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## Vertical limit pricing<sup>\*</sup>

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## ***Abstract***

*A new theory of limit pricing is provided which works through the vertical contract signed between an incumbent manufacturer and a retailer. We establish conditions under which the incumbent can obtain full monopoly profits, even if the potential entrant is more efficient. A key feature of the optimal vertical contract we describe is quantity discounting, typically involving three-part incremental-units or all-units tariffs, with a marginal wholesale price that is below the incumbent's marginal cost for sufficiently large quantities.*

**Key words:** *limit pricing, vertical contracts, multi-part tariffs.*

**JEL Classification:** L12, L42.

## ***Résumé***

*On propose une nouvelle théorie de la tarification limitée qui fonctionne par un contrat vertical signé entre un fabricant en exercice et un détaillant. Nous établissons des conditions dans lesquelles le fabricant peut obtenir des profits de monopole, même si le nouvel entrant potentiel est plus efficace. Une caractéristique essentielle du contrat vertical optimal décrit est la remise sur quantité, impliquant généralement des unités supplémentaires en trois parties ou le tarif sur toutes les unités, avec un prix de gros marginal qui est en dessous du coût marginal du fabricant en exercice pour des quantités suffisamment importantes.*

**Mots clés:** *prix limité, contrat vertical, tarif des plusieurs parties.*

**Classification JEL:** L12, L42.

# 1 Introduction

Existing theories of limit pricing and predation treat buyers as final consumers, focusing on the retail price charged by an incumbent firm to consumers, and whether this price is set low enough to keep out (or drive out) a rival. In contrast, in many important cases of limit pricing and predation, the incumbent is actually a manufacturer that offers a non-linear wholesale price schedule (i.e. a non-linear tariff) to downstream buyers. For example, in the landmark predatory pricing case *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*, 509 U.S. (1993), the firms in question were manufacturers of cigarettes and the alleged predatory pricing related to volume discounts given for large wholesale purchases of generic cigarettes by distributors. Similarly, in the ongoing dispute between *Intel* and *AMD*, the firms manufacture microprocessors that they sell to competing computer-makers like Dell and Hewlett-Packard. One of AMD's complaints is that Intel offers computer-makers substantial discounts for large purchases through an all-units quantity discounting scheme, so that in some cases the price of incremental purchases to computer-makers is below Intel's own marginal cost (sometimes being zero or even negative) and that "Intel's practices exacerbate normal impediments to entry and expansion".<sup>12</sup> Such cases naturally raise the question of whether limit pricing and predation can still work in a vertical setting, and if so, how the mechanism behind limit pricing and predation might differ from that traditionally studied.

This paper offers an answer to these questions by providing a new theory of limit pricing, one which works through the vertical contract signed between an incumbent manufacturer and a retailer (i.e. its distributor) to the detriment of a potential entrant. We call this "vertical limit pricing".<sup>3</sup> Unlike standard theories of limit pricing and predation, such as those based on signaling (as introduced by Milgrom and Roberts, 1982), the theory we propose does not rely on any asymmetric information between the incumbent and entrant. Despite this, in our theory the incumbent sets a low (wholesale) price, necessarily below its own marginal cost over some range of output. The result is that efficient entry is deterred. Profit is recovered from the retailer either through a fixed fee or high initial wholesale prices. We establish conditions under which the incumbent can obtain full monopoly

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<sup>1</sup>Civil Action No. 05-441-JJF, US District Court (Delaware), filed 27 June 2005, available at [http://www.amd.com/us-en/assets/content\\_type/DownloadableAssets/AMD-Intel\\_Full\\_Complaint.pdf](http://www.amd.com/us-en/assets/content_type/DownloadableAssets/AMD-Intel_Full_Complaint.pdf)

<sup>2</sup>Other similar examples include *Barry Wright v. ITT Grinnell* (724 F.2d 227: 1983) and *Tetra Pak v. Commission* (ECR I-5951: 1996).

<sup>3</sup>In case the vertical contract is used to induce the exit of a rival, say where the rival needs to incur additional fixed costs to stay in the market, it could likewise be called "vertical predation".

profits, even if the rival is more efficient, even if entry costs are trivial, and even if the incumbent's vertical contract is only observed with a small probability.

A key feature of the optimal vertical contract we describe is quantity discounting or declining *marginal* wholesale prices. We establish that both of the common forms of quantity discounting used in practice, incremental-units or all-units quantity discounting (Munson and Rosenblatt, 1998) are optimal. For low levels of purchases, the retailer purchases at a wholesale price set above the incumbent's marginal cost, thereby providing a way for the manufacturer to extract the retailer's profit (alternatively, a fixed fee can be used for this purpose). For purchases in some intermediate range, the retailer purchases at a wholesale price set equal to the incumbent's marginal cost, thereby ensuring the retailer sets the correct monopoly price when it is indeed a monopolist. For purchases beyond some yet higher level, the retailer purchases at a wholesale price set below the incumbent's marginal cost, thereby ensuring that in the face of competition, the retailer will want to compete aggressively, in such a way that the rival will not want to enter. Three-part tariffs are therefore the simplest optimal tariffs.

We consider both the case in which firms are homogenous price competitors and the case in which the goods are imperfect substitutes (competing in prices or quantities). In both cases, there are situations where to deter entry the incumbent must give its retailer a wholesale schedule so that the retailer is willing to price at a point where marginal revenue is negative. In the absence of entry, the incumbent's retailer could take advantage of this fact by disposing of some units to move back up its monopoly revenue function. To prevent this, the incumbent must leave the retailer with some rent in equilibrium. We call this the retailer's "disposal-rent". In case goods are imperfect substitutes, a different type of rent may also be required in order that the incumbent's retailer is willing to choose the monopoly quantity in equilibrium rather than the out-of-equilibrium entry deterring quantity, which we call the retailer's "incentive-rent". Such rents turn out to help limit the incumbent's incentive to renegotiate its vertical contract with its retailer, thereby ensuring it can still sometimes profitably deter entry even when its contract can be freely renegotiated.

A powerful feature of the optimal contract we discuss is that it allows the incumbent to indirectly condition its contract on entry. The non-linear nature of the incumbent's vertical limit pricing contract exploits the fact the quantity purchased by the downstream firm will differ depending on whether it faces competition or not. This avoids the incumbent monopolist having to explicitly write a contingent contract in which its wholesale price is lowered below cost in case entry occurs, since doing so is likely to be deemed to be anticompetitive. Instead, the type of

quantity discounting contracts we propose may be used to engage in traditional predation, but in a less obvious way. Thus, for instance, an incumbent manufacturer that wanted to build a reputation for toughness (along the lines of Kreps and Wilson, 1982), can use the seemingly standard quantity discounting contract we propose, which ensures its retailer only “fights” when necessary, while reducing the likelihood of antitrust action that might otherwise result from *shifting* to a more aggressive wholesale pricing schedule (involving a wholesale price below cost) in the face of entry.

From a policy viewpoint, our theory provides a particular setting which supports the use of a predatory pricing standard for dealing with wholesale price discounts in single-product cases. In our theory, marginal wholesale prices must fall below a firm’s own marginal cost for sufficiently large quantities in order to deter entry. Where there are no efficiency justifications for below-cost wholesale prices, such contracts are therefore anticompetitive. Forcing the incumbent to raise its marginal wholesale price to be no less than marginal cost will encourage efficient entry and increase welfare in our setting. More generally, vertical limit pricing provides a rationale for competition authorities to be concerned about vertical contracts which involve declining marginal wholesale prices which become very low for high quantities, especially when they are employed by dominant firms in the face of possibly efficient entry.

The rest of the paper proceeds as follows. The related literature is discussed in section 2. Section 3 presents a benchmark model in which firms are homogeneous price competitors. Our main results are presented in Section 4. Section 5 discusses a variety of extensions including allowing for partially unobservable contract offers, upfront fees, renegotiation possibilities, and imperfect substitutes (including quantity competition). Finally, section 6 concludes with some directions for future research.

## 2 Related literature

Our theory is related to a substantial body of work that studies the commitment benefits of vertical contracts. A standard result in this literature is that manufacturers can soften price competition if they can commit to contracts with retailers in which wholesale prices are inflated above cost. Examples of papers in this line include Bonanno and Vickers (1988), Irmen (1998), Kühn (1997), Rey and Stiglitz (1988, 1995), Sklivas (1987), and Vickers (1985). We explore a previously overlooked implication of the commitment effects of vertical contracts, which is to deter entry.

An incumbent manufacturer writes a vertical contract with a retailer in which it commits to provide goods at a low cost, in fact, below its marginal cost for purchases beyond a certain level.

Quantity discounting naturally arises with vertical limit pricing, reflecting the underlying concavity of a monopolist's revenue function. As such, our theory is complementary to the theory of Kühn (1997), who provides a related explanation for such quantity discounting. His theory is also based on commitment effects through vertical contracts, but in a context where retailers compete in quantities and there is demand uncertainty. In his symmetric environment, concave tariffs are used by each manufacturer to commit their retailers to be more aggressive competitors, since retailers will face lower marginal costs if they sell a lot. Without the possibility of entry deterrence, quantity discounting ends up hurting manufacturers in equilibrium. However, in the face of price competition, he shows the opposite result holds. To soften competition, manufacturers will offer convex contracts with increasing marginal wholesale prices. Thus, in contrast to Kühn's results, our results can explain quantity discounting even if firms compete in prices, and moreover, even if inter-brand competition takes the homogenous Bertrand form (in which no such softening of competition is possible).

Vertical limit pricing relates to the large literature studying exclusive dealing. When the incumbent can commit to a wholesale pricing schedule as part of its initial exclusive deal and buyers are downstream competitors, the setting is quite similar to that in our paper.<sup>4</sup> This has been considered by Simpson and Wickelgren (2001), Stefanadis (1998), and Appendix B of Fumagalli and Motta (2006). For instance, Fumagalli and Motta show the incumbent manufacturer will commit to a low wholesale price (to deter entry), extracting the surplus enjoyed by retail buyers paying this low wholesale price through an upfront fee which it receives when the exclusive deal is signed. This enables the incumbent to deter entry. Our results imply the incumbent can do better, often obtaining the full monopoly profit, with a contract involving quantity discounting but which often does not require an upfront fee or exclusionary terms. Our results also suggest that such exclusive deals (i.e. involving commitments to low wholesale prices) may actually be better understood as a form of vertical limit pricing or vertical predation rather than as a form of exclusive dealing. Whether the entrant is denied access to retailers or not may not matter much if the incumbent's retailers enjoy low wholesale prices.

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<sup>4</sup>In other theories in which the incumbent uses exclusive (or partially exclusive) contracts as a barrier to entry (e.g. Aghion and Bolton, 1987, Rasmusen *et al.*, 1991, and Segal and Whinston, 2000), contracts are signed directly with final consumers and the mechanisms at work are very different to ours.

