A Search-Theoretic Model of Bureaucracy and Corruption*

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Abstract

We analyze bureaucracy and corruption in a market with decentralized exchange and "lemons." Exchange is modelled as a sequence of bilateral, random matches. Agents have private information about the quality of goods they produce and can supplement trade with socially inefficient bribes. Bureaucracy is modelled as a group of agents similar to private agents, but who enjoy centralized production and consumption. Transaction patterns between the bureaucracy and the private sector are fully endogenous. Our main finding is that centralized production and consumption in the bureaucracy also gives rise to low power incentives for the individual bureaucrats. As a result, we find conditions under which private agents bribe bureaucrats, while they do not bribe each other. An equilibrium with corruption and an equilibrium without corruption can co-exist. We discuss some welfare implications of the model.

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1. Introduction

The seminal work of Becker (1968) has stimulated a vast literature that analyzes bureaucracy and corruption as economic phenomena (see Rose-Ackerman, 1999 for references). This literature models a bureaucracy as a provider of public goods and/or a rent-seeking organization which controls some aspects of production, distribution, or inspection of goods and services. In this paper we abstract from any such advantages that might be exogenously given to a bureaucracy. We also deviate from the literature in the specification of a bureaucracy as we do not assume that it is the sole provider of some goods and services.[1] Instead, we model a bureaucracy as a group of agents with centralized production and consumption of goods, who have access to *the same* technologies as the private sector in the production and monitoring of product quality. We then ask: (i) can bribery and corruption arise? (ii) given the possibility of corruption, can the bureaucracy improve social welfare despite its lack of any exogenous advantage?

Our analysis starts with an economy without a bureaucracy. Decentralized exchanges are modelled as a sequence of bilateral, random matches. There is a large number of infinitely-lived private agents. Each agent can produce either a high-quality storable good that yields no utility to himself/herself but positive utility to other agents, or a low-quality storable good that yields no utility to anyone. As in Akerlof (1971), a lemons problem arises since agents do not always know the quality of other agents' goods. Upon meeting a trading partner, an agent recognizes the quality of his partner's good with some probability. Agents can choose to supplement the trade of storable goods with services that cost more to produce than the utility of consuming them. We call such services *bribes*. A trade occurs only if the two agents both agree to trade the storable goods, and if they ask for a bribe that is no higher than what the partner offers. We find that, under restrictions on the size of the bribe, private agents in the economy without bureaucracy do

[1] An exogenously imposed interaction between a bureaucracy and the private sector seems to us to be conducive to corruption. In addition, an exogenous role of a bureaucracy is not always realistic. For example, a bureaucracy may provide public housing, rationed goods, or product safety certificates, but agents can also obtain close substitutes for these goods and services in the market. We believe that to understand why corruption may arise within a benevolent bureaucracy, we need a model in which both the role of the bureaucracy *and* the trading patterns between a (possibly corrupt) bureaucracy and the private sector are endogenous.

not bribe each other.

We then introduce a bureaucracy which produces the same set of goods and services as the private sector. In contrast to private agents, however, bureaucrats produce the storable goods collectively, and every bureaucrat consumes the same amount of storable goods in each period. In order to focus on the bureaucrats' trading (as opposed to production) decisions, we assume that the bureaucracy always produces high-quality goods. The bureaucracy instructs its members to follow the following trading rules: (i) to not accept or give bribes, and (ii) to trade with a private agent if and only if the bureaucrat can recognize the private agent's storable good as a high quality good. However, a bureaucrat can choose to deviate from these trading rules. *Corruption* is an exchange between a private agent and a bureaucrat in which the private agent offers, and the bureaucrat accepts, a bribe. Individual bureaucrats' decisions on whether to accept bribes affect social welfare, because bribes involve exchange of socially inefficient services. However, individual bureaucrats ignore such an effect. In this sense the bureaucracy creates an externality.

The monitoring technology is imperfect. The bureaucracy cannot observe the type of match a bureaucrat was in or the bribe the bureaucrat might have received; instead, the bureaucracy can observe, with some probability, the quality of the storable good that each bureaucrat accepts in exchange. When a bureaucrat is found to hold a low-quality good or no good at all, he is expelled from the bureaucracy forever. We are interested in whether a bureaucracy will be corrupt and if so, whether a corrupt bureaucracy can be welfare improving.

