Twin Engines of Growth

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Abstract

We develop an endogenous growth model in which new technology and new skills are *bounded complements* — they complement each other up to a point, but beyond this the impact of each factor is constrained by the level of the other. As a result, *both* technological progress and human capital accumulation are *necessary* for sustained productivity growth, but neither alone is sufficient. Rapid technological progress generates increased returns to education and encourages each generation to spend more time in school. Rapid human capital accumulation increases the feasibility and profitability of innovation and encourages the private business sector to allocate more resources towards R&D. Our model has important implications for the effectiveness of alternative growth–promoting policies, for the interpretation of the empirical relationship between growth and schooling, and for the relationship between growth and intergenerational wage dispersion.

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1. Introduction

Technological progress is driven by individuals' self interested attempts to profit from their ingenuity. These innovations do not materialize from thin air; they are the result of costly investments, often requiring extensive specialized technical knowledge and much accumulated skill. However, the desire to push forward the knowledge frontier is not the primary motivation for household investments in human capital. In fact, people typically acquire skills to enable themselves to implement technologies that already exist. New technologies expand the set of what can be feasibly produced and generate incentives for people to acquire new skills. Similarly, skill acquisition reduces the costs of implementing existing technologies and generates incentives for new technologies to be developed.

Although both innovation and skills are widely recognized as being important factors in the growth process, the recent growth literature leaves two important questions largely unanswered. First, what is the relative importance of technological progress versus human capital formation in driving sustained productivity growth? Second, how do these two forms of knowledge accumulation endogenously interact in driving productivity growth? Early endogenous growth theories focussed on either innovation or human capital as the primary engine of growth. More recently, growing evidence of strong complementarity between new technology and skill (e.g. Bartel and Lichtenberg, 1987, Goldin and Katz, 1998) has motivated a greater emphasis on the interaction between technological change and human capital accumulation. However, in most cases, only one of these forces can be thought of as the primary engine of endogenously sustainable growth. The other is either exogenous or is driven by direct "spillovers" from the primary engine.[1]

Although technology and skills are complementary, it is clear that there are limits to this relationship. In particular, that simply combining an increasingly educated labor force with a fixed set of technologies could enhance labor productivity without bound seems implausible. In this paper, we construct a model in which skills and technologies are *bounded complements* at the aggregate level — they are complements up to a point, but the marginal productivity of each factor is ultimately constrained by the level of the other. Complementarity generates endogenous interactions between technology, skills and growth that we argue are broadly consistent with the evidence. The fact that

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[1] We discuss the associated literature in more detail below.

this complementarity is bounded implies that growth cannot proceed without technical progress *and* aggregate human capital accumulation. Moreover, in contrast to previous analyses, our approach allows us to accommodate the observed complementarity between technology and skills in a model in which both engines of growth are truly endogenous.[2]

Our model features continuously overlapping generations of agents who *must* acquire skills to both implement and invent technologies. Skills can be acquired in two ways: by investing in education and training, and by learning while on the job.[3] Because knowledge learned on–the–job is specialized and cannot be transferred costlessly to new generations (even between members of the same household), there is an endogenous demand for education as a vehicle for transferring skills between generations. A dynamic externality arises because educational investments transform previously specialized knowledge into general knowledge that can be costlessly transferred to future generations. However, education is not primarily a source of *new* knowledge. New technologies arise as a result of profit–seeking innovative activities by agents whose productivity in the introduction of new ideas and technologies requires high levels of specialized skill some of which is acquired through work experience.[4] Thus, we distinguish between frontier knowledge, generated by the innovation sector, and the knowledge embodied in humans, acquired through education and experience.

The endogenous growth process that arises has several key predictions for the interactions between technological progress, human capital accumulations and productivity growth:

• *Technological change is necessary for sustained growth in per capita income* — In models of endogenous growth based solely on human capital accumulation (e.g.. Lucas, 1988), human capital raises the productivity of the available fixed technology and grows without bound. In our model, human capital accumulation may raise aggregate productivity to a limited extent, by relaxing skill constraints on the efficient use of technologies, but growth cannot be sustained without the introduction of new technologies.

In an influential article, Mankiw, Romer and Weil (1992) claim that most (around 80%) of the international variation in per capita incomes can indeed be explained by differences in physical and

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[2] That is, where both are driven by private incentives to allocate labor effort away from production.

[3] The absorption of knowledge is privately and publicly costly (see Jovanovic, 1997). Mincer (1994) estimates that schooling costs are over 10% of US GNP, whereas job training and learning costs are approximately 3%.

[4] As Jovanovic (1997, p. 329) notes, " ... it is by now clear that more educated and more skilled workers have a comparative advantage in implementing new technology."

human capital alone. However, more recent evidence suggests that this claim is exaggerated. For example, Klenow and Rodriguez–Clare (1997) show that the result is sensitive to the definition of the human capital variable. Moreover, Howitt (2000) shows that the assumption that productivity differences are uncorrelated with investment rates is generally invalid and leads them to overestimate the impact of physical and human capital. Another kind of evidence casting doubt on the view that productivity differences are determined by human capital alone is that workers with apparently similar human capital levels have widely different productivities in different countries (see Trefler, 1995 and Hendricks, 1999).[5]

• *Growth in the aggregate stock of human capital is necessary for sustained growth in per capita income* — In models of endogenous growth based solely on innovation (e.g. Romer, 1990, Grossman and Helpman, 1991 and Aghion and Howitt, 1992), new technologies raise the productivity of a given stock of human capital and do so without bound. In our model, the frontier knowledge generated by the R&D sector is not immediately and costlessly implementable, and only sufficiently skilled workers are productive in R&D. As a result, the aggregate stock of human capital must grow in order to sustain the profitable introduction of new technologies in the long run.

Early empirical studies (e.g. Benhabib and Spiegel, 1994), using average years of schooling in the working population as a measure of the stock of human capital, seem to support the view that the initial level of human capital and not its growth determines productivity growth. However, more recently, several authors (e.g. Bils and Klenow 1999, Jones 1996) have argued that it is more reasonable to interpret years of schooling as a measure of the investment in human capital rather than the stock itself. Moreover, recent empirical work (e.g. Krueger and Lindahl, 2000) has found modelling the relationship between human capital and schooling along the lines of Mincerian wage regressions results in a preferable specification. Our model of human capital formation mirrors that of Bils and Klenow (1999) — an individual's human capital depends on the time spent in school and acquiring experience, as in the Mincerian model.

• *Rapid technological change generates high net returns to skill which in turn generates investment in education and training* — Complementarity between skills and new technologies implies that more rapid innovation drives up anticipated future earnings relative to the current opportunity

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[5] Although this prediction would also arise in human capital based models that include an aggregate externality, recent evidence (e.g. Acemoglu and Angrist, 1999 and Heckman and Klenow 1997), suggests that such an externality is unlikely to be large.

costs of labor effort in education. It is this disparity which stimulates the increased investment in education necessary to support greater technological progress in the long run.

Nelson and Phelps (1966) argue that when human capital lags technology growth this will generate a "catch up" process whereby human capital grows more rapidly.[6] In our model, this "catch up" effect arises endogenously in response to the incentives generated by increased wage dispersion. The feedback effect from technical progress to human capital accumulation is consistent with numerous pieces of evidence. First, the feature that more rapid technical change generates rising returns to skill is consistent with recent experience in the U.S. (see Berman, Bound and Griliches 1994) and other countries (e.g. Foster and Rosenzweig 1996, Bartel and Lichtenberg 1987). Second, there is also evidence that agents do respond to such incentives, although there are often "bottlenecks". At the micro level, Mincer (1994) argues that "in both analyses of school education and of job training, the evidence shows that investments in human capital respond positively to profitability."[7] At the macro level, Heckman and Klenow (1997) argue that countries that have high levels of schooling do so *because* of high levels of technology.[8]

● *Rapid skill accumulation generates high net returns to innovation which in turn generates investment in R&D* — Complementarity also implies that faster human capital growth lowers future production costs relative to current production costs, thereby raising the future returns to innovation relative to the current opportunity cost of the labor effort used in R&D. It is this disparity that stimulates the increase in innovation necessary to make human capital accumulation valuable in the long run.

Goldin and Katz (1999) document that the slowdown in the growth rate of skilled relative to unskilled labor in the US between the 1970s and 1980s was a key contributor to the increase in the skill premium.[9] Relatedly, Lloyd–Ellis (1999) argues that the slowdown in the growth rate of average educational attainment experienced in the US after 1970 was responsible for the rise in the returns to unobserved skill, and that this led to a slowdown in the rate of innovation. Through this

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[6]   In a similar reduced form way, Jones (1996) exhibits this catch up feature.

[7]   He adds "Yet the supply of the accumulated stock has not as yet (1991) begun to reduce current profitabilities which are high by historical standards. Lags in the educational pipeline, growing costs, and perverse demographics represent delays and impediments to timely supply effects. It is also very likely that the poor performance of elementary and high school students represents a major bottleneck for the supply adjustment." Mincer (1994, p. 5)

[8]   When they include lifespan in a macro–Mincer regression, as a proxy for future income, they find that it accounts for much of the impact on output — that is, macro regressions attribute too large an output affect to schooling.

[9]   They argue that technological change has been skill biased throughout this century and find no evidence that it has accelerated in recent decades.

channel, technological progress is intimately related to the extent to which the knowledge disseminated by the education system lags behind the frontier knowledge generated by the R&D sector. Rosenberg (2000), for example, argues that the post–war growth success of the U.S. relative to other countries was related to the responsiveness of its universities to the technological frontier in both subject matter and curriculum.[10]

Our model also has several other implications which we discuss in the paper. First, while the existence of two engines of growth expands the menu of growth–enhancing policy instruments, the effectiveness of each instrument is reduced. Second, our model suggests an alternative interpretation of the findings of Bils and Klenow (1999) regarding the causal relationship between schooling and per capita GDP growth. Our model also endogenizes the so called "catch–up" effect posited by Nelson and Phelps (1966) and Jones (1999), provides a simple interpretation of the relationship between growth and intergenerational wage dispersion, and highlights the various roles of lifespan in determining schooling decisions.

The remainder of the paper is structured as follows. We conclude this section with a summary of the related literature, distinguishing our contribution from others. In Section 2, we develop a model of production with minimum skill–requirements and an endogenous skill distribution. In Section 3, we embed this production structure in a dynamic general equilibrium model and characterize the resulting steady–state growth paths. In Section 4, we detail the implications of the model for the effectiveness of growth policy and the interpretation of the results of recent empirical work on cross–country growth. In Section 5, we show how the model can be generalized to allow for rising intermediate quality, positive population growth and a capacity to absorb new ideas that declines with age. Section 6 contains some concluding remarks.

1.1   Related Literature

Using a similar structure of minimum skill requirements to ours, Stokey (1991) develops a model of technology adoption driven by the endogenous accumulation of human capital. Lucas (1993) and Parente (1994) develop related adoption models which emphasize the role of experience with existing technologies as an adoption cost. Parente and Prescott (1994) demonstrate the potential role

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[10]   He argues (p. 46) that "... intellectual innovations were introduced into US universities with remarkable speed, where they were further systemized and entered into the teaching curriculum as soon as their potential utility become apparent."

of adoption costs in accounting for cross-country growth experiences. However, these models assume an exogenous menu of existing productive technologies, and have no role for an independent sector that generates these technologies.[11] In a model featuring endogenous technological change, Young (1995) also emphasizes the importance of skill accumulation in implementing new technologies. However, because he assumes that labor learns serendipitously about technologies, neither the private incentives for human capital accumulation, nor the costs of absorption can be explored.

Grossman and Helpman (1991, ch. 5) extend Romer's (1990) model by endogenizing the level of human capital. However, in their framework the schooling decision is independent of the rate of innovation. The idea that the rate of technical change affects the incentives to acquire skills is emphasized by Chari and Hopenhayn (1991), Greenwood and Yorukoglu (1997), Violante (1998), and Caselli (1999). However, in these models, technological change is exogenous. Eicher (1996), Galor and Tsiddon (1998) and Galor and Moav (1998) develop two–period OLG models in which technological change is a pure spillover from human capital accumulation by previous generations. However, in these models private incentives to invent do not matter. Aghion and Howitt (1998) develop a model in which the accumulation of physical/human capital and innovation can be interpreted as twin engines of growth.[12] However, their formulation requires a human capital production technology identical to that of other goods (as in Mankiw, Romer and Weil, 1992). In contrast, we follow the recent growth literature (e.g. Jones 1996, Bils and Klenow 1999) by modeling human capital formation in a way that is consistent with the Mincerian model.

In response to Jones' (1995) critique that there is no "scale effect", several authors (e.g. Smulders and van de Klundert 1995, Young 1998, Howitt 1999, Segerstrom 1999) have now developed new variants of the ideas–based growth models in which the rate of technological change depends on the growth rate rather than the level of human capital (i.e. population). However, in these models this growth rate of human capital is exogenous. An exception is that of Arnold (1999) who extends Sergestrom's (1999) model, by introducing a Lucas–type human capital accumulation process. However, his model does not feature any of the interactions between technological change and human capital emphasized here.

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[11] Moreover, the rate of adoption does not respond to changes in the *rate* of technological change.
[12] They assume that new technologies are increasingly capital intensive and that innovation becomes increasingly costly over time.

2. Technological Change and Skill Acquisition as Bounded Complements

In this section, we develop a model of production with minimum skill requirements and an endogenous skill distribution. We characterize the resulting labor market equilibrium taking the allocations of labor effort to education and R&D as exogenously given. We show that the interaction between skills and technology at the aggregate level exhibits bounded complementarity and detail the implications of this for growth. In Section 3, we endogenize both labor allocations and characterize the self–sustaining balanced growth path.