Another mechanism to deter entry that has been studied in the literature is the use of divisionalization, following the work of Schwartz and Thompson (1986). They establish that an incumbent may deter an equally efficient rival by (costlessly) creating independent competing divisions that emulate the behavior of the rival and therefore do not allow it to recover its fixed cost of entry. Their mechanism is akin to delegating production to competing downstream firms with a vertical contract in which the wholesale price is fixed at the incumbent’s marginal cost of production (and profits recovered through a profit sharing agreement). In our setting, such an approach would not work given we assume the rival is more efficient. Nevertheless, the idea of committing downstream divisions or firms to be more aggressive to deter entry is the same.

Our study of vertical limit pricing is somewhat less related to a recent line of literature exploring the ability of bundled rebates, market-share discounts and all-units discounts to have exclusionary effects. As Tom *et al.* (2000, p.615) note “*The traditional analysis governing exclusive dealing arrangements has focused on a manufacturer’s requirement that its distributors deal exclusively with it. In recent years, however, some manufacturers have begun to use subtler arrangements in which incentives replace requirements ...*”. Our paper differs in two respects. First, much of this literature has focused on contracts with end-users (or a retailer representing end-users’ interests) which is not the case in our theory. Second, our paper provides a predatory-type purpose for quantity discounting. Whether or not volume discounts also include exclusivity provisions, their purpose in our theory is to commit the incumbent to price below cost where this is necessary to drive out the (potential) rival. They are not simply replicating exclusive deals that are designed to prevent distributors sourcing inputs from competing manufacturers so as to block (or soften the effect of) the manufacturers’ entry.

Finally, our theory relates to the literature on contingent contracts. Katz (2006) provides a nice analysis of the power of contingent vertical contracts in delegation games. In a framework where contracts are directly contingent on the rival’s contract he obtains a “folk theorem” result. The mechanism at work in our paper, that the optimal non-linear contract allows the incumbent to indirectly condition its contract on entry, is similar to the taxation principle in common agency (see Martimort and Stole, 2002) where two principals compete for one agent through non-linear schedules. As in our entry deterrence framework, the “punishment” to the other principal (e.g. the entrant) is carried out through the agent (e.g. the incumbent’s retailer).

### 3 Benchmark model

We focus on a model in which firms sell an identical good and set prices (i.e. homogenous Bertrand competition). There are two upstream firms, i.e. manufacturers. There is an incumbent upstream firm, which we will denote as  $I$ , which faces constant marginal costs of  $c_I$ . A potential upstream entrant, denoted  $E$ , faces lower marginal costs of  $c_E < c_I$  but some fixed cost of entry  $F$ . Each upstream firm (or manufacturer) can sell through a downstream firm (or retailer). We assume that there is a perfect competition on the retailers' market. Therefore, if say  $I$  contracts with some  $D$  it may design the contract which leads to zero profit for  $D$ . For simplicity downstream firms face no costs other than those determined by the manufacturers' wholesale tariffs. Whichever retailer (or upstream firm if it decides to sell by itself) sets the lower price obtains the entire market demand at that price.<sup>5</sup> *To proceed, we employ a standard vertical chains structure, following Rey and Stiglitz (1988, 1995). denoted respectively  $D$  and  $D'$ . Retailers set final prices. They if retailers set the same price, the existence of an equilibrium will require one of the retailers to obtain the entire market (i.e. in standard cases, this will be the retailer facing the lower marginal cost).*<sup>6</sup>

Market demand  $Q(P)$ , where  $P$  is the market price, is assumed to be non-negative, continuously differentiable and strictly decreasing in price.<sup>7</sup> The inverse demand function is denoted  $P(Q)$ .

We assume that the revenue function  $R(Q) = P(Q)Q$  is strictly concave in  $Q$ . The monopoly price given any constant marginal cost  $w$  is denoted

$$P_M(w) = \arg \max_P (P - w) Q(P).$$

For notational convenience, define  $Q_M(w) = Q(P_M(w))$ .

The incumbent's monopoly price and quantity are defined as  $P_M = P_M(c_I)$  and  $Q_M = Q_M(c_I)$ , with corresponding monopoly profit  $\Pi_M = (P_M - c_I) Q_M$ .<sup>8</sup>

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<sup>5</sup>If retailers set the same price, the existence of an equilibrium will require one of the retailers to obtain the entire market (i.e. in standard cases, this will be the retailer facing the lower marginal cost).

<sup>6</sup>This particular vertical structure is not crucial for our results. The analysis would still apply if more than one retailer competed to be a manufacturer's distributor or if retailers could sign with both manufacturers, provided  $U_1$  can still sign a binding contract with at least one of the retailers before  $U_2$  has to decide whether to incur its fixed costs of entry.

<sup>7</sup>As usual  $Q(P)$  is defined over the relevant price interval (i.e. the interval of prices from the lowest non-negative price at which demand is defined up to the point where demand just becomes zero, if indeed such a maximum price exists) and is zero thereafter.

<sup>8</sup>Assume  $P(0) > c_I$  which ensures that if the incumbent is a monopolist it will produce a positive output (and so can obtain a positive profit).



To make things interesting we assume that fixed cost of entry  $F$  satisfies

$$0 \leq F < (c_I - c_E) Q(c_I). \quad (1)$$

Note that if  $F \geq (c_I - c_E) Q(c_I)$ , then  $I$  is able to deter entry without using delegation to  $D$ .<sup>9</sup> Thus, if (1) is satisfied it will always be profitable for  $E$  to enter if it competes directly with  $I$ .

Define the limit (break-even) price  $\underline{P}$  for  $E$  to be (the lowest value of)  $p_2$  that solves

$$\underline{P} = \min \{P \text{ such that } (P - c_E) Q(P) = F\}. \quad (2)$$

This  $\underline{P}$  exists and satisfies  $c_E \leq \underline{P} < c_I$  given the strict concavity of the revenue function and since (1) implies  $(P - c_E) Q(P) > F$  when  $P = c_I$  and  $(P - c_E) Q(P) \leq F$  when  $P = c_E < c_I$ .

Initially, we assume  $R'(Q(\underline{P})) \geq 0$  so that the market revenue function is non-decreasing at  $E$  break-even price. This condition holds for standard demand specifications such as constant elasticity and logit demand where the monopolist's revenue function  $R(Q)$  is always increasing in  $Q$ . For demand specifications such as linear and exponential, where the revenue function can decrease, the condition requires the price elasticity of the market demand  $Q(P)$  to be greater than unity (in magnitude) at  $Q(\underline{P})$ . We will subsequently discuss what happens without this condition.

The timing of the game is as follows:

- Stage 1 (Incumbent's contracting),  $I$  offers a contract to  $D$ , which accepts or not. If  $D$  rejects the offer  $I$  supplies the market directly.
- Stage 2 (Entry), after observing  $I$ 's contract and  $D$ 's decision,  $E$  can decide whether to enter the market (incurring the cost  $F$ ), and if it does it offers a contract to  $D'$  which accepts or not.<sup>10</sup> If  $D'$  rejects the offer  $E$  supplies the market directly.
- Stage 3 (Renegotiation),  $I$  can secretly renegotiate the contract with  $D$ .
- Stage 4 (Downstream competition), In the last stage the contracts are observed and active retailers (or manufacturers if they fail to contract with retailers) set prices and pay wholesalers based on the amount they order from them.

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<sup>9</sup> $E$  is assumed to only enter if it makes positive profit (which is the reason we can include the case in which fixed costs are assumed to be zero).

<sup>10</sup>As will be seen, the incumbent will want to disclose its contract publicly if any such device exists. In section 5 we expand on this point and show that our results are robust to contracts only being observed with some, possibly small, probability.

*Contract space.* Upstream firms can only observe how much their downstream firm buys from them, but not anything else. In particular, the contract cannot depend on the amount the downstream firm sells or the price it sets (as in revenue or profit sharing contracts), or on the rival’s decisions or outcomes. This setup also assumes downstream firms only observe a rival’s contract after it has been signed.<sup>11</sup>

This informational structure implies the tariff  $T_I(q_I)$  (or  $T_E(q_E)$ ) depends only on  $q_I$  (or  $q_E$ ), the amount purchased by downstream firm  $D$  (or  $D'$ ). For the most part we restrict contracts further by considering tariff schedules without upfront fees, either a fee paid prior to stage 3 (e.g. a franchise fee) or a fee paid in stage 3 even if  $D$  does not purchase any units. Such upfront fees make it substantially easier for an incumbent to obtain monopoly profits through vertical limit pricing since they provide a further first-mover advantage to the incumbent. Large upfront fees may also not be feasible or efficient in practice, for reasons outside of this model. Taken literally, they may imply a very large upfront payment so that the manufacturer can extract the ongoing monopoly profits of the retailer. Such payments may be infeasible in practice due to credit constraints or may be inefficient if they put too much risk on the retailer. Later, we will explain how our results change allowing for such upfront fees.<sup>12</sup>

Without loss of generality, tariffs in the allowed class are defined as  $T_i(q_i) = T_i \mathbf{I}_{q_i > 0} + W_i(q_i)$  on  $[0, \infty)$ , where  $T_i$  is a possible “fixed fee” that is only paid if  $D_i$  makes a positive purchase from  $U_i$  (i.e.  $\mathbf{I}_{q_i > 0}$  is an indicator variable which is 0 if  $q_i = 0$  and 1 if  $q_i > 0$ ) and  $W_i(q_i)$  corresponds to some wholesale price schedule with  $W_i(0) \leq 0$ . Note  $D_i$  is allowed to buy more units than it sells and freely dispose of the extra units, to take advantage of any wholesale schedule with decreasing tariffs. This constrains the type of contracts that can be profitably offered. Define the class of such tariffs as  $\mathcal{T}$ .

Even tariffs within the class  $\mathcal{T}$  may be far more complicated than can reasonably be used in practice. An important focus of our analysis will be to see whether the incumbent’s optimal contract can be implemented with plausible or realistic tariff schedules. For instance, one possible concern arises if the optimal contract involves very high tariffs for certain quantities, implying the incumbent is effectively engaged in quantity forcing, which could be illegal.<sup>13</sup> Another concern might arise

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<sup>11</sup>These informational assumptions are standard in the literature (see Kühn, 1997 and Rey and Stiglitz, 1995; Rey and Vergé, 2004 call this interim observability).

<sup>12</sup>Note the absence of a fixed fee in stage 3 is implied if  $D_1$  can walk away from any contract which it finds unprofitable ex-post (i.e. after observing firm 2’s contract) by not buying anything from  $U_1$  and not paying anything to  $U_1$ . In other words, while we assume that the upstream firm can commit to its contract, we do not require the downstream firm can do the same.

<sup>13</sup>See the “Guidelines on Vertical Restraints,” Commission Notice of 13 October 2000, COM

if the contract is required to be very complicated. In practice, tariff schedules are not presented as non-linear functions, but rather schedules of wholesale prices that apply for different quantities purchased, possibly with a fixed fee.<sup>14</sup> This is the class of tariffs that are piece-wise linear.

