Competing Premarital Investment

Michael Peters and Aloysius Siow*
Department of Economics
University of Toronto

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Abstract

This paper studies pre-marital parental investments in their children's wealth where spousal wealth is a public good in marriage. By investing in their children's wealth, parents increase the wealth of their children and the quality of the spouses that their children can marry. In large marriage markets, the hedonic return to investment internalizes all the external benefits of pre-marital investment in wealth so that the competitive equilibrium is efficient. Marriage market competition also increases investments in small marriage markets relative to no competition, but equilibrium investments are not efficient.

1   Introduction

In marriage, an individual derives utility from own pre-marital investment and the pre-marital invesment of his or her spouse. Much of these investments are human capital investments made by altruistic parents. Since pre-marital

1

investment is a public good in marriage, parents may under invest in their children.[1]

This literature typically assumes that the child's marriage partner is fixed and independent of any investment that the family makes in the child. In this sense the arguments leave out the important effects that competition for spouses will have on ex ante investment incentives. In this paper, we study pre-marital investments when children use these investments to compete for spouses. We are primarily interested in the implications of *assortative matching* equilibria which occur when wealthy individuals are matched with wealthy partners. Then altruistic parents take into account the additional utility their children will enjoy from wealthier partners, and this will increase their incentive to invest in their children on the margin.

Our first model considers the case where the number of families is very large. We study a *competitive* equilibrium in which all families on the same side of the market believe that they face the same non-stochastic *return* to their investment in their children. This return function adjusts until families beliefs are fulfilled in equilibrium.

Perhaps the most remarkable property of investment in the competitive equilibrium is the fact that the externalities associated with families' investments in their children are completely internalized by this return function. Any pair of families whose children match on the competitive equilibrium path, will make investments that are bilaterally pareto optimal. Despite the fact that neither family can directly compensate the other family for the investment that it makes in its child, the marriage market and the assortative matching that occurs there forces each family to compensate the other indirectly through the investment that it makes in its own child.

This efficiency result is an application of Rosen's hedonic pricing approach to large matching problems (Rosen (1974)). The market return function provides what is essentially a hedonic value for every investment level that a family might consider making. Families on the other side of the market need to provide these hedonic values in order to attract partners with specific investment levels. In equilibrium, each family's indifference curve (in the

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[1]For a discussion of the incentive for private provision of public goods see Bergstom, Blume and Varian (1986) and references cited therein. More specifically, MacLeod and Malcomson (1993) explore investment incentives in bilateral matching problem, and in particular discuss the impact of outside options on these incentives. For a discussion of investment incentives in marriage with outside options but without marriage market competition, see Konrad and Lommerud (2000).

space of investments) will be tangent to this hedonic return function, and consequently, families whose children match will have indifference curves that are tangent to each other. The investments that families undertake will then be *bilaterally efficient* in the sense that there will not be another pair of investments that will make both of the matched families better off at the same time. Since the joint payoffs that we employ are supermodular (Becker (1973),Smith (1996)) assortative matching along with bilateral efficiency are sufficient to guarantee that the distribution of investments for the economy is efficient.

Matters are more complicated in small marriage markets. When the number of families and children is small, assortative matching among children will raise families incentive to invest and at the same time make families investments less predictable. If there are significant wealth disparities on the other side of the market, parents may find that they can increase the wealth of their child's partner significantly by raising their investment only slightly. This makes parental payoff functions discontinuous, which rules out pure strategy equilibria in some situations. In the mixed strategy equilibria that do prevail, parental investment is stochastic. Though rich families will invest more in their children on average than poor families do, there will be a positive probability that the poor families will invest more than rich families so that their children move up the wealth distribution. This creates endogenous intergenerational mobility.

The small numbers case is perhaps not so interesting in the context of the family matching problem in which large market arguments seem quite natural. However, our methods apply to a variety other bilateral matching markets where small numbers are more important. For example workers and firms face a similar difficulty with ex ante investments in physical and human capital, even though a wider variety of contractual remedies for holdup problems are available in that context. Some problems where small numbers are likely to be more important are co-authorship in academics, sports teams, business partnerships or other forms of collaborative activity.[2] We have not been able to provide a complete link between the small and large numbers case, but we can give complete characterization of equilibrium for a special case which illustrates most of the issues involved.

Ex ante investments in labor markets have been analyzed in a couple of papers. In an interesting paper, Shi (1997) allows firms to choose which

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[2]These examples were suggested by a referee.

3

of several market segments to join and then to pick a capital investment optimal for the workers who are available in that segment. Since the workers human capital endowments are exogenous, and matching is random within a segment, his results are not easy to compare with ours. The primary difference is that workers and firms cannot move across market segments once their investments have been made, so workers do not compete against one another for partners and the matching return to investment that we describe does not arise.

