilibria are block recursive. (ii) There exists a unique BRE. (iii) The BRE is socially efficient in the sense that: (a)* $\theta(x_u(y), r_u(y), y) = \theta_u^*(y)$, *and* $r_u(y) = r_u^*(y)$; *(b)* $d(z, y) = d^*(z, y)$; *(c)* $\theta(x_e(z, y), r_e(z, y), y) = \theta_e^*(z, y)$, *and* $r_e(z, y) = r_e^*(z, y)$.

The equilibrium is block recursive because searching workers are endogenously separated in different markets and, as in the social planner's problem, such separation is possible only when search is directed. To explain why directed search induces workers to separate endogenously, note that workers choose in which submarket to search in order to maximize the product between the probability of finding a new match and the value of moving from their current employment position to the new match. For a worker in a low-value employment position (unemployment or employment in a low quality match), it is optimal to search in a submarket where the probability of finding a new match is relatively high and the value of the match is relatively low. For a worker in a high-value employment position (i.e., employment in a high quality match), it is optimal to search in a submarket where the probability of finding a new match is relatively low and the value of the match is relatively high. Overall, workers in different employment positions choose to search in different submarkets. As a result of the self-selection of workers, a firm that opens a vacancy in submarket (x, r) knows that it will only meet one type of worker. For this reason, the expected value to the firm from meeting a worker in submarket (x, r) does not depend on the entire distribution of workers across employment states and, because of the free entry condition (16), the probability that a firm meets a worker in submarket (x, r) has the same property. Since the meeting probability across different submarkets is independent from the distribution of workers across employment states, it is easy to see from (12) and (14) that the value of unemployment and the joint value of a match will also be independent from the distribution.

If we replaced the assumption of directed search with random search, the equilibrium could not be block recursive. Under random search, workers in high and low-value employment positions all have to search in the same market. When this is the case, the firm's expected value from meeting a worker depends on how workers are distributed across different employment positions, as this distribution determines the probability that the employment contract offered by the firm will be accepted by a randomly selected worker. In turn, the free-entry condition implies that the probability that a firm meets a worker must also depend on the distribution of workers. Since the meeting probability between firms and workers depends on the distribution, so do all of the agents' value and policy functions.[6]

It is important to clarify that the assumption of bilaterally efficient contracts is not necessary for establishing the existence of a block recursive equilibrium. In fact, in some of our work (Shi 2009, Menzio and Shi 2010 *a*, 2010 *b*), we have shown that block recursive equilibria exist also in economies where the contract space is so limited that bilateral efficiency cannot be attained (e.g., economies in which contracts can only specify a wage that remains constant over the entire duration of an employment relationship).

However, we use of the assumption of bilaterally efficient contracts in order to establish the equivalence between the block recursive equilibrium and the social plan, and to rule out equilibria that are not block recursive. When contracts are bilaterally efficient, the joint value of a match to the firm and the worker satisfies the equilibrium condition (14). After solving the free-entry condition (16) for x and substituting the solution into (14), we get

$$\begin{aligned}V_e(z,\psi) &= y+z\\ &+\beta\mathbb{E}\max_{(d,\theta,r)}\Big\{dV_u(\hat{\psi})-(1-d)\lambda_e k\theta+(1-d)\lambda_e V_e(z,\hat{\psi})\\ &+(1-d)\lambda_e p(\theta)\textstyle\sum_{s\geq r}\Big[\alpha V_e(s,\hat{\psi})+(1-\alpha)\mathbb{E}_z V_e(z,\hat{\psi})-V_e(z,\hat{\psi})\Big]f(s)\Big\}.\end{aligned}\tag{17}$$

One can easily verify that (17) is satisfied not only by the joint value of a match to the firm and the worker, $V_e(z,\psi)$, but also by the value of an employed worker to the planner, $y+z+\beta\mathbb{E}W_e(z,\hat{y})$. Moreover, one can easily verify that the functional equation (17) is a contraction mapping and, hence, it admits a unique solution. Therefore, the joint value of a match to the firm and the worker must be equal to the value of an employed worker to the

[6] One should clearly distinguish block recursivity from the property that the market tightness is independent of unemployment in simple models of random search (e.g. Pissarides 1985, Mortensen and Pissarides 1994). The latter feature arises only when searching workers are identical, so that a vacancy knows exactly the type of worker it will meet. In fact, when there is on-the-job search or when searching workers are heterogeneous ex ante, random search will cause the market tightness to depend on their distribution.

planner. Similarly, one can establish the equivalence between the value of unemployment to a worker, $V_u(\psi)$, and the value of an unemployed worker to the planner, $b + \beta \mathbb{E} W_u(\hat{y})$. The equivalence between the value functions of individual agents and the component value functions of the planner is sufficient for establishing that any equilibrium is efficient and block recursive.

4 Calibration

In the previous two sections, we have developed a directed search model of workers' transitions between employment, unemployment and across different employers. In this section, we calibrate the parameters of the model using data on the movements of workers across employment states in the US labor market. In the next two sections, we will use the calibrated model to measure the effect of aggregate productivity shocks on unemployment, vacancies and workers' transition rates. We carry out this quantitative analysis for the version of the model in which matches are pure experience goods (i.e. $\alpha = 0$) and for the one in which matches are pure inspection goods (i.e. $\alpha = 1$).

Households' preferences are described by the discount factor β and the value of leisure b. Firms' technology is described by the vacancy cost k, the distribution of match-specific productivity f, the stochastic process for the aggregate component of productivity ϕ, and the exogenous match-destruction probability δ. We restrict f to be a 200 point approximation of a Weibull distribution with mean μ_z, shape ν_z, and scale σ_z.[7] We also restrict the stochastic process for aggregate productivity to be a 3-state Markov process with unconditional mean μ_y, autocorrelation ρ_z, and standard deviation σ_y. The matching process is described by the search probabilities λ_u and λ_e, the meeting probability p, and the precision of the signal about the quality of a new match, α. As in most of the related literature (e.g. Shimer 2005 and Mortensen and Nagypál 2007), we restrict $p(\theta)$ to be of the form $\min\{\theta^\gamma, 1\}$, $\gamma \in (0, 1)$.

In order to calibrate the parameters of the model, we use data on the transitions of workers across employment states in the US labor market (see Appendix D for details). We choose the model period to be one month[8]. We normalize λ_u to 1 and choose the parameters

[7] The Weibull density function is:

$$f(z) = \frac{\nu_z}{\sigma_z} \left(\frac{z - \mu_z}{\sigma_z} + \Gamma\left(\frac{1}{\nu_z} + 1\right) \right)^{\nu_z - 1} \exp\left[- \left(\frac{z - \mu_z}{\sigma_z} + \Gamma\left(\frac{1}{\alpha_z} + 1\right) \right)^{\nu_z} \right],$$

where Γ is the gamma function. The parameters ν_z and σ_z control respectively the shape and the variance of the distribution. In particular, the shape of the Weibull distribution is similar to the shape of the exponential distribution for $\nu_z = 1$, to the lognormal distribution for $\nu_z = 2$, to the normal distribution for $\nu_z = 4$, and to a left-skewed version of a normal distribution for $\nu_z = 10$. To keep the calibration manageable, we restrict attention to these four values of ν_z.

[8] In our benchmark calibration, workers can only change employment status once a month. However, in

λ_e, k, and δ so that the average UE, EU and EE rates are the same in the model as in the data. We set the value of γ so that the model matches the empirical elasticity of the UE rate with respect to the vacancy-to-unemployment ratio. We set the value of b so that the model matches the empirical ratio of labor productivity at home and in the market as measured by Hall and Milgrom (2008). We normalize μ_z to 0 and choose the values of ν_z and σ_z that minimize the distance between the distribution of workers across tenure lengths generated by the model and its empirical counterpart. Finally, we normalize μ_y to 1 and choose ρ_y and σ_y to match the empirical autocorrelation and standard deviation of average labor productivity.

Most of the calibration strategy outlined above is standard (see e.g. Shimer 2005). The main novelty is to calibrate the shape and scale of the distribution of match-specific productivities using the empirical tenure distribution.[9] Let us briefly explain why these two distributions are related. In the model, matches with different idiosyncratic productivity have a different probability of surviving from one year to the next. In particular, a low productivity match has a lower survival probability than a high productivity match because a worker employed in a low productivity match is more likely to move into unemployment and into a new match. This implies that the distribution of the match-specific productivity among newly affects the fraction of matches that survives for t years and, consequently, the cross-sectional tenure distribution. Figure 1 shows the fit of the empirical tenure distribution obtained with the experience and inspection versions of the model.