2.1 Production and the Demand for Skills

Time, indexed by t, is continuous. A single final good is produced by a continuum of industries, indexed by $i \in [0, I]$. For each industry i, there is a continuum of actual and potential ideas and technologies indexed by $s(i) \in [0, \infty)$. All technologies $s(i) \in [0, n(i)]$ are actual technologies that can be used in production in industry i. In industry i, competitive firms use intermediate goods and services, each of which uses one of the actual technologies currently available. Intermediates are used within each industry according to the constant elasticity of substitution (CES) production function

$$Y(i) = \left[\int_0^{n(i)} x(i, s)^\alpha ds \right]^{\frac{1}{\alpha}}, \tag{1}$$

where $\alpha \in (0, 1)$ and $x(i, s)$ is the quantity of the intermediate used in industry i whose production requires technology s. Intermediate input producers specialize in goods and services sold to industry i. They have infinitely–lived patents over their intermediate good and are monopolists in output markets. However, they must compete against intermediate producers supplying other industries for labor with the same skill requirement. One unit of labor with skill $s' \geq s$, can produce one unit of intermediate (i, s).[13]

Profit maximization by final goods producers yields constant elasticity demand functions for intermediate goods and services:

$$x(i, s) = p(i, s)^{-\frac{1}{1-\alpha}} Y(i). \tag{2}$$

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[13]   The simplifying assumption, implicit in (1), that intermediate productivity does not rise with skill, can be dropped without changing the qualitative nature of the results (see Section 5).

Whatever the quantity of labor hired, intermediate producers earn monopoly profits by setting prices at a mark–up over the competitive wage:

$$p(i, s) = \frac{w(s)}{\alpha}. \tag{3}$$

It follows that the derived demand for labor from the production of intermediates that require at least skill level s is

$$l_d(s) = \int_0^I x(i, s) di = \left(\frac{\alpha}{w(s)} \right)^{\frac{1}{1-\alpha}} Y, \tag{4}$$

where $Y = \int_0^I Y(i) di$.

Following Romer (1990) and Grossman and Helpman (1991), we assume that the introduction of new productive technologies per unit time in industry i, $\dot{n}(i)$, is equal to $l_R(i) K_n(i)/a$, where a is a productivity parameter, $l_R(i)$ is the measure of labor employed in R&D, and $K_n(i)$ is the cumulative stock of industry–specific knowledge in the innovative activity. The latter is assumed to equal $n(i)$, so that $\dot{n}(i) = l_R(i) n(i)/a$. In equilibrium, all industries are identical, so we simplify the notation by dropping the argument i. Thus, we can denote the rate of technical change as

$$g = \frac{\dot{n}}{n} = \frac{l_R}{a}, \tag{5}$$

where $l_R = \int_0^I l_R(i) di$. As in Lloyd–Ellis (1999), we assume that only those workers who have acquired the skills necessary to implement *at least* the most recently introduced technology $n(i)$ are productive in the R&D sector. We let w_R denote the wage paid to agents working in the R&D sector.

2.2   Households and the Supply of Skills

The economy is populated by a measure N continuum of infinitely–lived households. Each household is composed of an infinite stream of continuously overlapping generations. Each generation is the same size, and each individual lives for T units of time. Hence, the size and demographic composition of the population are constant over time and size of the population is NT.[14] Workers are distinguished by the range of technologies that they are capable of implementing. This capability is cumulative: to acquire the skills needed to implement technology $s'(i)$, a worker must be able to implement all technologies $s(i) < s'(i)$. We therefore index a worker's skill by the most recently

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[14]   The model can be adapted to allow for population growth without changing any of the results (see Section 5).

learned technology.

2.2.1 Education and Intergenerational Knowledge Transfer

As in Stokey (1991) and Bils and Klenow (1999) the skills acquired through schooling are a function of the time spent in school and a positive externality from the investments in knowledge made by previous generations. Specifically, we assume that an agent who spends z periods in school and graduates at time t acquires a skill level given by

$$H(t) = e^{\theta(z)} H(t-m) \tag{6}$$

where

$$\theta(z) = \frac{\beta z^{1-\gamma}}{1-\gamma} \tag{7}$$

and $H(t-m)$ denotes the skill level of the generation that graduated m periods earlier. In effect we assume that teachers are m periods older than their students. The function $\theta(\cdot)$ is the same as that discussed by Bils and Klenow (1999), and the parameters m, β and γ reflect the ease with which knowledge is transferred between generations.[15]

If schooling is constant for all cohorts, the minimum skill level in the labor force (i.e. that of the most recent graduates) then grows at the constant rate

$$\lambda = \frac{\dot{H}(t)}{H(t)} = \frac{\theta(z)}{m}. \tag{8}$$

2.2.2 On–the–Job Learning and Specialization

Upon graduation, workers have the capacity to implement technologies requiring a level of skill $s(i) \leq H$. However, higher level skills $s(i) > H(t)$ represent industry–specific knowledge that must be acquired on the job.[16] We assume that the measure of industry–specific skills that can be acquired per period on–the–job is limited by the worker's range of skills at time t:

$$\dot{s}(i,t) = \mu s(i,t). \tag{9}$$

Thus workers' absorptive capacities — the rate at which they can absorb new ideas — are assumed

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[15] In Section 3, we derive restrictions on these parameters that ensure the existence of a unique optimal schooling choice.
[16] The learning we emphasize here is learning about new technologies, not learning to become more productive with a given technology as emphasized by Lucas (1993), Parente (1994) and Young (1995).

to be identical across industries. These specialized skills are not transferrable to future generations except through the education system. As a consequence, μ is not exogenously driving growth. Indeed, the fact that agents learn and acquire skills while working does not, on its own, result in a positive aggregate growth rate — the skills being learned are already known by others. However, on–the–job learning does play a key role in determining the incentives to acquire human capital and is crucial for the existence of an equilibrium balanced growth path.[17]

2.2.3 The Distribution of Skills

Consider a growth path along which the rates of change of frontier knowledge, g, and of transferrable knowledge, λ, are constant and have been for some time. The time t skill level of a worker with experience $a - z$ is

$$s(a; z, t) = e^{\mu(a-z)} H(t + z - a) = e^{(\mu-\lambda)(a-z)} H(t), \tag{10}$$

where $H(t)$ is the human capital of the cohort who graduate at time t. It follows that the time t distribution of skills has support $[H(t), e^{(\mu-\lambda)(T-z)} H(t)]$. A change of variable yields that the skill distribution is given by

$$F(s, t) = \frac{1}{(\mu - \lambda)(T - z)} \ln\left(\frac{s}{H(t)}\right), \tag{11}$$

and has density,

$$f(s) = \frac{1}{(\mu - \lambda)(T - z)} \frac{1}{s}. \tag{12}$$

The fact that the density is declining in skill reflects the fact that more specialized skills are relatively scarce. Since skills accumulate exponentially, more experienced cohorts become more finely divided across tasks or more specialized within their industries.[18]

2.3 Labor Market Equilibrium

Households allocate their labor effort to the market that offers the highest wage for their range of skills. At time t the population is allocated between three activities: education, R&D and production so that

$$l_R + \int_0^n l_d(s)ds = N(T - z) = L, \tag{13}$$

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[17] In Section 5, we discuss how the model can be generalized to allow for learning to slow with age.
[18] With rising intermediate productivity, more experienced workers would become more productive as well as more specialized (see Section 5).

where L denotes the total labor force. The restriction that only those workers with skill levels greater than n can be used in the R&D sector implies that the rate of innovation is bounded from above by the proportion of such agents:

$$l_R = ag \le L[1 - F(n)], \tag{14}$$

where $F(\cdot)$ is the distribution of skills in the labor force at time t. The equilibrium wage of these research workers must be no less than the maximum wage in production, $w(n)$:

$$w_R \ge \overline{w} = w(n). \tag{15}$$

In addition, the market for each skill level must clear. The following Proposition gives the implied conditions:

Proposition 1: *Given $g = l_R/a$ and $\lambda = \theta(z)/m$, labor market equilibrium is characterized by*

$$l_d(s) = \begin{cases} Lf(\underline{\sigma}) & \text{if } s \in [H, \underline{\sigma}) \\ Lf(s) & \text{if } s \in [\underline{\sigma}, \sigma) \\ Lf(\sigma) & \text{if } s \in [\sigma, e^{(\mu - \lambda)(T-z)}H] \end{cases} \tag{16}$$

where $\underline{\sigma}$ and σ are given by

$$Lf(\underline{\sigma}) = \frac{LF(\underline{\sigma})}{\underline{\sigma}} \tag{17}$$

and

$$Lf(\sigma) = \frac{L - ag - LF(\sigma)}{n - \sigma} \tag{18}$$

This equilibrium characterization is discussed in some detail by Lloyd–Ellis (1999). Briefly, given that the steady–state skill distribution is single–peaked, as long as the supply of workers at a particular skill level, s, exceeds the available supply at lower skill levels, $[H, s)$, averaged over lower skill markets, the competitive wage for that skill level cannot rise above that of lower skill levels. Similarly, so long as the supply of workers at a particular skill level s, is less than the available supply of higher skill levels $(s, e^{(\mu - \lambda)T}H]$ averaged over higher skill markets, the competitive wage for that skill level cannot fall below that of higher skill levels. Equations (17) and (18) define two critical skill levels $\underline{\sigma}$ and σ, where $H < \underline{\sigma} \le \sigma \le n$, such that for all $s < \underline{\sigma}$ and for all $s > \sigma$ wages do not rise with skill. Between these two critical skill levels, the wage rises with s reflecting the increasing scarcity of specialized skills.

At any point in time, the labor market equilibrium together with (30) yields a steady–state wage

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profile over skills given by

$$
w(s) = \begin{cases} \alpha \left[y/f(\underline{\sigma}) \right]^{1-\alpha} & \text{if } s \in [H, \underline{\sigma}) \\ \alpha \left[y/f(s) \right]^{1-\alpha} & \text{if } s \in [\underline{\sigma}, \sigma) \\ \alpha \left[y/f(\sigma) \right]^{1-\alpha} & \text{if } s \in [\sigma, e^{(\mu-\lambda)(T-z)}H] \end{cases}
\tag{19}
$$

This steady–state wage profile over skills implies a particular age–wage profile (see Figure 1). Early in agents' lives their skill level is low, and they receive a correspondingly low wage $\underline{w}$ relative to output — this also reflects the relatively low opportunity cost of schooling. Once their skill level grows beyond $\underline{\sigma}$, their wages begin to grow at a rate exceeding overall productivity growth, reflecting the additional specialized skills acquired on the job. Finally, once their skill level has risen beyond σ their wages cease to grow faster than the economy.[19]

— **FIGURE 1 GOES HERE** —

Equilibrium labor productivity is given by

$$
y = \frac{Y}{L} = \left[\underline{\sigma} f(\underline{\sigma})^\alpha + \int_{\underline{\sigma}}^\sigma f(s)^\alpha ds + (n-\sigma)f(\sigma)^\alpha \right]^{\frac{1}{\alpha}}.
\tag{20}
$$

Note that intermediates are not used symmetrically in production, but instead diffuse throughout the economy as the skills required to implement them evolve. As the following proposition illustrates, the impact of technology and skill on aggregate labor productivity exhibits bounded complementarity:

Proposition 2: *In the short–run, labor productivity growth may be driven by* <u>either</u> *technological progress* <u>or</u> *rising human capital. However, sustained growth in labor productivity requires* <u>both</u> *technological progress* <u>and</u> *human capital accumulation.*

To illustrate, suppose we shut down the education sector, so that $\lambda = z = 0$, but let $g > 0$. As long as the maximum skill in the economy exceeds the skill requirement of the most recently introduced technology, $n < e^{\mu T}H$, technological progress will result in rising labor productivity due to increasing returns to specialization. However, eventually it will be the case that $n = e^{\mu T}H$. Beyond this point the skills needed to implement further innovations no longer exist and growth

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[19] Note that $\underline{\tau}(t)$ represents the time period when cohort t starts to experience rising *relative* wages, and $\overline{\tau}(t)$ represents the time period when cohort t stops experiencing rising *relative* wages. Wages rise throughout their lifetime, but just rise relatively rapidly in the between these two time periods.

must cease. Conversely, suppose we shut down the R&D sector so that $g = 0$, but let $\lambda > 0$. In this case, skill accumulation may raise labor productivity by allowing a more efficient usage of existing intermediate technologies in final production. However, as H grows relative to n, it will eventually be the case that the skill–constraints on the use of intermediates in final production no longer bind, and their marginal products are equalized.[20] Beyond this point, production efficiency cannot be improved any further and growth must cease.

3. Endogenous Growth

In this section, we embed the production structure described in Section 2 in a dynamic general equilibrium model of endogenous growth. In particular, we characterize the balanced growth paths that arise when labour is allocated optimally across activities. In Section 4, we illustrate the implications of this model for the effectiveness of growth–promoting policies and the interpretation of recent empirical analyses of growth.

3.1   Intertemporal Optimization

We consider a closed economy in which households have identical preferences over a single final good given by the discounted flow

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} \log c(\tau) d\tau, \tag{21}$$

where $\rho \in (0, 1)$ is the rate of pure time preference and $c(\tau)$ is their time τ consumption. The final good is the numeraire. As described above, each household is composed of an infinite stream of continuously overlapping generations. Households can borrow or lend freely at the equilibrium instantaneous interest rate $r(\tau)$, there is no utility of leisure and children do not inherit the human capital of their parents. It follows that although consumption and schooling decisions are made by the household, we can break the optimization problem down into two steps.