Closer to our treatment is a paper by Felli and Roberts (2000) in which workers offer to work for firms at wages that depend on their human capital investments. Each worker offers a wage that makes its target firm just indifferent between accepting the offer and accepting the offer of the next most skilled worker. The logic of their equilibrium is reminiscent of the logic used to show that bidders bid their true valuations in second price auctions. The profit a worker needs to offer a firm is determined by the human capital investment of a worker that the firm does not hire on the equilibrium path. The worker that the firm does hire gets all the residual, and so has the correct ex ante investment incentives. Their procedure differs from ours in two ways. First, utility is transferable and the return to investment is a monetary payment that the worker extracts from the firm instead of a matching return. Though their procedure resolves the holdup problem for workers, it does not achieve full efficiency since firms investment incentives are not inefficient.

The closest work to ours is Cole, Malaith and Postlewaite (1998) and Cole, Malaith and Postlewaite (2000). They consider a family matching problem with transfers in which investments are followed by a cooperative matching process that selects partners and redistributes income across children. They show that efficient investments can be supported as an equilibrium outcome, though other inefficient equilibria are also possible. Efficient investments occur because the cooperative matching process generates an explicit monetary return to investment. In our model there is no monetary return to investment, and marginal incentives are created solely by the expectation that higher investment will improve match quality.

Han (2001) extends the worker firm investment matching model to environments where traders disagree about who the most desirable partners are. In addition, he provides a survey and comparison of the equilibrium outcomes of the papers listed above.

Siow and Zhu (1998) also study a large marriage market and pre-marital

4

investment problem with transferable utility and two wealth classes on each side of the market. They also study multigenerational equilibria. Acemoglu (1997) studies a two side matching investment model with workers and firms. He obtains underinvestment because due to potential random matching, workers and firms are unable to fully capture the returns to their pre-employment investments.

While this paper focuses on the marriage market, our analysis applies to other partnerships in which the share of surplus in the partnership is not conditioned on the level of pre-partnership investment. Members of amateur sports teams and co-authors in economics usually do not divide surplus according to their levels of pre-partnership investments. In most of these markets, agents invest in pre-partnership human capital and then compete for partners. The results in this paper should be useful for thinking about those markets as well.

2 Preliminaries

Families begin with an endowment of wealth y which can be used partly as current consumption, and partly as an investment in children. Let w be the amount invested in the child. If the child subsequently matches with a partner whose wealth level is $\widetilde{w}$ then utility for the parents is given by

$$V\left(y - w\right) + z \cdot \left(w + \widetilde{w}\right) , \; z > 0$$

and utility of the child is

$$z \cdot \left(w + \widetilde{w}\right)$$

If the family invests w and the child is not expected to match, then we assume that the child has utility zw. The actual value of the childrens' utility when no match occurs is unimportant as long as both the child and family are at least weakly better off when a match occurs than if it doesn't.[3]

Assumption The function $V\left(\cdot\right)$ is monotonically increasing, strictly concave, differentiable, has bounded marginal utilities and satisfies

$$\lim_{x \to 0} V\left(x\right) = -\infty$$

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[3]It is possible that children might strictly prefer not to match if the best available partner is too poor. This creates problems for our methodology, but these are not particularly relevant for the issues we wish to discuss.

The bilateral Nash, or non-competitive investment levels for each family are given by the solutions to

$$V'(y_i - w_i^*) = z \tag{1}$$

These are the investment levels that the families would make if they believed (for whatever reason) that their children's match partner is independent of parental investment. In the case where there are only two families, one on each side of the market, the investment of the family on the other side of the market would be fixed and equilibrium investment would satisfy 1. Investment would be inefficient in this case because 1 does not take account of the positive effect that the family's investment has on the family on the other side of the market.

After the families have made their investments, the children compete for partners in the marriage market. We will first study the investment and matching problem in a large marriage market. Then we will investigate properties of small marriage markets.

3   Large Marriage Markets

This section considers a large marriage market with a continuum of families on each side of the market. We refer to families with female children as families 'in F' and similarly, families with male children are families 'in M'. Let the total measure of female families be F and the total measure of male families be M where $F \geq M$. Let G and H be measures of sets of families in M and F respectively. Interpret $G(B)$ to be the measure of the set of families whose endowments lie in the set B and similarly for H. Suppose that the wealth levels of families in M are distributed on a closed interval Y, while families in F have their wealth distributed on a closed interval $\tilde{Y}$.

Let $g(w)$ represent the wealth of the wife that each family in M expects to match with from an investment of w in their son. If families expectations are realized, then $g^{-1}(\widetilde{w})$ will represent the wealth of the groom that each family in F expects to match with from an investment of $\widetilde{w}$ in their daughter. Since higher wealth attracts better partners with assortative matching, $g(w)$ will be non-decreasing in w.