Table 2 summarizes the calibration outcomes. Notice that, for the version of the model in which matches are experience goods, search on the job and match heterogeneity (the two central elements of our model) are both quantitatively important. The search probability for an employed worker, λ_e, is 73 percent per month, nearly as high as the search probability for an unemployed worker. The scale of the distribution of match-specific productivity σ_z is 0.95, which implies that a match at the 90th percentile of the distribution is nearly twice as productive as a match at the 10th percentile. For the version of the model in which matches are inspection goods, the calibrated value of λ_e is even higher, while the calibrated value of σ_z is lower.

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the data, some workers experience multiple changes in their employment status within a month. As pointed out by Shimer (2005), this discrepancy between the model and the data may lead to biased estimates of the parameters of the model and to a mis-measurement of the causes of business cycle fluctuations. In order to address this potential concern, we calibrated and simulated a biweekly version of our model. We found that aggregate productivity shocks have a similar effect on workers' transition rates, unemployment and vacancies whether we use the biweekly or the monthly version of the model.

[9] This identification strategy has a precedent in Moscarini (2003), who considers a model of random search on the job in which workers and firms learn over time the quality of their match by observing their output. He uses the empirical tenure distribution to identify the precision of output as a signal of match quality.

5 Experience model

In this section, we study the effect of aggregate productivity shocks on unemployment, vacancies and workers' transition rates for the version of the model in which matches are experience goods. In this version of the model, workers and firms have no information about the quality of their match before starting production. Hence, every time a worker and a firm meet, they match. An unemployed worker is hired as soon as he meets a firm. After the worker is hired, he begins production and observes the quality of the match with his employer. If the quality of the match is sufficiently low, the worker returns into unemployment. If the quality of the match is sufficiently high, the worker stays in the match and stops searching (i.e. he searches in submarkets without vacancies). If the quality of the match takes on intermediate values, the worker stays in the match but continues searching and moves to another employer as soon as he meets one.

5.1 The effect of aggregate productivity shocks

We examine the response of the economy to a positive shock to the aggregate component of productivity (henceforth, y-shock). Specifically, we carry out the following experiment. The economy at time $t = 0$ is at the steady state associated with the average realization of aggregate productivity. That is, at time $t = 0$, the aggregate component of productivity y is given by μ_y and the distribution of workers across employment states (u, g) is given by the ergodic distribution associated with μ_y. At time $t = 1$, aggregate productivity jumps up by 1 percent and, afterwards, remains at this higher level. Figures 2 through 4 illustrate the response of unemployment, vacancies and workers' transition rates to this aggregate productivity shock.

To better understand these responses, it is useful to discuss the effect of the y-shock on the policy functions r_d, θ_u and θ_e. The y-shock lowers r_d, the cutoff on the idiosyncratic component of productivity below which matches are endogenously destroyed. Intuitively, an increase in aggregate productivity raises the social value of employment relative to unemployment, and so it lowers the threshold on the idiosyncratic productivity below which it more efficient to break up a match than to maintain it. The y-shock increases θ_u, the tightness of the submarket where unemployed workers look for jobs. Intuitively, an increase in aggregate productivity increases the social value of moving workers out of unemployment and into employment and, hence, it increases the efficient vacancy-to-applicant ratio of the submarket visited by unemployed workers.

The effect of the y-shock on $\theta_e(z)$, the tightness of the submarket where workers employed in matches of quality z look for new jobs, is more complicated. In fact, the y-shock increases

$\theta_e(z)$ for low values of z, and it lowers $\theta_e(z)$ for high values of z. It is easy to explain this phenomenon. A positive shock to y raises the social value of a high quality match relative to a low quality match, because a worker employed in a better match is more likely to be employed in the future and, hence, more likely to take advantage of the increase in y. For this reason, a positive shock to y increases (*decreases*) the social value of moving a worker from a low (*high*) quality match to a new match, and so it increases (*decreases*) the efficient vacancy-to-applicant ratio in the submarket visited by workers who are currently employed in low (*high*) quality matches.

Figure 2 shows the response to the y-shock of the UE rate, $h^{ue} = \theta_u^\gamma$, the EU rate, $h^{eu} = \left[\sum d(z)g(z)\right]/(1-u)$, and EE rate, $h^{ee} = \left[\sum(1-d(z))\lambda_e\theta_e(z)^\gamma\right]/(1-u)$. The UE rate goes up because the increase in θ_u raises the probability than an unemployed worker finds a job. The EU rate falls because the decline in r_d lowers the fraction of new matches that are destroyed after their idiosyncratic productivity is revealed. On impact, the EE rate increases because of an increase in the average tightness of the submarkets where employed workers look for new jobs. Over time, the EE rate continues to grow because the distribution of employed workers shifts towards matches with lower idiosyncratic productivity, which have a higher probability of terminating with a job-to-job transition. Quantitatively, the 1 percent increase in y leads to a 2 percent increase in the steady state UE rate, a 4 percent decline in the steady state EU rate, and to a 4 percent increase in the steady state EE rate. As a result of both the increase in the UE rate and the decline in the EU rate, the steady state unemployment rate falls by 6 percent.

Figure 3 shows the response to the y-shock of the number of vacancies created for unemployed workers, $v_u = u\theta_u$, the number of vacancies created for employed workers, $v_e = \sum(1-d(z))\lambda_e\theta_e(z)g(z)$, and the total number of vacancies in the economy, $v = v_u + v_e$. On impact, v_u increases because of the increase in the number of vacancies that are created for each unemployed worker. Over time, as the number of unemployed workers falls towards its new steady state value, v_u returns to its initial level and then falls below it. The response of v_e is different. On impact, v_e jumps up because of the increase in the average number of vacancies created for each employed worker. Over time, as the number of employed workers grows towards its new steady state value, v_e continues to increase. Quantitatively, the 1 percent increase in y leads to a 2.5 percent decline in the steady state value of v_u and to a 5 percent increase in the steady state value of v_e. Since $v = v_u + v_e$ and $v_u \sim v_e$, the steady state number of vacancies increases by 2 percent.

Figure 4 shows the response of the average idiosyncratic productivity, $\overline{z} = \left[\sum zg(z)\right]/(1-u)$, and the average labor productivity, $\pi = y + \overline{z}$. The y-shock has two opposing effects on $\overline{z}$. On the one hand, the y-shock tends to lower $\overline{z}$ because it lowers the endogenous destruction cutoff z_d. On the other hand, the y-shock tends to increase $\overline{z}$ because it increases the probability that a worker employed in a low quality match finds a better job. In practice, the first effect dominates the second one, and the 1 percent increase in y leads to a 0.3 percent decline in the steady state value of $\overline{z}$. Since $\pi = y + \overline{z}$ and $\overline{z} \sim y/3$, the 1 percent increase in y leads to a 0.7 percent increase in the steady state value of average labor productivity.

Let us summarize our findings. According to the version of our model in which matches are experience goods, an aggregate productivity shock induces unemployment to move in the opposite direction than the UE and EE rates, vacancies and labor productivity, and in the same direction as the EU rate. Table 1 shows that this is exactly the same pattern of comovement that is observed in the US labor market at the business cycle frequency. Moreover, according to our model, an aggregate productivity shock induces movements in unemployment, vacancy and workers' transition rates that are large relative to the movement in the average productivity of labor π. Specifically, the response of unemployment is approximately 8 times larger than the response of π. The response of vacancies is 3 times larger than the response of π. And the response of the UE, EU and EE rates is respectively 2, 6 and 5 times larger than the response of π. Table 1 shows that the volatility generated by the aggregate productivity shock constitutes a large fraction of the overall volatility observed in the US labor market at the business cycle frequency. In the US data, unemployment is 10 times more volatile than π, vacancies are 11 times more volatile than π, and the UE, EU and EE rates are approximately 5 times more volatile than π. Finally, Table 3 shows that the implications of our model are substantially the same if, instead of looking at the response to a y-shock, we simulate the stochastic economy and compute the volatility of the model-generated time series for unemployment, vacancies and workers' transition rates.

