University of Toronto
Department of Economics

Working Paper 298

Pricing and Signaling with Frictions

By Alain Delacroix and Shouyong Shi

September 12, 2007

Pricing and Signaling with Frictions*

Alain Delacroix
Département des Sciences Économiques
Université du Québec à Montréal

Shouyong Shi
Department of Economics
University of Toronto

September 2007

Abstract

In this paper, we introduce private information into a market with search frictions and evaluate the relative efficiency of two pricing mechanisms, price posting and bargaining. Each seller chooses investment that determines the quality of the good. This quality is the seller's private information before matching and it will be observed in a match. Sellers enter a search market competitively and can choose either to post prices or to bargain. In this environment, a pricing mechanism affects efficiency through the choice of quality *and* the number of trades. Bargaining induces the efficient choice of quality but an inefficient number of trades because the division of the match surplus is generically inefficient. By directing buyers' search, posted prices internalize search externalities and induce the constrained efficient outcome in the case of public information. However, when the quality is private information, this role of posted prices in directing search can conflict with their role in signaling quality. Focusing on this conflict, we find that bargaining could yield higher efficiency than price posting. We characterize the parameter regions in which each of the two mechanisms dominates in efficiency.

JEL classifications: D8, C78, E24.
Keywords: Directed search; Signaling; Bargaining; Efficiency.

* We have received useful comments from the participants of the Society for Economic Dynamics meeting (Prague, 2007). The second author would like to acknowledge financial support from the Bank of Canada Fellowship and from the Social Sciences and Humanities Research Council of Canada. The opinion expressed here is the authors' own and it does not reflect the view of the Bank of Canada.

1. Introduction

In this paper, we evaluate the relative efficiency of price posting versus bargaining in a search market with private information. On one side of the market, each agent chooses investment that determines the quality of the good. This quality is the agent's private information before matching and it will be observed in a match. Agents enter a search market competitively and can choose either to post prices or to bargain. A posted price can signal the quality of the good, as well as directing the search by agents on the other side of the market. We characterize the equilibrium under the two pricing mechanisms separately, compare their relative efficiency, and then make predictions about which mechanism will arise in the market. We find that with private information, bargaining can dominate price posting in efficiency in a positively measured subset of parameter values, in contrast with the public information case (Acemoglu and Shimer, 1999).

The market outlined above has features that are common to all "search goods", whose qualities are observed only after match.[1] One example is the labor market, where firms can create jobs that differ in amenities and working conditions. Workers can find out the quality of a particular job only after visiting the firm. Another example is the market of goods such as a piece of furniture. A seller can incur a cost to improve the quality of the furniture, but buyers do not know the quality unless they visit the particular seller and inspect the furniture. Let us refer to the agents who undertake the investment as "sellers" and the agents on the other side of the market as "buyers". Sellers can choose either to post prices or bargain over prices. A posted price can signal the quality of the good.

The key insight of our analysis is that a pricing mechanism affects efficiency in two dimensions: the quality of goods and the number of trades. With bargaining, sellers always choose the efficient quality: because a buyer will observe the quality of the good after meeting a seller, it is optimal for the seller to choose the quality to maximize the joint surplus of the match, regardless of the bargaining power. However, because bargained prices do not direct buyers' search, they produce generically inefficient division of the matched surplus between buyers and sellers. This inefficient division leads to inefficient entry of sellers into the market and, hence, an inefficient number of trades in the equilibrium. The extent of this inefficiency under bargaining can be measured by the deviation from the Hosios (1990) condition, i.e., by the difference between sellers' bargaining power, denoted σ, and the share of sellers' contribution to matches. The entry of sellers is excessive when

[1] They contrast with the so-called "experience goods" whose qualities can be discovered only after consumption, such as a cup of coffee.

this difference is positive and deficient when this difference is negative.

With price posting, in contrast, each seller can direct buyers' search in the following sense: by changing the posted price, the seller understands that he will affect directly the number of buyers whom he will attract and, hence, the probability of selling the good. This directed nature of search endogenizes sellers' share of the match surplus. When quality is public information, this endogenous share satisfies the Hosios condition; that is, directed search internalizes matching externalities and generates the efficient division of the match surplus. In this case, the amount of entry of sellers and the number of trades are efficient. When quality is private information, however, the ability of posted prices to direct search may be compromised by the need to set prices to signal quality. This compromise can generate either inefficient entry of sellers and/or an inefficient choice of quality.

A large part of this paper is devoted to the analysis of the signaling equilibrium under price posting, since the equilibrium under bargaining is straightforward. Under reasonable restrictions on buyers' beliefs about sellers' types, we show that there is a unique equilibrium, where all sellers in the market produce the same quality in the signaling equilibrium. The quality level in the equilibrium and the extent of the potential inefficiency depend on the ratio of low to high quality, denoted as ρ.

To evaluate the relative efficiency of the two pricing mechanisms, we use a social welfare function which is defined as the sum of expected match surpluses of all the agents in the market. Because sellers make zero expected profit under competitive entry, and because agents are risk neutral, the social welfare function is equal to the sum of buyers' expected surpluses. The efficient allocation requires that only high-quality goods be produced and that the amount of entry of sellers be efficient. When quality is public information, price posting is efficient and hence dominates bargaining for almost all parameter values, as in Acemoglu and Shimer (1999). When quality is private information, the relative efficiency of the two pricing mechanisms depends on two parameter values: sellers' bargaining power, σ, and the relative quality of low-quality goods, ρ. For almost all values of (σ, ρ), the two mechanisms can be ranked in efficiency and only the mechanism with higher efficiency will exist in the equilibrium.

The ranking, described below, is directly related to the two dimensions of efficiency described above. When ρ is sufficiently low, price posting can efficiently signal high-quality investment and therefore dominates bargaining generically. When ρ is moderately large, however, posted prices cannot fulfill both their search-directing and signaling roles: prices must be below a certain level to direct search efficiently, but such prices would be too low for high-quality sellers to signal their quality. This conflict between the two roles of posted prices generates inefficiency. In this case, the ranking in efficiency between the two pricing mechanism can take one of the following forms: (i) price posting with high quality still dominates, but with too much entry; (ii) price posting with low quality dominates, resulting in too little entry and inefficient quality; (iii) bargaining with efficient quality dominates, but with generically inefficient entry. Thus, with private information, bargaining dominates price posting when sellers' bargaining power is close enough to the Hosios condition and when the difference between low and high quality is small. Otherwise, price posting dominates bargaining.

The two main ingredients in our analysis are directed search and signaling. Each ingredient has been analyzed in the literature separately but not together. In the literature on prices as signals of quality, either search frictions are absent (e.g., Wolinsky, 1983, and Rogerson, 1988), or search is not directed (e.g., Bester, 1993). On the other hand, the growing literature on directed search often omits private information, e.g., Peters (1984, 1991), Moen (1997), Acemoglu and Shimer (1999), and Burdett et al. (2001). Putting the two ingredients together not only captures important features of realistic markets, but also yields non-trivial comparisons between the two pricing mechanisms. For example, while Acemoglu and Shimer (1999) show that price posting dominates bargaining in efficiency for almost all parameter values, we show that bargaining can dominate price posting for a positively measured subset of parameter values.

A few recent papers have incorporated private information into models of directed search. Analyzing the goods market, Peters and Severinov (1997) construct a model where buyers have independent values of the goods and sellers use auctions to direct buyers' participation. Forand (2007) analyzes a related problem where sellers compete by offering a probability with which buyers can realize the true valuation of the object after visiting the seller. Guerrieri (2005), Menzio (2007), Michelacci and Suarez (2006), and Shimer and Wright (2007) analyze directed search in the labor market with private information. One main difference between all these papers and ours is that signaling through prices is unimportant in these papers but critical in our analysis. In these papers, except Menzio (2007), the agents who offer pricing mechanisms do not have private information before setting prices and so, signaling is irrelevant. Menzio (2007) allows firms to have private information on the quality of the vacancies they want to fill, but he excludes price signaling by assuming that wages are determined ex post by Nash bargaining.

There are other specific differences between each of these papers and ours. Menzio (2007) focuses on how firms can direct workers' search by making cheap talks, i.e., pre-match announcements that do not constitute any contractual obligation. Guerrieri (2005) focuses on dynamic inefficiency in the labor market which arises from the externality that recruiting firms in one period generate on recruiting firms in the next period. In her model, private information (in a worker's disutility of working) is created after, rather than before, a match is formed. Peters and Severinov (1997) and Forand (2007) examine auctions. Shimer and Wright (2007) analyze a model with two-sided private information (worker's effort and firm's productivity), where firms can direct workers' search by offering contracts. Again, firms do not know their productivity levels at the time of offering contracts. In common, these papers fix one pricing mechanism and, hence, do not compare price posting with bargaining. Michelacci and Suarez (2006) compare the two pricing mechanisms, but their model is one of adverse selection rather than signaling. At the end of section 5, we will compare their results with ours in detail.

The remainder of this paper is organized as follows. In section 2, we will describe the environment of the economy. In section 3, we will assume that there is no private information and analyze the efficient allocation, the equilibrium under price posting, and the equilibrium under bargaining. In section 4, we will analyze the equilibrium with price posting under private information. Section 5 will compare the two pricing mechanisms in efficiency. Section 6 will discuss related topics. We will conclude in section 7 and provide necessary proofs in the Appendix.

2. The Model

To simplify the terminology, we will describe the market as one for goods, although it also captures some aspects of the labor market. The economy has one period. There are a large number of identical buyers, whose mass is normalized to 1. Each buyer wants to consume one unit of good. The utility of consumption is equal to the quality of the good, denoted k. There are also a large number of potential sellers. The number of active sellers in the market per buyer, denoted n, is endogenously determined by competitive entry. Refer to n as the economy-wide tightness of the market. A seller must produce the good when entering the market. The cost of a good of quality k is $\psi(k)$, where $\psi(0) > 0$, $\psi' > 0$ and $\psi'' < 0$. The assumption $\psi(0) > 0$ is intended to capture the cost of entering the market.

The quality of the good is the seller's private information before a match. However, after meeting the seller, a buyer observes the quality of the good immediately. Thus, we focus on "search goods" and abstract from "experience goods". As in the literature (e.g., Bester, 1993), this abstraction enables us to focus on the interaction between search frictions and pricing mechanisms.

To simplify the analysis, let $k \in \{k_H, k_L\}$, where $k_H = 1$ and $k_L = \rho \in (0, 1)$.[2] Label the goods with quality k_H as high-quality goods and the goods with quality k_L as low-quality goods. The sellers producing these goods are called high-quality sellers and low-quality sellers, respectively. Denote the average cost of quality as $a(k) = \psi(k)/k$. We maintain the following assumption:

Assumption 1. *(i) $a(k) < 1$ for all $k \leq 1$; (ii) $a'(k) < 0$ for all $k \leq 1$.*

With part (i) of this assumption, a good of quality k can be produced in a price posting equilibrium. Part (ii) ensures that producing a high-quality good is always more efficient than producing a low-quality good if the quality is public information. Thus, if low-quality goods are produced in an equilibrium, it is an inefficiency generated by private information.

