Dynamic Spatial Competition Between Multi-Store Firms

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First Version: November 2004
This Version: August 2006

Abstract

We propose a dynamic model of an oligopoly industry characterized by spatial competition between multi-store firms. Firms compete in prices and decide where to open or close stores depending on demand conditions and the number of competitors at different locations, and on location-specific private-information shocks. We provide an algorithm to compute Markov Perfect Equilibria (MPE) in our model. We conduct several numerical experiments to study how the propensity of multi-store retailers to spatial preemptive behavior depends on the magnitude of entry costs, exit value and transportation costs.

Keywords: Spatial competition; Market dynamics; Sunk costs; Spatial preemptive behavior.

JEL classifications: C73, L13, L81, R10, R30.

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*We thank the National Science Foundation for financial support (grant SES-0241943).

1 Introduction

Multi-store retailers account for more than 60% of sales in food retailing, drugstores, and bookstores (Hollander and Omura, 1989). Geographic location is in many cases the most important source of product differentiation for these firms. It is also a forward-looking decision with significant non-recoverable entry costs, mainly due to capital investments which are both firm- and location-specific. Thus, sunk costs, and the dynamic strategic behavior associated with them, are important forces behind the configuration of the spatial market structure that we observe in retail markets.

Despite its relevance, there have been very few studies analyzing spatial competition as a dynamic game. Existent models of industry dynamics often lack an explicit account of spatial competition. Although useful applications have emerged from the seminal work by Ericson and Pakes (1995), none have explicitly incorporated the spatial and multi-store features which are prevalent in many retailing industries.[1] The literature on spatial competition often restricts the treatment of time. Models based on the seminal work of Hotelling (1929) describe a two- or three-period framework where firms choose locations and then compete in the product market.[2] Eaton and Lipsey (1975), Schmalensee (1978), and Bonanno (1987) study the multi-store monopolist under the threat of entry. They find that, depending on the magnitude of entry costs, the monopolist will either proliferate or strategically locate its stores to deter entry, successfully preempting the competition. Judd (1985) notes that the aforementioned models place strong assumptions on firms' level of commitment. These papers assume that entry and location decisions are completely irreversible, with no possibility of exit or relocation. Judd shows that when there is strong substitutability among stores (*i.e.* proximity in space), *allowing for exit* may result in non-successful spatial preemption by the incumbent. Potential entrants know the incumbent firm may prefer to have a monopoly in a single location rather than being a monopolist in a location and a duopolist in another nearby location. Therefore, spatial preemption and entry deterrence by the incumbent is not a credible (i.e., equilibrium) strategy.

Judd's paper emphasizes that models of spatial competition between multi-store firms need to incorporate dynamics to its full extent, allowing for endogenous firm entry, store proliferation, exit, and forward-looking strategies. That is the intention of this paper. In this context, the contribution of this paper is threefold. First, we propose a dynamic model of an oligopoly industry

[1] Examples of applications of the Ericson-Pakes framework are Pakes and McGuire (1994), Gowrisankaran's (1999) study of mergers, Markowitz's (2003) study of network effects, and Benkard's (2004) study of the commercial aircraft industry. Ellickson and Beresteanu (2005) is also in that framework. They endogenize supermarkets' "store density," i.e., the number of stores per capita a firm owns in a market. However, spatial competition per se is not accounted for.

[2] Hay (1976), Prescott and Visscher (1977), Lane (1980), and Neven (1987) use the concept of subgame-perfect equilibrium to study pre-specified sequential entry by single-store firms. Firms incur entry costs but relocation is prohibitively expensive. They find that strategic product positioning is pursued by earlier movers to either deter entry or take profitable locations first. For a compilation of spatial competition models see Anderson *et al.* (1992).

characterized by spatial competition between multi-store firms. In this model, firms compete in prices and decide where to open or close stores depending on the location profile of competitors, demand conditions, and location-specific private-information shocks. We define and characterize a Markov Perfect Equilibria (MPE) in this model. Our framework is a useful tool to study multi-store competition issues that involve spatial and dynamic considerations. Some examples of topics within this class are: the evaluation of the welfare effects of possible mergers between multi-store firms; understanding the main factors that explain the patterns in the evolution of multi-store retailers such as Wal-Mart, Starbucks, or McDonalds; or studying the conditions under which spatial preemptive behavior is an equilibrium strategy.[3]

A second contribution of this paper is to provide an algorithm to compute an equilibrium of the model. The algorithm exploits simulation techniques that have been recently proposed for solving single-agent dynamic models (Rust, 1997), and for the estimation of dynamic games (Bajari, Benkard, and Levin, 2006). We apply and extend these ideas to the computation of equilibria in dynamic games of spatial competition.[4] The main idea behind our algorithm is that when firms calculate the expected value associated with a possible action, they do it by integrating only through the most likely paths of future exogenous state variables. Many low-probability paths of these state variables are not taken into account. This assumption reduces very substantially the cost of calculating these expected values and of computing an equilibrium. The algorithm provides an approximation to the actual MPE. However, an alternative interpretation is that the algorithm obtains the actual MPE of a model where firms face computational costs and use simulation techniques to minimize these costs when making decisions.

To illustrate the model and the algorithm, we present several numerical examples that analyze how the propensity of multi-store retailers to spatial preemptive behavior depends on the magnitude of sunk costs. With this purpose, we have to start with a useful definition of spatial preemption. Previous definitions have been based on simple three-period games where firms move sequentially, and in equilibrium there either is or there is not pre-emption. Instead we consider spatial preemptive behavior (both in practice and in the context of our model) to be a matter of degree, and propose an index that measures the intensity of firms' preemptive behavior in a market. This index can be calculated using information on firms' decisions of where to open/close their stores. In our numerical examples we calculate this index using simulated data from the equilibrium of the model. Then, we look at how this index varies when we modify some parameters of the model, i.e., entry costs, exit value and transportation costs. We also look at the relationship between the spatial preemption index and several market outcomes such as number of stores, profits and markups.

[3] The model in sections 5 and 6 of Judd (1983) is the closest to ours in the literature. However, Judd's model is still highly stylized relative to our model.

[4] The software for this algorithm, in GAUSS language, can be downloaded from the authors' web pages. In a companion paper we provide a manual that describes in detail the programs and procedures in this software (see Aguirregabiria and Vicentini, 2006).

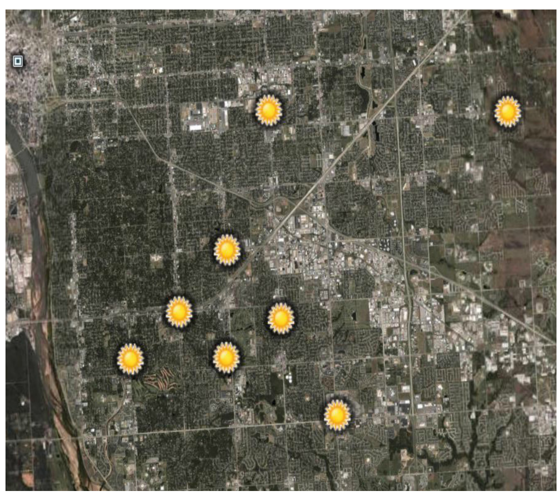
The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes the algorithm used to solve for equilibria. Section 4 presents the numerical exercises on spatial preemptive behavior. Section 5 summarizes and concludes.

2 Model

2.1 The Market

Consider a local market of a differentiated retail product (e.g., retail banking, supermarkets). From a geographic point of view the market is a compact set $\mathbb{C}$ in the Euclidean space $\mathbb{R}^2$. The distance between two points in the market, say a and b, is the Euclidean distance denoted by $\|a-b\|$. There is a finite set of L pre-specified locations where it is feasible for firms to operate stores. Let $\{z_1, z_2, ..., z_L\}$ be the set of geographical coordinates of these feasible locations, where $z_\ell \in \mathbb{C}$. We call each of this business locations a *submarket*. As a real life example, figure 1 presents the location of eight shopping malls (circles) in the city of Tulsa, Oklahoma.[5] These malls can be thought as the feasible locations for retailers such as large department stores.