5.2 Role of match heterogeneity and search on the job

In section 4, we showed that the model needs match heterogeneity and search on the job in order to fit the main acyclical features of worker reallocation in the US labor market. Here, we show that these two features of the model are also needed in order to properly measure the effect of aggregate productivity fluctuations on the US labor market. To make this point precise, we calibrate and simulate two constrained versions of the model. First, we calibrate and simulate a version of the model in which the parameters λ_e and σ_z are constrained to be zero and, hence, matches are homogeneous and search only takes place off the job. We refer to this version of our model as P-00 because it is equivalent to the textbook model by Pissarides (2000, Chapter 1). Second, we calibrate and simulate a version of the model in which σ_z is allowed to be positive but λ_e is constrained to be zero and, hence, search on the job is ruled out. We refer to this version of the model as MP-94 because it is very similar to the classic model by Mortensen and Pissarides (1994).

The top half of Table 4 presents a statistical summary of the effect that aggregate productivity shocks have on unemployment, vacancies and workers' transition rates in P-00. Note that y-shocks generate nearly ten times less unemployment volatility in P-00 than in our model. This striking difference is due to the fact that y-shocks generate much less volatility in both the UE and EU rates in P-00 than they do in our model. It is easy to explain why the volatility of the UE rate is lower in P-00. The elasticity of the UE rate with respect to y is given by the product between the elasticity of the job finding probability with respect to the vacancy-to-applicant ratio, γ, and the elasticity of the vacancy-to-applicant ratio in the submarket visited by unemployed workers with respect to y, i.e. $d\log(v_u/u)/d\log y$. The value of $d\log(v_u/u)/d\log y$ is similar in the two models. However, the value of γ is much smaller in P-00 than in our model. To understand this, remember that γ is chosen so that the elasticity of the UE rate with respect to the vacancy-to-applicant ratio is the same as in the data (0.27). That is, γ is chosen so that

$$\frac{d\log h^{ue}}{d\log(v/u)}/=0.27 \Longrightarrow \gamma = 0.27\frac{d\log(v/u)}{d\log(v_u/u)}. \tag{18}$$

In our model, $d\log(v/u)$ is twice as large as $d\log(v_u/u)$ because the elasticity of the number of vacancies created for employed workers is higher than the elasticity of the number of vacancies created for unemployed workers. Hence, in our model, $\gamma = 0.6$. In P-00, $d\log(v/u)$ equals $d\log(v_u/u)$ because, without search on the job, there are no vacancies created for employed workers. Hence, in P-00, $\gamma = 0.27$.

It is also easy to explain why the volatility of the EU rate is lower in P-00 than in our model. In our model, a shock to the aggregate component of productivity affects the EU rate because it affects the cutoff r_d on the idiosyncratic component of productivity below which a match is endogenously destroyed. Quantitatively, the effect on the EU rate is large because, according to the calibration, the distribution of idiosyncratic productivity has a high density around the steady state value of r_d. In P-00, the distribution of idiosyncratic productivity is constrained to be degenerate at $z = 0$, and so an aggregate productivity shock has no effect on the EU rate.

In light of these observations, it is clear why models that abstract from match heterogeneity and search on the job typically predict that the response of the unemployment rate to aggregate productivity shocks is implausibly small (e.g. Shimer 2005) unless additional amplification mechanisms are introduced (e.g. training costs in Mortensen and Nagypál 2007, countercyclical vacancy costs in Shao and Silos 2009, exogenous wage rigidity in Hall 2005 and Gertler and Trigari 2009, endogenous wage stickiness in Menzio 2005, Kennan 2010, and

Menzio and Moen 2010).

The bottom half of Table 4 presents a statistical summary of the effect that aggregate productivity shocks have on unemployment, vacancies and workers' transition rates in MP-94. In MP-94, the volatility of the UE rate is nearly 3 times smaller than in our model and the volatility of the EU rate is approximately the same as in our model. As a result, the volatility of the unemployment rate is approximately 30 percent lower in MP-94 than in our model. These findings are easy to explain. The volatility of the EU rate is similar in the two models because the calibrated distribution of the idiosyncratic productivity is similar and because the aggregate productivity shocks have a similar effect on the destruction cutoff r_d. The volatility of the UE rate is lower in MP-94 because the calibrated value of γ is smaller (for exactly the same reason why it is smaller in P-00).

Next, note that the correlation between total vacancies and labor productivity is negative in MP-94, while it is positive in our model. Let us explain this difference. In our model, a positive shock to y generates a decline in the number of vacancies created for unemployed workers, v_u, and an increase in the number of vacancies created for employed workers, v_e. Since the second effect dominates the first one, a positive y-shock leads to an increase in the total number of vacancies in the economy, v. In MP-94, a positive y-shock also generates a decline in v_u. This is because the fall in the EU rate is so large that the increase in the number of vacancies created for each unemployed worker, θ_u, is dominated by the decline in the number of unemployed workers, u. However, in MP-94, a positive y-shock has no effect on v_e because, without search on the job, firms do not create any vacancies for employed workers. Hence, in MP-94, the total number of vacancies in the economy falls in response to a positive aggregate productivity shock.

These observations explain why models that abstract from search on the job tend to predict a positive correlation between vacancies and unemployment whenever the EU rate is strongly countercyclical either for endogenous reasons (e.g. movements in the endogenous destruction cutoff r_d as in Mortensen and Pissarides 1994 and Merz 1995) or for exogenous reasons (e.g. shocks to the match destruction probability as in Shimer 2005).[10]

[10] In a paper contemporaneous to ours, Ramey (2008) makes a similar point. Specifically, using a model of random search on the job, Ramey shows that the correlation between unemployment and vacancies generated by aggregate productivity shocks is positive when employed workers are not allowed to search, and it is negative when employed workers search as frequently as unemployed workers. There are two differences between this result and ours. First, our result is obtained using a model of directed search, while Ramey's result is obtained using the random search model by Mortensen (1994). As we discussed in the introduction, there are important economic differences between these two models. Second, Ramey shows that the correlation between unemployment and vacancies is negative when employed workers search as frequently as unemployed workers. This arbitrary assumption might drive Ramey's result as it is likely to overestimate the importance of search on the job. In contrast, in this paper, the frequency at which employed workers get the opportunity to search is calibrated to match the average EE rate observed in the data.

6 Inspection Model

In this section, we study the effect of a 1 percent increase in aggregate productivity on unemployment, vacancies and workers' transition rates for the version of the model in which matches are inspection goods. In this version of the model, workers and firms receive a perfectly informative signal about the quality of their match before starting production. Hence, an unemployed worker searches off the job until he finds a match that is more valuable than unemployment. Similarly, an employed worker searches on the job until he finds a match that is more valuable than the one he has with his current employer. Note that, since the value of all matches that are created is greater than the value of unemployment, employed workers move back into unemployment only when their match is hit by the exogenous destruction shock δ.

In the version of the model where matches are inspection goods, the +1% shock to y increases θ_u, the tightness of the submarket where unemployed workers look for matches, and it lowers r_u, the cutoff on the idiosyncratic productivity above which a match between an unemployed worker and a firm is created. Intuitively, an increase in aggregate productivity increases the social value of employment relative to unemployment and, for this reason, it increases the efficient vacancy-to-applicant ratio and it lowers the efficient creation cutoff in the submarket visited by unemployed workers. For the same reason, the positive shock to y lowers r_d, the cutoff on the idiosyncratic productivity below which a match is endogenously destroyed.

In contrast, the y-shock has no effect on $\theta_e(z)$ and $r_e(z)$, the tightness and the creation cutoff in the submarket where workers employed in matches of quality z look for new jobs. It is easy to explain this effect. An increase in y raises the social value of matches with different quality by exactly the same amount, because workers employed in matches with different quality have exactly the same probability of being employed in the future (i.e. $1-\delta$) and, hence, the same probability of taking advantage of the increase in y. For this reason, an increase in y has no effect on the social value of moving a worker from a match of quality z to a new match, and so it has no effect on the efficient vacancy-to-applicant ratio, $\theta_e(z)$, and the efficient creation cutoff, $r_e(z)$.