Definition 1 *The return function $g(w)$ is a* rational expectations equilibrium *if there exist investment strategies $\sigma(y)$ and $\tilde{\sigma}(\tilde{y})$ for families in M and F respectively such that*

1. $\sigma(y) \in \arg\max_x \{V(y - x) + z(x + g(x))\}$ for each $y \in Y$ and $\tilde{\sigma}(\tilde{y}) \in \arg\max_x \{V(\tilde{y} - x) + z(x + g^{-1}(x))\}$ for each $\tilde{y} \in \hat{Y}$;

2. for each w, $G\{y : \sigma(y) \geq w\} = H\{\tilde{y} : \tilde{\sigma}(\tilde{y}) \geq g(w)\}$.

(1) in the definition above says that each family optimally chooses a wealth level for their child given the return function $g(w)$. (2) is the marriage market clearing condition. It requires that the measure of the set of families in M who (optimally) invest w or more is exactly equal to the measure of the set of families in F who invest $g(w)$ or more. By assortative matching, this means that every family in M who invests w or more will be able to find a partner for their child whose investment is $g(w)$ or more.

Figure 1 illustrates a rational expectations equilibrium. The investment levels for families in M are given along the horizontal axis, while investments for families in F are along the vertical axis. The dark curve illustrates the equilibrium matching function. The lighter curves that are convex upward are indifference curves for families in M, those that are convex downward are indifference curves for families in F. A family in M who invests w^{**} should expect their child to match with someone whose wealth is $g(w^{**})$. In equilibrium, if a family chooses investment w^{**} then their indifference curve should be tangent to the curve $g(w)$ at the point $(w^{**}, g(w^{**}))$. The reason is that the family thinks that $g(w)$ represents the market return function that they face. Similarly, any family in F who chooses to invest $g(w^{**})$ should expect return w^{**}. In equilibrium this family must have an indifference curve tangent to the market trade-off function $g^{-1}(\cdot)$ at the point $(w^{**}, g(w^{**}))$. This implies that the indifference curves of the families of every pair of children who match in equilibrium will be tangent to each other. So every pair of families who matches will choose investments that are bilaterally pareto optimal - the under investment problem disappears.

As the picture is drawn, the family from M who attains the indifference curve II is the one with the lowest endowment, while the family from F who attains the indifference curve $I'I'$ touching the market return line at point A is the family with the lowest endowment who actually succeeds in matching. In equilibrium, this family will have to be just indifferent between making a positive investment and matching with the poorest family in M, and investing Nash and having no partner at all. That is why the indifference curve $I'I'$ just touches the vertical axis.

To see this more formally, focus on the case where both G and H are monotonic, and let $\alpha(y)$ satisfy $M - G(y) = F - H(\alpha(y))$. By assortative
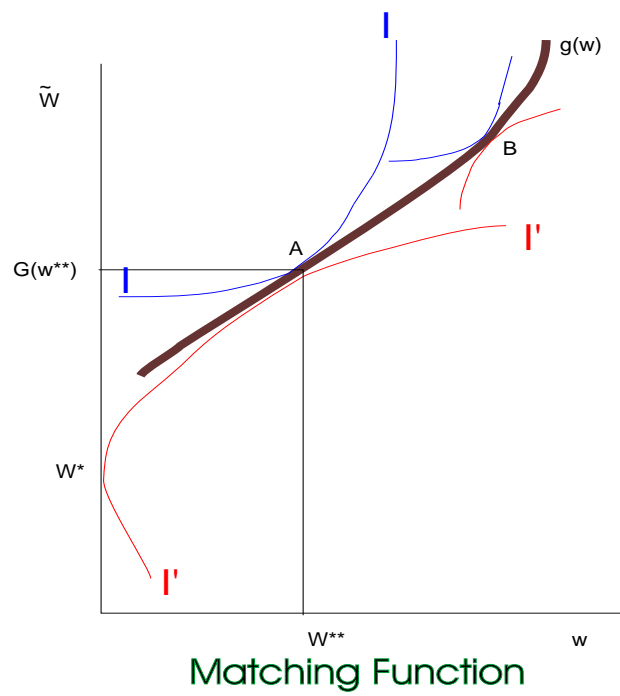
Figure 1: Matching Function

matching, a family in M with income y should end up matching with a family from F whose income is $\alpha(y)$ provided that each families investment is an increasing function of its endowment. A necessary condition for optimality is that a family from M whose income is y should prefer to invest $\sigma(y)$ than to invest $\sigma(y')$ as they would if their income were y' instead of y. In equilibrium with assortative matching, each family's payoff can be written

$$V(y - \sigma(y)) + z(\sigma(y) + \tilde{\sigma}(\alpha(y)))$$

This can be no smaller than $V(y - \sigma(y')) + z(\sigma(y') + \tilde{\sigma}(\alpha(y')))$. So differentiating this latter expression with respect to y' and setting the resulting derivative evaluated at $y' = y$ equal to zero gives the necessary condition

$$\sigma'(y)[-V'(y - \sigma(y)) + z] = -z\tilde{\sigma}'(\alpha(y))\alpha'(y) \qquad (2)$$

