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Qualified Equal Opportunity and Conditional Mobility: Gender Equity and Educational Attainment in Canada

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Abstract

"Equalizing Opportunity" is the policy objective underlying interest in generational mobility. Absent any other objective, such a policy is counterfactually symmetric in effect simultaneously increasing the upward mobility of those poorly endowed and the downward mobility of those richly endowed. However, the inclusion of a pseudo "Pareto-Utilitarian" policy imperative (That the inheriting generation should not be made worse off in a first order stochastic dominance sense) implies a "Qualified Equal Opportunity" imperative yielding a "Conditional Mobility" policy more akin to common observation, since it is asymmetric in effect focusing on improving the mobility of the poorly endowed without diminishing outcomes of the richly endowed. In terms of generational regression models, such policies "convexify" the relationship and induce a particular form of heteroskedastic error structure. In terms of Markov transition matrices they generate structures quite different from those characterized by independence. These two ideas are explored in the context of *Gender Equity* in educational attainment in Canada for cohorts born between the 1920s and the 1970s, and the results are largely consistent with such a program with the exception of the very poorest segment of society.

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1 Introduction

> "*The conception of social justice held by many, perhaps most, citizens of the Western democracies is that of equality of opportunity. Exactly what that kind of equality requires is a contested issue, but many would refer to the metaphor of "leveling the playing field", or setting the initial conditions in the competition for social goods as to give all, regardless of their backgrounds an equal chance of achievement. A central institution to implement that field leveling is education, meaning education that is either publicly financed or made available to all at affordable costs.* " Roemer (2006)

With roots in recent egalitarian political philosophy[1], the *Equal Opportunity Imperative* sees differential outcomes as ethically acceptable when they are the consequence of individual choice and action but not ethically acceptable when they are the consequence of circumstances beyond the individual's control. To the extent that an individual's circumstances have to do with their gender and the parents they were blessed with, equal opportunity policies have to address the degree to which a child's status when adult is related to their gender and the status of their parents at a similar stage in their life cycle. As Kanbur and Stiglitz (1986) observed, in essence the issue is one of generational mobility and the manner in which it engenders a dynastic aspect to poverty and wealth. The imperative has provoked considerable empirical interest in the extent of generational mobility (or the degree to which a child's parental circumstance conditions her outcomes), however evidence for complete mobility (independence of outcome from circumstance) is at best mixed (see for example Corak (2004) and references therein).

When *Equal Opportunity* is not the sole imperative there would probably be trade-offs or qualifications for the policy maker to consider. Piketty (2000) noted as much in his interpretation of the conservative − right wing view that, if generational mobility is low (because of the high inheritability of ability) and the distortionary costs of welfare redistributions are high, it is reasonable to argue that low mobility is acceptable[2]. Friedman (2005) argues the other side of this coin in conjecturing (with a considerable amount of supporting evidence)

[1]See Arneson (1989), Cohen (1989), Dworkin (1981a), Dworkin (1981b), and Dworkin (2000).

[2]Indeed the pursuit of an equal opportunity goal has not been unequivocal, Cavanagh (2002) expresses some philosophical reservations, Jencks and Tach (2006) question whether an equal opportunity imperative should require the elimination of "..all sources of economic resemblance between parents and children. Specifically ...(it)... does not require that society eliminate the effects of either inherited differences in ability or inherited values regarding the importance of economic success relative to other goals.". In a similar vein

that economic growth has facilitated the equalizing of opportunities (amongst other improvements in social justice) in effect allowing the poor to catch up without disadvantaging the rich.

If other societal aspirations are at play in mediating the intergenerational mobility objective, then societies may be distinguished by the extent to which equal opportunity is the only or primal policy goal. Here a distinction is made between societal ambitions for mobility which are not conditioned on a child's socio-economic status and those which are. When societal mobility ambitions are free of concern for socio-economic status, it will pursue policies which break both the good connects (productive parents producing productive children) and the bad connects (unproductive parents producing unproductive children). The success or failure of such policies is readily evaluated using statistical techniques which reflect degrees of dependence.

On the other hand, a *conditional* or *qualified* equal opportunity program could be characterized as a policy of an affirmative action flavour, focusing on breaking the "bad" connects only. These policies incorporate normative objectives that weighs policies in the favour of "poorly" endowed, and focuses on improving the life chances of the "inherited poor" rather than diminishing the life chances of the "inherited rich"[3]. In focusing only on elevating the prospects of the poorly endowed, the policy maker is in effect responding to a second imperative, a sort of Pareto condition wherein the lot of the poorly endowed is improved without diminishing the lot of the richly endowed. Indeed under such a Utilitarian mandate, just breaking the "bad" connects elevates overall wellbeing. In essence increased mobility of those poor in circumstance is revealed to have greater societal value than increased mobility of those rich in circumstance (which in the face of constraints will almost by definition be increased downward mobility which lowers aggregate material wellbeing under this mandate). Here in terms of societal wellbeing, consuming resources in disinheriting the well endowed (or destroying inherited human capital) and the concomitant distortionary wellbeing costs is considered too high.

Dardanoni et al. (2006) question how demanding the pursuit of equal opportunity should be in terms of the feasibility of such a pursuit.

[3]As a matter of casual empiricism, equal opportunity programs observed in "Liberal" societies do seem to be of this flavour. For example, when questioned on the widening gap between the rich and poor, the British Prime Minister responded that "... the issue is not in fact whether the very richest person ends up being richer. The issue is the poorest person is given the chance they don't otherwise have. The most important thing is to level up, not level down." Interview with the Prime Minister on BBC News Newsnight on June 4, 2001. Transcript available from `http://news.bbc.co.uk/2/hi/events/newsnight/1372220.stm`

Should the policy maker follow the dual mandates of equal opportunity guided by a Utilitarian imperative, a qualified equal opportunity program emerges with asymmetric mobility aspirations for increasing the mobility of the poorly endowed and not increasing the mobility of the well endowed when it involves a loss of their wellbeing. The extent to which these objectives are fulfilled is bounded by the capacity in the system. Such policies can no longer be characterized as unqualified moves towards the independence of outcomes and circumstances for all groups. They are rather equivocal moves, modifying the joint distribution of outcomes and circumstances differentially toward independence for the poor in circumstance and independence for the rich in circumstance only if their material wellbeing is not diminished. Evaluating their success or failure requires rethinking the way current empirical mobility measures (generational regression coefficients and transition matrix structures) are used and interpreted since generally they attach equal weight to both the poor and rich in circumstance.

Implications of intergenerational mobility have been examined in van de Gaer, Schokkaert and Martinez (2001) which demonstrates the axiomatic incompatibility of three possible motivations for examining intergenerational mobility: (1) mobility as *movements* of the constituents of a society, (2) as an indicator of *equal opportunity* and (3) as an *indicator of life chances*. Their aim was to develop a measure which could distinguish between these disparate ambitions. They first construed definition (1) as a preference for changes in economic status across generations or movement (which they characterized by an empty diagonal in the transition matrix). They define (2) as the equalizing of opportunity of attaining a socio-economic status by children across socio-economic groups (which is the context examined in this paper). They argue that both these interpretations require contentious ordering of variables (usually income in intergenerational studies) and propose (3) as a definition which eliminates the need for such an ordering. This paper augments van de Gaer et al.'s (2001) discussion by first providing a simple model that illustrates the implications of mobility in terms of *unqualified* or *unconditional* movements of the constituents of society, distinguishing it from the distribution generated when mobility is *qualified* in nature. We then show that if the researcher is concerned with *qualified* mobility, it has implications for the heteroskedasticity of the error terms in the generational regression techniques commonly employed in examining intergenerational mobility.

In Section 2 it is formally shown that when the policy maker faces the Pareto improvement constraint of not making the children of specific socioeconomic groups materially worse off under an equal opportunity policy, a *Qualified* Equal Opportunity Policy emerges. The extent to which this can be achieved is limited by the degree of flexibility in the system (represented in the model presented by potential growth, much along the lines of Friedman (2005)). Mobility improvements are qualified by their circumstance source in some sense and implications for the way in which such mobility is measured are then developed. A means of evaluating the success of mobility policies differentially is developed in Section 3 where a *Qualified* or *Conditional* Mobility measure is proposed which is simple to employ and permits the identification and examination of group specific mobility changes in the sense that the mobility of the "poor" or "rich" in circumstance can be addressed separately. Implications for the way in which conventional measures of mobility are used and interpreted are also examined.

To illustrate the concepts and their measurement, Statistics Canada's General Social Survey Cycle 19 (2005) is used to examine the closing gender gap in educational attainment that occurred in Canada[4] (Blau et al. 2006) in section 4. One of the preoccupations of Sen's considerable body of work on social justice is the achievement of *gender justice* (See Nussbaum (2006), Sen (1990), Sen (1995) for example). This could have been achieved quite swiftly by a transfer of resources from the investments in male human capital to investments in female human capital. Had that been so, an improvement in the achievements of females accompanied by deterioration in the achievements of males would have been observed. However it will be shown that, while male academic achievements did not deteriorate, the narrowing gender gap is characterized by an increased generational mobility of women relative to men. Furthermore the source of this increased mobility was the daughters of parents with lower educational attainments (which may be construed as a "good" since it implies upward mobility) rather than the daughters of parents with high educational attainments (which may be construed as a "bad" since it implies downward mobility and the attrition of inherited wellbeing). Indeed it appears that the increased mobility of women has come about as a consequence of a reduction in the dependence of their educational outcomes on those of their mothers especially at the lower end of the maternal educational attainment spectrum. However increasing immobility was observed in the lowest inheritance class. Finally, a brief discussion of the results is provided in the section 5.

