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Bid-Ask Spreads and Volume:The Role of Trade Timing

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Abstract

I formulate a stylized Glosten-Milgrom model of financial market trading in which people are allowed to time their trading decision. The focus of the analysis is to understand people's timing behavior and how it affects bid- and offer-prices and volume. Assuming heterogeneous quality of information, not all informed traders choose to trade immediately but some chose to delay, although they expect public expectations to move against them. Compared to a myopic, no-timing setting, first movers with timing have *better* quality information. Contrary to casual intuition this behavior *lowers* bid-ask spreads early on and increases them in later periods. Price-variability and total volume in both periods combined decrease. A numerical analysis shows that with timing the spreads are very stable (though decreasing), and that volume is increasing over time. Moreover, with timing the probability of informed trading (PIN) increases between periods.

JEL Classification: C70, D80, D82, D84, G14.

Keywords: Microstructure, Sequential Trade, Trade timing.

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1 Introduction

One persistent finding in empirical market microstructure is that volume and spreads display intra-day patterns. These patterns have different shapes across markets and across the time periods studied, but all in all systematic patterns exist and persist,[1] the most common being that spreads decline through the day and that volume increases towards the end of the day. While there are some (theoretical) explanations for these patterns, no published paper employs a Glosten and Milgrom (1985) (henceforth GM) style formulation to study timing and bid-ask spread patterns — even though this kind of model should be the natural choice to study spreads. In this paper I will demonstrate that traders' behavior in such a GM model with strategic timing of trades naturally generates a large part of the commonly observed patterns.

In my simple framework with endogenous timing of unit trades investors not only choose whether to buy or sell, but also when to trade. The purpose of the analysis is to understand the timing behavior of individuals and that effects that this behavior has on the observables, namely prices and volume. I employ a stylized version of Glosten and Milgrom's sequential trading model with two periods, two investors, two liquidation values, and a continuum of signals. Prices are set by competitive market makers.

In equilibrium better informed investors trade early, and less-well informed trade late. This implies that people with private information delay even though they expect the public expectation to move against them (e.g. someone with favorable information expects the public expectation to rise). Moreover, compared to a no-timing scenario, fewer people trade early (and thus more delay). This behavior affects the observables by *decreases* in the bid-ask-spread early on, and by increasing the spread later on which thus goes hand-in-hand with a reduction in price variability. Total volume is lower in the setting with timing, and the pattern of volume across time is reversed: with timing, volume is low early-on and large later-on, without timing it is the reverse.

Patterns in trading variables suggest non-stationary behavior. For my study of trade-timing, I employ the most frequently used, stylized formulation of the GM sequential

[1] The most common pattern for NYSE is that spreads and volume are U- or reverse J-shaped; see, for instance, Jain and Joh (1988), Brock and Kleidon (1992), McInish and Wood (1992), Lee, Mucklow, and Ready (1993), or Brooks, Hinich, and Patterson (2003). There is some recent evidence, however, that the spread-pattern may have morphed to an L-shape after decimalization; see Serednyakov (2005). On Nasdaq spreads are L-shaped and volume is U-shaped; see, for instance, Chan, Christie, and Schultz (1995). On the London Stock Exchange, spreads are L-shaped and volume reverse-L-shaped, with two small humps during the day; see Kleidon and Werner (1996) or Cai, Hudson, and Keasey (2004). Other world markets, for instance, the Taiwan and the Singapore Stock exchanges have L-shaped spreads and reverse L-shaped volume/number of transactions; for Taiwan, see Lee, Fok, and Liu (2001); for Singapore see Ding and Lau (2001).

trading model and amend it slightly to allow non-stationarity, strategic behavior in that some people are allowed to time their actions. In said standard models traders arrive according to a random sequence, and trade exactly at the time of their arrival. Some traders are informed, having received a single private signal about the true value of the underlying asset, others are noise and trade for reasons outside of the model such as liquidity.[2] A risk-neutral, perfectly competitive market maker sets a bid-price at which she is willing to sell and an ask price at which she is willing to buy. Each of these prices anticipates the informational content of the upcoming trade, thus generating a spread between the bid- and the ask price.

In the model presented in this paper, *two* traders enter the market before the first period and they can then choose whether to trade in period one or two. This allows me to study a stylized timing decision for a very short-run setting. Loosely, one could take the two periods of trading as the morning and afternoon sessions. Also, the presence of competition between traders suggests information that has a relatively short half-life (e.g. it is based on an upcoming announcement).

Next, most GM models use a single kind of binary signal; while in my model informed traders also receive binary signals, these signals come in a continuum of precisions and an informed trader is thus associated with the precision (or quality) of his binary signal.[3] This effectively induces a continuous signal structure and allows a simple and concise characterization of the equilibrium by *marginal trading types*. These marginal types are indifferent between trading immediately and delaying.[4] Employing symmetry with respect to the prior distribution of the security values and the signal distribution (as is common in the literature), I then establish the existence of a trading equilibrium.

At the heart of the paper is the comparison of the size of bid-ask-spreads and overall volume for cases with and without timing. Since bid-ask spreads are employed in most measures of market liquidity, it is of some importance to understand how traders time their trading decision in an attempt to exploit liquidity and to manage transaction costs. As the no-timing benchmark I employ a hypothetical setting in which agents do not time their trade and instead behave purely myopically.[5]

[2]See, e.g., Easley and O'Hara (1987), or Avery and Zemsky (1998).

[3]This information structure is commonly used in informational learning models, see, for instance, Smith and Sørensen (2000).

[4]On a theoretical level, this is a 'purification' approach: standard stylized GM models use only a single signal. To study meaningful information-induced delay, I would have to include at least two kinds of signal (e.g. high and low quality; as also used in Easley and O'Hara (1987)). This, however, would invariably force me to describe behavior by mixed strategies. The continuous signal space not only allows me to circumvent this complication, but it also delivers cleaner insights (mixed strategies can be difficult to interpret).

[5]When prompted to trade, such traders simply ignore the possibility of delay. For instance, such

In equilibrium, there are two forces at play: first, informed traders believe that the market will move against them. Traders with favorable information expect the public expectation (the market) to rise, traders with unfavorable information expect the public expectation to fall.[6] Delaying for this reason can be interpreted as causing a type II error (not buying a valuable security, not selling an overpriced security). The public expectation is not, however, the measure that determines traders' payoffs — trades occur at bid- and offer-prices (which underlines to importance of using a GM formulation). This gives rise to the second force: Suppose, for the sake of the argument, that in Period 1 people with very high quality information buy or sell and that people with low quality information choose not to trade.[7] In Period 2, the market maker would know that she is only facing agents with low quality information, and so there is less of a reason to defend against the potential informational advantage of the remaining informed traders. Consequently, the bid-ask-spread in the second period would be smaller. In equilibrium there is a unique marginal type who is indifferent between trading in Period 1 and 2.

In stationary binary states Glosten-Milgrom models, the bid- and ask prices merely separate people with favorable information (who buy) from those with unfavorable information (who sell). With timing, the adjustments of the spread across time also separate people with stronger and weaker information on the same side of the market.[8]

To understand the impact of timing on the equilibrium behavior I then compare the timing equilibrium with a situation in which traders have no strategic timing considerations. The first result is that the marginal trader in the no-timing (myopic) situation who is indifferent between trading early or never, strictly prefers to delay with timing. In other words, in equilibrium the marginal trading type in a myopic setting has a *lower* signal quality than in the setting with strategic timing. Consequently, in the equilibrium with timing the average signal quality of informed traders who move early is larger, and thus traders need relatively better information in order to trade early.

It may appear at first sight that with timing the bid-ask spread in the first period should be larger relative to the no-timing situation. This intuition, however, is misleading: the size of the bid-ask spread depends on the extent of informed trading. While each informed trade is more informative, people who have this high quality signal are

traders may arrive at the market in the morning, check prices, and decide not to trade. In the afternoon, they take a second look, and then may reconsider, even though they hadn't planned on it.

[6]More technically, the public expectation follows a martingale process, but the expectation of any trader with favorable information about this public expectation follows a *sub*martingale, that of a trader with unfavorable information follows a *super*martingale.

[7]Those who trade then have either a very favorable or a very unfavorable opinion about the asset.

[8]As Malinova and Park (2008) show, different trade sizes in an anonymous market, for instance, cannot easily accomplish this feat.

rare. And so for any trade, the relative likelihood of informed trading relative to uninformed trading is smaller in the timing setting. This causes the timing bid-ask-spread in the first period to be smaller than the myopic spread. In the second period, however, the opposite occurs: the bid-ask-spread with timing is wider than without timing. Since the spread between periods declines, prices with timing are actually more stable. Moreover, with timing total volume will be smaller because the overall proportion of informed insiders who trade is smaller.

While short of an analytical result, patterns in volume are unambiguously reflected in numerical simulations.[9] Volume patterns with and without timing are strikingly different: with timing, volume is small early and large later, whereas without timing it is the other way round. The intuition behind the volume patterns can be deduced from the bid-ask-spreads: in the timing equilibrium, early- and late-moving incentives must be balanced for a marginal type. This balancing ensures that, by-and-large, spreads are similar in both periods. Also, spreads are, loosely speaking, proportional to the product of the average signal quality with the probability mass of traders who have this signal quality. Since the average quality in Period 1 is high and in Period 2 lower, the probability mass has to be low first and then high, giving rise to the volume pattern. With myopic behavior, spreads screen out a similar proportion of signal qualities in each period; since the size of remaining set declines, so will the volume. While my model is too stylized to make sweeping claims about being a full explanation for volume patterns, it tells a conclusive story as to why an increase in volume over time may arise as an equilibrium phenomenon. My results are thus an important first step in understanding such patterns theoretically.

The volume-spread simulations can straightforwardly be extended to capture another common empirical microstructure measure: the probability of informed trading, PIN. The concept was first employed by Easley, Kiefer, O'Hara, and Paperman (1996) and it estimates the probability that a trade is initiated by an informed insider. In line with the results outlined above, analytically, PIN in Period 1 is larger in the myopic case. This alone gives no testable implication, but simulations provide a new testable hypothesis: simulations show that with timing PIN actually *increases* from Period 1 to Period 2, whereas with myopic behavior, PIN *decreases* from Period 1 to Period 2.[10]

[9] These simulations were run for a quadratic and a symmetric Beta distribution over signal qualities. The computation was run in Maple; the underlying code is available upon demand.

[10] Several authors, e.g. Lin, Sanger, and Booth (1995), Madhavan, Richardson, and Roomans (1997), or Serednyakov (2005), have ventured to empirically decompose the spread into informational, order processing, and inventory costs (for NYSE) and they find that the informational cost is higher early on whereas inventory costs are high later on. Lee, Fok, and Liu (2001) find that insiders and noise traders submit more trades at the open and the close. Comerton-Forde, O'Brian, and Westerholm (2004), on

Related Theoretical Literature on Timing in Markets. There is a substantial body of literature that studies repeated trading in either Kyle-type or in rational expectations equilibria (REE) type settings. Neither of these frameworks, however, is well-suited to study bid-ask-spreads. Moreover, on a conceptual level these models capture a different market microstructure, namely (batch) auctions or order-driven markets with simultaneous moves; GM models, on the other hand, capture quote-driven markets.[11] There is also a reasonably large body of literature on the timing of investments, e.g. Chamley and Gale (1994), Gul and Lundholm (1995) Chari and Kehoe (2004), Lee (1998), Abreu and Brunnermeier (2003), but these papers are not concerned with standard market microstructure measures such as volume, the probability of informed trading, liquidity, or bid-ask-spreads.

To the best of my knowledge, there are only two published papers that extend Glosten and Milgrom's seminal model to allow endogenous timing: Back and Baruch (2004) and Chakraborty and Yilmaz (2004a). Both of these, however, model a *single* informed insider who can trade repeatedly. Back and Baruch's focus is to show that in the limit, the equilibrium from a Glosten-Milgrom setting converges to the equilibrium in a Kyle model. Chakraborty and Yilmaz analyze if the single insider can effectively manipulate the market. In my paper, the competition between two potentially informed traders is the mechanism that creates the delay/early-play trade-offs, because rational traders anticipate the direction of future prices. Studying manipulative strategies, such as those described in Chakraborty and Yilmaz (2004a), would go beyond the scope of this paper and thus manipulation plays no role here.

Finally, Smith (2000) has a similar financial market setting, but without a market maker who sets bid- and ask-prices and without the informed trade having an impact on the price. Then in Smith, an investor with private information trades early. The intuition is, of course, that someone with, say, a favorable signal expects transaction prices to increase (for him it is a sub-martingale). Casual intuition then suggests, that profits from speculation are largest early so that investors should invest rather earlier

the other hand, find that the proportion of informed traders tend to increase throughout the day. I am not aware, however, of any study that estimates intra-day patterns in PIN directly.

[11]In Kyle (1985), a market maker sets the price after observing the aggregate order flow (which consists entirely of market orders), and so this procedure rather mimics the opening sessions at NYSE or Deutsche Börse. Likewise, REE setups, most famously Admati and Pfleiderer (1988), require that traders submit (complete) demand-supply schedules and so an REE setting mimics opening sessions as held at the TSX or Paris Bourse; in an REE setup, however, spreads are not modeled explicitly. To explain patterns in volume and spreads, there are also models that are not information-based in which one determines the optimal behavior against exogenous, periodic occurrences of the supply of trading parties (see Brock and Kleidon (1992)). At the same time, one usually uses a GM sequential trading framework when modeling continuous trading.

than later. My result is somewhat surprising as it indicates the opposite: the type who is the marginal buyer without timing would *delay* the purchase with timing because he would expect the future ask-prices to *fall* — despite his favorable information. The difference between our setups are that Smith has no bid-ask-spread and that market prices in his model do not account for the (timing) behavior of the informed agent.