The following results emerge from our framework. First, there is an equilibrium in which bureaucrats accept bribes. Because such bribes are not exchanged between private agents, this demonstrates that bureaucrats are more susceptible to bribery than private agents. It should be emphasized that this is so not because the bureaucracy has a monopoly in the goods or services that it offers but, rather, because production and consumption for bureaucrats are centralized. Since each bureaucrat bears a small fraction of the production cost and consumption depends little on the bureaucrat's own trading outcome, it is tempting for a bureaucrat to accept bribes. Thus, our main finding is that while the bureaucracy may create a positive externality which cannot be easily reproduced by the private sector, this same feature makes bureaucrats susceptible to bribery since the externality gives rise to a low power incentive scheme.

Second, an equilibrium without corruption can co-exist with an equilibrium with corruption. We find this result interesting for the following reason. It is common to attribute the different levels of corruption among countries to different monitoring technologies and punishment schemes. Our model suggests that two economies that have the same fundamentals may have very different levels of corruption since corruption deterrence depends on beliefs as well as on fundamentals. If agents believe that bureaucrats accept bribes, the bureaucracy is inefficient, and so the loss from being expelled from the bureaucracy is small. In this case, bureaucrats indeed accept bribes. However, if agents believe that bureaucrats do not accept bribes, the bureaucracy is efficient, and the benefit from belonging to the bureaucracy is large enough to deter corruption. The two equilibria differ in welfare and the size of the bureaucracy. The equilibrium without corruption Pareto dominates the equilibrium with corruption, thus, corruption reduces welfare.[2]

Third, when the bureaucracy is small and agents are patient, an increase in the size of the bureaucracy increases the welfare of both private agents and bureaucrats. Thus, if a bureaucracy can commit to producing only high-quality goods, an economy with a corrupt bureaucracy can still be better off than an economy without a bureaucracy. As the size of the bureaucracy increases, the public provision of high-quality goods crowds out private provision, but not one for one. The total supply of high-quality goods in the economy increases, leading to higher welfare. In addition, centralized production and consumption can make bureaucrats more inflexible in trade; for example, they find it easier to refuse to trade when a private agent's good is of unknown quality. This inflexibility improves welfare by reducing the number of lemons in the economy.

Finally, in the presence of corruption, an improvement in the ability to recognize the quality of goods does not necessarily improve welfare, due to the effect on the size of the bureaucracy.

[2] Interestingly, although corruption does speed up exchange, it speeds up only socially inefficient exchange, thus encouraging the production of lemons. This is in contrast to Lui (1985).

Because the improved ability enables the bureaucracy to catch and expel corrupt bureaucrats with a higher probability, the bureaucracy becomes smaller in a stationary equilibrium, which leads to a lower total supply of high-quality goods and hence lower welfare. For an improvement in the monitoring technology to increase welfare, either the economy must be in an equilibrium without corruption, or the improvement must be sufficiently large.

The literature on corruption is too voluminous to be surveyed here; we refer to Rose-Ackerman (1999) for references. Some features or results of our model exist in the literature. For example, the role of market failure is emphasized by Rose-Ackerman (1978) and Lui (1985) in a queuing model and by Cadot (1987) in a model with asymmetric information. Also, multiple equilibria exist when the level of punishment on corruption is endogenous (Lui, 1986, and Cadot, 1987), or when the returns to rent-seeking activities do not dissipate as quickly as those to regular production activities (Murphy et al., 1993). In contrast to these models, our paper develops a search-theoretic foundation for corrupt exchanges by determining the exchange patterns endogenously. This allows us to demonstrate clearly how corruption arises from the *same* frictions that make the Walrasian market unattainable, without resorting to the assumption that the bureaucracy controls the production, distribution, or inspection of certain goods/services. Another advantage of our model is that it permits a coherent welfare analysis because it takes into account how the exchange patterns may respond to policies.[3]

As an illustration of these differences, we contrast the welfare role of a bureaucracy in our paper with that in Acemoglu and Verdier (2000). Assuming that the bureaucracy controls the inspection of project quality, Acemoglu and Verdier show that the bureaucracy can increase welfare by forcing private agents to adopt high-quality projects. In our model, the bureaucracy does not have such capacity; instead, it affects private agents' behavior only indirectly through its non-negligible size in the market. To put it differently, the bureaucrats in our model improve

[3]The welfare analyses in existing models are subject to the following version of the Lucas critique. The bureaucracy's policies may change transaction patterns and/or the extent to which the bureaucracy controls certain elements of the economy. In this sense, assuming that these elements are fixed makes policy evaluation unreliable. A model in which transaction patterns are meaningful and endogenous provides a first step towards addressing this criticism. The search model used in this paper offers such a setup.

welfare by cleaning up their *own* act rather than forcing private agents to clean up theirs as in Acemoglu and Verdier's model.