The goods market has two submarkets and agents can choose which one to participate in. In one submarket, sellers post prices; in the other, prices are determined by bargaining. At the beginning of the period, potential sellers choose whether to enter a particular submarket. Upon entering, a seller chooses a quality level and produces the goods. In the submarket with price posting, each seller must post and commit to a price when entering the submarket. After observing all posted prices and the measure of sellers in each submarket, buyers choose which submarket to enter. Each buyer can choose to visit at most one seller.[3] After matching, a seller randomly chooses one of the visiting buyers to trade with at the posted price. In the submarket with bargaining, buyers and sellers are randomly matched in pairs and, in each match, the two agents trade at a price that is determined by generalized Nash bargaining. After trade, buyers consume and the economy ends. The value of an unsold good is zero.

In both submarkets, search frictions can be described by the matching probabilities. Let q denote the queue length of buyers at a particular seller, i.e., the expected number of buyers whom the seller will receive. Assume that a buyer's matching probability is $F(q)$ and a seller's matching probability is $qF(q)$. Search frictions are reflected by the feature that these matching probabilities are less than one, which we will assume below.

[2] It is straightforward to extend our analysis to the economy where the set of k contains more than two levels, provided that the set is discrete. However, the analysis will be significantly more complicated if k lies in an interval. In this case, the refinement criterion that we will use in section 4 will not be sufficient to produce a unique signaling equilibrium under price posting. Much stronger criteria, such as the universal divinity, will be needed.

[3] The assumption that a buyer cannot visit two or more sellers simultaneously captures the fact that a buyer cannot physically be at two stores at the same time. Because a buyer must visit a seller in order to find the quality of the seller's good, the assumption is realistic.

The two mechanisms differ in two critical aspects. First, posted prices cannot directly be a function of the quality of the good in a match but bargained prices can. This assumption is common in the literature (e.g., Bester, 1993, and Michelacci and Suarez, 2006), and it is necessary for the two mechanisms to potentially induce different choices of quality. A justification for the assumption is that it is difficult for a third party to verify the quality of the good, and so a posted price as a function of the quality cannot be enforced.

Second, posted prices can *direct buyers' search* but bargained prices cannot. With price posting, a seller anticipates that he can affect buyers' search decisions by changing the price and, thereby, affect the number of buyers visiting him. In this case, each seller regards q as a function of the price he posts, which will be derived later.[4] In contrast, with bargaining, each seller takes q as given because changes in a bargained price cannot affect the probability with which that match is formed. In this case, the queue length for each seller is equal to the economy-wide tightness of the market, which is $1/n$. To emphasize this feature, we sometimes write q under bargaining as Q.

The matching probabilities above imply that the matching technology has constant returns to scale, because the probabilities depend only on the queue length. In addition, we require the matching technology to satisfy the following standard assumption:

Assumption 2. *(i)* $F(q), qF(q) \in [0,1]$ *for all* q, $\lim_{q\to 0} F(q) = \lim_{q\to+\infty} qF(q) = 1$ *and* $\lim_{q\to+\infty} F(q) = \lim_{q\to 0} qF(q) = 0$ *; (ii)* $F' \leq 0$, $F + qF' \geq 0$*; (iii)* $qF'' \leq -2F'$*; and (iv)* $\lim_{q\to+\infty} q[F(q) + qF'(q)] = 0$. *All the inequalities are strict if* $F(q)$ *and* $qF(q)$ *lie in the interior of* $(0,1)$.

Part (i) of this assumption is evident. Part (ii) requires that the matching probability should decrease for a buyer and increase for a seller as the number of buyers per seller increases. Part (iii) requires a seller's matching probability to be a concave function of q. Part (iv) is required for existence of a solution to the canonical price posting equilibrium.

Part (ii) of the above assumption implies non-trivial trade-offs between prices and matching probabilities under price posting. When a seller posts a price higher than others', he should expect that fewer buyers will visit him and, hence, his matching probability will be lower. However, if he gets a match, he sells the good for a higher price. Conversely, when a seller posts a price lower than others, he should expect an increase in the matching

[4] We assume that the function $F(.)$ is exogenous, as in the formulations of directed search by Moen (1997) and Acemoglu and Shimer (1999). Some other models of directed search derive the function F from the strategic game among a finite number of agents and then take the limit when the number of agents goes to infinity (e.g., Peters, 1991, and Burdett et al., 2001). This difference in the modelling is not very important for the main results of our analysis.

probability, at the cost of a lower ex post profit. Similarly, buyers face the trade-off between price and the matching probability. These trade-offs do not exist under bargaining.

The following well-known matching functions satisfy the above assumption:

Example 2.1. *One example of the matching function is the so-called urn-ball matching function, which yields $F(q) = (1 - e^{-q})/q$. This matching technology can be derived from micro-foundations (Burdett et al., 2001) and satisfies all conditions in Assumption 2. Another example is the CES matching function, which yields $F(q) = F_0 \left[\lambda + (1-\lambda) q^{-\theta}\right]^{1/\theta}$, where $\lambda \in (0,1)$ and $F_0 > 0$. Truncate the function and restrict F_0 suitably to satisfy (i) of Assumption 2. The function also satisfies (ii) and (iii) of the assumption. Part (iv) of the assumption is satisfied when $\theta < 0$.*

3. The Economy with Public Information

Assuming that quality is public information in this section, we characterize the efficient allocation, the equilibrium with price posting, and the equilibrium with bargaining. Throughout this paper, we focus on equilibria that are symmetric in the sense that all buyers make the same decisions, including their responses to a seller's deviation. Then, we compare the two pricing mechanisms in efficiency.

3.1. Efficient Allocation under Public Information

Let us first examine the socially efficient allocation under public information. This will provide a reference point to which we compare the efficiency levels of the two pricing mechanisms. For this purpose, imagine that a social planner faces the same search frictions as the market does. An implication of this restriction is that the planner must treat symmetrically all buyers and all sellers with the same quality. Such symmetric allocations require that $q = 1/n$. The social planner maximizes a social welfare function, which is defined as the sum of net values in the economy. Because the sum of expected utility of buyers' is $F(q)k$ and the sum of production costs is $n\psi(k)$, where $n = 1/q$ is the measure of sellers, then the social welfare function is:

$$W(k,q) = F(q)k - \frac{\psi(k)}{q}. \tag{3.1}$$

Price does not enter here because it is a transfer between agents. The social planner's choices are the quality of goods, k, and the tightness of the market, q (or equivalently, n).

Under the maintained assumptions on $F(.)$, the welfare function has a single peak in q. The peak occurs at $q = q^*(k)$, where $q^*(k)$ is the solution to the following first-order condition:

$$-q^2 F'(q) = a(k). \tag{3.2}$$

Because the left-hand side is an increasing function under part (iii) of Assumption 2, the solution to the equation is unique, if it exists. Existence is ensured under Assumptions 1 and 2.[5] Then, part (ii) of Assumption 1 implies that the function $W(k, q^*(k))$ is increasing in k for all $k \leq 1$ $(= k_H)$. Thus, the efficient choice of quality is $k = 1$.

To simplify notation, let us denote $q_H^* = q^*(1)$ and $q_L^* = q^*(\rho)$. Note that part (ii) of Assumption 1 and part (iii) of Assumption 2 imply that $q_H^* < q_L^*$. We summarize the above results as follows:

Lemma 3.1. *Under public information, the efficient allocation is $k = 1$ and $q = q_H^*$.*

The efficient allocation requires both that all goods have high quality and that the number of sellers be efficient. In turn, efficiency of sellers' entry requires a particular price if every seller makes zero net expected profit, i.e., a particular level of transfer from a buyer to a seller upon selling the good. For any given quality k, denote this efficient price level as $p^*(k)$. Because net expected profit of a seller who sells a good of quality k at price p is $[qF(q)p - \psi(k)]$, then the efficient price level is:

$$p^*(k) = \frac{a(k)\,k}{q^*(k)\,F(q^*(k))}. \tag{3.3}$$

Because the efficient quality is $k_H = 1$, the efficient price is $p^*(1)$. Denote $p_H^* = p^*(1)$ and $p_L^* = p^*(\rho)$. It can be shown that $p_H^* > p_L^*$.

To interpret the efficient price more clearly, let us rewrite it as $p_H^* = \gamma$, where

$$\gamma \equiv G(q_H^*) \quad \text{and} \quad G(q) \equiv \frac{-qF'(q)}{F(q)}. \tag{3.4}$$

Then, the efficient price satisfies the so-called Hosios (1990) condition, which requires the share of the match surplus given to one side of the market should be equal to that side's share of contribution to the number of matches. To see that the above price satisfies this condition, note that $G(q)$ is sellers' share of contribution to matches.[6] The surplus in a

[5] From part (ii) of Assumption 2, we know that $0 \leq -q^2F'(q) \leq qF(q)$ so that $\lim_{q\to 0}[-q^2F'(q)] = 0$. Part (iv) ensures that $\lim_{q\to+\infty}[-q^2F'(q)] = 1$.

[6] The total number of matches is $nqF(q) = F(q)$, since $n = 1/q$. Sellers' share of contribution to matches is defined as $d\ln F(q)/d\ln(n)$, which is equal to $G(q)$.

match of quality k is k (because the cost of production is already sunk at the time of a match), of which the seller gets a share p/k. The above price equates this share to the sellers' share of contribution to matches.

For future use, let us use the definition of γ and (3.2) to express the equation for q_H^* as:

$$q_H^* F\left(q_H^*\right) = a\left(1\right)/\gamma. \tag{3.5}$$

In the next two subsections, we will first examine each submarket separately by shutting down the other submarket. Then we will allow agents to choose between the two submarkets and briefly describe the equilibrium outcome in section 3.4.

3.2. Equilibrium with Bargaining

Suppose that the submarket with bargaining is the only market. The equilibrium outcome is the same regardless of whether the quality of a good is private or public information. This is because a buyer in a match observes the quality of the good before bargaining. Given the quality, k, and the price, p, the buyer's surplus is $(k-p)$ and the seller's surplus is p. Clearly, for a trade to take place, it is necessary that $p \in [0, k]$. Assume that the bargaining power of the seller is $\sigma \in [0, 1]$, Nash bargaining solves:

$$\max_{p\in[0,k]} \; p^{\sigma}\left(k-p\right)^{1-\sigma}.$$

The solution is $p = \sigma k$ and the expected value to the seller is $J(k) = \sigma QF(Q)k$.