FIGURE 1
Shopping Malls in Tulsa, Oklahoma



Time is discrete. At time t the market is populated by a continuum of consumers. Each consumer is characterized by a geographical location $z \in \mathbb{C}$. The geographical distribution of consumers at period t is given by the absolute measure $\phi_t(z)$ such that $\int_{\mathbb{C}} \phi_t(dz) = M_t$, where M_t is the size of the market. This measure ϕ_t evolves over time according to a discrete Markov process. Let Ω be the discrete set of possible ϕ_t's.

[5] Satellite picture constructed using the free software `Google Earth`, available at `http://earth.google.com/`.

There are I multi-store firms that can potentially operate in the market. We index firms by i and use $\Upsilon = \{1, 2, ..., I\}$ to represent the set of firms. At the beginning of period t a firm's network is represented by the vector $n_{it} = (n_{i1t}, n_{i2t}, ..., n_{iLt})$, where $n_{i\ell t}$ is the number of stores that firm i operates in location ℓ at period t. For simplicity, we assume that a firm can have at most one store in a location, such that $n_{i\ell t} \in \{0, 1\}$. The model can be easily generalized to the case with a maximum of $\bar{n} > 1$ stores per location and firm.[6] Overlapping of stores from different firms at the same location is allowed. The *spatial market structure* at period t is represented by the vector $\mathbf{n}_t = (n_{1t}, n_{2t}, ..., n_{It}) \in \{0, 1\}^{IL}$. A store in this market is identified by a pair (i, ℓ) where i represents the firm, and ℓ identifies the location.

We conclude this sub-section providing a big picture of the structure of the model. The details are in sections 2.2 to 2.4. Every period t, firms observe the spatial market structure $\mathbf{n}_t$, the state of the demand ϕ_t, and some location- and firm-specific shocks in entry, exit and fixed costs, which are private information of each firm. Given this information, incumbent firms compete in prices. Prices can vary over stores within the same firm. This spatial Bertrand game is static because current prices do not have any effect on future demand or profits. Furthermore, private information shocks affect fixed operating costs and entry costs but not the demand or variable costs. Therefore, these shocks do not have any influence in equilibrium prices. The resulting Bertrand prices determine equilibrium variable profits for each firm i at period t. At the end of period t, firms decide simultaneously their network of stores for next period. This choice is dynamic because of partial irreversibility in the decision to open a new store, i.e., sunk costs. Firms are allowed to open or close at most one store per period. Exogenous changes in the spatial distribution of demand (i.e., changes in ϕ_t), as well as firms' location-specific shocks to costs, generate simultaneous entry and exit at different locations and changes over time in the spatial market structure. Firms may grow over time in the geographic context and expand their network of stores, and possibly become a dominant player.

2.2 Consumer Behavior[7]

A consumer is fully characterized by a pair (z, υ), where z is her location in space and $\upsilon \in \mathbb{R}^{IL}$ is a vector representing her idiosyncratic preferences over all possible stores. Consumer behavior is static and demand is unitary. At every period t, consumers know all active stores with their respective locations and prices. A consumer decides whether to buy or not a unit of the good, and from which firm and store to buy it. The indirect utility of consumer (z, υ) patronizing store (i, ℓ) at time t is:

$$u(i, \ell) = \omega_i - p_{i\ell t} - \tau \ \ \|z - z_\ell\| + \upsilon_{i\ell} \tag{1}$$

[6] Some retail industries are characterized by firms having more than one store in a particular submarket. Starbucks often has multiple coffee shops in large malls.

[7] De Palma *et al.* (1985) consider a similar demand system but in a linear city.

ω_i is the quality of the product offered by firm i, common across its locations. All consumers agree on this measure. $p_{i\ell t}$ is the mill price charged by store (i, ℓ) at time t. The term $\tau \|z - z_\ell\|$ represents consumer's transportation costs, where τ is the unit transportation cost. Finally, $\upsilon_{i\ell}$ captures consumer idiosyncratic preferences for store (i, ℓ). The utility of the outside alternative (i.e., not purchasing the good) is normalized to zero.

A consumer purchases a unit of the good at store (i, ℓ) iff $u(i, \ell) \geq 0$ and $u(i, \ell) \geq u(i', \ell')$ for any (i', ℓ'). To obtain the aggregate demand at each store we have to integrate individual demands over the distribution of (z, υ). We assume that υ is independent of z and it has a Extreme Value distribution with dispersion parameter μ. The parameter μ measures the importance of horizontal product differentiation, other than spatial differentiation. Integrating over υ we obtain the *local demand* for store (i, ℓ) from consumers at location z:

$$d_{i\ell}(z, \mathbf{n}_t, \mathbf{p}_t) = \frac{n_{i\ell t} \ \exp\left\{(\omega_i - p_{i\ell t} - \tau \ \|z - z_\ell\|) / \mu\right\}}{1 + \sum_{i'=1}^{I} \sum_{\ell'=1}^{L} n_{i'\ell' t} \ \exp\left\{(\omega_{i'} - p_{i'\ell' t} - \tau \ \|z - z_{\ell'}\|) / \mu\right\}} \tag{2}$$

Integrating these local demands over the spatial distribution of consumers we obtain the *aggregate demand* for store (i, ℓ) at time t:

$$D_{i\ell}(\mathbf{n}_t, \mathbf{p}_t, \phi_t) = \int_{\mathbb{C}} d_{i\ell}(z, \mathbf{n}_t, \mathbf{p}_t) \ \phi_t(dz) \tag{3}$$

The aggregate *consumer surplus* is defined as:

$$CS(\mathbf{n}_t, \mathbf{p}_t, \phi_t) = \int_{\mathbb{C}} \mu \ \ln\left[1 + \sum_{i=1}^{I} \sum_{\ell=1}^{L} n_{i\ell t} \ \exp\left\{(\omega_i - p_{i\ell t} - \tau \ \|z - z_\ell\|) / \mu\right\}\right] \phi_t(dz) \tag{4}$$

And the aggregate *transportation costs* incurred by consumers is:

$$TC(\mathbf{n}_t, \mathbf{p}_t, \phi_t) = \sum_{i=1}^{I} \sum_{\ell=1}^{L} \int_{\mathbb{C}} d_{i\ell}(z, \mathbf{n}_t, \mathbf{p}_t) \ \tau \ \|z - z_\ell\| \ \phi_t(dz) \tag{5}$$

A few comments about this demand system are needed. Consumers' substitution patterns depend directly on the distance function $\|z - z_\ell\|$, so that a store competes more fiercely against closer stores. Stores' market areas are *overlapping* because of the unobserved heterogeneity of consumers, υ. Therefore a store serves consumers from all corners of the city $\mathbb{C}$, but more so the nearby patronage. Stores will always face a positive demand and can adjust prices without facing a perfectly elastic demand. Firms face the trade-off between strategic and market share effects. As stores locate closer to each other, the more intense price competition acts as a centrifugal force of dispersion (*strategic* effect). At the same time, firms wish to locate where transportation costs are minimum, which acts as a centripetal force of agglomeration (*market share* effect). An equilibrium spatial market structure would balance these forces, along with the effect of own-firm stores cannibalization.