The literature on operations management (see Munson and Rosenblatt, 1998) identifies two types of piece-wise linear tariffs that are studied by researchers and widely used by industry to quantity discount.<sup>15</sup> The first type of tariff is associated with incremental-units quantity discounting, which is a block declining tariff, in which the marginal wholesale prices declines at each increment. More generally, allowing for the possibility of a fixed fee  $T_i$  and not requiring quantity discounting, this is the case where the wholesale schedule  $W_i(q_i)$  is a continuous piece-wise linear function. To be more precise, define  $W_I^{(n)}(q) = W_I^{(n)}(q; w_1, \dots, w_{n-1}, S_1, \dots, S_{n-2})$  as

$$W_I^{(n)}(q) = \begin{cases} w_1 q & \text{if } 0 < q \leq S_1, \\ w_1 S_1 + w_2 (q - S_1) & \text{if } S_1 < q \leq S_2, \\ \dots & \dots \dots \\ \sum_{i=1}^{n-1} w_i [S_i - S_{i-1}] + \dots + w_n (q - S_{n-1}) & \text{if } S_{n-1} < q. \end{cases} \quad (3)$$

Then an  $n$ -part incremental-units tariff is either  $T_I^{(n)}(q) = T_I \mathbf{1}_{q>0} + W_I^{(n-1)}(q)$  in the case a positive fixed fee  $T_I > 0$  is used or  $T_I^{(n)}(q) = W_I^{(n)}(q)$  in the case of no fixed fee (i.e.  $T_I = 0$ ). The class of  $n$ -part incremental-units tariff is denoted  $\mathcal{T}_I^{(n)}$ . In general, we will refer to the class of contracts which belong to some  $\mathcal{T}_I^{(n)}$  as *incremental-units* contracts, denoted  $\mathcal{T}_I$ . Then  $\mathcal{T}_I = \cup_{n=1}^{\infty} \mathcal{T}_I^{(n)}$ .

The second type of tariff is associated with all-units quantity discounting, in which wholesale prices decline at each increment, but the lower wholesale price applies to all units purchased rather than just marginal units. More generally, again allowing for the possibility of a fixed fee and no longer requiring quantity discounting, this is the case where the wholesale schedule is a special type of piece-

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(2000/C291/01) which relates quantity forcing to exclusivity issues, noting some competition concerns that arise from quantity forcing. We have already ruled out direct quantity forcing in which the upstream firms restrict the range of quantities that can be chosen by downstream firms. Section 4 notes how the optimal contract can be implemented if quantity forcing is allowed.

<sup>14</sup>Munson and Rosenblatt (1998) survey contracts offered by manufacturers or received by wholesalers and retailers and find (p.364) “None of the participants have seen continuous schedules in practice.” Simplicity of the contract was a key concern expressed by interviewees.

<sup>15</sup>According to the survey of Munson and Rosenblatt, incremental-units quantity discounting was used by 37% of their sampled firms. Even more commonly used was all-units quantity discounting (used by 95% of sampled firms). In addition to the use of fixed fees by 29% of the interviewees, these were the only forms of quantity discounting identified by the authors. Kolay *et al.* (2005) also note all-units quantity discounting is widely used in intermediate-goods markets, with the list of companies known to have used them including Coca-Cola, British Airways, and Michelin.

wise linear function with discontinuities associated with the different price-breaks. More precisely, define  $W_A^{(n)}(q) = W_A^{(n)}(q; w_1, \dots, w_{n-1}, S_1, \dots, S_{n-2})$  as

$$W_A^{(n)}(q) = \begin{cases} w_1 q & \text{if } 0 < q \leq S_1, \\ w_2 q & \text{if } S_1 < q \leq S_2, \\ \dots & \dots \quad \dots \\ w_n q & \text{if } S_{n-1} < q. \end{cases} \quad (4)$$

Then an  $n$ -part all-units tariff is either  $T_A^{(n)}(q) = T_A \mathbf{I}_{q>0} + W_A^{(n-1)}(q)$  in the case a fixed fee  $T_A$  is used or  $T_A^{(n)}(q) = W_A^{(n)}(q)$  in the case it is not. The class of  $n$ -part all-units tariff is denoted  $\mathcal{T}_A^{(n)}$ . In general, we will refer to the class of contracts which belong to some  $\mathcal{T}_A^{(n)}$  as *all-units* contracts, denoted  $\mathcal{T}_A$ . Then  $\mathcal{T}_A = \cup_{n=1}^{\infty} \mathcal{T}_A^{(n)}$ . In what follows we will drop the subscript which characterizes the type of the tariff. Instead, we will indicate the set to which this tariff belongs. For example, the notation  $T_1(q) = T_1 \mathbf{I}_{q>0} + W_1(q) \in \mathcal{T}_I^{(n)}$  means that the tariff  $T_1(q)$  is a  $n$ -part incremental-units tariff.

In the next section, in addition to characterizing optimal contracts, we will be interested in whether incremental-units tariffs and/or all-units tariffs can achieve optimality, and if they can, the properties of the simplest of such tariffs (those where  $n$ , the number of parts, is lowest). These questions will be addressed both with and without the use of a fixed fee.

## 4 Results

In this section we characterize optimal contracts. Different restrictions on the upstream firms' tariffs will be considered. Actually, these restrictions are only required on the incumbent's tariff. The results would remain the same if the rival was allowed to use any tariff from the general class  $\mathcal{T}$  or sell directly to consumers. However, to avoid giving the false impression that the results depend on the rival having access to a wider tariff class, all our results will be stated assuming both upstream firms have the same restrictions on the tariffs they can write.

Before considering non-linear tariffs, it is useful to point out that by restricting to linear tariffs (which in the terminology of section 3 is just a one-part tariff without a fixed fee), the incumbent manufacturer cannot prevent entry. Trivially, we have that:

**Proposition 1** *If upstream firms are restricted to using linear tariffs, then the incumbent never makes any sales or profit. The potential entrant always enters.*

**Proof.** So as to cover its costs for any level of sales,  $I$  must set its wholesale price at or above  $c_I$ .  $E$  can always undercut with a slightly lower wholesale price (if necessary), so that given (1),  $D'$  will profitably take the whole market.  $I$  ends up with no sales or profit. ■

This result motivates the use of non-linear tariffs. Naturally, one wonders whether simple two-part tariffs can do better. Here we consider two-part tariffs belonging to the classes  $\mathcal{T}_I$  or  $\mathcal{T}_A$ . This includes a standard two-part tariff in which  $T_i > 0$  and  $W_i(q_i)$  is linear, or tariffs in which there is no fixed fee but two different regions with different wholesale prices applying. As the next proposition establishes, such contracts, while sometimes enabling entry deterrence, cannot lead to full monopoly profit for the incumbent.

**Proposition 2** *If upstream firms are restricted to using a two-part tariff belonging to the class  $\mathcal{T}_I$  or  $\mathcal{T}_A$ , then the incumbent can profitably take the whole market by setting a wholesale price below its own cost (so that the potential entrant stays out) but its profit is strictly less than monopoly profit.*

**Proof.** We consider each of the three possible forms of tariffs in turn.

(i) Consider first a two-part tariff belonging to the classes  $\mathcal{T}_I$  or  $\mathcal{T}_A$  in which a fixed fee is used. This is a standard two-part tariff. The best  $I$  can do by offering a two-part tariff of the form  $T_1(q_1) = T_1 \mathbf{1}_{q_1 > 0} + w_1 q_1$  is to set  $T_1 = (\underline{P} - w_1) Q(\underline{P})$  and  $w_1 < \underline{P} < c_I$ . With this offer,  $D$  will be willing to price down to  $\underline{P}$  to take the whole market, since by doing so it will sell  $Q(\underline{P})$  units and obtain a profit of  $(\underline{P} - w_1) Q(\underline{P}) - (\underline{P} - w_1) Q(\underline{P}) = 0$ . Observing this offer,  $E$  will not enter since it cannot make a positive profit given  $D'$  can only sell units at a price below  $\underline{P}$ , its break-even price. Given  $E$  will not enter,  $I$ 's profit is

$$\Pi_1 = (\underline{P} - w_1) Q(\underline{P}) - (c_I - w_1) Q_M(w_1)$$

and  $D$ 's profit is

$$\pi_1 = (P_M(w_1) - w_1) Q_M(w_1) - (\underline{P} - w_1) Q(\underline{P}).$$

Clearly,  $\pi_1 \geq 0$  since by definition  $P_M(w_1)$  maximizes  $(p - w_1) Q(p)$ . For  $I$  to deter entry requires there exists  $0 \leq w_1 < \underline{P}$  with  $\Pi_1 > 0$ . If such a  $w_1$  exists, then we have  $\Pi_1 \leq \Pi_1 + \pi_1 = (P_M(w_1) - c_I) Q_M(w_1) < (P_M - c_I) Q_M$  since  $P_M$  maximizes  $(p - c_I) Q(p)$ . That is,  $I$ 's profit is strictly less than monopoly profit. (An example of this case is  $Q(p) = 1 - p$ ,  $c_I = 12/20$ ,  $\underline{P} = 11/20$ ,  $w_1 = 7/20$ . If such a  $w_1$  does

not exist, then  $I$  cannot deter entry with such a two-part tariff. (For instance, when  $c_I$  is increased to  $13/20$  in the previous example.)

(ii) Consider a two-part tariff belonging to the class  $\mathcal{T}_I$  without a fixed fee. Denote the first wholesale price  $w_1$  which applies to purchases within  $0 < q_1 < S_1$  and the second wholesale price  $w_2$  which applies to marginal purchases when  $q_1 \geq S_1$ . First, consider the contract designed so  $D$  chooses some  $q_1 < S_1$  in equilibrium. Then the best  $I$  can do is to set  $w_1 = \arg \max_w (w - c_I) Q_M(w) > c_I$ . To deter entry, set  $w_2 = R'(Q(\underline{P}))$  and  $S_1$  to solve  $w_1 S_1 + w_2 (Q(\underline{P}) - S_1) = \underline{P} Q(\underline{P})$ . This ensures that  $D$  is willing to price down to  $\underline{P}$  if required. As a result,  $D$  will be a monopolist. Since  $w_1$  is chosen so that  $Q_M(w_1) > 0$ ,  $D$  can always choose a price sufficiently close to  $w_1$  such that  $R(q_1) > W_1(q_1)$  for some positive quantity. This ensures  $D$  will choose to sell a positive quantity. Moreover,  $D$ 's choice of  $q_1$  is less than  $S_1$  since  $W_1(q_1) \geq R(q_1)$  for  $q_1 \geq S_1$  (this follows given  $W_1(q_1) = R(q_1)$  at  $Q(\underline{P})$ ,  $W_1(q_1)$  is linear for  $q_1 \geq S_1$ , and  $R(q_1)$  is strictly concave). Since  $D$  chooses  $q_1 < S_1$ , this means  $I$  obtains the positive profit  $\max_w (w - c_I) Q_M(w)$ , which is less than the full monopoly profit  $\max_w (w - c_I) Q(w)$ .