At the entry stage, the seller chooses $k \in \{\rho, 1\}$ to maximize net expected profit $\pi(k) = J(k) - \psi(k)$, taking Q as given. In equilibrium, Q is such that $\pi\left(k\right) = 0$. Note that $J(k) \leq \sigma k$, because $QF\left(Q\right) \leq 1$ is a seller's matching probability. If $\sigma \leq a\left(1\right)$, then $J(k) < \psi\left(k\right)$ for all such Q that $QF\left(Q\right) < 1$. In this case, no seller will enter the market, because an entrant makes negative expected profit. If $\sigma > a\left(1\right)$ and if all sellers enter the market with high-quality goods, then competitive entry determines Q as follows:

$$QF(Q) = a\left(1\right)/\sigma. \tag{3.6}$$

The solution for Q to this equation, the quality choice $k = 1$, and the price level $p = \sigma k$ form the unique equilibrium under bargaining. To see this, first note that this outcome is an equilibrium. That is, if all other sellers choose the high quality, then an individual seller will make a loss by deviating to the low quality. Net expected profit from such a deviation is $\pi\left(\rho\right) = J\left(\rho\right) - \psi\left(\rho\right)$; substituting $J\left(\rho\right) = \sigma QF\left(Q\right)\rho$ and $QF\left(Q\right) = a\left(1\right)/\sigma$ yields $\pi\left(\rho\right) = \rho\left[a\left(1\right) - a\left(\rho\right)\right] < 0$, where the inequality follows from part (ii) of Assumption 1.

Similarly, there is no other equilibrium. That is, if there were any sellers in the market that offer low-quality goods and make zero net expected profit, then a seller would profit by deviating to the high-quality good.[7]

Therefore, bargaining generates the efficient quality, provided $\sigma > a(1)$. However, the equilibrium level of sellers' entry is inefficient for all $\sigma \neq \gamma$. This is evident from comparing (3.6) with the counterpart in the efficient allocation, (3.5). Because the function $qF(q)$ is an increasing function, the comparison reveals that $Q < q_H^*$ if and only if $\sigma > \gamma$. Because the measure of active sellers in the market is $n = 1/Q$, then the equilibrium with bargaining has excessive entry of sellers if $\sigma > \gamma$ and deficient entry if $\sigma < \gamma$.

3.3. Price Posting with Public Information

Now suppose that the submarket with price posting is the only market and that the quality of a good is public information. A seller chooses k and p simultaneously when entering the market. However, because quality is public information, the choice problem can be divided into two problems. The first problem is to choose a posted price given the choice of quality, and the second problem is to choose quality. For any given quality k, the price posting decision solves the following maximization problem:

$$J(k) = \max_{p \in [0,k],\, q} qF(q)p \quad \text{s.t. } F(q)(k-p) \geq D.$$

Here, $D \geq 0$ is the expected surplus that a buyer can get in the market. Since the market is large, each seller takes D as given (see Burdett et al., 2001, for a proof).

The constraint in the above problem is unique to directed search. It is necessary because, if the constraint is violated, then the seller will not be able to attract buyers at all. Moreover, the constraint must hold as equality in any equilibrium. To see this, suppose that the constraint holds as strict inequality; that is, a buyer obtains a strictly higher expected surplus from visiting the particular seller than from visiting any other seller. Because every buyer can observe all sellers' offers, all buyers will strictly prefer visiting the particular seller to any other seller. As a result, $q \to \infty$ for the particular seller. However, since $F(\infty) = 0$ by assumption, a buyer who visits the particular seller will obtain zero expected surplus. This contradicts the supposition because $D \geq 0$.

[7] For the low-quality sellers to make zero net expected profit, they must face a queue length $\hat{Q}$ that satisfies $\hat{Q}F(\hat{Q}) = a(\rho)/\sigma$. Given $\hat{Q}$, a deviation to the high quality yields the following net expected profit: $\sigma\hat{Q}F(\hat{Q}) - \psi(1) = a(\rho) - a(1) > 0$, where the inequality follows from part (ii) of Assumption 1.

To emphasize the feature that a seller's posted price directs buyers' search, we rewrite the equality form of the constraint in the above problem as follows:

$$p = P(q) \equiv k - \frac{D}{F(q)},$$

where we suppressed the dependence of p on D. The function $P(q)$ can be viewed as the demand function *facing the particular seller.* Note that $P(q)$ is a decreasing function, because $F(q)$ is decreasing. Thus, the higher the price posted by a seller, the smaller the expected number of buyers who will visit the seller. When a seller chooses prices, he takes into account this relationship between q and p. This feature is the defining characteristic of directed search.

Formally, we can substitute the function $P(q)$ to write the seller's problem as $J(k) = \max_q [qF(q)P(q)]$. The optimal choice yields:

$$p/k = G(q) \quad \text{and} \quad F(q) + qF'(q) = D/k, \tag{3.7}$$

where G is defined in (3.4).[8] Let $q(k, D)$ be the solution for q to the second equation in (3.7). The expected value for the seller is $J(k, D) = -kq^2F'(q)$, with $q = q(k, D)$.

At the entry stage, each seller chooses $k \in \{\rho, 1\}$ to maximize $\pi(k, D) = J(k, D) - \psi(k)$. The optimal choice of k is a function of D, say, $k^*(D)$. Then, determining an equilibrium amounts to determining D by the condition for competitive entry: $\pi(k^*(D), D) = 0$.

There is a unique equilibrium and the equilibrium allocation coincides with the efficient allocation. To establish these results, let us examine the dual problem of the above choice of k, where each seller chooses $k \in \{\rho, 1\}$ to maximize D subject to the constraint that $\pi(k, D) \geq 0$. Competitive entry ensures $\pi(k, D) = 0$, which can be rewritten as (3.2). Thus, $q = q^*(k)$. Substituting this function into (3.7), we can express D as a function of k and verify that D is maximized by the choice $k = 1$.[9] Thus, there is a unique equilibrium, where $q = q_H^*$, $k = 1$ and $p = p_H^* = \gamma$. This equilibrium allocation is the same as the efficient allocation. Therefore, price posting is more efficient than bargaining for all $\sigma \neq \gamma$. This is the result in Acemoglu and Shimer (1999).

3.4. Choice between the Two Submarkets

Now let agents choose between the two submarkets. In both submarkets, competitive entry of sellers drives down a seller's expected profit to zero. Recall that buyers choose between

[8] Because $G(q) \in [0, 1]$ under part (ii) of Assumption 2, the above solution for p indeed satisfies the constraint $p \in [0, k]$.

[9] Substituting $q = q^*(k)$ and $F'(q^*(k)) = -a(k) / [q^*(k)]^2$, we have: $D = k[F(q^*(k)) - a(k)/q^*(k)]$. This function increases in k under part (ii) of Assumption 1. Thus, D is maximized by the choice $k = 1$.

the submarkets after observing posted prices and the measure of sellers in each submarket. This timing of choices eliminates the possibility that sellers choose not to enter a submarket simply because they expect that no buyer will enter the submarket. In particular, a seller in the submarket with posted prices can always attract some buyers by posting a sufficiently low price, if it is optimal to do so. Thus, the only reason for a submarket to be missing in the equilibrium is that it generates a lower expected surplus for buyers than does the other submarket.

With zero expected profit for sellers, buyers' expected surplus in a submarket is equal to the expected joint surplus in the submarket. That is, $D = W(k, q)$ in each submarket. The analysis in the previous two subsections shows that, for all $\sigma \neq \gamma$, the submarket with posted prices generates strictly higher social welfare than the submarket with bargained prices. We summarize the results so far in the following lemma:

Lemma 3.2. *If the quality of goods is public information, then the equilibrium under price posting (and directed search) is socially efficient. For all $\sigma \neq \gamma$, price posting dominates bargaining in efficiency, and only the submarket with posted prices will exist. When $\sigma = \gamma$, price posting and bargaining are both efficient, in which case both submarkets exist.*

4. Price Posting with Private Information

Because the equilibrium with bargaining does not depend on whether the quality of a good is private information, we focus on the submarket with price posting in this section.

4.1. Beliefs and Payoffs

A seller may be able to signal the quality of his good by posting particular prices. To describe such signaling, let $\mu(p, M)$ be the belief (probability) that buyers view a seller who posts price p as a high-quality seller, given the distribution of prices posted by other sellers, M. We will suppress M whenever there is no confusion. As before, let D be the expected surplus that a buyer can get from the market. Let $J(p, \mu(p), D)$ be the expected value for a seller who posts price p, given D and the belief μ. Then,

$$J(p, \mu(p), D) = qF(q)\, p \quad \text{s.t. } q \text{ satisfies:}$$
$$F(q)\{\mu(p) + [1 - \mu(p)]\,\rho - p\} = D.$$

Here, the constraint holds as equality, as explained in section 3.3. Then, a seller's decision problem is as follows:

$$\pi(D, \mu(.)) = \max_{(k,p)} [J(p, \mu(p), D) - \psi(k)] \text{ s.t. } p \in [0, k].$$

Note that the seller chooses the quality and price at the same time, as we assumed in the description of the environment. This assumption allows us to impose a particular restriction on beliefs below.

Without restricting beliefs out of the equilibrium, the signaling game has many Bayesian equilibria. For example, take any arbitrary price p_a. Suppose that buyers have the following belief: All sellers who post p_a are high quality; if any seller posts $p \neq p_a$, then the seller is viewed as a low quality. With this belief, all sellers will indeed post only p_a. In this case, the belief that any seller who posts $p \neq p_a$ is a low-quality seller cannot be checked because the event occurs with probability zero in the equilibrium. However, this particular belief may not be "reasonable". To refine the set of equilibria, we need to impose restrictions on beliefs out of the equilibrium.

One such restriction is the intuitive criterion by Cho and Kreps (QJE, 1987). To illustrate it, take a supposed equilibrium as a reference point, where the sellers post price p_a. Consider a deviation from the supposed equilibrium: $p' \neq p_a$. This deviation is a credible signal for a type i seller if the following conditions are met: (i) p' is feasible to a type i seller; (ii) if buyers view a seller who deviates to p' as type i, then the deviation is profitable for a type i seller; and (iii) even if buyers view a seller who deviates to p' as type i, the deviation is not profitable for a type i' seller, where $i' \neq i$. In our model, the intuitive criterion implies the following restrictions on beliefs:

> **Restriction 1**. $p \geq \rho\ (= k_L)$ implies $\mu(p) = 1$; that is, if a seller posts $p \geq \rho$, then buyers should view the seller as a high type.
>
> **Restriction 2**. If two prices $p_1, p_2 \geq \rho$ satisfy $J(p_1, 1, D) > J(p_2, 1, D)$ for given D, then a high-quality seller should prefer posting p_1 to p_2.