The paper proceeds as follows: in the next section I outline the basic model, in particular, the information structure, and I discuss some key assumptions of the model. In Section 3 I derive the equilibria for the timing and the no-timing cases. The discussion and comparison of these equilibria follows in Section 4. Their respective price-, volume- and PIN-implications are discussed in Sections 5, 6, and 7. Section 8 discussed the results and concludes. Appendix A contains some further numerical robustness checks. Appendix B provides the more technical details of the signal structure. All proofs are delegated to Appendix C.

2 The Basic Setup

I formulate a stylized version of security trading in which informed and uninformed investors trade unit lots of a single asset with competitive market makers. There are two investors who can trade either in Period 1 or Period 2; each can trade at most once. Informed investors receive information about the true state of the asset before time 1, whereas uninformed investors trade for liquidity reasons. After time 2, the true value of the security will be revealed. Short positions are filled at the true value. Each market maker posts a bid-price at which she is willing to buy the security and an offer-price ('ask') at which she is willing to sell.[12] Market makers are competitive and thus set prices at which they make zero expected profits. After receiving their information, informed traders try to predict the transaction price in Period 2 and thus determine whether it is worthwhile to submit a market order in Period 1 (at the posted price) or whether they should delay until the second period. In every period, orders are submitted simultaneously. After Period 1, all trading activity is published. No trading occurs after Period 2.

2.1 The Security, Investors, and the Trading Mechanism

Security: There is a single risky asset with a liquidation value V from a set of two potential values $\mathbb{V} = \{\underline{V}, \overline{V}\} \equiv \{0, 1\}$. The two values are equally likely.

[12]Throughout the paper I will refer to market makers as female and investors as male.

Investors: There is an infinitely large pool of investors out of which two are drawn at random before Period 1. Each investor is equipped with private information with probability $\mu > 0$; if not informed, an investor becomes a noise trader (probability $1-\mu$). The informed investors (also referred to as insiders) are risk neutral and rational.

Noise traders have no information and trade randomly. These investors are not necessarily irrational, but they trade for reasons outside of this model, such as liquidity.[13] I assume that they buy and sell in every period with equal probability, e.g. the probability of a noise trader buy in each period is $(1-\mu)/4 =: \lambda$.[14]

I will use the terms 'trader' and 'investor' interchangeably; likewise I will use the terms 'informed investor', 'informed trader' and 'insider' interchangeably.

Each investor can buy or sell *one* round lot (one unit)[15] of the security at prices determined by the market maker, or he can be inactive ('hold'). As in Glosten and Milgrom (1985), each investor can trade only once. Investors can post only market orders. The possible actions are thus $\{\{$buy in 1, hold in 2$\}$, $\{$hold in 1, buy in 2$\}$, $\{$hold in 1, sell in 2$\}$, $\{$sell in 1, hold in 2$\}$, and $\{$hold in 1, hold in 2$\}$. Insiders choose an action to maximize their expected trading profits.

Market and timing: There are two trading periods, $t = 1, 2$. Loosely, one could understand these periods as the morning and the afternoon trading sessions. Both investors enter the market before time 1 and they leave the market after time 2. There is a continuum of market makers, all are risk neutral and competitive. Since there is a continuum of market makers, the probability of submitting the order to the same market maker is zero. I assume that the market makers post identical prices so that if two traders submit the same order at the same time, they also pay the same price. As a consequence, similarly to GM, when submitting their market order investors know the price at which their order will clear; this is also in line with the usual perception that market orders have no or hardly any price risk.

[13] Assuming the presence of noise traders is common practice in the literature on micro-structure with asymmetric information to prevent "no-trade" outcomes à la Milgrom-Stokey (1982).

[14] The results in this paper are robust to noise traders who abstain from trading entirely with positive probability or who trade with different probabilities in the two periods.

[15] This single unit assumption is standard in GM models and needed when with risk neutral traders. Allowing traders to act repeatedly would be contrary to the goal of the model of removing stationarity of behavior. To see this consider a risk-neutral traders with perfect information (see below). This trader would like to trade as much and as often as he can. Thus allowing traders to act in both periods would remove the timing motive and thus lead to a stationary model. In summary, one should think of traders as being sufficiently credit constrained so that indeed they can trade at most once.

2.2 Information

The structure of the model is common knowledge among all market participants.[16] The identity of an investor and his signal are private information, but everyone can observe past trades and transaction prices. The public information at the beginning of Period 2 lists the buys and sales in Period 1 together with the realized transaction prices.

Market maker. The market maker only has public information; she observes all trades and no-trades.

Insiders' information. I follow most of the GM literature and assume that investors receive a binary signal, h or l, about the true liquidation value V. These signals are independently distributed, conditional on the true value V. In contrast to most of the GM literature, I assume that these signals come in a *continuum* of qualities. Specifically, insider i is told one of two possible, statistically true statements: 'with chance q_i, the liquidation value is High/Low (h/l)'. This q_i is the *signal quality.* The distribution of qualities is independent of the asset's true value and can be understood as reflecting, for instance, the distribution of traders' talents to analyze securities. Figure 1 illustrates the procedure according to which people are sorted into informed and noise traders, and according to which the informed receive their signals.

Formally, the signal takes either value h or l, with conditional signal distribution

pr(signal|true value)	$V = 0$	$V = 1$
signal $= l$	q_i	$1 - q_i$
signal $= h$	$1 - q_i$	q_i

In the subsequent analysis it will be convenient to combine the signal and the signal quality in a single variable, which is the trader's *private belief* $\pi_i \in (0, 1)$ that the asset's liquidation value is high $(V = 1)$. This belief is the trader's posterior on $V = 1$ after he learns his quality and sees his private signal but *before* he observes the public history.

Of course this belief is simply obtained by Bayes Rule and trivially coincides with the signal quality if the signal is h, $\pi_i = \mathsf{pr}(V = 1|h) = q_i/(q_i + (1 - q_i)) = q_i$; or if the signal is l, it is $\pi_i = 1 - q_i$. In what follows I will use the distribution of these private beliefs. Let $f_1(\pi)$ be the density of beliefs if the true state is $V = 1$, and likewise let $f_0(\pi)$ be the density of beliefs if the true state is $V = 0$. Because of the underlying signal-quality distribution, the combined conditional distributions of beliefs trivially satisfy the Monotone Likelihood Ratio Property. Appendix B fleshes out how these densities are to be obtained from the underlying distribution of qualities.

[16] The formulation of information is similar to Smith and Sørensen (2000).

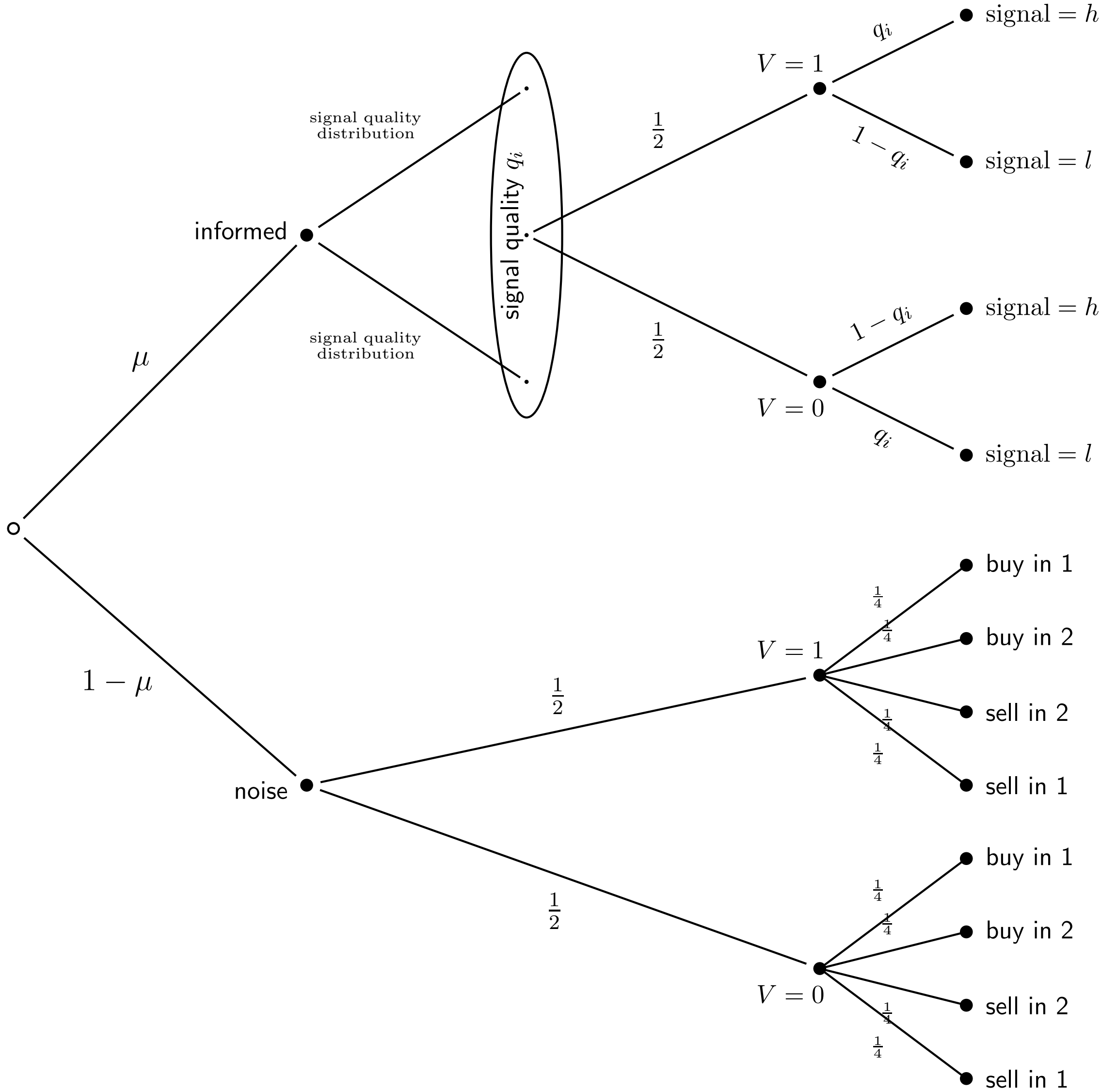

Figure 1: **Illustration of signals and noise.** This figure illustrates how signals are distributed to investors: first, for each investor it is determined whether or not this trader is informed (probability μ) or noise (probability $1-\mu$). If informed, each trader i receives a draw of the signal quality is determined according to the signal quality distribution. Depending on whether the state is high or low, the investor receives the correct signal with probability q_i and the wrong signal with probability $1 - q_i$. (Of course, the draw of the state V is identical for all agents.) If the trader is a noise trader, then he will buy or sell with equal probability in either period.

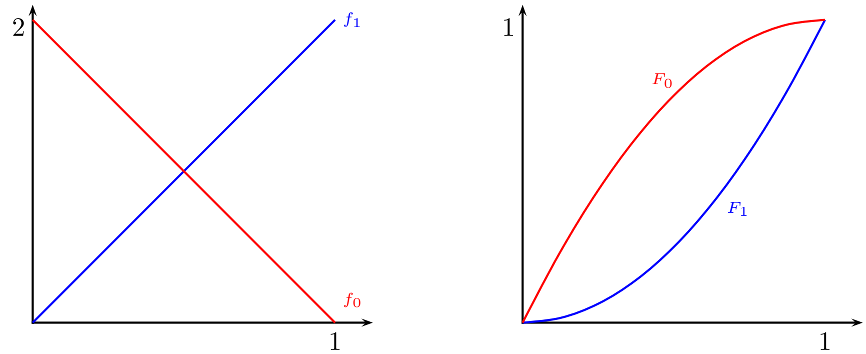

Figure 2: **Plots of belief densities and distributions.** *Left Panel:* The densities of beliefs for an example with uniformly distributed qualities. The densities for beliefs conditional on the true state being $V = 1$ and $V = 1$ respectively are $f_1(\pi) = 2\pi$ and $f_0(\pi) = 2(1 - \pi)$; *Right Panel:* The corresponding conditional distribution functions : $F_1(\pi) = \pi^2$ and $F_0(\pi) = 2\pi - \pi^2$.

Example of private beliefs. Figure 2 depicts an example where the signal quality q is uniformly distributed. Conditional densities are $f_1(\pi) = 2\pi$ and $f_0(\pi) = 2(1 - \pi)$, yielding distributions $F_1(\pi) = \pi^2$ and $F_0(\pi) = 2\pi - \pi^2$. The figure also illustrates the important principle that signals are informative: recipients in favor of state $V = 0$ are more likely to occur in state $V = 0$ than in state $V = 1$.

2.3 The Trading Equilibrium

I assume that simultaneously submitted orders clear at the same transaction price, and that investors who submit orders simultaneously do not observe each other's actions. Consequently, as in GM, when posting the order, an investor knows the price at which his order will transact.

Equilibrium concept. The game played is one of incomplete information, the appropriate equilibrium concept is thus the Perfect Bayesian Nash equilibrium. Henceforth an equilibrium refers to a profile of actions for each type of insider that constitutes a Perfect Bayesian equilibrium of the game. The price set by the market maker given insiders' action profiles is referred to as the equilibrium price. I will restrict attention to symmetric equilibria, where all insiders use the same threshold decision rule.

The pricing rule: Market makers are competitive and make zero expected profits. At each time t they post a bid- and an ask-price; the bid-price bid_t is the price at which they buy one unit of the security, the ask-price ask_t is the price at which they sell one unit. With zero expected profits, for that trade it must hold that

$$\mathsf{ask}_t = \mathsf{E}[V|\text{buy at } \mathsf{ask}_t, \text{ public info at } t], \quad \mathsf{bid}_t = \mathsf{E}[V|\text{sale at } \mathsf{bid}_t, \text{ public info at } t].$$

This equilibrium pricing rule is common knowledge. Since, ex post (upon observing the transaction price) an insider is better informed than the market maker, the latter makes an expected loss on trades with informed agents. To break even, she must profit in expectation on trades with liquidity investors.

