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ASSET PRICING, GROWTH, AND THE BUSINESS CYCLE WITH IRREVERSIBLE INVESTMENT

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Abstract

This paper advances a simple model that emphasizes the diversity of capital types, some of which are long lived, while others are highly specific. This modeling of capital implies that irreversibility constraints may be strongly binding, thus generating sizable and undiversifiable capital losses, even with moderate shocks and positive aggregate investment. The resulting riskiness of investing in capital has consequences for growth, the business cycle, and asset returns. Growth is affected since the representative consumer invests a larger portion of output as a form of self-insurance. The business cycle is affected since consumption becomes more variable. The asset returns are affected since the added risk raises its premium, especially in recessions. The focus of the paper is to evaluate the quantitative importance of these effects. When evaluated, the model is capable of matching the most prominent characteristics of U.S. output, consumption, and asset returns, including a wide equity premium. However, this is not a resolution to the equity premium puzzle because the paper does not address why the representative consumer has the high risk aversion necessary to match these observed time series.

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any shortcomings are my responsibility.

1. Introduction

Asset prices and asset returns provide invaluable information not only to prospective investors but also to economists modeling economic growth and the business cycle. Investors have never disregarded such information. However, most research on equilibrium theories of growth and the business cycle have ignored many lessons from asset pricing in their modeling of investment and production. Fortunately, there has been a recent surge of interest on learning from asset returns to improve the production structures of applied general equilibrium models. For example, Rouwenhorst (1995) and Tallarini (1997) show that the large variability of stock prices and the observed wide risk premium are inconsistent with a homogeneous and flexible stock of capital, even if the representative consumer is highly risk averse. Conversely, Beaudry and Guay (1996), Boldrin, Christiano, and Fisher (1995), Christiano and Fisher (1995), Coleman (1997), and Jermann (1998) demonstrate the importance of costs to either adjusting or relocating the capital stock to account for observed asset returns.[1] This paper follows the latter strain of the literature. Extending the "Ak" model in Barro (1990), a simple model is advanced that emphasizes the diversity of capital types. Some of these types are highly durable, while others are highly specific. When evaluated, this model is capable of matching the most prominent growth and business cycle characteristics of output, consumption, and asset returns.

The features of capital, responsible for inducing realistic paths for output, consumption, and

[1]The good performance of the investment based asset pricing model by Cochrane (1996) also gives support to this argument as it focuses on a crucial equilibrium condition of these models.

asset returns in the present model are summarized as follows. Capital is heterogeneous. Some forms of capital, such as basic human capital and buildings, are long-lived and versatile in regards to the occupations in which they may be employed. These forms of capital are labeled general capital. Other forms of capital, such as skills, equipment, and consumer durables, are relatively short-lived and specific to the occupations where they are useful. These forms of capital are labeled specific capital. With relocation shocks affecting the relative demand for capital across the various activities in the economy, a portion of the stock of specific capital is lost because units of this capital are less useful when employed in different occupations than in those for which they were initially created. Furthermore, this relocation has a negative impact on the demand for general capital for two reasons. First, households, seeking to smooth consumption, reduce the aggregate demand for investing in all types of capital as their income falls. Second, the marginal product of general capital falls because of its relative abundance with respect to specific capital. Consequently, even for shocks with moderate effects on output, the demand for general capital is sufficiently reduced for the irreversibility constraint of this type of capital to bind and its price to fall. Due to general capital's sectoral mobility, all sectors experience these capital losses. Hence, this modeling of capital implies that relocation shocks realistically induce simultaneous capital losses both across capital types and across sectors.

In their analysis of sectoral mobility in California at the end of the Cold War, Ramey and Shapiro (1988) document the realism of the distinction I make between general and specific capital. For example, they document that some pieces of capital, previously used by defense contractors, had to be sold to non-defense contractors at very low prices. Meanwhile, other pieces of capital retained most of their value in similar liquidations. This disparity of sell out prices is linked to the ease in which the pieces of capital could be relocated. The California experience also provides an example of relocation shocks leading to generalized capital losses for versatile pieces of capital, such as buildings, that closely match my idealized general capital. Indeed, the prices of most buildings in California, even those employed in expanding activities, were adversely affected in the aftermath of the Cold War.