To model decentralized exchange, we draw from the search monetary theory pioneered by Kiyotaki and Wright (1989). In this sense, the economy without a bureaucracy generalizes the work of Williamson and Wright (1994) and Wang (2000), who introduce the lemons problem into search monetary models but do not allow for bribery. While these authors proceed to study how money can reduce the severity of the lemons problem, we retain the non-monetary economy and study the role of a bureaucracy. Our approach of modelling bribery as an exchange of socially inefficient services follows from Engineer and Shi (1998). Finally, we model a bureaucracy as a non-negligible group of agents who have centralized consumption and production as in Li and Wright (1998). Li and Wright assume that the government agents always follow the government's trading rules. We abandon this assumption in order to analyze corruption.

Before describing the model, we wish to emphasize the following point. In what follows, the bureaucracy should be interpreted as a benevolent government organization and not as a large private coalition of agents. Given that the bureaucracy in our model does not control any aspect of the economy, and it produces the same goods/services that the private sector does, one may be tempted to interpret it as a large private firm (possibly a monopoly) and argue that such a firm can improve social welfare by internalizing the externality which we associate with the bureaucracy. Our model does not lead to this interpretation. In particular, the production and trading rules that our bureaucracy follows are aimed to improve social welfare, rather than to maximize private profits. While a benevolent government will find it optimal to adopt these rules, the rules are in general *not* the best responses of a private firm that maximizes its own present value. Thus, a bureaucracy that adopts such rules could *not* easily be interpreted as a private firm. We do not take a stand on the issue of whether any bureaucracy is truly benevolent in reality. We believe that it is interesting to study why corruption can exist in a benevolent bureaucracy and whether such a bureaucracy can improve welfare. This focus also explains why we assume that the bureaucracy always produces high-quality goods.

2. An Economy without Bureaucracy

In this section we describe an economy without a bureaucracy and show that no trade involves the use of inefficient services (bribes), provided that some restrictions on the size of the bribe are satisfied. This paves the way for an analysis of an economy with a bureaucracy, which appears in Sections 3 and 4.

2.1. The Environment

Time is discrete. There is no public record-keeping, so there is no credit or any other contractual arrangement. Agents live forever and discount the future with a discount factor $\beta \in (0, 1)$. Each agent can produce a perfectly storable and indivisible good, which has no consumption value for the agent himself, but can yield utility to other agents. The good can be either of high quality (H) or low quality (L, lemons). Producing a high-quality good costs $c > 0$ in utility and yields utility $u > c$ to the consumer. Producing a low-quality good costs nothing and yields no utility. The two agents in a match may not both have high-quality goods. Thus, the lack of double coincidence of wants arises not from the mismatch of the physical types of the goods, as in a search/money model, but rather from the differential quality of goods. The quality of a good is private information. With probability $\alpha \in (0, 1)$, nature determines whether an agent can discern the quality of his trading partner's good.

Each agent can also produce non-storable services, which can be consumed by all agents. Unlike the storable goods, services are socially inefficient to produce: the utility to a consumer of each unit of service is 1 but the unit cost of providing such services is $b > 1$. Thus, an agent will never produce services for himself. We refer to the exchange of services as "bribery." To simplify the analysis, we restrict attention to the case where the amount of bribes is either 0 or $B > 0$.[4]

[4]If services are divisible, the set of equilibria is much larger, but this does not add additional insights for the main issues studied here. Similarly, the cost function of service production can be extended to $b(q)$, with $b'(0) > 1$ and $b'' > 0$, as in Engineer and Shi (1998), who study a monetary model without private information regarding goods' quality.

Furthermore, we impose the following assumption.