The corresponding condition for the family $\alpha(y)$ from F is that

$$-z\sigma'(y)\frac{1}{\alpha'(y)} = \tilde{\sigma}'(\alpha(y))[-V'(\alpha(y) - \tilde{\sigma}(\alpha(y))) + z] \qquad (3)$$

Dividing (2) by (3) gives equality of the marginal rates of substitution between own investment and partner wealth (i.e., tangency of the families indifference curves in the Figure above).

This tangency condition means that there is no way that a pair of families whose children match can jointly change their investments to make both families better off. There remains only the possibility that families could be rematched, then adjust their investments in a way that will make everyone better off. The payoff function we employ has a simple single crossing property that rules this out. So an immediate consequence of bilateral efficiency is that

Proposition 2 *A rational expectation equilibrium is pareto optimal.*

Proof. We show that if families are rematched in a non-assortative fashion, then at least one family must be made worse off than they are in the rational expectations equilibrium. Figure 1 can be used to illustrate. Suppose that a pareto improvement can be obtained by rematching in such a way that family A in M (that is, the family who ends up at allocation A in Figure 1) is rematched with family B from F. Family A is the family in Figure 1 who chooses the point A while family B is the family from F

who chooses the outcome B. If matching family B with family A results in a pareto improvement, then investment levels need to be adjusted so that both families end up on higher indifference curves than they attain in the initial assortative matching equilibrium. Since family B chooses a higher investment level that $A's$ original partner, family B must have higher wealth than A's intial partner by assortative matching, so B's indifference curve will be too steep to cross family A's indifference curve and a pareto improving change in investment levels is impossible. ■

4 Existence of a Rational Expectations Equilibrium

Our notion of rational expectations equilibrium can be applied no matter what the properties of the underlying distributions G and H happen to be. For example, G and H could consist of atoms or agglomerations of families at specific wealth levels.[4] When a rational expectations equilibrium exists it will be pareto optimal as argued above. In some simple problems existence of a rational expectations equilibrium is immediate. For example, suppose that $G = H$. Then set $g(w) = w$. Then each family chooses the (efficient) level of investment w^{**} that equates the marginal utility of consumption and 2. For a variety of reasons, this is not a good example of the rational expectations solution concept. We return to it momentarily.

A more illuminating example occurs when G and H differ. Suppose as before that $F > M$ but that all the families in F have the same endowment. In figure 1, suppose that all families in F have indifference curves like $I'I'$. Define $g(x)$ so that it coincides with $I'I'$. Let the families in M choose the points on this indifference curve that they most desire. Families in F can then choose investment levels to match with families in M along this indifference curve. The market return function will have a closed form solution provided that the indifference curve can be represented in closed form.

To avoid making the mechanics too complicated, we sketch the argument for existence of equilibrium in the case where the distribution functions G and

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[4]One particularly simple case to imagine is the one in which all families on the same side of the market are identical. For example in Figure 1, all families in M could have indifference curves given by II while all families in F have indifference curves given by $I'I'$. The rational expectations return function drawn in that diagram would still work and provide and efficient rational expectations equilibrium.

H are both monotonic with differentiable inverse functions whose derivatives are bounded away from 0 and infinity. Define for each y in the support of G, $\gamma(y) = \{y' \in \operatorname{supp} H : F - H(y') = M - G(y)\}$. Since G and H are both monotonic and differentiable, so is γ. Furthermore, the derivative of γ is bounded away from 0 and ∞. The first order condition for the optimal investment for a family of income y is given by

$$-V'(y - w) + z(1 + g'(w)) = 0$$

Since V' is monotonic, it has an inverse. This implies that the income of the family in M who invests w must be equal to

$$V'^{-1}(z(1 + g'(w))) + w$$

The family in F who invests $g(w)$ has an income level such that $g(w)$ satisfies

$$-V'(y' - g(w)) + z(1 + \frac{1}{g'(w)}) = 0$$

In equilibrium, this family matches with the family in M who invests w. If $g(\cdot)$ is a rational expectations solution, the measure of families who are wealthier than this family from M must be equal to the measure of families who are wealthier that y'. This requires that

$$y' = \gamma\left(V'^{-1}z(1 + g'(w))\right) + w\right)$$

This yields the ordinary differential equation

$$-V'\left(\gamma\left(V'^{-1}(z(1 + g'(w))) + w\right) - g(w)\right) + z(1 + \frac{1}{g'(w)}) = 0$$

