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Competitive Effects of Vertical Integration with Downstream Oligopsony and Oligopoly*

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Preliminary Version

Abstract

We analyze the competitive effects of backward vertical integration by a partially vertically integrated firm that competes with non-integrated firms both upstream and downstream. We show that vertical integration is procompetitive under fairly general conditions. It can be anticompetitive only if the ex ante degree of integration is relatively large. Interestingly, vertical integration is more likely to be anticompetitive if the industry is less concentrated. These results are in line with recent empirical evidence. In addition, we show that even when vertical integration is procompetitive, it is not necessarily welfare enhancing.

Keywords: Vertical Integration, Downstream Oligopsony, Downstream Oligopoly, Competition Policy, Capacity Choice

JEL-Classification: D43, L41, L42

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1 Introduction

Over the last two decades substantial progress has been made on identifying pro- and anticompetitive effects of vertical mergers. After the Chicago School critique of the relatively aggressive enforcement policy on vertical mergers in the 1960's, several theories have emerged that base the potential competitive effects of vertical integration on more solid game-theoretic ground. Yet, there is no general consensus under which conditions a vertical merger is likely to benefit or harm consumers. The elimination of double marginalization has been identified as a major efficiency gain from vertical integration. On the other hand, the merging parties may be inclined to raise the input prices to their rivals and thereby induce market foreclosure. As a consequence of this trade-off, vertical mergers are often judged by antitrust authorities and courts on a case-by-case basis. General conclusions under which conditions either effect dominates are not easily gained.[1]

Moreover, many models that identify different effects of vertical mergers are not readily applicable for policy implications because they are unsatisfactory in two important aspects. First, merging parties often claim to merge because of efficiency gains in production that lead to cost reductions. This means that they not only avoid double marginalization but are also able to produce the output good in a different and more efficient way than without integration.[2] Second, firms that have oligopolistic market power in the downstream market often also exert oligopsonistic market power when buying the intermediate goods. However, these effects are not considered in many important and well-established models on vertical integration (e.g. Salinger, 1988; Ordover, Saloner, and Salop, 1990; Hart and Tirole, 1990; Choi and Yi, 2000; Chen, 2001).

A notable exception is Riordan (1998), who considers a model with a dominant firm and a competitive fringe in the downstream market. To produce the final good, firms need a fixed input, termed capacity, that is competitively offered on an upward sloping supply curve. The dominant firm exerts market power both downstream and on the input market. The more capacity a firms owns, the lower are its production costs of the final good. Therefore, the model is not open to the two criticisms above. If the dominant firm integrates backwards, it

[1] For recent surveys on the effects of vertical mergers, see Church (2008), Rey and Tirole (2007) and Riordan (2008).

[2] For example, Church (2008) argues that one of the reasons why vertical mergers are complicated to evaluate is that the incentives to integrate often arise because of non-price efficiencies (in contrast to the efficiency of internalizing price effects such as double marginalization) and are usually not attributable to market power effects.

acquires more capacity and so produces more output. On the other hand, since the demand for capacity increases, the price of capacity increases as well and so fringe firms are foreclosed. Riordan (1998) obtains the powerful result that the second effect always dominates and, thus, that vertical integration is anticompetitive, i.e. leads to a decrease in output and to an increase in the final good price. However, a drawback of Riordan's model is that it applies only to market structures where the final good market is comprised of a dominant firm facing a competitive fringe.

The present paper provides an analysis of a model with an oligopolistic downstream market where the structure is otherwise very close to Riordan (1998). We obtain the following results. First, vertical integration is procompetitive under a fairly wide array of circumstances. In the extreme, even monopolizing the downstream market can enhance consumer welfare because the integrated firm expands its quantity by a very large extent after integrating. Thus, the policy implications of the present paper differ from those of the dominant firm model in that they suggest a more permissive approach to vertical mergers. Second, we find that vertical integration is more likely to be procompetitive if the ex ante degree of integration of the integrating firm is relatively low and if the concentration of the industry is relatively high, i.e. if the number of competitors is small. The former result is intuitive and policy relevant. The latter contrasts with the common wisdom that vertical mergers are suspicious especially when firms have considerable market power.[3] Third, we show that in the limit as the number of competitors becomes large, vertical integration is always anticompetitive. Therefore, in the limit our model encompasses the one of Riordan (1998), which shows that his model is a good approximation for market structures in which competitors have only little market power. Fourth, even if it is procompetitive, vertical integration is not necessarily welfare increasing. Thus, procompetitive but welfare reducing mergers are possible. Last, there exist critical thresholds for input and output market shares for an integrating firm above which further vertical integration is anticompetitive. Both market shares are very similar. This is useful for antitrust policy because these market shares are typically relatively easy to observe for antitrust authorities. Since both critical thresholds fall in the number of competitors, antitrust authorities should be the more suspicious about vertical mergers the larger is the number of firms.

[3] For example, Lafontaine and Slade (2007) note that most empirical studies on vertical integration were conducted for highly concentrated markets because evidence for foreclosure is thought most likely to be found there.

The intuition behind our main result is the following. As in the dominant firm model, backward integration has a procompetitive efficiency effect because the integrated firm produces more output thanks to its lower production cost and an anticompetitive foreclosure effect because rival firms lower their capacity and produce less. However, in an industry with a dominant firm and a competitive fringe only the dominant firm has market power. Thus, the dominant firm has only little incentive to expand its quantity after a capacity increase because it wants to keep the output price high. Therefore, it utilizes its capacity less efficiently after integration. In contrast, in an oligopolistic market all firms exert market power, and so each of them is inclined to restrict its quantity relative to capacity. As a consequence, they all utilize their capacity less efficiently than a competitive fringe firm does, and so the quantity reduction of a non-integrated firm in oligopoly after foreclosure is smaller than the reduction resulting from exit of a fringe firm that has no market power. Therefore, the aggregate reaction of non-integrated competitors is relatively weak which renders vertical integration procompetitive for a broad range of circumstances.

The intuitions for our other results can now be grasped from the above explanation. If a firm becomes more integrated, its production costs fall and, therefore, it benefits to a larger extent from a higher final good price. Thus, it has an incentive to curb its quantity expansion and so it utilizes its capacity less efficiently. This explains why vertical integration is more likely to be anticompetitive if the merging firm is already integrated to a large extent. It also helps to understand why procompetitive but welfare reducing mergers can occur. Vertical integration changes the cost structure by shifting more capacity to the integrated firm that uses it less efficiently. This does not play a role when considering just the effect on final output but it is important for overall welfare. Therefore, welfare may fall although output rises. Finally, the aggregate reaction of competitors is larger, the more competitors are present. Thus, if there are many small firms, their aggregate capacity and quantity reduction as a reaction to vertical integration is larger. This explains why vertical integration is more likely to be anticompetitive if the industry is less concentrated, i.e. if the number of firms is large.

The empirical predictions of our model are consistent with recent evidence. For example, Hortaçsu and Syverson (2007) study vertical integration in the cement and ready-mixed concrete industries. They find that output rises and prices fall if vertical integration increases and show that this can be explained by the expansion of larger vertically integrated firms at the expense of non-integrated firms. In addition, they demonstrate that via vertically integrating a firm gets larger, i.e. produces more final output, but does not become more productive per se. Both of these results are in line with our findings that an integrated firm produces a larger quantity but utilizes its capacity less efficiently. Lafontaine and Slade (2007) present a comprehensive review of empirical studies on the effects of vertical integration for several industries. They show that the vast majority of these studies find only weak empirical evidence for the foreclosure effect but document strong efficiency effects. In particular, the efficiency effect dominates the foreclosure effect in almost all studies, and, therefore, vertical integration has led to a fall in the final good price in almost all cases.[4]

Our paper extends and complements Riordan's (1998) study. We show that his results are robust in that they obtain in the limit of the oligopolistic model when the number of competitors becomes large, but that the conclusions and policy implications differ when the competitors possess significant market power. As mentioned, most of the literature on vertical integration is concerned with the trade-off between avoidance of double marginalization and foreclosure. For example, Hart and Tirole (1990) or Ordover, Saloner, and Salop (1990), where no efficiency gains from vertical integration are present, are only concerned with the foreclosure motives. In Salinger (1988), Choi and Yi (2000) and Chen (2001) both effects are present but although the downstream market is comprised of an oligopoly or a duopoly, downstream firms have no market power in the intermediate good market. A recent model that incorporates both effects and additionally allows downstream firms to exert market power in the intermediate good market is Hendricks and McAfee (2009). However, when analyzing vertical mergers they keep the downstream price fixed and suppose that the market structure consists of no vertical integration at the outset. Under these assumptions they show that output increases with vertical mergers. In contrast, in our model the downstream price is flexible and, as argued above, we show that a crucial variable to determine the competitive effects of vertical integration is the degree to which the industry is already integrated. Nocke and White (2007) analyze a different aspect of vertical mergers, namely whether it facilitates upstream collusion. They show that this is indeed the case because vertical integration reduces the number of buyers for rival firms and, thus, they have less incentives to deviate from the collusive agreement by reducing their prices.

The remainder of the paper is organized as follows. Section 2 lays out the model and Section

[4]Cooper, Froeb, O'Brien, and Vita (2005) survey recent empirical evidence on vertical mergers and reach a similar conclusion, namely that there is strong support that vertical mergers are procompetitive and that instances where they are unambiguously negative are difficult to find.

3 presents the equilibrium. In Section 4 we derive the competitive effects of vertical integration. Section 5 analyzes the effects of vertical integration on social welfare. In Section 6 we discuss the empirical evidence and the policy implications of the model. Section 7 concludes. All proofs are relegated to the appendix.

2 The Model

The model is adapted from Riordan (1998) with some minor differences with respect to timing. The main difference is that we consider a downstream oligopoly in contrast to a dominant firm and a competitive fringe. There are two types of firms, one (partially) vertically integrated firm, which we index by I and $N \geq 1$ non-integrated firms. A typical non-integrated firm is indexed by j.[5]

All firms produce a homogenous good and compete à la Cournot on the downstream market, where the inverse demand function is $P(Q)$ with $P'(Q) < 0$. So $P(Q)$ is the market clearing price for aggregate quantity $Q \equiv q_I + \sum_{j=1}^{N} q_j$. The cost function of firm $j \in \{1, ..., N\}$ for production of q_j units is given by

$$c(q_j, k_j) = k_j C\left(\frac{q_j}{k_j}\right),$$

where k_j is firm j's production cost reducing capacity and $C'(q_j/k_j) \geq 0$ and $C''(q_j/k_j) > 0$.[6] The integrated firm I has a cost advantage of $\gamma \geq 0$ per unit of output.[7] Therefore, its cost function can be written as

$$c(q_I, k_I) = k_I C\left(\frac{q_I}{k_I}\right) - \gamma q_I.$$

As a consequence, marginal costs for all firms are increasing in the produced quantity for given capacity but $c(q_i, k_i)$ exhibits constant returns to scale in q_i and k_i, $i \in \{I, 1, ..., N\}$. This cost function is more general than most cost functions used in models of vertical integration since it allows a firm to vary its quantity for given capacity. In particular, it is more general than the widely used fixed proportions cost function which allows a firm to produce only a maximal number of output units given its number of inputs units.

Capacity is supplied competitively with an inverse supply function of $R(K)$, with $R'(K) > 0$ and $K \equiv k_I + \sum_{j=1}^{N} k_j$. Firm I's initial capacity endowment, i.e. its ex ante degree of vertical

[5] In Section 6.4 we briefly discuss the consequences of more than one firm being vertically integrated.

[6] This type of cost function was introduced by Perry (1978) and used e.g. by Perry and Porter (1985), Riordan (1998) and Hendricks and McAfee (2009). For an interpretation of the cost function see Perry (1978) and Riordan (1998).

[7] One can also interpret γ as a quality advantage in the integrated firm's product.

integration, is denoted by $\underline{k} \geq 0$.

The timing of the game is as follows: In the first stage, the capacity stage, all firms i choose simultaneously their level of capacity k_i. The ex ante degree of vertical integration $\underline{k}$ is exogenously given and common knowledge. Firm I buys $k_I - \underline{k}$ at the market price $R(K)$. Thus, the profit function of firm I at the capacity stage is given by

$$\Pi_I(q_I, k_I) = P(Q)q_I - k_I C\left(\frac{q_I}{k_I}\right) + \gamma q_I - (k_I - \underline{k})R(K),$$

and the one of a non-integrated firm j is $\Pi_j(q_j, k_j) = P(Q)q_j - k_j C(q_j/k_j) - k_j R(K)$. As in Riordan (1998), this implies that firm I has the opportunity to sell undesired capacity to an outside market, which occurs if $k_I < \underline{k}$. In the second stage, the quantity stage, all firms simultaneously choose their quantities after having observed all capacity levels $\mathbf{k} = (k_I, k_1, .., k_N)$. The aggregate quantity Q determines the market clearing price $P(Q)$ and payoffs are realized. We focus on symmetric subgame perfect equilibria, where symmetry means that the non-integrated firms play the same strategies.