First, household member j chooses the length of time in school, z, so as to maximize his/her contribution to household wealth. The resulting lifetime wealth of the jth successor of the current

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[20]   With rising intermediate productivity, the efficient allocation of intermediates requires that price differences reflect only productivity differences (see Section 5).

household member is given by

$$W(j) = \max_z \int_{Tj+z}^{T(j+1)} e^{-[R(\tau)-R(Tj)]} w(j,\tau) d\tau \tag{22}$$

where $R(\tau) = \int_0^\tau r(\nu) d\nu$ represents the discount factor, $w(j,\tau)$ is the time τ wage rate of the current household member's jth successor. A second–order condition must also hold (see below).

Second, a household whose current member was born at time $t = 0$ maximizes utility (21) subject to the intertemporal budget constraint:

$$\int_t^\infty e^{-[R(\tau)-R(t)]} c(\tau) d\tau \leq \int_t^T e^{-[R(\tau)-R(t)]} w(0,\tau) d\tau + \sum_{j=1}^\infty e^{-R(Tj)} W(j) + A(t) \tag{23}$$

where $A(t)$ is the current value of the household's assets and $\lim_{t\to\infty} e^{-\rho t} A(t) = 0$.

Household assets consist of claims to the profits streams of intermediate firms. The value of claims to the profits of intermediate producers in industry i using technology s, $\{\pi(i,s,\tau)\}_{\tau=t}^\infty$, is

$$V(i,s,t) = \int_t^\infty e^{-[R(\tau)-R(t)]} \pi(i,s,\tau) d\tau. \tag{24}$$

The reward to investment in R&D is a claim, with value $V(n,t)$, to the profits of intermediate firms that use the technology. From (32), the labor used to produce one new technology ($\dot{n}(t) = 1$) is $a/n(t)$ and each unit of labor costs w_R. With free entry into the sector, the equilibrium value of this claim must equal the cost of innovation:

$$V(n) = \frac{w_R a}{n}. \tag{25}$$

3.2 General Equilibrium

Given initial stocks of transferrable skill, $H(0)$, and frontier knowledge, $n(0)$, a **competitive equilibrium** for this economy satisfies the following conditions:

• Final goods producers choose intermediates to maximize profits, (2).

• Intermediate producers set prices and hire labor so as to maximize profits, (3) and (4).

• Workers allocate themselves to the labor market which offers the highest wage for their skill.

• Households choose the optimal amount of schooling for each cohort, (22).

• Households allocate consumption over time so as to maximize (21), subject to (22) and (23). With time separable log–utility and perfect capital markets, the consumption of each household,

and therefore aggregate consumption, C, optimally grows at the instantaneous rate:

$$\frac{\dot{C}}{C} = \frac{\dot{c}}{c} = r - \rho. \tag{26}$$

- Firms in the R&D sector earn zero profits, (25).

- The final goods market clears. Since the final good is the numeraire and the economy is closed, this implies that

$$C(t) = Y(t). \tag{27}$$

- The asset market clears. Differentiating (24) with respect to time yields the no arbitrage condition, which requires that the rate of return on all firm shares must equal the rate of interest (see Grossman and Helpman, 1991). In particular, for the most recently introduced intermediate,

$$\frac{\pi(n)}{V(n)} + \frac{\dot{V}(n)}{V(n)} = r, \tag{28}$$

where $\pi(n)$ is the total profit of intermediate firms using the most recently introduced technology, n.[21]

- The labor market clears: (13), (14) and (15) hold, and the market for each skill–level clears.

3.3 Stationary Growth Paths

We now characterize the stationary competitive growth paths for this economy. Along such a path the distribution of skills is given by (11), and the individual labor market clearing conditions are given by (16), (17) and (18).[22] A key variable in our analysis is an index of "labor market tightness" given by

$$\psi = \frac{z}{T} + \frac{1}{(\mu - \lambda)T} \ln\left(\frac{\sigma}{\underline{\sigma}}\right) = \frac{z}{T} + \frac{1}{(\mu - \lambda)T} \ln\left(\frac{\overline{w}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}. \tag{29}$$

The variable ψ measures the impact of the steady–state labor market equilibrium on the incentives both to invest in education and to invest in R&D. The first term is the portion of time each cohort spends in school and the second term reflects the growth rate in the wages during a worker's lifetime. The larger is ψ the greater is the incentive for individual households to invest in education. This is because (a) the more time individuals spend in school the fewer low-skilled workers in the labor

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[21] Note that it must be the case in equilibrium that $n(t) < e^{(\mu - \lambda)(T-z)} H(t)$.

[22] This distribution of skills and the associated labor market equilibrium conditions are correct *only* along a stationary growth path.

force and the greater the returns to scarce skills, and (b) the greater is the growth in wages the higher are the net returns from education. In contrast, the larger is ψ the less incentive firms have to invest in R&D. This is because (a) the more time individuals spend in school, the smaller is the labor force and, hence, the lower is the size of the market, and (b) the greater is the relative cost of skilled labor, the lower are the net profits from innovation. Note that increased on the job learning μ raises the opportunity costs of schooling, but slackens the constraints associated with skill, thereby reducing the opportunity cost of research.

Along a stationary growth path, g, λ and ψ are constant. This implies that

Proposition 3: *Along a balanced growth path, frontier knowledge must grow at the same rate as transferrable knowledge:*

$$g = \lambda. \tag{30}$$

Although the stock of frontier knowledge strictly exceeds that of transferrable knowledge (i.e. $n > H$), (30) implies that both frontier and transferrable knowledge must *grow* at the same rate in a steady state. If this were not the case, say if $g > \lambda$, the skill constraints would become more binding over time as the gap between new and transferrable knowledge expands. This would increase ψ, reflecting an increase in the returns to investment in human capital and an increase in the relative cost of R&D. It follows that λ would tend to increase, and g would tend to decrease over time. This strengthens the result in proposition 2: when labor allocations are endogenized, not only are both types of knowledge accumulation necessary for sustained growth, but both must grow at the same rate.

>From this, we also derive that despite the non–symmetric use of intermediates, labor productivity grows at a rate, g_y, which is proportional to the rate of innovation given by

$$g_y = \left(\frac{1-\alpha}{\alpha} \right) g. \tag{31}$$

This is also the growth rate of the wage associated with a given skill level over time.

3.4 The Unconstrained Growth Path

Along an **unconstrained growth path,** the labor market equilibrium is such that the measure of workers engaged in R&D is strictly less than the measure capable of doing so ($n > \sigma$). It follows

that (15) holds with a strict inequality. This also implies that the wage in research will be exactly equal to the highest wage in production ($w_R = w(\sigma)$). The unshaded area OXY in Figure 2 illustrates the combinations of g and ψ that are consistent with such a labor market equilibrium. Correspondingly, the boundary XY represents combinations of g and ψ which yield a labor market equilibria in which all workers who are capable of working in the R&D sector are doing so ($n = \sigma$). This boundary is described by

$$\psi = 1 - \frac{ag}{NT} - \frac{1}{(\mu - \lambda)T}. \tag{32}$$

Along this XY boundary, high growth is associated with low values of ψ because high rates of innovation can only be sustained by high levels of education that serve to compress the dispersion of skills.

Combining (4), (13), (25), (26), (27), (28) and (31) yields the stationary combinations of g and ψ such that the expected benefits from the marginal innovation are just equal to the labor costs of R&D, and equilibrium in the labor market obtains. This is given by

$$\psi = 1 - \frac{ag}{NT} - \frac{\alpha a(\rho + g)}{(1 - \alpha)NT}, \tag{33}$$

and is depicted as the RD curve in Figure 2. One can interpret the RD curve as being analogous to a relative demand curve for *growth* in new skills. The schedule is downward sloping reflecting the fact that, in equilibrium, investment in R&D, g, depends negatively on the cost of skilled labor relative to that used in production, which is reflected by ψ. Given ψ, the rate of innovation depends positively on the productivity of R&D labor, $1/a$, and the size of the population, NT.[23] An increase in either of these variables will cause the RD schedule to shift up and to the right. Note that if $\psi = 0$, implying no wage dispersion in production and no education sector, this equation would be identical to that in Grossman and Helpman (1991, p. 61).

Solving (22), and using (26) and (31), yields the household's optimal schooling condition in steady state:

$$z = \psi T - \frac{1}{\rho - (1 - \alpha)(\mu - \lambda)} \ln \left[\frac{\beta z^{-\gamma} - \mu}{\beta z^{-\gamma} - \mu - e^{\frac{\rho}{\mu - \lambda}} \left(\frac{\rho}{1 - \alpha} - (\mu - \lambda) \right)} \right]. \tag{34}$$

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[23] Note however that the model can easily be adapted to allow for population growth (see Section 5).

Rearranging (34) yields the stationary combinations of λ and ψ such that the optimal schooling equation is satisfied. It is described by

$$\psi = \frac{z(\lambda)}{T} - \frac{\ln\left[1 - \left(\frac{\rho}{(1-\alpha)} - (\mu - \lambda)\right)\frac{e^{\frac{\rho}{\mu-\lambda}}}{\beta z(\lambda)^{-\gamma} - \mu}\right]}{[\rho - (1-\alpha)(\mu - \lambda)]T} \tag{35}$$

where, from (7) and (8),

$$z(\lambda) = [m(1-\gamma)\lambda/\beta]^{\frac{1}{1-\gamma}} \tag{36}$$

This is depicted as the ET curve in Figure 2. One can interpret the ET curve as being analogous to a relative supply curve of *growth* in labor force skills.

Lemma 1: *Along the ET curve, ψ and λ are positively related.*

The ET curve is upward sloping because household increments to human capital, λ, depend positively on the expected value of future wages relative to the current cost of acquiring human capital. Given ψ, the growth rate in the supply of skills depends positively on the rate of absorption, μ, the productivity of education, $1/\gamma$ and β, and lifespan, T. An increase in any of these variables will cause the ET curve to shift up and to the left.

— **FIGURE 2 GOES HERE** —

Along the unconstrained growth path, the equilibrium rates of innovation and skill acquisition, $g^* = \lambda^*$, and labor market tightness, ψ^*, are determined by the intersection of the RD and ET curves, as illustrated in Figure 2. In equilibrium, it is the relative returns to education and to R&D which ensure that growth in both factors is equalized. The following proposition provides conditions which guarantee that the two curves intersect in the region OXY:

Proposition 4: *If*

$$\alpha a \rho < (1-\alpha)NT \tag{37}$$

and if the rate of knowledge growth g^* which solves (30), (33), (35) and (36) satisfies

$$\frac{1}{(\mu - g^*)} < \frac{\alpha a(\rho + g^*)}{(1-\alpha)N} \tag{38}$$

and

$$g^* < \frac{\gamma\mu}{\gamma + (1-\gamma)\rho m} \tag{39}$$

18

then an unconstrained steady–state growth path exists and is unique.

Uniqueness follows from the fact that the RD curve is downward sloping and the ET curve is upward sloping. The condition on parameters given in (37) ensures that the RD curve lies above zero when $g = 0$. Since the ET curve passes through the origin ($\theta'(0) = \infty$), the two curves must intersect in the positive quadrant. Condition (38) ensures that the intersection must be within the boundary XY and condition (39) ensures that the optimal schooling choice is a maximum.[24] Note that both the existence and the uniqueness of the balanced growth path stem from the bounded complementarity between technology and human capital implicit in our model.

3.5   The Constrained Growth Path

An unconstrained equilibrium does not exist for all parameter values. In particular, it is possible that, at the maximum production wage, the demand for R&D workers would exceed the supply of those who are capable of working in that sector. Equilibrium then requires that all sufficiently skilled workers work in the R&D sector ($n(t) = \sigma(t)$). This implies that (14) holds with equality, and that the R&D wage is bid up above the highest production wage, $w_R(t) > \overline{w}(t)$. Along such a **constrained growth path**, the RD curve in (33) is no longer relevant because for a given distribution of skills, the supply of agents with $s > n$ constrains the level of activity in the R&D sector. Instead, free entry into the R&D sector and asset market equilibrium together determine the wage in R&D relative to that in production:

$$\frac{w_R}{\overline{w}} = \frac{(1-\alpha)N}{\alpha a(\mu - \lambda)(\rho + g)}. \tag{40}$$

The supply curve for new skills growth (35) is replaced by an "adjusted" inverse relative supply curve of new skills, which captures the fact that that the greater is $\frac{w^R}{\overline{w}}$, the lower the maximum–minimum wage ratio in the production sector required to induce a given investment in human capital, λ. Note that R&D incentives affect the long–run growth rate *only* indirectly by raising the returns to human capital accumulation via $\frac{w_R}{\overline{w}}$ and thereby inducing a greater supply of skills.

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[24]   In fact, this condition is stronger than the sufficient condition derived in the appendix.

4. Implications

In this section, we highlight some of the implications of our model. Since our growth process nests those implied by other models, many standard results in the growth literature also apply here, at least qualitatively. Consequently, we focus only on implications that are specific to our framework. Not surprisingly, in contrast to models with a single primary engine of growth, factors that affect both R&D and human capital accumulation have effects on growth. However, the nature of these growth effects reflect the crucial role of bounded complementarity within our model. In order for a factor to increase growth, it must not only increase the rate of accumulation of one type of knowledge, but also induce an accompanying increase in the accumulation of the other type of knowledge. Without this induced response, higher growth could not be sustained. If, however, that were this complementarity not bounded, growth would be explosive. More precisely, factors which directly stimulate innovation (e.g. R&D productivity, $1/a$) raise the returns to investment in education, which induces the necessary supporting increase in education. Similarly, factors which directly stimulate increased investment in education (e.g. the quality of education, β) raise the returns to investment in R&D, thereby inducing the expansion of technology that gives the additional human capital value.