Models with flexible and homogeneous capital imply that the price of capital in terms of consumption is counter-factually constant. In principle, this unrealistic absence of capital losses is easily broken with irreversibility constraints or, more generally, with costs to disinvestment. However, in practice, generating realistic capital losses is not as easy. In single sector economies, shocks producing realistic paths for output and consumption also imply that gross investment is always positive.[2] Thus, the representative consumer is never faced with having to remove capital from production. Models with idiosyncratic or sectoral shocks potentially avoid this problem by having positive investment in some sectors and binding constraints in some others. However, for these models to be empirically successful they must incorporate the right combination of shocks, sectors, and constraints to relocate capital. Recent papers have initiated the search for these models.[3] A lesson from this search is that, with a unique type of capital, large undiversifiable capital losses are still hard to obtain for realistic shocks. This paper explores an alternative modeling strategy by incorporating multiple types of capital with diverse durability and specificity.

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[2]See Sargent (1980), Olson (1989), and Christiano and Fisher (1994) for stochastic versions of the neoclassical growth model with irreversibility constraints.

[3]See Dow and Olson (1992), Boldrin, Christiano, and Fisher (1995), Christiano and Fisher (1995), Coleman (1997), Veracierto (1997), and Ramey and Shapiro (1997) for examples of neoclassical growth models with multiple sectors or multiple firms and constraints to the relocation of capital.

The modeling of capital in this paper implies that investment is much riskier than predicted in a model with flexible and homogeneous capital. A piece of specific capital may become unnecessary in its present occupation and face costly relocation to other uses. The price of installed general capital may also fall because of a binding irreversibility constraint. Moreover, capital losses tend to move together both across capital types and across sectors. Hence, investing in capital is risky, and the risk cannot be diversified away. These effects prove to be quantitatively important with strong complementarities between general and specific capital. When this happens, the model generates a substantially riskier environment than in models with flexible capital, and as a result, generates a wider risk premium.

The effect of irreversibility on the risk premium is not a potential resolution to the equity premium puzzle.[4] Indeed, the puzzle contends that it is curious how a representative consumer with standard preferences requires such high risk aversion to account for observed asset returns and aggregate consumption, when the risk aversion of individual consumers appears to be fairly moderate. While there have been many attempts to resolve this puzzle, there is little consensus on which one is the most plausible. This paper is not a new attempt to resolve the puzzle. The focus of this paper is on the modeling of capital, which has no bearing on the fit of Euler equations relating observed patterns of consumption and asset returns.

In this paper, I conduct two complementary thought experiments to demonstrate how properly

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[4]See Mehra and Prescott (1985) for the original statement of the puzzle, and Kocherlakota (1996) for a recent survey of the literature seeking to solve it.

modeling capital can help to generate realistic patterns of output, consumption, and asset returns. In the first experiment, I assume a representative consumer with high risk aversion. A possible motivation for this consumer is the type of aggregation phenomena described by Constantinides and Duffie (1996), who show that, with incomplete insurance markets, one can reconcile the low risk aversion of individual consumers with the high risk aversion of the representative consumer. In this experiment, I show that an economy with this type of consumer endogenously generates a realistic pattern of output, consumption and asset returns if the diversity of capital types, their costly relocation, and their irreversibility are incorporated into the model. As Rouwenhorst (1995) and Tallarini (1997) have shown, such an enterprise leads to failure if capital is homogeneous and flexible. In the second experiment, I assume a representative consumer with the low degree of risk aversion normally associated with an individual consumer. I show that investment irreversibility lowers the risk-free rate and raises the risk premium in comparison to a flexible economy, but as expected these effects are much too small to account for the observed asset returns.