To ensure interior solutions and a unique equilibrium, we make some shape assumptions on the demand, supply and cost function. We suppose that $\lim_{Q\to\infty} P(Q) \leq 0$, that $P''(Q)$ is not too positive and that $P'''(Q)$, $C'''(q_i/k_i)$ and $R''(K)$ are not too negative. These assumptions, which are similar to the ones imposed by Riordan (1998), are relatively mild and guarantee that the second-order conditions are fulfilled. They are standard in two-stage games where firms have market power upstream and downstream.[8]

3 Equilibrium

We solve the game by backward induction.

3.1 The Quantity Stage (Stage 2)

At the quantity stage, $\mathbf{k}$ is already determined. Since $\underline{k}$ has a direct effect only on k_I but not on q_I, the first-order condition for a profit maximum for each firm i does not depend directly on $\underline{k}$. So, the first-order condition for a non-integrated firm $j \in \{1, ..., N\}$ in the subgame of the quantity stage is given by[9]

$$P(Q) + P'(Q)q_j = C'_j, \tag{1}$$

[8] Linear demand and supply functions and quadratic cost functions are obviously sufficient for these assumptions to be satisfied. While it is possible to spell out more general conditions on these functions, these more general conditions are rather complex and obscure.

[9] To shorten notation, in the following we abbreviate $P(Q)$ by P, $C(q_j/k_j)$ by C_j and $R(K)$ by R. We do so also for all derivatives.

while the first-order condition for firm I is given by

$$P(Q) + P'(Q)q_I = C_I' - \gamma. \tag{2}$$

It is easy to see that the second-order conditions are satisfied given that P'' is not too positive, which we assumed above. Our assumptions also imply that firm i's reaction function has a negative slope greater than -1. Therefore, every quantity-stage subgame has a unique equilibrium.

We denote by $Q^*(\mathbf{k})$ the aggregate equilibrium quantity given any vector of capacities $\mathbf{k}$, and by $q_i^*(\mathbf{k})$ the corresponding equilibrium quantity of firm i. The first-order conditions (1) and (2) imply that $q_i^*(\hat{k}_i, \mathbf{k}_{-i}) > q_i^*(k_i, \mathbf{k}_{-i})$ if and only if $\hat{k}_i > k_i$, where $\mathbf{k}_{-i}$ is the capacity vector of all firms other than i. That is, a firm's optimal quantity increases in its capacity independently of the type of the firm.[10] We then get the following lemma:

Lemma 1 $\frac{q_i^*(\mathbf{k})}{k_i}$ *decreases in* k_i $\forall i \in \{I, 1, ..., N\}$.

The same result obtains in Riordan (1998). As observed above, a firm with a larger capacity produces a larger quantity. But because it produces more inframarginal units, it benefits more from a price increase. Therefore, it has an incentive to lower its quantity relative to capacity, i.e. it utilizes its capacity to a smaller extent.

From the first-order conditions we get the following intuitive lemma.

Lemma 2

$$\frac{dq_i^*(\mathbf{k})}{dk_i} > 0 \quad \textit{and} \quad \frac{dq_i^*(\mathbf{k})}{dk_j} < 0 \quad \textit{for all} \quad i \neq j, i, j \in \{I, 1, ..., N\}. \tag{3}$$

Therefore, all own effects are positive and all cross effects are negative.

3.2 The Capacity Stage (Stage 1)

We now move on to the first stage of the game, the capacity choice game.

Using the envelope theorem we get that the first-order condition of a non-integrated firm j in the capacity stage is given by

$$\frac{\partial \Pi_j}{\partial k_j} = P' \frac{dQ_{-j}^*}{dk_j} q_j^* - C_j + C_j' \frac{q_j^*}{k_j} - R - k_j R' = 0, \tag{4}$$

[10] To see this, suppose to the contrary that $\hat{k}_i > k_i$ but $q_i^*(\hat{k}_i, \mathbf{k}_{-i}) \leq q_i^*(k_i, \mathbf{k}_{-i})$. But then the right-hand side of (1), respectively (2), is strictly smaller for $\hat{k}_i$ than for k_i because C_i is convex in q_i and decreasing in k_i while the left-hand side is (weakly) larger for $\hat{k}_i$ than for k_i, which is a contradiction. Conversely, if $q_i^*(\hat{k}_i, \mathbf{k}_{-i}) > q_i^*(k_i, \mathbf{k}_{-i})$, the left hand-side of (1), respectively (2), is smaller than the right-hand side. Since C_i is convex, $\hat{k}_i$ must be bigger than k_i.

where Q^*_{-j} is the equilibrium quantity of all firms but firm j. The first-order condition of the integrated firm I is given by

$$\frac{\partial \Pi_I}{\partial k_I} = P' \frac{dQ^*_{-I}}{dk_I} q^*_I - C_I + C'_I \frac{q^*_I}{k_I} - R - (k_I - \underline{k})R' = 0. \tag{5}$$

Showing that an equilibrium exists and, if it does, is unique is more involved in the capacity stage than in the quantity stage. This is the case because now a change in firm i's capacity has an effect on the equilibrium quantity of each firm in the second stage. This must be taken into account when considering the capacity reaction function of every firm other than i. Thus, the expression for the reaction function is more complicated than in a standard single stage game.[11] Nevertheless, using arguments based on Kolstad and Mathiesen (1987) the next lemma establishes that an equilibrium exists and is indeed unique.

Lemma 3 *There exists a unique equilibrium in the capacity stage. In this equilibrium,* k^*_I *and* k^*_j $\forall j \in \{1, ..., N\}$ *are determined by* (4) *and* (5).

From the two first-order conditions we can now derive the following lemma:

Lemma 4

$$\frac{dk^*_I}{d\underline{k}} > 0 \quad \textit{and} \quad \frac{dk^*_j}{d\underline{k}} < 0 \quad \forall j \in \{1, ..., N\}.$$

This result is intuitive. If $\underline{k}$ increases, firm I owns more capacity units. Thus, the number of inframarginal units for which it has to pay the capacity price R on the upstream market decreases. As a consequence, firm I finds it optimal to increase its overall amount of capacity. On the other hand, $\underline{k}$ does not directly influence the optimal capacity of the non-integrated firms. However, since k^*_I rises, the price for capacity increases, and so it is optimal for each non-integrated firm to acquire less capacity. Thus, capacities are strategic substitutes.

It follows immediately from Lemma 4 and equations (1) and (2) that $k^*_I > k^*_j$ if either $\underline{k} > 0$ or $\gamma > 0$ or both. Thus, if firm I is vertically integrated to some extent or has a cost advantage or both, its equilibrium capacity is larger than the one of the non-integrated firms. From Lemma 1 we know that this implies that its capacity utilization q^*_I/k^*_I is lower than for the non-integrated firms if γ is small.[12]

[11] Moreover, the game is not an aggregator game since the reaction of a non-integrated firm is different if firm I changes its capacity than if a non-integrated firm changes its capacity because this has different effects on the overall quantity produced in the second stage.

[12] This replicates the finding of Riordan (1998) who shows that the capacity utilization of the dominant firm is smaller than the one of fringe firms provided that the cost advantage is not too large.

4 Competitive Effects of Vertical Integration

In this section we analyze whether vertical integration is pro - or anticompetitive, i.e. whether a change in $\underline{k}$ increases or decreases the aggregate equilibrium quantity supplied in the downstream market. From above it follows that an increase in $\underline{k}$ has a direct positive effect on k_I and via that an indirect negative effect on all k_j.[13] This in turn leads to an increase in q_I and to a decrease in all q_j. Thus, $dQ/d\underline{k} > 0$ if and only if

$$\frac{dQ}{d\underline{k}} = \left(\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right)\frac{dk_I}{d\underline{k}} + N\left(\frac{dq_I}{dk_j} + \frac{dq_j}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right)\frac{dk_j}{d\underline{k}} > 0$$

or equivalently

$$\frac{\left(\frac{dk_j}{d\underline{k}}\right)}{\left(\frac{dk_I}{d\underline{k}}\right)} > -\frac{\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}}{N\left(\frac{dq_I}{dk_j} + \frac{dq_j}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right)}. \tag{6}$$

The left-hand side of (6) expresses the relative change of a non-integrated firm's capacity with $\underline{k}$ to the change in the integrated firm's capacity. We know from Lemma 4 that this relative change is negative. The right-hand side gives a benchmark against which to compare this term. The inequality says that if the relative change is small enough in absolute terms, then vertical integration is procompetitive. Intuitively, if k_j does not fall by too large an extent after firm I becomes more integrated, the positive effect resulting from the increase in q_I dominates the negative effect that stems from the decrease in q_j of all non-integrated firms.

Inserting the respective derivatives (derived in the proof of Lemma 2) in the term on the right-hand side of (6) and simplifying yields

$$\frac{\left(\frac{dk_j}{d\underline{k}}\right)}{\left(\frac{dk_I}{d\underline{k}}\right)} > -\frac{C_I'' \frac{q_I}{k_I}(C_j'' - k_j P')}{N C_j'' \frac{q_j}{k_j}(C_I'' - k_I P')}. \tag{7}$$

To gain some intuition for this formula suppose that both $\underline{k}$ and γ are zero. In this case all $N+1$ firms are the same and we have that $q_I = q_j$, $k_I = k_j$ and thus $C_I'' = C_j''$. As a consequence, the right-hand side of (7) simplifies to $-1/N$: At $\underline{k} = 0$ and $\gamma = 0$, all firms' capacity utilization is the same. Thus, to keep overall output constant, the aggregate capacity reduction of the non-integrated firms must be the same as the increase in the capacity of firm I. But since all N non-integrated firms are symmetric, this is the case if each of them lowers its capacity by $1/N$ of the increase in the integrated firm's capacity.

[13] To simplify notation here and in what follows we omit the superscript $*$ on equilibrium quantities and capacities.

Suppose now that $\gamma = 0$ but $\underline{k} > 0$. From the above lemmas we know that in this case $k_I > k_j$, $q_I/k_I < q_j/k_j$ and thus $C_I'' < C_j''$. Then, the right-hand side of (7) is in absolute value smaller than $1/N$. The reason is that in this case the integrated firm uses its capacity less efficiently than a non-integrated firm. As a consequence, if all non-integrated firms reduced their capacity in sum by the same amount as the capacity increase of the integrated firm, overall output would fall since capacity is shifted to the less efficient firm. Thus, to keep output constant the reduction has to be smaller and overall capacity must rise.

To characterize how vertical integration changes overall output, we begin with the case where $\underline{k}$ is small.

Proposition 1 *For any finite N there exists a $\underline{k}^* > 0$, such that for all $\underline{k} < \underline{k}^*$ vertical integration is procompetitive at the margin.*

If the ex ante degree of vertical integration is small enough, further integration is procompetitive at the margin. If $\underline{k}$ is small, firm I is more efficient than a non-integrated one or, in case γ is small, only slightly less efficient. But the reaction of the non-integrated firms to an increase in $\underline{k}$, i.e. the fall in Nk_j, is in sum always smaller than the increase in k_I. Thus, the aggregate capacity that is used increases and overall output rises.

Next we look at the opposite case where $\underline{k}$ is so high that the resulting k_I^* in equilibrium is large enough to induce $k_j^* = 0$ for all $j \in \{1, ..., N\}$. The following definition is useful. We define $\bar{\underline{k}}$ as the ex ante degree of vertical integration at which $k_j^* = 0$ and therefore $q_j^* = 0$, i.e. if $\underline{k} = \bar{\underline{k}}$, only the integrated is active and the market is monopolized.[14]

Proposition 2 *For any finite N there either exists a $\underline{k}^{**} < \bar{\underline{k}}$, such that vertical integration is anticompetitive at the margin for all $\underline{k} > \underline{k}^{**}$, or it is procompetitive at the margin for all $\underline{k}$ close to $\bar{\underline{k}}$.*

This result implies that even if rival firms are completely foreclosed by the integrated firm, this is not necessarily detrimental to consumer welfare. This is the case because our model does explicitly take into account efficiency gains in production beyond pure avoidance of double marginalization. If a firm has to acquire a very large amount of capacity, so that its competitors

[14]Such a $\bar{\underline{k}}$ necessarily exists since from Lemma 4 we know that $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$. In addition, variable production costs $c(q_j, k_j)$ are decreasing in k_j since $C''(q_j/k_j) > 0$. Thus, both production and capacity costs are increasing for a non-integrated firm j, while revenue is decreasing because q_j is decreasing and q_I is increasing. So if $\underline{k}$ and therewith k_I is large enough, j's costs are too high relative to $P(Q)$, and so it is optimal for firm j to stop producing.

stop producing, its production costs are so low that it may produce a quantity that is larger than the oligopoly quantity.