Finally, we note the importance of the parameters μ and τ that capture product differentiation. As $\mu \to 0$ the degree of non-spatial horizontal product differentiation becomes smaller and market areas become well defined and predictable, with a consumer strictly shopping at the store with lowest full price (quality-adjusted mill price plus transportation costs) from her location. At the limit we would observe market areas defined as Voronoi graphs (or Thiessen polygons) with well-defined market borders (see Eaton and Lipsey, 1975, or Tabuchi, 1994, among others). Higher transportation cost τ increases the importance of location and the isolation of consumers, serving as a shield for market power and creating incentives for firm dispersion.[8]

2.3 Price Competition

For notational simplicity we omit the time subindex in this subsection. Every period, firms compete in prices taking as given their network of stores, the state of the demand, and variable costs. Firms may charge different prices at different stores. This price competition is a game of complete information. A firm variable profit function is:

$$R_i(\mathbf{n}, \mathbf{p}, \phi) = \sum\nolimits_{\ell=1}^{L} (p_{i\ell} - c_i(d_i)) \; D_{i\ell}(\mathbf{n}, \mathbf{p}, \phi) \tag{6}$$

$c_i(d_i)$ is the unit variable cost of firm i, constant across its stores. This cost is an increasing function of d_i, the average distance between all the stores in the network of firm i.[9] This specification captures the existence of *economies of density* in marginal costs. For a given number of stores, it is more costly to operate a network the larger the distance between the stores, such that scope economies are positively related to the proximity of own-firm stores. Distribution costs are a source of economies of density. It seems plausible that most of these economies of density operate through fixed costs. We incorporate that feature in the model (see section 2.4 below). However, economies of density may also reduce marginal costs. For instance, unit inventory costs can be smaller when stores are closer to each other because it is easier for these stores to share inventories in case of stockouts. Note that with this structure of variable costs, firms pass part of the gains from economies of density to consumers in terms of lower prices. That is not the case when economies of density operate only through fixed costs. See the work by Holmes (2006) on economies of density and the implications on the dynamics of store location by multi-store firms.

Each firm maximizes its variable profit by choosing its best-response vector of prices. The best response of firm i can be characterized by the first-order condition for each price $p_{i\ell}$:

$$D_{i\ell} + (p_{i\ell} - c_i) \frac{\partial D_{i\ell}}{\partial p_{i\ell}} + \sum\nolimits_{\ell' \neq \ell} (p_{i\ell'} - c_i) \frac{\partial D_{i\ell'}}{\partial p_{i\ell}} = 0 \tag{7}$$

The first two terms are the price and output effects of $p_{i\ell}$ on its own store (i, ℓ), while the last term is the output effect of $p_{i\ell}$ on all other stores of firm i. In our demand system, stores of a same

[8] Besides computing equilibrium prices, our Bertrand algorithm computes demand price elasticities for each location and store at these prices. These elasticities help the researcher better understand what are the actual market areas in geographic space. The detection of the relevant geographical market area has long been debated among antitrust authorities (see Willig, 1991, and Baker, 1997).

[9] The average distance between stores is $d_i = \left[\sum_\ell \sum_{\ell' \neq \ell} n_{i\ell} \; n_{i\ell'} \, \|z_\ell - z_{\ell'}\|\right] / \left[\sum_\ell \sum_{\ell' \neq \ell} n_{i\ell} \; n_{i\ell'}\right]$.

firm are *gross substitutes*, i.e., $\partial D_{i\ell'}/\partial p_{i\ell} > 0$ and therefore the third term is always positive. This implies that, *ceteris paribus,* a multi-store firm will offer higher prices than a single-store firm.

Following Berry (1994) and Berry *et al.* (1995), we define a square matrix Λ of dimension $IL \times IL$ with elements:

$$\Lambda_{i\ell}^{i'\ell'} = \begin{cases} -\dfrac{\partial D_{i'\ell'}}{\partial p_{i\ell}} & \text{if } i' = i \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

We can write the entire system of best-response equations as $\mathbf{D}(\mathbf{p}) - \Lambda(\mathbf{p}) \cdot (\mathbf{p} - c) = 0$, or what is equivalent:

$$\mathbf{p} = c + \Lambda(\mathbf{p})^{-1} \cdot \mathbf{D}(\mathbf{p}) \tag{9}$$

A spatial Nash-Bertrand equilibrium is then a vector $\mathbf{p}^*$ that solves the fixed-point mapping (9). Given our assumptions on the distribution of consumer taste heterogeneity υ, the mappings $\mathbf{D}(\mathbf{p})$ and $\Lambda(\mathbf{p})$ are continuously differentiable. Furthermore, the vector of prices $\mathbf{p}$ belongs to a compact set. Every price is greater or equal than its corresponding unit variable cost, and it is smaller or equal than the monopoly price of a firm with L stores and maximum quality ω_i. Therefore, by Brower's fixed-point theorem, a Nash-Bertrand equilibrium exists. This proof of existence can be extended to other specifications of the distribution of the vector υ of consumer heterogeneous tastes, as long as the distribution of υ is such that the functions $\mathbf{D}(\mathbf{p})$ and $\Lambda(\mathbf{p})$ are continuous in $\mathbf{p}$.

The equilibrium is not necessarily unique. This is a problem if we want to use the model for comparative statics or to study the effects of public policies. To deal with this issue we assume a particular *equilibrium selection mechanism*. This criterion for equilibrium selection is incorporated in the algorithm that we use to compute the equilibrium. We select the Nash-Bertrand equilibrium that we converge to when the fixed-point algorithm is initialized with prices equal to marginal costs c. This implies that we select the equilibrium with the *lowest* equilibrium prices.[10] Let $\mathbf{p}^*(\mathbf{n},\phi)$ be the vector of equilibrium prices associated with a value $(\mathbf{n},\phi)$ of the state variables. Solving this vector into the variable profit function one obtains the *equilibrium variable profit function*:

$$R_i^*(\mathbf{n}, \phi) \equiv R_i(\mathbf{n}, \mathbf{p}^*(\mathbf{n},\phi), \phi) \tag{10}$$

2.4 Dynamic game

At the end of period t firms simultaneously choose their network of stores $\mathbf{n}_{t+1}$ with an understanding that they will affect their variable profits at future periods. We model the location choice as a game of *incomplete information*, so that each firm i has to form beliefs about other firms' choices

[10] Alternatively, we might select the Nash-Bertrand equilibrium that we converge to when the fixed-point algorithm is initialized with prices equal to monopoly prices. In that case, we would be selecting the equilibrium with the highest prices.

of networks.[11] More specifically, there are components of the entry costs and exit values of a store which are firm-specific and private information. We assume that a firm may open or close *at most one store* per period. Given that we can make the frequency of firms' decisions arbitrarily high, this is a plausible assumption that reduces significantly the cost of computing an equilibrium in this model.

Let a_{it} be the decision of firm i at period t such that: $a_{it} = \ell_+$ represents the decision of opening a new store at location ℓ; $a_{it} = \ell_-$ means that a store at location ℓ is closed; and $a_{it} = 0$ means the firm chooses to do nothing. Therefore, the choice set is $A = \{0, \ell_+, \ell_- : \ell = 1, 2, ..., L\}$. Some of the choice alternatives in A may not be feasible for a firm given her current network n_{it}. In particular, a firm can not close a store in a submarket where it has no stores, and it can not open a new store in a location where it already has a store. We incorporate these constraints in the specification of the profit function below. In particular, there is a cost K of taking this type of actions, where K is a very large number.