Re-arranging gives

$$g(w) = \gamma\left(V'^{-1}(z(1 + g'(w)) + w)\right) - V'^{-1}\left(z(1 + \frac{1}{g'(w)})\right)$$

Let $\phi(g', w)$ denote the expression on the right hand side of this equation. Since ϕ is monotonically decreasing in g' the inverse function $\phi^{-1}(\cdot, w)$ exists. So the market return function must satisfy

$$g'(w) = \phi^{-1}(g(w), w) \tag{4}$$

11

To see the initial condition, let y^* be the poorest family from F who successfully matches with some family from M in equilibrium (y^* satisfies $F - H(y^*) = M$). Let I^* be the highest indifference curve that the family with endowment y^* can attain when they do not match. Let $(\underline{w}, g(\underline{w}))$ be the point where this indifference curve is tangent to the indifference curve for a family in M who endowment is $\underline{y}$. The equilibrium market return function is then any solution to (4) with initial condition $(\underline{w}, g(\underline{w}))$. To ensure existence and uniqueness of the solution we need to impose additional restrictions on the problem to ensure that ϕ^{-1} satisfies the usual Lipshitz condition in g. We do not pursue these issues here.

5   A Hedonic Pricing Interpretation

The above model can be interpreted as a special case of Rosen's hedonic market model. To see this most easily, consider families in M as suppliers. Let y be the characteristic of a supplier. Let w be the level of output (premarital investment) that a supplier produces. Note that w also provides consumption value for the supplier and thus is not purely costly. $g(w)$ is the return that a son gets for supplying w. We may consider families in F as demanders. If a demander pays $\widetilde{w}$, the daughter will match with a supplier whose output is $g^{-1}(\widetilde{w})$. Unlike Rosen, demanders value paying $\widetilde{w}$. However this does not cause any analytic difficulty because a demander, that is matched, will pay a higher $\widetilde{w}$ than she is willing to pay if she is not matched. So as in the case of Rosen's firms, the demander will prefer to pay less $\widetilde{w}$ for her matched supplier if she could.

The existence of a market clearing wealth matching function $g(w)$ is useful for researchers who want to study parental investment decisions in the face of marriage market considerations. For an individual family, $g(w)$ summarizes the marriage market opportunities that that family will face. For example, Botticini and Siow (1999) assumes the existence of such a function to study the demand for dowries by individual families.

6   Small Marriage Markets

We continue to assume without further discussion that children match assortatively in investment during the matching process. It is not hard to

construct non-cooperative matching games where this occurs. One approach is to follow the approach associated with wage posting games (for example Shi (1997)) in which one side publicly announce the wealth that they bring to a match, the other side proposes to the partner they most prefer, then the respondents pick their favorite partner among those who have proposed to them. We do not pursue these details here.

It is also possible to show that the investment game among families that occurs prior to this assortative matching process always has at least one equilibrium which typically involves mixed strategies. The details of this argument can be found in the working paper.

We focus instead on the richest example in which the equilibrium can be fully characterized. Our point in doing this is twofold. First, this example is of independent interest in matching problems like co-authorship, or business partnership where the number of investors on each side of the market really is small. Secondly, this example illustrates the difficulties associated with connecting the rational expecations solution with exact equilibria in the family matching problem when the number of families is large but finite.

We consider the case where there are four families. One of the families from F and one from M have an initial endowment y_l while the other pair of families each have endowment y_h. Consider investment levels $w_l^f, w_h^f, w_l^m, w_h^m$, where, in an obvious notation the superscript f refers to children of families from F while the superscript m refers to children of families in M. In the case where the investments levels differ in the sense that for example $w_h^f > w_l^f$ the continuation equilibrium is straightforward. The wealthy child from F will propose to the wealthy child from M with probability 1 and this proposal will be accepted. Similarly for the less wealthy children. The poor F has no incentive to propose to the wealthy M because she expects the wealthy F to propose there with probability 1 and she knows that the wealthy male will always prefer her proposal.

If the wealth disparity of the families is large enough, it will never pay families to try to improve their children's match quality. In that case there will be an obvious equilibrium where families make non-cooperative investments and never mix. To avoid this we assume

Assumption

$$V\left(y_l^i - w_h^{*i}\right) + \frac{2w_h^{*i}}{z} > V\left(y_l^i - w_l^{*i}\right) + \frac{2w_l^{*i}}{z}$$

The assumption says that a poor family would be willing to raise investment to w_h^* if they believed that this would generate a match with a child from a wealthy family making the non-cooperative investment.