[4]This phenomena has also been observed in the United States see Buchmann and Diprete (2006), Dynarski (2007), Goldin et al. (2006) and Jacob (2002).

2 The Constrained Equal Opportunity Imperative

2.1 Implications of Unqualified Intergenerational Mobility Policy

To illustrate matters assume that parent-child characteristics have 4 discrete realizations (Though the model discussed can easily be generalized to any number of characteristic realizations and the case when the number of realizations for both parent and child differ in number. See Anderson and Leo (2008)). Consider a simple generational income class transition structure where the vector of parental incomes $\mathbb{x} = [1, 2, 3, 4]'$ transit to the vector of child incomes $\mathbb{y} = [1, 2, 3, 4]'$, denoting each element as x_i and y_k, $i, k \in \{1, 2, 3, 4\}$, respectively. That is x_i and y_k are realizations of random variables x (parental incomes) and y (child incomes) respectively. Let the vector of probabilities for parents be $\mathbb{p}$ with elements p_k for the probability of a parent being in income class x_k. Similarly, the vector of probabilities for children is $\mathbb{c}$ with elements c_i for the probability of a child being in income class y_i. In other words,

$$\mathbb{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \mathbb{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

Let $\mathbb{J}$ be the matrix of joint probabilities, with element $j_{i,k}$ corresponding with the probability of a parent-child observation being in income class x_k, and y_i respectively. More precisely,

$$\mathbb{J} = \begin{bmatrix} j_{1,1} & j_{1,2} & j_{1,3} & j_{1,4} \\ j_{2,1} & j_{2,2} & j_{2,3} & j_{2,4} \\ j_{3,1} & j_{3,2} & j_{3,3} & j_{3,4} \\ j_{4,1} & j_{4,2} & j_{4,3} & j_{4,4} \end{bmatrix}$$

where $p_k = \sum_{i=1}^{4} j_{i,k}$ and $c_i = \sum_{k=1}^{4} j_{i,k}$, where $i, k = \{1, 2, 3, 4\}$. Let $\mathbb{P} = \text{dg}(\mathbb{p})$, dg being the diagonal operator, then the conventional *transition matrix* $\mathbb{T}$ can be written as $\mathbb{T} = \mathbb{J}\mathbb{P}^{-1}$ whose i^{th}, k^{th} element is $\Pr(y = y_i | x = x_k) = j_{i,k}/p_k$ and yields the child's income class vector $\mathbb{c}$ from the equation $\mathbb{c} = \mathbb{T}\mathbb{p}$ (Noting that $\mathbb{P}^{-1}\mathbb{p} = \mathbb{1}$, where $\mathbb{1}$ is vector of ones). Let $\mathbb{J}^I$ be the joint density matrix in a *pure equal opportunity* environment, where $\mathbb{J}^I = \mathbb{c}\mathbb{p}'$, i.e. independence between parent-child outcomes which yields a transition matrix, $\mathbb{T}^I$ with common columns $\mathbb{c}$ reflecting the fact that a child's life chances are the same for all parental classes. Average parent and child incomes may be written as $\mathbb{p}'\mathbb{x}$ and $\mathbb{c}'\mathbb{y}$ respectively.

On the other hand, when child outcomes are positively correlated with adult outcomes the conditional distribution of the outcomes of children with low income parents will be stochastically dominated by that of higher income parents so that, in its strongest form:

$$\sum_{i=1}^{m} (j_{i,l} - j_{i,k}) \geq 0 \text{ for } l < k;\ l,\ k,\ m = \{1, ..., 4\}$$

A *pure equal opportunity* program is one which moves a joint density $\mathbb{J}$ towards $\mathbb{J}^I$. Note that a move toward $\mathbb{J}^I$ that preserves the children's socioeconomic status structure will necessarily make the children of one parental income group worse off while making the children of another better off. To see this, first suppose the population's joint density matrix is such that $\mathbb{J} \neq \mathbb{J}^I$, in other words the population exhibits some dependence in mobility. Consider the socioeconomic group denoted by the index $x_1 = 1$. Let the nature of dependence be such that $j_{1,1} = \max\{j_{1,1}, j_{2,1}, j_{3,1}, j_{4,1}\}$. Suppose the shift towards independence or mobility shifts the attainment of children towards higher attainment. Then by definition of raising mobility, it must necessarily be true that $j_{1,1} > j^I_{1,1} = c_1 p_1$. Then for socioeconomic group x_1, the following must be true,

$$\begin{aligned} \sum_{i=1}^{m} j^I_{i,1} &\leq \sum_{i=1}^{m} j_{i,1} \\ \Rightarrow \sum_{i=1}^{m} \left(j^I_{i,1} - j_{i,1}\right) &\leq 0 \end{aligned}$$

where $m \in \{1, 2, 3, 4\}$, which means that a shift towards independence leads to a stochastic dominant shift for socioeconomic group x_1. However, since it is assumed that $\mathbb{c}$ remains unchanged, this then implies that, $j_{1,q} < j^I_{1,q} = c_1 p_q$, for some $q \in \{2, 3, 4\}$, which in turn means that,

$$\begin{aligned} \sum_{i=1}^{m} j^I_{i,q} &\geq \sum_{i=1}^{m} j_{i,q} \\ \Rightarrow \sum_{i=1}^{m} \left(j^I_{i,q} - j_{i,q}\right) &\geq 0 \end{aligned}$$

that is consequent to the shift towards independence without any qualifying conditions on the policy, child outcomes of higher socioeconomic status families are necessarily made worse off. Put another way, the children's outcome distribution in the status quo state, for higher socioeconomic status families, first order dominates that of the equal opportunity distribution.

2.2 A Simple Model of the Qualified Mobility Problem Confronting the Social Planner

The social planner's problem is modelled as one of minimizing the "distance" between the targeted joint density matrix and that under perfect independence, namely:

$$\min_{j^*_{i,k}\in\mathbb{J}^*}\sum_{i=1}^{4}\sum_{k=1}^{4}\left(j^*_{i,k}-c^I_i p_k\right)^2$$

where $j^*_{i,k}$ is an element of the joint density matrix $\mathbb{J}^*$ the social planner is choosing, while $c^I_i p_k = j^I_{i,k}$ is the joint density matrix under *perfect independence.* c^I_i is an element of $\mathbb{c}^I$, the desired marginal density vector of child income, determined by the social planner. Another way to think about the choice of the social planner is that she is implicitly choosing the level of funding or assistance towards differing socioeconomic groups to achieve the desired parent-child joint density. It is clear that if there are no constraints, the optimal choice of the social planner is to simply set $\mathbb{J}^* = \mathbb{J}^I$, which is counter factual as noted above.

Consider augmenting the social planner's choice such that she faces two constraints. Firstly, she wants to meet a growth rate constraint, g, which gives $\mathbb{c}^I$. Secondly, she wants to promote equal opportunity but does not want the outcomes for children in any parental income class to deteriorate. Note next that the existing parental income distribution, $\mathbb{p}$, is fixed and immutable. With respect to the first constraint, let $\mathbb{J}$ correspond to the existing (i.e. pre-policy) transition matrix $\mathbb{T}$ which yields $\mathbb{c}$ with an average child outcome of $\mathbb{c}'\mathbb{y}$, and suppose that $\mathbb{J} \neq \mathbb{c}\mathbb{p}'$. Next let $\mathbb{J}^*$ correspond to the post policy transition matrix $\mathbb{T}^*$ which yields an average child outcome of as much as $\mathbb{c}'\mathbb{y}+g$. In effect $(\mathbb{T}^*\mathbb{p})'\,\mathbb{y} = (\mathbb{J}^*\mathbb{1})'\,\mathbb{y} \leq \mathbb{c}'\mathbb{y}+g$ is a constraint on the possible choices of $\mathbb{J}^*$ since, as demonstrated above, when g is 0 no move of the elements of $\mathbb{J}$ toward an equal opportunity structure is possible without making the children of at least one parental income group worse off. Noting again that *pure equal opportunity* corresponds to $\mathbb{J}^* = \mathbb{c}^I\mathbb{p}'$. We can rewrite the growth constraint as,

$$\begin{aligned}\mathbb{c}^{*\prime}\mathbb{y}-\mathbb{c}'\mathbb{y} &\leq g\\ \Rightarrow \sum_{i=1}^{4} y_i\left(\sum_{k=1}^{4} j^*_{i,k}-c_i\right) &\leq g\end{aligned}$$

where $\mathbb{c}^*$ is the corresponding vector of marginals from the constrained mobility policy chosen by the social planner.

The second utilitarian constraint constrains the rows of $\mathbb{J}^*$ to first order stochastically dominate the corresponding rows of $\mathbb{J}$ following the notion that the young generation should not be made worse off by the equal opportunity policy. Put another way, the new conditional density (conditional on the child's socioeconomic status) must first order stochastically dominate the status quo conditional density. In this simple stylized model, the social planner's objective function will be subject to three stochastic dominance criteria of $\sum_{i=1}^{q} c_i^* \leq \sum_{i=1}^{q} c_i$, $q = \{1, 2, 3\}$, and that $\sum_{i=1}^{4} c_i^* = 1$, noting that $\sum_{k=1}^{4} j_{i,k}^* = c_i^* \leq c_i^I$.