2.4.1 Specification of the profit function

Firm i's current profit is:

$$\pi_i \left(a_{it}, \mathbf{n}_t, \phi_t, \varepsilon_{it}\right) \quad = \quad R_i^* \left(\mathbf{n}_t, \phi_t\right) - FC(n_{it}) - EC_{it} + EV_{it} \tag{11}$$

$FC(n_{it})$ is the fixed cost of operating all the stores of firm i. EC_{it} is the entry cost of creating a new store. And EV_{it} is the exit value of closing a store. Fixed operating costs depend on the number of stores but also on their location.

$$FC(n_{it}) = \left(\sum_{\ell=1}^{L} \theta_\ell^{FC} \; n_{i\ell t}\right) + \theta^{ED} \left(d_{it}\right) \tag{12}$$

θ_ℓ^{FC} is the fixed cost of operating a store in submarket ℓ. d_{it} is the average distance between all the stores in the network of firm i, as defined above, and $\theta^{ED}(.)$ is an increasing real valued function that captures the existence of *economies of density*. For a given number of stores, it is more costly to operate a network the larger the distance between the stores. The specification of entry cost is:

$$EC_{it} \quad = \quad \sum_{\ell=1}^{L} I\{a_{it} = \ell_+\} \;\; \left(\theta_\ell^{EC} + K \; n_{i\ell t} + \varepsilon_{i\ell t}^{EC}\right) \tag{13}$$

$I\{.\}$ is the indicator function. θ_ℓ^{EC} is the entry cost at location ℓ, and K is an arbitrarily large number that accounts for the restriction that a firm cannot open more than one store in the same location. The variable $\varepsilon_{i\ell t}^{EC}$ represents a firm- and location-specific component of the entry cost. This idiosyncratic shock is private information of firm i. The specification of the exit value is:

$$EV_{it} \quad = \quad \sum_{\ell=1}^{L} I\{a_{it} = \ell_-\} \;\; \left(\theta_\ell^{EV} - K \; (1 - n_{i\ell t}) + \varepsilon_{i\ell t}^{EV}\right) \tag{14}$$

[11] See Doraszelski and Satterthwaite (2003) for an analysis of dynamic games of incomplete information.

θ_ℓ^{EV} is the scrapping or exit value of a store in location ℓ. The term $K(1 - n_{i\ell t})$ accounts for the restriction that it is not possible to close a store that does not exist. The variable $\varepsilon_{i\ell t}^{EV}$ is a firm- and location-specific shock in the exit value of a store.

The vector of private information variables for firm i at period t is $\varepsilon_{it} = \{\varepsilon_{i\ell t}^{EC}, \varepsilon_{i\ell t}^{EV} : \ell = 1, 2, ..., L\}$. We make two assumptions on their distribution. First, we assume the vectors ε_{it}'s are independent of demand condition ϕ_t, and independently distributed across firms and over time. Independence across firms implies that a firm cannot learn about other firms' ε's by using its own private information. And independence over time means that a firm cannot use other firms' histories of previous decisions to infer their current ε's. These assumptions simplify significantly the computation of an equilibrium in this dynamic game. Second, we assume the ε_{it}'s have support over the entire real line with a cumulative distribution increasing with respect to every argument. These two assumptions allow for a broad range of specifications for the ε_{it}'s, including spatially correlated shocks.

2.4.2 Markov Perfect Equilibrium

We consider that a firm's strategy depends only on its payoff relevant state variables $(\mathbf{n}_t, \phi_t, \varepsilon_{it})$. Let $\alpha \equiv \{\alpha_i(\mathbf{n}_t, \phi_t, \varepsilon_{it}) : i \in \Upsilon\}$ be a set of strategy functions, one for each firm, such that $\alpha_i : \{0,1\}^{IL} \times \Omega \times \mathbb{R}^{2L} \to A$. A Markov perfect equilibrium (MPE) in this game is a set of strategy functions such that each firm's strategy maximizes the value of the firm for each possible $(\mathbf{n}_t, \phi_t, \varepsilon_{it})$ and taking other firms' strategies as given.

To characterize a MPE in this model, we first describe a firm's best response function. Let $\psi_i(\mathbf{n}_t, \phi_t, \varepsilon_{it}; \alpha)$ be firm i's best response function that is defined as:

$$\psi_i(\mathbf{n}_t, \phi_t, \varepsilon_{it}; \alpha) = \arg\max_{a_{it} \in A} \left\{ \pi_i\left(a_{it}, \mathbf{n}_t, \phi_t, \varepsilon_{it}\right) + v_i^\alpha(a_{it}, \mathbf{n}_t, \phi_t) \right\} \tag{15}$$

where $v_i^\alpha(a_{it}, \mathbf{n}_t, \phi_t)$ is the expect future profits of firm i if his current decision is a_{it} and all the firms, including firm i, behave in the future according to their respective strategy functions in α. That is,

$$v_i^\alpha(a_{it}, \mathbf{n}_t, \phi_t) \equiv \sum_{j=1}^{\infty} \beta^j \; E\left\{\pi_i\left[\alpha_i\left(\mathbf{n}_{t+j}^\alpha, \phi_{t+j}, \varepsilon_{i,t+j}\right), \mathbf{n}_{t+j}^\alpha, \phi_{t+j}, \varepsilon_{i,t+j}\right] \mid a_{it}, \mathbf{n}_t, \phi_t\right\} \tag{16}$$

where the expectation is taken over all the possible future paths of $\{\phi_{t+j}, \varepsilon_{t+j}\}$. We use the super-index α in $\mathbf{n}_{t+j}^\alpha$ to emphasize that the evolution of future networks of stores depends on the strategy functions in α. Note that $\psi_i(\mathbf{n}_t, \phi_t, \varepsilon_{it}; \alpha)$ is a best response to other firms' strategies but also to the own firm strategy α_i. That is, this best response function incorporates a *'policy iteration'* in the firm's dynamic programming problem. The *Representation Lemma* in Aguirregabiria and Mira (2006) shows that we can use this type of best response functions to characterize every MPE in the model. That is, a set of strategy functions is a MPE in this model if and only if these strategies are a fixed point of the best response functions in (15).

DEFINITION: A set of strategy functions $\alpha^* \equiv \{\alpha_i^*(\mathbf{n}_t, \phi_t, \varepsilon_{it}) : i \in \Upsilon\}$ *is a MPE in this model if and only if for any firm* i *and any state* $(n_t, \phi_t, \varepsilon_{it})$ *we have that:*

$$\alpha_i^*(\mathbf{n}_t, \phi_t, \varepsilon_{it}) = \psi_i(\mathbf{n}_t, \phi_t, \varepsilon_{it}; \alpha^*) \tag{17}$$

Next, we describe the form of the best response function ψ_i. Taking into account the specification of the profit function in equations (11) to (14), we have that firm i's best response is to *open* a store at location ℓ (i.e., $\psi_i(\mathbf{n}_t, \phi_t, \varepsilon_{it}; \alpha) = \ell_+$) if the following conditions hold:

$$-\theta_\ell^{EC} - \varepsilon_{i\ell t}^{EC} + v_i^\alpha(\ell_+, \mathbf{n}_t, \phi_t) \begin{cases} \geq & -\theta_{\ell'}^{EC} - \varepsilon_{i\ell' t}^{EC} + v_i^\alpha(\ell'_+, \mathbf{n}_t, \phi_t) \quad \text{for any } \ell' \\ \\ \geq & v_i^\alpha(0, \mathbf{n}_t, \phi_t) \\ \\ \geq & \theta_{\ell'}^{EV} + \varepsilon_{i\ell' t}^{EV} + v_i^\alpha(\ell'_-, \mathbf{n}_t, \phi_t) \quad \text{for any } \ell' \end{cases} \tag{18}$$

The first condition states that submarket ℓ is the best location for firm i to open a new store. The other two conditions establish that opening a new store is better than doing nothing and better than closing a store, respectively. Similarly, we have that firm i's best response is to *close* a store at location ℓ (i.e., $\psi_i(\mathbf{n}_t, \phi_t, \varepsilon_{it}; \alpha) = \ell_-$) if the following conditions hold:

$$\theta_\ell^{EV} + \varepsilon_{i\ell t}^{EV} + v_i^\alpha(\ell_-, \mathbf{n}_t, \phi_t) \begin{cases} \geq & \theta_\ell^{EV} + \varepsilon_{i\ell' t}^{EV} + v_i^\alpha(\ell'_-, \mathbf{n}_t, \phi_t) \quad \text{for any } \ell' \\ \\ \geq & v_i^\alpha(0, \mathbf{n}_t, \phi_t) \\ \\ \geq & -\theta_{\ell'}^{EC} - \varepsilon_{i\ell' t}^{EC} + v_i^\alpha(\ell'_+, \mathbf{n}_t, \phi_t) \quad \text{for any } \ell' \end{cases} \tag{19}$$

The first condition establishes that submarket ℓ is the best one to close an existing store of firm i. The other two conditions state that closing a store is better than doing nothing and better than opening a new store, respectively.