Similarly,  $I$  will also not be able to obtain monopoly profit if it tries to deter entry with a contract designed such that as a monopolist  $D$  chooses  $q_1 \geq S_1$ . To deter entry we require  $w_1 S_1 + w_2 (Q(\underline{P}) - S_1) \leq \underline{P} Q(\underline{P})$ . This implies  $I$ 's profit  $(w_1 - c_I) S_1 + (w_2 - c_I) (Q_M(w_2) - S_1) \leq (\underline{P} - w_2) Q(\underline{P}) + (w_2 - c_I) Q_M(w_2)$ . If  $I$  tries to induce  $D'$  to choose the monopoly price, it must set  $w_2 = c_I$ , which given  $\underline{P} < c_I$  implies it will make a loss.

(iii) Consider a two-part tariff belonging to the class  $\mathcal{T}_A$  without a fixed fee. Denote the first wholesale price  $w_1$  which applies when  $0 < q_1 < S_1$  and the second wholesale price  $w_2$  which applies to all units when  $q_1 \geq S_1$ .  $I$  can always profitably deter entry obtaining the same profit as in (ii). Set  $S_1 = Q(\underline{P})$ ,  $w_1 = \arg \max_w (w - c_I) Q_M(w) > c_I$  and  $w_2 = \underline{P}$ . Then by construction,  $D$  will be willing to price down to  $\underline{P}$  to take the whole market. As a result,  $D$  will be a monopolist and will choose the same price as in (ii). Note this contract is the best  $I$  can do. Any contract designed such that as a monopolist  $D$  chooses  $q_1 \geq S_1$  can be ruled out since to deter entry it would require  $w_2 \leq \underline{P}$  which would imply a loss for  $I$ . ■

Proposition 2 shows  $I$  may be able to use a two-part tariff contract with  $D$  to prevent a more efficient firm entering. Such a contract involves using a wholesale price that is lower than  $I$ 's marginal cost. This illustrates, in the simplest possible setting, the ability of non-linear vertical contracts to be used by an incumbent to prevent entry, which we refer to as vertical limit pricing. This relies on the

commitment of the incumbent to make its downstream firm a tough competitor in the face of competition, which it does through a vertical contract. Pricing below cost at the wholesale level can be profitable since (i) by so doing, its retailer can deter entry, thereby remaining a monopolist so it can sustain a high retail price and (ii) some of the profits associated with this high retail price can be recovered through a fixed fee. Thus, both instruments of the two-part tariff (the low wholesale price and the positive fixed fee) are needed for vertical limit pricing to work.

As proposition 2 established, the incumbent has to give up some part of its profit in order to deter entry. For instance, with linear demand, it can easily be established that an upper bound on the incumbent's profit from using vertical limit pricing with two-part tariffs from the classes  $\mathcal{T}_I$  or  $\mathcal{T}_A$  is one half of its normal monopoly profit. This suggests the incumbent has strong incentives to make use of more sophisticated tariffs to deter entry, which is the case we consider next.

*To allow for the possibility that revenue is decreasing in output (as is the case with linear demand, for instance), we also define the following modified revenue function*

$$\tilde{R}(Q) = \max_{0 \leq q \leq Q} R(q), \quad (5)$$

*which is non-decreasing in  $Q$ .*

Once we allow for more complicated tariffs, even say the class of piece-wise linear tariffs, the dimensionality of the contract space becomes large relative to the problem that the incumbent solves, so not surprisingly the optimal contract is not uniquely determined (see Kühn, 1997 for a more general discussion of this type of problem). However, it turns out we can still say something useful about the incumbent's optimal tariff. We start by showing that the optimal tariff within the general class  $\mathcal{T}$  will always allow the incumbent to obtain its full monopoly profit.

**Proposition 3** *When upstream firms can set general non-linear tariff functions within the class  $\mathcal{T}$ , the incumbent will obtain full monopoly profits, deterring entry in the process.*

**Proof.** Consider  $I$ 's tariff  $T_1(q_1)$  from the contract space  $\mathcal{T}$ . First note that  $T_1(q_1) \geq 0$  for all  $q_1$  since otherwise there will be some output at which  $D$  obtains positive profit through a subsidy from  $I$  which it will strictly prefer to the equilibrium, given in equilibrium  $I$  has to extract the full monopoly profit from  $D$ . This implies  $T_1(0) = 0$  (since  $T_1(0) > 0$  has already been ruled out).

To allow  $I$  to obtain monopoly profits (i.e. its optimal outcome) we require that if  $D$  does not face competition from  $D'$ , then  $D$  should choose to price at  $P_M$  and buy the monopoly output  $Q_M$  from  $I$ . This incentive constraint requires  $D$  be better

off (or no worse off) buying the monopoly output compared to any other positive output level. Formally, we require

$$R(Q_M) - T_1 - W_1(Q_M) \geq \tilde{R}(q_1) - T_1 - W_1(q_1), \quad (6)$$

for all  $q_1 > 0$ .<sup>16</sup> Second,  $I$  should recover the full monopoly profits in equilibrium but leave  $D$  willing to participate (since it can always choose to buy nothing and pay nothing). This participation constraint requires

$$T_1 = R(Q_M) - W_1(Q_M). \quad (7)$$

Third, we require that if  $D'$  enters and sets a price of  $\underline{P}$  (so  $E$  can just cover its costs of entry),  $D$  will be willing to price so as to take the whole market.<sup>17</sup> By offering such a contract,  $I$  ensures that  $E$  will never want to enter in stage 2. Suppose  $D$  faces a rival pricing at  $\underline{P}$ . If  $D$  sets a higher price than  $\underline{P}$ , it will not sell anything, obtaining a payoff of zero. Alternatively,  $D$  can set a price which is the same or lower than  $D'$ 's price and take the whole market. This will imply  $D$  sells  $q_1 \geq Q(\underline{P})$  units. Given it chooses  $q_1$  optimally (through its choice of  $p_1$ ),  $D$  will get a profit of  $R(q_1) - T_1 - W(q)$  for any  $q \geq q_1$ , where the number of units it offers for sale  $q_1$  may be less than the number it purchases  $q$  due to free disposal. This “minimal deterrence” constraint requires

$$\max_{q_1 \geq Q(\underline{P})} \left( R(q_1) - T_1 - \min_{q \geq q_1} W(q) \right) \geq 0, \quad (8)$$

which ensures  $D$  does at least as well with this option as it can setting a higher price in which it sells nothing.

Suppose  $I$  offers a tariff with the following features: (i) it fixes any arbitrary  $T_1 \geq 0$ ; (ii) it sets  $W_1(Q_M) = R(Q_M) - T_1$ ; (iii) it sets  $W_1(Q(\underline{P})) = R(Q(\underline{P})) - T_1$ ; and (iv) it sets  $W_1(q_1)$  sufficiently high for any other  $q_1$  (for example, it is sufficient to set  $W_1(q_1) > \tilde{R}(q_1) - T_1$  for any other  $q_1$ , which prevents  $D$  from doing better with a different output when it is a monopolist). Substituting these conditions into the above constraints, it is clear (6)-(8) hold given the revenue function is assumed to be non-decreasing at  $Q(\underline{P})$ .

Given the construction of any such tariff,  $E$  cannot profitably enter. For  $E$  to want to enter it must expect  $D'$  to price above  $\underline{P}$  in stage 3 (so it can expect to make

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<sup>16</sup>The use of the modified revenue function  $\tilde{R}(q)$  arises since we allow for the possibility of free disposal.

<sup>17</sup>In fact, even if it attracts some demand at this price, this will be sufficient to deliver the result.



a profit from entry). However, if this were the case then given the above tariff,  $D$  would profitably undercut to take the whole market. For this reason,  $E$  will stay out. Facing the contract,  $D$  cannot do better than set the monopoly price  $P_M$ , generating monopoly profits of  $(P_M - c_I) Q_M$ , which  $I$  extracts through its wholesale schedule and/or initial fixed fee. Thus, we have that the incumbent can deter entry and obtain monopoly profit. ■

The optimal tariff in Proposition 3 can work regardless of whether the fixed fee is positive or zero. What is important is that as a monopolist,  $D$  chooses the correct monopoly price from  $I$ 's perspective (the optimal pricing constraint), and that facing a rival, it will be willing to undercut whenever the rival prices at a level which ensures  $E$  can cover its costs (the minimal deterrence constraint). Even with these constraints, there is considerable redundancy in any optimal tariff within the class  $\mathcal{T}$ , since there are some quantities for which the tariff's only purpose is to avoid  $D$  wanting to deviate from one of these two situations. This can always be achieved with any sufficiently high tariffs.<sup>18</sup> The key properties for optimality are  $T_1(0) = R(0)$ ,  $T_1(Q_M) = R(Q_M)$  and  $T_1(q_1) = R(q_1)$  for some  $q_1 \geq Q(\underline{P})$ , with  $T_1(q_1) \geq \tilde{R}(q_1)$  elsewhere.

To give  $D$  a strict incentive to choose the monopoly output when there is no entry, or to choose a high output in the face of entry, the optimal tariffs characterized in Proposition 3 can always be approximated arbitrarily closely by slightly lower tariffs which leave  $D$  with a positive profit in each case, but higher profit at  $Q_M$  than at  $Q(\underline{P})$  when  $D$  is a monopolist. Similar approximations can be applied to the results throughout the paper.

Next we wish to see whether tariffs within the more reasonable classes  $\mathcal{T}_I$  and  $\mathcal{T}_A$  can achieve optimality for  $I$ , and if so, what are the properties of the simplest such tariffs. First, considering the class  $\mathcal{T}_I$ , we find optimality can indeed be achieved, and the simplest such tariff is a three-part block declining tariff, in which the marginal wholesale prices declines at each increment.<sup>19</sup> The tariff is concave, exhibiting “incremental-units” quantity discounting.

**Proposition 4** *The incumbent can obtain full monopoly profit, deterring entry in the process, by using a three-part block declining tariff in which  $W_1(q_1)$  is continuous and concave. This is the simplest tariff within the class  $\mathcal{T}_I$  that allows the incumbent*

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<sup>18</sup>Quantity forcing would just be an extreme example of such an optimal contract, in which the retailer is only offered the two quantity choices  $Q_M$  and  $Q(\underline{P})$ .

<sup>19</sup>In the following two propositions,  $U_2$  is still assumed to be able to choose tariffs from the more general class  $\mathcal{T}$ . Given  $U_1$  obtains its optimal outcome, identical results also apply if  $U_2$  is restricted to the same (more restrictive) class of tariffs as  $U_1$ .

to extract the full monopoly profit. The lowest wholesale price in the tariff is below the incumbent's marginal cost.