The question arises whether the thresholds identified in Propositions 1 and 2 coincide. Put differently, is it possible to show that there is a unique $\underline{k}^* = \underline{k}^{**}$ in case this threshold exists or that $dQ/d\underline{k} > 0$ for all $k \leq \bar{k}$?[15] Since the expressions that are involved in the calculations are rather complicated and unwieldy, it is not possible to show uniqueness in general. However, we can show that the threshold, provided it exists, is indeed unique for two important subclasses of the general specification: The first class consists of models where R' is dominating the derivatives of the other functions.[16] The second class is the widely used linear-quadratic specification, i.e. the demand and supply functions are linear and the cost function is quadratic. The functions can then be written as $P(Q) = \alpha - \beta Q$, $R(K) = \delta K$ and

$$C(q_i/k_i) = \frac{c}{2}\left(\frac{q_i}{k_i}\right)^2 \forall i \in \{I, 1, ..., N\},$$

where α, β, c and δ are positive constants. The integrated firm still has a marginal cost advantage $\gamma \geq 0$.

Proposition 3 *Suppose either that (i) R' is dominating all other derivatives in absolute values or that (ii) the model is linear-quadratic. Then, for any finite N there either exists a unique $\underline{k}^* \in (0, \bar{k})$, such that vertical integration is procompetitive at the margin for all $\underline{k} < \underline{k}^*$ and anticompetitive at the margin for all $\underline{k} > \underline{k}^*$, or vertical integration is always procompetitive.*

A few comments are in order. The intuition for case (i) of the proposition is that if R' is large compared to all other derivatives, the capacity reaction of a non-integrated firm to a change in k_I, and therefore also to a change in $\underline{k}$, is independent of the value of $\underline{k}$. Therefore, $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ stays constant as $\underline{k}$ varies. But the right-hand side of (7) is in absolute terms constantly decreasing as $\underline{k}$ rises. This is the case because firm I utilizes its capacity less and less as $\underline{k}$ increases. Thus, there is at most one point of intersection between the left- and the right-hand side of (7). Case (ii) of the proposition is important because it shows that the threshold is unique (given that it exists) in the general linear-quadratic specification used in many industrial organization models. In addition, this indicates that the threshold is unique

[15] This would imply that the left- and the right-hand side of (7) cross either exactly once or never.

[16] A steeply increasing supply curve can be observed in many high technological industries. For example, dedicated fiber-optic cables or several semiconductor devices like customized integrated circuits that are produced in specialized plants exhibit large production costs that are steeply increasing once a plant produces close to its capacity limit.

also for specifications that are close to the linear-quadratic one and suggests that the threshold is likely to be unique even more generally.

Since our result that the efficiency gains of vertical integration are often larger than the foreclosure effects differ from the one of the dominant firm model, it is of interest to understand the economic reason behind this. In the dominant firm model vertical integration leads to foreclosure of fringe firms. As a consequence, some of them exit the market. But since fringe firms have no market power, they have no incentive to restrict their quantity. Therefore, they utilize their capacity efficiently. In contrast, under downstream oligopoly the competitors of the integrated firm also exert market power and restrict their output to keep the final good price high. Thus, as a consequence of the foreclosure through vertical integration, a rival firm lowers its quantity to a smaller extent than the exit of a fringe firm reduces the final output in the dominant firm model. Moreover, as vertical integration increases, each rival firm in the oligopoly case buys a smaller amount of capacity, and so its capacity utilization increases. As a result, in the dominant firm model the output contraction of fringe firms after foreclosure is larger than the reaction of rival firms under oligopoly.

Change in the Number of Firms We now consider the effect of a change in the number of firms on the competitive effects of vertical integration. First, we look at the general model when the number of downstream firms becomes large. This is of interest from a theoretical perspective because this limit corresponds to the model Riordan (1998) analyzes. Second, understanding how the competitive effects of vertical integration depend on the competitive structure of the industry is particularly relevant for antitrust policy implications.

Proposition 4 *If $N \to \infty$, then vertical integration is anticompetitive for all $0 \leq \underline{k} \leq \bar{k}$.*

So if the downstream market becomes perfectly competitive, vertical integration is always anticompetitive. Intuitively, the aggregate reaction of the non-integrated firms to an increase in $\underline{k}$ is the larger, the more firms are in the market. Therefore, the aggregate capacity reduction and, hence, the quantity reduction of the non-integrated firms increases in their number. As N goes to infinity this effect dominates any cost advantage of the integrated firm. Thus in the limit, as the market power of the non-integrated firms vanishes, we obtain the result of Riordan (1998). As the integrated firm has no first-mover advantage in our model but has one in Riordan's, Proposition 4 also shows that his strong result stems genuinely from the

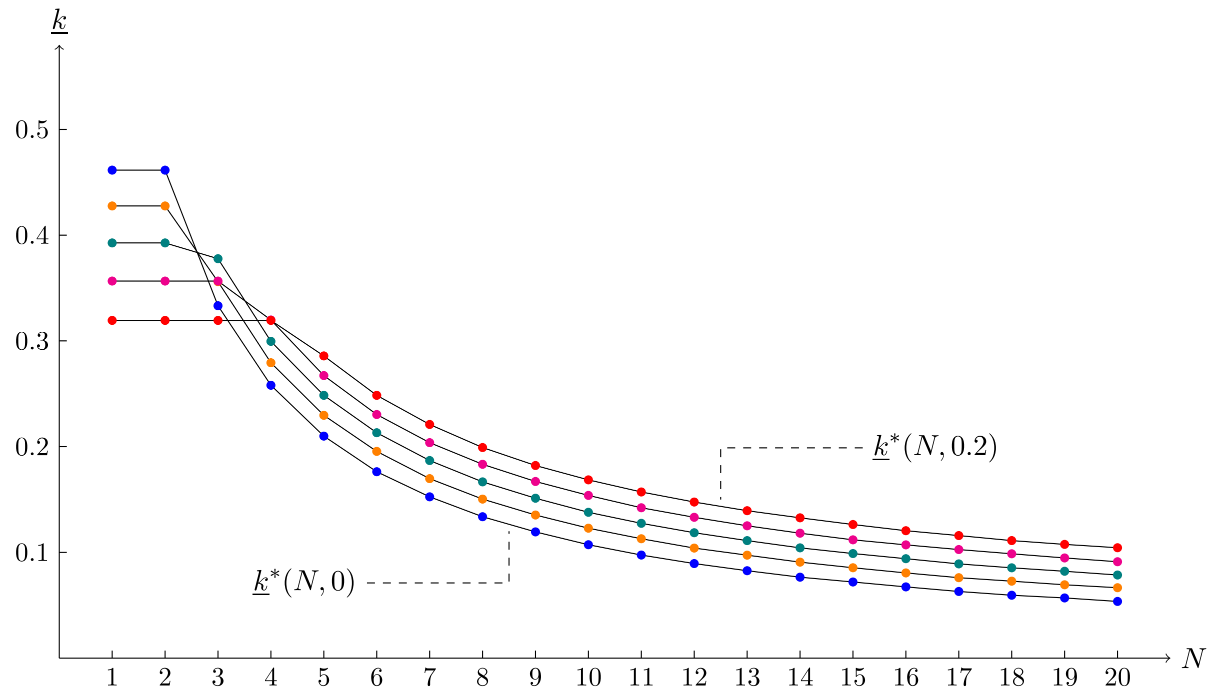

Figure 1: The threshold values $\underline{k}^*(N, \gamma)$ for $\gamma = 0, 0.05, 0.1, 0.15$ and $\gamma = 0.2$ in the linear-quadratic model.

dominant firm's market power rather than being an artefact of the first-mover advantage it has by assumption.

Let us now look at changes in N given that it is finite. We restrict our attention to the linear-quadratic case because, unfortunately, such a comparative static analysis is not possible in the model with general functions. In the linear-quadratic case introduced above, we can solve for the equilibrium capacities and quantities if we consider explicit numbers for the parameters α, β, c, δ and γ.

Using numerical computations we first analyze how the threshold $\underline{k}^*$ changes with N, provided $\underline{k}^* < \bar{\underline{k}}$. Figure 1 shows how $\underline{k}^*$ depends on N for five different values of γ.[17] It is evident from the downward sloping shape of the graphs that vertical integration is more likely to become anticompetitive as N increases. We have analyzed many variations of the model with different slopes of the demand function, the variable cost function and the capacity supply function. All results most strongly support the notion that in the linear-quadratic model $\underline{k}^*$ decreases in N. Since $\underline{k}^*$ decreases in N, these results also show that Riordan's (1998) dominant firm model becomes an increasingly better approximation as the downstream industry

[17]All simulations were done in Python and are available upon request. In Figure 1 we set the parameters α, β, c and δ equal to one.

becomes more and more competitive.

Surprisingly, Figure 1 also reveals that vertical integration is procompetitive for a larger set of $\underline{k}$ the larger is γ because increases in γ result in upward shifts of $\underline{k}^*$.[18] The intuition is that the integrated firm utilizes its capacity to a larger degree if its cost advantage is bigger. Therefore, capacity is shifted to the more efficient firm which makes vertical integration more likely to be procompetitive.

Figure 1 also illustrates that vertical integration even to monopoly can be procompetitive. For example, for $\gamma = 0.2$ vertical integration is procompetitive at the margin for *any* $\underline{k}$, provided $N \leq 4$. This is the case because for small N there exists no $\underline{k}^* < \bar{k}$ above which vertical integration reduces aggregate quantity. Notice that vertical integration to monopoly is procompetitive for a larger range of N the larger is γ. Interestingly, and to some extent ironically, these results show that vertical integration is more likely to be anticompetitive exactly when the industry is more competitive (i.e. when N is large). As we will discuss in Section 6.1 this is at odds with the common belief that foreclosure effects are present especially in concentrated markets but in line with the empirical observation that in these markets efficiency effects dominate foreclosure effects.

5 Welfare Effects of Vertical Integration

So far we have only looked at the competitive effects of vertical integration, i.e if vertical integration leads to an increase in overall quantity and thereby to an increase in consumer surplus. Since competition authorities both in Europe and in the U.S. base their decisions mainly on the effects on consumer surplus, this analysis is most relevant for competition policy. Yet, it is of equal importance to analyze the implications of vertical integration on social welfare, which can be expressed as

$$W = \int_0^Q P(x)dx - k_I C\left(\frac{q_I}{k_I}\right) + \gamma q_I - N k_j C\left(\frac{q_j}{k_j}\right) - \int_0^K R(y)dy.$$

The first term is the consumer surplus, the second and third term are the variable cost of the integrated firm while the fourth term represents the variable cost of all non-integrated firms.

[18] Each curve $\underline{k}^*(N, \gamma)$ also exhibits a flat segment initially. This flat part corresponds to the smallest value of $\underline{k}$ such that the non-integrated competitors stop production (in our notation $\bar{k}$), at which we stopped our simulations. For any $\underline{k} > \bar{k}$, vertical integration is procompetitive simply because it reduces the cost of the only active firm. The fact that the curves $\underline{k}^*(N, \gamma)$ intersect for small values of N does therefore not conflict with the statement that vertical integration is procompetitive for a larger set of $\underline{k}$ the larger γ.

The last term is the opportunity cost of capacity. Differentiating this expression with respect to $\underline{k}$ (and dropping arguments) yields that welfare is increasing in $\underline{k}$ if and only if

$$\frac{dW}{d\underline{k}} = P\frac{dQ}{d\underline{k}} - NC_j\frac{dk_j}{d\underline{k}} - Nk_jC_j'\left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - \tag{8}$$

$$-C_I\frac{dk_I}{d\underline{k}} - k_IC_I'\left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) + \gamma\frac{dq_I}{d\underline{k}} - R\frac{dK}{d\underline{k}} > 0.$$

Using the first-order conditions from the quantity and the capacity stage for all firms, we can rewrite the above expression to get

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} > -\frac{-P'\left(q_I\frac{dQ}{dk_I} + q_j\frac{dQ_{-I}}{dk_I}\right) + R'(k_I - \underline{k})}{N\left[-P'\left(q_j\frac{dQ}{dk_j} + (N-1)q_j\frac{dq_i}{dk_j} + q_I\frac{dq_I}{dk_j}\right) + R'k_j\right]}. \tag{9}$$

This inequality has a similar structure as (6). The left-hand side is the equilibrium ratio of the reaction of k_j in response to a change in $\underline{k}$ to the reaction of k_I. It is the same in both equations. The right-hand side is different because when considering social welfare we have to take into account that the cost structure and therefore the absolute value of the overall costs changes as $\underline{k}$ varies. Nevertheless, the result we obtain is similar to the one of the last section.

Proposition 5 *For any finite N there exists a $\underline{k}_W^* > 0$ such that for all $\underline{k} < \underline{k}_W^*$ vertical integration is welfare increasing at the margin. There also either exists a $\underline{k}_W^{**} < \bar{k}$ such that for all $\underline{k} > \underline{k}_W^{**}$ vertical integration is welfare decreasing at the margin, or it is welfare increasing at the margin for any $\underline{k}$ close to $\bar{k}$.*

The intuition behind this result is similar to the ones for Propositions 1 and 2. If the ex ante degree of vertical integration is low, further vertical integration increases final output and has the effect of shifting production to the more efficient firm. Therefore, it is welfare increasing. On the other hand, if $\underline{k}$ is already very large, the overall quantity may decrease and, in addition, the less efficient firm produces more, which rises production costs even for a given quantity.

As in the last section, the question arises under which conditions there is a unique threshold (given that it exists). We can show a result that is akin to the one of Proposition 3.