To see why the intuitive criterion implies Restriction 1, note that a buyer will see a seller's quality upon visiting the seller. If the good has low quality, the surplus to a buyer is non-positive at price $p \geq \rho$, and so the buyer will refuse to buy the good at such a price. Thus, posting $p \geq \rho$ can never be optimal for a low-type seller, even if such a price may (incorrectly) induce buyers to view the seller as a high type. In contrast, posting $p \geq \rho$ can be optimal for a high-quality seller if such prices induce buyers to believe that the seller has a high-quality good. Thus, the intuitive criterion requires that any seller who posts such prices should be viewed as a high type.

Restriction 2 is a result of iterated use of the intuitive criterion. Consider any two prices p_1 and p_2, with $p_1, p_2 \geq \rho$, that satisfy $J(p_1, 1, D) > J(p_2, 1, D)$. Because both prices are at least ρ, Restriction 1 implies $\mu(p_1) = \mu(p_2) = 1$. Thus, if other sellers post p_2, a particular seller can deviate to p_1 without inducing a change in buyers' beliefs. Because posting p_1 yields a higher expected value to the seller, such a deviation is profitable.

Restriction 2 implies that, among all possible choices $p \geq \rho$, a high-quality seller should consider only the particular price that maximizes the expected value. From the analysis in section 3.3, $J(p, 1, D)$ is maximized at the price level equal to $G(q(1, D))$, where $q(k, D)$ is defined as the solution to the second equation in (3.7). When the level of D induces zero net expected profit for each seller, $q(1, D) = q_H^*$ and $G(q(1, D)) = \gamma$, where q_H^* is defined in (3.5) and γ is defined in (3.4). If $\gamma \geq \rho$, then Restriction 2 above implies that high-quality sellers will post price γ and low-quality sellers will post the price $p_L^* < \rho$, where $p_L^* = p^*(\rho)$ is defined by (3.3) with $k = \rho$.

However, it is possible that $\gamma < \rho$, in which case high-quality sellers cannot separate themselves from low-quality sellers by posting price γ or any price $p < \rho$. In this case, we impose the following restriction:

Restriction 3: $p < \rho$ implies $\mu(p) = 0$.

To justify this restriction, note that the expected value for a seller, $J(p, \mu(p), D)$, does not depend directly on the quality of the good, provided that $k \geq p$. Thus, for any choice $p < \rho$ and any given $\mu(p)$ and D, a seller's net profit satisfies:

$$J(p, \mu(p), D) - \psi(\rho) > J(p, \mu(p), D) - \psi(1).$$

Recall that a seller chooses both k and p when entering the market. The above inequality implies that the choice (ρ, p) dominates the choice $(1, p)$ for all $p < \rho$ and all beliefs $\mu(p)$. It is reasonable to eliminate the choice $(1, p)$, where $p < \rho$, from a seller's set of strategies. Then, the only belief of buyers consistent with a seller's choice $p < \rho$ is $\mu(p) = 0$.

4.2. The Equilibrium Regions

Under Restrictions 1 – 3, we can divide the analysis into two cases: $\gamma \geq \rho$ and $\gamma < \rho$.

Case 1: $\gamma \geq \rho$. Because $p_H^* = \gamma \geq \rho > p_L^*$ in this case, sellers of each type i post the full-information price p_i^*, where $i = H, L$. Posted prices signal sellers' quality and, hence, separate the two types of sellers. Given this separation, it can be verified that entering the market with a low-quality good is not optimal (for the same reasons as in section 3.3). Therefore, all sellers in the market have high-quality goods and all post price γ. This equilibrium is the same as the one under public information, which is analyzed in section 3.3. Thus, private information does not generate any inefficiency in this case.

Case 2: $\gamma < \rho$. In this case, posting price γ would induce buyers to view a high-quality seller incorrectly as a low-quality seller (see Restriction 3). Private information generates inefficiency, which can appear in two forms. The first form is that prices are inefficiently high as they are bounded below by ρ and, hence, there is excessive entry of sellers into the market. Nevertheless, the market continues to provide the efficient quality of goods. The second form of inefficiency is that the quality of goods is inefficiently low in the market. In the remainder of this section, we find the conditions under which each of these two subcases occurs. We organize the analysis along a series of results.

> **Result 1**. Assume $\gamma < \rho$. If there is an equilibrium in which high-quality sellers enter the market, then all high-quality sellers post price ρ.

Suppose, to the contrary, that some high-quality sellers in the market post $p_a > \rho$ and make zero net expected profit. We show that it is profitable for a high-quality seller to deviate to a lower price. Let q_a be the queue length of buyers for each of the sellers who post p_a, and D_a be the expected surplus to a visiting buyer. Because (p_a, q_a) must deliver D_a to a visiting buyer, $p_a = 1 - D_a/F(q_a)$. The expected value to such a seller is:

$$J(p_a, 1, D_a) = q_a F(q_a) p_a = q_a [F(q_a) - D_a].$$

Because the seller makes zero net expected profit, then $J(p_a, 1, D_a) = a(1)$, which yields $D_a = F(q_a) - a(1)/q_a$. Substituting D_a into the expression for p_a, we can express the hypothesis $p_a > \rho$ as $q_a F(q_a) < a(1)/\rho$. Define q_0 and D_0 by:

$$q_0 F(q_0) = \frac{a(1)}{\rho}; \qquad D_0 = F(q_0) - \frac{a(1)}{q_0} = F(q_0)(1-\rho). \tag{4.1}$$

Because $[qF(q)]$ is an increasing function by Assumption 2, then $p_a > \rho$ if and only if $q_a < q_0$. Moreover, the hypothesis $\gamma < \rho$ implies $q_0 < q_H^*$ and $D_a < D_0$.[10]

Now consider a high-quality seller who deviates to a lower price p_b. This price is constructed so that it attracts a queue length of buyers, q_0, and that it delivers to each visiting buyer the same expected surplus, D_a, as other high-quality sellers do. These two requirements determine p_b as $p_b = 1 - D_a/F(q_0)$. Note that $p_b < p_a$ because $q_0 > q_a$ and

[10] To show $q_0 < q_H^*$, use (3.5) to substitute for $a(1)$ in the definition of q_0. Then $q_0 F(q_0) = \frac{\gamma}{\rho} q_H^* F(q_H^*)$. Because $\gamma < \rho$ in Case 2 and $[qF(q)]$ is an increasing function, then $q_0 < q_H^*$. To show $D_a < D_0$, note that the derivate of the function $[F(q) - a(1)/q]$ with respect to q has the same sign as that of $\left[a(1) + q^2 F'(q)\right]$. Because $a(1) = -[q_H^*]^2 F'(q_H^*)$ (see (3.2)) and $\left[q^2 F'(q)\right]$ is a decreasing function by Assumption 2, then $a(1) + q^2 F'(q) > 0$ for all $q < q_H^*$. Since $q_a < q_0 < q_H^*$, then $D_a < F(q_0) - a(1)/q_0 = D_0$.

because $F(q)$ is a decreasing function. Also, $p_b > \rho$ because $D_a < D_0$. The expected value for the deviating seller is: $q_0 F(q_0) p_b = q_0 F(q_0) - q_0 D_a \equiv J(q_0)$. We have:

$$J'(q_0) > q_0 F'(q_0) + F(q_0) - D_0 = \frac{1}{q_0}\left[(q_0)^2 F'(q_0) - (q_H^*)^2 F'(q_H^*)\right] > 0.$$

The first inequality follows from computing $J'(q_0)$ and substituting $D_a < D_0$, the equality from substituting D_0 from (4.1) and $a(1)$ from (3.2), and the last inequality from $q_0 < q_H^*$. Because $J(q)$ is strictly concave, the above result implies that $J'(q) > 0$ for all $q \leq q_0$. Then, $q_a < q_0$ implies $J(q_a) < J(q_0)$. That is, the deviation to p_b is profitable.

The reason why the deviation is profitable is that, when $\gamma < \rho$, prices above ρ are too high to be optimal. By reducing the price, a high-quality seller can increase the queue length of buyers and the probability of selling the good. This higher probability is more than compensating for the reduced price. Thus, the expected value to the seller increases. Note that the price in the profitable deviation, p_b, is still above ρ. Without private information problems, a seller would want to set price below ρ in the current case. In the presence of private information, however, reducing price below ρ would trigger buyers to change beliefs about the seller's quality from high-quality to low-quality. In this sense, maintaining high beliefs acts as a constraint on the sellers' problem.

The above analysis applies as long as some (not necessarily all) high-quality sellers post prices above ρ. Thus, when $\gamma < \rho$, no high-quality seller posts price above ρ in any equilibrium. On the other hand, if $p_a = \rho$ in the above analysis, then $q_a = q_0$ and $D_a = D_0$. In this case, a high-quality seller cannot reduce price further without triggering a change in buyers' beliefs. Therefore, Result 1 holds.

Result 1 describes high-quality sellers' optimal choice of price conditional on their entering the market. However, it is possible that a seller may not find it optimal to produce high-quality goods. To find the condition under which an equilibrium with high-quality goods exists, we examine the payoff to an individual seller who chooses the low quality, given that some other sellers will enter the market with high-quality goods, post price ρ and make zero net expected profit. By the above analysis, such a high-quality seller attracts a queue length q_0 of buyers and delivers the expected surplus D_0 to a visiting buyer. For the deviating seller who chooses the low quality, let p_1 be this seller's optimal choice of posted price and q_1 be the queue length of buyers for the seller. Clearly, $p_1 < \rho$, because this seller would not be able to sell the good by posting a price $p \geq \rho$. Under Restriction 3 on beliefs, $\mu(p_1) = 0$ and so the expected value to the seller is $J(p_1, 0, D_0)$.

Thus, $p_1 = \arg\max J(p, 0, D_0)$. That is, $p_1 = \rho G(q_1) < \rho$, where q_1 solves:[11]

$$F(q_1) + q_1 F'(q_1) = \frac{D_0}{\rho} = F(q_0)\left(\frac{1-\rho}{\rho}\right). \tag{4.2}$$

The expected value to the seller is $q_1 F(q_1) p_1 = -\rho q_1^2 F'(q_1)$. If this value is less than the cost of the good, $\psi(\rho)$, then entering the market with a low-quality good is not profitable. Thus, choosing the low quality yields negative expected profit if and only if

$$-q_1^2 F'(q_1) < a(\rho). \tag{4.3}$$

The following result describes not only existence, as established above, but also uniqueness of the equilibrium with high-quality sellers (see Appendix A for a proof):

> **Result 2**. Assume $\gamma < \rho$. If (4.3) holds, then there exists a unique equilibrium in which all sellers have high quality and all post price ρ.