The social planner's constrained problem can now be restated as,

$$\min_{j_{i,k}^* \in \mathbb{J}^*} \sum_{i=1}^{4} \sum_{k=1}^{4} \left(j_{i,k}^* - c_i^I p_k \right)^2 \tag{1}$$

subject to:

$$\sum_{i=1}^{l} \left(j_{i,k}^* - j_{i,k} \right) \leq 0, \forall l = \{1, 2, 3\} \tag{2}$$

$$\sum_{i=1}^{4} y_i \left(\sum_{k=1}^{4} j_{i,k}^* - c_i \right) \leq g \tag{3}$$

Note that $\sum_{i=1}^{4} j_{i,k}^* = p_k$, $\sum_{k=1}^{4} j_{i,k}^* = c_i^*$, and $j_{i,k}^* \in [0, 1]$ $\forall i$ and k. That is she wants to ensure that in choosing the matrix of joint densities, children of each socioeconomic group do not suffer a fall in welfare, and that growth in child outcomes is met at the same time. The question of equal opportunity phrased in this form highlights the competing considerations.

After some manipulation (See appendix A.1), the resultant Kuhn Tucker conditions are:

$$\frac{\partial L}{\partial j_{r,l}^1} = 4 \left(j_{r,l}^* - c_r^I p_l \right) + 2 \sum_{q=1, q \neq l}^{4} \left(j_{q,l}^* - c_q^I p_l \right) + \sum_{i=1}^{r} \lambda_{i,l} - \gamma \left(4 - y_r \right) = 0 \tag{4}$$

$$\frac{\partial L}{\partial \lambda_{r,l}} = \sum_{q=1}^{r} \left(j_{q,l}^* - j_{q,l} \right) \leq 0 \qquad \lambda_{r,l} \geq 0 \tag{5}$$

$$\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{3} (4 - y_i) \left(c_i - \sum_{k=1}^{4} j_{i,k}^* \right) - g \leq 0 \qquad \gamma \geq 0 \tag{6}$$

where $r = \{1, 2, 3\}$, $l = \{1, 2, 3, 4\}$. When the constraints do not bind ($\lambda_{r,l} = 0$ for $r = \{1, 2, 3\}$, $l = \{1, 2, 3, 4\}$ and $\gamma = 0$) the solution to (4) is the equal opportunity solution $j_{r,l}^* = c_r^I p_l$, $\forall r, l$. As the constraints successively bind the equal opportunity outcome is successively compromised with the solution being a combination of the initial and equal opportunity outcomes.

First note that the solution for the richest parental group ($l = 4$ in equation (4)) contains a compounding of the stochastic dominance shadow prices of each socioeconomic group. This implies that not meeting the stochastic dominance constraint at the lowest socioeconomic level implies costs at all socioeconomic levels. Thus suppose the initial state is one of complete immobility and $g > 0$[5], the social planner would reallocate the $j_{1,l}$'s to the extent that (6) does not bind and (5) does not bind for $l = 1$, thereby improving the mobility of the poorest children (note that increased mobility for the richest children would involve increased downward mobility making them worse off and conflicting with the dominance condition (5). Should there still be capacity for change, the $j_{2,l}$'s would next be reallocated and so on until the growth constraint is exhausted or complete equality of opportunity is achieved. Insofar as a move towards independence for children of higher socioeconomic status families implies a welfare reduction for them, a social planner abiding by the above program will not implement it. On the other hand, children of lower socioeconomic status families will see a shift towards independence, such that the post policy conditional density for them will first order stochastically dominate their pre-policy joint density. Finally, note that an implicit assumption in this model is that the cost of shifting children at various socioeconomic groups are constant, and the "distance" of the characteristic realization (in this case here income groups) are equidistant apart. Although relaxing the latter has no implication for this illustrative model, the former is substantial. If the costs of improving the stead of the children differ across socioeconomic groups, then improving the lot of those "high cost" children may impede the attainment of the desired level of growth in average income.

Now consider how this program affects within group variance, and its relative change vis-à-vis other socioeconomic groups. Let $l < q$ be two socioeconomic groups where l is affected by the social planner's policy towards qualified mobility, while q remains at the status quo, that is $j^*_{i,q} = j_{i,q}$, $\forall q \neq l$. Without lost of generality, let the only socioeconomic group affected by the policy be l, noting that by first order stochastic dominance $\sum_{i=1}^{4} \frac{j^*_{i,l}}{p_l} y_i = \mu^*_l > \mu_l = \sum_{i=1}^{4} \frac{j_{i,l}}{p_l} y_i$. Next let $\Delta_{r,l} = j^*_{r,l} - j_{r,l}$ and let $\overline{r}$ be the realization at or below which the density falls or remains the unchanged. After some manipulation (See appendix A.2) it

[5]Recall that if g were 0 no move toward an equal opportunity policy could be made without making some of the children in at least one of the income classes worse off.

may be shown that the variance falls if and only if,

$$\left\{\begin{array}{cc} & \sum_{r=2}^{\overline{r}}\sum_{i=1}^{r-1}\left(\Delta_{r,l}\Delta_{i,l}\right)y_i^2 \\ + & \sum_{r=\overline{r}+1}^{4}\sum_{i=\overline{r}+1}^{\max\{r-1,\overline{r}+1\}}\left(\Delta_{r,l}\Delta_{i,l}\right)y_i^2 \\ + & \sum_{r=\overline{r}+1}^{4}\sum_{i=\overline{r}+1}^{\max\{r-1,\overline{r}+1\}}\Delta_{i,l}j_{r,l}y_i^2 \\ + & \sum_{r=\overline{r}+1}^{4}\sum_{i=1}^{r-1}\Delta_{r,l}j_{i,l}y_i^2 \end{array}\right\} \leq -\left\{\begin{array}{cc} & \sum_{r=\overline{r}+1}^{4}\sum_{i=1}^{\overline{r}}\left(\Delta_{r,l}\Delta_{i,l}\right)y_i^2 \\ - & \sum_{r=2}^{4}\sum_{i=1}^{r-1}\Delta_{i,l}j_{r,l}y_i^2 \\ - & \sum_{r=\overline{r}+1}^{4}\sum_{i=1}^{\overline{r}}\Delta_{i,l}j_{r,l}y_i^2 \\ - & \sum_{r=2}^{\overline{r}}\sum_{i=1}^{r-1}\Delta_{r,l}j_{i,l}y_i^2 \end{array}\right\}$$

Note that the sign of the variance in child realizations is dependent on the degree of stochastic dominance shift (which in turn is dependent on the targeted distribution of child realizations, $\mathbb{c}^I$), the initial distribution, and the relative difference between the realizations of the outcome realizations. Intuitively, from a high level of dependence or immobility, a shift towards independence should yield an increase in variance.

To see the implications of the above discussion, consider the following example; suppose the pre and post qualified equal opportunity policy child-adult joint densities are $\mathbb{J}^0$ and $\mathbb{J}^1$ respectively, and were given by:

$$\mathbb{J}^0 = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix} \qquad \mathbb{J}^1 = \begin{bmatrix} 0.125 & 0 & 0 & 0 \\ 0.125 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

Pre-policy child outcomes are one to one with their parents, while post-policy there has been a "convexification" of this relationship with the children of the poorest parents now receiving an average outcome of 1.5 rather than a 1. Note that the conditional variance of low categories child outcomes will be greater than the pre-policy outcome (0.25 as opposed to 0) as well as the variance of the other socioeconomic groups. However, if $\mathbb{J}^1$ were the initial distribution, and $\mathbb{J}^2$ is the post-policy distribution such that,

$$\mathbb{J}^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

Here child outcomes have been further "convexified" and the lowest child outcome has been completely eliminated, noting the fall in variance across time, while variances across all socioeconomic groups remain the same.

To restate the key insights from this simple model; Firstly, mobility for the children of a particular socioeconomic group is achieved if and only if there is a possibility for a stochastic dominant shift. Secondly, under a qualified mobility policy the parent-child outcome relationship is "convexified". Thirdly, there would be an asymmetric change in the variance by socioeconomic group, so that empirical examination of mobility has to account for heteroskedasticity. For instance based on the previous example, beginning from a high level of immobility or dependence in the parent-child income relationship, we would expect variance to be falling by socioeconomic status, since the higher socioeconomic groups would not be affected by the qualified mobility program and experience no change in variance, while families in the lower socioeconomic groups would see an increase in variance. Finally, the change in variance among socioeconomic groups across time is dependent on the degree of pre-policy immobility[6].

3 Measuring Conditional or Qualified Mobility

3.1 Induced Hypotheses

Intergenerational mobility has often been examined via the regression coefficient (β) of a child's characteristic when adult (y) on the corresponding parental characteristic (x) (Solon 1992).

$$y = \alpha + \beta x + \gamma x^2 + \epsilon$$

where ϵ is the population error term (and $\gamma = 0$ for now). In effect that literature interpreted β as a mobility index, building upon Becker and Tomes (1979) to create a rich class of models highlighting the forces that determined the value of β, where it inferred *mobility* (*equal opportunity*) as $\beta \to 0$ and *immobility* (*unequal opportunity*) as $\beta \to 1$. Since Atkinson (1983) there has been interest in the nonlinearity of generational income elasticity ($\gamma < 0$) or asymmetry of mobility[7], largely stimulated by Becker and Tomes's (1986) conjecture that parent-child outcome relationships are concave due to asymmetries in borrowing constraints. Presumably theories of diminishing returns to human capital transfer and regression to the

[6] In the case of continuous variates, the problem of induced heteroskedasticity may be addressed by quantile regressions (See Grawe (2004a), Grawe (2004b)) without directly modelling heteroskedasticity, however in the case where the outcome variable is broadly categorized into socioeconomic groups the problem is likely to still exist.

[7] Behrman and Taubman (1990), Solon (1992), Mulligan (1999), Corak and Heisz (1999), Couch and Lillard (2004), Grawe (2004c) and Bratsberg et al. (2007) all being examples.

mean would also produce a similar conjecture. However here it is suggested that, whatever the initial generational regression relationship, a qualified equal opportunity program would reduce concavity and increase the extent to which conditional error heteroskedasticity of the child outcome is negatively related to adult income. It should be noted that an unqualified equal opportunity program in our model suggests that the stochastic errors associated with child outcomes would be homoskedastic with respect to socioeconomic status or parental outcome.