Then we have:

Theorem 3 *There exists a symmetric mixed strategy equilibrium in which the wealthy families both use mixture H_h while the poor families both use mixture H_l. These mixtures have the following properties:*

1. *H_l has an atom at w_l^* and H_h has an atom at w_h^* with $H_h\left(w_l^*\right) < H_l\left(w_h^*\right)$;*

2. *H_l and H_h both have smooth density functions on some interval $[w_h^*, w^{**}]$ with $H_l'\left(s\right) < H_h'\left(s\right)$ for all $s \in [w_h^*, w^{**}]$*

The proof of this theorem is constructive. The argument resembles the argument for a Bertrand competition with capacity constraints. If one family invests more than the other, that family's child will match with the wealthiest child on the other side of the market. In the mixed strategy equilibrium investments generate random returns since the investment level of the competing family is random. We choose the supports and distributions for the mixed strategies so that each family is indifferent to every investment level in the support of its equilibrium strategy. The major complication involved in this argument arises from the fact that family who invests most gets a random return equal to the first order statistic for investment levels on the other side of the market. The distribution of this order statistic is endogenous. So this apparently straightforward problem requires a fixed point argument to give a complete result.

The theorem illustrates nicely the inherent unpredictability of investment. Both kinds of families choose their bilateral Nash investment level with positive probability. However, they also use a strategy that involves investment at a level strictly above w_h^* with positive probability. It follows immediately that both families will invest, on average, strictly more than their bilateral Nash investment levels. From the position of the atoms, and the restriction on densities, it is immediate that the family with the highest endowment will invest more, on average, than the family with the low endowment. With randomization however, there is a strictly positive probability that the poor family will end up investing more than the wealthy family. Their child will

then be at the top of the wealth distribution. We interpret this as endogenous intergenerational mobility in wealth.

We have explored exact equilibria with more families, but apart from the general existence of mixed strategy equilibria, it is difficult to provide a good characterization of Nash equilibrium in investment. Unlike similar problems with discontinuous payoffs (for example Allen and Hellwig (1986)) the payoff associated with being first is itself endogenous. This makes it difficult to see what restrictions on payoffs are required to keep the equilibrium mixed strategies well behaved.

7   Discussion

There is also a close connection between the models in this paper and the directed search models of the labor market (for example Shi (1999) or Moen (1997)). To see this suppose that there is a measurable set of firms who invest in physical capital and workers who invest in human capital. Firms have different technologies parameterized by some variable $y \in \mathbf{R}$ with the marginal product of capital increasing in y. Workers differ according to a parameter y' that determined the cost at which the worker can acquire human capital. This cost is assumed to be decreasing as y' increases. Each firm has a single job to be filled and each worker wishes to fill one job. The total output produced by the firm is some increasing function of the physical capital w invested by the firm and the human capital w' invested by the worker who fills the job. Physical capital is purchased by the firm at a fixed price r while human capital is acquired by the worker according to a convex and increasing cost $e(w')$. For the moment, assume that when a firm and worker match, each receives a fixed share of the profit that is created. So if a firm who invests physical capital w is matched with a worker with human capital w', the profit of the firm is

$$\alpha f(w, w'; y) - rw$$

while the profit of the worker is

$$(1 - \alpha) f(w, w') - e(w'; y')$$

The functional form used in this example differs slightly from that used in the marriage market above, but otherwise the problems are identical. If we allow

the firms to advertize their capital stocks after they make their investments so that workers can apply to the firm that they like, all the equilibria will involve assortative matching exactly as in the marriage problem.

The hedonic value of the firms investment w is given by some function $g(w)$ that gives the human capital that will be embodied in the worker that the firm expects to be able to attract. Conversely, any worker who wants a job at a firm with physical capital w will have to provide the level $g(w)$ of human capital to get the job. In equilibrium, this hedonic value will ensure that firms and workers will invest efficiently. This is similar to the result in Moen (1997), though it generalizes that result by allowing firms and workers to differ, and by endogenizing the investments on both sides of the market. There are also some important differences. In the existing literature on directed search in labor markets, frictions generated by workers inability to coordinate their search decisions play an important role.[5] The hedonic value of any given wage that a firm offers to pay workers is then measured by the size of the queue of applicants that the firm attracts. The model here shows that when families or workers differ in equilibrium, the mixed strategy equilibria that support these frictions disappear - the matching equilibria that occur after wages are posted or capital stocks are chosen involve pure assortative matching - families use their own characteristics to coordinate their search decisions. Despite this, the hedonic interpretation in which the market responds to specific investments with a predictable return is supported.

The other major difference is that there are no side payments in the model studied here (in the labor market interpretation, firms do not offer wages but instead simply give workers an exogenously determined split of the profit). The case where workers and firms have multidimensional characteristics is certainly likely to support a hedonic interpretation, but so far models of this form have not been studied.

One of the predictions of the model studied here is certainly too strong - pure assortative matching. Clearly the model needs to be extended to allow for unobservable or match specific characteristics. The payoff to focussing on the case with perfect information is the simplicity of the model that it delivers. A synthesis of the directed search models of the kind discussed here and the random matching models that characterize the older literature

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[5]Frictions are generated by the fact that workers use mixed strategies when they choose which firms to apply to Shi (1999), or Peters (1999).

is clearly an important topic for future research.