The informed investor's optimal choice: An informed investor receives his private signal and observes all past trades, and he can only trade in either Period 1 or Period 2. In Period 2 he submits a buy order if he hasn't traded in Period 1 and if, conditional on his information, the expected transaction price is at or below his expectation of the asset's liquidation value; conversely for a sell order. He abstains from trading if he expects to make negative trading profits. I assume the tie-breaking rule that, in the case of indifference, agents always prefer to trade.

For behavior in Period 1 I will look at two settings. In the first, the timing case, the insider faces two questions: first, is trading at the current price profitable, i.e. is the current ask-price below his expectation (or the bid-price above it)? Second, if delaying by a period, would he expect to make a higher profit tomorrow? In the second, myopic setting the insider asks only whether or it is profitable to trade at the current prices. If so, then he trades.

For now let me restrict attention to monotone decision rules; in the next section I will show that this is indeed justified.[17] Namely, I assume that an insider uses a 'threshold' rule: he buys if his private belief π_i is at or above the time-t buy threshold π_b^t, $\pi_i \geq \pi_b^t$. He sells if $\pi_i \leq \pi_s^t$. And he abstains from trading otherwise.

In the subsequent discussion I will focus mainly on the buying decision; the selling decision follows analogously. To find the equilibrium, I proceed by backward induction: Suppose that in Period 1, the marginal buying type was π_b^1 and the marginal selling type was π_s^1, and suppose that all traders with beliefs higher than π_b^1 bought in Period 1 and all with beliefs smaller than π_s^1 sold. I then find the marginal trading types in Period 2, π_b^2, π_s^2; a trader who holds either of these beliefs is indifferent between trading in Period 2 and not trading at all. Everyone with belief higher than π_b^2 buys now, everyone with belief smaller than π_s^2 sells. The second step differs for the timing and the myopic case. With timing, given π_b^2, π_s^2, the marginal types π_b^1, π_s^1 are indifferent between trading in Period 1, and Period 2, given the Period 2 marginal types and given that bid- and offer-prices assume that they are the marginal types in Period 1. Without timing, the anticipated behavior of people in Period 2 is, of course, irrelevant.

I discuss the key assumptions (symmetry, unit lot trades, and the trading mechanism)

[17] The monotonic outcome is, of course, intuitive because all expectations are monotonic in signals, irrespective of equilibrium behavior.

and the robustness of the results to changes in these assumptions in Section 8.

Numerical Simulations. While some results on spreads and volume can be obtained analytically, others can only be obtained through simulations.[18] These simulations I present in what follows are based on two classes of quality distributions.[19] The first has a quadratic quality density:[20]

$$g^{\mathsf{quadratic}}(q) = \theta \left(q - \frac{1}{2}\right)^2 - \frac{\theta}{12} + 1, \quad q \in [0, 1]. \tag{1}$$

The feasible parameter space for θ is $[-6, 12]$.[21] Note that this class includes the uniform density ($\theta = 0$). The second distribution is the symmetric Beta distribution:

$$g^{\mathsf{Beta}}(q) = \frac{\Gamma(2\theta)}{\Gamma(\theta)^2} \, (q(1-q))^{\theta-1}, \quad q \in (0, 1),\ \theta > 0. \tag{2}$$

For $\theta = 1$ this is the uniform distribution; for $0 < \theta < 1$ the density is U-shaped, for $\theta > 1$ it is hill-shaped.

The quadratic quality distribution is either convex or concave on its support, whereas the Beta distribution is either convex-concave-convex (for $\theta > 1$ for Beta) or concave-convex-concave (for $\theta < 1$).

3 Equilibrium Analysis

The equilibrium will be described by the marginal types that buy and sell in each period. To construct the equilibrium, I proceed by backward induction: I first describe the trading equilibrium in Period 2, conditional on behavior in Period 1. I then use the anticipated Period 2 equilibrium prices to describe behavior in Period 1. In what follows, I will focus on the 'buy' decision; the 'sell' decision is analogous.

I will identify the marginal trading types for the timing case by a superscript T, and those for the myopic case by a superscript M. If superscripts are omitted, then the

[18] An analytical result could be obtained, for, say uniformly distributed qualities, but this special case is subsumed by the simulations provided here.

[19] I ran further simulations with a third class of distributions, power-4 polynomials. Since the insights coincided with those from the quadratic and Beta distribution, I am only reporting results from the most salient distributions here.

[20] It is computationally convenient to use a quality distribution over $[0, 1]$ instead of $[.5, 1]$; details are in Appendix B.

[21] This parameter set is exhaustive for quadratic distributions on $[0, 1]$ that are also symmetric around $1/2$. See Appendix B for a detailed description of the theory behind the signal distributions.

respective variable refers to a marginal trading type, irrespective of the timing setting. The probability of a buy in period $t = 1, 2$ and state $i \in \{0, 1\}$ will be denoted by β_i^t; similarly for σ_i^t which signifies a sale and γ_i^t for holds. Since we are considering threshold rules, these probabilities will depend on the marginal buying and selling types, but to simplify the exposition, I shall omit identifiers.

For now assume that everyone with private belief larger than π_b^1 buys in Period 1, and that everyone with private belief larger than π_b^2 and smaller than π_b^1 buys in Period 2. For the probability of the high value, I will write $p_t = \mathsf{pr}(V = 1|\text{public information at time } t)$.

Consider investor i with private belief $\pi_i = \pi$. This investor will buy in Period 2 if he has not traded in Period 1 and if his expectation exceeds the Period 2 ask price. The marginal trader who is indifferent between buying and abstaining in Period 2, must have the buy-threshold private belief π_b^2 that solves

$$\mathsf{E}\left[V|\pi_b^2, \text{public information at time } t = 2\right] = \mathsf{ask}_2(\pi_b^2 \text{ is the marginal buying type}). \quad (3)$$

Note that the Period 2 reasoning for the myopic and the timing case coincide.

In Period 1 in the *myopic* case an investor with belief π will buy if his expectation exceeds the Period 1 ask price. The marginal trader who is indifferent between buying and abstaining in Period 1 must have belief $\pi_b^{1,\mathsf{M}}$ which solves

$$\mathsf{E}\left[V|\pi_b^{1,\mathsf{M}}, \text{public information at time } t = 1\right] = \mathsf{ask}_1(\pi_b^{1,\mathsf{M}} \text{ is the marginal buying type}). \quad (4)$$

In Period 1 in the *timing* case, the marginal buyer in Period 1 must be indifferent between buying in Period 1 at the Period 1 offer price and delaying and then buying at the Period 2 offer price.[22] Threshold $\pi_b^{1,\mathsf{T}}$ must then solve

$$\mathsf{E}[V|\pi_b^{1,\mathsf{T}}] - \mathsf{ask}(\pi_b^{1,\mathsf{T}}) = \mathsf{E}\left[\mathsf{E}[V|\pi_b^{1,\mathsf{T}}, \text{public information at time } t = 2] - \mathsf{ask}_2(\pi_b^2(\pi_b^{1,\mathsf{T}}))\Big|\, \pi_b^{1,\mathsf{T}}\right].$$

This can be simplified immediately by applying the Law of Iterated Expectations, which yields $\mathsf{E}[V|\pi] = \mathsf{E}[\mathsf{E}[V|\pi, \text{public information at time } t = 2]|\pi]$ so that private expecta-

[22] In my setting a buyer would never change from buying to selling. The reason is that with two values, there always exists a 'neutral news' signal, which thus coincides with the public expectation. Moreover, in the signal quality setup, expectations are ordered in signals, so that one's expectation is either always above or always below the public expectation. And this precludes a switch from buying to selling.

tions can be dropped from the above equation. Thus π_b^1 solves

$$\mathsf{ask}_1(\pi_b^{1,\mathsf{T}} \text{ is the marginal buying type}) = \mathsf{E}\left[\mathsf{ask}_2(\pi_b^2(\pi_b^{1,\mathsf{T}}) \text{ is the marginal buying type})\middle|\, \pi_b^{1,\mathsf{T}}\right]. \tag{5}$$

In what follows I first argue for the existence of a unique threshold for the Period 2 problem, equation (3). I then show the existence of a solution for the myopic case for Period 1, equation (4), and finally for the timing case for Period 1, equation (5).

Step 1: The Insider's Trading Decision in Period 2.

The Period 2 ask price is given by

$$\mathsf{ask}_2 = \frac{\beta_1^2 p_2}{\beta_1^2 p_2 + \beta_0^2(1-p_2)}. \tag{6}$$

In contrast to standard sequential trading models, the probabilities of noise trading and informed trading change between periods. If a trader did not act in Period 1, then the conditional probability that this trader buys in Period 2 is

$$\beta_i^2 = \left[\lambda + \mu(F_i(\pi_b^1) - F_i(\pi_b^2))\right]/\gamma_i^1.$$

Assuming for now that thresholds in Period 1 are symmetric (I will show this below), it follows that $\gamma_1^1 = \gamma_0^1$, and so these γ_i^1 cancel in the ask price in (6). Moreover, the probabilities of holds, γ_i, cancel from public beliefs.

Consider investor i with private belief $\pi_i = \pi$. He computes expectations of the asset's value conditional on the public and his private information. First note that the market maker and an informed trader i who delays interpret the behavior of the other trader $-i$ in the same way. The reason is that signals are conditionally independent and thus conditional on the true value, $\mathsf{pr}(-i \text{ action in Period } 1|V, S_i) = \mathsf{pr}(-i \text{ action in Period } 1|V)$.

When forming the Period 2 public belief p_2, the market maker also has to account for the information that is revealed by i not trading. Here trader i has an informational advantage — but since the probability of holds, γ_i, cancels from public beliefs, this will not affect the solution. In other words, after the first period, all of trader i's informational advantage over the market maker is contained in i's private belief π. One can now expand and simplify equation (3) to

$$\frac{\pi_b^2 p_2}{\pi_b^2 p_2 + (1-\pi_b^2)(1-p_2)} = \frac{\beta_1^2 p_2}{\beta_1^2 p_2 + \beta_0^2(1-p_2)} \quad \Leftrightarrow \quad \pi_b^2 = \frac{\beta_1^2}{\beta_1^2 + \beta_0^2}. \tag{7}$$

Hence, in any equilibrium in Period 2, the threshold decision rules are independent of the public belief about the asset's liquidation value, p_2. In other words, *actions* in Period 1 do not affect actions in Period 2 — however, the marginal trader threshold π_b^2 does depend on the marginal trader threshold π_b^1.

Solving for the thresholds. Symmetry ensures that an investor is equally likely to buy when the liquidation value is high as he is to sell when the liquidation value is low: $\beta_1^2 = \sigma_0^2$, and $\beta_0^2 = \sigma_1^2$. For fixed π_b^1, one can then solve the second equation in (7) for π_b^2.

Monotonicity of the insider's decision rule. Thus far I have focused on the indifference thresholds. I will now argue that an insider's optimal action indeed increases in his private belief. Namely, for a given pair of marginal trading types $\pi_s \leq \pi_b$ any investor with private beliefs above π_b prefers to post a buy order, any investor with private beliefs below π_s prefers to post a sell order, and any investor with private belief in (π_s, π_b) refrains from trading.

The argument is, of course, quite simple. The signal quality setup trivially ensures that beliefs satisfy the monotone likelihood ratio property (and thus First Order Stochastic Dominance; see Appendix B). Expectations are then monotonic in beliefs: for $\pi_i > \pi_b^t$, $\mathsf{E}_t[V|\pi_i] > \mathsf{E}_t[V|\pi_b] = \mathsf{ask}_t$, and consequently profits from buying, $\mathsf{E}_t[V - \mathsf{ask}_t|\pi,\ i \text{ buys}]$, increase in π. Analogously for $\pi < \pi_s^t$.

Existence and Uniqueness. Trades in this setting are always informative[23] and so a buy order is a signal in favour of the high liquidation value $V = 1$, and a sell order is a signal in favour of the low liquidation value $V = 0$. I can now state the existence and uniqueness theorem for Period 2 equilibrium prices.

Theorem 1 (Symmetric Equilibrium in Period 2: Existence and Uniqueness) *Assuming thresholds are symmetric in Period 1, there exists a unique symmetric equilibrium with monotone decision rules in Period 2. Namely, for any $\pi_s^1, \pi_b^1 \in (0,1)$, there exist unique $\{\pi_s^2, \pi_b^2\}$ such that $0 \leq \pi_s^1 < \pi_s^2 \leq \pi_b^2 < \pi_b^1 \leq 1$, any investor with private belief $\pi \geq \pi_b^2$ buys, any investor with private belief $\pi \leq \pi_s^2$ sells, any investor with $\pi \in (\pi_s^2, \pi_b^2)$ does not trade, and thresholds are symmetric, $\pi_b^2 = 1 - \pi_s^2$.*

The intuition for existence is as follows: the ask-price is a function of the threshold belief, and it is hill-shaped, whereas the private expectation of agents is monotonic. If,

[23] To see that any trade is informative, consider the following argument: A trade is only uninformative if the marginal traders' beliefs would be either 0 or 1 for both buying and selling. So suppose that $\pi_s = \pi_b = 0$ in which case all insiders buy. A trade then reveals no information, and the market maker would set the price to equal the prior expectation 1/2. But then any insider with a private belief below 1/2 would post a sell-order, a contradiction. The same argument applies when $\pi_s = \pi_b = 1$.

hypothetically, the buying threshold were 1/2, then the ask-price is still bounded away from 1/2 whereas the private threshold expectation is 1/2. Likewise, if, hypothetically, the threshold were 1, then the ask price would be 1/2 (it is uninformative) whereas the private expectation of the marginal buying type is 1. Given continuity, the two intersect. The case for symmetry was made already, and uniqueness straightforwardly stems from the monotone likelihood ratio property of the underlying distributions. The remaining details are in the appendix.