The main conclusions of the paper are summarized as follows. The diversity of capital, with some types that are long lived and others that are highly specific, is an important feature of reality. This feature implies that irreversibility constraints may be strongly binding, thus generating large capital losses, even with moderate shocks and positive aggregate investment. These losses affect all the sectors in the economy, so they generate undiversifiable risk which has consequences for growth, the business cycle, and asset returns. The representative consumer reacts to this risk by investing a larger portion of output as a form of self-insurance, so the economy grows faster. Consumption becomes more volatile which affects the business cycle. The added risk raises the risk premium, so it modifies the structure of asset returns. This rise in the risk premium is strongest in recessions when irreversibility constraints are binding and capital prices are most variable. The focus of this paper is to evaluate the quantitative importance of these effects. A qualitative analysis of a less structured version of the model is found in Faig (1998). The rest of the paper is organized as follows. Section 2 describes the model. Section 3 evaluates the model numerically. Section 4 concludes with a brief summary of the main results.

2. The Model

In reality, many different shocks induce relocations of capital across economic activities. For example, a drop in defense spending induces a relocation away from the armaments industry. A tightening of the money supply induces a relocation away from firms dependent on bank credit. A rise of the price of oil induces a relocation away from energy intensive sectors. Other examples are not hard to find and include discoveries, shifts in fashion, and demographic changes. All these shocks have direct economic effects, in addition to the relocations of capital they induce. Methodologically, I want to abstract from these diverse direct effects to isolate the effects of the relocations of capital by themselves. For this reason, I advance a simple highly stylized model with pure relocation shocks.

The model economy is characterized by a representative consumer who either consumes or invests the output obtained from a stochastic production process. While output is homogeneous, capital is differentiated into two types. General capital, modeled to capture buildings and basic human capital, is versatile and long lived. Specific capital, modeled to capture equipment, skills, and consumer durables, is specific and relatively short lived. The versatility of general capital means that it can be relocated without cost. In contrast, a portion of specific capital is lost as it is relocated.

The amount of output produced in each period depends on the vector of capital stocks accumulated in the past and the portions of these stocks that survive the relocations that might take place at the beginning of the period:

$$y = F(k_1, zk_2, \zeta); \qquad \textbf{(1)}$$

where y = output;

k_1 = stock of general capital inherited from the past;

k_2 = stock of specific capital inherited from the past;

z = portion of specific capital that survives this period's relocation;

zk_2 = stock of specific capital put into production; and

? = stochastic state of nature.

The variable ? adopts positive integer values. When a shock does not occur, the value of ? remains the same as in the previous period. In this instance, the relocation of capital is unnecessary, so $z = z_1 = 1$. When a shock occurs, the value of ? is increased by one unit. In this instance, capital must be relocated to be of any use, so $z = z_2 < 1$. The value of ? has no direct effect on the amount of output produced. Moreover, the probability that ? will change next period depends only on whether ? has changed this period. Consequently, the variable z follows a two state Markov process.

In the allocation of output, the consumer faces a set of irreversibility constraints on each one of the capital stocks:

$$k_1' \geq \mu_1 k_1, \text{ and } k_2' \geq \mu_2 k_2; \tag{2}$$

where μ_1 = survival rate of general capital ($\mu_1 < 1$) ;

μ_2 = survival rate of specific capital ($\mu_2 < 1$) ; and

a prime on a variable denotes the value of the variable next period.

The survival rate of general capital is constant. In contrast, the survival rate of specific capital depends on the value of *z*. The loss of specific capital during a relocation not only has a transitory effect on output since less of this capital can be put into production in the current period, but also it has a permanent effect on the stock of this capital surviving to the future. Hence, in periods without relocation, the value of μ_2 is high, while in periods with relocation, the value of μ_2 is low. The ratio between the low and the high values of μ_2 is equal to z_2 (the portion of specific capital that survives a relocation). In addition to the irreversibility constraints, the consumer faces the standard resource constraint:

$$c + \left(k_1' - \mu_1 k_1\right) + \left(k_2' - \mu_2 k_2\right) \leq y. \tag{3}$$

The preferences of the consumer are recursive, homothetic, and independent across states for atemporal lotteries, but they are not necessarily time additive. Specifically, the consumer is endowed with the parametric version of Kreps and Porteus preferences introduced by Epstein and Zin (1989):[5]

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[5]This formula is valid for s … 1 and ? … 1. For s = 1, use a Cobb-Douglas aggregator. For ? = 1, use exp(*E* log(*u*N)) instead of (*E u*N$^{1-?}$)$^{1/1-?}$.

$$u = \left[(1-\beta)c^{1-\sigma} + \beta \left(Eu'^{1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}; \qquad \textbf{(4)}$$

where u is present utility; ? is the coefficient of relative risk aversion for atemporal lotteries; and s is the inverse of the inter-temporal elasticity of substitution along a deterministic path. Both s and ? are assumed positive. The expectation E is conditional on present information.