On the other hand, if (4.3) is reversed, then there exists a unique equilibrium in which all sellers have low quality. In this equilibrium, a seller posts price p_L^* and attracts a queue length of buyers q_L^*, where p_L^* and $q_L^* = q^*(\rho)$ are defined in section 3.1. The expected surplus to a visiting buyer is $D_L = \rho[F(q_L^*) + q_L^* F'(q_L^*)]$. The proof of existence and uniqueness is a straightforward modification of the proof of Result 2 and, hence, omitted.

Finally, there is a borderline case in which (4.3) is changed into equality. In this case, both high-quality sellers and low-quality sellers enter the market. A high-quality seller posts price ρ and attracts a queue length of buyers q_0, while a low-quality seller posts p_L^* and attracts a queue length of buyers q_L^*. Both types of sellers provide the same expected surplus to a visiting buyer and make zero net expected profit. The total measure of sellers in the market and the composition of the two types of sellers are indeterminate in this case.[12] We ignore this borderline case.

The following theorem expresses (4.3) in a different form and summarizes the above results (see Appendix B for a proof):

[11] We assume that $\lim_{q\downarrow 0}[F(q) + qF'(q)] > D_0/\rho$, so that (4.2) has a solution. If this condition does not hold, then for all pairs (p_1, q_1), the maximum expected surplus which the deviating seller can generate to a buyer is less than D_0. In this case, the deviation is clearly not profitable.

[12] If n is the total measure of sellers in the market and α_H is the fraction of high-quality sellers, then n and α_H must ensure that the total measure of buyers visiting all the sellers should add up to the given measure, 1. That is, $n[\alpha_H q_0 + (1-\alpha_H) q_L] = 1$. Changes in n are offset by changes in α_H, and so neither n nor α_H is determinate.

Theorem 4.1. *Maintain Restrictions 1 – 3. There exists* $\rho_0 \in (\gamma, 1)$ *such that (4.3) holds iff* $\rho \in (\gamma, \rho_0)$*. A unique equilibrium exists for all* $\rho \neq \rho_0$*, as characterized in Table 1:*

Table 1. The unique equilibrium under price posting

Cases	*existence region*	*quality of goods*	*posted prices*	*queue length*	*buyer's surplus*
Case 1	$0 \leq \rho \leq \gamma$	*high*	γ	q_H^*	$W(1, q_H^*)$
Case 2A	$\gamma < \rho < \rho_0$	*high*	ρ	$q_0 \ (< q_H^*)$	$W(1, q_0)$
Case 2B	$\rho_0 < \rho \leq 1$	*low*	$p_L^* < \rho$	$q_L^* \ (> q_H^*)$	$W(\rho, q_L^*)$

As stated before, the equilibrium is efficient in Case 1 but inefficient in Cases 2A and 2B. Clearly, the quality of goods is efficient in Case 2A but inefficient in Case 2B. To check how the amount of sellers' entry is inefficient in Cases 2A and 2B, note that there is only one type of sellers in the market in each of the cases listed in Table 1. Thus, the measure of sellers in the market is equal to $1/q$. Relative to the efficient amount of entry in Case 1, the equilibrium has excessive entry in Case 2A and deficient entry in Case 2B. The amount of inefficiency in a particular case can be measured by the difference in a buyer's surplus between the particular case and Case 1. Note that a buyer's surplus is equal to the level of social welfare, as discussed in section 3.4.

5. Comparing Efficiency between Price Posting and Bargaining

We evaluate efficiency of price posting versus bargaining, using the social welfare function $W(k, q)$. There are two cases in which the comparison is simple. The first is $\sigma \leq a(1)$. In this case, price posting is evidently superior to bargaining, because the market shuts down under bargaining. The second simple case is $\rho \leq \gamma$ (i.e., Case 1), where price posting is efficient and, hence, is superior to bargaining for all $\sigma \neq \gamma$. The following analysis assumes $\sigma > a(1)$ and focuses on the case $\rho > \gamma$. We organize the comparison according to Cases 2A and 2B listed in Table 1.

Case 2A: $\rho \in (\gamma, \rho_0)$. In this case, because price posting and bargaining both generate the efficient quality of goods, they differ from each other only in the amount of entry of sellers, i.e., in $1/q$. Price posting yields $q = q_0$ given by (4.1), while bargaining yields $q = Q$ given by (3.6). Because $[qF(q)]$ is increasing, $q_0 < Q$ if and only if $\rho > \sigma$. To translate this difference in the queue length into the difference in welfare, note that the welfare function $W(1, q)$ is increasing in q if and only if $q < q_H^*$. Because $q_0 < q_H^*$, entry is excessive in the equilibrium under posted prices. Under bargaining, entry can be either excessive (if $\sigma > \gamma$) or deficient (if $\sigma < \gamma$). Thus, we divide the analysis further into two subcases.

(a) $\sigma > \gamma$ and $\rho \in (\gamma, \rho_0)$. In this subcase, both price posting and bargaining generate excessive entry; i.e., both q_0 and Q are less than q_H^*. Thus, price posting yields higher welfare than bargaining if and only if $q_0 > Q$ and, hence, if and only if $\rho < \sigma$. Therefore, price posting is more efficient than bargaining if $\gamma < \rho < \sigma$, while bargaining is more efficient if $\gamma < \sigma < \rho$.

(b) $\sigma < \gamma$ and $\rho \in (\gamma, \rho_0)$. In this subcase, price posting generates excessive entry but bargaining generates deficient entry, i.e., $q_0 < q_H^* < Q$. To compare efficiency between the two mechanisms, let us define q_3 by $W(1, q_3) = W(1, q_0)$ with $W'(1, q_3) < 0$. The level q_3 is the image of q_0 on the other side of the hump of the welfare function, which has deficient entry rather than excessive entry of sellers.[13] By construction of q_3, price posting is more efficient than bargaining if and only if $W(1, q_3) > W(1, Q)$. Because both q_3 and Q are greater than q_H^*, then $W(1, q_3) > W(1, Q)$ iff $q_3 < Q$. This condition is equivalent to $a\,(1)\,/\sigma > q_3 F(q_3)$. Using (4.1) to substitute for $a\,(1)$, we can rewrite this condition as $\rho/\sigma > q_3 F\,(q_3)\,/\,[q_0 F\,(q_0)]$. Because $W(1, q_3) = W(1, q_0)$, we get:

$$F(q_3) = F(q_0) - \left(\frac{1}{q_0} - \frac{1}{q_3}\right) a\,(1) = F(q_0)\left[1 - \rho\left(1 - \frac{q_0}{q_3}\right)\right].$$

Then, price posting is superior to bargaining iff

$$\sigma < s\,(\rho) \equiv \left[1 + \left(\frac{1-\rho}{\rho}\right)\frac{q_3\,(\rho)}{q_0\,(\rho)}\right]^{-1},$$

where we have expressed q_0 and q_3 as functions of ρ. Therefore, if $\sigma < s(\rho)$ $(< \gamma < \rho)$, price posting is superior to bargaining; if $s(\rho) < \sigma$ $(< \gamma < \rho)$, bargaining is superior.

Case 2B: $\rho \in (\rho_0, 1)$. In this case, price posting generates an inefficient quality of goods as well as an inefficient amount of entry of sellers. The welfare level is $W\,(\rho, q_L^*)$. Thus, price posting is more efficient than bargaining if and only if $W\,(\rho, q_L^*) > W\,(1, Q)$. Taking into account the dependence of q_L^* on ρ, it can be verified that $W\,(\rho, q_L^*)$ is a strictly increasing function of ρ.[14] Thus, there exists $R\,(\sigma)$ such that price posting generates higher welfare than bargaining if and only if $\rho > R\,(\sigma)$. The existence of the function $R\,(\sigma)$ is ensured by the fact that $W(\rho, q_L^*)|_{\rho=1} = W(1, q_H^*) > W(1, Q)$.

[13] q_3 is well defined since $W(1, q)$ is decreasing to the right of q_H^* and $W(1, q) \to 0$ as $q \to +\infty$.

[14] Since welfare is equal to buyer's expected surplus, we can use the expression for D in (3.7) to compute the welfare level at $k = \rho$ and $q = q_L^*(\rho)$ as follows:

$$W(\rho, q_L^*(\rho)) = \rho\,[F(q) + qF'(q)]_{q=q_L^*(\rho)} = \rho\,[F(q_L^*(\rho)) - a(\rho)/q_L^*(\rho)]\,.$$

To obtain the second equality, we substituted $F'\,(q^*)$ from (3.2). Note that the partial derivative of the above expression with respect to q_L^* is zero (see (3.2)). Moreover, part (iii) of Assumption 2 implies that $\psi'\,(\rho) < a\,(\rho) = -\,(q_L^*)^2\,F'\,(q_L^*)$. Then, we can verify that $W(\rho, q_L^*(\rho))$ is strictly increasing in ρ.

To summarize the results, let us characterize an economy by the two parameters, $(\sigma, \rho) \in [0,1]^2$. Define three sets of economies as follows:

$$
\begin{aligned}
B1 &= \{(\sigma, \rho) : \gamma < \rho < \rho_0,\ s(\rho) < \sigma < \rho,\ \sigma > a(1)\},\\
B2 &= \{(\sigma, \rho) : \rho_0 < \rho < R(\sigma),\ \sigma > a(1)\},\\
B3 &= \{(\sigma, \rho) : \sigma = \gamma,\ 0 \leq \rho \leq \gamma\}.
\end{aligned}
$$

Let $B = B1 \cup B2$ and let $cl(B)$ be the closure of B in $[0,1]^2$. We summarize the comparison in the following theorem (see Appendix C for a proof):

Theorem 5.1. *Bargaining dominates price posting when the economy lies in B. On the other hand, price posting dominates bargaining when the economy lies outside $B3 \cup cl(B)$. The two pricing mechanisms have the same level of efficiency when the economy lies in $B3$ or on the boundary of B, the measure of which set in $[0,1]^2$ is zero. The boundaries of B are determined by the functions $s(\rho)$ and $R(\sigma)$ which have the following properties: (i) $s'(\rho) < 0$ and $s(\gamma) = \gamma$; (ii) $R(\sigma) < 1$ for all $\sigma \neq \gamma$, $R(\gamma) = 1$, and $R'(\sigma) > 0$ iff $\sigma < \gamma$; (iii) $R(\rho_0) = \rho_0$ and $R(s(\rho_0)) = \rho_0$.*

Figure 1 depicts the set B as the interior of the shaded region, for the case where the line $\sigma = a(1)$ (not drawn) lies below $s(\rho_0)$.[15] The set $B1$ is the shaded region on the left side of the vertical line $\rho = \rho_0$, while $B2$ is the shaded region on the right side. The set $B3$ is the segment of the line $\sigma = \gamma$ between $\rho = 0$ and $\rho = \gamma$. Note that the lower portion of the curve $\rho = R(\sigma)$ meets the curve $\sigma = s(\rho)$ at $\rho = \rho_0$. Similarly, the upper portion of the curve $\rho = R(\sigma)$ meets the line $\sigma = \rho$ at $\rho = \rho_0$.