Assumption 1. *A*1. *B is bounded above so that a producer of a low-quality good is willing to offer a bribe:*

$$B < u/b; \tag{2.1}$$

*A*2. *B is sufficiently large so that a producer of a high-quality good is unwilling to offer a bribe even when he is offered a bribe:*

$$B > \frac{u - \beta c}{b - 1}; \tag{2.2}$$

*A*3. *The interval for B given by (2.1) and (2.2) is non-empty; i.e.,*

$$b > \frac{u}{\beta c} \quad (> 1). \tag{2.3}$$

Next, we describe the exchange process. At the beginning of a period, each agent who does not hold a storable good decides the quality of good he produces. After production, agents are randomly matched in pairs and nature determines, with probability α, whether an agent observes the quality of the partner's good. The realization of this random event is private information, so an agent does not know whether his partner recognizes the quality of his good. The two agents then simultaneously announce three decisions: whether to trade the storable good, the quantity of services proposed to give to the partner, and the quantity of services to ask from the partner. A trade occurs between the two agents if and only if both choose to trade the storable good, and the quantity of services asked by each agent does not exceed the quantity offered by his partner. If a trade takes place, the two agents immediately produce, swap the durable goods, and give the quantity of services requested by the partner. They then consume and depart. If a trade does not take place, the two agents depart immediately. This trading mechanism keeps the analysis tractable and is imposed on the basis of its simplicity.[5]

An agent's trading decision depends on two types of information. One is the quality of his own storable good, denoted by a subscript $i \in \{H, L\}$. The other is the knowledge of the partner's

[5] In particular, we do not allow agents to re-bid. The latter mechanism, albeit realistic, would generate a large set of equilibria, while it is not clear how it would contribute to the main issues studied here. Sequential bargaining causes similar complexity because of the two-sided asymmetric information.

good determined by nature, denoted by a second subscript $j \in \{H, L, U\}$. The meanings of $j = H$ and $j = L$ are clear, while the case $j = U$ ("uninformed") refers to the situation where nature did not reveal the quality of the partner's good to the agent. We refer to an agent with information (i, j) as agent (i, j). The agent makes three trading decisions. The first is whether to trade the storable good, denoted by $s_{ij} \in \{T, NT\}$, where T means "trade" and NT "no-trade". We allow for mixed strategies and denote $x_{ij} = \Pr(s_{ij} = T)$. The second trading decision is the quantity of services that the agent asks the partner to provide, denoted by q^a_{ij}. The third decision is the quantity of services that the agent proposes to give, denoted by q^g_{ij}. Thus, the trading decisions by an agent with information (i, j) are $(x_{ij}, q^g_{ij}, q^a_{ij})$. The trading decisions of an arbitrary agent are denoted by $(X_{ij}, Q^g_{ij}, Q^a_{ij})$, where $X_{ij} = \Pr(S_{ij} = T)$.[6]

An agent (i, j) in a match with an agent (i', j') chooses to trade if and only if $s_{ij} = S_{i'j'} = T$, $q^g_{ij} \geq Q^a_{i'j'}$ and $q^a_{ij} \leq Q^g_{i'j'}$. Denote

$$\delta^{i'j'}_{ij} = \begin{cases} 1, & \text{if } q^g_{ij} \geq Q^a_{i'j'} \text{ and } q^a_{ij} \leq Q^g_{i'j'} \\ 0, & \text{otherwise.} \end{cases}$$

The trade takes place with probability $x_{ij}\delta^{i'j'}_{ij}X_{i'j'}$. The notation corresponding to δ for an arbitrary agent is Δ. As mentioned earlier, if trade takes place, the quantities of services traded are the asked quantities, q^a_{ij} and $Q^a_{i'j'}$.

An agent's production decision is denoted by $p \in [0, 1]$, which is the probability of the agent producing a high-quality good. The production decision of other agents is denoted by P. Each agent chooses (p, x, q^g, q^a) taking other agents' decisions (P, X, Q^g, Q^a) as given. We will focus on stationary, symmetric equilibria, those in which the choices are stationary and agents of the same type make the same choices.