4.1 Policy Implications

Suppose the government introduces subsidies to R&D and education and finances them through lump–sum taxation. If the government pays a fraction ϕ of R&D costs and a fraction ω of schooling costs, its balanced budget condition requires that it sets lump sum taxes b so that

$$bNT = \omega \underline{w} z N + \phi w_R l_R. \tag{41}$$

The free entry condition in the R&D sector (25) is now replaced by

$$V(n) = \frac{w_R a(1-\phi)}{n}, \tag{42}$$

and the household's schooling choice now solves

$$\max_{z} \int_{Tj}^{Tj+z} e^{-[R(\tau)-R(Tj)]}[\omega w(j,\tau) - b(\tau)]d\tau + \int_{Tj+z}^{T(j+1)} e^{-[R(\tau)-R(Tj)]}[w(j,\tau) - b(\tau)]d\tau. \tag{43}$$

20

Proceeding as before yields the following equilibrium relationships:

$$\psi = 1 - \frac{ag}{NT} - \left(\frac{\alpha a (1 - \phi)(\rho + g)}{(1 - \alpha)NT} \right) \tag{44}$$

$$\psi = \frac{z(\lambda)}{T} - \frac{\ln \left[1 - \left(\frac{\rho}{(1-\alpha)} - (\mu - \lambda) \right) \left(\frac{1-\omega}{\beta z(\lambda)^{-\gamma} - \mu} \right) e^{\frac{\rho}{\mu - \lambda}} \right]}{[\rho - (1 - \alpha)(\mu - \lambda)]T} \tag{45}$$

Growth responds positively to both types of subsidy. However, the fact that both technological change and human capital accumulation are necessary for growth means that, although the menu of possible policy instruments is expanded, the effectiveness of any individual instrument used in isolation is diminished:

Proposition 5:

(a) The long–run effectiveness of R&D subsidies is constrained by the responsiveness of education to the growth in demand for skills.

(b) The long–run effectiveness of education subsidies is constrained by the responsiveness of R&D to the growth in supply of skills.

These implications can be understood using Figure 3. In the left panel, an R&D subsidy causes the RD curve to shift to the right. In the standard innovation–based growth model, the ET curve would be horizontal, reflecting the perfect technological mobility of labor. The effect of the subsidy on growth would be to increase it from g_0 to g_2. However, in our model, absorptive capacity on the job is bounded, so that a long–run increase in growth can only be supported if human capital growth also rises. This is accomplished by the endogenous rise in the relative returns to schooling, but since this also raises the relative costs of R&D, the increase in growth is partially offset and the actual long–run growth rate increases only as far as g_1.

Conversely, in the right panel, a subsidy to education causes the ET curve to shift to the right. In a human capital–based growth model, the RD curve would be horizontal, reflecting the costless introduction of technology. The effect of the subsidy would be to increase growth from g'_0 to g'_2. However, in our model the increase in productivity that can be obtained by increasing human capital on a given set of technologies is bounded, so that a long–run increase in growth can only be sustained if new technologies are developed and adopted. This is accomplished by the endogenous increase in

21

the relative return to R&D, but since this also reduces the relative returns to schooling, the increase in growth is partially offset and the growth rate increase only as far as g_1'.

— **FIGURE 3 GOES HERE** —

4.2   The Empirical Link between Schooling and Growth

The expression for optimal schooling (34) is similar to that derived by Bils and Klenow (1999, equation 16). This should not be surprising given that they are both based on the same Mincerian model of schooling. However, there are some important differences. Theirs is a partial equilibrium analysis, with a fixed interest rate, in which expected productivity growth enters through its effect on discounting. In our general equilibrium analysis, under the assumption of log preferences, this effect is not present because the interest rate fully adjusts in response to high consumption growth: $r - g_y = \rho$. As a consequence, productivity growth affects schooling through its equilibrium impact on incentives via labor market tightness, ψ. Technological change affects schooling decisions not only through the valuation of future wages via discounting, but also through affecting wages and returns to education directly.[25] This more direct effect of technological change on educational investments is a consequence of bounded complementarity between education and technology.

The relationship between schooling and growth in our model also differs from that discussed by Bils and Klenow for other reasons. First, in our basic model, wages rise with skill only because higher skills are increasingly scarce.[26] This implies an obsolescence effect whereby an individual's wage growth depends on the rate at which he/she learns *relative* to the growth in the human capital of the most recent graduates $(\mu - \lambda)$. Second, firms have monopoly power so that households do not appropriate the full static returns from their investments $(\alpha < 1)$. Finally, individuals' wages grow relative to GDP for only a portion of an individual's lifetime $(\psi < 1)$.

4.3   The Catch–Up Effect

Nelson and Phelps (1966) posit that human capital accumulates faster, the further it is behind the technological frontier. Jones (1999) captures this idea in a reduced form way in accounting for

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[25]   In our model, we could allow for both effects by assuming CES preferences, in which case $r - g_y = \rho - (1 - \sigma) g_y$.
[26]   When intermediate productivity rises with skill, wages also depend on own human capital.

post–war US economic growth. The human capital accumulation and innovation processes that he specifies have these effects imposed exogenously. Indeed, because he treats the allocations of labor effort to education and R&D as exogenous, this is the only way such a catch up effect could arise. In contrast, the catch–up effect arises endogenously in our model both in human capital investment *and* innovation — rapid accumulation of one type of knowledge stimulates accumulation of the other type of knowledge via the distribution of wages. This effect again reflects the importance of endogenizing both schooling (as highlighted by Bils and Klenow, 1999) and innovation .

Relatedly, the model also implies that economies which grow fastest in steady state are also those with the smallest gap between the frontier knowledge generated by a given R&D technology, and the transferrable knowledge generated by the education sector. Rosenberg (2000) argues that the post–war growth success of the U.S relative to other countries was related to the responsiveness of its universities to the technological frontier in terms of subject matter and curriculum.

4.4 The Relationship Between Growth and Intergenerational Wage Dispersion

Our basic model has predictions regarding the relationship between intergenerational wage dispersion and the sources of growth differences across countries. If growth differences are predominantly due to differences in the quality of the educational system, β, faster growing economies should have flatter experience wage profiles. However, if they are predominantly due to differences in R&D productivity or differences in the ease of technology adoption, $1/a$, faster growing economies will tend to have steeper experience wage profiles.[27] Bils and Klenow (1999) find no significant relationship between growth and intergenerational wage dispersion. They interpret this as evidence against a large effect of variations in schooling quality on growth. However in our model with its second engine of growth, this finding is more difficult to interpret, since both differences in educational and R&D quality have conflicting consequences for the relationship between intergenerational wage dispersion and growth. In particular if, as seems plausible, high growth economies tend to dominate along both dimensions, the two effects would offset each other and be hard to disentangle empirically.

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[27] The latter prediction is, in part, a result of the assumption that individuals learn at a constant rate throughout their lives. In Section 5, we discuss how the results change if absorptive capacity declines with age.

4.5 The Role of Lifespan

Heckman and Klenow (1997) and Bils and Klenow (1999) argue that longer lifespan stimulates educational investment because it implies a longer time over which to reap the benefits. In our model, lifespan also has another general equilibrium effect — it generates a scale effect, raising the innovation rate and the returns to education. Increased lifespan increases the amount of knowledge that can be learned on the job. This additional skill accumulation increases the supply of skilled workers, depresses the cost of innovation, and ultimately speeds up growth. Although, with zero population growth, the effect of lifespan cannot be distinguished from the overall scale effect, when we adapt the model so as to mitigate the scale effect and allow for population growth, the distinct effect of lifespan becomes clear (see Section 5).

5. Extensions to the Basic Model

5.1 Rising Intermediate Productivity

In the basic model, output growth arises from increasing returns to specialization in intermediate goods. The only distinction between technologies is their skill requirement — more recently introduced technologies have higher marginal productivities only because there is a limited supply of specialized labor with the requisite skill. However, more realistically, for a given quantity of labor, newer technologies might be expected to be inherently more productive than older ones. Qualitatively speaking, all of the previous results generalize when we allow intermediate productivity to rise over time.

Suppose that technology s produces s^δ units of intermediate goods and services from each unit of sufficiently skilled labor, where $\delta > 0$. The resulting derived demand for labor with skill level s is then

$$l_d(s) = s^{\frac{\alpha\delta}{1-\alpha}} \left(\frac{\alpha}{w(s)}\right)^{\frac{1}{1-\alpha}} Y, \tag{46}$$

where aggregate output Y is given by

$$Y = \left[\int_0^n [s^\delta l_d(s)]^\alpha ds\right]^{\frac{1}{\alpha}}. \tag{47}$$

The labor market equilibrium conditions can be derived in a similar fashion to that previously de-

scribed, by simply re–formulating the arguments in terms of "productivity–adjusted" demand for labor with skill s:

$$\hat{l}(s) = \frac{l_d(s)}{s^{\frac{\alpha\delta}{1-\alpha}}} = \left(\frac{\alpha}{w(s)}\right)^{\frac{1}{1-\alpha}} Y. \tag{48}$$

For low skill levels, the demand for labor from intermediates using the associated technology, completely exhausts the available supply without driving the wage above that earned by higher–skilled workers. However, there exists a skill level σ above which the productivity adjusted supply of labor with skill index s, $f(s)/s^{\frac{\alpha\delta}{1-\alpha}}$ is less than the productivity–adjusted supply of labor with skills $[s, S]$ divided equally across technologies, $[\sigma, n]$. It follows that, in equilibrium, an equal productivity–adjusted quantity of labor is allocated to each technology $s \in [\sigma, n]$, and that the actual quantity of labor using technology s must be $(\frac{s}{\sigma})^{\frac{\alpha\delta}{1-\alpha}} f(\sigma)$. Since this exceeds $f(s)$, the remainder is drawn from the pool of workers with skills greater than n that are not being used in the R&D sector, $1 - F(n) - ag/L$. Labor market equilibrium is therefore characterized by

$$l_d(s) = \begin{cases} L\left(\frac{s}{\underline{\sigma}}\right)^{\frac{\alpha\delta}{1-\alpha}} f(\underline{\sigma}), & \text{if } s \in [H, \underline{\sigma}] \\ Lf(s), & \text{if } s \in (\underline{\sigma}, \sigma) \\ L\left(\frac{s}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}} f(\sigma), & \text{if } s \in [\sigma, e^{(\mu-\lambda)(T-z)}H] \end{cases} \tag{49}$$

where

$$\int_0^{\underline{\sigma}} \left[\left(\frac{s}{\underline{\sigma}}\right)^{\frac{\alpha\delta}{1-\alpha}} f(\underline{\sigma})\right] ds = F(\underline{\sigma}). \tag{50}$$

and

$$\int_{\underline{\sigma}}^n \left[\left(\frac{s}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}} f(\sigma)\right] ds + \frac{ag}{L} = 1 - F(\sigma). \tag{51}$$

The associated wage profile is qualitatively similar to (13), except that the elasticity of the wage with respect to skills $s \in (\underline{\sigma}, \sigma)$ is now $(1 - \alpha) + \alpha\delta$.

The balanced growth path with rising intermediate productivity satisfies (30) the following system of equations

$$\psi = 1 - \frac{ag}{NT} - \frac{\alpha a(\rho + g)}{(1 - \alpha + \alpha\delta)NT} \tag{52}$$

$$\psi = \frac{z(\lambda)}{T} - \frac{\ln\left[1 - \frac{e^{\frac{\rho}{\mu-\lambda}\left(\frac{1-\alpha}{1-\alpha+\alpha\delta}\right)}}{z(\lambda)^{-1} - \mu}\left(\frac{\rho}{1-\alpha+\alpha\delta} - (\mu - \lambda)\right)\right]}{[\rho - (1 - \alpha + \alpha\delta)(\mu - \lambda)]T} \tag{53}$$

$$g_y = \left(\frac{1-\alpha}{\alpha}\right) g + \delta\lambda. \tag{54}$$

This equilibrium growth path is very similar to that without rising intermediate productivity, and all of the implications remain qualitatively unchanged. An increase in the size of productivity increments raises the incentives to invest in R&D causing the modified RD schedule (52) to shift up. Similarly, it raises the return to an increment in the skill level shifting the modified ET schedule (53) to the right. The combined effect induce greater productivity growth. As (54) shows, the impact of technological change on productivity growth can be decomposed into two parts: an effect due to the returns to specialization associated with more technologies in production, and an effect due to the increasing productivity of these technologies.

5.2 Population Growth

Our basic model exhibits a standard scale effect — growth increases with the size of the population.[28] Although the scale effect diminishes as the population grows, because it is partly diverted into rising wage costs in R&D, sustained population growth still places the economy on an explosive growth path. However, it is straightforward to adapt the model to allow for population growth without changing the results qualitatively.

We assume that the cohort born at time 0 has measure $N(0) = 1$, and that growth in the number of agents born is given by ν. This implies that the size of the cohort born at time t is $N(t) = e^{\nu t}$. Then, the cohort of age a at time t has measure $N(t-a) = e^{-\nu a}N(t)$. It follows that the total labor force in the economy at time t is

$$L(t) = \int_z^T e^{-\nu a} N(t) da = \left(\frac{1 - e^{-\nu(T-z)}}{\nu}\right) e^{-\nu z} N(t), \tag{55}$$

and the experience distribution is described by

$$\Pr(\hat{a} - z < a - z) = \frac{1 - e^{-\nu(a-z)}}{1 - e^{-\nu(T-z)}}. \tag{56}$$

As before, in the steady state, the time t skill of an agent of age a is $s = e^{(\mu-\lambda)(a-z)}H(t)$, and a

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[28] Jones (1995) has argued that this kind of scale effect is inconsistent with the available evidence.

change of variable yields the distribution of skills given by

$$F(s) = \frac{1}{1 - e^{-\nu(T-z)}} \left(1 - \left(\frac{s}{H} \right)^{-\frac{\nu}{\mu-\lambda}} \right),$$ (57)

with density

$$f(s) = \frac{1}{1 - e^{-\nu(T-z)}} \left(\frac{\nu}{\mu-\lambda} \right) \left(\frac{s}{H} \right)^{-\frac{\nu}{\mu-\lambda}} \frac{1}{s},$$ (58)

and support $[H, e^{(\mu-\lambda)(T-z)}H]$. Note again that this is a single–peaked density function. In the basic model without population growth, more skilled workers are scarce because they are more special-ized and therefore spread over a broader range of technologies. Now, beyond this specialization, highly skilled workers become relatively more scarce as larger generations of young, less experi-enced workers enter the economy.