Proposition 6 *Suppose either that (i) R' is dominating all other derivatives in absolute values or that (ii) the model is linear-quadratic. Then, for any finite N there either exists a unique $\underline{k}_W^* \in (0, \bar{k})$ such that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}_W^*$ and welfare reducing at the margin for all $\underline{k} > \underline{k}_W^*$, or vertical integration is always welfare enhancing.*

The analysis so far resembles the one of the previous section. However, the threshold values of $\underline{k}$ obtained in the welfare analysis are different from the ones obtained for consumer surplus because, as mentioned, the variable costs of production and the opportunity costs of capacity change with an increase in $\underline{k}$. Since the rise in k_I caused by an increase in $\underline{k}$ is larger than the fall in aggregate capacity of non-integrated firms, K is increasing in $\underline{k}$ and so capacity costs are increasing. If, in addition, firm I utilizes its capacity less efficiently than a non-integrated firm, we know that overall production costs must increase. In this case the set of $\underline{k}$ for which vertical integration is welfare enhancing is smaller than the one for which it is procompetitive. The next proposition confirms that for the linear-quadratic specification such a case can indeed occur.

Proposition 7 *In the linear-quadratic case, there either exists a unique $\hat{\gamma}$ such that $\underline{k}_W^* < \underline{k}^*$ for all $\gamma < \hat{\gamma}$ and $\underline{k}_W^* > \underline{k}^*$ for all $\gamma > \hat{\gamma}$, or $\underline{k}_W^* < \underline{k}^*$ for all γ.*

This result implies that if the cost advantage of the integrated firm is small and the ex ante degree of integration is between $\underline{k}_W^*$ and $\underline{k}^*$, vertical integration benefits consumers but lowers social welfare. The intuition is that for small γ, firm I is always less efficient than a non-integrated firm at $\underline{k}^*$. As a consequence, vertical integration increases overall production costs at $\underline{k}^*$, where aggregate quantity stays constant. Thus, even if aggregate quantity increases slightly, the effect of increased production costs dominates and welfare falls. The result is interesting since it seems natural to conjecture that procompetitive vertical integration also improves welfare because firms' profits should rise as the market becomes more concentrated. However, what is missing in this reasoning is that vertical integration shifts production costs between firms. Proposition 7 shows that this effect can be so large that procompetitive but welfare reducing mergers are possible.

On the other hand, if the cost advantage of the integrated firm is sufficiently large, vertical integration may shift production to the more efficient firm. In this case, anticompetitive but welfare enhancing mergers occur if $\underline{k} \in (\underline{k}^*, \underline{k}_W^*)$. Although overall quantity decreases, this smaller quantity is now produced more efficiently. This result is also consistent with Riordan's (1998) finding that welfare increasing but anticompetitive vertical integration is possible if the cost advantage of the dominant firm is large. However, procompetitive but welfare reducing mergers cannot occur in the dominant firm model.

6 Discussion

6.1 Empirical Evidence

The main results of our analysis are that the efficiency gains from vertical integration are larger than the foreclosure effects for a fairly wide array of circumstances, and that vertical integration is more likely to be anticompetitive if the industry is less concentrated. We now briefly argue that our results are consistent with the recent empirical studies by Hortaçsu and Syverson (2007) and Lafontaine and Slade (2007).

Hortaçsu and Syverson (2007) provide a study of the cement and ready-mixed concrete industries during the years 1963 to 1997. In this time period the extent of vertical integration between both industries increased, especially between 1982 and 1992 when the fraction of vertically integrated cement plants rose from 32.5% to 49.5%. Hortaçsu and Syverson (2007) find little support for anticompetitive effects of vertical integration. Instead, vertical mergers between firms in the above industries have led to a rise in output and to a fall in the final good price. The rise in output stems from the expansion of more productive integrated firms and was to the detriment of smaller, less efficient producers. These results are in line with our findings. In addition, Hortaçsu and Syverson (2007) show, after controlling for firm size and productivity impacts, that efficiency of a firm cannot be explained by vertical integration. Instead, firms that are larger and more efficient at the outset tend to be vertically integrated, and increases in integration reflect the expansion of these more efficient producers. This is also consistent with our finding that the firm with a cost advantage increases its output after integration but integration itself makes it utilize its capacity less.[19]

Lafontaine and Slade (2007) present a comprehensive survey of empirical studies on vertical integration. The industries in these studies include several different sectors ranging from the steel industry, where a steel producer acquires an iron ore mine, to the gasoline industry, where some refiners and stations are integrated, while others are not.[20] Lafontaine and Slade (2007) observe that "[a]uthors have looked for detrimental effects from vertical mergers mostly in concentrated markets. [...] However, even though authors typically choose markets where they expect to find evidence for exclusion, half of the studies find no sign of it. And where they find

[19]Cooper, Froeb, O'Brien and Vita (2005) as well as Lafontaine and Slade (2007) in their extensive reviews of empirical studies on vertical integration also find that in the vast majority of cases vertical mergers are beneficial to consumers. Although foreclosure effects are present in some cases, the net effect appears to be positive because efficiency gains dominate.

[20]See Mullin and Mullin (1997) for an in-depth study of the steel industry and e.g. Barron and Umbeck (1984) or Blass and Carlton (2001) for studies of the gasoline industry.

evidence of exclusion or foreclosure, they also at times document efficiencies that arise from the same merger" (p. 671). Lafontaine and Slade (2007, p. 680) draw the conclusion that "[e]ven in industries that are highly concentrated so that horizontal considerations assume substantial importance, the net effect of vertical integration appears to be positive in many instances". This evidence and the conclusion are fully consistent with the predictions of our model. In addition, our model suggests that detrimental effects from vertical integration are more likely to be found exactly when markets are *less* concentrated. Here, the foreclosure effect is likely to dominate the efficiency effect.

6.2 Policy Implications

Let us now discuss some policy implications of our analysis. As a general theme, our results suggest a relatively permissive approach to vertical integration since we find that although the foreclosure effect on non-integrated firms is present, the efficiency gains are 'usually' larger.[21]

An important question is if our model permits conclusions about the welfare effects of vertical integration that are based on observable market conditions. For example, a nice feature of Riordan's (1998) dominant firm model is that it establishes an indicator about the welfare effects of vertical integration that holds for general functions and is based on the ratio of input to output market shares. Though such general conclusions cannot be drawn in the present paper, it is nonetheless possible to calculate in an easy way the critical input or output market share of the integrated firm from the thresholds $\underline{k}^*$ and $\underline{k}_W^*$. Beyond these critical market shares further vertical integration reduces consumer surplus and social welfare. This is particularly useful if data on market shares are easier to obtain than assessing the ex ante degree of vertical integration. For the linear-quadratic model Figure 2 illustrates this critical output market share of the integrated firm, denoted by s^* for different values of γ and N. As expected, these graphs closely resemble the results obtained above for the threshold value of the ex ante degree of integration. We also find for several parameter constellations that the critical input market share is almost identical to the output market share. Qualitatively, these results also hold for the critical market shares beyond which further vertical integration reduces social welfare.

Our analysis thus provides a rationale for the EU non-horizontal merger guidelines issued in 2007 which recognize safe harbors expressed as market shares of 30% both in the input and in

[21] This result is line with the conclusions by Lafontaine and Slade (2007) and Church (2008) who argue that the burden of proof that vertical integration is harmful should be placed on the competition authorities.

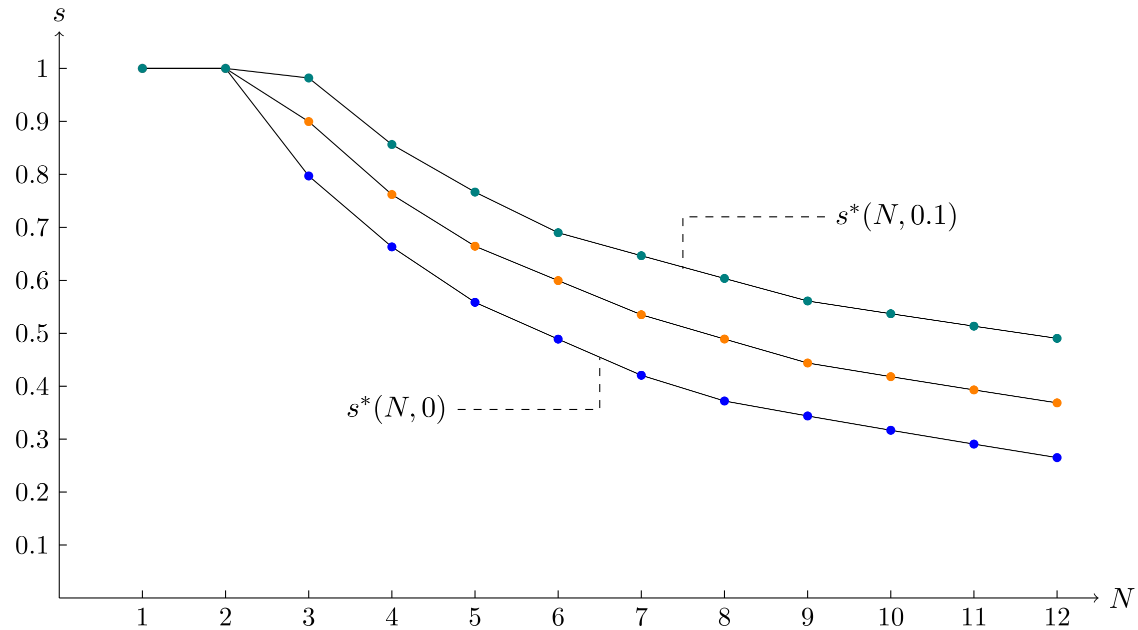

Figure 2: The critical output market share of the integrated firm $s^*(N, \gamma)$ for $\gamma = 0$, 0.05 and $\gamma = 0.1$ in the linear-quadratic model.

output market below which no investigation of vertical integration takes place.[22] However, our results also indicate that these thresholds decrease with the number of firms in the industry. The more competitors there are, the lower is the threshold above which vertical integration is anticompetitive. Overall, this suggests that it is reasonable to impose similar thresholds for input and output market shares but it may be useful to make them contingent on the number of firms in the industry.

6.3 Discrete Vertical Integration

Throughout the paper, we have treated vertical integration as a continuous variable. This is done for analytical tractability. In reality, however, vertical integration is rarely a continuous process. Instead, if a downstream firm merges with an upstream firm, it normally acquires a non-negligible fraction of the intermediate good market. Translated into our model this means that $\underline{k}$ increases in a discrete step. This is likely to shift the balance even more in favor of a more permissive approach to vertical integration. The reason is that the first units of vertical integration up to some threshold are always procompetitive. Suppose now that a firm is not or only slightly integrated at the outset. If this firm acquires an upstream firm even to such

[22]See European Union, Commission Notice, Guidelines on the assessment of non-horizontal mergers, p.5; available at: non-horizontal http://ec.europa.eu/competition/mergers/legislation/nonhorizontalguidelines.pdf.

an extent that the marginal unit of integration is anticompetitive, the merger as a whole may still be procompetitive because the first units are procompetitive. These units dominate if the threshold is not too small, and the merger not too large. So even if a firm can only merge with a significant part of the upstream industry, it seems plausible that the overall effect of vertical integration should result in an increase of final output.

6.4 Several Vertically Integrated Firms

In our analysis we have restricted attention to the case in which only one firm has the possibility to vertically integrate. Yet, since in the oligopoly model all firms have market power, it is conceivable that there is more than one integrated firm. Due to the complexity of the model it is impossible to treat this case analytically. However, numerical calculations confirm that all of our insights hold in this case as well. In fact, if there is a second integrated firm, say firm I_2, the threshold for $\underline{k}$ below which vertical integration of firm I is procompetitive is even larger. The reason is that the capacity reduction of firm I_2 as a reaction to the integration of firm I is smaller than the one of the non-integrated firms. This is the case because firm I_2 now owns some capacity units itself and is therefore less affected by an increase in the capacity price.

We can also compare the ex ante degree of vertical integration below which further marginal integration is procompetitive for the two firms. Here we find that the one of firm I, i.e. the firm with the cost advantage, is larger. The intuition is similar to the model with one integrated firm, namely that a cost advantage induces firm I to utilize its capacity more efficiently than firm I_2. In addition, the result that vertical integration is more likely to be procompetitive the larger is the number of competitors now holds for both integrated firms.

7 Conclusion

In this paper we show that vertical integration with downstream oligopoly and an increasing upstream supply curve is procompetitive under fairly wide circumstances. Whether it is procompetitive or anticompetitive depends on the ex ante degree of vertical integration. Only if this degree is high and the number of downstream competitors relatively large will vertical integration reduce final goods output and increase consumer prices. Otherwise, vertical integration lowers prices and thus benefits consumers. Nevertheless, this does not necessarily imply that social welfare increases as well, because final output may be produced less efficiently. Our analysis allows the determination of critical input and output market shares above which vertical integration is anticompetitive.