2.2. Value Functions

Let V_i be the value function of an agent holding storable good i, where $i \in \{H, L\}$ before matching takes place. The value function before the production decision is V_0. The production decision p

[6] Throughout the paper, most capital-case variables are per-capita or aggregate variables that individual agents take as given.

solves the following problem:

$$V_0 = \max_{p\in[0,1]} [V_L + p(V_H - V_L - c)]. \tag{2.4}$$

Therefore,

$$p \begin{cases} = 1, & \text{if } V_H - c > V_L; \\ = 0, & \text{if } V_H - c < V_L; \\ \in [0,1], & \text{if } V_H - c = V_L. \end{cases} \tag{2.5}$$

The value function V_H and the exchange decisions $(x_{Hj}, q^g_{Hj}, q^a_{Hj})_{j\in\{H,L,U\}}$ are given by:

$$\begin{aligned}(1-\beta)V_H = \alpha P \times & \max_{(x_{HH}, q^g_{HH}, q^a_{HH})} x_{HH} \left\{ \begin{array}{c} \alpha X_{HH}\delta^{HH}_{HH}[u + q^a_{HH} - bQ^a_{HH} + \beta(V_0 - V_H)] \\ +(1-\alpha)X_{HU}\delta^{HU}_{HH}[u + q^a_{HH} - bQ^a_{HU} + \beta(V_0 - V_H)] \end{array} \right\} \\ +\alpha(1-P) \times & \max_{(x_{HL}, q^g_{HL}, q^a_{HL})} x_{HL} \left\{ \begin{array}{c} \alpha X_{LH}\delta^{LH}_{HL}[q^a_{HL} - bQ^a_{LH} + \beta(V_0 - V_H)] \\ +(1-\alpha)X_{LU}\delta^{LU}_{HL}[q^a_{HL} - bQ^a_{LU} + \beta(V_0 - V_H)] \end{array} \right\} \\ +(1-\alpha) \times & \max_{(x_{HU}, q^g_{HU}, q^a_{HU})} x_{HU} \left\{ \begin{array}{c} P \left[\begin{array}{c} \alpha X_{HH}\delta^{HH}_{HU}[u + q^a_{HU} - bQ^a_{HH} + \beta(V_0 - V_H)] \\ +(1-\alpha)X_{HU}\delta^{HU}_{HU}[u + q^a_{HU} - bQ^a_{HU} + \beta(V_0 - V_H)] \end{array} \right] \\ +(1-P) \left[\begin{array}{c} \alpha X_{LH}\delta^{LH}_{HU}[q^a_{HU} - bQ^a_{LH} + \beta(V_0 - V_H)] \\ +(1-\alpha)X_{LU}\delta^{LU}_{HU}[q^a_{HU} - bQ^a_{LU} + \beta(V_0 - V_H)] \end{array} \right] \end{array} \right\}. \end{aligned} \tag{2.6}$$

The left-hand side of the above equation represents the gain from participating in the current exchange relative to holding onto the good until the next period. The three terms on the right-hand side specify expected gains from possible trades. There are three cases: (i) when the agent recognizes that his partner holds a high-quality good; (ii) when he recognizes that his partner holds a low-quality good; and (iii) when he does not recognize the quality of his partner's good. In the last case, the agent rationally believes that his partner holds a high-quality good with probability P, and a low-quality good with probability $1 - P$.

We explain the first case in detail next, and the other cases are similar. With probability αP the agent meets a partner with a high-quality good and recognizes the quality. The expected gain from trade in this case, specified by terms in the first bracket, arises in two situations. The first is when the partner also recognizes the quality of the agent's good, in which case the probability of trade is $X_{HH}\delta^{HH}_{HH}$. Trading in this situation gives the agent utility u from consuming the partner's storable good, utility q^a_{HH} from consuming the partner's services, and an opportunity to produce in the next period (which has a present value βV_0). The cost of trading comes from the disutility of producing services, bQ^a_{HH}, as well as the foregone value of the object, βV_H. The second situation is when the partner does not recognize the quality of the agent's good. The expected gain from trade can be explained similarly. Notice that in both situations the agent's trade decisions must be the same because the agent does not know which situation he is in. The value function V_L and the exchange decisions $(x_{Lj}, q^g_{Lj}, q^a_{Lj})_{j \in \{H,L,U\}}$ can be described similarly.