Step 2a: Establishing a Myopic Benchmark for Period 1.

The decision in Period 2 is not affected by timing considerations because the game ends after that period. Consequently, a holder of the threshold belief is indifferent between trading immediately or never.

The same reasoning applies to a myopic trader in Period 1: In a myopic equilibrium he is indifferent between trading immediately and never. The market maker is aware of this behavior and sets prices accordingly. Equation (4) can now be reformulated analogously to equation (7)

$$\pi_b^{1,\mathsf{M}} = \frac{\beta_1^1}{\beta_1^1 + \beta_0^1}. \tag{8}$$

Theorem 2 (Existence of a Symmetric Myopic Equilibrium in Period 1) *There exists a unique symmetric myopic equilibrium with monotone decision rules in Period 1. Namely, there exist unique* $(\pi_s^{1,\mathsf{M}}, \pi_b^{1,\mathsf{M}})$ *such that* $0 \leq \pi_s^{1,\mathsf{M}} \leq \pi_b^{1,\mathsf{M}} \leq 1$*, any myopic investor with private belief* $\pi \geq \pi_b^{1,\mathsf{M}}$ *buys, any myopic investor with private belief* $\pi \leq \pi_s^{1,\mathsf{M}}$ *sells, any investor with* $\pi \in (\pi_s^{1,\mathsf{M}}, \pi_b^{1,\mathsf{M}})$ *does not trade, and* $\pi_b^{1,\mathsf{M}} = 1 - \pi_s^{1,\mathsf{M}}$.

The proof and its intuition is analogous to that of Theorem 1 and omitted.

Step 2b: The Insider's Period 1 Trading Decision *with* Timing.

I now determine the marginal buyer in Period 1 when people take into account that they can trade at either time 1 or 2. Since private expectations are monotonic in the private belief π, it holds for $\pi > \pi_b^{1,\mathsf{T}}$ that $\mathsf{ask}_1(\pi_b^{1,\mathsf{T}}) < \mathsf{E}[\mathsf{ask}_2(\pi_b^2(\pi_b^{1,\mathsf{T}}))|\pi]$ and the reverse inequality is true for $\pi < \pi_b^{1,\mathsf{T}}$. The task is thus to find $\pi_b^{1,\mathsf{T}}$ that solves (5). For now let me assume again that the threshold rule in Period 1 is indeed symmetric (so that I can employ the solution for Period 2, which is based upon symmetry in Period 1).

From the perspective of a trader, between today and tomorrow, one of three events happens: the other trader either buys, or sells, or does not trade. Figure 3 illustrates

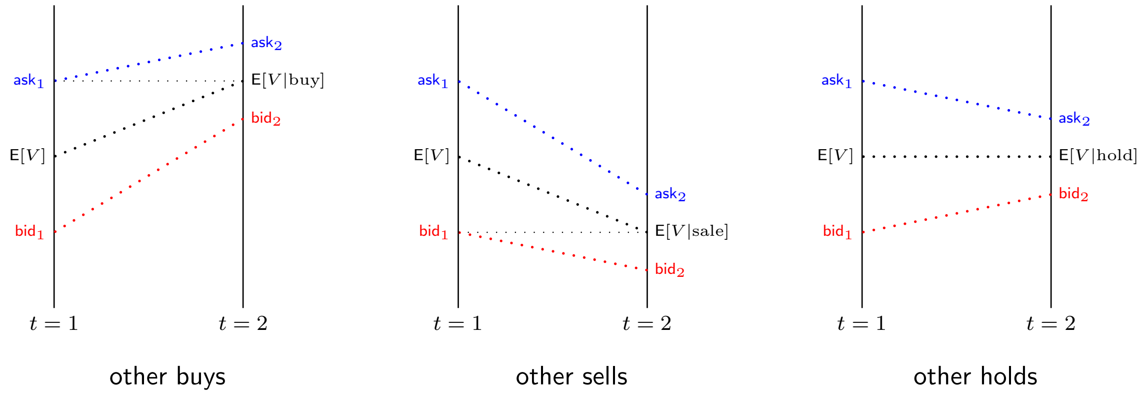

Figure 3: **Development of prices given the other trader's action.** If the other trader buys, all prices increase (and the public expectation coincides with the Period 1 ask-price); if the other sells, all prices decline (and the public expectation coincides with the Period 1 bid-price). When the other does not trade, the bid-ask-spreads get 'tighter'. These prices are, of course, illustrative; they depend on the Period 1 and 2 trading thresholds, which have to be computed in equilibrium.

the development of prices, given this action. Attaching the respective probabilities to these prices, the expected price tomorrow is

$$\mathsf{E}[\mathsf{ask}_2|\pi_b^{1,\mathsf{T}}] = \mathsf{pr}(\text{buy}|\pi_b^{1,\mathsf{T}})\ \mathsf{ask}_2(\text{buy}) + \mathsf{pr}(\text{sale}|\pi_b^{1,\mathsf{T}})\ \mathsf{ask}_2(\text{sale}) + \mathsf{pr}(\text{hold}|\pi_b^{1,\mathsf{T}})\ \mathsf{ask}_2(\text{hold}).$$

Expressing the probabilities and the prices explicitly, and simplifying and rearranging (details are in the Appendix), I use the above to rewrite equation (5) as

$$\frac{\beta_1}{\beta_0+\beta_1} - \pi_b^{2,\mathsf{T}} = \left(\frac{\pi_b^{2,\mathsf{T}}\beta_1}{\beta_1\pi_b^{2,\mathsf{T}} + \beta_0(1-\pi_b^{2,\mathsf{T}})} - \frac{\pi_b^{2,\mathsf{T}}\beta_0}{\beta_0\pi_b^{2,\mathsf{T}} + \beta_1(1-\pi_b^{2,\mathsf{T}})} \right) (\pi_b^{1,\mathsf{T}} - \pi_b^{2,\mathsf{T}})(\beta_1 - \beta_0). \tag{9}$$

The left hand side is the difference of today's ask-price and tomorrow's ask price, conditional on there being no trade. Absent trade, this is the benefit of delay induced by the market maker adjusting the spread.

The first term in brackets on the right hand side is, from the perspective of the insider with $\pi_b^{1,\mathsf{T}}$, the difference of the price in the worst case (there is a buy, so that the price rises) minus the price in the best case (there is a sale, prices drop). The difference $\beta_1 - \beta_0$ can be seen as the probability of a type II error, i.e. one would not be buying an asset that turns out to be valuable. Difference $\pi_b^{1,\mathsf{T}} - \pi_b^{2,\mathsf{T}}$ is the excess confidence of the marginal trading type $\pi_b^{1,\mathsf{T}}$ in Period 1 over that of the marginal trading type π_b^2 in Period 2. Taken together, the right hand side is the maximal cost of delay, given there is a trade, multiplied with the excess confidence and the probability of a type two error.

The equation for sales is analogous.

Theorem 3 (Existence of the Period 1 Timing Thresholds)
There exist thresholds $\pi_b^{1,\mathsf{T}}, \pi_s^{1,\mathsf{T}}$ *that solve (9) and* $\pi_b^{1,\mathsf{T}} = 1 - \pi_s^{1,\mathsf{T}}$*.*

To prove this theorem one needs to show that there exists a solution to equation (9). The proof then first argues that when setting $\pi_b^{1,\mathsf{T}} \equiv \pi_b^{1,\mathsf{M}}$, then the left hand side of (9) is larger than the right hand side. Likewise, when $\pi_b^{1,\mathsf{T}} \equiv 1$, then the right hand side is larger than the left hand side (it is 0, the LHS is negative). Since both the left and the right hand sides are continuous functions of $\pi_b^{1,\mathsf{T}}$, a solution exists.

4 Comparison of Marginal Trading Types

To compare the marginal trading types I will first derive two useful properties of the second period thresholds. First, the buy-threshold maximizes the ask price[24] and second, the Period 2 buy-threshold is an increasing function of the Period 1 buying threshold. Together these results are used to show that the timing buy-thresholds are always larger than the myopic ones; by analogy the opposite holds for the sell-threshold.

Proposition 1 (Thresholds maximize the Bid-Ask-Spread)
The Period 2 ask-price as a function of the marginal buying type is maximized at the equilibrium buying threshold π_b^2*. The same holds for the myopic Period 1 ask-price which is maximized by the equilibrium buy-threshold* $\pi_b^{1,\mathsf{M}}$*.*

The above result is quite intuitive: The ask price is set so that it averages signal qualities over a range. The trader who is indifferent between trading and abstaining is the marginal buying trader. So intuitively, in equilibrium the average quality (plus noise) must coincide with the marginal quality; and as in many economic problems this occurs when the average (i.e. the ask price as a function of the marginal trader's belief) is maximal.

Proposition 2 (Period 2 Bid-Ask-Spread Increases in Period 1 Thresholds)
The second period ask-price ask_2 *increases in the first-period buying threshold; the second period bid-price* bid_2 *decreases in the first-period selling threshold.*

[24] This result is similar to one in Herrera and Smith (2006) who derive it in a different context; they do not use it to study trade-timing. They kindly allowed me to study their private notes; I attempted a proof for my setting after I observed their result and I do not claim novelty; the proof techniques differ. The intuition for their result and mine coincides.

This proposition implies that the larger the fraction of informed traders who delay in Period 1, the larger the bid-ask-spread in Period 2; this result is irrespective of whether or not behavior is myopic.

I will now proceed to compare the marginal trading types and I will focus on the buy-side of the market. The marginal myopic buying threshold types in Period 1 and 2 are labeled $\pi_b^{1,\mathsf{M}}, \pi_b^{2,\mathsf{M}}$, the timing ones $\pi_b^{1,\mathsf{T}}, \pi_b^{2,\mathsf{T}}$.

Proposition 3 (Timing Marginal Trading Types are Larger)
Compared to the myopic scenario, with timing,
(a) the Period 1 buying-threshold is larger, $\pi_b^{1,\mathsf{T}} \geq \pi_b^{1,\mathsf{M}}$*,*
(b) the Period 2 buying-threshold is larger, $\pi_b^{2,\mathsf{T}} \geq \pi_b^{2,\mathsf{M}}$*.*

To see the first point, observe that when employing threshold $\pi_b^{1,\mathsf{M}}$ in expression (9), the left hand side of (9) becomes $\pi_b^{1,\mathsf{M}} - \pi_b^{2,\mathsf{T}}$. In the Proof of Theorem 3 I then argue that

$$\pi_b^{1,\mathsf{M}} - \pi_b^{2,\mathsf{T}} > \left(\frac{\pi_b^{2,\mathsf{T}} \beta_1}{\beta_1 \pi_b^{2,\mathsf{T}} + \beta_0 (1 - \pi_b^{2,\mathsf{T}})} - \frac{\pi_b^{2,\mathsf{T}} \beta_0}{\beta_0 \pi_b^{2,\mathsf{T}} + \beta_1 (1 - \pi_b^{2,\mathsf{T}})} \right) (\beta_1 - \beta_0)(\pi_b^{1,\mathsf{M}} - \pi_b^{2,\mathsf{T}}).$$

To see the inequality indeed goes this way, observe that the term $\pi_b^{1,\mathsf{M}} - \pi_b^{2,\mathsf{T}}$ cancels, and the first and second terms on the right hand side are smaller than 1; so the direction of the inequality is true. Applying the same interpretation as before, the price advantage of delay (the left hand side) is larger than the cost (measured by the worst case-best case price difference multiplied with the probability of a type-II error, adjusted for excess confidence). This implies that when taking the delay option into account, the marginal myopic buying type strictly prefers to delay.

Part (b) follows immediately from Proposition 2.

5 Bid-Ask Spreads and Price Variability

A major objective of Glosten-Milgrom type sequential trading models is to understand the role and the size of bid-ask-spreads, $\mathsf{ask}_t - \mathsf{bid}_t$. As the above analysis indicates, the equilibrium buying threshold with timing is always larger than without timing. At first sight, it appears that this should lead to larger bid-ask-spreads, because market makers are now dealing with informed traders that on average have better information than in a myopic equilibrium. Trading with people who have better information is costly, and so to defend themselves, one might think, market makers need to increase the spread. It turns out, that this intuition is incomplete.

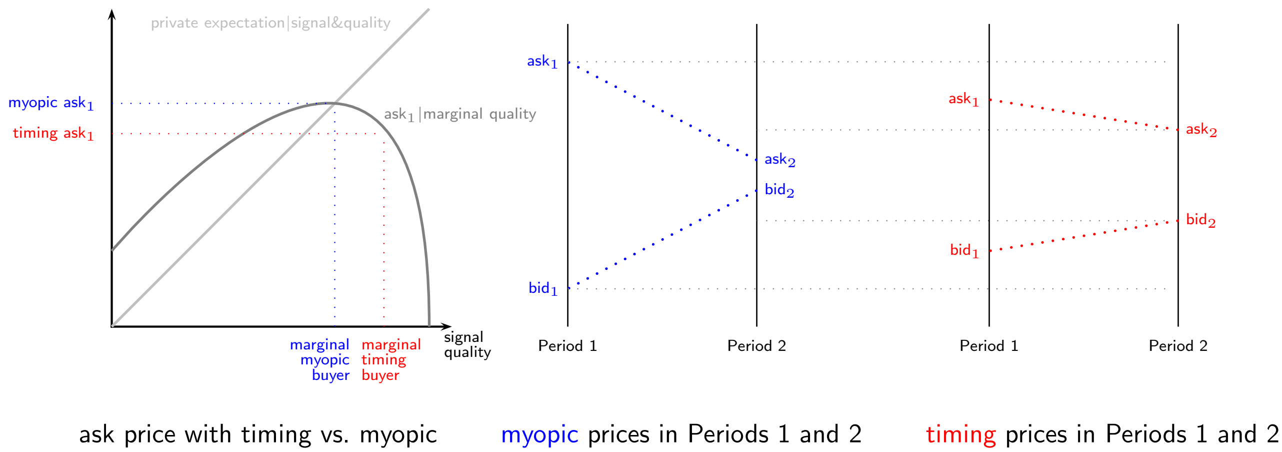

Figure 4: **Illustration of the Comparative Result: The Development of Spreads with and without Timing.** The left panel plots the private expectation of a trader who received a favorable signal as a function of this trader's signal quality (in the formal arguments in the text I use private beliefs and not signal qualities, but as explained before, for insiders with high signals, these two figures coincide). The panel also plots the ask price as a function of the marginal buying type's signal quality, i.e. the curve assumes that everyone with quality larger than the marginal quality trades. The figure then indicates that the myopic threshold maximizes the ask price. The marginal buyer with timing has a higher signal quality, and this implies that the timing ask-price is *lower* than the myopic ask-price. The middle panel plots the myopic spreads in Period 1 and 2. The right panel plots the timing spreads in Period 1 and 2. The spread with timing is tighter in Period 1 and wider in Period 2. This then implies a lower price-variability as the picture clearly indicates.