**Proof.** The proof is by construction. We will start with the case in which a fixed fee is used, so that  $T_1 > 0$ . It is helpful to start with this case since it is the limiting case of the equivalent tariff in which no fixed fee is used, which we will introduce in the proof below. Also a fixed fee is necessary if the demand function is such that  $P(0)$  is undefined, as it is with logit or constant elasticity demand. In such cases,  $R'(0)$  is arbitrarily high, so it is not possible to simultaneously satisfy  $T_1(0) = 0$  and  $T_1(q_1) > R(q_1)$  as  $q_1 \rightarrow 0$  with a piece-wise linear tariff that does not have a fixed fee.

Suppose  $I$  offers the three-part tariff, in which  $T_1 = (P_M - c_I) Q_M$ ,  $W_1(q_1) = c_I q_1$  if  $0 \leq q_1 < S_1$  and  $W(q_1) = c_I S_1 + w_1(q_1 - S_1)$  if  $q_1 \geq S_1$  where  $Q_M < S_1 < Q(\underline{P})$ . Set  $w_1 = R'(Q(\underline{P}))$ , in which case  $0 \leq w_1$  by assumption and  $w_1 = R'(Q(\underline{P})) < R(Q(\underline{P}))/Q(\underline{P}) = \underline{P}$  from the concavity of  $R(Q)$ . Then set  $S_1$  so that  $T_1(Q(\underline{P})) = R(Q(\underline{P}))$ ; i.e.

$$S_1 = \frac{(\underline{P} - R'(Q(\underline{P}))) Q(\underline{P}) - (P_M - c_I) Q_M}{c_I - R'(Q(\underline{P}))}.$$

The inequality  $S_1 < Q(\underline{P})$  holds given  $P_M > c_I$  and  $c_I > \underline{P}$ . The inequality  $Q_M < S_1$  holds given

$$(\underline{P} - R'(Q(\underline{P}))) Q(\underline{P}) > (P_M - R'(Q(\underline{P}))) Q_M,$$

which reflects that  $Q(\underline{P}) = \arg \max_{q_1} (R(q_1) - R'(Q(\underline{P})) q_1)$  and that  $R(q_1)$  is strictly concave. This establishes any such contract satisfies incremental-unit discounting.

It remains to check the conditions in proposition 3 hold for optimality. Recall a sufficient condition is  $T_1(0) = 0$ ,  $T_1(Q_M) = R(Q_M)$  and  $T_1(Q(\underline{P})) = R(Q(\underline{P}))$ , with  $T_1(q_1) > \tilde{R}(q_1)$  everywhere else. The equality conditions hold by construction. The inequality condition holds for  $0 < q_1 < S_1$  given that  $Q_M = \arg \max_{q_1} (R(q_1) - c_I q_1)$  and it holds for  $q_1 > S_1$  reflecting that the strictly concave function  $R(q_1)$  lies below the tariff line  $T_1(q_1)$  for  $q_1 > S_1$  which is tangent to  $R(q_1)$  at  $q_1 = Q(\underline{P})$ .

The same conditions for optimality can be maintained by shifting the tangency point to the right of  $Q(\underline{P})$ , so that  $w_1$  is lower and  $S_1$  is higher than those specified above. Specifically, for any  $Q \geq Q(\underline{P})$  such that  $R'(Q) \geq 0$ , then the proposition continues to hold with  $w_1 = R'(Q)$  and  $S_1 = ((P(Q) - R'(Q)) Q - (P_M - c_I) Q_M) /$

$(c_I - R'(Q))$ .

Now we show that the same conditions for optimality hold without the use of a fixed fee, provided market demand is such that  $P(0)$  is finite.  $I$  offers a tariff where the fixed fee is replaced with an extra step in the tariff schedule, with a high initial wholesale price. In particular, the following tariff satisfies all the conditions for optimality:  $T_1 = 0$ ,  $W(q_1) = R'(0)q_1$  for  $0 \leq q_1 < S_1 = (P_M - c_I)Q_M / (R'(0) - c_I)$  and  $W_1(q_1) = R'(0)S_1 + c_I(q_1 - S_1)$  for  $S_1 \leq q_1 < S_2$  and  $W_1(q_1) = R'(0)S_1 + c_I(S_2 - S_1) + w_2(q_1 - S_2)$  for  $q_1 \geq S_2$  where  $w_2$  and  $S_2$  are equal to  $w_1$  and  $S_1$  defined above for the tariff with a fixed fee. This works given the strict concavity of  $R(Q)$  provided  $R'(0) = P(0)$  is finite. This ensures the tariff is everywhere above the revenue function for positive quantities up to the point where this tariff schedule intersects with the tariff  $T_1(q_1)$  at  $S_1$ , with the rest of the proof as before.

Clearly the tariffs here are the simplest tariff within the class  $\mathcal{T}_I$ , since they involve just two price breaks (three parts). Proposition 2 established that a tariff in  $\mathcal{T}_I$  with just one price break (two parts) could not achieve optimality. ■

This proposition shows that quantity discounting, defined in a very standard way, can be used by the incumbent to deter entry and obtain its full monopoly profits. Incremental-units quantity discounting involves the downstream firm enjoying a progressively lower wholesale price as it purchases more. The optimal contract can thus be expressed in the form that the downstream firm pays a fixed fee to buy its initial units, paying  $c_I$  per unit for purchases up to  $S_1$  units, and thereafter  $w_1 < c_I$  per unit for additional units. No instrument in the tariff is redundant. Clearly the two different wholesale prices serve different purposes. The wholesale price of  $c_I$  ensures that  $D$  chooses the monopoly price, so that the monopoly level of quantity will be purchased. The wholesale price of  $w_1$  that applies if at least  $S_1$  units are purchased ensures that  $E$  does not find entry profitable (since  $D$  would be willing to price down to  $\underline{P}$  to serve the market). The constraint that at least  $S_1$  units be purchased for  $D$  to enjoy the lower wholesale price ensures that  $D$  does not want to move onto this lower wholesale price schedule in the absence of competition. The fixed fee is chosen to extract  $D$ 's monopoly profit in equilibrium.<sup>20</sup> Proposition 4 also establishes that (except for demand specifications where it takes an arbitrarily high price to drive demand to zero), the same outcome can be achieved by replacing the fixed fee with an additional step in the existing tariff schedule (so it remains a

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<sup>20</sup>This optimal three-part tariff is just the lower envelope of two two-part tariffs. One two-part tariff would be the tariff offered by a monopolist, which applies in equilibrium, and the other two-part tariff is designed to deter entry.  $D_1$  self selects the appropriate two-part tariff, depending on whether it faces entry or not.

three-part tariff), but with a sufficiently high initial wholesale price. The purpose of this very high initial wholesale price is exactly the same as the fixed fee, to extract  $D$ 's monopoly profit in equilibrium.

A similar result is obtained when looking at tariffs within the class  $\mathcal{T}_A$ . Again, the simplest such tariff that is optimal exhibits quantity discounting, in this case all-units quantity discounting.

**Proposition 5** *The incumbent can obtain full monopoly profit, deterring entry in the process, by using a three-part tariff from the class  $\mathcal{T}_A$  which exhibits all-units quantity discounting. This is the simplest tariff in the class  $\mathcal{T}_A$  that allows the incumbent to extract the full monopoly profit. The lowest wholesale price is below the incumbent's marginal cost.*

**Proof.** Suppose  $I$  offers the tariff in which  $D$  pays  $R'(0)$  per unit if  $q_1 < Q_M$ ,  $P_M$  per unit if  $Q_M \leq q_1 < Q(\underline{P})$  and  $\underline{P}$  per unit if  $q_1 \geq Q(\underline{P})$ . All-units discounting follows from the strict concavity of  $R(Q)$  which implies  $R'(0) > P_M > c_I > \underline{P}$ . By construction,  $T_1(0) = R(0)$ ,  $T_1(Q_M) = R(Q_M)$  and  $T_1(Q(\underline{P})) = R(Q(\underline{P}))$ . Given the strict concavity of  $R(q_1)$  we also have that  $T_1(q_1) > \tilde{R}(q_1)$  everywhere except at  $q_1 = 0$ ,  $q_1 = Q_M$  and  $Q(\underline{P})$ , so that the conditions in proposition 3 hold for optimality provided  $R'(0) = P(0)$  is finite. This tariff is the simplest tariff within  $\mathcal{T}_A$  that can satisfy the conditions for optimality without the use of a fixed fee.

Exactly as with optimal incremental-units tariffs, the optimal contract can also be achieved with a fixed fee  $T_1 = (P_M - c_I)Q_M$  replacing the first step in the tariff function. The use of this fixed fee will in fact be necessary if  $P(0)$  is not defined. Then  $W_1(q_1) = c_I q_1$  if  $0 \leq q_1 < Q(\underline{P})$  and  $W_1(q_1) = w_1 q_1$  with  $w_1 = \underline{P} - T_1/Q(\underline{P}) < c$  if  $q_1 \geq Q(\underline{P})$ . Note  $R'(Q(\underline{P})) \geq 0$  implies  $\underline{P}Q(\underline{P}) > P_M Q_M$  from the concavity of the revenue function, so  $w_1 > (Q_M/Q(\underline{P}))c_I > 0$  ensuring the wholesale price still remains positive. ■

Again, no instrument in the tariff is redundant. Here the high initial wholesale price of  $R'(0)$  for output levels below monopoly ensures  $D$  does not want to move into this region when it is indeed a monopolist. The wholesale price of  $P_M$  for intermediate levels of output starting from the monopoly output ensures the monopoly quantity level will be purchased and extracts the corresponding profit. The lower wholesale price of  $\underline{P}$  that applies if at least  $Q(\underline{P})$  units are purchased ensures that  $E$  does not find entry profitable. The constraint that at least  $Q(\underline{P})$  units be purchased to enjoy the lower wholesale price ensures that  $D$  does not want to move onto this lower wholesale price schedule in the absence of competition.

## 5 Extensions

In this section, we consider some important extensions of the above benchmark model. Section 5.1 shows that our results may continue to hold even if the incumbent's contract offer is only observed with a small probability. Section 5.2 explains how the ability to use upfront fees only makes it easier for an incumbent to deter entry. In particular, we show how an upfront fee can be used to extract the disposal-rent which must be left to the incumbent's retailer when the revenue function is decreasing at  $Q(\underline{P})$ . Section 5.3 discusses the commitment problem and renegotiation problem that arise from the incumbent's incentive to renege on its original vertical contract in case entry does arise (i.e. out-of-equilibrium) arguing this suggests some types of vertical contracts may be preferred to others (i.e. to limit the profitability of renegeing or renegotiating its contract). Section 5.4 explores an implication of this point, where a constraint on vertical contracts imposed by the incentive to renegotiate allows us to explain how limit pricing and quantity discounting can arise at the actual equilibrium quantity purchased by the incumbent's retailer (rather than just as part of the incumbent's offer). Finally, section 5.5 explains how the analysis changes when firms are no longer homogenous Bertrand competitors, in which we explain how the rent that must be left to the incumbent's retailer in equilibrium can help ensure that the incumbent's entry deterring contract is renegotiation-proof.