Figure 1 offers the following alternative way to express the relative efficiency of the two pricing mechanisms. For each given $\rho \in [0,1]$, define $b(\rho) = \{\sigma \in [0,1] : (b(\rho), \rho) \in cl(B)\}$. Then, $b(\rho)$ is the set of values of sellers' bargaining power with which bargaining weakly dominates price posting for the given ρ. The correspondence $b(.)$ is continuous. For all $\rho < \gamma$, $b(\rho)$ is empty. As ρ increases above γ, the set $b(\rho)$ enlarges first for $\rho < \rho_0$; that is, increases in ρ make it more and more likely that bargaining dominates price posting. For $\rho > \rho_0$, the opposite occurs. The economy with $\rho = \rho_0$ offers the highest chance for bargaining to dominate price posting in efficiency.

To explain the non-monotonic feature of the set $b(\rho)$, recall that ρ measures the gap between high and low quality levels. An increase in ρ has no effect on the equilibrium under bargaining, but it has two opposite effects on efficiency of price posting. First, it increases

[15] Recall that, when $\sigma \leq a(1)$, no equilibrium under bargaining exists and so price posting always dominates bargaining. Thus, if the line $\sigma = a(1)$ lies above $s(\rho_0)$, the lower part of the shaded region in Figure 1 is cut off by the line $\sigma = a(1)$.

the difficulty of signaling high quality and, hence, increases inefficiency. Second, when the quality of low-quality goods increases, the loss to buyers who consume low-quality goods falls, which increases efficiency. For $\rho < \rho_0$, only the first effect is present because the market has only high-quality goods. For $\rho > \rho_0$, only the second effect is present because the equilibrium involves no signaling. This is why the likelihood with which bargaining dominates price posting increases with ρ for $\rho < \rho_0$ and decreases with ρ for $\rho > \rho_0$.

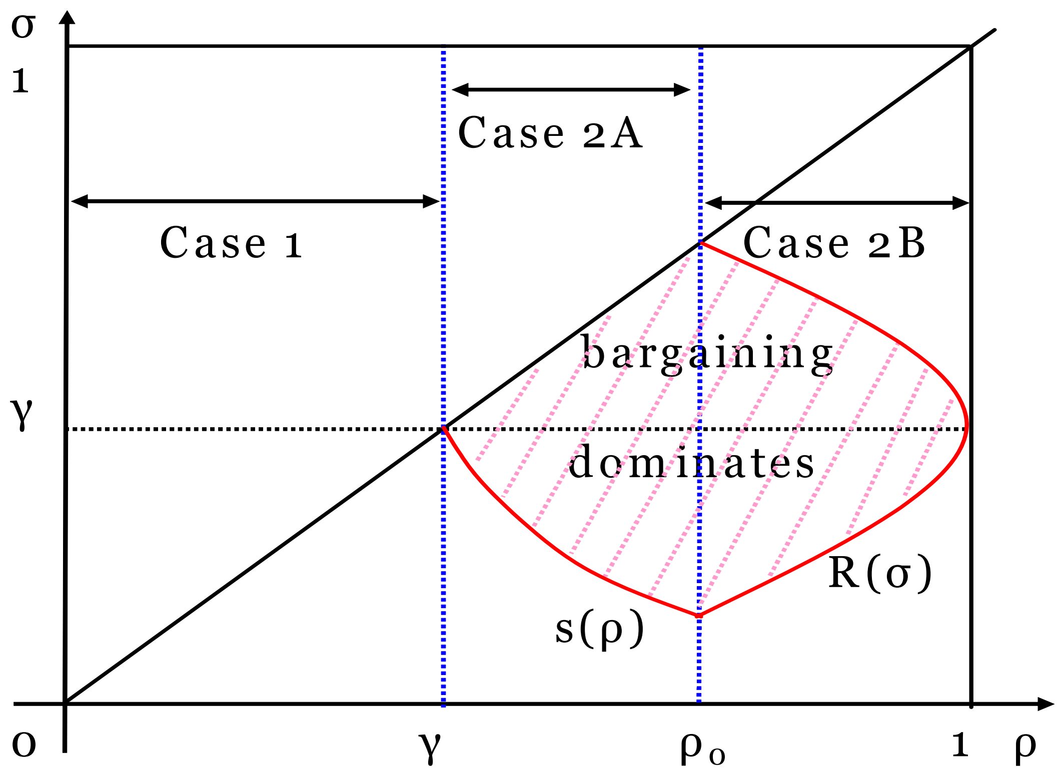

Figure 1. Comparing efficiency between price posting and bargaining

Now we can analyze agents' choices between the submarket with bargaining and the submarket with price posting. As in section 3.4, only the submarket with a higher social welfare level will exist in the equilibrium. Thus, when the economy lies in the interior of B, only the submarket with bargained prices exists; when the economy lies outside $B3 \cup cl\,(B)$, only the submarket with posted prices exists. The set of economies in which the two submarkets coexist has measure zero.[16]

Let us compare the results in Theorem 5.1 with some others in the literature. First, Acemoglu and Shimer (1999) incorporate directed search but abstract from private information. They show that bargaining can never dominate price posting in efficiency (see

[16]There is no systematic relationship between the mechanism that arises in the market and the price level. This is because inefficiency in bargaining can arise from both excessive and deficient entry of sellers.

Lemma 3.2). Incorporating private information, our model shows that bargaining can dominate price posting in a positively measured set of economies.

Second, Bester (1993) incorporates private information but abstracts from matching entirely. He models the search cost by time discounting, as a buyer must take one period to opt out from the described market to an outside good which has a uniform quality. To compare our results with Bester's requires us to set the discount factor to zero in Bester's model, because our model has one period and the payoff to an agent from not trading is zero. In this case, Bester's model predicts that bargaining is superior to price posting if and only if sellers' bargaining power (σ) is small.[17] In our model, bargaining is superior only when σ is moderate; moreover, when $\rho < \gamma$, bargaining is inferior to price posting for all $\sigma \neq \gamma$. These differences occur because the number of sellers is an important consideration for efficiency in our model, but it is exogenous in Bester's model.

Finally, Michelacci and Suarez (2006) compare price posting with bargaining in a directed search model of the labor market. In their model, workers' productivity is private information. Firms can either post wages to direct workers' search or participate in undirected search and ex post bargaining. They show that the equilibrium can be pure posting, pure bargaining or a mix of the two. In a positively measured subset of parameter values, wage posting and bargaining can coexist, in which wage-posting firms are more likely to attract low-productivity workers than do bargaining firms. On efficiency, they show that pure posting generates higher welfare than bargaining. The coexistence of the two pricing mechanisms and the unambiguous ranking of them in efficiency contrast sharply with our results in Theorem 5.1 above. The main causes for these differences are as follows. First, private information in our model lies on the side of the market that does pricing while, in Michelacci and Suarez, it lies on the side of market that searches. Thus, our model involves signaling while their model involves adverse selection. Second, in our model, the total supply and the composition of agents (sellers) who have private information are determined endogenously by competitive entry. In Michelacci and Suarez, these dimensions are fixed and, instead, competitive entry occurs on the side of the market that does not have private information.

[17] By this result, we refer to Bester's analysis on the set of stable pricing mechanisms, which is similar to our analysis of agents' choices between the two submarkets.

6. Discussion

In this section we discuss two assumptions of the model, their implications for the results, and possible modifications. The first assumption is that the cost of price posting is independent of the number of sellers who enter the price-posting submarket. The second assumption is that the good is a pure search good.

The above assumption on the cost of price posting is responsible for the result that price posting and bargaining do not coexist generically. The intuition is that, if the two mechanisms coexist, competitive free entry requires that sellers be indifferent between the two mechanisms; at the same time, buyers must obtain the same expected surplus from the two mechanisms. Generically, these two indifference conditions cannot hold simultaneously when the entry cost into a submarket is independent of the number of sellers in the submarket. However, in certain markets, the cost of entry may depend on the number of sellers in the market. For example, if price posting is associated with advertising, the cost of price posting is likely to increase with the number of sellers who choose to advertise. To incorporate this realistic feature, let $\alpha \in [0, 1]$ be the fraction of sellers who post prices. Let $c(\alpha)$ be the cost which a seller must incur in addition to ψ when entering the price-posting submarket, with $c(0) = 0$, $c(1) = \infty$ and $c'(\alpha) > 0$. We sketch below how this structure of entry cost allows for the coexistence of the two mechanisms.

To simplify the illustration, we consider only the case where the quality of the good is public information, but we will comment on how our main results would continue to hold under private information. Add a subscript a to the variables in the price posting (or advertising) submarket and a subscript $-a$ to the variables in the bargaining submarket. If a seller enters the price-posting submarket, he will choose price p_a and the implied queue size q_a according to (3.7), which are functions of (k_a, D). The expected value to this seller is $J_a(k_a, D) = -k_a q_a^2 F'(q_a)$, and the optimal investment k_a maximizes $J_a(k_a, D) - \psi(k_a)$. If a seller enters the bargaining submarket, he will choose $p_{-a} = \sigma k_{-a}$, which will generate $J_{-a} = \sigma Q_{-a} F(Q_{-a}) k_{-a}$ (see section 3.2). Use the buyer's participation constraint, $(1 - \sigma) k_{-a} F(Q_{-a}) = D$, to express Q_{-a} and J_{-a} as a function of (k_{-a}, D). Optimal investment, k_{-a}, maximizes $[J_{-a}(k_{-a}, D) - \psi(k_{-a})]$. A seller's choice between the two submarkets yields:

$$\alpha \begin{cases} \in (0,1), & \textit{if } J_a(k_a, D) - \psi(k_a) - c(\alpha) = J_{-a}(k_{-a}, D) - \psi(k_{-a}), \\ = 0, & \textit{if } J_a(k_a, D) - \psi(k_a) - c(\alpha) < J_{-a}(k_{-a}, D) - \psi(k_{-a}), \\ = 1, & \textit{if } J_a(k_a, D) - \psi(k_a) - c(\alpha) > J_{-a}(k_{-a}, D) - \psi(k_{-a}). \end{cases}$$

In equilibrium, aggregation requires that $\alpha q_a + (1 - \alpha) Q_{-a} = 1/n$, and free entry of sellers requires that the maximum of net expected profit in the two submarket be zero. Similar to the analysis in sections 3.2 and 3.3, it can be shown that $k_a = k_{-a} = 1$.