2.3. Equilibria

Agents' beliefs must be consistent with their strategies. If an agent believes that everyone else in the economy produces only high(low)-quality goods, then it is rational for the agent to treat a good of unknown quality as a high(low)-quality good. Thus, for all $i \in \{H, L\}$,

$$\begin{aligned} P = 1 \Longrightarrow (x_{iU}, q^g_{iU}, q^a_{iU}) = (x_{iH}, q^g_{iH}, q^a_{iH}), \\ P = 0 \Longrightarrow (x_{iU}, q^g_{iU}, q^a_{iU}) = (x_{iL}, q^g_{iL}, q^a_{iL}). \end{aligned} \tag{2.7}$$

A *symmetric equilibrium* consists of the value functions (V_H, V_L, V_0), individual agents' strategies $(p, x_{ij}, q^a_{ij}, q^g_{ij})$, and the representative agent's strategies $(P, X_{ij}, Q^a_{ij}, Q^g_{ij})$, $(i \in \{H, L\}$, $j \in \{H, L, U\})$, such that (i) for given value functions and strategies for the representative agent, each individual agent's production strategy solves the maximization problem in (2.4), and the trading strategies solve their maximization problems; (ii) agents' trading strategies satisfy the consistency requirements in (2.7); and (iii) strategies are symmetric; i.e., individuals' strategies are the same as the representative agent's strategies.

We focus on equilibria that have the following properties. First, there is a positive measure of high-quality goods in the market, i.e., where $P > 0$ (If $P = 0$, autarky is at least as good as any equilibrium with exchange). Second, some low-quality goods are exchanged for high-quality goods (i.e., $X_{HU} > 0$), since the informational asymmetry in this case generates a clear welfare cost. Finally, when the quality of the partner's good is unknown or observed to be of low quality, an agent trades only if he obtains a strictly positive surplus. This restriction, which can be rationalized by the existence of an arbitrarily small transaction cost, rules out spurious trading where two low-quality good holders swap their goods.

There are three possible types of symmetric equilibria. In all equilibria, an agent does not trade with another agent whose quality he observes to be low. Also, two agents always trade if both observe that their partner holds a high-quality good. The three types of equilibria differ in the values of P and X_{HU}. In the first type, which we term a *type a equilibrium*, every agent produces a high-quality good with probability 1 and always trades even when he does not discern the quality of his partner's good. That is, $P = X_{HU} = 1$. In the second type of equilibrium, termed a *type b equilibrium*, $P < 1 = X_{HU}$. In this equilibrium, even though a high-quality good holder does not discern the partner's quality and knows that there is positive probability that the partner's good is a lemon, he trades anyway. The third type of equilibrium, termed a *type c equilibrium*, has $P < 1$ and $X_{HU} < 1$.

Proposition 2.1. *There exists a type a ($P = 1$) equilibrium if and only if $\alpha > c/u$. In this equilibrium, agents do not request or offer bribes, and they trade with probability one.*

The proof appears in Appendix A. Intuitively, agents always produce only H and always trade with each other when it is easy to discern the quality of goods, in the sense that $\alpha > c/u$. Because anyone who deviates to produce a lemon will be caught with high probability and denied trade, a deviation is not profitable.

Since the lemons problem is not severe in a type a equilibrium, we shift our attention to other types of equilibria. In a type b or a type c equilibrium, $0 < P < 1$ and so $V_0 = V_L = V_H - c$. The following proposition characterizes these equilibria. The proof appears in Appendix B.

Proposition 2.2. *Equilibria with $0 < P < 1$ and $V_L > 0$ involve no bribe exchange. An agent with an L good always wants to trade. An agent with an H good trades if he discerns the partner's good as H; he does not trade if he discerns the partner's good as L; and he trades with a positive probability X_{HU} if he cannot discern the quality of the partner's good. In the last situation, $X_{HU} = 1$ in a type b equilibrium, while $X_{HU} < 1$ in a type c equilibrium. A type b equilibrium exists if and only if $c/u < \alpha < \alpha_2 \equiv u/[\beta(2u - \beta c)]$. A type c equilibrium exists if* *and only if* $\alpha_3 < \alpha < \alpha_2$*, where*

$$\alpha_3 \equiv \frac{1}{2}\left[\left(\frac{1}{\beta}-1\right)+\sqrt{\left(\frac{1}{\beta}-1\right)^2+\frac{4(1-\beta)c}{u-\beta c}}\right].$$