Equations (18) and (19) show that the strategies in α enter the best response function ψ_i only through the value function v_i^α. Therefore, we can write the best response function as $\psi_i(\mathbf{n}_t, \phi_t, \varepsilon_{it}; v_i^\alpha)$. This representation is useful to characterize a MPE in this model in terms of the value functions $\{v_i^\alpha : i \in \Upsilon\}$. And the characterization can be used to prove the existence of a MPE in this model and to compute an equilibrium. The following Proposition establishes this result.

PROPOSITION: Let $\alpha \equiv \{\alpha_i(n_t, \phi_t, \varepsilon_{it}) : i \in \Upsilon\}$ *be a set of strategy functions. And let* $v^\alpha \equiv \{v_i^\alpha : i \in \Upsilon\}$ *be the value functions associated with* α *as we have defined them in equation (16). Then,* α *is a MPE if and only if the vector of value functions* v^α *is a solution to the fixed-point problem* $v = \Gamma(v)$ *where* $\Gamma(v) = \{\Gamma_i(v) : i \in \Upsilon\}$ *and:*

$$\Gamma_i(v)(a_{it}, \mathbf{n}_t, \phi_t) \equiv \sum_{j=1}^{\infty} \beta^j \ E\left\{\pi_i \left[\psi_i \left(\mathbf{n}_{t+j}^{\psi(v)}, \phi_{t+j}, \varepsilon_{i,t+j}\right), \mathbf{n}_{t+j}^{\psi(v)}, \phi_{t+j}, \varepsilon_{i,t+j}\right] \mid a_{it}, \mathbf{n}_t, \phi_t\right\} \tag{20}$$

where the expectation is taken over all the possible future paths of $\{\phi_{t+j}, \varepsilon_{i,t+j}\}$*, and the notation* $n_{t+j}^{\psi(v)}$ *means that the evolution of future networks of stores is based on the strategy functions* $\psi_i(.,.,.;v)$.

Proof: Suppose that α is a fixed-point of the mapping (17). Therefore, the vector of value function v^α is equal to $v^{\psi(v^\alpha)}$, and this implies that v^α is a fixed point of the mapping Γ. Now, suppose that a vector of functions v is a fixed point of Γ and define the vector of strategy functions $\alpha = \{\psi_i(.,.,.;v) : i \in \Upsilon\}$. It is clear that α is an equilibrium of (17). ■

Given this Proposition, the proof of existence of a MPE is a straightforward application of Brower's theorem. Note that the variables $(a_{it}, \mathbf{n}_t, \phi_t)$ are discrete and take finite values. It follows that: (1) the set of value functions v^α can be represented as a vector in the Euclidean space; (2) this vector can take only a finite number of values; and (3) by the continuity of the profit function, these values are bounded. Therefore, v^α belongs to a compact set. Our assumptions on the distribution of the private information shocks imply that Γ is a continuous mapping. Thus, by Brower's theorem, an equilibrium exits. Given a vector of equilibrium values v^{α^*}, we can obtain the equilibrium strategy functions α^* using the characterization of the best response function in equations (18) and (19). We can also obtain the *equilibrium choice probability functions*:

$$P_i^{\alpha^*}(a_{it}|\mathbf{n}_t, \phi_t) \equiv \int I\{\alpha_i^*(\mathbf{n}_t, \phi_t, \varepsilon_{it}) = a_{it}\}\ dG(\varepsilon_{it}) \tag{21}$$

where $G(.)$ is the distribution function of the private information shocks. Knowledge of these functions, combined with the transition probabilities for the exogenous population distribution ϕ_t, allows the researcher to compute the probabilistic evolution of the city-economy given a particular initial condition, as well as the probabilistic steady-state features of this economy.

The model can have multiple equilibria. This is an issue when we use this model for comparative statics. We deal with this problem in the same way as we did with multiple equilibria in the Nash-Bertrand game: we impose an equilibrium selection mechanism. More specifically, we select the MPE that we converge to by iterating in the mapping Γ from the initial set of values $v^\alpha = \mathbf{0}$ (i.e., the values that correspond to firms' myopic behavior, $\beta = 0$).

3 Equilibrium algorithms

3.1 Computation of a Nash-Bertrand equilibrium

Recall the system of first-order conditions in the pricing game. For a given value of the state variables, define the mapping corresponding to that system as $\Delta(\mathbf{p}) \equiv c + \Lambda(\mathbf{p})^{-1} \cdot \mathbf{D}(\mathbf{p})$. Let $\Delta_{(i)}(\mathbf{p})$ be the elements of $\Delta(\mathbf{p})$ associated with the prices of firm i. Similarly, let $\mathbf{p}_{(i)}$ be the elements of the vector $\mathbf{p}$ associated with firm i. We implement a *Gauss-Siedel* fixed-point algorithm which iterates on firms using the mapping $\Delta(\mathbf{p})$. To obtain the lowest price equilibrium, we initialize the search with prices equal to marginal costs. *Step 0:* Start with the vector of prices $\mathbf{p}^0$ such that $\mathbf{p}^0_{(i)} = c_i$ for any $i \in \Upsilon$. *Step 1:* Compute aggregate demands $\mathbf{D}\left(\mathbf{p}^0\right)$ and the matrix of partial derivatives $\Lambda\left(\mathbf{p}^0\right)$ using quadrature integration (see below). *Step 2:* Starting with firm 1, obtain a new vector $\mathbf{p}^1_{(i)}$ for each firm i using Gauss-Siedel iteration: $\mathbf{p}^1_{(1)} = \Delta_{(1)}\left(\mathbf{p}^0\right)$, $\mathbf{p}^1_{(2)} = \Delta_{(2)}\left(\mathbf{p}^1_{(1)}, \mathbf{p}^0_{(2)}, ..., \mathbf{p}^0_{(I)}\right)$, ..., $\mathbf{p}^1_{(i)} = \Delta_{(i)}\left(\mathbf{p}^1_{(1)}, ..., \mathbf{p}^1_{(i-1)}, \mathbf{p}^0_{(i)}, ..., \mathbf{p}^0_{(I)}\right)$. *Step 3:* If $\left\|\mathbf{p}^1 - \mathbf{p}^0\right\|$ is smaller than a pre-fixed small constant, then $\mathbf{p}^* = \mathbf{p}^1$. Otherwise, proceed to *Step 1* with $\mathbf{p}^0 = \mathbf{p}^1$.

Therefore, the updating of the vector of prices for player i is done immediately after its computation, before computing a new vector $\mathbf{p}^1_{(i+1)}$ for the next player. Once the price equilibrium under an equilibrium selection rule is computed, we encode the equilibrium current variable profits of a firm given a particular state, $R_i(\mathbf{n}, \phi)$.

Given the logit assumption on the idiosyncratic tastes, the local demands have the closed form expression in (2). However, to obtain the vector of aggregate demands $\mathbf{D}(\mathbf{p})$ and the matrix of partial derivatives $\Lambda(\mathbf{p})$ we have to integrate local demands over the 2-dimensional city $\mathbb{C}$. We use a quadrature method with midpoint nodes (see Judd, 1998, ch. 7). We first divide $\mathbb{C}$ into a pre-specified number of mutually exclusive and adjacent rectangular cells, with each cell k having a representative node point $z_{(k)}$ in its center. For each location z in cell k we approximate the local demand $d_{i\ell}(z, \mathbf{n}_t, \mathbf{p}_t)$ and the density $\phi_t(z)$ using $d_{i\ell}(z_{(k)}, \mathbf{n}_t, \mathbf{p}_t)$ and $\phi_t\left(z_{(k)}\right)$, respectively. Therefore, we calculate aggregate demand for store (i, ℓ) as:

$$D_{i\ell}(\mathbf{n}_t, \mathbf{p}_t, \phi_t) = \textstyle\sum_k d_{i\ell}(z_{(k)}, \mathbf{n}_t, \mathbf{p}_t)\ \phi_t\left(z_{(k)}\right)\ area(k) \tag{22}$$

where $area(k)$ is the area of the rectangular cell k.