We can now address the issue of coexistence. For both submarkets to be active, it must be that sellers be indifferent between entering each submarket and that free entry drives net profits to zero. Thus,

$$J_a(1, D) - \psi(1) - c(\alpha) = 0 = J_{-a}(1, D) - \psi(1).$$

If $c(\alpha)$ is constant, the above two equations cannot generically be satisfied with the only variable, D, and so only one mechanism will exist in the equilibrium. However, if $c'(\alpha) > 0$, there can be values of (D, α) that satisfy the above equations, in which case the two mechanisms co-exist.[18]

Even with advertising costs, our main conclusion that bargaining may dominate price posting with private information still holds true. This is because bargaining is superior to price posting in regions where the search-directing and signaling roles of posted prices conflict. In these regions, price posting involves either (i) efficient investment, but excessive entry (case 2A), or (ii) inefficient investment and insufficient entry (case 2B). Bargaining on the other hand always features efficient investment, so that if the economy is close enough to the Hosios condition for efficient entry, bargaining dominates. Adding advertising costs can only increase the parameter region where this occurs.

Next, let us turn to the assumption that the good is a pure search good. This assumption can be relaxed by allowing for uncertain outcomes of a seller's investment and for buyers' signals. The details are as follows. At the beginning of the period, each seller chooses an amount of investment, m, in the good's quality. The outcome of the investment is a random variable distributed over $\{1, \rho\}$ according to the distribution $prob(k = 1) = \phi(m)$, where $\phi'(m) > 0$. Buyers observe each seller's investment but not the quality of the seller's good. Thus, before visiting a seller who invested m and posted price p, a buyer forms a prior belief, $\mu(p, m)$, on the quality of the good and decides whether to visit the seller. Once at the store, a buyer receives a signal about the quality of the good, $z \in \{\rho, 1\}$, which is drawn from the distribution $prob(z = k|k) = \nu > 1/2$. This signal is positively correlated with the good's true quality because the signal is correct more than half of the time. Let this signal be independent of the pricing decisions in the market and independent among the buyers (given k). The buyer will buy the good if $Prob(k = 1|z) + Prob(k = \rho|z) \cdot \rho - p \geq 0$. As long as $\nu < 1$, the good is not a pure search good, because a buyer cannot discover the

[18] Alternatively, we could assume that the advertising cost is a function of the market tightness n and solve for (D, n).

quality of the good for sure before the purchase. Moreover, each seller's payoff depends on quality k as well as p, since the quality affects the signal which influences the probability of a successful trade.

This extension enriches the basic framework in other ways too. In parameter regions where asymmetric information considerations do not prevent the efficient equilibrium to arise ($\rho < \gamma$), this extension is not different from the framework presented in the paper. However, when private information problems prevent high-type sellers from posting their "preferred" price, the market may be dominated by low-type sellers (case 2B in section 4.2), which occurs when ρ is high enough. In that case, because observable investment and posted prices both direct buyers' search, the parameter region where price posting by high-type sellers is an equilibrium may be larger than in the baseline model.

We leave a more complete treatment of these extensions for future research.

7. Conclusion

In this paper, we introduced private information into a market with search frictions and evaluated the relative efficiency of two pricing mechanisms, price posting and bargaining. In contrast to other models that have introduced private information into search markets, our model puts private information on the side of sellers so that sellers can use posted prices to signal the quality of goods. This role of posted prices in signaling quality may conflict with their role in directing buyers' search. Focusing on this conflict, we found that bargaining could yield higher efficiency than price posting, a result that reverses what Acemoglu and Shimer (1999) established for a search market with public information. We characterized the parameter regions in which each mechanism dominates in efficiency.

In the model, sellers choose a costly investment in the quality of the good, which remains private information until buyers visit the store. Sellers enter a search market competitively and can choose either to post prices in one submarket or to bargain in another submarket. With bargaining, private information does not affect the allocation because the quality is revealed prior to bargaining. In particular, bargaining induces sellers to produce the efficient quality of goods. But bargained prices do not direct buyers' search, and so they do not internalize search externalities. Posted prices direct search and, in the case of public information, they internalize search externalities and generate constrained efficiency. When the quality of a good is private information, posted prices both direct search and signal the quality. If the difference between different quality levels is large, the two roles do not conflict with each other, in which case price posting continues to be constrained efficient. However, if the difference between different quality levels is small, the price level that internalizes search externalities is too low to be able to signal high quality. In this case, the compromise between the two roles of posted prices can generate either inefficient entry of sellers and/or an inefficient choice of quality. This inefficiency can make price posting inferior to bargaining when sellers' bargaining power is close to the so-called Hosios condition.

We discussed several possible extensions of the model, including those that generate coexistence of the two pricing mechanisms and that allow for buyers to receive quality related signals. Another alley of research would be to apply the insights developed here to other markets such as the labor market. As mentioned in the introduction, job amenities would be a good example of the "quality" of a match that we examined. It might be interesting to investigate across labor markets about a link between wage determination and amenities or working conditions.

Appendix

A. Proof of Result 2

Existence of the equilibrium described in Result 2 follows from the analysis leading to (4.3). Note that (4.3) ensures that no seller will enter the market with low-quality goods provided that some high-quality sellers will be present in the market. For uniqueness of the equilibrium, it suffices to show that there is no equilibrium in which all sellers have low quality. To do so, suppose that such an equilibrium exists. Then, a low-type seller will post price p_L^* and will attract a queue length of buyers, q_L^*, where $p_i^* = p^*(k_i)$ is defined in (3.3) and $q_i^* = q^*(k_i)$ is defined in (3.2). The expected value to a visiting buyer is: $D_L = \rho[F(q_L^*) + q_L^* F'(q_L^*)]$. Because (4.3) implies that $-q_1^2 F'(q_1) < -(q_L^*)^2 F'(q_L^*)$, then $q_1 < q_L^*$. Since $[F(q) + qF'(q)]$ is a decreasing function, then

$$D_L < \rho[F(q_1) + q_1 F'(q_1)] = F(q_0)(1-\rho) < 1-\rho,$$

where the equality follows from (4.2).

Given this value D_L, it is profitable for a seller to enter the market with a high-quality good and post price ρ. By posting price ρ, this deviating seller can induce buyers to view him (correctly) as a high-quality seller. Use q_2 to denote the queue length of buyers whom this seller will attract. Since the expected surplus to a buyer who visits the deviating seller is $F(q_2)(1-\rho)$, which must be equal to D_L, then q_2 satisfies the following condition:

$$F(q_2) = \frac{D_L}{1-\rho} = \frac{\rho}{1-\rho}[F(q_L^*) + q_L^* F'(q_L^*)]. \tag{A.1}$$

The solution, q_2, is well-defined because $D_L < 1-\rho$ (see the above). The expected value for this seller is $q_2 F(q_2)\rho$. The entry of this seller is profitable if $q_2 F(q_2) > a(1)/\rho$. Substituting $a(1)$ from (4.1), we can rewrite the above condition as $q_2 > q_0$, i.e., $F(q_2) < F(q_0)$. Using (A.1) to substitute $F(q_2)$ and using (4.2) to substitute $F(q_0)$, we can rewrite this condition further as:

$$F(q_L^*) + q_L^* F'(q_L^*) < F(q_1) + q_1 F'(q_1).$$

That is, the deviation is profitable if $q_L^* > q_1$. Because $-(q_L^*)^2 F'(q_L^*) = a(\rho)$ and $[-q^2 F'(q)]$ is an increasing function, then $q_L^* > q_1$ iff (4.3) holds. Thus, under (4.3) and $\gamma < \rho$, the equilibrium described in Result 2 is the only equilibrium. QED

B. Proof of Theorem 4.1

The text preceding Theorem 4.1 established most of the statements, except the result that there exists $\rho_0 \in (\gamma, 1)$ such that (4.3) holds if and only if $\rho \in (\gamma, \rho_0)$. To prove this result, we rewrite (4.3) as a condition on ρ. Substitute $a(1)$ from (3.5) into the definition of q_0 in (4.1) to obtain:

$$q_0 F(q_0) = \frac{\gamma}{\rho} q_H^* F(q_H^*). \tag{B.1}$$

This solves q_0 as a function of ρ, which we denote as $q_0(\rho)$. The function $q_0(\rho)$ is well defined since $[qF(q)]$ is strictly increasing between 0 and 1 and since $\rho > \gamma$. Clearly, $q_0'(\rho) < 0$. Using the above equation to substitute for ρ in (4.2), we get:

$$F(q_1) + q_1 F'(q_1) = F(q_0(\rho))\left(\frac{1-\rho}{\rho}\right). \tag{B.2}$$

Solve this equation for q_1 as $q_1(\rho)$. (If equation (B.2) does not have a solution, set $q_1(\rho) = 0$.) Then, (4.3) can be expressed as $f(\rho) > 0$, where

$$f(\rho) \equiv a(\rho) + [q_1(\rho)]^2 F'(q_1(\rho)).$$

Next, we show that $f(\gamma) > 0$ and $f(1) < 0$. To show $f(\gamma) > 0$, note that $q_0(\gamma) = q_H^*$. Then, at $\rho = \gamma$, the right-hand side of (B.2) becomes:

$$\frac{F(q_H^*)}{\gamma}(1-\gamma) = \frac{1}{\gamma}\left[F(q_H^*) + q_H^* F'(q_H^*)\right] > F(q_H^*) + q_H^* F'(q_H^*).$$

Because $[F(q) + qF'(q)]$ is decreasing, (B.2) implies $q_1(\gamma) < q_H^*$. Hence,

$$f(\gamma) > a(\gamma) + (q_H^*)^2 F'(q_H^*) > a(1) + (q_H^*)^2 F'(q_H^*) = 0.$$

The first inequality follows from the result $q_1(\gamma) < q_H^*$, the second inequality from $a(\gamma) > a(1)$, and the equality from (3.5). To show $f(1) < 0$, note that $q_0(1)$ is bounded both below and above, and so $F(q_0(1)) \in (0,1)$. Then, (B.2) implies that $F(q_1) + q_1 F'(q_1) \to 0$ as $\rho \to 1$. This means $q_1(\rho) \to +\infty$ as $\rho \to 1$, by part (iv) of Assumption 2. Clearly, $q_1(1) > q_H^*$. We have:

$$f(1) = a(1) + [q_1(1)]^2 F'(q_1(1)) < a(1) + (q_H^*)^2 F'(q_H^*) = 0.$$

Because $f(\rho)$ is continuous, the two results, $f(\gamma) > 0$ and $f(1) < 0$, imply that there is at least one such value $\rho_0 \in (\gamma, 1)$ that satisfies $f(\rho_0) = 0$.