Figure 5 shows the response to the y-shock of the UE rate, $h^{ue} = \theta_u^{\gamma} m_u$, EU rate, $h^{ue} = \left[\sum d(z)g(z)\right]/(1-u)$, and EE rate, $h^{ee} = \left[\sum (1-d(z))\lambda_e \theta_e(z)^{\gamma} m_e(z) g(z)\right]/(1-u)$. The UE rate increases for two reasons. First, the increase in θ_u increases the probability that an unemployed worker meets a firm. Second, the decline in r_u increases the probability that a meeting between an unemployed worker and a firm turns into a match. Quantitatively, the steady state UE rate increases by 0.8 percent. The EE rate increases because of a composition effect. The shock affects neither $\theta_e(z)$ nor $r_e(z)$ and, hence, it affects neither the probability that a worker employed in a match of quality z meets a new firm, nor the probability that such a meeting turns into a match. However, because it lowers r_u, the y-shock shifts the distribution of employed workers towards matches with lower quality, which have a higher probability of terminating with a job-to-job transition. Quantitatively, the steady state EE rate increases by 0.05 percent. The EU rate does not respond to the shock. This result is intuitive. Since the quality of a match is perfectly observed before the match is created, a worker moves from employment to unemployment only when the match is hit by the destruction shock, an event which occurs with the exogenous and time-invariant probability δ. From the response of the workers' transition rates, it follows that the steady state unemployment rate falls by 0.75 percent.

Figure 6 shows how the y-shock affects the number of vacancies for unemployed workers, v_u, the number of vacancies for employed workers, v_e, and the total number of vacancies in the economy, v. The steady state value of v_u increases because the increase in the number of vacancies created per each unemployed workers, θ_u, is stronger than the decline in the number of unemployed workers, u. The steady state value of v_e increases both because of an increase in the average number of vacancies created for each employed worker, $\mathbb{E}\theta_e(z)$, and because of an increase in the number of employed workers, $1-u$. Quantitatively, the steady state value of v_u increases by 2.5 percent and the steady state value of v_e increases by 0.1 percent. Since $v = v_u + v_e$ and $v_u \sim v_e$, the steady state value of v increases by 2.75 percent. Finally, Figure 7 shows that the y-shock decreases the average quality of a match by 0.05 percent and increases the average productivity of labor by 0.99 percent.

Overall, the response to the y-shock of unemployment, vacancies and workers' transition rates is much smaller in the version of the model where matches are inspection rather than experience goods. In particular, the response of unemployment (relative to average productivity) is 10 times smaller when matches are inspection goods, due to both a smaller decline in the EU rate and a smaller increase in the UE rate. It is not surprising that the decline in the EU rate is muted when matches are inspection goods. However, it is somewhat surprising that the increase in the UE rate is smaller when matches are inspection goods considering that, in this version of the model, the y-shock increases not only the probability that an unemployed worker meets a firm but also the probability that the meeting turns into a match. There is a simple explanation for this result. The elasticity of the UE rate with respect to y is given by

$$\frac{d\log h^{ue}}{d\log y} = \frac{d\log m_u}{d\log y} + \gamma \frac{d\log \theta_u}{d\log y}. \tag{19}$$

The parameter γ is calibrated so as to match the empirical elasticity of the UE rate with respect to the vacancy-to-unemployment ratio, i.e.

$$\frac{d\log h^{ue}}{d\log(v/u)} = 0.27 \Longrightarrow \gamma = \frac{0.27\ d\log(v/u) - d\log m_u}{d\log(\theta_u)}. \tag{20}$$

Substituting (20) into (19), one obtains

$$\frac{d\log h^{ue}}{d\log y} = 0.27\frac{d\log(v/u)}{d\log y}. \tag{21}$$

The previous expression demonstrates that, once γ is calibrated, the elasticity of the UE rate with respect to y does not depend on the elasticity of the probability with which a meeting turns into a match. It only depends on the empirical elasticity of the UE rate with respect to v/u and on the elasticity of v/u with respect to y. And since the elasticity of v/u with respect to y is smaller when matches are inspection goods (due to the smaller response in v_e), so is the elasticity of the UE rate.

7 Conclusion

In this paper, we developed a model of directed search on the job in which the transitions of workers between employment, unemployment and across different employers are driven by heterogeneity in the quality of different firm-worker matches. In the theoretical part of the paper, we proved that the unique equilibrium is efficient, in the sense that it decentralizes the solution to the planner's problem, and block recursive, in the sense that the agents' value and policy functions depend on the aggregate state of the economy only through the realization of aggregate shocks and not through the entire distribution of workers across employment states (unemployment and employment in different matches). Because the equilibrium is block recursive, the model can be easily solved outside of the steady state and, hence, used for studying the cyclical dynamics of the labor market. In the empirical part of the paper, we first calibrated the model to match the frequency and pattern of the transition of individual workers across employment states. We then simulated the model to measure the effect that cyclical fluctuations in aggregate productivity have on the labor market. We found that, when matches are experience goods, aggregate productivity shocks account for the empirical pattern of comovement between unemployment, vacancies and workers' transition rates and for a large fraction of their empirical volatility. In contrast, when matches are inspection goods, aggregate productivity shocks can only account for a negligible fraction of the empirical volatility of the labor market.

Appendix

A Proof of Theorem 1

(i) Let $C(\Psi)$ be the set of bounded continuous functions $R:\Psi\to\mathbb{R}$ with the sup norm, $\|R\|=\sup_{\psi\in\Psi}R(\psi)$. Define the operator T on $C(\Psi)$ by

$$\begin{aligned}
&(TR)(\psi)=\max\nolimits_{(d,\theta_u,\theta_e,c_u,c_e)}F(d,\theta_u,\theta_e,c_u,c_e|\psi)+\beta\mathbb{E}R(\hat{\psi})\\
\text{s.t.}\quad &\hat{u}=u\left[1-\lambda_u p(\theta_u)m_u\right]+\textstyle\sum_z\left[d(z)g(z)\right],\\
&\hat{g}(z')=u\lambda_u p(\theta_u)\left[\alpha c_u(z')+(1-\alpha)m_u\right]f(z')\\
&\qquad +g(z')\left[1-d(z')\right]\left[1-\lambda_e p(\theta_e(z'))m_e(z')\right]\\
&\qquad +\textstyle\sum_z g(z)\left\{\left[1-d(z)\right]\left[\lambda_e p(\theta_e(z))\right]\left[\alpha c_e(z',z)+(1-\alpha)m_e(z)\right]f(z')\right\}\\
&d:Z\to[\delta,1],\ \theta_u\in\mathbb{R}_+,\ \theta_e:Z\to\mathbb{R}_+,\ c_u:Z\to[0,1],\ c_e:Z\times Z\to[0,1].
\end{aligned}\tag{A1}$$

The return function F is defined as

$$F(d,\theta_u,\theta_e,c_u,c_e|\psi)=-k\left\{\lambda_u\theta_u u+\textstyle\sum_z\left[(1-d(z))\lambda_e\theta_e(z)g(z)\right]\right\}+b\hat{u}+\textstyle\sum_z\left[(y+z)\hat{g}(z)\right].\tag{A2}$$

First, we prove that TR is bounded. Take an arbitrary $R\in C(\Psi)$. Since R is bounded, there exists $\underline{R}$ and $\overline{R}$ such that $\underline{R}\leq R(\hat{\psi})\leq\overline{R}$ for all $\hat{\psi}\in\Psi$. Hence, $(TR)(\psi)$ is bounded above by below by $(N(z)+1)\min\{b,y_1+z_1\}+\beta\underline{R}$ and it is bounded above by $(N(z)+1)\max\{b,y_{N(y)}+z_{N(z)}\}+\beta\overline{R}$. Now, we prove that TR is continuous in ψ. Let $\overline{\theta}$ be defined as

$$\overline{\theta}=k^{-1}\left\{(N(z)+1)\left[\max\{b,y_{N(y)}+z_{N(z)}\}-\min\{b,y_1+z_1\}\right]+\beta\left[\overline{R}-\underline{R}\right]\right\}.$$

Note that the maximand in (A1) is strictly smaller than $(N(z)+1)\min\{b,y_1+z_1\}+\beta\underline{R}$ for any $\theta_u>\overline{\theta}$ or for any $\theta_e(z)>\overline{\theta}$. Therefore, the problem in (A1) is equivalent to the problem in which the constraint $\theta_u\in\mathbb{R}_+$ is replaced with $\theta_u\in[0,\overline{\theta}]$, and the constraint $\theta_e:Z\to\mathbb{R}_+$ is replaced with $\theta_e:Z\to[0,\overline{\theta}]$. For the modified problem, the maximand is continuous in $(\psi,d,\theta_u,\theta_e,c_u,c_e)$ and set of feasible choices for $(d,\theta_u,\theta_e,c_u,c_e)$ is compact. Then it follows from the Theorem of the Maximum (Theorem 3.6 in Stokey, Lucas and Prescott, 1989) that TR is continuous in ψ. Hence: $T:C(\Psi)\to C(\Psi)$.