Following Young (1998) and Howitt (1999), we assume that the scale effect is mitigated by the proliferation of final goods. Specifically we assume that increments in the number of final products is proportional to the population:

$$\dot{I} = \gamma P$$ (59)

Since the population growth rate is constant, the number of workers per industry P/I converges monotonically to the constant: ν/γ. We assume that this convergence has already occurred, so that $P = \nu I/\gamma$. Increasing the number of final goods does not affect aggregate output, but does imply that labor must be more finely divided across industries. Since research is industry specific, the proportion of the labor force used in the R&D sector is given by

$$\frac{l_R}{L} = ag\frac{I}{L} = \frac{ag\gamma}{\nu e^{-\nu z}} \left(\frac{1 - e^{-\nu T}}{1 - e^{-\nu(T-z)}} \right).$$ (60)

With these changes, the stationary combinations of ψ and g which are consistent with free entry in the R&D sector are described by a modified RD curve. If population growth is small, this curve can be expressed as

$$\psi = -\frac{1}{\nu T} \ln \left[ag\gamma T + \frac{\alpha a}{1 - \alpha} (\rho + g) \gamma T + e^{-\nu T} \right].$$ (61)

Increased population growth unambiguously shifts this modified RD curve up and to the right. Stationary combinations of ψ and λ which are consistent with the optimal schooling decision are described by a modified ET curve. This modified ET schedule is quite similar to that without

27

population growth. Indeed if population growth is small, the schedule can be written as

$$\psi = \frac{z(\lambda)}{T} - \frac{\ln\left(1 - \left(\frac{\mu-\lambda}{z(\lambda)^{-1}-\mu}\right)\left[\frac{\rho-(1-\alpha)(\mu-\lambda+\nu)}{(1-\alpha)(\mu-\lambda+\nu)}\right]e^{\frac{\rho}{\mu-\lambda}}\right)}{[\rho-(1-\alpha)(\mu-\lambda+\nu)]T}, \tag{62}$$

where $z(\lambda)$ is given by (36). Increased population growth has an ambiguous impact of the ET curve.

Introducing population growth in the manner described here does not qualitatively change the results discussed in Section 4. For small population growth rates, productivity growth is increasing with population growth. However, for larger values of ν, the impact of population growth on per capita income growth is not unambiguously positive. This ambiguity arises since a fast growing population raises the relative cost of education by magnifying the relative scarcity of older more skilled workers, offsetting the effects of population *growth* on R&D. This result contrasts with other models of "growth without scale effects". Note finally that, as suggested earlier, a scale effect due to lifespan T remains, even though the cohort–size scale effect is removed.

5.3   Old Dogs and New Tricks

In our basic model we assume that, once they enter the work force, agents acquire further skills through on the job learning throughout their lives. Suppose instead that for an agent born at time t learning initially occurs at a rate μ, but that at some age a^* learning stops.[29] This captures the reduction in learning in a simple way. It follows that the time t skill level of a worker with experience $a - z$ is

$$s(a; z, t) = \begin{cases} e^{(\mu-\lambda)(a-z)}H(t) & \text{if } a \in (z, a^*) \\ e^{\mu(a^*-z)-\lambda(a-z)}H(t) & \text{if } a \in (a^*, T) \end{cases} \tag{63}$$

The resulting impact of experience on human capital can be viewed as a proxy for the quadratic term typically assumed in Mincerian wage equations. Although skills never actually decumulate with age, they do relative to the most recently graduated cohorts. The minimum skill level at time t is

$$\min[H(t), e^{\mu(a^*-z)-\lambda(T-z)}H(t)]. \tag{64}$$

Assume that $\mu(a^* - z) \geq \lambda(T - z)$, so that the oldest generation has learned enough to have higher skills relative to the youngest (this need not be the case). Then, the support of the time t distribution

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[29]   That the learning rate falls to zero simplifies the exposition, but we could alternatively have assumed some positive rate less than μ.

of skills is $[H(t), e^{(\mu-\lambda)(a^*-z)}H(t)]$ and the distribution of skills is

$$
F(s) = \begin{cases} \frac{1}{(\mu-\lambda)(T-z)} \ln\left(\frac{s}{H(t)}\right) & \text{if } s < e^{\mu(a^*-z)-\lambda(T-z)}H(t) \\ \frac{\mu}{\lambda(\mu-\lambda)(T-z)} \ln\left(\frac{s}{H(t)}\right) + 1 - \frac{\mu(a^*-z)}{\lambda(T-z)} & \text{otherwise} \end{cases}
\tag{65}
$$

with density

$$
f(s) = \begin{cases} \frac{1}{(\mu-\lambda)(T-z)}\frac{1}{s} & \text{if } s < e^{\mu(a^*-z)-\lambda(T-z)}H(t) \\ \frac{\mu}{\lambda(\mu-\lambda)(T-z)}\frac{1}{s} & \text{otherwise} \end{cases}
\tag{66}
$$

Note that the distribution of skills remains single–peaked, so that Proposition 1 is still applicable. In equilibrium, if we looked at a cross–section of ages in a given time period, wages would initially rise relative to GDP, and then fall with age. In this modified version of the model, R&D investment incentives are not much altered. Moreover, although an agent's lifetime wage profile is somewhat different, so long as on–the–job learning continues for a sufficient time period after graduation, their basic incentives are not that much altered either.[30] Consequently, one can show (see Appendix B) that an equilibrium growth path which is qualitatively similar to that characterized in Section 3 continues to exist.

6. Concluding Remarks

In this paper, we have constructed a model in which technologies and skills are bounded complements in the determination of the growth process. As a result, growth cannot proceed without *both* types of knowledge accumulation, and *both* innovation and skill acquisition depend on private incentives to make costly investments. Our model captures two important intuitions: that both the distribution of skills and wages are endogenous, generated by the response of individuals to the incentives generated by technological change, and that the amount of innovation depends on the relative scarcity and hence the cost of skill. In addition, to providing a general framework for understanding the interactions between technological progress, human capital formation and productivity growth, our model has unique implications for the effectiveness of growth–enhancing policies, for the interpretation of the link between growth and schooling, and for other features of the growth process.

---

[30]   The analysis is somewhat tedious, but follows the same steps as before. The experience wage profile now consists of 5 parts. Relative to GDP, wages are constant initialy, then grow plateauing at a maximum. After learning stops, the wage then falls relative to GDP, before becoming constant again.

In its most limited sense, our model might be interpreted as a model of a leading–edge developed economy. Less developed economies do not engage in much frontier R&D, but rather acquire existing technologies from abroad. Jovanovic (1997) argues that a model of adoption (e.g. Parente, 1994) is a more appropriate characterization for LDCs than innovation–based models of growth. However, the initial adoption of new technologies is costly.[31] That LDC governments are aware of this is evidenced by the significant allocation of resources towards research extension services (see Hoff, Braverman and Stiglitz, 1990). In a more general model with international technology spillovers (along the lines of Howitt, 2000), one could re–interpret our R&D sector as an initial adoption sector which procures technologies internationally and engages in their costly adoption. Once the initial adoption costs have been incurred, it becomes possible for the knowledge to disseminate throughout the economy, and eventually to find its way into the education system. This process exactly parallels the evolution of knowledge explored in this paper.

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[31] Grossman and Helpman (1991) emphasize that the costs of immitation may often be almost as high as those of innovation.

Appendix

Proof of Proposition 3: Substituting for the stationary distribution function into (17) yields

$$\frac{N}{(\mu - \lambda)}\frac{1}{\underline{\sigma}} = \frac{1}{\underline{\sigma}}\left[\frac{N}{(\mu - \lambda)}\ln\left(\frac{\underline{\sigma}}{H}\right)\right] \tag{A1}$$

$$\underline{\sigma} = eH \tag{A2}$$

Substituting for the stationary distribution function in (18)

$$\frac{N}{(\mu - \lambda)}\frac{1}{\sigma} = \frac{N(T - z) - ag - \frac{N}{(\mu - \lambda)}\ln\left(\frac{\sigma}{H}\right)}{n - \sigma} \tag{A3}$$

$$\frac{n}{\sigma} = 1 + (\mu - \lambda)(T - z - ag/N) - \ln\left(\frac{\sigma}{H}\right) \tag{A4}$$

Combining (A2) and (A4) yields:

$$\frac{n}{\sigma} = (\mu - \lambda)(T - ag/N - z) - \ln\left(\frac{\sigma}{\underline{\sigma}}\right) \tag{A5}$$

$$\frac{n}{\sigma} = (\mu - \lambda)(T - ag/N - T\psi) \tag{A6}$$

Since in a steady state, $\lambda = \dot{H}/H$ is constant, and it follows from (A2) that $\underline{\dot{\sigma}}/\underline{\sigma} = \lambda$. Along the steady state growth path the r.h.s. of (A6) is constant so that $\dot{\sigma}/\sigma = g$. From (29), if ψ is constant in the steady state it must be the case that $\dot{\sigma}/\sigma = \underline{\dot{\sigma}}/\underline{\sigma}$ and so $g = \lambda$.

Growth in Labor Productivity: Integrating over labor demands as follows yields the following expression for labor productivity

$$y = \frac{1}{L}\left[\sigma\left(\frac{N}{(\mu - \lambda)}\frac{1}{\underline{\sigma}}\right)^{\alpha} + \int_{\underline{\sigma}}^{\sigma}\left(\frac{N}{(\mu - \lambda)}\frac{1}{s}\right)^{\alpha}ds + (n - \sigma)\left(\frac{N}{(\mu - \lambda)}\frac{1}{\sigma}\right)^{\alpha}\right]^{\frac{1}{\alpha}} \tag{A7}$$

$$y = \frac{\sigma^{\frac{1-\alpha}{\alpha}}}{(\mu - \lambda)(T - z)}\left[\frac{\alpha}{1 - \alpha} + \frac{n}{\sigma} - \left(\frac{\alpha}{1 - \alpha}\right)\left(\frac{\underline{\sigma}}{\sigma}\right)^{1-\alpha}\right]^{\frac{1}{\alpha}} \tag{A8}$$

Since along the steady–state growth path both n/σ and $\underline{\sigma}/\sigma$ are constant and $\frac{\dot{\sigma}}{\sigma} = g$, it follows that $g_y = \left(\frac{1-\alpha}{\alpha}\right)g$.

The XY Boundary: If all agents that are available for R&D are being used in that sector, then $\sigma = n$ and

$$ag = N(T - z)[1 - F(n)] = N(T - z) - \frac{N}{(\mu - \lambda)}\ln\left(\frac{n}{H}\right) \tag{A9}$$

Now from (29), noting that $\sigma = n$ and that $\underline{\sigma} = eH$, we have

$$\psi = \frac{z}{T} + \frac{1}{(\mu - \lambda)T} \ln\left(\frac{n}{eH}\right) = \frac{z}{T} + \frac{1}{(\mu - \lambda)T} \ln\left(\frac{n}{H}\right) - \frac{1}{(\mu - \lambda)T} \tag{A10}$$

Substituting out $\frac{1}{(\mu - \lambda)} \ln\left(\frac{n}{H}\right)$ between (A9) and (A10) and rearranging yields (32).

The RD Schedule: The maximum wage in production is given by $\overline{w} = w(\sigma) = \alpha\left(y(\mu - \lambda)(T - z)\sigma\right)^{1-\alpha}$.
Hence, it grows at the rate

$$\frac{\dot{w}}{\overline{w}} = (1 - \alpha)\left(g_y + \frac{\dot{\sigma}}{\sigma}\right) = (1 - \alpha)\left(\left(\frac{1 - \alpha}{\alpha}\right)\lambda + \lambda\right) = \left(\frac{1 - \alpha}{\alpha}\right)\lambda = g_y. \tag{A11}$$

Differentiating (25) w.r.t. time therefore yields

$$\frac{\dot{V}}{V} = \frac{\dot{w}}{\overline{w}} - g = g_y - g. \tag{A12}$$

The total profit of the most recently introduced intermediate is

$$\pi(n) = \left(\frac{1 - \alpha}{\alpha}\right)\frac{\overline{w}N}{(\mu - \lambda)\sigma}. \tag{A13}$$

Noting that $w_R = \overline{w}$ and dividing (A13) by (25) yields

$$\frac{\pi(n)}{V(n)} = \left(\frac{(1 - \alpha)N}{\alpha a(\mu - \lambda)}\right)\frac{n}{\sigma}. \tag{A14}$$

Substituting (A12) and (A14) into the no–arbitrage condition (28) then yields

$$\left(\frac{(1 - \alpha)N}{\alpha a(\mu - \lambda)}\right)\frac{n}{\sigma} - g = \rho. \tag{A15}$$

Substituting for n/σ using (A6) and rearranging yields (33).