Finally, we show that ρ_0 is unique. Because $f(\gamma) > 0$ and $f(1) < 0$, uniqueness of ρ_0 means that $f(\rho) > 0$ if and only if $\rho \in (\gamma, \rho_0)$, as stated in the theorem. For uniqueness of ρ_0, it suffices to show that $f'(\rho_0) < 0$ whenever $f(\rho_0) = 0$ and $\rho_0 \in (\gamma, 1)$. Note first that the function $f'(\rho_0)$ is well defined as long as $q_1(\rho) > 0$ and that it is straightforward to verify that $q_1(\rho_0) > 0$. (ρ_0 is the value of investment such that if high-quality sellers play $[k = 1, q = q_0, p = \rho_0]$, then playing $[k = \rho_0, p_1, q_1]$ also earns zero profit. However, if $q_1(\rho_0) = 0$, then profits would be negative.) Now suppose $\rho_0 \in (\gamma, 1)$ is such that $f(\rho_0) = 0$. We have:

$$f'(\rho_0) = a'(\rho_0) + q_1'(\rho_0)\left[\frac{d}{dq_1}\left(q_1^2 F'(q_1)\right)\right]_{q_1 = q_1(\rho_0)}.$$

Because the derivative in [.] is negative, a sufficient condition for $f'(\rho_0) < 0$ is $q_1'(\rho_0) > 0$. Examine the equation (B.2), for $\rho = \rho_0$. Because the left-hand side of the equation is a decreasing function of q_1, then $q_1'(\rho_0) > 0$ if and only if the right-hand side of the equation is decreasing in ρ at $\rho = \rho_0$. From (B.1) we can compute:

$$q_0'(\rho) = -\frac{q_0(\rho) F(q_0(\rho))}{\rho \left[F(q_0(\rho)) + q_0(\rho) F'(q_0(\rho))\right]}.$$

Differentiate the right-hand side of (B.2) with respect to ρ, substitute the above result for $q_0'(\rho)$, and evaluate the derivative at $\rho = \rho_0$. Then,

$$\left[\frac{d}{d\rho}\text{RHS(B.2)}\right]_{\rho=\rho_0} = \frac{1}{(\rho_0)^2}\left[-\frac{F'(q_0)\,\rho_0 q_0 F(q_0)}{F(q_0) + q_0 F'(q_0)}\left(\frac{1-\rho_0}{\rho_0}\right) - F(q_0)\right]_{q_0=q_0(\rho_0)}.$$

We simplify this derivative. At $\rho = \rho_0$, we have:

$$-\left[q_1(\rho_0)\right]^2 F'(q_1(\rho_0)) = a(\rho_0) > a(1) = -\left(q_H^*\right)^2 F'\left(q_H^*\right).$$

The first equality comes from $f(\rho_0) = 0$, the first inequality from $\rho_0 < 1$, and the last equality from (3.5) and the definition of γ. Thus, $q_1(\rho_0) > q_H^*$. Also, (B.1) implies $q_0(\rho_0) < q_H^*$, because $\rho_0 > \gamma$. Thus, $q_1(\rho_0) > q_0(\rho_0)$. Because $[F(q) + qF'(q)]$ is a decreasing function, then $[F(q) + qF'(q)]_{q=q_1(\rho_0)} < [F(q) + qF'(q)]_{q=q_0(\rho_0)}$. Substituting this result and using (B.2) at $\rho = \rho_0$ to replace $(\frac{1-\rho_0}{\rho_0})$, we have:

$$\left[\frac{(1-\rho_0)/\rho_0}{F(q_0) + q_0 F'(q_0)}\right]_{q_0=q_0(\rho_0)} < \frac{1}{F(q_0(\rho_0))}.$$

Substituting this result and noticing $F' < 0$, we have:

$$\left[\frac{d}{d\rho}\text{RHS(B.2)}\right]_{\rho=\rho_0} = \left[-\frac{\rho_0 q_0 F'(q_0) + F(q_0)}{(\rho_0)^2}\right]_{q_0=q_0(\rho_0)} < 0.$$

The inequality follows from the fact that $\rho q F' + F > qF' + F > 0$. Therefore, there exists a unique $\rho_0 \in (\gamma, 1)$ such that (4.3) is satisfied if and only if $\rho \in (\gamma, \rho_0)$. QED

C. Proof of Theorem 5.1

The first part of theorem 5.1 summarizes the results of section 5. We prove below the properties of $s(\rho)$ and $R(\sigma)$.

We start by showing that $s'(\rho) < 0$ for $\gamma \le \rho \le \rho_0$. Denote by $Q(\sigma)$ the queue length in a bargaining equilibrium as a function of the sellers' bargaining power. By definition of $s(\rho)$, $W(1, q_0(\rho)) = W(1, Q(s(\rho))$. Under part (b) of case 2A, $\sigma < \gamma$, so that any queue size Q relevant for this case satisfies $q_0(\rho) < q_H^* < Q$. By definition of q_H^*, the welfare function $W(1, q)$ is increasing in q for all $q < q_H^*$. Since $q_0(\rho) < q_H^*$ and $q_0'(\rho) < 0$, $W(1, q_0(\rho))$ is a decreasing function of ρ. Therefore, so is $W(1, Q(s(\rho)))$. Because $Q > q_H^*$, however, $Q(s(\rho))$ must be increasing in ρ and thus from (3.6), $s'(\rho) < 0$.

To show that $s(\gamma) = \gamma$. Notice that at $\rho = \gamma$, posting $[k = 1, p = \rho = \gamma]$ is socially efficient. We know that the bargaining equilibrium corresponds to the socially efficient allocation if and only if $\sigma = \gamma$. Thus, $s(\gamma) = \gamma$.

By definition of $R(\sigma)$, $W(R(\sigma), q^*(R(\sigma))) = W(1, Q(\sigma))$. It is immediate that $R(\sigma) < 1$ for $\sigma \neq \gamma$ and that $R(\gamma) = 1$ since the bargaining equilibrium corresponds to the socially efficient allocation if and only if the Hosios condition applies.

To show that $R'(\sigma) > 0$ iff $\sigma < \gamma$, we notice that the function $W(\rho, q^*(\rho))$ is strictly increasing in ρ. When $\sigma < \gamma$, an increase in sellers' bargaining power improves efficiency of the bargaining equilibrium and thus $W(1, Q(\sigma))$ increases, implying that $R'(\sigma) > 0$. Similarly, $R'(\sigma) < 0$ if $\sigma < \gamma$.

We now show that $R(\rho_0) = \rho_0$ and that $R(s(\rho_0)) = \rho_0$. Combining (B.1) expressed at $\rho = \rho_0$ and (3.6) expressed at $\sigma = \rho_0$, we obtain that $Q(\rho_0) = q_0(\rho_0)$. The function $R(\sigma)$ has been defined as such that the welfare of the price posting equilibrium at $\rho = R(\sigma)$ is equal to that of the bargaining equilibrium with sellers' bargaining power equal to σ. Equality of welfare implying equality of buyers' surplus, it follows that at $\sigma = \rho_0$,

$$F(q_0(\rho_0))(1 - \rho_0) = R(\rho_0)[F(q^*(R(\rho_0))) + q^*(R(\rho_0))F'(q^*(R(\rho_0)))]. \tag{C.1}$$

We can rewrite the left-hand side of (C.1) as

$$\text{LHS(C.1)} = \rho_0[F(q_1(\rho_0)) + q_1(\rho_0) F'(q_1(\rho_0))] = \rho_0[F(q^*(\rho_0)) + q^*(\rho_0) F'(q^*(\rho_0))].$$

The first equality is obtained from writing (B.2) at $\rho = \rho_0$, and the second equality from the fact that $f(\rho_0) = 0$. Since the function $\rho[F(q^*(\rho)) + q^*(\rho) F'(q^*(\rho))]$ is strictly increasing, it implies that $R(\rho_0) = \rho_0$.

By definition of $s(\rho)$, $s(\rho_0)$ satisfies $W(1, Q(s(\rho_0)) = W(1, q_0(\rho_0))$. By definition of $R(\sigma)$, $W(1, Q(s(\rho_0)) = W(R(s(\rho_0)), q^*(R(s(\rho_0))))$. Since $f(\rho_0) = 0$, it implies that $q_1(\rho_0) = q^*(\rho_0)$. Finally, expressing (B.2) at $\rho = \rho_0$ and recognizing equality of welfare and surplus, we obtain that $W(\rho_0, q^*(\rho_0)) = W(1, q_0(\rho_0))$. Since the function $W(\rho, q^*(\rho))$ is strictly increasing in ρ, it follows that $R(s(\rho_0)) = \rho_0$. QED

References

[1] Acemoglu, D. and Shimer, R. (1999), "Holdups and efficiency with search frictions", *International Economic Review* 40, 827-850.

[2] Bester, H., 1993, "Bargaining versus pricing competition in markets with quality uncertainty," *American Economic Review* 83, 278-288.

[3] Burdett, K., Shi, S. and R. Wright, 2001, "Pricing and matching with frictions," *Journal of Political Economy* 109, 1060-1085.

[4] Cho, I.K. and D.M. Kreps, 1987, "Signaling games and stable equilibria," *Quarterly Journal of Economics* 102, 179-221.

[5] Forand, J.G., 2007, "Competing through information provision," manuscript, University of Toronto.

[6] Guerrieri, V., 2005, "Efficiency of competitive search under asymmetric information," manuscript, University of Chicago.

[7] Hosios, A., 1990, "On the efficiency of matching and related models of search unemployment," *Review of Economic Studies* 57, 279-298.

[8] Menzio, G., 2007, "A theory of partially directed search," manuscript, University of Pennsylvania.

[9] Michelacci, C. and J. Suarez, 2006, "Incomplete wage posting," *Journal of Political Economy* 114, 1098-1123.

[10] Moen, E.R., 1997, "Competitive search equilibrium," *Journal of Political Economy* 105, 385-411.

[11] Peters, M., 1984, "Bertrand equilibrium with capacity constraints and restricted mobility," *Econometrica* 52, 1117-1129.

[12] Peters, M., 1991, "Ex ante price offers in matching games: Non-steady state," *Econometrica* 59, 1425-1454.

[13] Peters, M. and S. Severinov, 1997, "Competition among sellers who offer auctions instead of prices," *Journal of Economic Theory* 75, 141-179.

[14] Rogerson, W.P., 1988, "Price advertising and the deterioration of product quality," *Review of Economic Studies* 55, 215-229.

[15] Shimer, R. and R. Wright, 2007, "Competitive search equilibrium with asymmetric information," manuscript, University of Chicago.

[16] Wolinsky, A., 1983, "Prices as signals of product quality," *Review of Economic Studies* 50, 647-658.