The representative consumer maximizes (4) subject to (1) to (3). A solution to this optimal growth problem exists, given certain restrictions on F (see Epstein and Zin [1989]) which are assumed to be satisfied throughout the paper. A proof similar to Epstein and Zin (1989) shows that there is a value function V which maps the vector (k_1, k_2, z) onto the consumer's maximized utility. Standard recursive dynamic programming arguments imply that, with respect to (k_1, k_2), V is continuously differentiable, concave, and linearly homogeneous.

An optimal growth path must satisfy the following conditions. As long as the irreversibility constraint for capital i (i = 1 and 2) is not binding, one unit of output must bring the same utility to the representative consumer either if it is immediately consumed, or if it is invested in capital i. Once the irreversibility constraint for capital i is binding, the marginal value of one unit of capital i, denoted q_i, is the ratio between the utility that this unit of output brings to the consumer if invested and the utility it brings if consumed. This ratio never exceeds one. The Euler equation that summarizes this condition is:

$$q_i(1-\beta)c^{-\sigma} = \beta\left(EV'^{1-\gamma}\right)^{\frac{\gamma-\sigma}{1-\gamma}} EV'^{-\gamma}\partial_i V';\ q_i \le 1;\ \text{and } (q_i - 1)(k_i' - \mu k_i) = 0. \quad \textbf{(5)}$$

Moreover, the utility that a unit of capital *i* yields to the consumer is the marginal utility of consuming the sum of the marginal product of this capital and the value of the capital left after production:

$$\partial_i V = (1-\beta)c^{-\sigma}V^{\sigma}(\partial_i F + \mu_i q_i). \quad \textbf{(6)}$$

Equations (5) and (6) are easily computed using standard numerical methods.

The next section reports a numerical evaluation of the model. The rest of this section constructs an asset pricing equation to be used in this evaluation. Multiplying to both sides of (6) by k_i and adding over i, i 0 {1, 2}, yields:

$$V = \left[(1-\beta)c^{-\sigma}\tilde{x}\right]^{\frac{1}{1-\sigma}};\ \text{where } \tilde{x} = y + q_1\mu_1 k_1 + q_2\mu_2 k_2. \quad \textbf{(7)}$$

The variable $\tilde{x}$ measures the consumer's wealth at market prices. This equation, once rearranged, can be interpreted as a consumption rule that relates consumption to utility and wealth. Using the definition of utility (4), this equation is transformed into:

$$(1-\beta)c^{-\sigma}(\tilde{x} - c) = \beta\left(EV'^{1-\gamma}\right)^{\frac{1-\sigma}{1-\gamma}}. \quad \textbf{(8)}$$

Using the resource constraint (3) and the last equality in (5), the allocation of $\tilde{x}$ satisfies:

$$\tilde{x} - c = q_1 k_1' + q_2 k_2'. \quad \textbf{(9)}$$

The value of the market portfolio invested at the end of the period is $q_1 k'_1 + q_2 k'_2$. The gross return of this market portfolio next period will be $\tilde{x}'$, so the gross rate of return of the market portfolio is:

$$\tilde{R} = \frac{\tilde{x}'}{q_1 k_1' + q_2 k_2'}\,. \tag{10}$$

Using (6) and (8), the first order condition (5) can be rewritten as:

$$q_i = \beta^{\frac{1-\gamma}{1-\sigma}} \left[(1-\beta) c^{-\sigma} (\tilde{x} - c)\right]^{\frac{\gamma-\sigma}{1-\sigma}} E V'^{\,\sigma-\gamma} (\partial_i F' + q_i' \mu_i') \tag{11}$$