8 Appendix

8.1 Proof of Theorem 3:

Restrict attention for the moment to families in M. The proof is constructive. Let $\underline{w}$ and $\overline{w}$ be the ex post expected level of investment of the poorest and wealthiest child in F. Let H_w and H_l denote the probability mixtures used by the wealthy and rich families in M respectively and suppose that these satisfy the hypothesis of the theorem. In other words, H_l and H_h have atoms at w_l^* and w_h^* respectively, and are otherwise smooth on some interval $[w_h^*, w^{**}]$.

Consider the wealthy family first. If it turns out that it's child has the highest investment level ex post, then he will match with the wealthiest child in F who will have expected investment $\overline{w}$. If he is poorest ex post, he will match with the poorest child in F, gaining a partner whose expected wealth is $\underline{w}$. The only tie we need to worry about occurs when the poor family invests exactly w_h^* and this is matched by the rich family. In this case we assume that the wealthy child from F proposes to the male from the poor family for sure.[6] Under these conditions, the expected payoff when the family invests $w \in [w_h^*, w^{**}]$ is

$$H_l(w)\left\{V(y_h - w) + \frac{w + \overline{w}}{z}\right\} + (1 - H_l(w))\left\{V(y_h - w) + \frac{w + \underline{w}}{z}\right\} \quad (5)$$

To support the equilibrium this must be constant along $[w_h^*, w^{**}]$ and equal to

$$V(y_h - w_h^*) + \frac{w_h^* + (1 - H_l(w_h^*))\,\underline{w} + H_l(w_h^*)\,\overline{w}}{z}$$

to induce the wealthy family in M to make the investment. If the function is constant, it's derivative should be almost everywhere 0, or

$$-V'(y_h - w) + \frac{1}{z} + H_l'(w)\frac{\overline{w} - \underline{w}}{z} = 0$$

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[6]This is for notational convenience only - our results are unaffected by the tie breaking rule. If the rich child from F randomizes in some fashion, the payoff to playing w_h^* exactly will still be smaller than the limit of the payoffs associated with playing slightly more than w_h^* and this is all that is required to support our equilibrium.

which gives

$$H'_l(w) = \frac{V'(y_h - w) - \frac{1}{z}}{\frac{\overline{w} - \underline{w}}{z}}$$

The function H_l is then determined (up to a constant) by integrating

$$H_l(w) - H_l(w_h^*) = \frac{\int_{w_h^*}^{w} \left\{ V'(y_h - s)\, ds - \frac{1}{z} \right\} ds}{\frac{\overline{w} - \underline{w}}{z}}$$

$$= \frac{V(y_h - w_h^*) - V(y_h - w) - \frac{w - w_h^*}{z}}{\frac{\overline{w} - \underline{w}}{z}} \tag{6}$$

where $H_l(w_h^*)$ is the probability with which the low wealth family invests w_l^*. This atom, and the value of w^{**} are determined below.

The poor family faces a similar problem. Let H_h be the distribution of investments by the wealthy family. The poor family's payoff is

$$H_h(w)\left\{ V(y_l - w) + \frac{w + \overline{w}}{z} \right\} + (1 - H_h(w))\left\{ V(y_l - w) + \frac{w + \underline{w}}{z} \right\}$$

and this should be constant on the interval $[w_h^*, w^{**}]$ and equal to

$$H_h(w_h^*)\left\{ V(y_l - w_h^*) + \frac{w_h^* + \overline{w}}{z} \right\} + (1 - H_h(w_h^*))\left\{ V(y_l - w_h^*) + \frac{w_h^* + \underline{w}}{z} \right\} \tag{7}$$

where $H_h(w_h^*)$ is the probability with which the wealthy family invests w_h^*.

The poor family will choose the investment level w_l^* on the equilibrium path, so the atom in $H_h(w_h^*)$ should be chosen to make the poor family indifferent between the investment levels w_l^* and w_h^* conditional on the assumption that if the poor family invests w_h^* it will match with the rich family on the other side of the market in the event of ties. To accomplish this, assign the atom $H_h(w_h^*)$ so that

$$H_h(w_h^*)\left\{ V(y_l - w_h^*) + \frac{w_h^* + \overline{w}}{z} \right\} + (1 - H_h(w_h^*))\left\{ V(y_l - w_h^*) + \frac{w_h^* + \underline{w}}{z} \right\} =$$

$$V(y_l - w_l^*) + \frac{w_l^* + \underline{w}}{z}$$

18

or

$$H_h(w_l^*) = \frac{V(y_l - w_l^*) + \frac{w_l^*}{z} - V(y_l - w_h^*) - \frac{w_h^*}{z}}{\frac{\overline{w} - \underline{w}}{z}} \qquad (8)$$