3.2 Computation of a MPE

A MPE is a fixed point of the equilibrium mapping Γ that we defined in equation (20). The evaluation of this equilibrium mapping requires one to integrate over all possible future paths of the exogenous variables $\{\varepsilon_t, \phi_t\}$. We use Monte Carlo simulation to deal with this high-dimension multiple-integration problem. Our algorithm computes a fixed point of an equilibrium mapping $\tilde{\Gamma}_R$ that is a simulated version of the mapping Γ. The subindex R represents the number of Monte Carlo simulations used to approximate expected values. The main feature of $\tilde{\Gamma}_R$ is that firms calculate the expected value associated with an action by using Monte Carlo simulation to integrate over the path of future exogenous state variables. This means that only R future paths are assigned a positive probability, while the rest of the paths receive zero probability. As R goes to infinity, the simulated mapping $\tilde{\Gamma}_R$ converges to Γ. But for finite R the mapping $\tilde{\Gamma}_R$ can still be a good approximation to Γ. We might also consider that $\tilde{\Gamma}_R$ is the actual equilibrium mapping because, due to computational costs, firms calculate their best responses by using Monte Carlo simulation.

First, we simulate R *independent* sequences of length T for the exogenous state variables ε_{it} and ϕ_t. We index simulated sequences by r, such that $\{\varepsilon_{it}^r : t = 1, 2, ..., T\}$ is the r^{th} simulated path for the private information shocks of firm i, and $\{\phi_t^r : t = 1, 2, ..., T\}$ is the r^{th} simulated path of the demand conditions.[12] Very importantly, these simulated sequences remain constant over the iteration of the algorithm. Define $\{n_t^r(a_0, \mathbf{n}_0, \phi_0, v) : t = 1, 2, ..., T\}$ as the sequence of networks given: (1) the r^{th} simulated sequence of exogenous state variables; (2) the initial conditions $(a_0, \mathbf{n}_0, \phi_0)$; and (3) that every firm behaves according to their best response functions given v. These simulated sequences of networks do change over the iteration of the algorithm because the vector of value function v changes. Now, we can describe the simulated equilibrium mapping $\tilde{\Gamma}_R(v)$ as $\{\tilde{\Gamma}_{R,i}(v) : i \in \Upsilon\}$ such that:

$$
\begin{aligned}
& \tilde{\Gamma}_{R,i}(v)(a_{i0}, \mathbf{n}_0, \phi_0) \\
\equiv \quad & \frac{1}{R} \sum_{r=1}^{R} \left\{ \sum_{t=1}^{T} \beta^t \ \pi_i \left[\left(\psi_i \left(n_t^r(a_0, \mathbf{n}_0, \phi_0, v), \phi_t^r, \varepsilon_{it}^r; v \right), n_t^r(a_0, \mathbf{n}_0, \phi_0, v), \phi_t^r, \varepsilon_{it}^r \right) \right] \right\}
\end{aligned} \tag{23}
$$

Step 0: We initialize the algorithm with a vector of values v^0 such that $v_i^0(a_{i0}, \mathbf{n}_0, \phi_0) = 0$, for any state, player, and action. *Step 1:* Then, we calculate a new vector of values v^1 using a Gauss-Jacobi iteration in the mapping $\tilde{\Gamma}_R$. That is, for any player i, any state $(\mathbf{n}_0, \phi_0)$ and any action a_{i0}, we calculate v^1 as $v_i^1(a_{i0}, \mathbf{n}_0, \phi_0) = \tilde{\Gamma}_{R,i}(v^0)(a_{i0}, \mathbf{n}_0, \phi_0)$. *Step 2:* If $\left\| v^1 - v^0 \right\|$ is smaller than a pre-fixed small constant, then $v^{\alpha^*} = v^1$. Otherwise, proceed to *Step 1* with $v^0 = v^1$, and conduct another round of Gauss-Jacobi iterations.

This algorithm computes value functions at every possible value of the state variables $(\mathbf{n}_0, \phi_0)$. The number of possible values in the space of $\mathbf{n}_0$ is 2^{IL}. This can be a huge number for values of IL greater than 20. When the number of firms is relatively large, an assumption that reduces significantly the number of values in the space of $\mathbf{n}_0$ is *symmetry among firms*. If firms have the same quality of their products and marginal costs (i.e., $\omega_i = \omega$ and $c_i = c$ for any firm i) then, the vector of arguments in the value function v_i is (a_i, n_i, N_{-i}, ϕ), where n_i is the network of firm i, and N_{-i} is the vector $\{N_{-i\ell} : \ell = 1, 2, ..., L\}$ where $N_{-i\ell} \equiv \sum_{j \neq i} n_{j\ell}$ is the number of stores in location ℓ from firms other than i. The number of values in the space of (n_i, N_{-i}) is $(2I)^L$, which is a smaller number than 2^{IL} and it is not exponential in the number of firms. However, the assumption of symmetric firms is only useful when the number of firms is greater than 2 and the number of locations is small. Otherwise, it does not imply any important reduction in computational cost. However, some interesting applications involve two firms over many possible locations in a city or region. For these cases, the use of interpolation can be useful. The idea is to compute the values $\tilde{\Gamma}_{R,i}(v)(a_i, \mathbf{n}, \phi)$ over a grid of points of $\mathbf{n}$ and then interpolate these values to approximate the mapping $\tilde{\Gamma}_{R,i}(v)(a_i, \mathbf{n}, \phi)$ over the values of $\mathbf{n}$ which are not in the grid.

[12] When ϕ_t is autocorrelated, the simulated sequence for this variable depends on the initial condition ϕ_0. In that case, we should generate a different simulated sequence for each possible value of the initial condition ϕ_0.

4 Spatial preemption

We apply the previous model to study spatial preemptive behavior. Spatial preemptive behavior cannot be defined or characterized in terms of primitives of the model. It is a characteristic of firms' equilibrium behavior in some dynamic games of entry in spatial markets. We start with a definition of what we mean by preemptive behavior in the context of our model.

DEFINITION: An equilibrium in our model presents spatial preemptive behavior if, in the absence of economies of density, equilibrium choice probabilities have the following properties.

> *(1) (Preemption): A firm's probability of entry to become a monopolist in a local market is larger when that firm is already a monopolist in a nearby local market.*
> *(2) (Entry Deterrence): Suppose that a firm is a monopolist in a local market. The probability that other firm enters in a nearby market is smaller if the monopolist is also a monopolist in a nearby market.*
> *(3) (Credibility): A firm's probability of exit from a duopoly in a local market is smaller when that firm is a monopolist in a nearby local market.*

A key feature of spatial preemptive behavior is that there is a relationship between firms' behavior in a local market and the (predetermined) market structure in nearby local markets. That is, spatial preemptive behavior is a form of strategic interaction that is both spatial and dynamic. Note that the definition includes the clause "*in the absence of economies of density.*" Economies of density may generate some of these spatial-dynamic behavioral patterns. However, in the case of economies of density, these patterns are not explained by strategic behavior but by simple cost reductions.[13] Here we concentrate in preemptive behavior and ignore economies of density.[14]

To illustrate more formally the previous definition, consider a simple version of the model with only two firms and two locations (i.e., $I = L = 2$) and where the demand conditions ϕ_t are constant over time. The market structure in a local market can take four possible values: zero firms; monopoly of firm 1; monopoly of firm 2; and duopoly. We represent these market structures using the symbols $\varnothing$, M_1, M_2, and D, respectively. Following the definitions of the equilibrium choice probabilities in section 2.4.2, let $P_i(\ell_+|n_\ell, n_{-\ell})$ be the probability that firm i enters in market ℓ given that the market structures in the two local markets are n_ℓ and $n_{-\ell}$. Similarly, $P_i(\ell_-|n_\ell, n_{-\ell})$ is the probability that firm i exits from market ℓ conditional on the initial market structures n_ℓ

[13] For instance, suppose that there is only one multi-store firm in the city and that the costs of this monopolist present economies of density. It is clear that condition (1) should be a characteristic of the optimal behavior of this monopolist.