When comparing the bid-ask-spread in Period 1 with that in Period 2 I assume that in both cases the probability of each value $V = 0, 1$ is $1/2$, for this allows the cleanest comparison.

Proposition 4 (Bid-Ask-Spreads and Price Variability)

(*a*) *Timing bid-ask-spreads relative to myopic bid-ask-spreads are* smaller *in Period 1 and* larger *in Period 2.*

(*b*) *Bid-ask-spreads decline from Period 1 to Period 2.*

(*c*) *Price variability decreases with timing.*

Without timing, the Period 1 thresholds signify the maximal ask and the minimal bid price; consequently, any other threshold implies that the ask-price is smaller and the bid-price larger. The Period 2 result follows by Proposition 3. The results in (*b*) and (*c*) are straightforward consequences of the differences in the size of the spreads: Figure 4 illustrates the point.

Proposition 4 outlines the dynamic behavior of the bid-ask spread: myopic spreads are larger early on and smaller later-on than timing spreads. With timing the market maker sets a spread that makes one type ($\pi_b^{1,\mathsf{T}}$) indifferent between trading in Period 1 and 2. Thus with timing spreads should not vary dramatically between periods.

The following observations are based on numerical computations for the quality distribution in expressions (1) and (2) confirm this intuition.

Numerical Observation 1 (Spreads) *Without timing the change in the size of the spreads is larger than with timing.*

Figure 5 plots these Period 1 and Period 2 spreads for both the myopic (right panels) and the timing case (left panels). The top row plots spreads for the quadratic quality the feasible parameter space for θ; the middle row for hill-shaped Beta quality ($\theta \geq 1$); and the bottom row for U-shaped Beta-quality ($\theta < 1$). With the quadratic quality distribution, the timing spreads are almost identical for both periods. With the Beta distribution timing spreads are not as similar across period as with the quadratic distribution, but the change in timing-spreads between periods is still much smaller than with myopic-spreads.

Bid-ask spreads are the most basic form of transaction costs. Their size typically depends on market liquidity — the more liquid the market, the smaller the spread. Thus empirically one usually measures liquidity by the size of bid-ask-spreads.[25] Ceteris paribus, a smaller spread indicates that there is an *absolute* increase in liquidity trading. The above result highlights a second feature: changes in the size of the bid-ask spread can also account for a change in the *relative* importance of liquidity trading, i.e. in the ratio of informed to uninformed trading. While with timing, 'early' on the average quality of informed trading is larger than in the myopic case, the probability that the market maker actually encounters such an informed trader is smaller.

6 The Dynamics of Volume

With two players a natural proxy for (average) volume is the probability that a given market participant trades, i.e. the probability of a buy plus the probability of a sale. Since the model is symmetric, this is, of course, the same as $\mathcal{V}_t = \beta_1^t + \beta_0^t$, where in abuse of notation I shall use $\beta_i^2 = \lambda + \mu(F_i(\pi_b^1) - F_i(\pi_b^2))$. I can then show:

[25] Aitken and Comerton-Forde (2003) contains an extensive literature review on measures of liquidity.

Proposition 5 (Total Volume) *Compared to the myopic scenario, with timing,*
(a) Period 1 volume is lower and
(b) total average volume measured across both periods is lower.

This follows because total volume depends on the fraction of informed investors who trade and if thresholds are more extreme, as is the case with timing, then volume is smaller.

To understand intraday volume patterns it is useful to understand how volume changes from Period 1 to Period 2. Simplifying and rearranging it is straightforward to see that

$$\mathcal{V}_1 - \mathcal{V}_2 \text{ has the same sign as } 1 - G(\pi_b^1) - (G(\pi_b^1) - G(\pi_2^b)). \tag{10}$$

In other words, the difference of volume in Period 1 and 2 is proportional to the difference of the probability that an informed investor trades in Period 1 or 2. For instance, when quality is uniformly distributed, the change in volume between periods from (10) depends on the sign of $(1+\pi_b^2)/2-\pi_b^1$. Consequently, volume in Period 1 is larger than in Period 2 if and only if the average belief of all people who trade exceeds the marginal belief of people trading in Period 1.

The bid-ask-spread in each period is proportional to the average signal quality of trading agents multiplied with the probability that traders with this signal quality are actually present, since

$$\beta_1^1 = \lambda + \mu \cdot (1 - F_1(\pi_b^1)) \text{ has the same sign as } \int_{\pi_b^1}^1 q \cdot g(q)\, dq = (1 - G(\pi_b^1))\, \mathsf{E}[q | q \geq \pi_b^1].$$

With timing, the market maker sets prices to ensure that the spread does not change dramatically between periods. Since the average signal quality between Period 1 and 2 must decline, the probability of someone trading must increase in Period 2 relative to Period 1. As a consequence, volume increases in from Period 1 to Period 2.

In the myopic case, on the other hand, the spread intuitively screens out a, loosely, equally large proportion of the remaining quality types in every period. Since there is simply a larger fraction of types in Period 1 than in Period 2, volume will be decreasing from Period 1 to Period 2.

Numerical Observation 2 (Early vs. Late Volume) *In the myopic case volume decreases from Period 1 to 2, whereas with timing volume rises from Period 1 to 2.*

Figure 6 has six panels: the left panels plot the Period 1 and Period 2 volume proxy for the timing case, the right panels plot the Period 1 and Period 2 volume for the myopic case. The top row uses the quadratic quality distribution from (1) over the feasible parameter space for θ, with the level of insider trading $\mu \in [0.1, 0.9]$; the middle row plots volumes for the beta-distribution from (2) for parameter $\theta \in [1, 10]$ and $\mu \in [.25, .75]$;[26] and the bottom row plots volume for beta-distribution from (2) for parameter $\theta \in [1/10, 1]$ and $\mu \in [.25, .75]$. In all cases, with timing the Period 1 volume is smaller than the Period 2 volume, and the reverse holds without timing.

Empirically, the pattern of volume differs across markets. For NYSE and Nasdaq volume is usually said to be U- or reverse J-shaped.[27] On London Stock Exchange, on the other hand, recent evidence shows that volume is reverse L-shaped, with two small humps, one in the morning and the other in the early afternoon.[28] Other world markets, for instance, the Taiwan and the Singapore Stock exchanges have reverse L-shaped volume/number of transactions.[29] Spreads are L-shaped on most markets.[30]

The patterns of volume and spreads that my model predicts would intuitively remain consistent even if there would be more periods or more traders, i.e. myopic spreads would fall stronger than timing spreads and volume would be increasing with and decreasing without timing. So since my formulation predicts monotonic volume and thus cannot possibly capture U-shaped volume, what is it that it can say? First one should note that public information that appears overnight is usually reflected in the behavior during the opening sessions. Markets then react to the information revealed by the behavior in the opening sessions and this leads to active behavior early in the day. This reaction to the opening session is nicely documented by the behavior on the LSE where around the opening of the North American exchanges on the East Coast there is a 'hump' of activity. Most information that affects the opening session, however, is public, whereas information that spreads throughout the trading day is probably private or at least has a private component (otherwise, with public information, there would be a trading halt). Arguably, my model is applicable to exactly these situations, where information is obtained during the day and *after* the opening session.

[26] For smaller and larger μ the rounding error gets too large for the Beta distribution.

[27] See, for instance, Jain and Joh (1988), Brock and Kleidon (1992), McInish and Wood (1992), Lee, Mucklow, and Ready (1993), or Brooks, Hinich, and Patterson (2003), for Nasdaq see, for instance, Chan, Christie, and Schultz (1995).

[28] The classic reference is Kleidon and Werner (1996), the more recent one Cai, Hudson, and Keasey (2004).

[29] For Taiwan, see Lee, Fok, and Liu (2001); for Singapore see Ding and Lau (2001).

[30] Earlier papers find that spreads are U-shaped on NYSE, but recent evidence (Serednyakov (2005)) suggests that the pattern of spreads has morphed to an L-shape after decimalization.

My analysis is thus consistent with the volume and spread declines that occur towards the end of the trading day. Importantly my GM-model with timing displays these patterns without having to assume that certain types of traders exogenously cluster at specific times of the day. A GM model with timing thus reconciles that informed agents would trade early, inducing an equilibrium separation of better and less-well informed traders that leads to precisely the volume-spread-patterns that are observed on most exchanges towards the end of the trading day. Since behavior in GM models is fully rational, the volume-spreads patterns towards the end of the trading day should no longer be considered surprising or puzzling.

7 PIN: The Probability of Informed Trading

Smaller spreads are usually said to indicate that there is less information based trading. This continues to hold here: although with timing each informed trade is on average performed by better informed investors, it is less likely that such an informed trader exists. The (empirical) measure that expresses this "probability of informed trading" was introduced by Easley, Kiefer, O'Hara, and Paperman (1996) and it is most often labeled PIN. It measures the probability that a given trade is informed and is usually computed when estimating Glosten-Milgrom models. It is commonly defined as the ratio of the probability of an informed trade to the probability of a trade. For instance, in the simplest case when there is only one signal that reveals the true state perfectly, PIN $= \mu/(2\lambda + \mu)$.[31] In my setting, however, one would distinguish PIN for Periods 1 and 2, where as with spreads I assume an equal prior for both periods to obtain a clean comparison.

In Period 1, an informed trade can be a buy or a sale, and it can occur if the value is 0 or 1. The total probability of a trade, as ,above is $\beta_1 + \beta_0$, the total probability of an informed trade is $\beta_1 + \beta_0 -$noise. Thus in Period 1, the probability of an informed trade is

$$\text{PIN} = \frac{\beta_1 + \beta_0 - 2\lambda}{\beta_1 + \beta_0}. \tag{11}$$

It is straightforward to show that this expression in decreasing in π_b^1, which means that PIN is lower with timing than in the myopic case. For PIN in the second period, there is no analytical solution. Numerical simulations, however, deliver the following testable implication:

Numerical Observation 3 (PIN) *In the myopic case, PIN* decreases *from Period 1*

[31] See Hasbrouck (2007), Ch. 6; he has an additional parameter, α, which in my setting is 1.

to Period 2; with timing, PIN increases *from Period 1 to 2.*

Figure 5 plots PIN for Period 1 and Period 2 for both the myopic (right panels) and the timing case (left panels). The top row plots PIN for the quadratic quality over the feasible parameter space for θ; the middle row plots PIN for the hill-shaped Beta quality ($\theta \geq 1$); and the bottom row plots PIN for the U-shaped Beta-quality ($\theta < 1$).

One could loosely interpret the first period in my setting as the morning session, and the second period as the afternoon session. If timing does occur as outlined in my model, then PIN should be higher in the afternoon than in the morning — despite the fact the better informed traders move in the morning. With myopic trading, the situation is reversed.

8 Discussion

Discussion of some key assumptions. In my opinion, the model has three critical assumptions. First, priors are symmetric. This assumption is crucial to solve the problem analytically: one property that turns out to simplify the solution greatly is that the types of the marginal buyer and seller are 'symmetric', i.e. the marginal signal quality for buying is the same as for selling. Without symmetric priors, these thresholds will no longer be symmetric, and it is then no longer possible to solve the model analytically. In Appendix A I discuss numerical simulations which show that the *qualitative* features described in this paper remain valid with asymmetric priors.

Next, I allow people to trade at most once. The reason is simple: I want to focus on the investment-disinvestment timing decisions. With multiple trades, one has to consider also manipulative strategies,[32] which is beyond the scope of this paper.

Finally, there is the trading mechanism. Usually, in sequential trading models prices adjust after every trade; at the same time, however, one also assumes that in any given period there is only one trader who trades. In my formulation here, there are potentially two traders who submit orders simultaneously.[33]

The cleanest formulation would have the market maker specify a complete contingent plan, e.g. "if there is 1 buy and 1 hold the price is ...". There are three arguments against such a specification: (1) it is a fundamental deviation from the institutions that sequential trading frameworks aim to describe; (2) it becomes analytically intractable; and (3) numerically, it makes hardly any difference to the current formulation.

[32] Chakraborty and Yilmaz (2004b) formulate such strategies for a setting with a *single* insider.

[33] In Period 1, there are two traders, and in Period 2, there can also be two traders, provided no one has traded in Period 1. In each case, I assume that the bid- and ask-price accounts for only one trade.

To elaborate, GM frameworks are the purest formulations of quote-driven markets; a complete contingent plan, however, more closely resembles a market which is usually described as order-driven. Moreover, in such an order-driven market, it would no longer be possible to describe a meaningful bid-ask-spread, because there would be no unique quotes for bid- and ask-prices.[34] Traders also would not know at which price their market orders clear, and so such a setting would rather resemble a batch auction.