### 5.1 Partially unobservable offers

Thus far, we have assumed the incumbent's contract can be observed by the entrant before it decides whether to enter. Indeed the incumbent is interested in disclosing its contract, since this is necessary to prevent entry. If there is some mechanism to disclose its true contract, it would want to use it. Even if no such mechanism exists, the vertical limit pricing strategy may still characterize an equilibrium, provided there is at least some small chance the incumbent's contract will be observed by the rival.

Specifically, suppose  $E$  observes  $I$ 's contract with some positive probability before deciding whether to enter in stage 2 and consider  $I$  making an optimal contract offer as in propositions 3-5. If  $E$  expects  $I$  to make such an offer, it will prefer to stay out even if the offer is unobserved. This finding is in contrast to the existing literature on competing vertical chains in which the commitment benefits of delegation rely on the observability of vertical contracts. Given contracts are not observed (or observed with an arbitrarily small probability), a manufacturer always does best selling to its retailer at its true cost and recovering the maximum profit possible through its fixed

fee (Katz, 1991). Note, though, for our results to hold in a robust way, it is necessary that there be at least some small probability of  $I$ 's contract being observed. If not, then the vertical limit pricing strategy would be weakly dominated by a simple two-part tariff in which the wholesale price equals  $c_I$  and the fixed fee extracts monopoly profit (which gives the same profit in equilibrium but would avoid the possibility that  $I$  has to supply units below cost in case of entry).

Partially unobserved offers do, however, open up the possibility of other equilibria arising. Entry deterrence is no longer the unique equilibrium outcome. For instance, as the probability of observing  $I$ 's contract becomes small, another equilibrium arises in which  $E$  chooses to be active and offers a tariff with a linear wholesale price at (or if necessary) just below  $c_I$ . Since  $c_I > \underline{P}$ , this ensures that  $E$  more than covers its fixed costs if it takes the market. Given this offer is expected by  $I$ , it is not profitable for  $I$  to offer a contract which induces  $D$  to undercut given its contract will most likely not be observed. It cannot do better than to set a wholesale price of  $c_I$ . Thus, in cases in which contracts are most likely to be unobserved, entry deterrence is not inevitable, although it is still possible.

## 5.2 Upfront fees and the “disposal-rent”

In our analysis up to this point we have restricted attention to a certain class of contracts in which upfront fees (fixed fees paid at the time the retailers accept their respective contracts) are not allowed but only fixed fees that apply when some positive quantity is purchased by the retailer are considered. We argued in section 3, large upfront fees (e.g. equal to the ongoing monopoly profit of the industry) may not be feasible or efficient in practice. However, even if such upfront fees are possible, the existing contracts characterized by propositions 3-5 remain optimal. In other words,  $E$  can still not profitably enter even if it can use upfront fees (while  $I$  does not). Given the contracts characterized by propositions 3-5, for  $D'$  to capture any share of the market it must price at or below  $\underline{P}$ , meaning there is no way for  $E$  to make a profit. Given  $I$  already obtains the monopoly profit, it has no reason to use upfront fees. Nevertheless, the ability to use upfront fees as opposed to the fixed fees analyzed up until now can make it easier for  $I$  to deter entry. As we will show, they are in fact necessary to ensure optimality if we allow the possibility that  $R'(Q(\underline{P})) < 0$ .

Upfront fees can make it easier for an incumbent to obtain monopoly profits through vertical limit pricing since they provide a further first-mover advantage to the incumbent, whose offer is accepted first. In equilibrium,  $D$  is a monopolist and  $I$  extracts the expected monopoly profit in equilibrium through its upfront fee. This

means if  $D$  does face competition, this upfront fee is a sunk cost for  $D$ , allowing  $I$  to collect more in total from  $D$  while still ensuring it will undercut  $E$  or  $D'$  as is required to prevent entry. This also means, with upfront fees,  $D$  may regret signing its contract with  $I$ , in the case there is entry. Despite this difference, the existing optimal contracts continue to work as in propositions 3-5 if the fixed fee  $T_1$  is simply replaced by an upfront fee of equal magnitude instead. Moreover, even if  $I$  uses the upfront fee to recover the maximum amount possible (so  $D$  will regret signing its contract if  $D'$  also enters), the same types of contracts as analyzed above will still exhibit quantity discounting since they will still have slope  $c_I$  for intermediate quantity levels and a lower slope (less than  $\underline{P}$ ) for sufficiently high quantities (in order that the wholesale tariff passes through  $R(Q)$  at  $Q(\underline{P})$ ). With this upfront fee, the wholesale price that applies at the margin for large purchases can be increased, although it must still remain less than  $\underline{P}$ .

An upfront fee becomes necessary to achieve optimality when the assumption  $R'(Q(\underline{P})) \geq 0$  does not hold. Without upfront fees,  $I$  cannot do better than to offer a three-part all-units tariff. However, unlike the case before in which  $D$  is left with no profit in equilibrium, here  $I$  must leave some rent for  $D$ . The reason is that if there is no entry (as will be the case in equilibrium),  $D$  can always make a positive profit buying  $Q(\underline{P})$  units for  $T(Q(\underline{P}))$  but then selling fewer units so as to obtain a higher revenue given  $T(Q(\underline{P})) = R(Q(\underline{P}))$  and  $R'(Q(\underline{P})) < 0$ , freely disposing of the additional units.<sup>21</sup> In fact,  $D$  will optimally sell only the revenue maximizing number of units; i.e. it will sell  $Q_R = \arg \max_Q R(Q)$  units. To prevent this situation,  $I$  will offer  $D$  a rent  $r_D$  for selling the monopoly output level  $Q_M$  which must equal  $R(Q_R) - R(Q(\underline{P})) = \tilde{R}(Q(\underline{P})) - R(Q(\underline{P}))$ . We call this rent  $D$ 's "disposal-rent", the rent  $D$  can obtain in equilibrium given it can freely dispose of the good. Thus, the incumbent may still deter entry, but its monopoly profit will be reduced by the size of this rent. If an upfront fee is possible, it will then allow  $I$  to extract this rent. The resulting optimal tariff involves an upfront fee plus a three-part all-units tariff (in other words, it has four parts).

### 5.3 Commitment and renegotiation

All our results to this point have illustrated that quantity discounting is a somewhat generic property of optimality contracts, reflecting that the optimal contract is pinned down by the underlying concave revenue function. Apart from counter-examples relying on unnecessarily complicated tariffs, there is one simple counter-

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<sup>21</sup>If instead the marginal cost of disposing of units is at least  $|R'(Q(\underline{P}))|$ , then no such rent is needed when  $R'(Q(\underline{P})) < 0$ .

example to quantity discounting that arises within the class of piece-wise linear tariffs, but only when an upfront fee can be used. It is the special case of an incremental-units tariff in which the price-break is set at  $Q_M$ , such that the wholesale price is less than  $c_I$  up to  $Q_M$  and above  $c_I$  for quantities exceeding  $Q_M$ . This also satisfies the key properties of optimality, but only because the fixed fee is upfront. As a monopolist,  $D$  will buy  $Q_M$  units, given additional units cost more than  $c_I$ . The tariff works rather like quantity forcing in equilibrium. Facing competition for  $D'$  and with its upfront fee sunk,  $D$  will be willing to price down to  $\underline{P}$  given the wholesale tariff passes through  $R(Q)$  at  $Q(\underline{P})$ . Since the initial wholesale price (up to  $Q_M$ ) is lower than  $c_I$ , this contract requires an upfront fee which exceeds the full monopoly profit (previously, we argued collecting the full monopoly profit through an upfront fee may be unreasonable). For this reason,  $I$  would like to renege on the contract after collecting the upfront fee, *in equilibrium* as well as out-of-equilibrium (i.e. regardless of whether the rival enters or not).

As the previous contract illustrated, our results rely on the assumption that the incumbent can commit to its vertical contract. (Note we do not require downstream firms to commit to the contract.) Some contracts require more commitment power than others in that the incentives for  $I$  to renege or renegotiate are stronger. The previous contract is one such case. More generally, the commitment problem arises since if the rival does enter, the incumbent would prefer not to provide goods according to its original tariff schedule given this would involve supplying its retailer below its own marginal cost. Knowing the incumbent would want to renege on its promise to supply goods below cost, entry becomes profitable.

To avoid any such commitment problem, we have implicitly treated the vertical contract as a commercial contract which is enforceable by law. Any breach of contract by the incumbent would give rise to penalties. Provided these are sufficient, our theory still applies. This is, of course, also a common requirement in the existing literature on vertical contracts. Alternatively, if a third party is available which can help enforce the contract, then upfront fees may be used to avoid the commitment problem. The upfront fee could be paid to the third party upon signing the contract. The fee could then only be passed onto the incumbent if it honors its initial contract. This can make it profitable for the incumbent to supply its retailer even in the face of entry, since the amount it receives in total (with the upfront fee) is  $\underline{P}Q(\underline{P}) + (P_M - c_I)Q_M$  which can exceed its costs  $c_I Q(\underline{P})$  of supplying this many units even though  $\underline{P} < c_I$ .

Even if the incumbent can commit to make good on its original contract offer, this still leaves open the problem of renegotiation given that the incumbent and



retailer will be jointly better off renegotiating in the face of a more efficient entrant. This possibility will be analyzed in section 5.4 for the case with homogenous goods (when renegotiation is costly) and in section 5.5 with imperfect substitutes (even if renegotiation is costless). Alternatively, in practice, the incentive to renegotiate may be limited by the incumbent’s incentive to keep a reputation for toughness in a multiperiod environment, along the lines of Kreps and Wilson (1982). In contrast to using a very low linear wholesale price to build such a reputation in a vertical setting, the incumbent can do better using a contract such as the one we propose, since it ensures the retailer only “fights” when necessary and otherwise extracts the monopoly profit on behalf of the incumbent. Even if wholesale prices can be conditioned on entry directly, doing so is more likely to raise antitrust concerns compared to the seemingly standard quantity discounting contract we have proposed.

## 5.4 Equilibrium limit pricing and renegotiation

For the most part, our theory of limit pricing is one in which limit pricing would not actually be observed in equilibrium. Although  $D$  is offered a tariff which involves units that can be purchased at below  $I$ ’s marginal cost over some range, in equilibrium  $D$  will not actually purchase any units at a price below  $I$ ’s marginal cost. Limit pricing can arise in equilibrium when  $I$  is constrained in the tariffs it can offer, such as when it can only offer two-part tariffs. To deter entry requires  $I$  offers a contract which in equilibrium will leave  $D$  with some rent. This means  $I$  can no longer extract  $D$ ’s monopoly profit in equilibrium.  $I$  will then choose a wholesale price below marginal cost which optimally trades-off the cost of increasing its quantity above its preferred monopoly level  $Q_M$  with the reduction in rent it must leave  $D$ .