We obtained the results in a framework with homogeneous goods in the downstream market. An interesting direction for further research is how the strength of the efficiency effect and the foreclosure effect changes when firms produce differentiated goods. In this case, the effect of vertical integration on the downstream market interaction between firms is smaller. This indicates that the reduction in quantity of a non-integrated firm is smaller as well. However, since firms have more market power downstream, it is not obvious how a non-integrated firm reacts with its capacity choice if the capacity price increases. In general, since downstream competition is lower than with homogeneous goods, it seems likely that the overall effect is procompetitive for a large range of parameters as well.

An important challenge for future research is to endogenize the downstream market structure by allowing firms to enter and exit conditional on the degree of vertical integration. This might be accomplished by imposing fixed costs of entry and adding an entry stage that precede the capacity stage in the current model. Since the non-integrated firms can now not only reduce their capacity but also exit as a reaction to further integration, the range for anticompetitive vertical integration is likely to become larger. Nevertheless, if the integrating firm is only integrated to a moderate extent, it seems plausible that the efficiency effect is still dominating, which would yield similar policy implications as the present analysis. As such an exercise is most likely to be a difficult one, we leave it for further research.

A Appendix

To simplify notation, we omit the superscript $*$ on equilibrium quantities and equilibrium capacities throughout this appendix.

A.1 Proof of Lemma 1

We know that $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i})$ if and only if $\hat{k}_i > k_i$. But this implies that the left-hand side of (1), respectively (2), is smaller for $\hat{k}_i$ than for k_i. As a consequence, the right-hand side must be smaller as well. Since C_i is convex, it follows that $q_i(\hat{k}_i, \mathbf{k}_{-i})/\hat{k}_i < q_i(k_i, \mathbf{k}_{-i})/k_i$. The only if part can be proved by following the steps in the opposite direction. ■

A.2 Proof of Lemma 2

Let $j \neq i, j \neq I$ and $i \neq I$. Totally differentiating (1) with respect to k_j yields[23]

$$P' \frac{dQ}{dk_j} + P' \frac{dq_j}{dk_j} + P'' q_j \frac{dQ}{dk_j} = -C_j'' \frac{q_j}{k_j^2} + C_j'' \frac{1}{k_j} \frac{dq_j}{dk_j}. \tag{10}$$

We can write dQ/dk_j as $dQ/dk_j = dq_I/dk_j + \sum_{i \neq j} dq_i/dk_j + dq_j/dk_j$, which under the symmetry assumption that $k_i = k_j$ for all $i, j \in \{1, ..., N\}$, becomes

$$\frac{dQ}{dk_j} = \frac{dq_I}{dk_j} + (N-1) \frac{dq_i}{dk_j} + \frac{dq_j}{dk_j}.$$

Therefore, (10) can be written as an equation that depends on the three variables dq_i/dk_j, dq_j/dk_j and dq_I/dk_j, which we wish to determine.

Totally differentiating the first-order condition of firm i, which is analogous to (1), with respect to k_j yields

$$P' \frac{dQ}{dk_j} + P' \frac{dq_i}{dk_j} + P'' q_i \frac{dQ}{dk_j} = C_i'' \frac{1}{k_i} \frac{dq_i}{dk_j}, \tag{11}$$

and, analogously, differentiating the first-order condition for I, equation (2), with respect to k_j yields

$$P' \frac{dQ}{dk_j} + P' \frac{dq_I}{dk_j} + P'' q_I \frac{dQ}{dk_j} = C_I'' \frac{1}{k_I} \frac{dq_I}{dk_j}. \tag{12}$$

The unique solution to the system of three equations (10), (11) and (12), which are linear in the three unknowns dq_i/dk_j, dq_j/dk_j and dq_I/dk_j, is

$$\frac{dq_I}{dk_j} = \frac{C_j'' q_j k_I (P' + P'' q_j)}{\eta k_j} < 0 \quad \text{for} \quad j \neq I, \tag{13}$$

$$\frac{dq_i}{dk_j} = \frac{C_j'' q_j [(C_I'' - P' k_j)(P' + P'' q_j)]}{\eta (C_j'' - P' k_j)} < 0 \quad \text{for} \quad j \neq i \tag{14}$$

[23] Notice that the proof holds for any capacities, not only for equilibrium values.

and

$$\frac{dq_j}{dk_j} = \frac{C_j''q_j[(P')^2k_jk_I(N+1)+P'(P''k_jk_I(q_I+(N-1)q_j)-2C_j''k_I-C_I''k_jN)]}{\eta k_j(C_j''-P'k_j)} \tag{15}$$
$$+ \frac{C_j''q_j[C_j''C_I''-P''(C_j''k_Iq_I+(N-1)C_I''k_jq_j)]}{\eta k_j(C_j''-P'k_i)} > 0,$$

where $\eta \equiv \{(P')^2(N+2)k_Ik_j+P'[P''k_jk_I(q_I+Nq_j)-C_I'k_j(N+1)-2k_IC_j'']+C_I''C_j''-P''(C_j''q_Ik_I+C_I''q_jk_jN)\} > 0$. The inequality sign follows from the assumption that P'' is negative or not too positive.

Totally differentiating the first-order conditions of firm I and j with respect to k_I yields, respectively,

$$P'\frac{dQ}{dk_I}+P'\frac{dq_I}{dk_I}+P''q_I\frac{dQ}{dk_I} = -C_I''\frac{q_I}{k_I^2}+C_I''\frac{1}{k_I}\frac{dq_I}{dk_I} \tag{16}$$

and

$$P'\frac{dQ}{dk_I}+P'\frac{dq_j}{dk_I}+P''q_i\frac{dQ}{dk_I} = C_j''\frac{1}{k_j}\frac{dq_j}{dk_I}, \tag{17}$$

where under symmetry $dQ/dk_I = dq_I/dk_I + Ndq_j/dk_I$. Using the last equation to replace dQ/dk_I in (16) and (17) yields a system of two linear equations in the two unknowns dq_I/dk_I and dq_j/dk_I. The solution is

$$\frac{dq_j}{dk_I} = \frac{C_I''q_Ik_j(P''q_j+P')}{\nu} < 0 \tag{18}$$

and

$$\frac{dq_I}{dk_I} = -\frac{C_I''q_I[k_j(P'q_j(N+1)+P''Nq_j)-C_j'']}{\nu} > 0, \tag{19}$$

where $\nu \equiv k_I\{(P')^2k_Ik_j(N+2)+P'[P''k_jk_I(q_I+Nq_j)-C_I''k_j(N+1)-2C_i''k_I]+C_I''C_j''-P''[C_j''q_Ik_I+C_I''q_jk_jN]\} > 0$. Again, the inequality sign follows from P'' not being too positive.
■

A.3 Proof of Lemma 3

Differentiating (4) with respect to k_j and (5) with respect to k_I yields second-order conditions of

$$\frac{\partial^2\Pi_j}{\partial k_j^2} = P'\frac{dq_j}{dk_j}\left[\frac{dq_I}{dk_j}+(N-1)\frac{dq_i}{dk_j}\right]+P'q_j\left[\frac{d^2q_I}{dk_j^2}+(N-1)\frac{d^2q_i}{dk_j^2}\right]+ \tag{20}$$
$$+P''q_j\left[\frac{dq_I}{dk_j}+(N-1)\frac{dq_i}{dk_j}\right]\left[\frac{dq_j}{dk_j}+\frac{dq_I}{dk_j}+(N-1)\frac{dq_i}{dk_j}\right]+C_j''\frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j}-\frac{q_j}{k_j}\right)-2R'-k_jR''<0$$

and

$$\frac{\partial^2\Pi_I}{\partial k_I^2} = P'\frac{dq_I}{dk_I}N\frac{dq_j}{dk_I}+P'q_IN\frac{d^2q_j}{dk_I^2}+ \tag{21}$$

$$+P''q_I N\frac{dq_j}{dk_I}\left[\frac{dq_I}{dk_I}+N\frac{dq_j}{dk_I}\right]+C_I''\frac{q_I}{k_I^2}\left(\frac{dq_I}{dk_I}-\frac{q_I}{k_I}\right)-2R'-(k_I-\underline{k})R''<0.$$

In the following we show that (20) is indeed fulfilled when the first-order conditions are satisfied. The second-order condition for the integrated firm can then be shown to be fulfilled in exactly the same way.

In Lemma 2 we determined the equilibrium expressions for dq_i/dk_j, $i \in \{I, 1, ..., N\}$, that we need in (20). To determine the sign of $\partial^2\Pi_j/\partial k_j^2$ we still have to determine d^2q_I/dk_j^2 and d^2q_i/dk_j^2. To do so we again calculate dq_I/dk_j and dq_i/dk_j but now explicitly distinguish between q_j and q_i and between k_j and k_i, $i \neq j$, $i, j \in \{1, ..., N\}$. This gives us

$$\frac{dq_I}{dk_j}=\frac{C_j''q_jk_I(P'+q_IP'')(C_i''+P'k_i)}{k_j\rho}$$

and

$$\frac{dq_i}{dk_j}=\frac{C_I''q_jk_i(P'+q_iP'')(C_I''+P'k_I)}{k_j\rho},$$

with

$$\rho=-k_Ik_jk_i(N+2)(P')^3+(3C_i''k_jk_I+k_Ik_j(N+1)C_j''+k_ik_j(N+1)C_I''-P''k_Ik_ik_j((N-1)q_i+q_I+q_j))(P')^2+$$

$$((C_I''k_ik_I(q_I+(N-1)q_i)+C_j''k_ik_j(q_i+(N-1)q_i)+C_i''k_Ik_j(q_j+q_I))P''-Nk_iC_I''C_j''-2k_IC_j''C_i''-2k_jC_I''C_i'')P'$$

$$-((N-1)C_I''C_j''q_ik_i+C_i''(q_jk_jC_j''+q_Ik_IC_j''))P''+C_i''C_I''C_j''.^{24}$$

Differentiating both formulas with respect to k_j, using dq_i/dk_j, dq_j/dk_j and dq_I/dk_j from the proof of Lemma 2, and inserting the resulting expressions into the second-order condition yields

$$\frac{\partial^2\Pi_j}{\partial k_j^2}=-\frac{q_j^2\left(\sum_{s=1}^{9}(P')^s(\sum_{t=1}^{3}\kappa_{st}(P'')^t+\kappa_{s4}P'''+\kappa_{s5}C_j'''+\kappa_{s6}C_I'''+\kappa_{s7})\right)}{\varrho}-2R'-k_jR'',$$

with

$$\varrho=k_j^2(C_j''-k_jP')^3\Big[k_Ik_j(N+2)(P')^2+$$

$$+\left(k_Ik_j(q_I+Nq_j)P''-2k_IC_j''-(N+1)k_jC_I''\right)P'-\left(NC_I''k_jq_j+C_j''k_Iq_I\right)P''+C_I''C_j''\Big]^3>0,$$

where we have used that in equilibrium $q_i = q_j$, $k_i = k_j$ and $C_i'' = C_j''$.

In this equation $\kappa_{sh} = \kappa_{sh}(q_j, k_j, q_I, k_I, C_j'', C_I'', P', P'', N)$, $s \in \{1, ..., 9\}$ and $h \in \{1, ..., 7\}$. We do not specify the exact expressions for κ_{sh} here since they stand for rather complex expressions consisting of several terms. Yet, the sign of these expressions is easy to determine

[24] One can easily check that if $q_i = q_j$, $k_i = k_j$ and, therefore, $C_i'' = C_j''$ (which is the case in equilibrium), these formulas yield (13) and (14).

in each case and this is the only point of relevance for our purpose. These signs are the following: For $h = \{1, 2, 3\}$ $\kappa_{sh} \geq 0$, if both s and h are either even or odd and $\kappa_{sh} \leq 0$ if one is even and the other one is odd. κ_{s4}, κ_{s5}, $\kappa_{s6} \geq 0$ for s even and κ_{s4}, κ_{s5}, $\kappa_{s6} \leq 0$ for s odd. $\kappa_{s7} > 0$ for s even and $\kappa_{s7} < 0$ for s odd. Thus, the numerator in the fraction is positive because P'' is not too positive and P''' and C''' are not too negative. Since R'' is not too negative as well, we get that $\partial^2\Pi_j/\partial k_j^2 < 0$. In exactly the same way we can show that the second-order condition for firm I is satisfied. Thus, the profit function of each firm is quasiconcave in its own capacity and we have an interior equilibrium.