Finally, using (7), (9), and (10), this equation simplifies into the desired pricing equation:

$$q_i = E\psi(\partial_i F' + q_i'\mu_i'); \text{ where } \psi = \left[\beta \left(\frac{c'}{c}\right)^{-\sigma} \tilde{R}^{\frac{\sigma-\gamma}{1-\gamma}}\right]^{\frac{1-\gamma}{1-\sigma}}. \tag{12}$$

The value of capital *i* is equal to the expected present value of its gross return next period. The variable ψ denotes the contingent price of output next period in terms of output this period. This contingent price depends on the rate of consumption growth and the gross return on the market portfolio. Simple extensions of (12) can also be used to price the market portfolio as well as a risk-free bond. By simply multiplying both sides of (12) by k_i' and summing over i, $i \in \{1, 2\}$, the market portfolio can be priced. Similarly, by setting q_i equal 1 and $(\partial_i F' + q_i' \mu_i' k_i')$ equal to the gross interest rate of the bond, we obtain the risk-free bond price. Further discussion of the properties of (12) is found in Epstein and Zin (1989) and Weil (1989). The novelty of the present model is how consumption and asset returns are related to relocation costs and irreversibility constraints.

3. Numerical evaluation

This section conducts two numerical evaluations of the model.[6] The first evaluation calibrates the model to match prominent features of the United States data for growth, the business cycle, and asset returns. These features include consumption and the equity premium. With the preference structure adopted, fitting actual data on consumption and the equity premium implies a coefficient of relative risk aversion (?) in the neighborhood of 18 (see Kocherlakota [1996]). Therefore, the model can only be successful in replicating the main features of the historical data for the United States with values of ? around 18. Such a high coefficient of relative risk aversion is thought to be implausabily high for an average consumer whose behavior is normally modeled with values of ? around 2. This contradiction between the risk aversion coefficients of the average and the representative consumer is the equity premium puzzle. A major contribution to resolving this puzzle is the work of Constantinides and Duffie (1996), who show that one can reconcile the low risk aversion of individual consumers with the high risk aversion of the representative consumer with incomplete insurance markets. To date, despite encouraging results by Storesletten, Telmer, and Yaron (1996) and others, Constantinides and Duffie's work has not been successful in their empirical implementation. For this reason, I contrast the performance of the model with high and low values of ? by conducting a second evaluation of the model where ? is equal to 2 and the risk

[6]The computational algorithm used is similar to the one used by Coleman (1997), and it is described in Christiano and Fisher (1995) as "the colocation method".

premium is not matched to the data.

The numerical evaluations of the model are performed in two stages. The first stage sets the basic environment, while the second stage calibrates the remaining parameters. In the first stage, I set the simulated length of periods, the survival rates of the two types of capital, the frequency and duration of recessions, and the functional form for the production technology. The period length is set to 1 year. This permits calibrating the model with a long sample from the United States that dates back to 1871. The survival rate for general capital is set to 0.983 which is an intermediate value between the standard survival rates for basic human capital 0.98 (50 years duration) and structures 0.9875 (80 years duration). The average survival rate for specific capital is set to 0.9 (10 years duration) which is a typical rate for equipment, consumer durables, and skills.[7] Periods with relocations are set to occur every 4.9 years (on average) and last for 1.4 years (on average).[8] This matches the frequency and duration of recessions in the United States when a recession is defined to be a set of consecutive years with negative annual growth. Finally, the functional from of the production technology is CES:

$$y = A\left[a k_1^{-r} + (1-a)(zk_2)^{-r}\right]^{-\frac{1}{r}}. \qquad \textbf{(13)}$$

[7]The rates of depreciation consistent with these two survival rates are 0.017 for general capital and 0.1 for specific capital. These rates of depreciation appear low compared to the typical 0.08 assumed for aggregate physical capital in the standard neoclassical model. However, bear in mind that in the standard neoclassical model human capital is assumed to have a 0 rate of depreciation. Extended national accounts that include human capital accumulation find average propensities of consumption consistent with a 0.04 aggregate rate of depreciation.

[8]This implies that the probability of a recession next period is 0.2041 if this period is a boom and is 0.2857 if the period is a recession.