Other types of constraints may lead to similar results. One constraint on tariffs may arise from the incentive for  $I$  to renegotiate tariffs in the face of entry. To illustrate the point, consider an optimal tariff such as the three-part incremental-units tariff characterized in Proposition 4. Suppose  $I$ ’s perceived cost of renegeing on the contract is  $\kappa$ , so that  $I$  will renege on a contract if it expects a loss of more than  $\kappa$  from continuing to supply according to the contract at any point.<sup>22</sup> Given the information structure we have assumed, up until the equilibrium quantity  $Q_M$ ,  $I$  will expect to sell  $Q_M$  units. Beyond this, it will face a loss equal to  $(R(Q_M) - c_I Q_M) - (R(Q(\underline{P})) - c_I Q(\underline{P}))$ . If this is greater than  $\kappa$ ,  $I$  will have to reduce this loss in order to avoid the ex-post incentive to renege in the case of entry. It can do so by

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<sup>22</sup>The cost of renegotiation  $\kappa$  may be related to the transaction costs of achieving a new agreement (monetary or time cost) or, in a repeated setting, to the loss of the incumbent’s reputation for toughness (as in Kreps and Wilson, 1982).

shifting some of the loss to the left of the equilibrium point. Since  $I$  only expects to sell the equilibrium quantity, this loss reduces the profit  $I$  expects in equilibrium but increases the incremental profit out-of-equilibrium. Shifting the loss to the left of the equilibrium point may be achieved by reducing the marginal wholesale price to be below cost before the equilibrium point is reached, giving rise to limit pricing in equilibrium.

To see why limit pricing may arise in equilibrium, note that  $\kappa$  together with  $Q(\underline{P})$  determine the loci of equilibrium choices for different tariffs. These various equilibrium choices lie on a line with slope  $c_I$  that is below  $R(Q_M)$  at  $Q_M$ . The amount by which the line is below  $R(Q_M)$  just depends on  $\kappa$ . By construction, the equilibrium choices all give  $I$  the same equilibrium profit *and* the same loss from continuing to supply beyond the equilibrium point (equal to  $\kappa$ ). One such tariff is the optimal three-part tariff with a fixed fee with the wholesale price  $w_1 = c_I$  but with the switching point  $S_1$  shifted to the right so that  $(c_I - w_2)(Q(\underline{P}) - S_1) = \kappa$ , where  $w_2 = R'(Q(\underline{P})) < c_I$ . The corresponding profit of  $I$  for this tariff is

$$\Pi_1 = R(Q(\underline{P})) - c_I S_1 - w_2(Q(\underline{P}) - S_1) = -(c_I - \underline{P})Q(\underline{P}) + \kappa. \quad (9)$$

Obviously, we require  $\kappa > (c_I - \underline{P})Q(\underline{P})$  to ensure that  $I$  can make a profit in equilibrium.

Given the cost of reneging, no other tariff can do better. Other three-part tariffs (with different  $w_1$ ) which imply the same loss from reneging also imply the same equilibrium profits as in (9) but can involve  $w_1 < c$  so that equilibrium limit pricing arises. This would also be true even without a fixed fee. Moreover, the simplest optimal tariff given  $\kappa > 0$  is actually a two-part tariff which exhibits equilibrium limit pricing given  $w_1 < c_I$  must hold so that the equilibrium point lies on the loci described above. Provided the cost of reneging is neither too high or too small, such a two-part tariff does just as well as the optimal three-part tariff described above.<sup>23</sup>

## 5.5 Imperfect substitutes

In this extension, we show that tariffs with quantity discounting deter entry and may still lead to full monopoly profit even when goods are imperfect substitutes.<sup>24</sup> The analysis in the case with imperfect substitutes introduces some new elements.

<sup>23</sup>If the cost of reneging is too large, then  $U_1$  will do better offering a three-part tariff given two-part tariffs constrain the maximum profit that can be extracted by  $U_1$  (e.g. in the case of linear demand, to half the monopoly profit). Nor can the cost of reneging be too small, since it must exceed  $(c_1 - \underline{P})Q(\underline{P})$  for  $U_1$  to make a profit.

<sup>24</sup>A more complete analysis of different possible demand specifications is given in a supplementary appendix.

Obtaining the monopoly profit while blocking entry may no longer be possible since for some demand specifications such as logit, neither retailer can be driven from the market. In this case, if  $I$  tried to extract the monopoly revenue then the rival could always profitably enter knowing that  $D$  would want to exit given it cannot recover the monopoly revenue facing such competition. Even though the incumbent makes a take-it-or-leave-it offer to its retailer and this contract deters entry, the incumbent has to leave some profit with its retailer to satisfy ex-post participation by  $D$ . Furthermore, since  $I$  can no longer obtain monopoly profit, sometimes it may prefer not to deter entry, preferring instead to share the market. Finally, we will see that when goods are not too close substitutes, entry-detering contracts can be renegotiation-proof.

To proceed, we retain the timing of the game and the structure of tariffs proposed for homogenous goods. We assume that the demands  $q_i(p_i, p_j)$  are downward sloping, goods are imperfect substitutes and the system of demand functions can be inverted to obtain inverse demands. The monopoly demand for  $D_i$  is denoted by  $Q_i(p_1)$ . We focus first on price competition. At the end of this section, we note how our analysis extends to the case of quantity competition.

We assume that when  $D$  and  $D'$  compete and they face linear tariffs,  $w = (w^1, w^2)$ , there always exists a unique Nash equilibrium which we denote by  $p(w) = (p_1(w), p_2(w))$ . The residual revenue function, defined as  $R_1(q_1, p_2) = p_1(q_1, p_2)q_2$ , is assumed to be strictly concave in  $q_1$ , and likewise for  $R_2(q_2, p_1)$ . We denote by  $M(p_2)$  the limiting quantity  $q_1$  which maximizes  $R_1(q_1, p_2)$  when it does not coincide with the monopoly revenue function  $R_1(q_1)$ , i.e.

$$M(p_2) = \sup \{q_1 \text{ s.t. } R_1(q_1, p_2) < R_1(q_1)\}.$$

We impose the following mild assumption on the family of residual revenue functions  $\{R_1(\cdot, p_2)\}_{p_2 \geq 0}$ . For any  $p'_2 \leq p''_2$  and for any  $q'_1 \leq q''_1 \leq M(p''_2)$ , the following holds:

$$R_1(q''_1, p''_2) - R_1(q'_1, p''_2) \geq R_1(q''_1, p'_2) - R_1(q'_1, p'_2). \quad (10)$$

This assumption is less restrictive than the usual increasing differences property (Milgrom and Shannon, 1994) since it only requires increasing differences over a restricted range of quantities (corresponding to  $D'$  being left with positive demand). When the quantity  $M(p_2)$  is infinite for all  $p_2$ , as it is for logit demand, this inequality is equivalent to the increasing differences property. For linear demand functions, the quantity  $M(p_2)$  is finite and the condition (10) still holds. However, the linear demand functions do not satisfy the usual increasing differences property (without

a restriction on quantities). To allow for a direct comparison with the benchmark analysis in section 4, we assume that  $M(p_2)$  is finite.<sup>25</sup>

To keep matters simple, we focus on all-units tariffs in what follows. This is sufficient for our purpose since the outcome for any entry deterring tariff can be replicated by a tariff from  $\mathcal{T}_A$ . Specifically, we consider tariffs of the form  $T_1(q_1) = W_A^{(3)}(q_1; w_1, w_2, w_3, S_1, S_2) \in T_A$ , where  $w_i$  are (average) wholesale prices and  $S_i$  are break points in the tariff. We set  $S_2 \leq M(\underline{P})$ , where the break-even price  $\underline{P}$  is defined by (2) and  $w_3 = p_1(S_2, \underline{P})$ . Wholesale prices  $w_1$  and  $w_2$  are chosen in such a way that when  $D$  competes with  $D'$  it chooses the part of  $T_1(q_1)$  corresponding to the quantity  $q_1 \geq S_2$  and wholesale price  $w_3$ . If  $D$  chooses the price  $p_1(q_1, p_2)$  in response to  $p_2$  such that  $q_1 < S_2$  then its profit is  $p_1(q_1, p_2)q_1 - wq_1$ , where  $w \in \{w_1, w_2\}$ . On the other hand, if  $D$  chooses  $p_1(S_2, p_2)$  its profit is  $p_1(S_2, p_2)S_2 - p_1(S_2, \underline{P})S_2$ . Assuming that  $S_2 \leq M(p_2)$ ,<sup>26</sup> then from (10), the latter profit dominates the former if  $w \geq p_1(q_1, \underline{P})$ , which is satisfied if we set  $w_1 \geq \partial_1 R_1(0, \underline{P})$  and  $w_2 \geq p_1(S_1, \underline{P})$ . Therefore, when competing with  $D'$ ,  $D$  chooses the part of the tariff  $T_1(q_1)$  with linear tariff  $w_3$ , and the maximum that  $E$  can obtain is

$$\Pi_2(S_2) = \max_{w^2} \Pi_2(p_1(w_3, w^2), p_2(w_3, w^2))$$

$$\text{such that } q_1(p_1(w_3, w^2), p_2(w_3, w^2)) \geq S_2.$$

Thus, if  $\Pi_2(S_2) \leq F$  then  $E$  cannot recover the cost of entry when facing the tariff  $T_1(q_1)$ .

The choice of parameters of the tariff  $T_1(q_1)$  is further restricted by optimality conditions and constraints (participation and incentive compatibility) for  $I$  and  $D$ . The analysis differs depending on the shape of the residual revenue function.

*Case 1.* When  $R_1(M(\underline{P})) = R_1(M(\underline{P}), \underline{P})$  and  $R'_1(M(\underline{P})) \geq 0$  the firm  $D$  can take the whole market for any  $p_2 \geq \underline{P}$ . Therefore,  $\widehat{\Pi}_2(S_2) = 0$  and the logic of vertical limit pricing in this case is exactly as in the homogenous setting.  $E$  will not enter even if its cost of entry is negligibly small. As a monopolist,  $D$  is willing to buy exactly  $Q_M$  units as is required for optimality. Full monopoly profit is possible.