We now turn to the question of uniqueness. From Kolstad and Mathiesen (1987) and Vives (1999) we know that the equilibrium is unique if and only if the Jacobian determinant of minus the marginal profits is positive. In our case this determinant is given by

$$|J| = \begin{vmatrix} -\frac{\partial^2\Pi_j}{\partial k_j^2} & -\frac{\partial^2\Pi_j}{\partial k_j \partial k_i} & \cdots & -\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} \\ -\frac{\partial^2\Pi_i}{\partial k_i \partial k_j} & -\frac{\partial^2\Pi_i}{\partial k_i^2} & \cdots & -\frac{\partial^2\Pi_i}{\partial k_i \partial k_I} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial^2\Pi_I}{\partial k_I \partial k_j} & -\frac{\partial^2\Pi_I}{\partial k_I \partial k_i} & \cdots & -\frac{\partial^2\Pi_I}{\partial k_I^2} \end{vmatrix}, \tag{22}$$

with $i \neq j$, $i, j \in \{1, ..., N\}$. The terms that determine this determinant are given by the second-order conditions, (20) and (21), and the terms $\partial^2\Pi_j/(\partial k_j \partial k_I)$, $\partial^2\Pi_j/(\partial k_j \partial k_i)$, $\partial^2\Pi_i/(\partial k_i \partial k_j)$, $\partial^2\Pi_I/(\partial k_I \partial k_j)$ and $\partial^2\Pi_I/(\partial k_I \partial k_i)$. We know that in equilibrium $\partial^2\Pi_i/(\partial k_i \partial k_j) = \partial^2\Pi_j/(\partial k_j \partial k_i)$ and $\partial^2\Pi_I/(\partial k_I \partial k_i) = \partial^2\Pi_I/(\partial k_I \partial k_j)$ because of symmetry. The remaining terms can be written as

$$\begin{aligned}
\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} &= P'\frac{dq_j}{dk_I}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_j dk_I} + (N-1)\frac{d^2q_i}{dk_j dk_I}\right] \qquad (23)\\
&+ P''q_j\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right]\left[\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right] + C_j''\frac{q_j}{k_j^2}\frac{dq_j}{dk_I} - R' - k_jR'', \\
\frac{\partial^2\Pi_j}{\partial k_j \partial k_i} &= P'\frac{dq_j}{dk_i}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_j dk_i} + (N-2)\frac{d^2q_k}{dk_j dk_i} + \frac{d^2q_i}{dk_j dk_i}\right] \qquad (24)\\
&+ P''q_j\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right]\left[\frac{dq_i}{dk_i} + \frac{dq_I}{dk_i} + (N-1)\frac{dq_j}{dk_i}\right] + C_j''\frac{q_j}{k_j^2}\frac{dq_j}{dk_i} - R' - k_jR'', \\
\frac{\partial^2\Pi_I}{\partial k_I \partial k_j} &= P'\frac{dq_I}{dk_j}N\frac{dq_j}{dk_I} + P'q_I\left[\frac{d^2q_j}{dk_j dk_I} + (N-1)\frac{d^2q_i}{dk_j dk_I}\right] + \qquad (25)\\
&+ P''q_IN\frac{dq_I}{dk_j}\left[\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right] + C_I''\frac{q_I}{k_I^2}\frac{dq_I}{dk_I} - R' - (k_I - \underline{k})R'',
\end{aligned}$$

The second derivatives that appear in these expressions can be derived in the same way as above where we checked that the second-order conditions are satisfied.

Proceeding in a similar way as Kolstad and Mathiesen (1987), i.e. subtracting the first column in (22) from the other columns, and then dividing the m-th row by

$$\frac{\partial^2 \Pi_m}{\partial k_m \partial k_j} - \frac{\partial^2 \Pi_m}{\partial k_m^2},$$

with $m \in \{I, 1, ..., N\}$, yields

$$|J| = \begin{vmatrix} -\frac{\frac{\partial^2 \Pi_j}{\partial k_j^2}}{\frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} - \frac{\partial^2 \Pi_j}{\partial k_j^2}} & -1 & -1 & \dots & -\frac{-\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} + \frac{\partial^2 \Pi_j}{\partial k_j^2}}{\frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} - \frac{\partial^2 \Pi_j}{\partial k_j^2}} \\ -\frac{\frac{\partial^2 \Pi_j}{\partial k_i \partial k_i}}{\frac{\partial^2 \Pi_i}{\partial k_i \partial k_j} - \frac{\partial^2 \Pi_i}{\partial k_i^2}} & 1 & 0 & 0 & \dots - \frac{-\frac{\partial^2 \Pi_i}{\partial k_i \partial k_I} + \frac{\partial^2 \Pi_i}{\partial k_i \partial k_j}}{\frac{\partial^2 \Pi_i}{\partial k_i \partial k_j} - \frac{\partial^2 \Pi_i}{\partial k_i^2}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{\frac{\partial^2 \Pi_I}{\partial k_I \partial k_j}}{\frac{\partial^2 \Pi_I}{\partial k_I \partial k_j} - \frac{\partial^2 \Pi_I}{\partial k_I^2}} & 0 & 0 & \dots & 1 \end{vmatrix}.$$

We can then calculate the determinant in a relatively straightforward way. Cumbersome but otherwise routine manipulations show that this determinant is unambiguously positive and, therefore, that the equilibrium of the capacity stage is unique. ■

A.4 Proof of Lemma 4

Differentiating (4) and (5) with respect to $\underline{k}$ yields

$$\frac{\partial^2 \Pi_I}{\partial k_I^2} \frac{dk_I}{d\underline{k}} + N \frac{\partial^2 \Pi_I}{\partial k_I \partial k_j} \frac{dk_j}{d\underline{k}} + \frac{\partial^2 \Pi_I}{\partial k_I \partial \underline{k}} = 0$$

and

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} \frac{dk_j}{d\underline{k}} + (N-1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} \frac{dk_i}{d\underline{k}} + \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{dk_I}{d\underline{k}} = 0.$$

Using the fact that in equilibrium $\frac{dk_i}{d\underline{k}} = \frac{dk_j}{d\underline{k}}$ for $i, j \neq I$ we get

$$\frac{dk_j}{d\underline{k}} = \frac{\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{\partial^2 \Pi_I}{\partial k_I \partial \underline{k}}}{\frac{\partial^2 \Pi_j}{\partial k_j^2} \frac{\partial^2 \Pi_I}{\partial k_I^2} + (N-1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} \frac{\partial^2 \Pi_I}{\partial k_I^2} - N \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{\partial^2 \Pi_I}{\partial k_I \partial k_j}} \tag{26}$$

and

$$\frac{dk_I}{d\underline{k}} = -\frac{\frac{\partial^2 \Pi_I}{\partial k_I \partial \underline{k}} \left(\frac{\partial^2 \Pi_j}{\partial k_j^2} + (N-1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} \right)}{\frac{\partial^2 \Pi_j}{\partial k_j^2} \frac{\partial^2 \Pi_I}{\partial k_I^2} + (N-1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} \frac{\partial^2 \Pi_I}{\partial k_I^2} - N \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{\partial^2 \Pi_I}{\partial k_I \partial k_j}}. \tag{27}$$

The terms that appear in these expressions are given by (20), (21), (23), (24), (25) and by

$$\frac{\partial^2 \Pi_I}{\partial k_I \partial \underline{k}} = R' > 0.$$

Tedious but routine calculations then show that all terms in (23), (24) and (25) have a negative sign. Thus, the numerators of the fractions on the right-hand side of (26) and (27) are both negative. The denominator in these fractions is the same in both equations. It is easy to show that $|\partial^2\Pi_j/\partial k_j^2| > |\partial^2\Pi_j/(\partial k_j \partial k_i)|$ which implies that

$$\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} > \frac{\partial^2\Pi_j}{\partial k_j \partial k_i}\frac{\partial^2\Pi_I}{\partial k_I^2}.$$

In addition one can also easily show $|\partial^2\Pi_I/\partial k_I^2| > |\partial^2\Pi_I/(\partial k_I \partial k_j)|$ and that

$$\frac{\partial^2\Pi_j}{\partial k_j \partial k_i}\frac{\partial^2\Pi_I}{\partial k_I^2} > \frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}.$$

This implies that the denominator is positive. As a consequence, we get that $dk_j/d\underline{k} < 0$ and $dk_I/d\underline{k} > 0$. ■

A.5 Proof of Proposition 1

We start with the right-hand side of equation (7), i.e.

$$-\frac{C_I'' \frac{q_I}{k_I}(C_j'' - k_j P')}{N C_j'' \frac{q_j}{k_j}(C_I'' - k_I P')}.$$

Suppose first that $\gamma = 0$. As mentioned above, if $\underline{k} = 0$, we have $k_I = k_j$, $q_I = q_j$, and $C_I'' = C_j''$. Therefore, the right-hand side of (7) simplifies to $-1/N$.

We now turn to the left-hand side of equation (7). From (26) and (27) we obtain that

$$\frac{\left(\frac{dk_j}{d\underline{k}}\right)}{\left(\frac{dk_I}{d\underline{k}}\right)} = -\frac{\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}}{\frac{\partial^2\Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_i}}.$$

At $\gamma = 0$ and $\underline{k} = 0$, we know that there is no difference between firm I and any of the non-integrated firms. This implies that $\partial^2\Pi_j/(\partial k_j \partial k_i) = \partial^2\Pi_j/(\partial k_j \partial k_I)$. To determine if $(dk_j/d\underline{k})\,/\,(dk_I/d\underline{k})$ is bigger or smaller than $-1/N$ at $\gamma = 0$ and $\underline{k} = 0$, it remains to compare $\partial^2\Pi_j/\partial k_j^2$ with $\partial^2\Pi_j/(\partial k_j \partial k_I)$. This yields

$$\frac{\partial^2\Pi_j}{\partial k_j^2} - \frac{\partial^2\Pi_j}{\partial k_j \partial k_I} = -\frac{q_j^2 P' \xi}{k_j^2(C_j'' - (2+N)k_j P' - (1+N)k_j q_j P'')(C_j'' - P' k_j)^3},$$

where

$$\xi = k_j^2\left(q_j C_j''' N + k_j C_j''(3N+2)\right)(P')^3 +$$

$$+\left(q_j k_j^2 (k_j(1+3N)C_j'' + q_j C_j''' N)P'' - k_j C_j''(3k_j(N+2)C_j'' + q_j C_j''')\right)(P')^2 +$$

$$+\left(-q_j k_j C_j''(k_j C_j''(3N+4) + q_j C_j''')P'' + 5(C_j'')^3 k_j\right)P' - (C_j''')^3(C_j'' - 3q_j P'' k_j) < 0.$$

But since the denominator is positive and P' and ξ are negative, we get

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} - \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} < 0,$$

i.e. $\partial^2 \Pi_j / \partial k_j^2$ is larger in absolute terms than $\partial^2 \Pi_j / (\partial k_j \partial k_I)$. As a consequence, $(dk_j/d\underline{k}) / (dk_I/d\underline{k}) > -1/N$. Thus, at $\gamma = 0$ and $\underline{k} = 0$ vertical integration is procompetitive at the margin.

We now turn to $\gamma > 0$. From (4) and (5) we know that if $q_I = q_j$, we have $k_I = k_j$ at $\underline{k} = 0$. But since $\gamma > 0$, at $k_I = k_j$ we in fact get $q_I > q_j$. From (4) and (5) this in turn implies that $k_I > k_j$. But one can show that nevertheless $q_I/k_I > q_j/k_j$ because $q_I > q_j$ is a first-order effect. Thus, at $\underline{k} = 0$ and $\gamma > 0$, firm I utilizes capacity more efficiently. This implies that a shift in capacity to firm I is also procompetitive for $\gamma > 0$. By continuity it follows that vertical integration is also procompetitive at the margin for all $\underline{k}$ below a certain, positive threshold denoted by $\underline{k}^*$. ■

A.6 Proof of Proposition 2

Let $\underline{k} = \bar{k}$, so that $k_j = 0$ for all $j \in \{1, ..., N\}$. We first have to determine q_j/k_j in this case. Because C_j is strictly convex, C_j' is invertible and equation (1) can be written as

$$q_j = k_j \left(C_j'\right)^{-1} \left(P(Q) + P'(Q) q_j\right). \tag{28}$$

It follows directly from (28) that if $k_j = 0$ we also have $q_j = 0$.

Observe that the inverse $(C_j')^{-1}(.)$ is strictly increasing and that it is zero if and only if its argument is zero. By using the rule of L'Hôpital we get that

$$\frac{q_j}{k_j} = \left(C_j'\right)^{-1} (P(q_I)) > 0, \quad \forall j \in \{1, ..., N\}$$

if $q_j = 0$ and $k_j = 0$. To simplify notation in the following we denote $(C_j')^{-1}(P(q_I)) \equiv \rho$.

We now turn to equation (7). The right-hand side of (7) in the case of $\underline{k} = \bar{k}$ can then be written as

$$-\frac{C_I'' \frac{q_I}{k_I}}{\rho(C_I'' - k_I P')N}.$$

On the other hand, the left-hand side of (7) in case of $q_j = k_j = 0$ can be calculated from (26) and (27). We obtain that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{C_I'' P' \rho \frac{q_I}{k_I} + \sigma}{(N+1)\left(\rho^2 P'(C_I'' - k_I P') + \sigma\right)}, \tag{29}$$

with $\sigma \equiv R' k_I \left(2P' k_I + P'' k_I q_I - C_I''\right) < 0$.