The second stage of the evaluation calibrates the remaining parameters of the model (ß, s , ?, A, a , ?, and z_2), so that moments of the invariant distribution of the model match those from the historical records of the United States.[9] The first evaluation calibrates seven parameters to match six moments. The second evaluation calibrates six parameters (? is fixed at 2) to match five moments. In either case, there is one degree of freedom which is used in the following manner. The search algorithm first attempts to fit the moments implied by the model with those from the historical records with time separable preferences (that is, with s = ?). If this leads to values of ß higher than 1, the value of ß is set to 0.9999 (the maximum value considered), and the search continues with unconstrained values of s and ?.[10]

The moments sought to match from the historical records are the following. The mean and the standard deviation of the growth rate of output (0.0183, and 0.0547), the standard deviation of the growth rate of consumption (0.0379), the average propensity to consume (0.515), the average return on capital (0.081), and (in the first evaluation) the average risk premium of the market portfolio (0.06). The means and the standard deviations of the growth rates of output and consumption are from Cecchetti, Lam, and Mark (1990) for the period 1871-1985. These rates of growth involve output and consumption as conventionally measured in the national accounts. However, these conventional measures present a major a problem when constructing the average propensity to consume consistent with this model. In this model,

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[9]The model has an invariant distribution for the rates of growth of these variables and the ratios among them, but not for output, consumption, and capital stocks in levels.

[10]The conventional practice of constraining ß to values lower than 1 is somewhat arbitrary because, with time separable preferences, equilibria may exist with ß > 1 and s > 1 as long as ß is not too high. However, attempts at using values of ß higher than 1 while keeping ? = s were unsuccessful in fitting all the sought moments.

I assume that all forms of capital are accumulated as an act of investment. However, conventional national accounts assign human capital accumulation expenses such as child rearing, health care, and education, to consumption instead of investment. For this reason, I use the comprehensive and corrected measures of Kendrick (1976, p. 236) for the most recent year available (1973) to construct the average propensity to consume. The average return on capital is set equal to the average return on equity (0.07) from Kocherlakota (1995) for the period 1889-1978 corrected for the estimated upward bias on inflation when using the consumer price index (0.011) from Boskin, Dulberger, Gordon, Griliches, and Jorgenson (1998). Even with this correction, the resulting rate (0.081) is at the low end of the measures of the average return for comprehensive capital in Kendrick. The average risk premium of the market portfolio is set equal to the equity premium on the return of 3-month treasury bills, also from Kocherlakota.

The upper part of Table 1 presents the benchmark evaluation of the model with costs to relocate specific capital and constraints to dis-invest general capital. As documented in this table, these features enable the intended match of the model with historical moments from the United States including the average risk premium. As Rouwenhorst (1995) and Tallarini (1997) carefully document, this endeavor leads to failure when capital is flexible. With full flexibility, the type of shock considered in this paper would be irrelevant, so the standard deviations of the growth rates of output and consumption would be zero. Following Rouwenhorst and Tallarini, full flexibility could be combined with shocks to total factor productivity. The problem with this strategy is that for shocks that match the variability of output at the business cycle frequency, the market portfolio return is extremely safe as no capital losses are incurred. Thus, both the variability of consumption and the risk premium are minute. As long as capital remains flexible, this fact is robust to variations in the parameters of either the present model or the models by Rouwenhorst and Tallarini. Introducing costs to relocate specific capital increases the riskiness of the market portfolio, and thus increases the variability of consumption and the risk premium. However, these relocation costs by themselves are insufficient to achieve the intended match. Introducing the irreversibility constraint on general capital but maintaining shocks to total factor productivity is also insufficient. In this instance, with reasonable survival rates, the irreversibility constraint on general capital is either not binding or only weakly binding. Only when the costs to relocate specific capital are combined with the irreversibility constraint on general capital, the intended match is possible.