Thus the notion of a *Qualified Equal Opportunity* program induces several testable hypotheses. Firstly, it induces a positive dependence of the child characteristic on the parental characteristic since a qualified mobility policy would leave children of higher socioeconomic status families at the status quo (immobility) while improving the mobility for children of lower socioeconomic status families (mobility). Secondly, as a result of the qualified mobility policy, the effect of income/socioeconomic status would become less concave, implying a fall in $|\gamma|$. Finally, as noted in the previous section, depending on the initial conditional variance of child outcomes with respect to adult, there would be a change in conditional variance across socioeconomic groups and time, generally towards increased negative dependence.

These hypothesized changes maybe illustrated by considering the same example as in the previous section with $\mathbb{J}$ and $\mathbb{J}^*$ as the pre- and post-policy dependence structure respectively. In the pre-policy state, the generational regression has a coefficient of one (complete immobility) and a conditional variance of 0. The move toward independence between parent and child outcomes among low socioeconomic status families induces a reduction in concavity in the generational regression, flattening out the regression line (as observed in the results in Bratsberg et al. (2007)). In addition, for the low socioeconomic status group, the conditional variance of child outcomes will be greater than the pre-policy outcome (0.25 as opposed to 0) and greater than at higher socioeconomic groups where greater dependence between parent-child outcomes will be exhibited. This suggests that, in addition to a falling β, the residuals in a generational mobility regression may exhibit increased negative dependence on the parental status variable, if initial dependence is high, as a consequence of a qualified mobility policy.

3.2 Alternative Approach to Examining Intergenerational Mobility

Mobility interpretations of β are to some extent limited by its connection to the linear correlation coefficient ρ_{yx} ($\beta = \rho_{yx}(\frac{\sigma_y}{\sigma_x})$), and that statistic's ability to reflect general dependency. They are further circumscribed by the degree to which the parent-child outcome relationship is homogeneously linear across all strata of the outcome in question. Alternatively the *transition matrix*, $\mathbb{T}$, between the common quantiles of the marginal density vectors $\mathbb{p}$ and $\mathbb{c}$ can be more informative as to the nature of the dependence when it is non-linear. This has given rise to the application of techniques derived from Markov Chain processes and the development of mobility indices, some based upon the nature of the transition matrix directly, some based upon other concepts[8], but all of them reflecting to varying degrees the extent to which the underlying variables, x and y, are independent. With complete mobility the columns of the *transition matrix* would be identical (corresponding to independence between parent and child outcomes) while with complete immobility the leading diagonal would have as its elements 1.

For the present discussion, assume that the realizations are continuous and let $x \in X = [\underline{x}, \overline{x}] \subset \{0\} + \mathbb{R}^+$ and $y \in Y = [\underline{y}, \overline{y}] \subset \{0\} + \mathbb{R}^+$. Let $j(x, y)$ be the joint density function of the parent-child realization, and let $p(x)$ and $c(y)$ be the marginal density functions of the realizations for parent and child respectively. Following Anderson, Ge and Leo (2009), the degree of mobility is assessed via the joint distribution of x and y (namely $j(x, y)$) since such an approach is amenable to evaluating mobility conditional on particular ranges of parental outcome x (in other words socioeconomic group(s) of interest). The approach is based on the notion that if x and y are independent for a particular range of x and y, say $a_x < x < b_x$ and $a_y < y < b_y$ then:

$$\int_{a_y}^{b_y}\int_{a_x}^{b_x} j(x,y)dxdy - \int_{a_x}^{b_x} p(x)dx \int_{a_y}^{b_y} c(y)dy = 0 \tag{7}$$

This relation provides the basis of the contingency table test which examines whether or not $\Pr(a_x < x < b_x, a_y < y < b_y) = \Pr(a_x < x < b_x)\Pr(a_y < y < b_y)$ for the set of intervals $\{(\mathbb{a}_x, \mathbb{b}_x) \in X\}$ and $\{(\mathbb{a}_y, \mathbb{b}_y) \in Y\}$, where $\mathbb{a}_x$ and $\mathbb{a}_y$ are vectors of lower integral limits, and

[8] Bartholemew (1982), Blanden et al. (2004), Chakravarty (1995), Dearden et al. (1997), Hart (1983), Maasoumi (1986), Maasoumi (1986), Prais (1955), Shorrocks (1978), have all produced mobility indices many of which are discussed in Maasoumi (1996).

$\mathbb{b}_x$ and $\mathbb{b}_y$ are vectors of upper integral limits for x and y respectively, and they delineate mutually exclusive and exhaustive intervals in X and Y respectively.

An overall mobility index (Anderson et al. 2009) may be constructed from a sum of the terms

$$\min\left\{\int_{a_y}^{b_y}\int_{a_x}^{b_x} j(x,y)dxdy, \int_{a_x}^{b_x} p(x)dx \int_{a_y}^{b_y} c(y)dy\right\} \tag{8}$$

over the collections of intervals. This index is a measure of the extent to which the empirical joint density and the joint density implied by independence, overlap or coincide. The index has a support of $[0,1]$, where 1 indicates complete independence (mobility), with lower values indicating relative dependence (immobility). Further, the value of the statistic is asymptotically normally distributed (Anderson et al. 2009), consequently permitting simple statistical comparison of mobility states.

Note that condition (7) could be equally well written as

$$\frac{\int_{a_y}^{b_y}\int_{a_x}^{b_x} j(x,y)dxdy}{\int_{a_x}^{b_x} p(x)dx} - \int_{a_y}^{b_y} c(y)dy = 0 \tag{9}$$

This relation asks if the conditional probability of a child's outcome given its parent's outcome is equal to the marginal probability of the child's outcome. Conditional or qualified mobility may be examined by considering the sum of terms of the form:

$$\min\left\{\frac{\int_{a_y}^{b_y}\int_{a_x}^{b_x} j(x,y)dxdy}{\int_{a_x}^{b_x} p(x)dx}, \int_{a_y}^{b_y} c(y)dy\right\} \tag{10}$$

In this case the sum is taken over $(\mathbb{a}_y, \mathbb{b}_y)$ that exhaust the range of y. Such a statistic measures the proximity of the conditional distribution to its corresponding marginal distribution where the conditioning region is the range of the parental characteristic of interest. It has the same numeric and statistical properties as the overall mobility statistic outlined above and is more informative in the sense that mobility conditional upon a particular inherited circumstance can be examined.

Finally, note that these techniques can be easily generalized to examine questions involving more than 2 variables. For example, suppose the researcher wishes to examine how parental education (w) and income (x) is transmitted to their child's education(z) and income(y). Let $(\mathbb{a}_i, \mathbb{b}_i)$ where $i \in \{w, x, y, z\}$, be the interval under consideration, then a similar hypothesis of independence can be examined for,

$$\int_{a_w}^{b_w}\int_{a_x}^{b_x}\int_{a_y}^{b_y}\int_{a_z}^{b_z} j(w,x,y,z)dwdxdydz = \int_{a_w}^{b_w}\int_{a_x}^{b_x} p(w,x)dwdx \int_{a_y}^{b_y}\int_{a_z}^{b_z} c(y,z)dydz$$

(see Anderson et al. (2009)).

4 An Example: Narrowing the Educational Gender Gap in Canada

One profound change in the latter part of the 20$^{\text{th}}$ century was the emancipation of women and the declining significance of gender in labour and consequently educational outcome (Blau et al. 2006). The introduction of the pill, abortion rights and legislation against gender discrimination in the workplace improved the wellbeing and status of women in those years (Pezzini 2002, Goldin and Katz 2002, Siow 2002). One dimension in which this found expression is in the narrowing gap in academic achievement of men and women (Dynarski 2007). To study this phenomenon in light of the hypothesized qualified mobility mandate, the educational achievements of successive cohorts of Canadian individuals and their parents are compared. Relating to our previous discussion, the educational outcome of children here is the variable y, while that of the parent's is x. A priori under a qualified mobility policy, we should see an improvement in mobility of children of lower socioeconomic status families regardless of gender. Further, the circumstances favouring women implies that the gains to them over the years should also be greater than it was for men.

4.1 Summary of Data

The data on academic achievements of children and their parents in Canada are drawn from Statistics Canada's *General Social Survey Cycle 19* (2005). Table 1 outlines the attainment index which associates integers 1 through 5 with the highest academic achievements of individuals aged 25 and above and their parents in 2005.

Table 2 summarizes the proportion of individuals in each educational attainment category and the corresponding proportion of observations with their parents in those categories by the

Table 1: Attainment Definition

Index/Year	2005
1	Some Secondary or Elementary or No Education
2	High School Diploma
3	Some University
4	Trade or Technical Diploma or Certificate
5	Bachelors or Masters or Doctorate Degree

individual's gender and cohort (decade in which they were born). Note that for individuals born in the 1940s and earlier, the upper attainment levels are dominated by males, but this changes in favour of females in later cohorts, corresponding with the increased female labour force participation in the post World War II decades.

Table 3 presents a comparison of male and female academic attainment distributions across the cohorts, highlighting the turnaround in the academic achievements of males and females over time. Interpreting the continuous child outcome y from our previous discussion as education attainment, we can then denote $A(y)$ as the monotonically increasing educational attainment index function. Let the distribution function of attainment for males and females be $C_m(y)$ and $C_f(y)$ respectively. Then a necessary and sufficient condition for $E[A(y)]$ to be greater for males than females is $C_m(y) \leq C_f(y)$ for all y, the first order dominance criterion. For the cohorts born before 1940, we see three significantly negative differences at 5% level of significance and no significantly positive differences revealing that male attainment outcomes first order stochastically dominate that of their female counterparts. However tracking upwards in table 3, it is clear that the attainment gap narrowed across the cohorts, since the difference between male and female distributions disappeared by the 1950s, and in fact the trend of the pre-1940 years were completely reversed by the 1970s (Noting that there were three significant positive differences for the 1970s cohort).