It follows that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I''-k_IP')N}$$

if and only if

$$-\left(\frac{N}{1+N}\right)\left(\frac{C_I''P'\rho\frac{q_I}{k_I}+\sigma}{\rho^2P'(C_I''-k_IP')+\sigma}\right) < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I''-k_IP')}. \tag{30}$$

But the left-hand side of (30) can either be larger or smaller than the right-hand side. To see this suppose first that σ is small in absolute terms. In this case, the second term of the left-hand side is approximately the same as the right-hand side. But since $-N/(1+N) > -1$, the left-hand side is larger. On the other hand, suppose that N is very large. In this case, $N/(1+N)$ is close to 1. We then have that vertical integration is anticompetitive at the margin if

$$-\frac{C_I''P'\rho\frac{q_I}{k_I}+\sigma}{\rho^2P'(C_I''-k_IP')+\sigma} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I''-k_IP')}. \tag{31}$$

Obviously the left-hand side equals the right-hand side if $\sigma = 0$. But since $\sigma < 0$ and $\rho^2P'(C_I''-k_IP') < P'C_I''\rho\frac{q_I}{k_I} < 0$, the inequality in (31) is fulfilled.

By continuity, there exists either a $\underline{k}^{**} < \bar{k}$ such that for all $\underline{k} > \underline{k}^{**}$ vertical integration is anticompetitive at the margin or vertical integration is procompetitive at the margin even for $\underline{k}$ close to $\bar{k}$. ■

A.7 Proof of Proposition 3

Case (i):

From the proof of Proposition 1 we know that

$$\frac{\left(\frac{dk_j}{d\underline{k}}\right)}{\left(\frac{dk_I}{d\underline{k}}\right)} = -\frac{\frac{\partial^2\Pi_j}{\partial k_j\partial k_I}}{\frac{\partial^2\Pi_j}{\partial k_j^2}+(N-1)\frac{\partial^2\Pi_j}{\partial k_j\partial k_i}}.$$

If R' is dominating all other derivatives in absolute values, we get, after inserting (20), (23) and (24), that $(dk_j/d\underline{k})/(dk_I/d\underline{k}) = -1/(N+1)$. Thus, the left-hand side of (7) does not vary with $\underline{k}$.

We now analyze how the right-hand of (7) changes with $\underline{k}$. Differentiating it with respect to $\underline{k}$ reveals that this derivative has the same sign as

$$\begin{aligned}&-C_j''C_I''(C_I''-k_IP')(C_j''-k_jP')\left(\frac{d\left(\frac{q_I}{k_I}\right)}{d\underline{k}}\frac{q_j}{k_j}-\frac{d\left(\frac{q_j}{k_j}\right)}{d\underline{k}}\frac{q_I}{k_I}\right)-\\&-P'C_j''C_I''\frac{q_j}{k_j}\frac{q_I}{k_I}\left((C_j''-k_jP')\frac{dk_I}{d\underline{k}}-(C_I''-k_IP')\frac{dk_j}{d\underline{k}}\right)-\end{aligned} \tag{32}$$

$$-P'\frac{q_j}{k_j}\frac{q_I}{k_I}\left(\frac{dC''_j}{d\underline{k}}C''_I k_j(C''_I - k_I P') - \frac{dC''_I}{d\underline{k}}C''_j k_I(C''_j - k_j P')\right) + P''\frac{dQ}{d\underline{k}}C''_j C''_I \frac{q_j}{k_j}\frac{q_I}{k_I}(k_j C''_I - k_I C''_j).$$

From Lemma 4 we know that $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$. Because of Lemma 1 this implies that

$$\frac{d\left(\frac{q_I}{k_I}\right)}{d\underline{k}} < 0 \quad \text{and} \quad \frac{d\left(\frac{q_j}{k_j}\right)}{d\underline{k}} > 0.$$

Therefore, the first two terms in (32) are positive.

Now let us turn to the third term. Since C''' is positive or not very negative, we get that $dC''_j/d\underline{k}$ is also positive or not very negative while $dC''_I/d\underline{k}$ is negative or not very positive. Therefore, the third term is either positive, or, if it is negative, then only slightly so. As a consequence, the sum of the first three terms in (32) is positive.

Now let us look at the fourth term. We know that $dQ/d\underline{k} = 0$ at any intersection between the left- and the right-hand side. Therefore, the fourth term is zero at an intersection. But this implies that the right-hand side is strictly increasing in $\underline{k}$ at an intersection, and, therefore, it can cross the left-hand side only from below. Since we know that the right-hand side is smaller that the left-hand side at $\underline{k} = 0$, there can at most be one intersection between the two sides, which proves the result.

Case (ii):

We first solve for the equilibrium in the linear-quadratic case. The profit function of the integrated firm in this case can be written as

$$\Pi_I = \left[\alpha - \beta q_I - \beta\sum_{i=1}^{N} q_i\right] q_I - \frac{c q_I^2}{2k_I} + \gamma q_I - \delta(k_I - \underline{k})(k_I + \sum_{i=1}^{N} k_i), \tag{33}$$

and the one of a non-integrated firm j as

$$\Pi_I = \left[\alpha - \beta q_j - \beta q_I - \beta\sum_{i=1, i\neq j}^{N} q_i\right] q_j - \frac{c q_j^2}{2k_j} - \delta k_j(k_j + k_I + \sum_{i=1, i\neq j}^{N} k_i). \tag{34}$$

Differentiating with respect to q_I and q_j and solving for the equilibrium quantities yields

$$q_I = \frac{(\beta\,(\alpha + (N+1)\gamma)\,k_j + c\,(\gamma + \alpha))\,k_I}{\beta(\beta k_j(N+2) + 2c)k_I + c^2 + k_j\beta c(N+1)} \quad \text{and} \quad q_j = \frac{(\beta k_I(\alpha - \gamma) + c\alpha)\,k_j}{\beta(\beta k_j(N+2) + 2c)k_I + c^2 + k_j\beta c(N+1)}.$$

After substituting these quantities into the respective profit functions, we can take derivatives of Π_I with respect to k_I and of Π_j with respect to k_j.[25] The equilibrium capacities k_I and k_j

[25]As before, we have to differentiate between k_j and k_i $i \neq j$, $i, j \in \{1, ..., N\}$. Of course, in equilibrium we will have $k_i = k_j$.

are then implicitly defined by

$$\left(c^2(c+k_j(1+N)\beta)\right)\left(c^2(\gamma+\alpha)^2+2c((\gamma+\alpha)\beta(\alpha+\gamma(1+N))-N\delta c^2)k_j\right. \tag{35}$$

$$\left.+\beta((\alpha+\gamma(1+N))^2\beta-4N(1+N)\delta c^2)k_j^2-2N\delta c\beta^2(1+N)^2k_j^3\right)=\sum_{t=1}^{4}k_I^t\theta_t-\theta_0\underline{k},$$

with

$$\theta_0=2\delta(\beta^2(2+N)k_jk_I+\beta(N+1)k_jc+2\beta ck_I+c^2)^3,$$

$$\theta_1=(6\beta^4\delta Nc(2+N)(1+N)^2k_j^4-\beta^3((2+3N)(\alpha+(N+1)\gamma)^2\beta-4\delta c^2(1+N)(7N^2+11N+1))k_j^3-$$

$$-2\beta^2c((\alpha+(N+1)\gamma)(3\alpha(N+1)+\gamma(3+4N))\beta-3\delta c^2(7N^2+10N+2))k_j^2-$$

$$-\beta c^2((\gamma+\alpha)((7N+6)\gamma+3\alpha(N+2))\beta-12\delta c^2(2N+1))k_j-2c^3(\beta(\alpha+\gamma)^2-2\delta c^2)),$$

$$\theta_2=(6\beta^5\delta Nc(1+N)(2+N)^2k_j^4+6\beta^4\delta c^2(2+N)(N^2+10N+2)k_j^3+$$

$$+24\beta^3\delta c^3(2N+3)(2N+1)k_j^2+12\beta^2\delta c^4(7N+6)k_j+24\beta\delta c^5),$$

$$\theta_3=2\beta^2\delta(k_j^2\beta^2N(2+N)+2\beta k_jc(4N+3)+6c^2)(k_j\beta(N+2)+2c)^2,$$

$$\theta_4=4\beta^3\delta(k_j\beta(N+2)+2c)^3,$$

and

$$c^3(2k_I\beta+c)(\beta(-8c^2\delta+\beta(\alpha-\gamma)^2)k_I^2+2c(\beta\alpha(\alpha-\gamma)-c^2\delta)k_1+c^2\alpha^2-8\beta^2c\delta k_I^3)$$

$$=\Big((8\beta^4c\delta(8+3N)k_I^4-\beta^3((\alpha-\gamma)^2(6+N)\beta-16c^2\delta(7+4N))k_I^3- \tag{36}$$

$$-\beta^2c((\alpha-\gamma)((14+3N)\alpha-\gamma(N+2))\beta-18c^2\delta(3N+4))k_I^2+$$

$$+c^2(k_I\alpha(-(3N+10)\alpha-2(N+2)\gamma)\beta^2+c(2(9N+10)ck_I\delta-\alpha^2(N+2))\beta+2c^3\delta(N+1)))+\sum_{t=1}^{4}k_j^t\tau_t\Big)k_j,$$

with

$$\tau_1=c\beta(12\beta^4c\delta(N+4)(N+2)k_I^4+\beta^3(-(\alpha-\gamma)^2(2+3N)\beta+2c^2\delta(116N+104+27N^2))k_I^3+$$

$$+\beta^2c((\alpha-\gamma)((3N-1)\gamma-3(3+N)\alpha)\beta+6c^2\delta(39N+28+12N^2))k_I^2+$$

$$+\beta c^2(\alpha(2(3N-1)\gamma-9N\alpha)\beta+12c^2\delta(3N+5)(N+1))k_I+c^3(((1-3N)\alpha^2)\beta+2c^2\delta(3N+4)(N+1))).$$

$$\tau_2=2\beta^2\delta c((2+N)k_I\beta+(N+1)c)(\beta^3(N+8)(N+2)k_I^3+c\beta^2(8N^2+45N+40)k_I^2+$$

$$+2c^2\beta(5N+14)(N+1)k_I+3c^3(N+2)(N+1)),$$

$$\tau_3=2\beta^3\delta((2\beta^2(1+N))k_I^2+c\beta(N+9)(N+1)k_I+c^2(N+4)(N+1))((2+N)k_I\beta+(1+N)c)^2k_j^4,$$

$$\tau_4 = 2\beta^4\delta(N+1)((N+2)k_I\beta + (N+1)c)^3.$$

We now turn to the competitive effects of a change in $\underline{k}$. Since $Q = q_I + Nq_j$, we can insert the above explicit solutions for the quantities and differentiate Q with respect to $\underline{k}$. From this we get that $dQ/d\underline{k} > 0$ if and only if

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} > -\frac{(k_j\beta + c)(\beta(\gamma(N+1)+\alpha)k_j + c(\gamma+\alpha))}{N(k_I\beta + c)((\beta(\alpha-\gamma))k_I + c\alpha)}. \tag{37}$$

Via differentiating (35) and (36) with respect to $\underline{k}$, taking into account that k_I and k_j vary with $\underline{k}$, we can calculate the left-hand side of (37). Subtracting the right-hand side from the left-hand side yields an expression that has the following structure:

$$\sum_{u=0}^{6}\sum_{z=0}^{5} k_j^u k_I^z \upsilon_{uz}, \tag{38}$$

where $\upsilon_{uz} = \upsilon_{uz}(\alpha, \beta, \gamma, \delta, c, N)$. We do not spell out the exact expressions for υ_{uz}, $u \in \{1, ..., 6\}$, $z \in \{1, ..., 5\}$ because they are rather complicated. As will become clear, we are mainly interested in determining their signs and compare them, which is relatively easily possible.

Differentiating (38) with respect to $\underline{k}$ we get

$$\sum_{u=1}^{6}\sum_{z=1}^{5} \upsilon_{uz}\left(zk_j^u k_I^{z-1}\frac{dk_I}{d\underline{k}} + uk_j^{u-1}k_I^z\frac{dk_j}{d\underline{k}}\right) + \sum_{z=1}^{5}\upsilon_{0z} z k_I^{z-1}\frac{dk_I}{d\underline{k}} + \sum_{u=1}^{6}\upsilon_{u0} u k_j^{u-1}\frac{dk_j}{d\underline{k}},$$

where, from Lemma 4, $dk_j/d\underline{k} < 0$ and $dk_I/d\underline{k} > 0$.

First, one can show that all $\upsilon_{uz} > 0$ if $u > z$. Then calculating $\upsilon_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$ for $u > z$ reveals that all these expressions are negative. The expressions for υ_{uz} with $u < z$ can have different signs. So let us first take each term $\upsilon_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$, where $z = z_a > u_a = u$. Now we compare it with the corresponding expression where $u = z_a$ and $z = u_a$. One can then show that the latter expression is larger than the former in absolute values in any comparison. Therefore, the sum of each of the comparisons is negative. Finally, we have to look at terms with $u = z$. Again, υ_{uz} can be positive or negative, i.e. $\upsilon_{uz} > 0$ for $u = z = 1, 2, 3$, $\upsilon_{uz} < 0$ for $u = z = 4$ and $\upsilon_{uz} = 0$ for $u = z = 5$. Now for any of these expressions $\upsilon_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$ with $u = z$ we can find a previous comparison, to which we can add the expression and the resulting sum still stays negative. Thus, equation (38) is strictly decreasing in $\underline{k}$. Since at $\underline{k} = 0$, the left-hand side of (37) is larger than the right-hand side, we know that there exists either a unique intersection or no intersection between the terms on the two sides. ■

A.8 Proof of Proposition 4

We first show that $q_j \to 0$ and $k_j \to 0$, $j \in \{1, ..., N\}$, as $N \to \infty$. Suppose to the contrary that $q_j > 0$. But since $Q = q_I + Nq_j$ and $P(Q) \leq 0$, as $N \to \infty$, the first-order condition for firm j given by (1) cannot be satisfied if $q_j > 0$, since the right-hand side would be positive while the left-hand side would be negative. Therefore, $q_j \to 0$, as $N \to \infty$. Given this, suppose now that $k_j > 0$. But then in the first-order condition of the capacity stage, (4), the left-hand side would be negative while the right-hand side is zero. In order to fulfill this condition we must have $k_j \to 0$. Therefore, as $N \to \infty$, $q_j \to 0$ and $k_j \to 0$.