As the upper part of Table 1 reports, the model fits the intended moments from historical records without any major anomaly in other dimensions. Admittedly, the standard deviations of the risk-free rate and the market portfolio are lower than the standard deviations of the real returns on equity (0.165) and treasury bills (0.055). However, this is expected since equity represents a small portion of the capital stock, and treasury bills are not free of inflation risk. The almost constant risk-free rate in the model is consistent with the low variability of the return on indexed bonds.[11] The expected return of the market portfolio and the risk premium are both counter-cyclical. This is consistent with the mean reversion of asset prices documented in Fama and French (1988). It is also consistent with a counter-cyclical risk premium found in Ferson and Harvey (1991). In this model, the risk premium rises in recessions because the return on capital is more volatile when the irreversibility constraint is binding and not because risk aversion rises

[11]Unfortunately, the market for indexed bonds is too small and recent for using the rates of return on these bonds instead of the regular treasury bills in calibrating the model.

during recessions, as is the case in models with habit persistence (e.g. Campbell and Cochrane [1999]). Finally, the value of the coefficient of relative risk aversion is almost identical to the value that Kocherlakota (1996) finds to be necessary to provide an exact fit of the representative consumer's Euler equation to actual data. Thus, the degree of risk suffered by the consumer in the model is not all that different from the risk that a single representative consumer would suffer in the United States.

The lower part of Table 1 describes the model with the same parameters without imposing the irreversibility constraint on general capital. Once the irreversibility of general capital is relaxed, the capital losses on this form of capital disappear. Thus, the market portfolio becomes less risky and the variability of consumption falls. This implies a rise in the risk-free rate and a fall in the risk premium. Also, without the irreversibility constraint on general capital, the rate of growth of the economy falls from 0.0183 to 0.0172. Behind this drop is the rise of the average propensity to consume and the associated fall in the average propensity to invest. As discussed in Faig (1998), as long as the inter-temporal elasticity of consumption is less than one, irreversibility encourages capital creation because of the negative wealth effect it has on consumption.

Table 2 reports the second evaluation of the model. When the coefficient of relative risk aversion ? is fixed at 2, the model can still fit the targeted historical moments except for the risk premium. With this low risk aversion, the irreversibility constraint on general capital matters much less in lowering the risk-free rate and in magnifying the risk premium for two reasons. First, the direct and obvious reason is that a lower risk aversion coefficient implies a lower price of risk. Second, the indirect and paradoxical reason is that lower risk aversion implies a less risky market portfolio. With lower risk aversion, the representative consumer feels less compelled to invest as a form of self-insurance which tends to slow down growth. For the model to continue reproducing the historical rate of growth of the US economy, the inter-temporal elasticity of substitution must then be higher (s = 0.2257 for ? = 2, whereas s = 0.1078 for ? = 17.79). With a higher inter-temporal elasticity of substitution, the consumer becomes more tolerant to fluctuations in consumption and less eager to deplete the capital stock to avoid a large down fall on consumption in bad times. Consequently, the capital losses associated with the irreversibility constraint on general capital become less pronounced with low risk aversion, and so the standard deviation of the market portfolio falls from 0.0868 when ? = 17.79 (upper part of Table 1) to 0.0610 when ? = 2 (upper part of Table 2).

4. Conclusion

The diversity of capital with some types that are long-lived, such as basic human capital and buildings, and others that are highly specific, such as skills, equipment, and consumer durables, is an important feature of reality. This feature implies that irreversibility constraints may be strongly binding, thus generating sizable and undiversifiable capital losses, even in the presence of moderate shocks and positive aggregate investment. The resulting riskiness of investing in capital has consequences for growth, business cycles, and asset returns. Growth is affected as the representative consumer invests a larger portion of output as a form of self-insurance. The business cycle is affected as consumption becomes more variable. Asset returns are affected as the added risk raises its premium, especially in recessions. The strength of these consequences increases when the consumer is less tolerant to risk and fluctuations in consumption.

In this model, tolerance for risk and fluctuations in consumption are conveniently controlled with two parameters. However, similar results could be obtained with other preference structures, such as habit formation, if we were to maintain the diversity of capital as modeled in this paper.

Table 1

Total factor productivity $A = 0.1914$.
Production elasticity of substitution $(1+?)^{-1} = 0.2687$.
Distribution parameter a = 0.9633.
Average survival rates: $\mu_1 = 0.983$ for k_1 and $\mu_2 = 0.9$ for k_2 .
Realized values of z: $z_1 = 1$ in expansions and $z_2 = 0.7555$ in recessions.
Expected length of states: 4.9 years for expansions and 1.4 years for recessions.
Subjective pure discount factor ß = 0.9999.
Inter-temporal elasticity of substitution $s^{-1} = 0.1078$.
Coefficient of relative risk aversion ? = 17.79.