4.2 The Generational Regression Approach

In analyzing educational mobility in the context of generational regressions, the model considered is of the form:

$$y_{i,k} = \alpha_k + \beta_{1,k} x_{i,k} + \beta_{2,k} x_{i,k}^2 + \epsilon_{i,k} \tag{11}$$

Table 2: Summary Statistics by Gender and Cohort

Decade	Gender	No. of Obs.	Variable	Dropout	High School	Some College	Technical Education	University
70s	Male	895	Own	0.060335	0.15531	0.16425	0.31173	0.30838
			Father's	0.28492	0.3095	0.059218	0.1162	0.23017
			Mother's	0.20447	0.40894	0.056983	0.13184	0.19777
	Female	1187	Own	0.04802	0.11542	0.14575	0.32266	0.36816
			Father's	0.32098	0.27548	0.068239	0.14827	0.18703
			Mother's	0.27885	0.34709	0.073294	0.16428	0.13648
60s	Male	1039	Own	0.081809	0.14918	0.12801	0.35226	0.28874
			Father's	0.43503	0.30318	0.032724	0.07026	0.15881
			Mother's	0.38499	0.36959	0.029836	0.087584	0.12801
	Female	1340	Own	0.052985	0.14179	0.15149	0.34104	0.31269
			Father's	0.4791	0.24776	0.050746	0.08209	0.1403
			Mother's	0.43358	0.30522	0.049254	0.10672	0.10522
50s	Male	995	Own	0.1206	0.17286	0.15075	0.26533	0.29045
			Father's	0.60905	0.19397	0.036181	0.044221	0.11658
			Mother's	0.50151	0.33166	0.030151	0.056281	0.080402
	Female	1201	Own	0.076603	0.18068	0.14488	0.32889	0.26894
			Father's	0.58701	0.22315	0.037469	0.045795	0.10658
			Mother's	0.54621	0.26978	0.036636	0.079933	0.067444
40s	Male	659	Own	0.13505	0.1563	0.13809	0.23672	0.33384
			Father's	0.6434	0.22003	0.028832	0.028832	0.078907
			Mother's	0.58574	0.26859	0.021244	0.054628	0.069803
	Female	884	Own	0.15271	0.18439	0.1267	0.28959	0.24661
			Father's	0.67647	0.17308	0.030543	0.041855	0.078054
			Mother's	0.65271	0.20023	0.024887	0.062217	0.059955
$\leq$ 30s	Male	569	Own	0.32689	0.15641	0.11775	0.14587	0.25308
			Father's	0.73111	0.15114	0.040422	0.02109	0.056239
			Mother's	0.68366	0.19156	0.031634	0.045694	0.047452
	Female	887	Own	0.34724	0.18602	0.1195	0.20068	0.14656
			Father's	0.7283	0.14431	0.027057	0.036077	0.064262
			Mother's	0.71477	0.16234	0.020293	0.049605	0.052988

Table 3: Males vs. Females Cumulative Densities and First Order Dominance Results

Decade	Gender	Statistic	Dropout	High School	Some University	Technical Education	University
	Male	CDF	0.08452	0.24911	0.41637	0.74110	1.00000
	Female	CDF	0.07226	0.20109	0.34833	0.67962	1.00000
70s		Difference	0.01226	0.04802	0.06804	0.06148	
		σ	0.01070	0.01661	0.019260	0.01786	
		P(Z≤z)	0.87409	0.99808	0.99979	0.99971	
	Male	CDF	0.12700	0.29028	0.42598	0.75980	1.00000
	Female	CDF	0.07988	0.24310	0.39942	0.73793	1.00000
60s		Difference	0.04711	0.04717	0.02655	0.02186	
		σ	0.01108	0.01598	0.01776	0.01561	
		P(Z≤z)	0.99999	0.99843	0.93260	0.91939	
	Male	CDF	0.15589	0.35201	0.50144	0.76940	1.00000
	Female	CDF	0.13067	0.33454	0.46521	0.78523	1.00000
50s		Difference	0.02522	0.01747	0.03623	-0.01583	
		σ	0.01278	0.01728	0.01817	0.01515	
		P(Z≤z)	0.97580	0.84401	0.97693	0.14802	
	Male	CDF	0.25280	0.41698	0.53265	0.76772	1.00000
	Female	CDF	0.22996	0.42621	0.54231	0.82171	1.00000
40s		Difference	0.02284	-0.00923	-0.00966	-0.05399	
		σ	0.01757	0.02025	0.02045	0.01662	
		P(Z≤z)	0.90314	0.32425	0.31832	0.00058	
	Male	CDF	0.44424	0.58003	0.68525	0.83723	1.00000
	Female	CDF	0.44261	0.61818	0.72443	0.90454	1.00000
≤ 30s		Difference	0.00163	-0.03815	-0.03918	-0.06731	
		σ	0.01903	0.01879	0.01753	0.01310	
		P(Z≤z)	0.53413	0.02118	0.01272	0.00000	

where $E(\epsilon_{i,k}) = 0$ and $E(\ln \epsilon_{i,k}^2) = \gamma + \phi x_{i,k}$ where $i = \{1, 2, ..., n_k\}$, $k = \{male, female\}$. As before y corresponds to the child and x the parent's outcome (in terms of educational attainment) and heteroskedasticity is modeled in terms of the log squared error being a linear function of parental attainment. The results are reported in tables 4 and 5. At the outset it should be noted that the generational transfer technology appears to be concave i.e. it appears to exhibit diminishing returns to parental ability.

Examining the coefficients of the regression for all the female cohorts from table 4, note first that for females both maternal and paternal effects are highest for cohorts born in the 1940s, but gradually declining with each cohort. Table 5 reports the same results for males and exhibits a similar pattern of falling effect due to parental educational attainment, all of which are evidence of increased educational mobility within both genders. Further, examining the coefficient for heteroskedasticity for each gender in turn, note that all the coefficients are all negative and statistically significant, affirming the prediction of the model that variances should be decreasing across socioeconomic groups (in terms of parental education attainment). In addition, for both child genders, the maternal effect was stronger, and heteroskedasticity seem to be greatest among the 1950s, post World War II cohorts, reflecting the dependence of changes in heteroskedasticity on prior levels of mobility or dependence.

Table 4: Mobility OLS and an Examination of Heteroskedasticty by Cohort, Female Children

	Father	Mother	Father	Mother	Father	Mother	Father	Mother	Father	Mother
	1970s Cohort		1960s Cohort		1950s Cohort		1940s Cohort		$\leq$ 1930s Cohort	
Intercept	3.0850	3.0323	3.0178	2.9444	2.7769	2.5677	2.4334	2.3333	2.0438	2.0559
	(36.3055)	(32.0526)	(38.4946)	(35.4313)	(31.9278)	(29.0950)	(26.0298)	(23.7804)	(26.5908)	(26.3233)
Parent's	0.4909	0.4864	0.4273	0.5022	0.5426	0.7881	0.8104	0.8630	0.5386	0.5127
Education	(7.3014)	(6.7228)	(6.6026)	(7.5242)	(7.6256)	(10.9567)	(9.7828)	(10.0323)	(7.4610)	(7.0241)
(Parent's	-0.0479	-0.0421	-0.0337	-0.0500	-0.0491	-0.0953	-0.0954	-0.1037	-0.0468	-0.0458
Education)^2	(-3.7857)	(-3.1192)	(-2.7056)	(-3.8448)	(-3.4767)	(-6.4503)	(-5.4720)	(-5.7141)	(-2.9196)	(-2.7774)
	Heteroskedasticity									
Parent's	-0.1397	-0.1480	-0.1656	-0.2663	-0.2346	-0.3613	-0.1692	-0.2036	0.1112	0.1329
Education	(-4.8040)	(-4.3470)	(-5.6973)	(-7.5670)	(-8.7535)	(-11.0595)	(-4.1489)	(-4.8055)	(3.4174)	(3.8232)
R^2	0.1382	0.1338	0.1114	0.102	0.1263	0.1537	0.1491	0.1539	0.1081	0.0964
$\overline{R}^2$	0.1317	0.1273	0.1057	0.0963	0.1204	0.1481	0.142	0.1469	0.1025	0.0907
σ^2	1.3402	1.347	1.4123	1.4271	1.606	1.5556	1.8101	1.7999	1.8248	1.8489
No. of Obs.s	1467	1467	1740	1740	1653	1653	1335	1335	1760	1760

t-statistics are in parentheses.

Nine Provincial Indicators were included in each main regression.