The complete-contingent plan setting is also analytically far less tractable: first, since traders do not know at which price their order will clear, they have to compute their expectation of the Period 1 transaction price. Next, when computing the Period 2 expected (ask- or bid-) price, if there is a hold, then, again, in Period 2 one has to compute the expected transaction price. Conceptually, this is not a problem, but there is no analytical solution. Finally, comparing numerical simulations of the complete-contingent price-plan with single-price settings, it turns out that the numerical difference in the thresholds is minor.

Another way to think about the formulation that I propose is to suppose that market makers first post their limit orders (i.e. their bid and offer prices) simultaneously and that in doing so, several will post identical orders, so that there will be standing limit orders at identical prices.[35] With a fast electronic trade-through system, simultaneously submitted orders would then be cleared so quickly that the limit orders cannot be adjusted.

The purpose of this study is to understand the timing decision of informed traders who potentially face 'informational' competition. I choose the simplest possible formulation that allows insights into this problem. The key element is the anticipated information content of present and future trades and its impact on prices. The results do not hinge on the assumptions of symmetry or a single price for simultaneous orders.

What would happen if there are 'strategic' noise traders? Suppose some fraction of the $1 - \mu$ noise traders can choose when to trade. Such people would be uninformed traders who have to trade for liquidity reasons but who can choose when to trade. So when would they? Let me start with making an out-of-equilibrium observation: Having no information is equivalent to receiving a signal of quality 1/2; the corresponding belief is $\pi = 1/2$. By monotonicity, traders with belief below π_b^1 delay in Period 1 because they believe that 'things will get better' in Period 2. Thus delaying is a dominant strategy

[34]In particular, there would be a price for 'matched' trades, i.e. when the number of simultaneous buys and sales coincide.

[35]This is commonly observed; Level 2 views of the Market Book offered by stock exchanges (e.g. the TSX) usually provide views based on orders ("market-by-order") and price ("market-by-price"). The latter view aggregates all limit orders for identical prices, and usually there are multiple orders for such prices.

for this uninformed (liquidity) trader. If there are strategic delays of noise traders then there would an equilibrium effect on the marginal trading types. The reason is that when the fraction of noise traders who act in Period 1 declines, ceteris paribus, then spreads widen. By the same token, when there are more noise traders in Period 2, then the spread tighten. Yet despite these two effects, in equilibrium spreads must still shrink between periods because of the belief monotonicity. As a consequence, strategic noise traders would *always* trade in Period 2.

Discussion of the Results. In 'classic' models of sequential trade, agents cannot choose the time of market entry. In such models private and public beliefs converge, thus the bid-ask spread gets smaller. While in my model spreads also decrease between periods, the trajectory with timing is flatter (with the reservation that, obviously, a two period model is not fit to project limit arguments).

Furthermore, as Smith (2000) points out, an investor with a favorable signal expects transaction prices to increase (for him it is a sub-martingale). Casual intuition then suggests, that profits from speculation are largest early – thus investors should invest rather earlier than later. My result is somewhat surprising as it indicates the opposite: the type who is the marginal buyer without timing would *delay* the purchase with timing because he would expect future ask-prices to *fall* — despite his favorable information.

My model is, of course, very restrictive; in real markets, it is quite possible that there are combinations of myopic and timing behavior. For instance, the large volume at and just after the open may result from traders who act on new information that arrived during the night, and these traders may act myopically. At the same time, the volume upswing towards the close could be forced by strategic timers who receive information during the day. The restriction to two periods and two traders, however, is not a strong one: the intuition for behavior would carry through to settings with more trading periods and more traders.

A Appendix: Numerical Robustness with Asymmetric Priors

The numerical computations presented in this appendix employ the example from Section 2 with signal densities $f_1(\pi) = 2\pi,\ f_0(\pi) = 2(1-\pi)$ as plotted in Figure 2. Recall that these distributions together lead to an ex-ante uniform distribution of beliefs.

Asymmetric Priors. For the main results of this paper I have assumed an equal prior for the liquidation values V. I did this, because it simplifies the exposition and allows analytical results. Moreover, the prior actually does affect the computation of the myopic thresholds, although it does affect the timing-related thresholds. With an asymmetric prior the model no longer affords analytical results, but numerically an asymmetric prior presents no difficulties. Simulations then show the qualitative insights remain unaffected (although, of course, the trading thresholds change). I now briefly review the results from numerical simulations with unequal priors.

The most interesting question concerning asymmetric priors is whether one needs higher or lower quality information when the prior favors one's private information. In other words, compare prior 1/4 to prior 3/4: does the marginal trader have a higher (thus more favorable) belief than when the prior is 1/4? Moreover, what happens as the prior traverses from favoring the low to the high value? Will the marginal buyers' belief change monotonically?

It turns out that the direction of change in the threshold is non-monotonic: as the prior traverses from 0 to 1, the Period 1 buying threshold first declines and then increases (similarly for the Period 2 buying threshold); note that the switch from 'decreasing' to 'increasing' does not occur at 1/2 but for values of p smaller than 1/2.

The non-monotonicity is documented in Figure 8: the left panel plots the Period 1 and 2 buying thresholds respectively, the right panels plots the sign of the first differences of the thresholds (e.g. thresholds for prior 0.9 minus threshold for prior 0.8), where the sigh for Period 2 is multiplied with 1/2 to allow a representation in the same picture.

Numerical Observation 4 (Asymmetric Priors and Thresholds)
The relationship between the level of the threshold and the prior is ambiguous: As the prior p increases from 0 to 1, the Period 1 and 2 buying thresholds decrease for small p and increase for larger p.

B Appendix: Quality and Belief Distributions

I assume that each informed trader's signal quality is distributed over $[0,1]$ according to distribution $G(q)$ with continuous density $dG(q) = g(q)$, and I assume that g is symmetric around 1/2. Note that with this quality specification, signal qualities q and $1-q$ are equally useful for the individual: if someone receives signal h and has quality 1/4, then this signal has 'the opposite meaning', i.e. it has the same meaning as receiving signal l with quality 3/4. Assuming symmetry around 1/2 is thus without loss of generality. Signal qualities are assumed to be independent across agents, and independent of the asset's liquidation value V.

Beliefs are derived, given signals and signal-qualities, by Bayes Rule. I can then express the belief densities as follows: in state $V = 1$, the generalized density of individuals with private belief π is $f_1(\pi) = \pi[dG(\pi) + dG(1-\pi)]$, while analogously, in the 'low' state, $V = 0$, it is $f_0(\pi) = (1-\pi)[dG(\pi) + dG(1-\pi)]$.

Smith and Sørensen (2000) prove the following property of private beliefs:

Lemma 1 (Symmetric beliefs, Smith and Sørensen (2000))
With the above the signal quality structure, private belief distributions satisfy $F_1(\pi) = 1 - F_0(1-\pi)$ *for all* $\pi \in (0, 1)$.

The signal densities satisfy the monotone likelihood ratio property. This is straightforward to see because the likelihood ratio, expressed through the ratio of the belief densities,

$$\frac{f_1(\pi)}{f_0(\pi)} = \frac{\pi[dG(\pi) + dG(1-\pi)]}{(1-\pi)[dG(\pi) + dG(1-\pi)]} = \frac{\pi}{1-\pi}$$

is increasing in π.

C Omitted Proofs

Proof of Theorem 1 (Existence, Uniqueness, and Symmetry)

We have already argued the symmetry of Period 2 thresholds, monotonicity of trading rules and informativeness of trades. I have shown that the Period 2 threshold, π_b^2, is

$$\pi_b^2 = \frac{\beta_1^2}{\beta_1^2 + \beta_0^2} \quad \Leftrightarrow \quad \pi_b^2 = \frac{\lambda + \mu(F_1(\pi_b^1) - F_1(\pi_b^2))}{\lambda + \mu(F_1(\pi_b^1) - F_1(\pi_b^2)) + \lambda + \mu(F_0(\pi_b^1) - F_0(\pi_b^2))}. \tag{12}$$

Trade informativeness immediately implies that π_b^2 must be at least 1/2. What remains to show is existence and uniqueness of $\pi_b^2 \in [1/2, \pi_b^1)$.

From equation (12) we know that the threshold quality for buying depends on the relative probability of high correct to high wrong beliefs. We will now reformulate (12) to obtain a more manageable relation: First,

$$F_1(\pi) = \int_0^\pi f_1(s)\; ds = \int_0^\pi s \cdot (g(s) + g(1-s))\; ds = 2\int_0^\pi s \cdot g(s)\; ds,$$

where the last step follows from the symmetry of g around 1/2. Performing integration by parts, we obtain

$$F_1(\pi) = 2\int_0^\pi s \cdot g(s)\; ds = 2sG(s)|_0^\pi - 2\int_0^\pi G(s)\; ds = 2\pi G(\pi) - 2\int_0^\pi G(s)\; ds.$$

Then

$$F_1(\pi) + F_0(\pi) = 2\int_0^\pi s \cdot g(s)\ ds + 2\int_0^\pi (1-s)\cdot g(s)\ ds = 2\int_0^\pi \cdot g(s)\ ds = 2G(\pi).$$

Further

$$\begin{aligned} \beta_1^2 + \beta_0^2 &= \lambda + \mu(F_1(\pi_b^1) - F_1(\pi_b^2)) + \lambda + \mu(F_0(\pi_b^1) - F_0(\pi_b^2)) \\ &= 2\lambda + 2\mu(G(\pi_b^1) - G(\pi_b^2)). \end{aligned} \tag{13}$$

Thus, $\pi_b^2 = \beta_1^2/(\beta_1^2+\beta_0^2)$ can be rewritten as

$$\Leftrightarrow \quad \pi_b^2(2\lambda + 2\mu(G(\pi_b^1) - G(\pi_b^2))) = \lambda + 2\mu\left(\pi_b^1 G(\pi_b^1) - \pi_b^2 G(\pi_b^2) - 2\int_{\pi_b^2}^{\pi_b^1} G(s)\ ds\right).$$

By further simplifying I obtain the following simple expression

$$G(\pi_b^1)(\pi_b^1 - \pi_b^2) - \frac{\lambda}{2\mu}\ (2\pi_b^2 - 1) = \int_{\pi_b^2}^{\pi_b^1} G(s)\ ds. \tag{14}$$

Both sides of (14) are continuous in π_b^2. At $\pi_b^2 = \pi_b^1$, the right-hand side of (14) is 0, while the left-hand side is $-(1-\mu)/4/(2\mu) < 0$. Now consider $\pi_b^2 = 1/2$. Then

$$G(\pi_b^1)(\pi_b^1 - 1/2) < \int_{\frac{1}{2}}^{\pi_b^1} G(s)ds,$$

The left-hand side term, $G(\pi_b^1)(\pi_b^1 - 1/2)$, describes the area of a rectangle with side lengths $G(\pi_b^1)$ and $\pi_b^1 - 1/2$. The right-hand side describes the area under an increasing function (the distribution function of qualities) over the range $[1/2, \pi_b^1]$.

Thus the ordering with $\pi_b^2 = 1/2$ is reversed compared to $\pi_b^2 = \pi_b^1$: the right-hand side is smaller than the left-hand side. Since both left-hand side and right-hand side are continuous in π_b^2, there exists a π_b^2 that warrants equality.

To prove *uniqueness* it suffices to show that the left hand side of (14) is more steeply decreasing than the right hand side for all $\pi_b^2 \in (1/2, \pi_b^1)$. The slope of the left hand side in π_b^2 is simply $-G(\pi_b^1) - (1-\mu)/4/(2\mu)$. The slope of the right hand side is $-G(\pi_b^2) > -G(\pi_b^1) - \lambda/\mu$ for all $\pi_b^2 \in (1/2, \pi_b^1)$.