In the proof of Proposition 4 we already calculated the case of $q_j \to 0$ and $k_j \to 0$. Taking in addition $N \to \infty$ we get from (30) that vertical integration is anticompetitive if

$$-\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_I P')},$$

where ρ and σ are defined in the proof of Proposition 2. But we already showed in this proof that the inequality is fulfilled. Therefore, vertical integration is anticompetitive if $N \to \infty$. ■

A.9 Proof of Proposition 5

We start with the case where $\underline{k} = 0$ and $\gamma = 0$. In the proof of Proposition 1 we calculated the left-hand of (9). To determine the right-hand side of (9) we first insert $dQ/dk_I = dq_I/dk_I + Ndq_j/dk_I$, $dQ_{-I}/dk_I = Ndq_j/dk_I$ and $dQ/dk_j = dq_I/dk_j + dq_j/dk_j + (N-1)dq_j/dk_I$ into the right-hand side and then use equations (13), (14), (15), (18) and (19) from the proof of Lemma 1, i.e. the derivatives of q_i with respect to k_j, $i, j \in \{I, 1, ..., N\}$. Knowing that at $\underline{k} = 0$ and $\gamma = 0$ we have $q_I = q_j$, $k_I = k_j$ and $C_I'' = C_j''$, the right-hand side simplifies to $-1/N$. But from the proof of Proposition 1 we know that the left-hand side is larger than $-1/N$ at $\underline{k} = 0$ and $\gamma = 0$. Therefore, marginal vertical integration is welfare increasing at this point. In the same way as in the proof of Proposition 1 we can show that it is also welfare increasing for $\gamma > 0$. By continuity there exists a threshold $\underline{k}_W^*$ such that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}_W^*$.

Now we turn to the case where $\underline{k} = \bar{k}$. From the proof of Proposition 2 we know that in this case

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{C_I''P'\alpha\frac{q_I}{k_I} + \beta}{(N+1)\left(\alpha^2 P'(C_I'' - k_I P') + \beta\right)}.$$

Proceeding in the same way as above to determine the right-hand side of (9) but now inserting

$\underline{k} = \bar{\underline{k}}$, $q_j = k_j = 0$ yields

$$-\frac{P' q_I^2 C_I'' - k_I(C_I'' - k_I(2P' + q_I P'')(k_I - \underline{k})}{P' q_I k_I^2 N(P' + q_i P'')}. \tag{39}$$

Subtracting (39) from $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ then yields

$$-k_I^2 \left(-2k_I P' - k_I P'' q_I + C_I''\right)^2 (1+N)(R')^2 +$$

$$+P'\left(-2\,k_I P' - k_I P'' q_I + C_I''\right)\left(\left(q_I^2 + k_I^2\alpha^2\right)(1+N)\,C_I'' - k_I^2\left(q_I^2 N P'' + P'\left(k_I\alpha^2 + kIN\alpha^2 + Nq_I\right)\right)\right) R' -$$

$$-C_I''(P')^2 q_I^2 \alpha\,\left(-k_I P'\left(\alpha + N\alpha + N\right) - k_I N P'' q_I + \alpha C_I''\,(1+N)\right) + \tag{40}$$

$$+R' kI\,\left(2\,k_I P' + k_I P'' q_I - C_I''\right)\left(R' q_I k_I P'' - kI(P')^2\alpha^2 + 2\,k_I R' P' + P'\alpha^2 C_I'' - R' C_I''\right)(1+N)\,\bar{\underline{k}}.$$

The first three terms in this expression are negative while the last term is positive. Therefore, if the ex ante capacity that is needed to induce the non-integrated firms to stop producing, $\bar{\underline{k}}$, is small, the expression is negative and welfare is decreasing at $\underline{k} = \bar{\underline{k}}$. By continuity there then exists a $\underline{k}_W^{**}$ such that for all $\underline{k} > \underline{k}_W^{**}$ vertical integration is reducing welfare at the margin. If instead $\bar{\underline{k}}$ is large, the expression is positive and vertical integration is welfare enhancing at the margin. ■

A.10 Proof of Proposition 6

Case (i):

From (9) we know that vertical integration enhances welfare if

$$N\left[-P'\left(q_j\frac{dQ}{dk_j} + (N-1)q_j\frac{dq_i}{dk_j} + q_I\frac{dq_I}{dk_j}\right) + R'k_j\right]\left(\frac{dk_j}{d\underline{k}}\right) + \tag{41}$$

$$+\left(-P'\left(q_I\frac{dQ}{dk_I} + q_j\frac{dQ_{-I}}{dk_I}\right) + R'(k_I - \underline{k})\right)\left(\frac{dk_I}{d\underline{k}}\right) > 0.$$

If R' is dominating all other derivatives, we can calculate $dk_j/d\underline{k}$ and $dk_I/d\underline{k}$ from (26) and (27) to get

$$\frac{dk_j}{d\underline{k}} = -\frac{1}{N+2} \quad \text{and} \quad \frac{dk_I}{d\underline{k}} = \frac{N+1}{N+2}. \tag{42}$$

Inserting this into the last expression and using the fact that R' is dominating all other derivatives yields

$$\frac{Nk_j + (N+1)(\underline{k} - k_I)}{N+2} R' > 0.$$

Differentiating the left-hand side of the last equation with respect to $\underline{k}$ and using (42) yields

$$\frac{d\left(\frac{Nk_j + (N+1)(\underline{k} - k_I)}{N+2} R'\right)}{d\underline{k}} = -\frac{1}{(N+2)^2} R' < 0.$$

Therefore, the term that determines the sign of (41) is strictly decreasing in $\underline{k}$. Since welfare is increasing in $\underline{k}$ at $\underline{k} = 0$, there is either a unique intersection point or none.

The proof for case (ii) proceeds along the same lines as the proof of case (ii) in Proposition 3 and is therefore omitted. ■

A.11 Proof of Proposition 7

From (8) we know that welfare is increasing in $\underline{k}$ if and only if

$$P\frac{dQ}{d\underline{k}} - NC_j\frac{dk_j}{d\underline{k}} - Nk_jC_j'\left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - \tag{43}$$

$$-C_I\frac{dk_I}{d\underline{k}} - k_IC_I'\left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) + \gamma\frac{dq_I}{d\underline{k}} - R\frac{dK}{d\underline{k}} > 0.$$

The first term on the left-hand side, $PdQ/d\underline{k}$, has the same sign as the condition for pro- or anticompetitive vertical integration. Therefore, we know that it is zero at $\underline{k}^*$. As a consequence, if the rest of the left-hand side is negative at $\underline{k}^*$, this would imply that $\underline{k}_{WF}^* < \underline{k}^*$.

We start with the case of $\gamma = 0$. In this case the term $\gamma(dq_I/d\underline{k}) = 0$. The term $-R(dK/d\underline{k})$ is negative since overall capacity is increasing in $\underline{k}$. Thus, if the terms

$$-NC_j\frac{dk_j}{d\underline{k}} - Nk_jC_j'\left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - C_I\frac{dk_I}{d\underline{k}} - k_IC_I'\left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) \tag{44}$$

are negative at $\underline{k}^*$, we have established that $\underline{k}_{WF}^* < \underline{k}^*$ at $\gamma = 0$. We can now use the respective expressions for the cost functions and the equilibrium values of q_j and q_I in the linear-quadratic case that we calculated in the proof of Proposition 3, case (ii). Inserting them into (44) yields that the sign of this expression is given by the sign of

$$-\left[N\alpha^2(c+\beta k_I)\left(k_I^2\beta^2(2c-\beta k_j(N+2)) + k_Ic\beta(c-\beta k_j(2N+5)) + c^2\alpha(c-\beta k_j(N+1))\right)\right]\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} - \tag{45}$$

$$-\alpha^2\left[c+k_j\beta\right]\left[k_j^2\left(\beta^2c(N+1) - k_I\beta^3(N+2)\right) + k_jc\beta\left(c(2-N) - k_I\beta(3N+4)\right) - 2k_Ic^2\beta + c^3\right].$$

From (37) we know that $dQ/d\underline{k} = 0$ at $\gamma = 0$, if $\underline{k}$ implies equilibrium values of k_I and k_j such that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{(k_j\beta + c)\alpha(\beta k_j + c)}{N(k_I\beta + c)\alpha(\beta k_I + c)}.$$

Inserting the last equation into (45) and simplifying gives

$$-\frac{2c\beta(k_I - k_j)(c+\beta k_j)\left(k_I\beta(2c + k_j\beta(N+2)) + c(c + k_j\beta(N+1))\right)}{(c+\beta k_I)},$$

which is negative because $k_I > k_j$ at $\underline{k}^*$. Thus, we have shown that $\underline{k}^*_{WF} < \underline{k}^*$ at $\gamma = 0$.

Now we turn to the case in which $\gamma \neq 0$. We know that $dQ/d\underline{k} = 0$ at $\underline{k} = \underline{k}^*$. We can then write (43) under the linear-quadratic specification for the case of $\underline{k} = \underline{k}^*$ as

$$\begin{gathered}
-\frac{k_j^2 k_I^2 \varrho}{\left(k_I\beta(2c + k_j\beta(N+2)) + c(c + k_j\beta(N+1))\right)^3} + \\
+c\gamma \frac{(c + \beta k_j(N+1))(c(\alpha+\gamma) + \alpha\beta k_j + \beta\gamma k_j(N+1)) - k_I\beta N(\alpha c + \beta k_I(\alpha-\gamma))\left(\frac{dk_j}{d\underline{k}} \Big/ \frac{dk_I}{d\underline{k}}\right)}{\left(k_I\beta(2c + k_j\beta(N+2)) + c(c + k_j\beta(N+1))\right)^2} - \\
-d(k_I + Nk_j)(N\left(\frac{dk_j}{d\underline{k}} \Big/ \frac{dk_I}{d\underline{k}}\right) + 1),
\end{gathered} \tag{46}$$

with

$$\begin{gathered}
\varrho \equiv \Big[N(\alpha c + \beta k_I(\alpha-\gamma))\left(k_I^2\beta^2(\alpha-\gamma)(2c - \beta k_j(N+2)) + \right. \\
\left. + k_I c\beta(c(\alpha - 3\gamma) - \beta k_j(\alpha(2N+5) + \gamma(N+1))) + c^2\alpha(c - \beta k_j(N+1))\right)\Big]\left(\frac{dk_j}{d\underline{k}} \Big/ \frac{dk_I}{d\underline{k}}\right) + \\
+ \left[c(\alpha+\gamma) + k_j\alpha\beta + \beta\gamma k_j(N+1)\right]\Big[k_j^2(\alpha + \gamma(N+1))\left(\beta^2 c(N+1) - k_I\beta^3(N+2)\right) + \\
+ k_j c\beta\left(c(2(\alpha+\gamma) - N(\alpha - 2\gamma)) - k_I\beta(\alpha(3N+4) + \gamma(N+4))\right) - 2k_I c^2\beta(\alpha+\gamma) + c^3(\alpha+\gamma)\Big].
\end{gathered}$$

From (37) we have that $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ at $\underline{k} = \underline{k}^*$ is given by

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{(k_j\beta + c)(\beta(\gamma(N+1) + \alpha)k_j + c(\gamma+\alpha))}{N(k_I\beta + c)((\beta(\alpha-\gamma))k_I + c\alpha)}.$$

Inserting this into (46), differentiating the resulting expression with respect to γ and using the fact that $dk_I/d\gamma > 0$ and $dk_j/d\gamma < 0$ reveals that the expression is strictly increasing in γ. But from the first part of the proof we know that (46) evaluated at $\underline{k} = \underline{k}^*$ is negative at $\gamma = 0$ which implies that $\underline{k}^*_{WF} < \underline{k}^*$. Therefore, we have shown there exists either a unique value of γ denoted by $\hat{\gamma}$ such that $\underline{k}^*_{WF} < \underline{k}^*$ for all $\gamma < \hat{\gamma}$ and $\underline{k}^*_{WF} > \underline{k}^*$ for all $\gamma > \hat{\gamma}$, or no such value exists because (46) turns positive only at such high values of γ at which the non-integrated firms are not active. In the latter case $\underline{k}^*_{WF} < \underline{k}^*$ for all γ. ■

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