The ET Schedule: The time τ equilibrium wage of an agent born at time t is given by

$$w(\tau, t) = \begin{cases} \alpha\left[\Omega y(\tau)\underline{\sigma}(\tau)\right]^{1-\alpha} & \text{if } \tau \in [t + z, t + \underline{\tau}) \\ \alpha\left[\Omega y(\tau)e^{\mu(\tau - z - t) + \theta(z)}\overline{H}\right]^{1-\alpha} & \text{if } \tau \in [t + \underline{\tau}, t + \overline{\tau}) \\ \alpha\left[\Omega y(\tau)\sigma(\tau)\right]^{1-\alpha} & \text{if } \tau \in [t + \overline{\tau}, t + T] \end{cases} \tag{A16}$$

where $\Omega = (\mu - \lambda)(T - z)$, $\overline{H} = H(z + t - m)$ denotes the dynamic spillover, and $\underline{\tau}$ and $\overline{\tau}$ are defined by

$$e^{\mu(\underline{\tau} - z) + \theta(z)}\overline{H} = \underline{\sigma}(t + \underline{\tau}) = e^{\lambda\underline{\tau}}\underline{\sigma}(t) \tag{A17}$$

$$e^{\mu(\overline{\tau} - z) + \theta(z)}\overline{H} = \sigma(t + \overline{\tau}) = e^{\lambda\overline{\tau}}\sigma(t) \tag{A18}$$

The present value of the lifetime steady–state earnings of an agent born at time t is thus given by

$$W(z,t) = \int_{t+z}^{t+\underline{\tau}} e^{-r(\tau-t)} w(\underline{\sigma}(\tau),\tau)d\tau + \int_{t+\underline{\tau}}^{t+\overline{\tau}} e^{-r(\tau-t)} w(s(\tau),\tau)d\tau + \int_{t+\overline{\tau}}^{t+T} e^{-r(\tau-t)} w(\sigma(\tau),\tau)d\tau \quad (A19)$$

Substituting using (A16) and differentiating with respect to z, yields

$$
\begin{aligned}
W'(z,t) &= -e^{-rz} w(\underline{\sigma}(z+t)) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A20)\\
&\quad +(1-\alpha)\left(\theta'(z)-\mu\right)\int_{t+\underline{\tau}}^{t+\overline{\tau}} e^{-r(\tau-t)}\alpha\left[y(\tau)\Omega e^{\mu(\tau-z-t)+\theta(z)}\overline{H}\right]^{1-\alpha}d\tau\\
&= w(\underline{\sigma}(t))\left(-e^{-\rho z}+(1-\alpha)\left(\theta'(z)-\mu\right)\left(\frac{e^{\theta(z)-\mu z}\overline{H}}{\underline{\sigma}(t)}\right)^{1-\alpha}\int_{t+\underline{\tau}}^{t+\overline{\tau}} e^{[(1-\alpha)(\mu-\lambda)-\rho](\tau-t)}d\tau\right) (A21)
\end{aligned}
$$

But along the steady–state growth path $e^{\theta(z)}\overline{H} = H(t+z) = e^{\lambda z}H(t)$. Hence, the first–order condition can be written as

$$(1-\alpha)\left(\theta'(z)-\mu\right)\left(\frac{H(t)}{\underline{\sigma}(t)}\right)^{1-\alpha} e^{-(1-\alpha)(\mu-\lambda)z}\int_{t+\underline{\tau}}^{t+\overline{\tau}} e^{[(1-\alpha)(\mu-\lambda)-\rho](\tau-t)}d\tau = e^{-\rho z}. \qquad (A22)$$

Substituting for $\underline{\sigma}(t)$ using (A2) and integrating yields

$$\frac{(1-\alpha)\left(\theta'(z)-\mu\right)e^{-(1-\alpha)[1+(\mu-\lambda)z]}\left[e^{[(1-\alpha)(\mu-\lambda)-\rho]\overline{\tau}}-e^{[(1-\alpha)(\mu-\lambda)-\rho]\underline{\tau}}\right]}{(1-\alpha)(\mu-\lambda)-\rho} = e^{-\rho z}. \qquad (A23)$$

From (A17) we have that, along the steady–state growth path, $e^{\mu(\underline{\tau}-z)}H(t) = e^{\lambda(\underline{\tau}-z)}\underline{\sigma}(t) = e^{\lambda(\underline{\tau}-z)}eH(t)$. It follows that

$$\underline{\tau} = \frac{1}{\mu-\lambda} + z. \qquad (A24)$$

From (A18) we have $e^{\mu(\overline{\tau}-z)}e^{\lambda z}H(t) = e^{\lambda\overline{\tau}}\sigma(t) = e^{\lambda\overline{\tau}}e^{(\mu-\lambda)(\psi T-z)}\underline{\sigma}(t) = e^{\lambda\overline{\tau}}e^{(\mu-\lambda)(\psi T-z)}eH(t)$. Hence,

$$\overline{\tau} = \frac{1}{\mu-\lambda} + \psi T \qquad (A25)$$

Substituting (A24) and (A25) into (A23) yields

$$\frac{(1-\alpha)\left(\theta'(z)-\mu\right)\left[e^{[(1-\alpha)(\mu-\lambda)-\rho]\left(\frac{1}{\mu-\lambda}+\psi T\right)}-e^{[(1-\alpha)(\mu-\lambda)-\rho]\left(\frac{1}{\mu-\lambda}+z\right)}\right]}{(1-\alpha)(\mu-\lambda)-\rho} = e^{(1-\alpha)[1+(\mu-\lambda)z]}e^{-\rho z}$$

$$(A26)$$

Dividing both sides by $e^{1-\alpha-\frac{\rho}{\mu-\lambda}}$ and rearranging yields

$$e^{[(1-\alpha)(\mu-\lambda)-\rho]\psi T} - e^{[(1-\alpha)(\mu-\lambda)-\rho]z} = \left(\frac{(1-\alpha)(\mu-\lambda)-\rho}{(1-\alpha)(\theta'(z)-\mu)}\right) e^{[(1-\alpha)(\mu-\lambda)-\rho]z} e^{\frac{\rho}{\mu-\lambda}} \qquad (A27)$$

Rearranging yields (35).

The Second–order Condition for Optimal Schooling: Differentiating (A21) w.r.t. z yields

$$W''(z;t) = w(\underline{\sigma}(t))\rho e^{-\rho z} + w(\underline{\sigma}(t))(1-\alpha)\theta''(z)\left(\frac{e^{\theta(z)-\mu z}\overline{H}}{\underline{\sigma}(t)}\right)^{1-\alpha}\int_{t+\underline{\tau}}^{t+\overline{\tau}} e^{[(1-\alpha)(\mu-\lambda)-\rho](\tau-t)}d\tau \quad (A28)$$

$$+w(\underline{\sigma}(t))(1-\alpha)^2(\theta'(z)-\mu)^2\left(\frac{e^{\theta(z)-\mu z}\overline{H}}{\underline{\sigma}(t)}\right)^{1-\alpha}\int_{t+\underline{\tau}}^{t+\overline{\tau}} e^{[(1-\alpha)(\mu-\lambda)-\rho](\tau-t)}d\tau$$

$$+w(\underline{\sigma}(t))(1-\alpha)(\theta'(z)-\mu)\left(\frac{e^{\theta(z)-\mu z}\overline{H}}{\underline{\sigma}(t)}\right)^{1-\alpha}\left[e^{[(1-\alpha)(\mu-\lambda)-\rho]\overline{\tau}}\frac{d\overline{\tau}}{dz} - e^{[(1-\alpha)(\mu-\lambda)-\rho]\underline{\tau}}\frac{d\underline{\tau}}{dz}\right]$$

Now from (A17) and (A18) we have $\frac{d\underline{\tau}}{dz} = \frac{d\overline{\tau}}{dz} = -\frac{\theta'(z)-\mu}{\mu-\lambda}$. Also we know that $e^{[(1-\alpha)(\mu-\lambda)-\rho]\overline{\tau}} - e^{[(1-\alpha)(\mu-\lambda)-\rho]\underline{\tau}} = [(1-\alpha)(\mu-\lambda)-\rho]\int_{t+\underline{\tau}}^{t+\overline{\tau}} e^{[(1-\alpha)(\mu-\lambda)-\rho](\tau-t)}d\tau$. It follows that we can write (A28) as

$$W''(z;t) = w(\underline{\sigma}(t))\rho e^{-\rho z} + w(\underline{\sigma}(t))(1-\alpha)\left(\frac{e^{\theta(z)-\mu z}\overline{H}}{\underline{\sigma}(t)}\right)^{1-\alpha}\int_{t+\underline{\tau}}^{t+\overline{\tau}} e^{[(1-\alpha)(\mu-\lambda)-\rho](\tau-t)}d\tau \quad (A29)$$

$$\left[\theta''(z) + (1-\alpha)(\theta'(z)-\mu)^2 - (\theta'(z)-\mu)\left(\frac{\theta'(z)-\mu}{\mu-\lambda}\right)[(1-\alpha)(\mu-\lambda)-\rho]\right]$$

Substitution using (A22), allows us to express this as

$$W''(z;t) = w(\underline{\sigma}(t))e^{-\rho z}\left(\rho + \frac{\theta''(z)}{\theta'(z)-\mu} + (1-\alpha)(\theta'(z)-\mu) - \left(\frac{\theta'(z)-\mu}{\mu-\lambda}\right)[(1-\alpha)(\mu-\lambda)-\rho]\right)$$

$$= w(\underline{\sigma}(t))e^{-\rho z}\left(\frac{\theta''(z)}{\theta'(z)-\mu} + \rho\left(\frac{\theta'(z)-\lambda}{\mu-\lambda}\right)\right)$$

$$= w(\underline{\sigma}(t))e^{-\rho z}\left(-\frac{\beta\gamma z^{-\gamma-1}}{\beta z^{-\gamma}-\mu} + \rho\left(\frac{\beta z^{-\gamma}-\lambda}{\mu-\lambda}\right)\right) \qquad (A30)$$

Thus, a sufficient condition for a maximum is

$$\rho\left(\frac{\beta z^{-\gamma}-\lambda}{\mu-\lambda}\right) < \frac{\beta\gamma z^{-\gamma-1}}{\beta z^{-\gamma}-\mu} \qquad (A31)$$

34

This condition will certainly hold if

$$\rho \left(\frac{\beta z^{-\gamma}}{\mu - \lambda} \right) < \frac{\beta \gamma z^{-\gamma - 1}}{\beta z^{-\gamma}} \tag{A32}$$

$$\frac{\rho}{\mu - \lambda} < \frac{\gamma}{\beta z^{1-\gamma}} = \frac{\gamma}{(1 - \gamma) m \lambda} \tag{A33}$$

where the last equality follows from (36). Rearranging yields (39).

Proof of Lemma 1:

<u>Case 1:</u> Suppose $\mu - \lambda < \frac{\rho}{1-\alpha}$. Then, we can write (35) as

$$\psi = \frac{z(\lambda)}{T} + \frac{1}{(1 - \alpha) T . A(\lambda) \ln (1 - A(\lambda) B(\lambda))} \tag{A34}$$

where $A(\lambda) = \frac{\rho}{1-\alpha} - \mu + \lambda > 0$ and $B(\lambda) = \left(\frac{1-\omega}{\beta z(\lambda)^{-\gamma}} \right) e^{\frac{\rho}{\mu - \lambda}} > 0$. Note that $A'(\lambda) > 0$ and $B'(\lambda) > 0$. Also observe that existence requires that $A(\lambda) B(\lambda) < 1$. Let

$$\phi = A(\lambda) \ln (1 - A(\lambda) B(\lambda)) \tag{A35}$$

Then

$$\frac{d\phi}{d\lambda} = \left[\ln (1 - AB) - \frac{AB}{1 - AB} \right] A'(\lambda) - \frac{A^2}{1 - AB} B'(\lambda) < 0 \tag{A36}$$

since $\ln (1 - AB) < 0$. Since $z'(\lambda) > 0$, it follows that along the ET curve $d\psi/d\lambda > 0$.

<u>Case 2:</u> Suppose $\mu - \lambda > \frac{\rho}{1-\alpha}$. Then the original expression can be written

$$\psi = \frac{\ln \left(1 + \hat{A}(\lambda) B(\lambda) \right)}{(1 - \alpha) T . \hat{A}(\lambda)} + \frac{z(\lambda)}{T} \tag{A37}$$

where $\hat{A}(\lambda) = \mu - \lambda - \frac{\rho}{1-\alpha} > 0$. Note that $\hat{A}'(\lambda) < 0$. Differentiating yields

$$\frac{d\psi}{d\lambda} \bigg|_{ET} = \frac{1}{(1 - \alpha) T} \left(\left[\frac{\hat{A} B}{1 + \hat{A} B} - \ln \left(1 + \hat{A} B \right) \right] \frac{\hat{A}'(\lambda)}{\hat{A}^2} + \frac{B'(\lambda)}{(1 + \hat{A} B)} \right) + \frac{z'(\lambda)}{T} \tag{A38}$$

Since $z'(\lambda) > 0$ and $B'(\lambda) > 0$, a sufficient condition for $\frac{d\psi}{d\lambda} > 0$ is that the term in square brackets is non–positive. Let $x = \hat{A} B > 0$. Then $\frac{d\psi}{d\lambda} > 0$ if

$$x \leq (1 + x) \ln(1 + x) \ \forall \ x > 0. \tag{A39}$$

Observe first that this holds with equality if $x = 0$. Let $z = (1 + x) \ln(1 + x)$. The $\frac{dz}{dx} = 1 + \ln(1 + x) > 1$ and $\frac{d^2 z}{dx^2} = \frac{1}{1+x} > 0$. It follows that (A39) must hold with strict inequality for all $x > 0$.

Hence, in this case as well $d\psi/d\lambda > 0$.

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Appendix B: Extensions

This appendix provides the mathematical details for the extensions of the basic model discussed in Section 5. The steps involved are very much the same as for the basic model.