Then reasoning as above, we have that

$$H_h(w) = H_h(w_h^*) + \frac{V(y_l - w_h^*) - V(y_l - w) - \frac{w - w_h^*}{z}}{\frac{\overline{w} - \underline{w}}{z}}$$

$$= \frac{V(y_l - w_l^*) - V(y_l - w) - \frac{w - w_h^*}{z}}{\frac{\overline{w} - \underline{w}}{z}} \qquad (9)$$

Finally, there can be no atoms at the top of the distribution of investments because of the discontinuous increase in expected wealth that this creates. So the atom at $H_l(w_h^*)$ (i.e., the probability that the poor family chooses investment w_l^*) should be chosen so that the top of the supports of H_l and H_h coincide. The top of the support of H_h is given by the solution to

$$\frac{V(y_l - w_l^*) - V(y_l - w^{**}) - \frac{w^{**} - w_l^*}{z}}{\frac{\overline{w} - \underline{w}}{z}} = 1 \qquad (10)$$

This determines

$$H_l(w_h^*) = 1 - \frac{V(y_h - w_h^*) - V(y_h - w^{**}) - \frac{w^{**} - w_h^*}{z}}{\frac{\overline{w} - \underline{w}}{z}} \qquad (11)$$

Note that since the density of the distribution H_h is uniformly higher on the interval $[w_h^*, w^{**}]$ than the density of H_l, it follows that the atom $H_l^*(w_h^*)$ is strictly larger than the atom $H_h(w_l^*)$ which verifies the two properties of the distributions mentioned in the theorem. One implication of this is that the mean investment of the wealthy family exceeds the mean investment of the poor family.

Conditional on the mean payoff levels, $\overline{w}$ and $\underline{w}$ it is straightforward to show that neither family can profitably deviate from this strategy. The poor family is indifferent between investing w_l^* and any investment level in the support $[w_h^*, w^{**}]$ by construction. Investment levels between w_h^* and w_l^* guarantee a match with a partner whose expected wealth is $\underline{w}$. Since this

19

outcome is the same for every investment level on the interval $[w_l^*, w_h^*)$ the poor family's expected utility is strictly higher when they invest w_l^* than it is when they invest any amount in (w_l^*, w_h^*) by the strict concavity of V. Similarly, the quality of the poor family's match is independent of it's investment level if it tries to invest more.

References

Acemoglu, D. (1997), 'Training and innnovation in an imperfect labour market', *Review of Economic Studies* **64**, 445–464.

Allen, B. and Hellwig, M. (1986), 'Bertrand-edgeworth oligopoly in large markets', *Review of Economic Studies* **53**, 175–205.

Becker, G. (1973), 'A theory of marriage, part i,', *Journal of Political Economy* **81**, 813–846.

Bergstom, T., Blume, L. E. and Varian, H. R. (1986), 'On the private provision of public goods', *Journal of Public Economics* **29**, 25–49.

Botticini, M. and Siow, A. (1999), Why dowries, University of Toronto working paper.

Cole, H., Malaith, G. and Postlewaite, A. (1998), Efficient non-contractible investments in a finite economy, Technical report, Caress Working Paper no. 98-13, University of Pennsylvania. to appear in the Quality Rated Journal.

Cole, H., Malaith, G. and Postlewaite, A. (2000), Efficient non-contractible investments in large economies, to appear in the Journal of Economic Theory.

Felli, L. and Roberts, K. (2000), Does competition solve the holdup problem?, London School of Economics working paper.

Han, S. (2001), Pre-contractual investment competition in a large matching market, Technical report, University of Toronto. Working Paper.

Konrad, K. and Lommerud, K. E. (2000), 'The bargaining family revisited', *Canadian Journal of Economics*.

MacLeod, W. B. and Malcomson, J. M. (1993), 'Investment, holdup, and the form of market contracts', *American Economics Review* **83**, 811–837.

Moen, E. (1997), 'Competitive search equilibrium', *Journal of Political Economy* pp. 385–411.

Peters, M. (1999), 'Limits of exact equilibria for capacity constrained sellers with costly search'. to appear in Journal of Economic Theory.

Rosen, S. (1974), 'Hedonic prices and implicit markets: Product differentiation', *Journal of Political Economy* **82**(1), 34–55.

Shi, S. (1997), A model of wage differentials between firms of different sizes, Queens University, unpublished manuscript.

Shi, S. (1999), 'Frictional assignment'. Queens University ms.

Siow, A. and Zhu, X. (1998), Differential fecundity and gender biased parental investments, University of Toronto working paper.

Smith, L. (1996), The marriage model with search frictions, University of Michigan Working Paper.