The above proposition, together with Proposition 2.1, illustrates two main features of the equilibria in our model. First, no bribe is exchanged in any equilibrium without a bureaucracy. The precise proof of this result is involved, as shown in Appendix B, but an informal argument is straightforward. First, an agent who holds a high-quality good will not trade if he discerns that the partner holds a low-quality good, even if his partner offers a bribe. The surplus from trading away a high-quality good for a lemon plus a bribe is $B - \beta(V_H - V_0)$ and, because $V_H - V_0 = c$ in all equilibria with $P \in (0, 1)$, this surplus is negative under (2.1) and (2.3). Second, an agent who holds a high-quality good will not offer a bribe, regardless of his knowledge of the partner's good. Even if the partner's good is observed to be of high quality, offering a bribe yields the agent a surplus no more than $u - bB - \beta c$, which is negative under (2.2). Thus, only agents who hold low-quality goods may offer bribes. Third, a high-quality good holder does not ask for a bribe if he cannot discern the quality of his partner's good. In such a match, a high-quality good holder trades away the storable good with positive probability, hoping that the partner holds a high-quality good as well. But if the partner indeed holds a high-quality good, requesting a bribe is not optimal as it will surely result in no trade since a high-quality good partner never offers a bribe. Thus, a high-quality good holder never requests a bribe. Similarly, if an agent with a low-quality good wants to trade in a match with an agent who holds a high- or unknown-quality good, the agent should never ask for a bribe. Thus, no trade involves the exchange of bribes.

Another implication of the model is that different equilibria can co-exist. For example, whenever a type b equilibrium exists, a type a equilibrium exists as well. Which one of these two equilibria occurs depends on the agents' beliefs about other agents' production decisions, and these beliefs are self-fulfilling. If a producer believes that other agents will produce high-quality goods, then it is optimal for him to produce a high-quality good since producing a low-quality good would significantly reduce his trading opportunities. But if a producer believes that other agents will produce high-quality goods with a probability strictly less than 1, then it is optimal for him to do the same since always producing a high-quality good would incur a high production cost which would be wasted in trade with partners who hold (unknown) low-quality goods.[7]

In a type c equilibrium, the value functions and X_{HU} are as follows:

$$\left.\begin{aligned} (1-\beta)V_H &= P[\alpha + (1-\alpha)X_{HU}]^2(u-\beta c) - (1-\alpha)\beta c X_{HU}(1-P), \\ (1-\beta)V_L &= (1-\beta)(V_H - c) = P(1-\alpha)uX_{HU}, \end{aligned}\right\} \tag{2.8}$$

$$X_{HU} = \frac{1}{1-\alpha}\left[\frac{(1-P)\beta c}{P(u-\beta c)} - \alpha\right], \tag{2.9}$$

In the remainder of this paper, we will restrict our attention to the parameter region $\alpha \in (0, c/u)$. This serves two purposes, in addition to simplifying the analysis of a bureaucracy. First, the restriction captures the severe information asymmetry which is the type of market failure we wish to focus on. Second, the restriction will facilitate the comparison between an economy with a bureaucracy and the economy without. Under this restriction, the type c equilibrium is the only possible equilibrium in the economy without a bureaucracy, but there can be multiple equilibria in an economy with a bureaucracy. Thus, we can attribute the multiplicity exclusively to the distinct features of the bureaucracy.

3. A Bureaucracy of Exogenous Size

In this section, we introduce a bureaucracy but keep the size of the bureaucracy and the level of punishment on corruption exogenous. We demonstrate that bribery can exist in an economy with a bureaucracy and study how the level of corruption depends on the size of the punishment. This is a preliminary step toward Section 4, in which we endogenize both the size of the bureaucracy and the level of punishment.

Here, the population consists of two groups, private agents and bureaucrats. The mass of bureaucrats is $\gamma \in (0,1)$ and that of private agents is $1-\gamma$. Private agents are as described in

[7] Propositions 2.1 and 2.2 show that the results in Williamson and Wright (1994, section III) are robust to the possibility of bribery, provided that the size of bribes satisfies Assumption 1.

the previous section, and each behaves according to his own incentives. In contrast, bureaucrats have the following two distinct characteristics:

- The bureaucracy centralizes production and consumption of storable goods. All bureaucrats jointly produce high-quality goods at the beginning of each period and allocate one unit to each member who has traded the storable good away in the previous period. No bureaucrat alone can produce storable goods although he is able to produce services. We assume that the bureaucracy can produce only high-quality goods. With regards to consumption, all bureaucrats pool their holdings from exchange at the end of the period, and every bureaucrat consumes the same amount regardless of whether he succeeded in trade.[8]

- The bureaucracy prescribes the following trading instructions for its members: (i) to not accept bribes, and (ii) to trade if and only if their trading partner is known to hold a high-quality good. A bureaucrat can deviate from these trading instructions. If he does so and is caught, he is punished with an exogenous loss of utility $R > 0$ but is allowed to stay in the bureaucracy (in Section 4, such a defector is expelled from the bureaucracy).