The operator T maps the set of bounded continuous function $C(\Psi)$ into itself, and one can easily verify that it satisfies the monotonicity and discounting hypotheses in Blackwell's sufficient conditions for a contraction (Theorem 3.3 in Stokey, Lucas and Prescott, 1989). Hence, the operator T is a contraction mapping and it admits one and only one fixed point $R^*\in C(\Psi)$. Since $\lim_{t\to\infty}\beta^t R^*(\psi)=0$ for all $\psi\in\Psi$, it follows from Theorem 4.3 in Stokey, Lucas and Prescott (1989) that R^* is equal to the planner's value function W.

(ii) Let $C'(\Psi) \subset C(\Psi)$ be the set of functions $R : \Psi \to \mathbb{R}$ that are bounded, continuous and linear in the measure of unemployed workers, u, and in the measure of workers employed in matches of type z, $g(z)$. Clearly, $R \in C'(\Psi)$ if and only if there exist two functions $R_u : Y \to \mathbb{R}$ and $R_e : Z \times Y \to \mathbb{R}$ such that

$$R(\psi) = R_u(y)u + \textstyle\sum_z R_e(z,y)g(z). \tag{A3}$$

Consider an arbitrary function R in $C'(\Psi)$. Then, after substituting the constraints into the maximand of (A1), we obtain

$$(TR)(\psi) = \hat{R}_u(y)u + \textstyle\sum_z \hat{R}_e(z,y)g(z), \tag{A4}$$

where $\hat{R}_u(y)$ is given by

$$\begin{aligned} \hat{R}_u(y) = \quad & \max_{(\theta_u, c_u)} \{-k\lambda_u\theta_u + (1-\lambda_u p(\theta_u)m_u)\,[b + \beta\mathbb{E}R_u(\hat{y})] \\ & \quad + \lambda_u p(\theta_u) \textstyle\sum_{z'} [\alpha c_u(z') + (1-\alpha)m_u]\,[y + z' + \beta\mathbb{E}R_e(z',\hat{y})]\, f(z')\} \\ & \text{s.t. } \theta_u \in \mathbb{R}_+, \quad c_u : Z \to [0,1]. \end{aligned} \tag{A5}$$

and $\hat{R}_e(z,y)$ is given by

$$\begin{aligned} \hat{R}_e(z,y) = \quad & \max_{(d,\theta_e,c_e)} \{d\,[b + \beta\mathbb{E}R_u(\hat{y})] - (1-d)\,k\lambda_e\theta_e \\ & \quad + (1-d)\,(1-\lambda_e p(\theta_e)m_e)\,[y + z + \beta\mathbb{E}R_e(z,\hat{y})] \\ & \quad + (1-d)\lambda_e p(\theta_e) \textstyle\sum_{z'} [\alpha c_e(z') + (1-\alpha)m_e]\,[y + z' + \beta\mathbb{E}R_e(z',\hat{y})] f(z')\} \\ & \text{s.t. } d \in [\delta, 1], \quad \theta_e \in \mathbb{R}_+, \quad c_e : Z \to [0,1]. \end{aligned} \tag{A6}$$

Since R is an arbitrary function in $C'(\Psi)$, (A4) implies that $T : C'(\Psi) \to C'(\Psi)$. Moreover, since $C'(\Psi)$ is a closed subset of $C(\Psi)$ and $T : C'(\Psi) \to C'(\Psi)$, Corollary 1 to Theorem 3.2 in Stokey, Lucas and Prescott (1989) implies $W \in C'(\Psi)$.

(iii) Let $C''(\Psi) \subset C'(\Psi)$ be the set of functions $R : \Psi \to \mathbb{R}$ such that the associated component R_e is nondecreasing in z. Let R be an arbitrary function in $C''(\Psi)$. From part (ii), it follows that $TR \in C'(\Psi)$ and the associated components $\hat{R}_u$ and $\hat{R}_e$ satisfy the equations (A5) and (A6). Since the maximand in (A6) is nondecreasing in z and the feasible set in (A6) is independent of z, $\hat{R}_e$ is nondecreasing in z. Hence, $T : C''(\Psi) \to C''(\Psi)$. Since $C''(\Psi)$ is a closed subset of $C(\Psi)$, Corollary 1 to Theorem 3.2 in Stokey, Lucas and Prescott (1989) implies that $W \in C''(\Psi)$.

(iv) From part (ii), it follows that the policy correspondences (θ_u^*, c_u^*) solve the maximization problem (A5) for $(R_u, R_e) = (W_u, W_e)$. Since the maximand and the constraints in (A5) do not depend on (u,g), (θ_u^*, c_u^*) depend on ψ only through y and not through (u,g). Similarly, the policy correspondences (d^*, θ_e^*, c_e^*) solve the maximization problem (A6) for $(R_u, R_e) = (W_u, W_e)$. Since the maximand and the constraints in (A6) do not depend on (u,g), (θ_u^*, c_u^*) depend on ψ only through y and not through (u,g). ■

B Proof of Proposition 2

For any $y \in Y$, $\theta_u^*(y)$ and $c_u^*(z,y)$ are the solutions to the maximization problem

$$\begin{aligned} &\max_{(\theta_u, c_u)} \{-k\lambda_u\theta_u + (1-\lambda_u p(\theta_u)m_u)\,[b + \beta\mathbb{E}W_u(\hat{y})] \\ &\qquad\qquad +\lambda_u p(\theta_u)\textstyle\sum_{z'}\left[\alpha c_u(z') + (1-\alpha)m_u\right][y + z' + \beta\mathbb{E}W_e(z',\hat{y})]\, f(z')\} \qquad \text{(A7)} \\ &\text{s.t. } \theta_u \in [0, \bar{\theta}], \quad c_u : Z \to [0,1]. \end{aligned}$$

which can be rewritten as

$$\begin{aligned} &\max_{\theta_u \in [0,\bar{\theta}]} \{-k\lambda_u\theta_u + b + \beta\mathbb{E}W_u(\hat{y}) + \lambda_u p(\theta_u)\times \\ &\quad \times \max_{c_u : Z \to [0,1]} \textstyle\sum_{z'} \{[\alpha c_u(z') + (1-\alpha)m_u]\,[y + z' - b + \beta\mathbb{E}\left(W_e(z',\hat{y}) - W_u(\hat{y})\right)] f(z')\} \Big\}. \end{aligned} \tag{A8}$$

First, consider the inner maximization problem in (A8). The maximand is linear in c_u and its derivative with respect to $c_u(s)$ is given by

$$\alpha\,[y + s + \beta\mathbb{E}W_e(s,\hat{y})] + (1-\alpha)\mathbb{E}_{z'}[y + z' + \beta\mathbb{E}W_e(z',\hat{y})] - b - \beta\mathbb{E}W_u(\hat{y}). \tag{A9}$$

Hence, the solution to the maximization problem is $c_u^*(s,y) = 1$ if (A9) is positive, and $c_u^*(s,y) = 0$ if (A9) is strictly negative. Therefore, $c_u^*(s,y)$ is unique. Moreover, since (A9) is increasing in s, $c_u^*(s,y)$ is increasing in s. Therefore, there exists $r_u^*(y)$ such that $c_u^*(s,y) = 1$ if $s \geq r_u^*(y)$, and $c_u^*(s,y) = 0$ else. This completes the proof of parts (i) and (iii) of the proposition for c_u^*.