As for *symmetry*: The Period 2 thresholds must satisfy

$$\pi_b^2 = \frac{\beta_1^2}{\beta_1^2 + \beta_0^2}, \quad \text{and} \quad \pi_s^2 = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_0^2}. \tag{15}$$

The probabilities of trades are computed as follows: Suppose for now that monotonicity of decisions holds and let π_b, π_s denote the threshold values so that traders in Period 2 with private beliefs above π_b buy and those with private beliefs below π_s sell if they have not traded in Period 1. Then

$$\beta_i^2 = \frac{\lambda + \mu(F_i(\pi_b^1) - F_i(\pi_b))}{\gamma_i^1}, \ \ \sigma_i^2 = \frac{\lambda + \mu(F_i(\pi_s) - F_i(\pi_s^1))}{\gamma_i^1}, \ \ \gamma_i^1 = 2\lambda + \mu(F_i(\pi_b^1) - F_i(\pi_s^1)).$$

For now assume that symmetry holds in Period 1, i.e $\pi_b^1 = 1 - \pi_s^1$. The quality paradigm thus yields

$$F_1(\pi_b^1) = 1 - F_0(1 - \pi_b^1) \quad \Leftrightarrow \quad F_1(\pi_b^1) = 1 - F_0(\pi_s^1).$$

Suppose that also symmetry in Period 2 prevails. Then this ensures consistency

$$\begin{aligned} 1 - \pi_b^2 &= 1 - \frac{\beta_1^2}{\beta_1^2 + \beta_0^2} = \frac{\beta_0^2}{\beta_1^2 + \beta_0^2} = \frac{\lambda + \mu(F_0(\pi_b^1) - F_0(\pi_b^2))}{\lambda + \mu(F_1(\pi_b^1) - F_1(\pi_b^2)) + \lambda + \mu(F_0(\pi_b^1) - F_0(\pi_b^2))} \\ &= \frac{\lambda + \mu(F_1(\pi_s^2) - F_1(\pi_s^1))}{\lambda + \mu(F_1(\pi_s^2) - F_1(\pi_s^1)) + \lambda + \mu(F_0(\pi_s^2) - F_0(\pi_s^1))} = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_0^2} = \pi_s^2. \end{aligned}$$

Deriving Equation (9) from equation (5)

With symmetry I can express the probabilities for buys and sales in Period 1 by

$$\begin{aligned} &\beta_i^1 = \lambda + \mu(1 - F_i(\pi_b^1)), \ \sigma_i^1 = \lambda + \mu F_i(\pi_s^1) = \beta_{|i-1|}^1, \\ &\gamma_i^1 = 2\lambda + \mu(F_i(\pi_b^1) - F_i(\pi_s^1)) = 1 - (\beta_0^1 + \beta_1^1). \end{aligned}$$

The insider with belief π_b^1 then assigns the following probabilities to these events:

$$\begin{aligned} &\mathsf{pr}(\text{buy}|\pi_b^1) = \pi_b^1\beta_1^1 + (1 - \pi_b^1)\beta_0^1, \ \mathsf{pr}(\text{sale}|\pi_b^1) = \pi_b^1\beta_0^1 + (1 - \pi_b^1)\beta_1^1, \\ &\mathsf{pr}(\text{no trade}|\pi_b^1) = 1 - (\beta_1^1 + \beta_0^1). \end{aligned}$$

By symmetry $\gamma_1^1 = \gamma_0^1$, so that the price in Period 2, given π_b^1 is found by solving the following for π_b^2.

$$\frac{\beta_1^2(\pi_b^1, \pi_b^2)}{\beta_1^2(\pi_b^1, \pi_b^2) + \beta_0^2(\pi_b^1, \pi_b^2)} = \pi_b^2.$$

As a consequence, I can expand each ask price in $\mathsf{E}[\mathsf{ask}_2|\pi_b^1]$ by substituting each β_1^2 in Period 2's price with the threshold π_b^2 and each β_0^2 with the threshold $1 - \pi_b^2$. Omitting superscripts $t = 1$ on β_i,

$$\mathsf{ask}_2(\text{buy}) = \frac{\beta_1\pi_b^2}{\beta_1\pi_b^2 + \beta_0(1 - \pi_b^2)}, \ \ \mathsf{ask}_2(\text{sale}) = \frac{\beta_0\pi_b^2}{\beta_0\pi_b^2 + \beta_1(1 - \pi_b^2)}, \ \ \mathsf{ask}_2(\text{hold}) = \pi_b^2.$$

Since $\mathsf{ask}_1 = \beta_1/(\beta_0 + \beta_1)$, equation (5) expands to

$$\frac{\beta_1}{\beta_1+\beta_0} = \frac{\pi_b^1\beta_1 + (1-\pi_b^1)\beta_0}{\beta_1\pi_b^2 + \beta_0(1-\pi_b^2)}\,\beta_1\pi_b^2 + \frac{\pi_b^1\beta_0 + (1-\pi_b^1)\beta_1}{\beta_0\pi_b^2 + \beta_1(1-\pi_b^2)}\beta_0\pi_b^2 + (1-(\beta_1+\beta_0))\pi_b^2. \quad (16)$$

One can rearrange and simplify this equation to yield equation (9)

$$\Leftrightarrow \frac{\beta_1}{\beta_0+\beta_1} - \pi_b^2 = \left(\frac{\pi_b^2\beta_1}{\beta_1\pi_b^2 + \beta_0(1-\pi_b^2)} - \frac{\pi_b^2\beta_0}{\beta_0\pi_b^2 + \beta_1(1-\pi_b^2)}\right)(\pi_b^1 - \pi_b^2)(\beta_1 - \beta_0). \quad (17)$$

Proof of Theorem 3 (Existence of Period 1 Timing Thresholds)

I focus only on the buy-threshold (the sale threshold argument is analogous) and I proceed constructively: Suppose $\pi_b^1 = 1$. Then $\beta_1 = \beta_0$ and thus the LHS of (9) is $1/2 - \pi_b^2 \leq 0$ whereas the RHS is 0. Next, let $\pi_b^1 = \pi_b^{1,\mathsf{M}}$, where $\pi_b^{1,\mathsf{M}}$ solves the myopic problem. Then the LHS of (9) is $\pi_b^1 - \pi_b^2$ and we can reduce the LHS to 1 and the RHS, using $\pi_b^{1,\mathsf{M}} = \beta_1/(\beta_1 + \beta_0)$ to

$$\left(\frac{\pi_b^2\pi_b^1}{\pi_b^1\pi_b^2 + (1-\pi_b^1)(1-\pi_b^2)} - \frac{\pi_b^2(1-\pi_b^1)}{(1-\pi_b^1)\pi_b^2 + \pi_b^1(1-\pi_b^2)}\right)(\beta_1 - \beta_0). \quad (18)$$

Naturally, both terms are smaller than 1 and thus the above expression is smaller than 1. Since LHS and RHS of (9) are both continuous in π_b^1, a solution exists.

Symmetry obtains straightforwardly: reformulate equation (16) for bid-prices, hypothesizing a symmetric threshold. With a symmetric prior, $\mathsf{pr}(\text{buy}|\pi_b^1) = \mathsf{pr}(\text{sale}|1-\pi_b^1)$, $\mathsf{ask}_1 = 1 - \mathsf{bid}_1$ etc. Substituting this into (9) immediately yields the desired equality.

Proof of Proposition 1 (Equilibrium Threshold Beliefs Maximize the Spread)

Considering first order conditions, $\frac{\partial}{\partial\pi}\mathsf{E}[V|H_t, \text{buy with threshold } \pi] = 0$ is equivalent to

$$\frac{\partial}{\partial\pi}\frac{\beta_1(\pi)p_t}{\beta_1(\pi)p_t + \beta_0(\pi)(1-p_t)} = 0 \quad \Leftrightarrow \quad \frac{\beta_0(\pi)}{\beta_1(\pi)} = \frac{\beta_0'(\pi)}{\beta_1'(\pi)}. \quad (19)$$

Now recall that $\beta_i^2(\pi) = (\lambda + \mu(F_i(\pi_b^1) - F_i(\pi)))/\gamma_i$. Thus the right hand side of the second relation above is

$$\frac{\beta_0'(\pi)}{\beta_1'(\pi)} = \frac{f_0(\pi)}{f_1(\pi)}.$$

In Appendix B I have argued that $f_1(\pi) = \pi[dG(\pi) + dG(1-\pi)]$, and $f_0(\pi) =$

$(1-\pi)[dG(\pi)+dG(1-\pi)]$. With this we have

$$\frac{\beta_0'(\pi)}{\beta_1'(\pi)}=\frac{1-\pi}{\pi}. \tag{20}$$

Next, consider the equilibrium condition $\mathsf{E}_t[V|\pi]=\mathsf{ask}_2(\pi)$. Rearranging and simplifying, threshold π must satisfy

$$\frac{\beta_0(\pi)}{\beta_1(\pi)}=\frac{1-\pi}{\pi}. \tag{21}$$

Combining equations (19) and (20) delivers the same relation as equation (21) and thus the equilibrium condition is identical to the First Order Condition for the ask price as a function of threshold π.

Proof of Proposition 2 (Period 2 Spread increases in Period 1 Threshold)

The equilibrium condition for the Period 2 buying-threshold, equation (7), can be reformulated to $\beta_1(\pi_b^1,\pi_b^2)/\beta_0(\pi_b^1,\pi_b^2)=\pi_b^2/(1-\pi_b^2)$. To prove the statement of the proposition it thus suffices to compute the derivative of $\beta_1(\pi_b^1,\pi_b^2)/\beta_0(\pi_b^1,\pi_b^2)$ with respect to π_b^1 and show that it is positive:

$$\frac{\partial}{\partial\pi_b^1}\frac{\beta_0(\pi_b^1,\pi)}{\beta_1(\pi_b^1,\pi)}<0 \;\Leftrightarrow\; f_0(\pi_b^1)\beta_1^2-f_1(\pi_b^1)\beta_0^2<0 \;\Leftrightarrow\; \frac{f_0(\pi_b^1)}{f_1(\pi_b^1)}<\frac{\beta_0^2}{\beta_1^2}.$$

Using the above insights and the Period 2 equilibrium condition, this relation says that $(1-\pi_b^1)/\pi_b^1<(1-\pi_b^2)/\pi_b^2$, which certainly holds as $\pi_b^1>\pi_b^2$.

Proof of Proposition 3 (Benchmark Comparison)

(a): To show $\pi_b^{1,\mathsf{T}}>\pi_b^{1,\mathsf{M}}$ I will argue that $\pi_b^{1,\mathsf{T}}\not\leq\pi_b^{1,\mathsf{M}}$.

First, one can apply similar arguments as in Theorem 1 to show that $\pi_b^{1,\mathsf{M}}$ is unique; the proof for this is omitted. Next, recall that $\pi_b^{1,\mathsf{M}}$ maximizes the ask-price. Also, $\mathsf{ask}(1)=1/2$ whereas the private expectation for $\pi_b=1$ is 1. Consequently, it must hold that for any $\pi<\pi_b^{1,\mathsf{M}}$, $\pi<\beta_1(\pi)/(\beta_1(\pi)+\beta_0(\pi))$. So for every $\pi<\pi_b^{1,\mathsf{M}}$ the left hand side of (9) satisfies

$$\frac{\beta_1(\pi)}{\beta_1(\pi)+\beta_0(\pi)}-\pi_b^2>\pi-\pi_b^2.$$

By the same arguments as used in the proof of Theorem 3, the RHS of the above remains larger than

$$\left(\frac{\pi_b^2\beta_1}{\beta_1\pi_b^2+\beta_0(1-\pi_b^2)}-\frac{\pi_b^2\beta_0}{\beta_0\pi_b^2+\beta_1(1-\pi_b^2)}\right)(\beta_1-\beta_0),$$

because both terms in the latter are smaller than 1. Thus there cannot be a solution to (9) for $\pi < \pi_b^{1,\mathsf{M}}$.

(b) follows immediately from (a) and Proposition 2.

(c) follows from (a) and (b) because higher thresholds imply that a smaller fraction of informed agents trades.

Proof of Proposition 4 (Bid-Ask Spreads and Price Variability)
(a): The result follows from Propositions 1 and 3: without timing, the Period 1 thresholds signify the maximal ask and the minimal bid price; consequently, any other threshold implies that the ask-price is smaller and the bid-price larger.

With symmetry, the bid-ask spread in Period 2 with timing can be expressed by threshold beliefs and it is thus $\pi_b^{2,\mathsf{T}} - \pi_s^{2,\mathsf{T}}$; in the myopic case it is $\pi_b^{2,\mathsf{M}} - \pi_s^{2,\mathsf{M}}$. By Proposition 3 (b), $\pi_b^{2,\mathsf{T}} > \pi_b^{2,\mathsf{M}}$ and $\pi_s^{2,\mathsf{T}} < \pi_s^{2,\mathsf{M}}$.

(b): The result is immediately clear in the myopic case. In the timing case, observe that the marginal buyer expects tomorrow's ask price to be the same as today's. Since beliefs are monotonic, traders with higher beliefs consider tomorrow's ask price to be larger (because they consider buys more likely), and traders with beliefs lower than the Period 1 buying threshold believe that prices will drop. In particular this holds for traders with belief 1/2, which is the same as the public (unconditional) expectation. Consequently, the ask price in Period 1 is unconditionally larger than the ask price in Period 2; a symmetric result applies to the bid-prices. Thus the spread declines.

(c): Variability follows from the spread relation: with timing one moves from a smaller to a larger spread. At the same time, the overall spread declines (in both cases), and thus the change in prices is larger for the myopic case.