*Case 2.* When  $R_1(M(\underline{P})) = R_1(M(\underline{P}), \underline{P})$  and  $R'_1(M(\underline{P})) < 0$ , the choice  $S_2 = M(\underline{P})$  leads to positive disposal-rent  $r_D = \widetilde{R}_1(S_2) - R_1(S_2)$ . This rent prevents  $D$  from preferring to buy  $S_2 = M(\underline{P})$  units as a monopolist (to get the low wholesale price) and disposing of the additional units to move to the maximum of the monopoly

<sup>25</sup>The supplementary appendix analyzes the case where  $M(\underline{P})$  may be infinite. Note that  $M(p_2)$  may be infinite only when the residual revenue function  $R_1(q_1, p_2)$  is unbounded and  $D_1$  cannot take the whole market even if  $D_1$  sets an arbitrarily low price  $p_1$ .

<sup>26</sup>See the supplementary appendix for other cases.

revenue function. With an upfront fee this rent can be captured by  $I$  so that it obtains the full monopoly profit if  $r_D$  is not too large. The condition for optimality is  $R_1(Q_M) - R_1(Q_M, \underline{P}) \geq r_D$ .

*Case 3.* In the cases 1 and 2,  $D$  would prefer to take the whole market if  $D'$  enters and sets any price at or above  $\underline{P}$ . This is because at  $S_2 = M(\underline{P})$  we have  $R_1(M(\underline{P})) = R_1(M(\underline{P}), \underline{P})$ . In the cases where  $R_1(M(\underline{P})) > R_1(M(\underline{P}), \underline{P})$  and  $R'_1(M(\underline{P})) < 0$  together with the disposal rent, a new rent  $r_I$  is introduced which we call the incentive-rent. This rent arises since the tariff required to deter entry is below the monopoly revenue function at  $S_2 = M(\underline{P})$ . When  $D$  does not face entry, it can earn a profit equal to the difference between the monopoly revenue function and the tariff at  $S_2$ , which is the amount  $r_I = R_1(S_2) - R_1(S_2, \underline{P})$ . To give  $D$  an incentive to choose the monopoly quantity  $Q_M$  in equilibrium,  $I$  must leave  $D$  at least  $r_I$ . In addition,  $I$  must also leave  $D$  with some disposal-rent  $r_D$  due to the fact that the monopoly revenue function is decreasing at  $S_2$ . Thus, the total rent that must be left is  $r = r_D + r_I = \tilde{R}_1(S_2) - R_1(S_2, \underline{P})$ .

$I$  can obtain full monopoly profit if, in addition to the given tariff, it can use an upfront fee to extract the rent that  $D$  must be left. However, if  $R_1(Q_M) - R_1(Q_M, \underline{P}) < r$  and, therefore, it is no longer optimal to induce  $D$  to choose the monopoly quantity in equilibrium (i.e. when entry is deterred) since  $D$  has an incentive to deviate by choosing  $Q_M$  instead of  $S_2$  when it competes with  $D'$ . The equilibrium quantity in this case is  $S_1$ , where  $R_1(S_1) - R_1(S_1, \underline{P}) = r$ . Thus  $I$  still chooses a three-part tariff.

*Case 4.* In the remaining case,  $R_1(M(\underline{P})) > R_1(M(\underline{P}), \underline{P})$  and  $R'_1(M(\underline{P})) \geq 0$ . The difference  $R_1(q_1) - R_1(q_1, \underline{P})$  increases for  $q_1 \leq M(\underline{P})$  by (10) and it is not possible to find the break points  $S_1$  and  $S_2$  such that the rent at  $S_2$  can be paid at  $S_1$ . Thus, the optimal entry-detering contract is two-part.

With linear demand,  $M(\underline{P})$  is always finite. This example can be used to confirm the general intuition that when goods are close substitutes it is easier to find the optimal entry deterring tariff. On the other hand, for distant substitutes the rent paid to  $D$  in order to deter entry is large and it is more likely that the optimal choice of  $D$  will be distorted away from the monopoly level. As a result, in some cases considered above,  $I$  obtains less than the full monopoly profit. Thus,  $I$  may do better by offering a contract which accommodates entry even though entry deterrence is possible. With price competition, this contract would involve softening competition as in the existing literature.

What about renegotiation possibilities? Denote by  $\Pi_1^*$  the equilibrium profit of  $I$  if it optimally accommodates entry and by  $\Pi_1$  the equilibrium profit of  $I$  if it

optimally deters entry (allowing here for the possibility of an upfront fee so that  $I$  can extract any rent  $r$ ). Then  $I$  prefers to deter entry if the profit it obtains is greater than the profit when it accommodates entry

$$\Pi_1 \geq \Pi_1^*. \quad (11)$$

When (11) is satisfied,  $E$  will not enter if  $I$  can commit not to renegotiate the contract.  $I$  does not want to renegotiate in the face of entry if

$$\Pi_1' \geq \Pi_1^* - r, \quad (12)$$

where  $\Pi_1'$  is the profit of  $I$  in case it keeps its entry deterring tariff as above and  $E$  enters. On the other hand, if  $I$  switches to the optimal entry accommodating tariff, it must leave  $D$  with the rent  $r$  which is what  $D$  can get if it does not renegotiate.

Conditions (11) and (12) together guarantee that  $T_1(q_1)$  is renegotiation-proof. When goods are close substitutes then it is hard to have a renegotiation-proof contract since the rent in (12) is too small (even zero as in Case 1 above). In the supplementary appendix a specific example of a renegotiation-proof entry-detering tariff is given. In the example given, goods are sufficiently distant substitutes and the fixed cost of entry is sufficiently high.  $I$  does not obtain the full monopoly profit. Still it is optimal for it to deter entry and not to renegotiate the contract. The crucial point in constructing this contract is that  $D$  has a high rent. This rent plays the role of a new reservation payoff (instead of zero) in the renegotiation stage. Therefore, when contemplating renegotiation,  $I$  is left with less profit than the standard entry-accommodating profit (i.e. that in Rey and Stiglitz, 1995). This can preclude renegotiation even when there is no cost of renegotiation.

A key property of the optimal tariffs is each involves quantity discounting, that is the wholesale price is declining at each break point in the tariff and the lowest marginal wholesale price is strictly less than  $I$ 's marginal cost  $c_I$ . This result can be generalized. Allowing for more general forms of the residual revenue function satisfying our assumptions (e.g. we can allow for a residual revenue function which is always increasing, as is the case with logit, or with a maximum of the residual revenue function to the left of the maximum of the monopoly revenue function), it can be established<sup>27</sup>:

**Proposition 6** *The optimal entry deterring tariff in the class of all-units three-part tariffs exhibits quantity discounting, with the marginal wholesale price declining at*

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<sup>27</sup>Since the proof contains many different cases, the supplementary appendix contains the formal details and proof.

each break point in the tariff and the lowest marginal wholesale price is (weakly) less than  $I$ 's marginal cost  $c_I$ .

The analysis above can be straightforwardly extended to quantity competition. The equivalent of assumption (10) is then that for any  $q'_2 \leq q''_2$  and for any  $q'_1 \leq q''_1 \leq M(q''_2)$ , the family  $\left\{ \widehat{R}_1(q_1, q_2) \right\}_{q_2 \geq 0}$ , where  $\widehat{R}_1(q_1, q_2) = p_1(q_1, q_2) q_1$ , satisfies the usual decreasing differences property (Milgrom and Shannon, 1994). The construction of the optimal entry deterring tariff is essentially the same as for the case of price competition. Since firms are less competitive under quantity competition than under price competition, to deter entry in this case  $D$  has to be induced to be even more competitive than it otherwise would be. This makes entry deterrence somewhat more difficult. On the other hand, renegotiation issues may be less severe under quantity competition since even when accommodating entry,  $I$  can benefit by committing to make its downstream firm more aggressive. That is, under quantity competition a “top-dog” strategy as in Fudenberg and Tirole (1984) is optimal for both entry deterrence and entry accommodation.

## 6 Conclusions

The key new idea developed in this paper is that commonly used forms of wholesale contracts involving quantity discounting can have entry deterring effects. An upstream incumbent can use such contracts to commit its downstream distributor or retailer to be more aggressive in the face of competition. In a benchmark setting, with homogenous price-setting firms, the simplest optimal contract is a three-part tariff. For low levels of purchases, the retailer purchases at a wholesale price set above the incumbent's marginal cost, thereby providing a way for the manufacturer to extract the retailer's monopoly profit (alternatively, a fixed fee can be used for this purpose). For purchases in some intermediate range, the retailer purchases at a wholesale price set equal to the incumbent's marginal cost, thereby ensuring the retailer sets the correct monopoly price when it indeed does not face competition. For purchases beyond some yet higher level, the retailer purchases at a wholesale price set below the incumbent's marginal cost, thereby ensuring that in the face of competition, the retailer will want to compete aggressively, in such a way that the rival will not want to enter. Thus, we provide a new explanation of limit pricing (or predation), one which does not depend on asymmetric information.

The benchmark model we have provided can be extended in numerous directions. Several natural extensions have been analyzed in this paper, most significantly to

the case with imperfect substitutes. In this and some other cases, conditions are provided under which the incumbent still deters entry using a vertical contract although often this requires the incumbent to leave its retailer with some rent. We showed such rents can help make the incumbent's vertical contract renegotiation-proof, thereby ensuring it can sometimes still profitably deter entry even when its contract can be freely renegotiated. We also explored what happens under alternative informational assumptions. The key assumptions necessary to establish our results are that the incumbent can commit, at least partially, to a wholesale contract and that this contract is observed, at least some fraction of the time, prior to a potential entrant making a decision about incurring fixed costs to enter the market (or in the case of predation, prior to an existing rival making a decision about incurring additional fixed costs to stay in the market).

An interesting direction for future research would be to explore whether a similar theory can be constructed when there is more than one incumbent firm, in which case the design of the optimal tariff is likely to be more complicated given the tension between deterring entry and softening competition between incumbents in the case of price competition (this tension may be less of a constraint under quantity competition). Also of interest is to consider a dynamic version of the vertical limit pricing story, in which downstream firms make a sequence of purchase decisions. We discussed such a possibility informally when analyzing optimal contracts in the face of costly renegotiation. A dynamic version of our vertical limit pricing story should be able to formally explain the use of rebates to deter entry or drive existing rivals out. In particular, it could be used to formalize the reputation story we gave, in which the incumbent's incentive to keep a reputation for toughness in a multiperiod or multiple-entrant environment provided an additional reason why it may not want to renegotiate its contract in case of entry.

Finally, related to this last point, a very natural extension of the established literature would be to modify the standard signaling and reputation stories of limit pricing and predation based on asymmetric information so as to incorporate the fact that the incumbent sells to retailers rather than final consumers. In such a theory, a low wholesale price might signal that the incumbent has low cost, thereby deterring entry. However, an aggressive wholesale pricing schedule can also have a direct entry deterring effect, in addition to its signaling effect, along the lines considered in this paper. Moreover, in such a setting, the nature of limit pricing and predation could be quite different if rivals only observe retail prices rather than wholesale contracts. In other words, the analysis of signaling and reputation building in vertical settings is likely to make for interesting future research.



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