1. Rising Intermediate Productivity

Labor Market Equilibrium: Substituting in the relevant distribution functions, we have

$$\frac{N}{(\mu - \lambda)} \left(\frac{1}{\underline{\sigma}}\right)^{\frac{\alpha\delta}{1-\alpha}+1} \int_0^{\sigma} s^{\frac{\alpha\delta}{1-\alpha}} ds \;=\; \frac{N}{(\mu - \lambda)} \ln \frac{\sigma}{H} \tag{A40}$$

$$\underline{\sigma}(t) \;=\; H(t) e^{\frac{1-\alpha}{1-\alpha+\alpha\delta}} \tag{A41}$$

and

$$\frac{N}{(\mu - \lambda)} \left(\frac{1}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}+1} \int_{\underline{\sigma}}^{n} s^{\frac{\alpha\delta}{1-\alpha}} ds \;=\; N(T - z) - ag - \frac{N}{(\mu - \lambda)} \ln \frac{\sigma}{H} \tag{A42}$$

$$\left(\frac{1 - \alpha}{1 - \alpha + \alpha\delta}\right) \left(\frac{n}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}+1} \;=\; \frac{1 - \alpha}{1 - \alpha + \alpha\delta} + (\mu - \lambda)(T - \frac{ag}{N} - z) - \ln \frac{\sigma}{H} \tag{A43}$$

Combining (A41) and (A43) yields:

$$\left(\frac{1 - \alpha}{1 - \alpha + \alpha\delta}\right) \left(\frac{n}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}+1} \;=\; (\mu - \lambda)(T - ag/N - z) - \ln \left(\frac{\sigma}{\underline{\sigma}}\right) \tag{A44}$$

$$\left(\frac{n}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}+1} \;=\; \left(\frac{1 - \alpha + \alpha\delta}{1 - \alpha}\right) (\mu - \lambda)(T - ag/N - T\psi), \tag{A45}$$

where the second step uses (29)

Labor Productivity: Integrating over labor demands as before as follows yields

$$y = \frac{\sigma^{\frac{1-\alpha}{\alpha}+\delta}}{(\mu - \lambda)(T - z)} \left[\frac{\alpha - \alpha\delta}{1 - \alpha + \alpha\delta} + \frac{n}{\sigma} - \left(\frac{\alpha - \alpha\delta}{1 - \alpha + \alpha\delta}\right) \left(\frac{\underline{\sigma}}{\sigma}\right)^{1-\alpha+\alpha\delta}\right]^{\frac{1}{\alpha}} \tag{A46}$$

Differentiating w.r.t. time yields (54).

The RD Schedule: The maximum wage in production is now $\overline{w} = \alpha \left[(\mu - \lambda)(T - z)y\right]^{1-\alpha} \sigma^{1-\alpha+\alpha\delta}$. Hence, it grows at the rate

$$\frac{\dot{w}}{w} = (1 - \alpha + \dot{\alpha}\delta) \left(\left(\frac{1 - \alpha}{\alpha}\right)\lambda + \lambda\right) = \left(\frac{1 - \alpha}{\alpha} + \delta\right)\lambda = g_y. \tag{A47}$$

The total profit of the most recently introduced intermediate is

$$\pi(n) = \left(\frac{1-\alpha}{\alpha}\right)\frac{\overline{w}N}{(\mu-\lambda)n}\left(\frac{n}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}+1} \tag{A48}$$

Dividing (A48) by (25) and substituting the result, together with (26) and (A12) into the no–arbitrage condition (28) then yields

$$\left(\frac{(1-\alpha)N}{\alpha a(\mu-\lambda)}\right)\left(\frac{n}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}+1} - g = \rho \tag{A49}$$

Substituting for $\left(\frac{n}{\sigma}\right)^{\frac{\alpha\delta}{1-\alpha}+1}$ using (A45) and rearranging yields (52)

The ET Schedule: The time τ equilibrium wage of an agent born at time t is given by

$$w(\tau,t) = \begin{cases} \alpha\left[\Omega y(\tau)\right]^{1-\alpha}\underline{\sigma}(\tau)^{1-\alpha+\alpha\delta} & \text{if } \tau \in [t, t+\underline{\tau}) \\ \alpha\left[\Omega y(\tau)\right]^{1-\alpha}\left(e^{\mu(\tau-t-z)}e^{\theta(z)}H\right)^{1-\alpha+\alpha\delta} & \text{if } \tau \in [\underline{\tau}, t+\overline{\tau}) \\ \alpha\left[\Omega y(\tau)\right]^{1-\alpha}\sigma(\tau)^{1-\alpha+\alpha\delta} & \text{if } \tau \in [t+\overline{\tau}, t+T] \end{cases} \tag{A50}$$

where $\Omega = (\mu-\lambda)(T-z)$, $\overline{H} = H(z+t-m)$ and $\underline{\tau}$ and $\overline{\tau}$ are defined as in (A17) and (A18). Following the same steps as for the basic model, and noting that along the balanced growth path $e^{\theta(z)}H(t+z-m) = H(z+t) = e^{\lambda z}H(t)$, the first–order condition may be expressed as

$$\frac{(1-\alpha+\alpha\delta)(\theta'(z)-\mu)\left[e^{[(1-\alpha+\alpha\delta)(\mu-\lambda)-\rho]\overline{\tau}} - e^{[(1-\alpha+\alpha\delta)(\mu-\lambda)-\rho]\underline{\tau}}\right]}{(1-\alpha+\alpha\delta)(\mu-\lambda)-\rho} = e^{[1-\alpha+(1-\alpha+\alpha\delta)(\mu-\lambda)z]}e^{-\rho z}, \tag{A51}$$

where

$$\underline{\tau} = \frac{1}{\mu-\lambda}\left(\frac{1-\alpha}{1-\alpha+\alpha\delta}\right) + z \tag{A52}$$

and

$$\overline{\tau} = \frac{1}{\mu-\lambda}\left(\frac{1-\alpha}{1-\alpha+\alpha\delta}\right) + \psi T \tag{A53}$$

Substituting (A52) and (A53) into (A51) and rearranging as before yields (53).

2. Population Growth

Labor Market Equilibrium: Substituting the stationary distribution function in (57) into (16) yields

$$\left(\frac{\nu}{\mu-\lambda}\right)\left(\frac{\underline{\sigma}}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\frac{1}{\underline{\sigma}} = \frac{1}{\underline{\sigma}}\left(1-\left(\frac{\underline{\sigma}}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\right) \tag{A54}$$

$$\underline{\sigma}(t) = H(t)\left[\frac{\nu+\mu-\lambda}{\mu-\lambda}\right]^{\frac{\mu-\lambda}{\nu}}. \tag{A55}$$

Substituting (55) and the distribution function (57) into (17) yields

$$\frac{1}{1-e^{-\nu(T-z)}}\left(\frac{\nu}{\mu-\lambda}\right)\left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\left(\frac{n-\sigma}{\sigma}\right) = 1 - \frac{1}{1-e^{-\nu(T-z)}}\left(1-\left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\right) \quad \text{(A56)}$$
$$-\frac{ag\gamma}{\nu e^{-\nu z}}\left(\frac{1-e^{-vT}}{1-e^{-v(T-z)}}\right),$$

which after some rearrangement can be written as

$$\left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\frac{n}{\sigma} = \left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\left(\frac{\nu+\mu-\lambda}{\nu}\right) - \left(\frac{\mu-\lambda}{\nu}\right)\left(e^{-\nu(T-z)}+ag\gamma\left(\frac{1-e^{-\nu T}}{\nu e^{-\nu z}}\right)\right) \quad \text{(A57)}$$

Combining (51) and (52) yields:

$$\left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\frac{n}{\sigma} = \left(\frac{\mu-\lambda}{\nu}\right)\left[\left(\frac{\sigma}{\underline{\sigma}}\right)^{-\frac{\nu}{\mu-\lambda}} - e^{-\nu(T-z)} - ag\gamma\left(\frac{1-e^{-\nu T}}{\nu e^{-\nu z}}\right)\right] \quad \text{(A58)}$$

$$\left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\frac{n}{\sigma} = \left(\frac{\mu-\lambda}{\nu}\right)\left[e^{\nu z-\psi\nu T} - e^{-\nu(T-z)} - ag\gamma\left(\frac{1-e^{-\nu T}}{\nu e^{-\nu z}}\right)\right], \quad \text{(A59)}$$

where ψ is defined as before (29).

The XY Boundary: In a constrained equilibrium we know that $l_R/L = 1 - F(n)$. Substitution using (57) and (60) yields

$$\frac{ag\gamma}{\nu e^{-\nu z}}\left(\frac{1-e^{-vT}}{1-e^{-v(T-z)}}\right) = 1 - \frac{1}{1-e^{-\nu(T-z)}}\left(1-\left(\frac{n}{H}\right)^{-\frac{\nu}{\mu-\lambda}}\right). \quad \text{(A60)}$$

Rearranging gives

$$\left(\frac{n(t)}{H(t)}\right)^{-\frac{\nu}{\mu-\lambda}} = e^{-\nu(T-z)} + \frac{ag\gamma}{\nu e^{-\nu z}}\left(1-e^{-vT}\right). \quad \text{(A61)}$$

Since $n=\sigma$, the expression of ψ in (29) can be written as

$$\psi = \frac{1}{\nu T}\ln\left(\frac{n}{\underline{\sigma}}\right)^{\frac{\nu}{\mu-\lambda}} + \frac{z}{T} = \frac{1}{\nu T}\ln\left[\frac{\left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu-\lambda}}}{\left(\frac{n}{H}\right)^{-\frac{\nu}{\mu-\lambda}}}\right] + \frac{z}{T}. \quad \text{(A62)}$$

Substitution using (A55) and (A61) then yields the following expression for the XY boundary:

$$\psi = \frac{1}{\nu T}\ln\left[\frac{\mu-\lambda}{\nu+\mu-\lambda}\right] - \frac{1}{\nu T}\ln\left[e^{-\nu T} + \frac{ag\gamma}{\nu}\left(1-e^{-vT}\right)\right]. \quad \text{(A63)}$$

The RD Schedule: The total profit of the most recently introduced intermediate is

$$\pi(n) = \left(\frac{1-\alpha}{\alpha}\right)\overline{w}Lf(\sigma)/I = \left(\frac{1-\alpha}{\alpha}\right)\overline{w}f(\sigma)e^{-\nu z}\nu/\gamma \quad \text{(A64)}$$

$$= \left(\frac{1-\alpha}{\alpha\gamma}\right) \frac{\nu e^{-\nu z}}{1 - e^{-\nu T}} \left(\frac{\nu}{\mu - \lambda}\right) \left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu - \lambda}} \frac{\overline{w}}{\sigma}, \qquad (A65)$$

Dividing (A65) by (25) and substituting the result, together with (26) and (A12) into the no–arbitrage condition (28) then yields

$$\left(\frac{(1-\alpha)\nu}{\alpha a(\mu - \lambda)\gamma}\right) \frac{\nu e^{-\nu z}}{1 - e^{-vT}} \left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu - \lambda}} \frac{n}{\sigma} - g = \rho. \qquad (A66)$$

Substituting for $\left(\frac{\sigma}{H}\right)^{-\frac{\nu}{\mu - \lambda}} \frac{n}{\sigma}$ using (A59) yields

$$\left(\frac{(1-\alpha)}{\alpha a\gamma}\right) \frac{\nu e^{-\nu z}}{1 - e^{-\nu T}} \left[e^{\nu z - \nu\psi T} - e^{-\nu(T-z)} - ag\gamma \left(\frac{1 - e^{-\nu T}}{\nu e^{-\nu z}}\right)\right] = \rho + g. \qquad (A67)$$

Re–arrangement yields the RD schedule given by

$$\psi = \frac{1}{\nu T} \ln\beta - \frac{1}{\nu T} \ln\left(ag\gamma + \left(\frac{\alpha a\gamma}{1-\alpha}\right)(\rho + g) + \delta\right) \qquad (A68)$$

where $\beta = \frac{\nu}{1 - e^{-\nu T}}$ denotes the birth rate and $\delta = \frac{\nu e^{-\nu T}}{1 - e^{-\nu T}}$ denotes the death rate. Letting $\nu \to 0$ so that $1 - e^{-\nu T} \simeq \nu T$ yields (61).