	Expansion	Recession	Mean	Std.
COSTLY RELOCATION OF k_2 AND IRREVERSIBILITY OF k_1				
q	1	0.8888	0.9741	0.0543
$E \ln \tilde{R}$	0.0646	0.1383	0.0810	0.0868
$\ln r$	0.0173	0.0339	0.0210	0.0106
$E \ln \tilde{R} - \ln r$	0.0473	0.1044	0.0600	0.0831
c / y	0.5107	0.5305	0.5150	0.0091
k'_2 / k'_1	0.3605	0.3255	0.3527	0.0165
g_k	0.0196	0.0139	0.0183	0.0290
g_y	0.0459	-0.0782	0.0183	0.0547
g_c	0.0381	-0.0510	0.0183	0.0379
COSTLY RELOCATION OF k_2 AND FLEXIBILITY OF k_1				
q	1	1	1	0
$E \ln \tilde{R}$	0.0780	0.0722	0.0767	0.0287
$\ln r$	0.0578	0.0505	0.0562	0.0031
$E \ln \tilde{R} - \ln r$	0.0202	0.0217	0.0205	0.0286
c / y	0.5214	0.5568	0.5293	0.0148
k'_2 / k'_1	0.3289	0.3200	0.3269	0.0037
g_k	0.0185	0.0129	0.0172	0.0285
g_y	0.0473	-0.0880	0.0172	0.0627
g_c	0.0339	-0.0411	0.0172	0.0313

Note: The ex-post growth rates of the initial capital stock, output, and consumption (g_k, g_y, and g_c) are annual and continuously compounded. The remaining variables are: q = average price of capital in units of consumption, $E \ln \tilde{R}$ = expected market return conditional on the present period information (continuously compounded), $\ln r$ = risk-free interest rate (continuously compounded). c/y = average propensity to consume, and k'_2 / k'_1 = ratio of capital stocks at the end of the period.

Table 2

Total factor productivity A = 0.1967.
Production elasticity of substitution $(1+?)^{-1}$ = 0.1914 .
Distribution parameter a = 0.9848.
Average survival rates: μ_1 = 0.983 for k_1 and μ_2 = 0.9 for k_2 .
Realized values of z: z_1 = 1 in expansions and z_2 = 0.7786 in recessions.
Expected length of states: 4.9 years for expansions and 1.4 years for recessions.
Subjective discount factor ß = 0.9999.
Inter-temporal elasticity of substitution s^{-1} = 0.2257.
Coefficient of relative risk aversion ? = 2.

	Expansion	Recession	Mean	Std.
COSTLY RELOCATION OF k_2 AND IRREVERSIBILITY OF k_1				
q	1	0.9332	0.9846	0.0341
$E \ln \tilde{R}$	0.0723	0.1116	0.0810	0.0611
$\ln r$	0.0686	0.1009	0.0757	0.0173
$E \ln \tilde{R} - \ln r$	0.0037	0.0107	0.0053	0.0579
c / y	0.5107	0.5304	0.5150	0.0090
k'_2 / k'_1	0.4082	0.3724	0.4002	0.0172
g_k	0.0196	0.0139	0.0183	0.0288
g_y	0.0458	-0.0780	0.0183	0.0547
g_c	0.0381	-0.0510	0.0183	0.0379
COSTLY RELOCATION OF k_2 AND FLEXIBILITY OF k_1				
q	1	1	1	0
$E \ln \tilde{R}$	0.0802	0.0745	0.0789	0.0293
$\ln r$	0.0785	0.0724	0.0771	0.0025
$E \ln \tilde{R} - \ln r$	0.0017	0.0021	0.0718	0.0292
c / y	0.5112	0.5380	0.5171	0.0112
k'_2 / k'_1	0.4006	0.3970	0.3998	0.0015
g_k	0.0194	0.0136	0.0181	0.0292
g_y	0.0452	-0.0767	0.0181	0.0555
g_c	0.0347	-0.0401	0.0181	0.0312

See note in Table 1.

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