Table 5: Mobility OLS and an Examination of Heteroskedasticty by Cohort, Male Children

	Father	Mother	Father	Mother	Father	Mother	Father	Mother	Father	Mother
	1970s Cohort		1960s Cohort		1950s Cohort		1940s Cohort		≤ 1930s Cohort	
Intercept	2.9950	3.0173	2.8551	2.8272	2.7213	2.6538	2.3847	2.5445	2.0318	2.1087
	(29.0205)	(27.8419)	(30.5550)	(30.5137)	(27.7411)	(27.9917)	(20.7241)	(22.0155)	(19.7722)	(20.5475)
Parent's	0.2941	0.3470	0.5143	0.5847	0.5266	0.6409	1.0118	0.8339	0.6733	0.7182
Education	(3.7405)	(4.3857)	(6.8010)	(7.9760)	(6.3012)	(8.3503)	(10.5623)	(8.8184)	(6.9507)	(7.3131)
(Parent's	-0.0057	-0.0217	-0.0415	-0.0577	-0.0505	-0.0720	-0.1360	-0.1125	-0.0491	-0.0815
Education)^2	(-0.3932)	(-1.4983)	(-2.8964)	(-4.0500)	(-3.0845)	(-4.5121)	(-6.7923)	(-5.4738)	(-2.2273)	(-3.5751)
	Heteroskedasticity									
Parent's	-0.1505	-0.1907	-0.2409	-0.2562	-0.1764	-0.2045	-0.0376	-0.1068	0.0979	0.1676
Education	(-3.9561)	(-4.9216)	(-8.6583)	(-8.2600)	(-4.5017)	(-5.8717)	(-0.8183)	(-2.6851)	(2.0506)	(3.6398)
R^2	0.1446	0.109	0.1479	0.1407	0.0959	0.1097	0.1676	0.1324	0.1342	0.1096
$\overline{R}^2$	0.1361	0.1002	0.141	0.1338	0.0887	0.1026	0.1589	0.1234	0.1255	0.1007
σ^2	1.3869	1.4446	1.5544	1.5676	1.7885	1.7611	1.9688	2.0519	2.1154	2.1755
No. of Obs.s	1124	1124	1378	1378	1392	1392	1072	1072	1112	1112

t-statistics are in parentheses.

Nine Provincial Indicators were included in each main regression.

Table 6: Standard Normal Tests for Concavity Changes Across Cohort

	Male		Female	
	Father	Mother	Father	Mother
70s-60s	-1.7596	-1.7690	0.8019	-0.4194
	(0.0392)	(0.0384)	(0.7887)	(0.3375)
70s-50s	-2.0526	-2.3302	-0.0629	-2.6567
	(0.0201)	(0.0099)	(0.4749)	(0.0039)
70s-40s	-5.277	-3.6077	-2.2034	-2.7229
	(0.0000)	(0.0002)	(0.0138)	(0.0032)
70s-30s	-1.6470	-2.2113	0.0533	-0.1705
	(0.0498)	(0.0135)	(0.5213)	(0.4323)
60s-50s	-0.4150	-0.6674	-0.8194	-2.303
	(0.3391)	(0.2523)	(0.2063)	(0.0107)
60s-40s	-3.8384	-2.1906	-2.8805	-2.4070
	(0.0000)	(0.0142)	(0.0020)	(0.0080)
60s-30s	-0.2893	-0.8844	-0.6475	0.2015
	(0.3862)	(0.1883)	(0.2586)	(0.5796)
50s-40s	-3.3039	-1.5570	-2.0624	-0.3601
	(0.0005)	(0.0597)	(0.0196)	(0.3593)
50s-30s	0.0520	-0.3413	0.1068	2.2389
	(0.5207)	(0.3664)	(0.5425)	(0.9874)
40s-30s	2.9179	1.0106	2.0498	2.3649
	(0.9982)	(0.8439)	(0.9798)	(0.9910)

p-values in parenthesis.

Tables 6 and 7 tests the "convexification" and heteroskedasticity comparisons across the five cohorts respectively. Through the five cohorts, there seem to have been a significant decline in concavity of the "production function", somewhat more pronounced for males than females. For males born in the 1970s, the quadratic term was in fact not significant in terms of transmission from both fathers and mothers. Concerning the heteroskedasticity parameter, it appears to have become substantially more negative when the comparison is made between the female cohort born in the 1950s against earlier cohorts. The patterns of declining heteroskedasticity is likewise noted for males throughout the cohorts from the earliest year to the cohort born in the 1960s. Taken together, the above findings highlight the increase in mobility across the decades for both genders, emphasizing the primary point made by the qualified mobility program hypothesis that it will not impinge on the progress or lack of mobility for the well endowed (here the well endowed being male children).

Table 7: Standard Normal Tests for Heteroskedasticity Changes Aross Cohort

	Male		Female	
	Father	Mother	Father	Mother
60s-70s	-1.9192	-1.3191	-0.6309	-2.4152
	(0.0275)	(0.0936)	(0.2641)	(0.0079)
50s-70s	-0.4737	-0.2638	-2.3998	-4.5190
	(0.3179)	(0.3960)	(0.0082)	(0.0000)
50s-60s	1.3441	1.1094	-1.7440	-1.9779
	(0.9105)	(0.8664)	(0.0406)	(0.0240)
40s-70s	1.8913	1.5123	-0.5892	-1.0226
	(0.9707)	(0.9348)	(0.2778)	(0.1532)
40s-60s	3.7831	2.9635	-0.0713	1.1380
	(0.9999)	(0.9985)	(0.4716)	(0.8724)
40s-50s	2.2967	1.8487	1.3398	2.9467
	(0.9892)	(0.9677)	(0.9099)	(0.9984)
30s-70s	4.0688	5.9538	5.7493	5.7732
	(1.000)	(1.0000)	(1.0000)	(1.0000)
30s-60s	6.1310	7.6332	6.3443	8.0706
	(1.0000)	(1.0000)	(1.0000)	(1.0000)
30s-50s	4.4406	6.4447	8.2027	10.3603
	(1.0000)	(1.0000)	(1.0000)	(1.0000)
30s-40s	2.0448	4.5098	5.3745	6.1401
	(0.9796)	(1.0000)	(1.0000)	(1.0000)

p-values are in parenthesis

4.3 The Overlap Measure

The "Qualified Equal Opportunity" hypothesis suggests that the conditional density of child attainment for lower socioeconomic groups should be a closer match to the marginal density of child attainment relative to the children from higher socioeconomic status groups since a qualified mobility policy would leave the latter group largely untouched. Section 3.2 provides a test that could easily be performed, which intuitively measures the degree of overlap between two densities. Specifically, the discrete realization analog of the measures in (10) is

$$\min\left\{\frac{j_{i,k}}{p_k}, c_i\right\} \tag{12}$$

The Overlap measure between the conditional density and the marginal density for each parental attainment (socioeconomic group) is then

$$\sum_{i=1}^{m} \min\left\{\frac{j_{i,k}}{p_k}, c_i\right\} \tag{13}$$

for each $k \in \{1, 2, ..., n\}$. If child outcomes and parental circumstances are independent, the Overlap measure will record values close to 1. To the extent that they are not independent the statistic will record a value less than 1. The results of this measure for each parental attainment outcome by gender of the children are reported in table 8. Since the measure is asymptotically normal (Anderson et al. 2009), we can examine how the measure differs across each cohort (reported in table 9), parental attainment groups which we use as a proxy for socioeconomic group status (reported in table 10), and across gender of the children (reported in table 11). Tables 9 to 11 then essentially detail the direction and evolution, and the statistical significance of the changes.

From table 8, according with expectations, note the strong tendency of the Overlap measure to move towards 1 among children of parents with High School education to Technical training for both genders. This pattern is strongest when comparisons are made between the cohorts born in the 1960s and 1970s against the earlier cohorts and it is stronger (in terms of the change in the Overlap measure) among females. This pattern is not mimicked by children with parents with University education and particularly parents who did not complete their education. The former accords with our Qualified Mobility Policy conjecture, since a high dependence between parent-child outcomes in the status quo would render these children outside the sphere of influence of this policy. All measures are significantly different from 1 suggesting that a pure equal opportunity imperative has not been pursued or achieved. Finally, note that maternal effects were greater than paternal for both genders.

The drive toward higher mobility can be examined by comparing cohorts *within a particular parental attainment class*, with successful policies rendering statistically significantly higher mobility measures with successively younger cohorts. However, from the perspective of the qualified equal opportunity program, the comparison should be between particular parental attainment groups *within a particular cohort* where such programs would result in statistically significantly lower mobility coefficients in higher attainment groups. These comparisons are reported in Tables 9 and 10 respectively, which look specifically at daughters of mothers and sons of fathers comparisons[9].

[9]The other comparisons did not differ in substance from these and have been omitted for space reasons