[14] For studies that analyze empirically spatial preemption and economies of density, see West (1981), Holmes (2006) and Vicentini (2006).

and $n_{-\ell}$.[15] Our definition of spatial preemptive behavior implies the following inequalities between equilibrium choice probabilities (for $i \neq j$): (1) preemption, $P_i(\ell_+|\varnothing, M_i) > P_i(\ell_+|\varnothing, \varnothing)$; (2) entry deterrence, $P_j(\ell_+|M_i, M_i) < P_j(\ell_+|\varnothing, M_i)$; and (3) credibility, $P_i(\ell_-|D, M_i) < P_i(\ell_-|D, \varnothing)$.

According to this, we define the following index, which should be positive under condition (1):

$$\textit{Preemption Index} \quad \equiv \quad P_i(\ell_+|\varnothing, M_i) - P_i(\ell_+|\varnothing, \varnothing) \tag{24}$$

For models with two firms but more than two locations, we construct the index in the following way. For any location ℓ, let ℓ^* be the business location in the city that is closer to ℓ. When looking at entry/exit probabilities in location ℓ, we represent the vector of networks $\mathbf{n}$ as $(n_\ell, n_{\ell^*}, n_{-(\ell,\ell^*)})$, where n_ℓ and n_{ℓ^*} have the same definition as before, and $n_{-(\ell,\ell^*)}$ represents the vector with the spatial market structure at locations other than ℓ and ℓ^*. Then, we defined the index as:

$$\textit{Preemption Index} \quad \equiv \quad \textit{Average}\left\{ \; P_i(\ell_+|\varnothing, M_i, n_{-(\ell,\ell^*)}) - P_i(\ell_+|\varnothing, \varnothing, n_{-(\ell,\ell^*)}) \; \right\} \tag{25}$$

where the average is over all the locations ℓ and over all the possible values of $n_{-(\ell,\ell^*)}$.

As mentioned in the introduction, the importance of Judd (1985)'s article was to emphasize that, in order for an incumbent firm to effectively spatially preempt a market through multi-store positioning, a full consideration of store substitutability, entry costs, and exit values is needed. Once exit is allowed the potential entrant recognizes that upon entry into one of the incumbent submarkets, the incumbent's self-cannibalization is exacerbated by the augmented price competition in the duopoly submarket, which may lead it to leave that market if exit costs are low enough. This forward-looking behavior of the potential entrant impacts the credibility of the preemptive motive by the incumbent. In sum, the credibility of preemptive behavior can in principle be parametrized by entry costs, exit values, and the distance (substitutability) amongst locations (submarkets).

However, it is complicated to establish ex-ante for which specifications of the primitives of the model preemptive behavior is more likely. To investigate the relationship between some structural parameters and spatial preemption, we perform some numerical experiments. In particular, we compute the MPE for different levels of entry costs, exit values, and consumer transportation costs. For each equilibria we measure preemptive behavior by using our preemption index. We also analyze the implications of preemptive behavior on market outcomes by looking at market structure, profitability of firms, and price markup.

4.1 Benchmark model

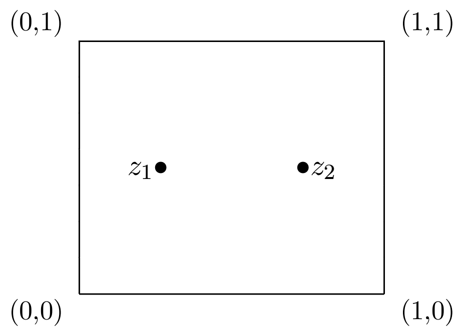
The following parameters are constant over our experiments.

(a) The Market. The city is a unit square, $\mathbb{C} = [0, 1]^2$, and consumers are uniformly distributed on $\mathbb{C}$ with population size equal to 10. Both the geographical distribution of consumers and population

[15] Of course, the probability of entry is zero if firm i is already an incumbent in market ℓ, and the probability of exit is zero if firm i is not active in that market.

size are constant over time. The transportation cost parameter τ will vary in our experiments. We consider a simple specification for the space of feasible business locations. There are two locations ($L = 2$) with $z_1 = (0.2, 0.5)$ and $z_2 = (0.8, 0.5)$. Therefore the market and the locations are symmetric. Figure 2 presents the spatial configuration of these business locations.

FIGURE 2
Feasible Business Locations



(b) Firms. There are two multi-stores firms ($I = 2$) which are potential entrants in the local markets of this city. These firms are identical in terms of the quality of their products, which is set at $\omega_1 = \omega_2 = 1$. The parameter μ, that measures the importance of horizontal product differentiation, is equal to 0.25. Firms are also identical in their cost structures. Marginal costs are equal to one.[16] Fixed operating costs are normalized to zero. The common knowledge component of entry costs and exit values are constant across markets. For the benchmark model we set entry costs at $\theta^{EC} = 1$ and exit values at $\theta^{EV} = 0$ for every location. However, we analyze the effects of changes in both of these parameters. The private information parts of entry costs and exit values have standard normal distributions which are independently distributed across firms and locations and over time. The discount factor is set to $\beta = 0.9$.

We obtain the MPE for different values of exit values, entry costs, and consumer transportation costs. In particular, we consider a grid for exit values of $\theta^{EV} \in \{-3.0, -2.5, ..., 2.0\}$, for entry costs of $\theta^{EC} \in \{0, 0.2, ..., 2.0\}$, and for transportation costs of $\tau \in \{0.0, 0.5, 1.0\}$. For each simulation path we consider $T = 15$ time periods and we simulate $R = 700$ paths.

[16] To give an idea of the range of price cost margins associated with different values of ω, μ, and c, note that with zero transportation costs the unique Nash-Bertrand equilibrium (see the proof of equilibrium uniqueness in the appendix of chapter 7 in Anderson *et al* (1992)) is such that the equilibrium price cost margin m solves the nonlinear equation:

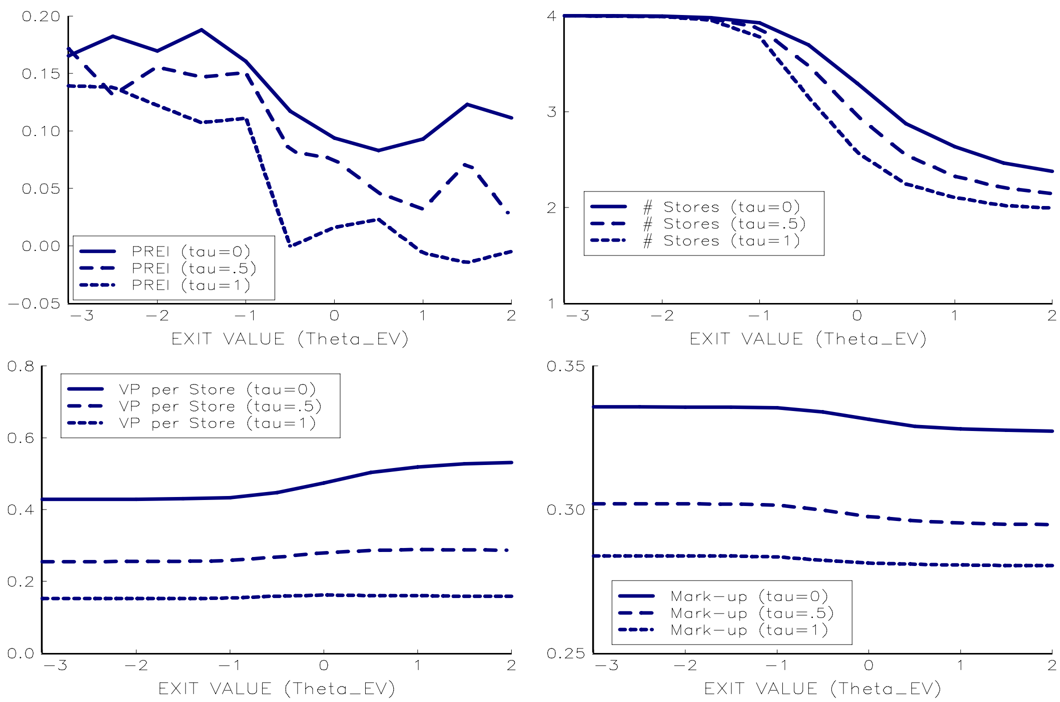
$$m = \mu \left(\frac{1 + I \ \exp\{(\omega - c - m)/\mu\}}{1 + (I-1)\exp\{(\omega - c - m)/\mu\}} \right)$$

With $I = 2$, $\omega = 1$, $\mu = 0.25$, and $c = 1$, the solution is $m = 0.307$, which is a 30.7% markup.