Next, consider the outer maximization problem in (A8). The derivative of the maximand with respect to θ_u is given by

$$-k + p'(\theta_u) \sum_{s \geq r_u^*(y)} \left\{ \begin{array}{l} \alpha\,[y + s - b + \beta\mathbb{E}\left(W_e(s,\hat{y}) - W_u(\hat{y})\right)] \\ +(1-\alpha)\mathbb{E}_{z'}\left[y + z' - b + \beta\mathbb{E}\left(W_e(z',\hat{y}) - W_u(\hat{y})\right)\right] \end{array} \right\} f(s). \tag{A10}$$

The expression above is strictly decreasing in θ_u because $p''(\theta_u) < 0$, and it is strictly negative at $\theta_u = \bar{\theta}$ because $p'(\bar{\theta}) = 0$. Hence, the solution to the maximization problem, $\theta_u^*(y)$, is unique and such that

$$k \geq p'(\theta_u^*(y)) \sum_{s \geq r_u^*(y)} \left\{ \begin{array}{l} \alpha \left[y + s - b + \beta \mathbb{E} \left(W_e(s, \hat{y}) - W_u(\hat{y})\right)\right] \\ +(1-\alpha)\mathbb{E}_{z'} \left[y + z' - b + \beta \mathbb{E} \left(W_e(z', \hat{y}) - W_u(\hat{y})\right)\right] \end{array} \right\} f(s), \quad \text{(A11)}$$

and $\theta_u^*(y) \geq 0$, with complementary slackness. This completes the proof of part (i) of the proposition for θ_u^*. The proofs of parts (i) and (iii) for c_e^*, θ_e^* and d^*, as well as the proofs of parts (ii) and (iv) are omitted for the sake of brevity. ■

C Proof of Theorem 4

(i)-(ii) Let $(\theta, V_u, V_e, x_u, r_u, d, x_e, r_e)$ be an equilibrium. We take five steps to prove that the equilibrium is unique and block recursive.

Step 1. Unify the notation for V_u and V_e. Let the function $V : \{0,1\} \times Z \times \Psi \to \mathbb{R}$ be defined as $V(0, z, y) = V_u(\psi)$ for all $(z, \psi) \in Z \times \Psi$, and $V(1, z, y) = V_e(z, \psi)$ for all $(z, \psi) \in Z \times \Psi$. Given the definition of V, we can rewrite the equilibrium conditions (12) and (14) as

$$\begin{aligned} & V(a, z, \psi) \\ = \quad & a \left\{ y + z + \beta \mathbb{E} \max_{(d,x,r)} \left\{ \begin{array}{l} dV(0, z, \hat{\psi}) + (1-d)V(1, z, \hat{\psi}) \\ +(1-d)\lambda_e p\left(\theta(x, r, \psi)\right) m(r) \left[x - V(1, z, \hat{\psi})\right] \end{array} \right\} \right\} \qquad \text{(A12)} \\ + \quad & (1-a) \left\{ b + \beta \mathbb{E} \max_{(x,r)} \left\{ \begin{array}{l} V(0, z, \hat{\psi}) + \\ \lambda_u p(\theta(x, r, \hat{\psi})) m(r) \left[x - V(0, z, \hat{\psi})\right] \end{array} \right\} \right\}. \end{aligned}$$

Step 2. Express the value offered in submarket x as a function of the tightness θ, the reservation signal r, and the aggregate state of the economy ψ. Let $x(\theta, r, \psi)$ denote the value offered to a worker in a submarket with tightness $\theta(x, r, \psi) = \theta > 0$. From the equilibrium condition (16), it follows that

$$x(\theta, r, \psi) = \frac{1}{m(r)} \left\{ \sum\nolimits_{s \geq r} \left\{ \left[\alpha V(1, s, \psi) + (1-\alpha)\mathbb{E}_z V(1, z, \psi)\right] f(s) \right\} - \frac{k}{q(\theta)} \right\}. \quad \text{(A13)}$$

In any submarket with $\theta(x, r, \psi) = 0$, the value offered to a worker cannot be expressed uniquely as a function of (θ, r, ψ). However, the value offered to a worker in these submarkets is irrelevant because the worker meets a vacancy with zero probability. Hence, without loss in generality, let $x(\theta, r, \psi) = 0$ in all submarkets with tightness $\theta(x, r, \psi) = \theta = 0$.

Step 3. Reformulate the equilibrium condition for V. Substituting x with $x(\theta, r, \psi)$ and

$\theta(x, r, \psi)$ with θ, we can rewrite (A12) as

$$
\begin{aligned}
& V(a,z,\psi) \\
= \quad & a \left\{ y + z + \beta \mathbb{E} \max_{(d,\theta,r)} \left\{ \begin{array}{l} dV(0,z,\hat{\psi}) - (1-d)\lambda_e k\theta + (1-d)(1-\lambda_e p(\theta) m(r)) V(1,z,\hat{\psi}) \\ +(1-d)\lambda_e p(\theta) \sum_{s\geq r} \left[\alpha V(1,s,\hat{\psi}) + (1-\alpha)\mathbb{E}_z V(1,z,\hat{\psi}) \right] f(s) \end{array} \right\} \right\} \\
+ \quad & (1-a) \left\{ b + \beta \mathbb{E} \max_{(\theta,r)} \left\{ \begin{array}{l} -k\lambda_u \theta + (1-\lambda_u p(\theta) m(r)) V(0,z,\hat{\psi}) + \\ \lambda_u p(\theta) \sum_{s\geq r} \left[\alpha V(1,s,\hat{\psi}) + (1-\alpha)\mathbb{E}_z V(1,z,\hat{\psi}) \right] f(s) \end{array} \right\} \right\}.
\end{aligned} \tag{A14}
$$

Step 4. Establish the uniqueness of V and its independence from (u, g). Let $\Omega = \{0, 1\} \times Z \times \Psi$ and let $C(\Omega)$ denote the space of bounded continuous functions $R : \Omega \to \mathbb{R}$, with the sup norm. Let $T : C(\Omega) \to C(\Omega)$ denote the operator associated with (A14). It is straightforward to verify that: (i) $R, R' \in C(\Omega)$ and $R \leq R'$ implies $T(R) \leq T(R')$; (ii) $R \in C(\Omega)$ and $\epsilon \geq 0$ implies $T(R + \epsilon) = TR + \beta\epsilon$. Therefore, by Blackwell's sufficient conditions, it follows that the operator T is a contraction and that it admits a unique solution. Hence, V is unique. Next, notice that if R depends on $\hat{\psi}$ only through $\hat{y}$, then $T(R)$ depends on ψ only through y. Hence, the fixed point of the operator T depends on ψ only through y. That is, $V(a, y, \psi) = V(a, z, y)$.

Step 5. Establish the uniqueness of the policy functions $(\theta, x_u, r_u, d, x_e, r_e)$ and their independence from (u, g). Since $V(a, z, \psi)$ only depends on ψ through y, we can rewrite the equilibrium condition (16) as

$$
k \geq q(\theta(x,r,\psi)) \sum_{s\geq r} \left\{ \left[\alpha V_e(s,y) + (1-\alpha)\mathbb{E}_z V_e(z,y) - x \right] f(s) \right\} \tag{A15}
$$

and $\theta(x, r, \psi) \geq 0$, with complementary slackness. It is easy to verify that $\theta(x, r, \psi)$ is unique and only depends on ψ through y; that is, $\theta(x, r, \psi) = \theta(x, r, y)$. Since $V(a, z, \psi)$ and $\theta(x, r, y)$ only depend on ψ through y, we can rewrite the equilibrium condition (12) as

$$
V_u(y) = b + \beta \mathbb{E} \max_{(x,r)} \left\{ V_u(\hat{y}) + \lambda_u p(\theta(x,r,\hat{y})) m(r) \left[x - V_u(\hat{y}) \right] \right\}. \tag{A16}
$$

Since the maximization problem in (A16) only depends on $\hat{\psi}$ through $\hat{y}$, the associated policy functions $(x_u(\hat{\psi}), r_u(\hat{\psi}))$ only depend on $\hat{\psi}$ through $\hat{y}$. That is $(x_u(\hat{\psi}), r_u(\hat{\psi})) = (x_u(\hat{y}), r_u(\hat{y}))$. Similarly, we can show that the policy functions $d(\hat{\psi})$ and $(x_e(\hat{\psi}), r_e(\hat{\psi}))$ only depend on $\hat{\psi}$ through $\hat{y}$. That is, $d(\hat{\psi}) = d(\hat{y})$ and $(x_e(\hat{\psi}), r_e(\hat{\psi})) = (x_e(\hat{y}), r_e(\hat{y}))$. This completes the proof that there exists a unique equilibrium and that this equilibrium is block recursive.