Table 8: Qualified Mobility

	Mother's Attainment					Father's Attainment				
	Female Child									
	Drop Out	High School	Some College	Technical Education	University	Drop Out	High School	Some College	Technical Education	University
1970s Cohort	0.8437	0.9871	0.8183	0.8933	0.7393	0.8364	0.9680	0.8793	0.9021	0.7859
	(0.0189)	(0.0053)	(0.0400)	(0.0212)	(0.0338)	(0.0188)	(0.0096)	(0.0349)	(0.0221)	(0.0270)
1960s Cohort	0.9112	0.9733	0.8294	0.8703	0.7606	0.9141	0.9380	0.8246	0.8073	0.7128
	(0.0112)	(0.0077)	(0.0437)	(0.0272)	(0.0350)	(0.0108)	(0.0130)	(0.0451)	(0.0363)	(0.0324)
1950s Cohort	0.9223	0.9271	0.6800	0.8411	0.7888	0.9267	0.9232	0.8615	0.8670	0.6419
	(0.0099)	(0.0138)	(0.0695)	(0.0352)	(0.0440)	(0.0095)	(0.0159)	(0.0509)	(0.0435)	(0.0419)
1940s Cohort	0.9200	0.8855	0.8562	0.7690	0.7390	0.9310	0.8579	0.8362	0.7283	0.6447
	(0.0105)	(0.0229)	(0.0675)	(0.0523)	(0.0572)	(0.0101)	(0.0276)	(0.0644)	(0.0695)	(0.0568)
$\leq$ 1930s Cohort	0.924	0.7779	0.7282	0.7176	0.6872	0.9219	0.7658	0.8620	0.7679	0.7337
	(0.0098)	(0.0331)	(0.0928)	(0.0596)	(0.0609)	(0.0101)	(0.0365)	(0.0610)	(0.0651)	(0.0532)
	Male Child									
	Drop Out	High School	Some College	Technical Education	University	Drop Out	High School	Some College	Technical Education	University
1970s Cohort	0.8820	0.9417	0.8568	0.9366	0.7827	0.84353	0.9389	0.8730	0.8537	0.7347
	(0.0232)	(0.0119)	(0.0460)	(0.0215)	(0.0303)	(0.02219)	(0.0140)	(0.04336)	(0.0332)	(0.0303)
1960s Cohort	0.9027	0.9578	0.8914	0.8341	0.7611	0.8675	0.9222	0.8524	0.7882	0.7126
	(0.0143)	(0.0100)	(0.0519)	(0.0382)	(0.0364)	(0.0152)	(0.0148)	(0.0575)	(0.0460)	(0.0344)
1950s Cohort	0.9168	0.9611	0.8318	0.7933	0.7441	0.9191	0.9400	0.8881	0.8859	0.6700
	(0.0120)	(0.0103)	(0.0661)	(0.0527)	(0.0485)	(0.0106)	(0.0166)	(0.0492)	(0.0425)	(0.0427)
1940s Cohort	0.9267	0.8973	0.8470	0.8255	0.8429	0.9116	0.8344	0.8213	0.8754	0.7127
	(0.0128)	(0.0220)	(0.0873)	(0.0624)	(0.0525)	(0.0128)	(0.0297)	(0.0817)	(0.0721)	(0.0594)
$\leq$ 1930s Cohort	0.9227	0.8319	0.7096	0.6476	0.7245	0.9129	0.7965	0.7295	0.6192	0.6204
	(0.0131)	(0.0350)	(0.107)	(0.0858)	(0.07777)	(0.0130)	(0.0407)	(0.0825)	(0.1178)	(0.0798)

Note: Standard Errors are in Parenthesis.

Table 9: Standard Normal Tests of Qualified Mobility Differences Across Cohorts

	Mother's Attainment (Female Child)					Father's Attainment (Male Child)				
	Drop Out	High School	Some College	Technical Education	University	Drop Out	High School	Some College	Technical Education	University
60s-70s	3.07346	-1.46651	0.186821	-0.66926	0.43799	0.892696	-0.81934	-0.28536	-1.15459	-0.48282
	(0.99894)	(0.071254)	(0.574099)	(0.25166)	(0.66930)	(0.81399)	(0.20629)	(0.38768)	(0.12413)	(0.3146)
50s-70s	3.6876	-4.0440	-1.7245	-1.27288	0.89222	3.0724	0.051138	0.23078	0.59709	-1.2357063
	(0.99989)	(0.000026)	(0.042312)	(0.10153)	(0.81386)	(0.99894)	(0.52039)	(0.591262)	(0.72478)	(0.10828)
50s-60s	0.74861	-2.9152	-1.8187	-0.65721	0.50186	2.7852	0.80071	0.47141	1.5610	-0.77615
	(0.77295)	(0.0017774)	(0.034479)	(0.25552)	(0.69212)	(0.99733)	(0.78835)	(0.68132)	(0.94074)	(0.21883)
40s-70s	3.5244	-4.3165	0.48225	-2.2051	-0.0033131	2.6565	-3.1857	-0.55874	0.27315	-0.33071
	(0.99979)	(0.0000079)	(0.68518)	(0.013722)	(0.49868)	(0.99605)	(0.00072)	(0.28817)	(0.60763)	(0.37043)
40s-60s	0.57025	-3.631	0.33284	-1.7198	-0.32097	2.219	-2.6478	-0.31136	1.0198	0.0010195
	(0.71574)	(0.000141)	(0.63037)	(0.042735)	(0.37412)	(0.98676)	(0.0040511)	(0.37776)	(0.84610)	(0.50041)
40s-50s	-0.16745	-1.5551	1.8174	-1.1444	-0.68908	-0.45141	-3.1071	-0.70052	-0.12586	0.58282
	(0.43351)	(0.059965)	(0.96542)	(0.12624)	(0.24539)	(0.32585)	(0.00094469)	(0.24180)	(0.44992)	(0.71999)
30s-70s	3.7794	-6.2454	-0.89167	-2.7772	-0.74821	2.6969	-3.3092	-1.5397	-1.9165	-1.3396
	(0.99992)	(0.000000)	(0.18629)	(0.0027412)	(0.22717)	(0.99650)	(0.00047)	(0.061815)	(0.027649)	(0.090184)
30s-60s	0.86495	-5.7553	-0.98622	-2.33000	-1.0452	2.2702	-2.9024	-1.2222	-1.3370	-1.0610
	(0.80647)	(0.000000)	(0.16201)	(0.00990)	(0.14796)	(0.98840)	(0.0018514)	(0.11081)	(0.090609)	(0.14435)
30s-50s	0.11757	-4.1637	0.41627	-1.7832	-1.3523	-0.37354	-3.2657	-1.6512	-2.1305	-0.54789
	(0.5468)	(0.00002)	(0.66139)	(0.037276)	(0.088143)	(0.35437)	(0.00054593)	(0.049346)	(0.016563)	(0.2919)
30s-40s	0.28275	-2.6749	-1.1148	-0.64743	-0.62106	0.06857	-0.75110	-0.79093	-1.8556	-0.92736
	(0.61132)	(0.0037381)	(0.13246)	(0.25868)	(0.26728)	(0.52733)	(0.22629)	(0.21449)	(0.031754)	(0.17687)

Note: $\Pr(Z \leq z)$ in Parenthesis.

From Table 9, observe that excepting “Drop Out” parents, all of the significant changes across cohorts are increasing mobility changes, predominantly among children with “High School” parents (and then more so with females than males as adjudged from table 10). There are a few significant increases among the daughters of “Technical Education” parents but no significant mobility changes across cohorts in the children (of either gender) of University Graduates, all of which is consistent with a Qualified Mobility program. What is at odds with the Qualified Mobility scenario is the significant reductions in mobility experienced by the younger cohorts in the “Drop Out” parent category. This suggests a forgotten segment of the populace that public policy has neglected. In the stylized model, it has implicitly been assumed that the cost of advancing children across the distribution is the same but in all probability this is not the case. A more appropriate model would explicitly include the cost to the social planner of affecting the different cells of the density vector. Intuitively, if the cost of improving the mobility of the lowest socioeconomic group is relatively the highest, then it is those children that might be left behind. The results of Table 10, reporting the within cohort across parental attainment category comparisons, are equally supportive of a Qualified Mobility paradigm. Again excluding the “Drop Out” category, mobility is significantly higher in the lower attainment categories and is more so in the recent as compared to the older cohorts.

Finally a comparison of the qualified mobility of daughters of mothers with that of sons of fathers reported in Table 11 reveals that with one exception (among children in the 1950s cohort, with parents with some college education), all of the significant differences relate to higher mobility of daughters in more recent cohorts. Furthermore the advances have taken place among children of parents with high school education. No significant differences were identified in the $\leq$ 1930s cohort and only one significant difference was observed in the 1940s cohort at 10% level of significance. This signals the advances that females have made over males in the last half century.

Table 10: Standard Normal Tests of Qualified Mobility Differences Across Parental Attainments

	Mother's Attainment (Female Child)					Father's Attainment (Male Child)				
	1970s Cohort	1960s Cohort	1950s Cohort	1940s Cohort	$\leq$ 1930s Cohort	1970s Cohort	1960s Cohort	1950s Cohort	1940s Cohort	$\leq$ 1930s Cohort
High School	7.30220	4.57230	0.28040	-1.36540	-4.23950	3.63340	2.57580	1.05920	-2.39030	-2.72490
-Drop Out	(1.00000)	(1.00000)	(0.61040)	(0.08610)	(0.00000)	(0.99990)	(0.99500)	(0.85520)	(0.00840)	(0.00320)
Some College	-0.57380	-1.81270	-3.45100	-0.93340	-2.09900	0.60400	-0.25440	-0.61590	-1.09240	-2.19610
-Drop Out	(0.28300)	(0.03490)	(0.00030)	(0.17530)	(0.01790)	(0.72710)	(0.39960)	(0.26900)	(0.13730)	(0.01400)
Some College	-4.18370	-3.24170	-3.48600	-0.41170	-0.50430	-1.44670	-1.17420	-0.99860	-0.15040	-0.72940
-High School	(0.00000)	(0.00060)	(0.00020)	(0.34030)	(0.30700)	(0.07400)	(0.12020)	(0.15900)	(0.44020)	(0.23290)
Technical Edu.	1.75020	-1.39240	-2.22430	-2.83110	-3.41620	0.25470	-1.63860	-0.75800	-0.49470	-2.47860
-Drop Out	(0.96000)	(0.08190)	(0.01310)	(0.00230)	(0.00030)	(0.60050)	(0.05070)	(0.22420)	(0.31040)	(0.00660)
Technical Edu.	-4.29810	-3.64870	-2.27660	-2.04160	-0.88380	-2.36060	-2.77350	-1.18510	0.52660	-1.42350
-High School	(0.00000)	(0.00010)	(0.01140)	(0.02060)	(0.18840)	(0.00910)	(0.00280)	(0.11800)	(0.70070)	(0.07730)
Technical Edu.	1.65870	0.79490	2.06730	-1.02080	-0.09610	-0.35220	-0.87140	-0.03340	0.49670	-0.76700
-Some College	(0.95140)	(0.78670)	(0.98060)	(0.15370)	(0.46170)	(0.36240)	(0.19180)	(0.48670)	(0.69030)	(0.22160)
University	-2.69800	-4.10400	-2.96150	-3.11190	-3.84160	-2.90030	-4.11990	-5.65570	-3.27280	-3.61800
-Drop Out	(0.00350)	(0.00000)	(0.00150)	(0.00090)	(0.00010)	(0.00190)	(0.00000)	(0.00000)	(0.00050)	(0.00010)
University	-7.24840	-5.94300	-2.99920	-2.37790	-1.30940	-6.12460	-5.59400	-5.88760	-1.83210	-1.96680
-High School	(0.00000)	(0.00000)	(0.00140)	(0.00870)	(0.09520)	(0.00000)	(0.00000)	(0.00000)	(0.03350)	(0.02460)
University	-1.51020	-1.22920	1.32190	-1.32340	-0.37010	-2.61470	-2.08500	-3.34470	-1.07530	-0.95020
-Some College	(0.06550)	(0.10950)	(0.90690)	(0.09280)	(0.35570)	(0.00450)	(0.01850)	(0.00040)	(0.14110)	(0.17100)
University	-3.86640	-2.47880	-0.92850	-0.38630	-0.35750	-2.64730	-1.31680	-3.58260	-1.74210	0.00870
-Technical Edu.	(0.00010)	(0.00660)	(0.17660)	(0.34960)	(0.36040)	(0.00410)	(0.09390)	(0.00020)	(0.04070)	(0.50350)

Note: $\Pr(Z \leq z)$ in Parenthesis.