The ET Schedule: The time τ equilibrium wage of an agent born at time t is given by

$$w(\tau, t) = \begin{cases} \alpha \left[\Omega y(\tau) H(\tau)^{-\frac{\nu}{\mu - \lambda}} \underline{\sigma}(\tau)^{\frac{\nu + \mu - \lambda}{\mu - \lambda}}\right]^{1-\alpha} & \text{if } \tau \in [t, t + \underline{\tau}) \\[2ex] \alpha \left[\Omega y(\tau) H(\tau)^{-\frac{\nu}{\mu - \lambda}} \left[e^{\mu(\tau - t - z)} e^{\theta(z)} \overline{H}(t)\right]^{\frac{\nu + \mu - \lambda}{\mu - \lambda}}\right]^{1-\alpha} & \text{if } \tau \in [t + \underline{\tau}, t + \overline{\tau}) \\[2ex] \alpha \left[\Omega y(\tau) H(\tau)^{-\frac{\nu}{\mu - \lambda}} \sigma(\tau)^{\frac{\nu + \mu - \lambda}{\mu - \lambda}}\right]^{1-\alpha} & \text{if } \tau \in [t + \overline{\tau}, t + T] \end{cases} \qquad (A69)$$

where $\Omega = (L/N)(\mu - \lambda)$, $\overline{H}(t) = H(t + z - n)$ and $\underline{\tau}$ and $\overline{\tau}$ are defined as in (A17) and (A18). It follows that the agent's optimal education decision satisfies

$$W'(t) = -e^{-rz} w(\underline{\sigma}(z + t)) + (1 - \alpha)\left(\frac{\nu + \mu - \lambda}{\mu - \lambda}\right)(\theta'(z) - \mu) \int_{\underline{\tau}}^{\overline{\tau}} e^{-r(\tau - t)} w(\tau, t) d\tau = 0 \quad (A70)$$

Following the same steps as before, the first–order condition along the balanced growth path can be written as

$$\frac{(1-\alpha)\left(\frac{\nu + \mu - \lambda}{\mu - \lambda}\right)(\theta'(z) - \mu)\left[e^{[(1-\alpha)(\mu - \lambda + \nu) - \rho]\overline{\tau}} - e^{[(1-\alpha)(\mu - \lambda + \nu) - \rho]\underline{\tau}}\right]}{(1 - \alpha)(\mu - \lambda + \nu) - \rho} \qquad (A71)$$

$$= e^{[(1-\alpha)(\nu + \mu - \lambda) - \rho]z} \left(\frac{\nu + \mu - \lambda}{\mu - \lambda}\right)^{(1-\alpha)\left(\frac{\nu + \mu - \lambda}{\nu}\right)}$$

where
$$\underline{\tau} = \frac{1}{\nu} \ln \left(\frac{\nu + \mu - \lambda}{\mu - \lambda} \right) + z \tag{A72}$$

and
$$\overline{\tau} = \frac{1}{\nu} \ln \left(\frac{\nu + \mu - \lambda}{\mu - \lambda} \right) + \psi T \tag{A73}$$

Substituting (A72) and (A73) into (A71) and re–arranging yields the modified ET schedule

$$\psi = \frac{z(\lambda)}{T} - \frac{\ln \left(1 - (\frac{1}{z(\lambda)^{-\gamma} - \mu}) \left[\frac{\rho - (1-\alpha)(\mu - \lambda + \nu)}{(1-\alpha)\left(\frac{\mu - \lambda + \nu}{\mu - \lambda}\right)} \right] \left[\frac{\mu - \lambda + \nu}{\mu - \lambda} \right]^{\frac{\rho}{\nu}} \right)}{[\rho - (1-\alpha)(\mu - \lambda + \nu)]T} \tag{A74}$$

When ν is small, then
$$\lim_{\nu \to 0} \left(\frac{\mu - \lambda + \nu}{\mu - \lambda} \right) \approx e^{\frac{\nu}{\mu - \lambda}}. \tag{A75}$$
and the ET schedule can be expressed as in (62).

3. Old Dogs and New Tricks

Labor Market Equilibrium:

$$\frac{\mu}{\lambda(\mu - \lambda)} = \frac{\mu}{\lambda(\mu - \lambda)} \ln \left(\frac{\sigma}{H(t)} \right) + (T - z) - \frac{\mu(a^* - z)}{\lambda} \tag{A76}$$

$$\ln \left(\frac{\sigma}{H(t)} \right) = 1 - \frac{\lambda(\mu - \lambda)(T - z)}{\mu} + (\mu - \lambda)(a^* - z) \tag{A77}$$

$$\frac{\mu}{\lambda(\mu - \lambda)} \left(\frac{n - \sigma}{\sigma} \right) = -\frac{\mu}{\lambda(\mu - \lambda)} \ln \left(\frac{\sigma}{H(t)} \right) + \frac{\mu(a^* - z)}{\lambda} - \frac{ag}{N} \tag{A78}$$

$$\frac{n}{\sigma} = \frac{\lambda(\mu - \lambda)}{\mu} \left(T - z - \frac{ag}{N} \right) - \ln \left(\frac{\sigma}{\underline{\sigma}} \right) \tag{A79}$$

Let
$$\psi = \frac{z}{T} + \frac{\mu}{\lambda(\mu - \lambda)T} \ln \left(\frac{\sigma}{\underline{\sigma}} \right) \tag{A80}$$

Then
$$\frac{n}{\sigma} = \frac{\lambda(\mu - \lambda)T}{\mu} \left(1 - \frac{ag}{NT} - \psi \right) \tag{A81}$$

The RD Schedule: The total profit of the most recently introduced intermediate is

$$\pi(n) = \left(\frac{1 - \alpha}{\alpha} \right) \frac{\overline{w} N \mu}{(\mu - \lambda)\lambda\sigma}. \tag{A82}$$

43

Noting that $w_R = \overline{w}$ and dividing (A82) by (25) yields

$$\frac{\pi(n)}{V(n)} = \left(\frac{(1-\alpha)N\mu}{\alpha a(\mu-\lambda)\lambda}\right)\frac{n}{\sigma}. \tag{A83}$$

Substituting into the no–arbitrage condition (28) then yields

$$\left(\frac{(1-\alpha)N\mu}{\alpha a(\mu-\lambda)\lambda}\right)\frac{n}{\sigma} = \rho + g. \tag{A84}$$

Substituting for n/σ using (A81) and rearranging yields the same RD schedule as before (33).

The ET Schedule: The time τ equilibrium wage of an agent born at time t is given by

$$w(\tau,t) = \begin{cases} \alpha\left[\Omega y(\tau)\underline{\sigma}(\tau)\right]^{1-\alpha} & \text{if } \tau \in [t+z, t+\underline{\tau}) \\ \alpha\left[\Omega y(\tau)e^{\mu(\tau-z-t)+\theta(z)}\overline{H}\right]^{1-\alpha} & \text{if } \tau \in [t+\underline{\tau}, t+\underline{u}) \\ \alpha\left[\Omega y(\tau)\sigma(\tau)\right]^{1-\alpha} & \text{if } \tau \in [t+\underline{u}, t+\overline{u}] \\ \alpha\left[\Omega y(\tau)e^{\mu(a^*-z)+\theta(z)}\overline{H}\right]^{1-\alpha} & \text{if } \tau \in [t+\overline{u}, t+\overline{\tau}] \\ \alpha\left[\Omega y(\tau)\underline{\sigma}(\tau)\right]^{1-\alpha} & \text{if } \tau \in [t+\overline{\tau}, t+T] \end{cases} \tag{A85}$$

where $\Omega = (\mu-\lambda)\lambda(T-z)/\mu$, $\overline{H} = H(z+t-m)$ denotes the dynamic spillover, and $\underline{\tau}$ and $\overline{\tau}$ are defined by

$$e^{\mu(\underline{\tau}-z)}H(z+t) = \underline{\sigma}(t+\underline{\tau}) = e^{\lambda\underline{\tau}}\underline{\sigma}(t) \tag{A86}$$

$$e^{\mu(\underline{u}-z)}H(z+t) = \sigma(t+\underline{u}) = e^{\lambda\underline{u}}\sigma(t) \tag{A87}$$

$$e^{\mu(a^*-z)}H(z+t) = \sigma(t+\overline{u}) = e^{\lambda\overline{u}}\sigma(t) \tag{A88}$$

$$e^{\mu(a^*-z)}H(z+t) = \underline{\sigma}(t+\overline{\tau}) = e^{\lambda\overline{\tau}}\underline{\sigma}(t) \tag{A89}$$

The present value of the lifetime steady–state earnings of an agent born at time t is thus given by

$$\begin{aligned} W(z,t) &= \int_{t+z}^{t+\underline{\tau}} e^{-r(\tau-t)}w(\underline{\sigma}(\tau),\tau)d\tau + \int_{t+\underline{\tau}}^{t+\underline{u}} e^{-r(\tau-t)}w(s(\tau),\tau)d\tau + \int_{t+\underline{u}}^{t+\overline{u}} e^{-r(\tau-t)}w(\sigma(\tau),\tau)d\tau \\ &+ \int_{t+\overline{u}}^{t+\overline{\tau}} e^{-r(\tau-t)}w(s(\tau),\tau)d\tau + \int_{t+\overline{\tau}}^{t+T} e^{-r(\tau-t)}w(\underline{\sigma}(\tau),\tau)d\tau \end{aligned} \tag{A90}$$

Substituting using (A85) and differentiating with respect to z, yields

$$W'(z,t) = -e^{-rz}w(\underline{\sigma}(z+t)) + (1-\alpha)(\theta'(z)-\mu)\int_{t+\underline{\tau}}^{t+\underline{u}} e^{-r(\tau-t)}\alpha\left[y(\tau)\Omega e^{\mu(\tau-z-t)+\theta(z)}\overline{H}\right]^{1-\alpha}d\tau$$

44

$$+(1-\alpha)\left(\theta'(z)-\mu\right)\int_{t+\underline{u}}^{t+\overline{\tau}}e^{-r(\tau-t)}\alpha\left[y(\tau)\Omega e^{\mu(a^*-z)+\theta(z)}\overline{H}\right]^{1-\alpha}d\tau \tag{A91}$$

$$= -w(\underline{\sigma}(t))e^{-\rho z}+w(\underline{\sigma}(t))(1-\alpha)\left(\theta'(z)-\mu\right)\left(\frac{e^{\theta(z)-\mu z}\overline{H}}{\underline{\sigma}(t)}\right)^{1-\alpha}\int_{t+\underline{\tau}}^{t+\underline{u}}e^{[(1-\alpha)(\mu-\lambda)-\rho](\tau-t)}d\tau$$

$$+w(\underline{\sigma}(t))(1-\alpha)\left(\theta'(z)-\mu\right)\left(\frac{e^{\theta(z)-\mu z}\overline{H}}{\underline{\sigma}(t)}\right)^{1-\alpha}e^{(1-\alpha)\mu a^*}\int_{t+\overline{u}}^{t+\overline{\tau}}e^{[-\rho-(1-\alpha)\lambda](\tau-t)}d\tau \tag{A92}$$

But along the steady–state growth path $e^{\theta(z)}\overline{H} = H(t+z) = e^{\lambda z}H(t)$. Hence, the first–order condition can be written as

$$(1-\alpha)\left(\theta'(z)-\mu\right)\left(\frac{e^{-(\mu-\lambda)z}H(t)}{\underline{\sigma}(t)}\right)^{1-\alpha}\left(\int_{t+\underline{\tau}}^{t+\underline{u}}e^{[(1-\alpha)(\mu-\lambda)-\rho](\tau-t)}d\tau + e^{(1-\alpha)\mu a^*}\int_{t+\overline{u}}^{t+\overline{\tau}}e^{[-\rho-(1-\alpha)\lambda](\tau-t)}d\tau\right) = e^{-\rho z}. \tag{A93}$$

Substituting for $\underline{\sigma}(t)$ using (A77) and integrating yields

$$e^{-\rho z} = (1-\alpha)\left(\theta'(z)-\mu\right)e^{-(1-\alpha)\left[1-\frac{\lambda(\mu-\lambda)(T-z)}{\mu}+(\mu-\lambda)(a^*-z)+(\mu-\lambda)z\right]}\times \tag{A94}$$

$$\left(\frac{e^{[(1-\alpha)(\mu-\lambda)-\rho]\underline{u}}-e^{[(1-\alpha)(\mu-\lambda)-\rho]\underline{\tau}}}{[(1-\alpha)(\mu-\lambda)-\rho]}-\frac{e^{(1-\alpha)\mu a^*}}{\rho+(1-\alpha)\lambda}\left[e^{[-\rho-(1-\alpha)\lambda]\overline{\tau}}-e^{[-\rho-(1-\alpha)\lambda]\overline{u}}\right]\right).$$

$$e^{(1-\alpha)\left[1-\frac{\lambda(\mu-\lambda)(T-z)}{\mu}+(\mu-\lambda)a^*\right]}e^{-\rho z} \tag{A95}$$

$$= (1-\alpha)\left(\theta'(z)-\mu\right)\left(\frac{e^{[(1-\alpha)(\mu-\lambda)-\rho]\underline{u}}-e^{[(1-\alpha)(\mu-\lambda)-\rho]\underline{\tau}}}{(1-\alpha)(\mu-\lambda)-\rho}-\frac{e^{(1-\alpha)\mu a^*}}{\rho+(1-\alpha)\lambda}\left[e^{[-\rho-(1-\alpha)\lambda]\overline{\tau}}-e^{[-\rho-(1-\alpha)\lambda]\overline{u}}\right]\right).$$

Along the steady–state growth path, $e^{\mu(\underline{\tau}-z)}H(t) = e^{\lambda(\underline{\tau}-z)}\underline{\sigma}(t) = e^{\lambda(\underline{\tau}-z)}eH(t)$. It follows that

$$\underline{\tau} = \frac{1}{\mu-\lambda}-\frac{\lambda}{\mu}(T-z)+a^* \tag{A96}$$

$$\underline{u} = \frac{1}{\mu-\lambda}-\frac{\lambda}{\mu}(T-\psi T)+a^* \tag{A97}$$

$$\overline{u} = -\frac{1}{\lambda}+\frac{(\mu-\lambda)}{\mu}(T-\psi T)+a^* \tag{A98}$$

$$\overline{\tau} = -\frac{1}{\lambda}+\frac{(\mu-\lambda)}{\mu}(T-z)+a^* \tag{A99}$$

45

Substituting (A96) through (A99) into (A95) yields the ET curve:

$$
\begin{aligned}
& \frac{e^{-\rho\left(\frac{1}{\mu-\lambda}-\frac{\lambda}{\mu}T\right)}e^{[(1-\alpha)(\mu-\lambda)-\rho]\left(\frac{\lambda}{\mu}z\right)}}{(1-\alpha)(\mu-\lambda)-\rho}\left(e^{[(1-\alpha)(\mu-\lambda)-\rho]\frac{\lambda}{\mu}(\psi T-z)}-1\right) \\
& \quad -\frac{e^{-\rho\left(-\frac{1}{\lambda}+T-\frac{\lambda}{\mu}T\right)}e^{[-\rho-(1-\alpha)\lambda]\left(-\frac{(\mu-\lambda)}{\mu}z\right)}}{\rho+(1-\alpha)\lambda}\left[1-e^{[-\rho-(1-\alpha)\lambda]\left(-\frac{(\mu-\lambda)}{\mu}(\psi T-z)\right)}\right] \\
& = \frac{e^{\rho(a^*-z)}}{(1-\alpha)\left(\theta'(z)-\mu\right)}. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{(A100)}
\end{aligned}
$$

RD ET   ET

RD

g_0 g_1 g_2 g_0 g_1 g_2