(iii) To establish the equivalence between the equilibrium and the planner's allocation, we rewrite the component value functions (5) and (6). Recall that, in the planner's allocation, a match is formed if and only if the signal s is greater than or equal to the cutoff level $r_u^*(y)$ for an unemployed worker and $r_e^*(z,y)$ for a worker employed in a type-z match (see Proposition 2). Using r_u and r_e as the choices instead of (c_u, c_e), we can rewrite (5) as

$$\begin{aligned} W_u(y) = \max_{(\theta_u, r_u)} \quad & \{-k\lambda_u\theta_u + (1-\lambda_u p(\theta_u)m_u)\left[b + \beta\mathbb{E}W_u(\hat{y})\right] \\ & +\lambda_u p(\theta_u) \textstyle\sum_{s \geq r_u} \left\{\left[\alpha W_e(s,y) + (1-\alpha)\mathbb{E}_{z'}\left(y + z' + \beta\mathbb{E}W_e(z',\hat{y})\right)\right] f(s)\right\}\right\}. \end{aligned} \tag{A17}$$

Similarly, we can rewrite (6) as

$$\begin{aligned} & W_e(z,y) \\ = \quad & \max_{(d,\theta_e,r_e)} \{d\left[b + \beta\mathbb{E}W_u(\hat{y})\right] - (1-d)\, k\lambda_e\theta_e \\ & +(1-d)\,(1-\lambda_e p(\theta_e)m_e)\left[y + z + \beta\mathbb{E}W_e(z,\hat{y})\right] \\ & +(1-d)\lambda_e p(\theta_e) \textstyle\sum_{s \geq r_e} \left\{\left[\alpha\left(y + s + \beta\mathbb{E}r_e(s,\hat{y})\right) + (1-\alpha)\mathbb{E}_z\left(y + z + \beta\mathbb{E}r_e(z,\hat{y})\right)\right] f(s)\right\}\right\}. \end{aligned} \tag{A18}$$

Using these equations, we can verify that (A14) is satisfied by the function $W'(a,z,y)$ defined as $W'(0,z,y) = b + \beta\mathbb{E}W_u(\hat{y})$ and $W'(1,z,y) = y + z + \beta\mathbb{E}W_e(z,\hat{y})$. Since V is the unique solution to (A14), it follows that $V_u(y) = b + \beta\mathbb{E}W_u(\hat{y})$ and $V_e(z,y) = y + z + \beta\mathbb{E}W_e(z,\hat{y})$. Finally, notice that the equilibrium allocation solves the maximization problems in (A14), while the efficient allocation solves the maximization problems in (A17) and (A18). With the relations $V_u(y) = b + \beta\mathbb{E}W_u(\hat{y})$ and $V_e(z,y) = y + z + \beta\mathbb{E}W_e(z,\hat{y})$, it is not difficult to see that the two sets of allocations coincide. ■

D Data and calibration

We choose the model period to be one month. We set β so that the real interest rate in the model is 5 percent per year. We choose k and δ so that the average UE and EU transition rates are the same in the model and in the data. In the model, the UE rate is given by $h^{ue} = p(\theta_u)m_u$, and the EU rate is given by $h^{eu} = \left[\sum d(z)g(z)\right]/(1-u)$. In the data, we measure these transition rates following the methodology developed by Shimer (2005).[11] Specifically, we measure the UE rate in month t as $h_t^{ue} = u_{t+1}^s/(1-u_t)$, where u_t is the CPS unemployment rate in month t, and u_{t+1}^s is the CPS short-term unemployment rate in month $t+1$. Similarly, we measure the EU rate in month t as $h_t^{eu} = 1 - (u_{t+1} - u_{t+1}^s)/u_t$.

[11] There are two differences between the cyclical measures of the UE and EU rates constructed by Shimer (2005) and ours. First, Shimer multiplies the short-term unemployment rate by 1.1 in every month after February 1994 in order to correct for the fact that the 1994 redesign of the CPS changed the way in which unemployment duration is measured. In this paper, we follow Elsby et al. (2009) who argue that the short-term unemployment rate should be multiplied by 1.15 not 1.1. Second, Shimer computes the cyclical component of the log of quarterly workers' transition rates by using an HP-filter with a smoothing parameter of 100,000. In this paper, we use an HP-filter with the more standard smoothing parameter of 1600.

We normalize λ_u to 1, and we choose λ_e so that the average EE transition rate is the same in the model as in the data. The EE rate in the model is given by $h^{ee} = \left[\sum(1-d(z))\lambda_e p(\theta_e(z))m_e(z)g(z)\right]/(1-u)$. The EE rate in the data has been measured by Nagypál (2008) using the CPS microdata. Specifically, Nagypál measures the EE rate in month t as $h_t^{ee} = s_t/e_t$, where s_t is the number of workers who are employed at different firms in months t and $t+1$, and e_t is the number of workers who are employed in month t.

We choose γ so that the elasticity of the UE rate with respect to the vacancy-to-unemployment ratio is the same in the model as in the data. In the model, the vacancy-to-unemployment ratio is given by v/u, where the aggregate measure of vacancies v is given by the sum of $\lambda_u\theta_u u$ and $\sum(1-d(z))\lambda_e\theta_e(z)g(z)$. In the data, the vacancy-to-unemployment ratio is measured as the ratio of the Conference Board Help-Wanted Index and the CPS unemployment rate.

We normalize μ_z to 0. We choose the scale ν_z and shape σ_z parameters in the distribution of the idiosyncratic productivity to minimize the distance between the tenure distribution generated by the model and its empirical counterpart. In the model, the tenure distribution is defined as the fraction of workers who are employed and have been in the same match for t years. In the data, the analogous distribution is measured by Diebold, Neumark and Polsky (1997) using the 1987 CPS tenure supplement.[12]

We normalize μ_y to 1, and choose ρ_y and σ_y so that the average productivity of labor in the model has the same autocorrelation and standard deviation as in the data. In the model, the average productivity of labor is measured as $\pi = \left[\sum(y+z)g(z)\right]/(1-u)$. In the data, average labor productivity is measured as the CPS output per worker in the non-farm business sector. Note that, because the distribution of workers across matches with different idiosyncratic productivity may vary over time, the autocorrelation and standard deviation of average labor productivity need not be the same as ρ_y and σ_y. Finally, we choose b so that the ratio of the value of leisure to the average productivity of labor is 0.71, the value recently estimated by Hall and Milgrom (2008).

[12] Diebold, Neumark and Polsky (1997) also show that the empirical tenure distribution is stable over time. For this reason, it is appropriate to compare the empirical tenure distribution observed in 1987 with the tenure distribution generated by the steady-state of the model.

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Table 1: Summary Statistics, Quarterly U.S. Data

x		u	v	h^{ue}	h^{eu}	h^{ee}	π
$std(x)/std(\pi)$		9.56	10.9	5.96	5.48	5.98	1
$autocorr(x)$		.872	.909	.822	.698	.597	.760
	u	1	-.902	-.916	.778	-.634	-.283
	v	—	1	.902	-.778	.607	.423
$corr(\cdot, x)$	h^{ue}	—	—	1	-.677	.669	.299
	h^{eu}	—	—	—	1	-.301	-.528
	h^{ee}	—	—	—	—	1	.208
	π	—	—	—	—	—	1

Notes: The seasonally adjusted unemployment rate, u, is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The seasonally adjusted help wanted advertising index, v, is constructed by the Conference Board. The UE and EU rates, h^{ue} and h^{eu}, are constructed from the seasonally adjusted unemployment rate and the short-term unemployment rate as explained in Appendix D. The EE rate, h^{ee}, is constructed by Nagypál (2007) from the CPS microdata as explained in Appendix D. The variables u, v, h^{ue}, h^{eu} and h^{ee} are quarterly averages of monthly series. Average labor productivity, π, is seasonally adjusted real average output per worker in the non-farm business sector constructed by the BLS. The series for u, v, h^{ue}, h^{eu} and π cover the period 1951(I)-2006(II). The series for h^{ee} covers the period 1994(I)-2006(II). The standard deviation of h^{ee} is expressed relative to the standard deviation of π over the period 1994(I)-2006(II), and the correlation of h^{ee} with u, v, h^{ue}, h^{eu} and π refers to the period 1994(I)-2006(II). All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

Table 2: Calibration Outcomes

	Description	EXP	INS	P-00	MP-94
β	discount factor	.996	.996	.996	.996
b	home productivity	.907	.716	.710	.739
λ_u	off the job search	1	1	1	1
λ_e	on the job search	.735	.904	0	0
γ	elasticity of p wrt θ	.600	.250	.270	.270
k	vacancy cost	1.55	2.37	1.85	1.89
δ	exogenous destruction	.012	.026	.026	.012
μ_z	average idiosyncratic prod.	0	0	0	0
σ_z	scale idiosyncratic. prod.	.952	.008	0	.467
α_z	shape idiosyncratic prod.	4	10	—	10

Notes: Calibrated parameters for different versions of the model. The column EXP refers to the version of the model in which matches are experience goods. The column INS refers to the version of the model in which matches are inspection goods. The column P-00 refers to a version of the experience model in which the parameters λ_e and σ_z are constrained to be equal to zero. The column MP-94 refers to a version of the experience model in which the parameter λ_e is constrained to be equal to zero.