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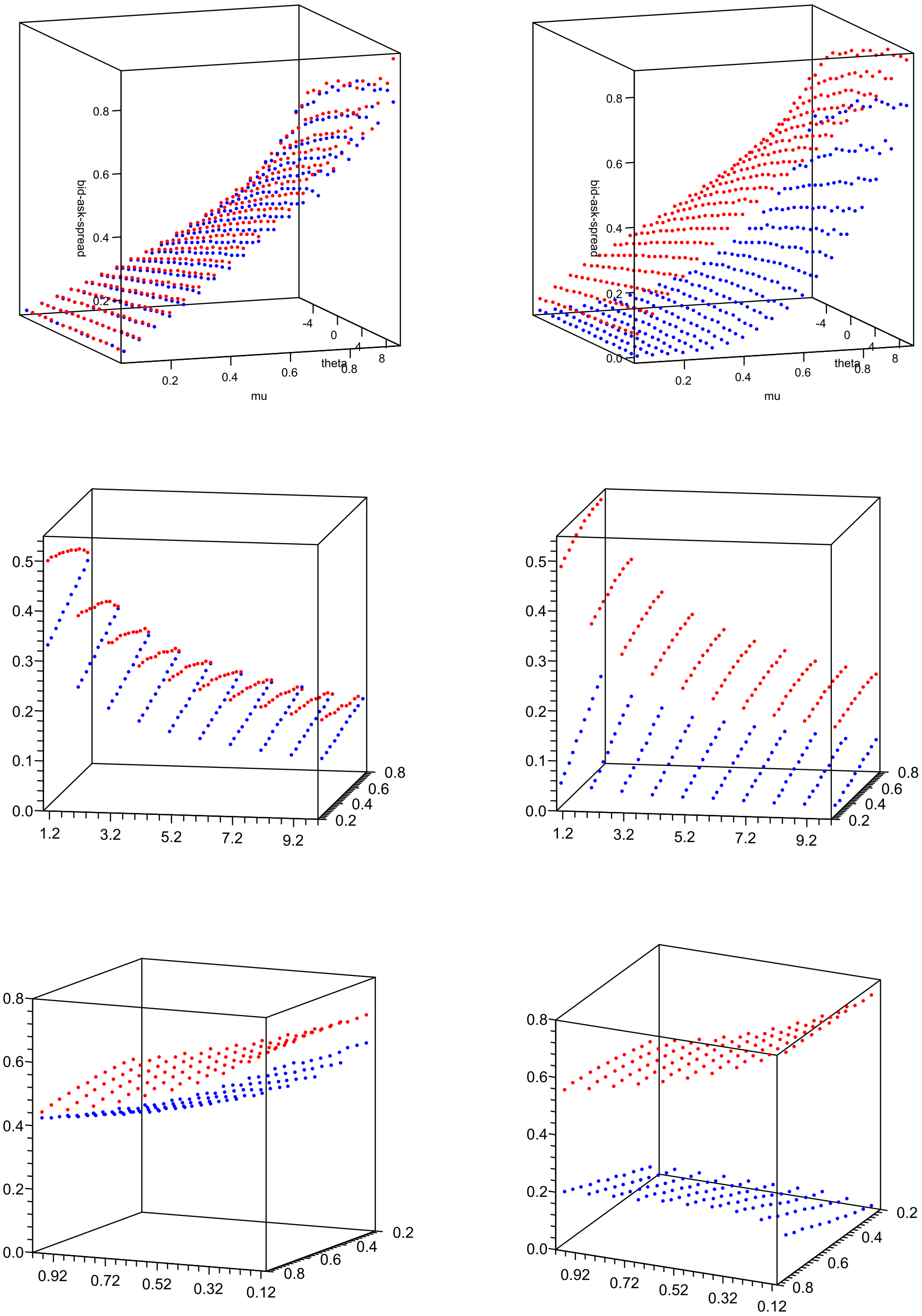
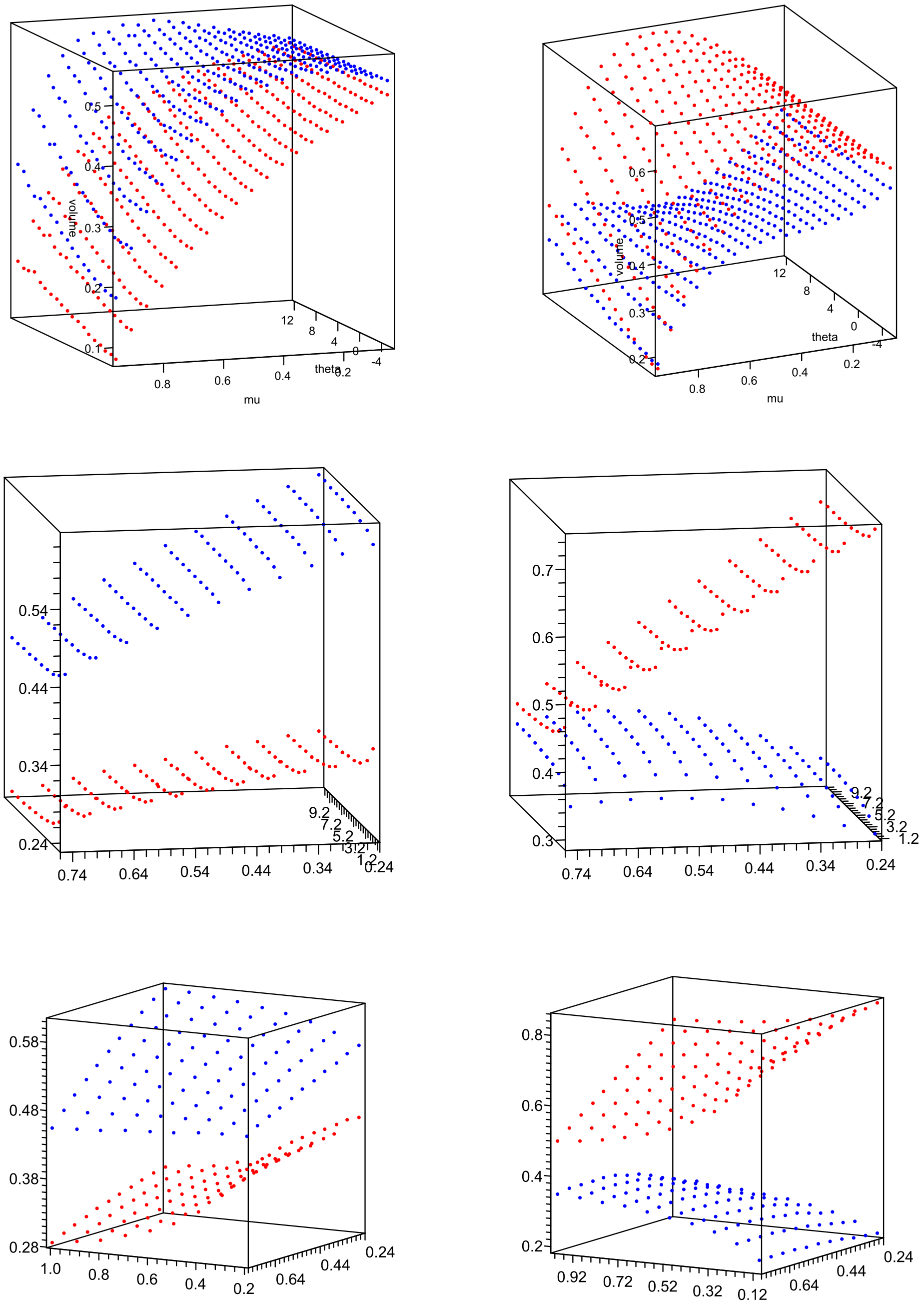


Timing Bid-Ask-Spreads

Myopic Bid-Ask-Spreads

Figure 5: The left panels plot the bid-ask-spread for the timing case, the right ones for the myopic case. Red refers to Period 1, blue to Period 2. The top row of panels is for the quadratic quality distribution, the middle row for the hill-shaped Beta distribution, and the bottom row for the U-shaped Beta distribution. For the top row of panels, the proportion of noise traders μ is on the horizontal axis, the spread is on the vertical axis, and the distribution parameter θ on the remaining axis. For the middle and bottom row, distribution parameter θ is on the horizontal axis, the spread is on the vertical axis, and the proportion of informed traders μ is on the remaining axis. As can be seen, in the timing case, in the top row, the spreads are almost of the same size in both periods whereas in the myopic case, the spread in the first period is much larger than in the second. The spreads are relatively wider for the Beta distribution than for the quadratic one, but timing spreads are still much smaller than myopic ones.



Timing Volume

Myopic Volume

Figure 6: The left panels plot the volume for the timing case, the right ones for the myopic case. Red refers to Period 1, blue to Period 2. The top row of panels is for the quadratic quality distribution, the middle row for the hill-shaped Beta distribution, and the bottom row for the U-shaped Beta distribution. For the top and middle row of panels, the proportion of noise traders μ is on the horizontal axis, the volume is on the vertical axis, and the distribution parameter θ on the remaining axis. For the bottom row, distribution parameter θ is on the horizontal axis, the volume is on the vertical axis, and the proportion of informed traders μ is on the remaining axis. As can be seen, in all timing cases, the volume in Period 1 is smaller than in Period 2; in all myopic cases, it is the reverse.

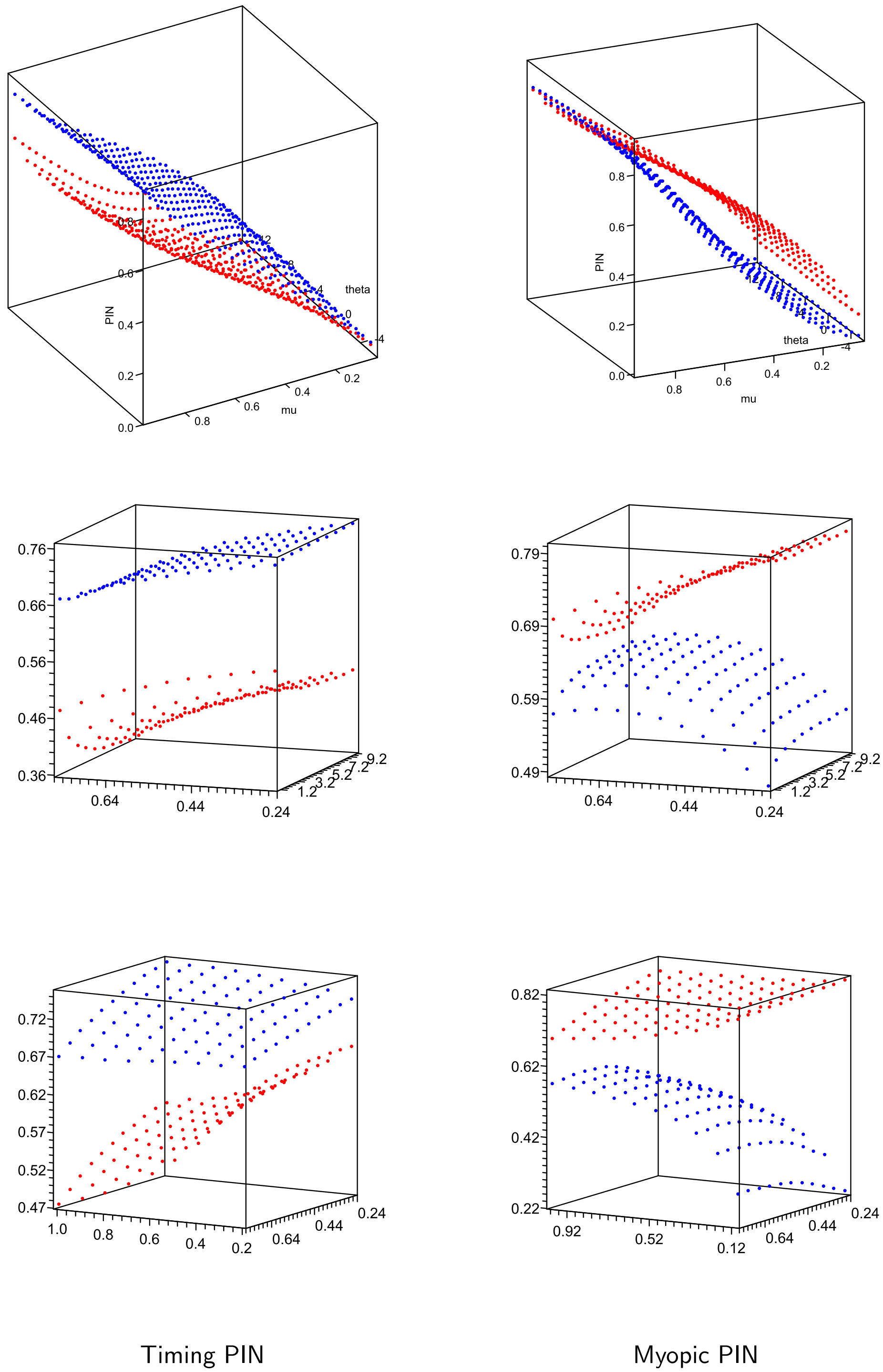
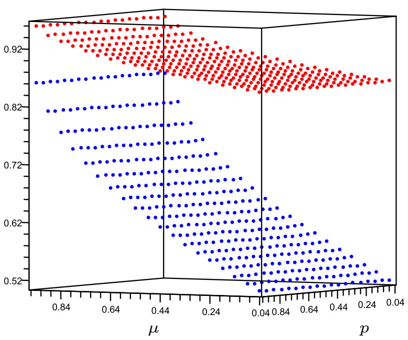
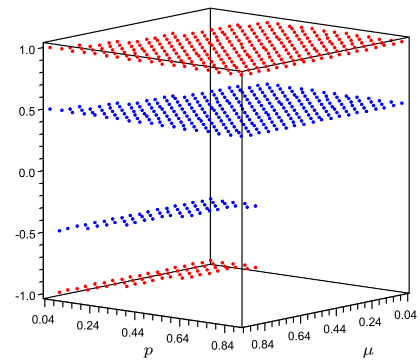


Figure 7: The left panels plot the probability of informed trading (PIN) for the timing case, the right ones for the myopic case. Red refers to Period 1, blue to Period 2. The top row of panels is for the quadratic quality distribution, the middle row for the hill-shaped Beta distribution, and the bottom row for the U-shaped Beta distribution. For the top and middle row of panels, the proportion of noise traders μ is on the horizontal axis, on the vertical axis is PIN as outlined in equation (11), and the distribution parameter θ on the remaining axis. For the bottom row, distribution parameter θ is on the horizontal axis, PIN is on the vertical axis, and the proportion of informed traders μ is on the remaining axis. As can be seen, in all timing cases, PIN in Period 1 is smaller than in Period 2; in all myopic cases, it is the reverse.



Period 1 and 2 Buying-Thresholds
with asymmetric priors
as function of prior and noise



Signs of the first differences
of buying thresholds
for Period 1 and 2

Figure 8: The LEFT panel plots the Period 1 and Period 2 buying thresholds when the prior $p = \mathsf{pr}(V = 1)$ is asymmetric. Red refers to thresholds in Period 1, blue to thresholds in Period 2. Free parameters are $p \in (0, 1)$, and the level of informed trading $\mu \in (0, 1)$. Clearly, the less noise there is, the higher the thresholds are. The RIGHT panel then illustrates how changes in the prior affect the Period 1 and Period 2 buying thresholds: I compute the threshold for priors p and $p - 0.05$, subtract the latter from the former, determine the sign. For the period 2 case, I multiply the sign with 1/2 to be able to fit the plots for both Period 1 and Period 2 into the same panel. As can be seen, the thresholds are not monotonic: they do not unambiguously increase as the prior is increases. In particular for low values of μ, when the prior traverses from 0 to 1, the threshold first declines (one needs lower beliefs/quality information to trade) and then increases (the more the prior favors one's opinion, the better quality signals one needs to trade). The Period 2 thresholds behave similarly to the Period 1 thresholds.