Table 11: Mobility Differences Daughters of Mothers - Sons of Fathers

	Parental Attainment				
	Drop Out	High School	Some College	Technical Education	University
1970s Cohort	0.0051	3.2198	-0.9266	1.0055	0.1001
	(0.5021)	(0.9994)	(0.1771)	(0.8427)	(0.5399)
1960s Cohort	2.3160	3.0624	-0.3185	1.5373	0.9775
	(0.9897)	(0.9989)	(0.3750)	(0.9379)	(0.8359)
1950s Cohort	0.2246	-0.5940	-2.4426	-0.8129	1.9354
	(0.5889)	(0.2763)	(0.0073)	(0.2081)	(0.9735)
1940s Cohort	0.5035	1.3646	0.3290	-1.1952	0.3198
	(0.6927)	(0.9138)	(0.6289)	(0.1160)	(0.6254)
$\leq$ 1930s	0.6883	-0.3558	-0.0099	0.7458	0.6651
Cohort	(0.7544)	(0.3610)	(0.4960)	(0.7721)	(0.7470)

Note: $\Pr(Z \leq z)$ are in parenthesis

5 Conclusions

It has been demonstrated that in the absence of sufficient flexibility or capacity in a society, the unqualified pursuit of an equal opportunity goal results in some people being made worse off while others are made better off. If some sort of Pareto-Utilitarian goal is also an objective of the policy maker (in effect a maintenance of the status of the well endowed) in a constant cost world, a qualified equal opportunity outcome emerges in which the most disadvantaged are addressed first. With such a program, complete independence of outcome from circumstance will not be observed across all socioeconomic groups and conventional measures of mobility will not record complete mobility. However such policies have predictable consequences for generational regressions and suggest ways that mobility measures could be re-interpreted. Qualified equal opportunity policies will induce a reduction in concavity in the prevailing generational regression relationship as well as inducing heteroskedasticity in the corresponding error process which is negatively related to the conditioning variable. Alternatively evaluating conditional mobility policies via the transition matrix or joint distribution of outcomes and circumstance requires indices which identify changes in mobility by subgroup or conditional mobility measurement.

To illustrate the concept and the associated indices, the success of various equal opportunity policies pursued either implicitly or explicitly in the emancipation of women was evaluated in terms of how they narrowed the gender gap in educational attainment in Canada.

Hypotheses relating to generational regressions that are consistent with a qualified equal opportunity program are not rejected for daughters whereas they are for sons. From the conditional mobility indices comparisons, the gender gap appears to have been narrowed by an increase in the mobility of the daughters of parents of lower educational status, without any change in the mobility of daughters or sons in the highest parental educational attainment category. All of which is what would have been expected from a Qualified Equal Opportunity or Conditional Mobility Policies.

It also appears that there is a segment of children, both males and females, of dropout parents whom society has neglected in that their mobility has diminished. It is conjectured that, contrary to what is implicitly assumed in the model here presented, the cost of improving the stead of the deprived are not the same as those associated with other better endowed segments of the populace. If those cost are significantly higher, the social planner may be less inclined to improve their mobility in the first instance.

A Appendix

A.1 The Lagrangian for the Planner's Problem

Since $\sum_{i=1}^{4} j^*_{i,k} = p_k$, we can rewrite the objective function as;

$$\min_{j^*_{i,k} \in \mathbb{J}^1, i \neq 4} \sum_{i=1}^{3} \sum_{k=1}^{4} 2 \left(j^*_{i,k} - c^I_i p_k \right)^2 + \sum_{k=1}^{4} \left\{ 2 \sum_{i=1}^{3} \left[(j^*_{i,k} - c^I_i p_k) \sum_{l=i^-} (j^*_{l,k} - c^I_l p_k) \right] \right\} \tag{A-1}$$

Similarly, the growth constraint can be simplified into

$$\sum_{i=1}^{3} (3 - y_i)\,(c_i - c^*_i) \quad \leq \quad g \tag{A-2}$$

$$\Rightarrow \sum_{i=1}^{3} (3 - y_i) \left(c_i - \sum_{k=1}^{4} j^*_{i,k} \right) \quad \leq \quad g \tag{A-3}$$

The Lagrangian in terms of the unconstrained $j_{i,k}$'s for this problem may be written as:

$$L = \left\{ \begin{array}{l} \sum\limits_{i=1}^{3} \sum\limits_{k=1}^{4} 2 \left(j^*_{i,k} - c^I_i p_k \right)^2 + \sum\limits_{k=1}^{4} \left\{ 2 \sum\limits_{i=1}^{3} \left[(j^*_{i,k} - c^I_i p_k) \sum\limits_{l=1, l \neq i}^{3} (j^*_{l,k} - c^I_l p_k) \right] \right\} \\ + \sum\limits_{l=1}^{3} \sum\limits_{k=1}^{4} \lambda_{l,k} \sum\limits_{i=1}^{l} (j^*_{i,k} - j_{i,k}) + \gamma \left[\sum\limits_{i=1}^{3} (4 - y_i) \left(c_i - \sum\limits_{k=1}^{4} j^*_{i,k} \right) - g \right] \end{array} \right\} \tag{A-4}$$

A.2 Change in Variance

The change in variance of child realizations for socioeconomic group l is

$$\begin{aligned} & p_l^{-1} \sum_{i=1}^{4} \left(j^*_{i,l} \left(y_i - \mu^*_l \right)^2 - j_{i,l} \left(y_i - \mu_l \right)^2 \right) \\ = & \left\{ \left[\sum_{i=1}^{4} \frac{j^*_{i,l}}{p_l} y_i^2 - \left(\sum_{i=1}^{4} \frac{j^*_{i,l} y_i}{p_l} \right)^2 \right] - \left[\sum_{i=1}^{4} \frac{j_{i,l}}{p_l} y_i^2 - (\mu_l)^2 \right] \right\} \\ = & p_l^{-2} \sum_{r=2}^{4} \sum_{i=1}^{r-1} \left(j^*_{r,l} j^*_{i,l} - j_{r,l} j_{i,l} \right) y_i^2 \\ = & p_l^{-2} \sum_{r=2}^{4} \sum_{i=1}^{r-1} \left([\Delta_{r,l} + j_{r,l}] \, [\Delta_{i,l} + j_{i,l}] - j_{r,l} j_{i,l} \right) y_i^2 \\ = & p_l^{-2} \sum_{r=2}^{4} \sum_{i=1}^{r-1} \left(\Delta_{r,l} \Delta_{i,l} + \Delta_{r,l} j_{i,l} + \Delta_{i,l} j_{r,l} \right) y_i^2 \end{aligned}$$

where $\Delta_{r,l} = j^*_{r,l} - j_{r,l}$. Let $\overline{r}$ be the realization at and below which the density falls or remains unchanged (without lost of generality, assume that the stochastic dominance shift induces a single intersection of the two distributions), such that $\sum_{i=1}^{\overline{r}} \Delta_{i,l} = \sum_{i=\overline{r}+1}^{4} \Delta_{i,l}$. We can then write the change in variance as,

$$\sum_{r=2}^{\overline{r}} \sum_{i=1}^{r-1} (\Delta_{r,l}\Delta_{i,l})\, y_i^2 + \sum_{r=\overline{r}+1}^{4} \sum_{i=1}^{\overline{r}} (\Delta_{r,l}\Delta_{i,l})\, y_i^2 + \sum_{r=\overline{r}+1}^{4} \sum_{i=\overline{r}+1}^{\max\{r-1,\overline{r}+1\}} (\Delta_{r,l}\Delta_{i,l})\, y_i^2$$
$$+ \sum_{r=2}^{\overline{r}} \sum_{i=1}^{r-1} \Delta_{i,l} j_{r,l} y_i^2 + \sum_{r=\overline{r}+1}^{4} \sum_{i=1}^{\overline{r}} \Delta_{i,l} j_{r,l} y_i^2 + \sum_{r=\overline{r}+1}^{4} \sum_{i=\overline{r}+1}^{\max\{r-1,\overline{r}+1\}} \Delta_{i,l} j_{r,l} y_i^2$$
$$+ \sum_{r=2}^{\overline{r}} \sum_{i=1}^{r-1} \Delta_{r,l} j_{i,l} y_i^2 + \sum_{r=\overline{r}+1}^{4} \sum_{i=1}^{r-1} \Delta_{r,l} j_{i,l} y_i^2$$

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