Table 3: Experience Model

x		u	v	v_u	v_e	h^{ue}	h^{eu}	h^{ee}	π
$std(x)/std(\pi)$		7.88	2.54	4.29	8.21	2.51	6.23	5.59	1
$autocorr(x)$		.850	.637	.748	.824	.799	.772	.823	.762
	u	1	-.807	.841	-.980	-.976	.972	-.979	-.977
$corr(\cdot, x)$	v	—	1	-.380	.855	.897	-.898	.858	.894
	π	—	—	-.729	.984	.999	-.979	.983	1

Notes: Summary statistics of the last 6,000 month of a 9,000 month long time series for u, v, v_u, v_e, h^{ue}, h^{eu}, h^{ee}, and π generated by the experience model with aggregate productivity shocks. Section 4 provides details on the stochastic process for productivity. All variables are quarterly averages of monthly series. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

Table 4: P-00 and MP-94 Models

P-00 Model						
x		u	$v = v_u$	h^{ue}	h^{eu}	π
$std(x)/std(\pi)$		0.82	2.69	0.91	0	1
$autocorr(x)$		.815	.677	.994	1	.745
	u	1	-.932	-.936	0	-.972
$corr(\cdot, x)$	v	—	1	.990	0	.990
	π	—	—	.999	0	1
MP-94 Model						
$std(x)/std(\pi)$		5.98	4.55	0.83	6.61	1
$autocorr(x)$		.674	.453	.740	.397	.736
	u	1	.726	-.737	.906	-.732
$corr(\cdot, x)$	v	—	1	-.267	.481	-.259
	π	—	—	.998	-.583	1

Notes: Summary statistics of the last 6,000 month of a 9,000 month long time series for u, v, v_u, h^{ue}, h^{eu} and π generated by a version of the experience model in which the parameters λ_e and σ_z are constrained to be equal to zero. All variables are quarterly averages of monthly series. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

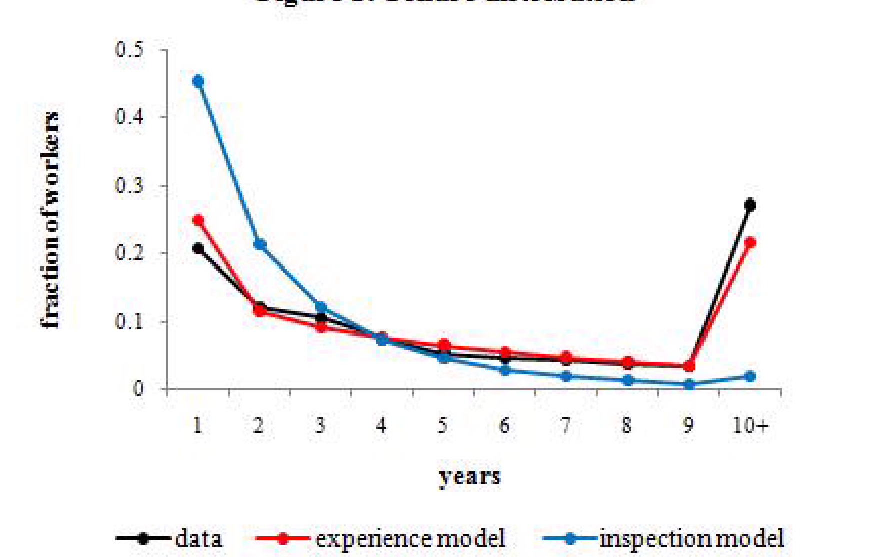

Figure 1: Tenure distribution

Figure 2: Transition rates, Experience model

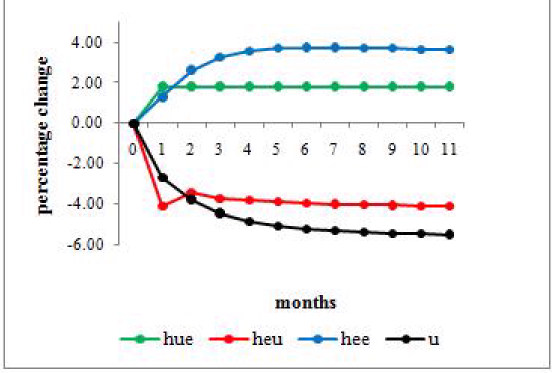

Figure 3: Vacancies, Experience model

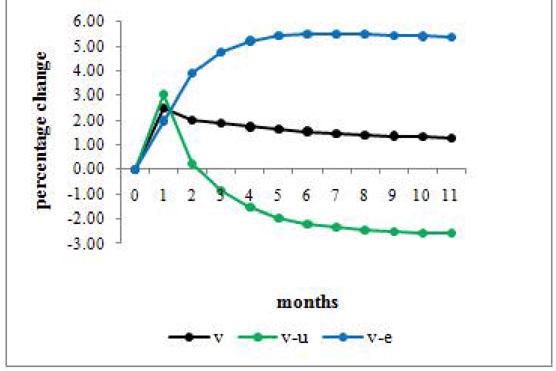

Figure 4: Productivities, Experience model

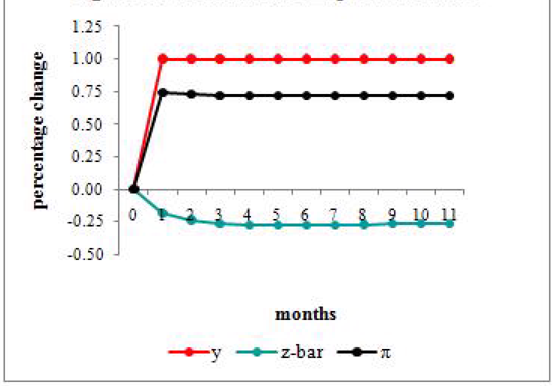

Figure 5: Transition rates, Inspection model

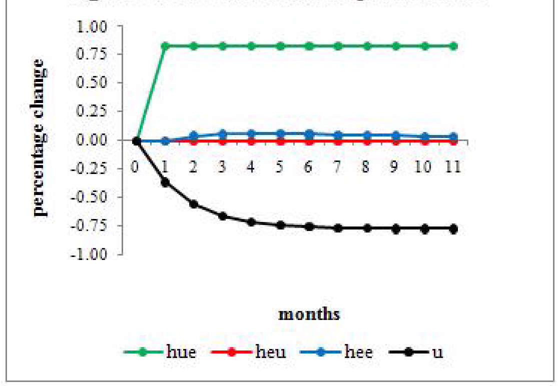

Figure 6: Vacancies, Inspection model

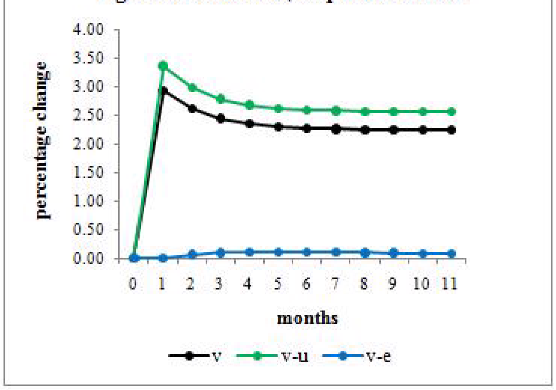

Figure 7: Productivities, Inspection model