4.2 Results

First, we analyze the effect of changes in exit values given three different values for the transportation cost parameter. We compute the MPE's associated with the values of $\theta^{EV} \in \{-3.0, -2.5, ..., 2.0\}$ and $\tau \in \{0.0, 0.5, 1.0\}$. The value of the entry cost is held fixed at $\theta^{EC} = 1$. Figure 3 displays the average values of the spatial preemption index, the number of total stores, variable profits per store, and markup, based on simulations from each equilibria.

FIGURE 3
MPE Features for Different Values of θ^{EV} and τ



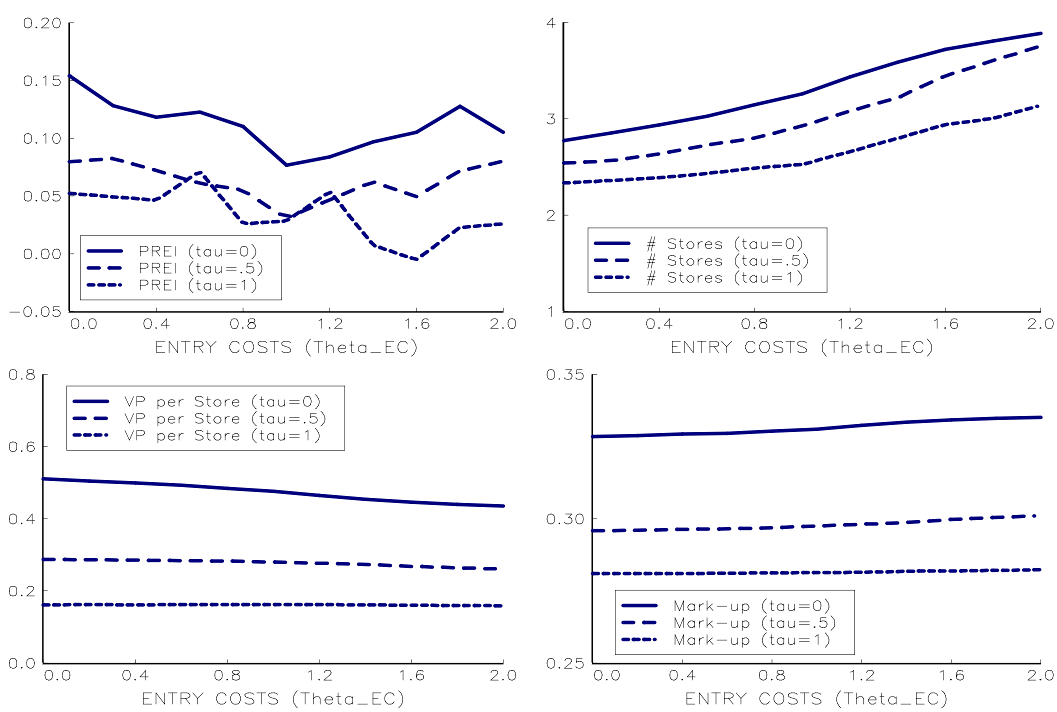
For ranges of θ^{EV} where exit is very expensive (*i.e.*, below -1), the preemption index is the highest, as expected from the analysis in Judd (1985). In this case, entry into a submarket entails commitment to that location, and the probability of exit is very small given the prohibitive exit costs, as the top right panel shows the number of stores in equilibrium to be basically 4. The preemptive motive declines steadily for the most part as exit costs decline (or exit values increase), which is a sign of the link between exit costs and the pursuit of preemption in equilibrium. For exit values greater than one, the preemption index is very small and this means that there is not any dynamic-spatial link between the two submarkets. That is, with that level of the exit value, spatial preemption is not a credible equilibrium strategy.

It is also interesting to note that the higher the transportation costs, the lower the preemptive behavior in equilibrium. With $\tau = 0$ and $\tau = 0.5$ the two business locations are very close substitutes, and the strategic interaction *across locations* matters most, with firms attempting to take the second location before the competitor does. With higher transportation costs ($\tau = 1$), the two submarkets become markets on their own, since the substitutability between the two locations declines, and the equilibrium behavior in a local market is no longer so dependent on the market structure in nearby local markets.

The upper right panel in figure 3 shows the number of stores. Very interestingly, the number of stores declines with the exit value θ^{EV}. Note that a "partial equilibrium" analysis, that ignores strategic interactions, would predict that the larger the exit value, the larger the number of stores. However, once we include dynamic strategic interactions and the preemption motive, this prediction does not hold. With a low exit value, firms show preemptive behavior and this leads to a large number of stores in equilibrium. For high exit values, this is no longer true. Now, firms do not have a preemptive motive and they end up having, on average, just one store each.

Next, we analyze the effect of changes in *entry costs* given three different values for τ. We compute the MPE's associated with the values of $\theta^{EC} \in \{0, 0.2, ..., 2.0\}$ and $\tau \in \{0.0, 0.5, 1.0\}$. The exit value is held fixed at $\theta^{EV} = 0$, so that the level of entry costs are equivalent to *sunk costs*. Figure 4 displays the equilibrium preemption index, number of total stores, variable profits per store, and markup.

FIGURE 4
MPE Features for Different Values of θ^{EV} and τ



The preemption index declines for entry costs between 0 and 1, but then it increases for values above 1, especially for low values of transportation costs. Therefore the value of commitment to a location, as parametrized by θ^{EC}, seems to increase the index when the substitutability among stores is greater (lower τ), which is similar to the findings in figure 3 for exit values. In fact, the level of commitment and sunk costs, as parametrized by either very negative values of θ^{EV} in figure 3 (as one moves towards the *left* of its panels), or high positive values of θ^{EC} in figure 4 (as one moves towards the *right* of its panels), seem to dictate the market structure, conduct, and strategic interaction in equilibrium. Moreover, the degree to which such commitment affects the equilibrium behavior is accentuated by *lower* levels of transportation costs, since then the competition among firms is fiercer and so are strategic interactions such as preemptive behavior.

5 Conclusion

This paper proposes a dynamic model of an oligopoly industry characterized by spatial competition between multi-store firms. Firms compete in prices and decide where to open or close stores depending on the spatial market structure. We define and characterize a Markov Perfect Equilibria (MPE) in this model. Our framework is a useful tool to study multi-store competition issues that involve spatial and dynamic considerations. An algorithm to compute an equilibrium of the model is proposed. The algorithm exploits simulation techniques recently proposed in Rust (1997) and in Bajari, Benkard, and Levin (2006). Its main idea is that when firms calculate the expected value associated with a possible action, they do it by integrating only through the most likely paths of future exogenous state variables. We illustrate the model and the algorithm with numerical experiments that analyze how the propensity of multi-store firms to spatial preemptive behavior depends on the magnitude of entry costs, exit value, and transportation costs. We find that higher levels of commitment to a particular location, as parametrized by sunk costs, and higher substitutability between locations, as parametrized by lower consumer transportation costs, entails higher propensity by multi-store firms to engage in spatial preemption.

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