Centralized production and consumption captures an important aspect of actual bureaucracies. That is, the benefit of belonging to the bureaucracy derives not so much from individual bureaucrats' actions alone but rather from all bureaucrats' actions together. This feature cuts both ways in a bureaucrat's decision on whether to deviate from the bureaucracy's trading instructions. On the one hand, centralized consumption generates the inflexibility typically associated with a bureaucracy, which increases efficiency. Because a bureaucrat receives the same level of consumption of storable goods regardless of whether he succeeds in trade, he may refuse to trade for an unknown quality good. On the other hand, bribes may entice a bureaucrat to trade the good away because an individual bureaucrat bears little of the production cost. This is a novel implication of our model that we believe also captures a main aspect of some bureaucracies. The

[8] More precisely, the bureaucracy uses a lottery to allocate the goods received from the exchange to the bureaucrats. Every bureaucrat has the same probability of winning one unit of the good regardless of whether he succeeded in trade in that period.

bureaucracy creates a positive externality which cannot be easily reproduced by the private sector, but which, at the same time, makes bureaucrats susceptible to bribery due to a low power incentive scheme.

The assumption that the bureaucracy produces only high-quality goods may not be realistic. Similarly, a bureaucracy in reality may not adopt the trading instructions specified above. We make these assumptions because we try to analyze the behavior of a benevolent bureaucracy. These production and trading rules maximize social welfare and, hence, can be thought of as the optimal policies of a benevolent bureaucracy. We emphasize, however, that they are typically *not* the best responses of a private firm that maximizes its present value: under the maintained restriction that $\alpha < c/u$, a private firm will choose $P < 1$ instead and will trade with a positive probability when the trading partner's good is of unknown quality. Therefore, the bureaucracy in our model fits the description of a government bureaucracy and not that of a private firm.

To complete the description of the bureaucracy, we need to describe how it enforces its trading instructions. The enforcement technology is imperfect. The bureaucracy cannot directly observe the type of the match or the bribe that each bureaucrat might have received; instead, it can only observe the quality of a bureaucrat's storable good with probability α. Thus, we assume that the bureaucracy's detection technology is the same as the private sector's. A bureaucrat is punished only if the bureaucracy finds that he holds a low-quality good, or that he holds no good at all.

To simplify the analysis, we make three additional assumptions. First, bureaucrats must pool the storable goods received from the exchange before consuming them. This rules out embezzlement. Second, each bureaucrat faces the same matching rate as a private agent does. Third, when two bureaucrats meet, they simply swap their inventories and, after swapping, the goods become consumption goods. The last two assumptions ensure that a bureaucrat is not at a disadvantage in the number of possible trades relative to a private agent.

To analyze agents' decisions, let us use a subscript G to denote variables related to a bureaucrat. An individual bureaucrat's decisions are $(y_{Gj}, q^g_{Gj}, q^a_{Gj})_{j=H,L,U}$, where y_{Gj} denotes the probability with which a bureaucrat agrees to exchange the storable good with a private agent who holds a type j good, and (q^g_{Gj}, q^a_{Gj}) denote the corresponding quantities of services offered and requested. Let δ^{iG}_{Gj} indicate whether the quantities of services by the bureaucrat and the private agent are consistent with each other ($\delta = 1$), or not ($\delta = 0$), where $i \in \{H, L\}$ and $j \in \{H, L, U\}$. Let a representative bureaucrat's decisions be $(Y_{Gj}, Q^g_{Gj}, Q^a_{Gj})_{j=H,L,U}$, and the corresponding indicator function be Δ^{iG}_{Gj}. Similarly, for a private agent in a match with a bureaucrat, the decisions are $(x_{iG}, q^g_{iG}, q^a_{iG})$, where $i = H, L$, and the corresponding decisions by a representative private agent are $(X_{iG}, Q^g_{iG}, Q^a_{iG})$.

As discussed before, we restrict $\alpha < c/u$ so as to focus on equilibria with $0 < P < 1$ and $0 < X_{HU} < 1$. Then $V_0 = V_L = V_H - c$. The exchanges between two private agents are the same as described in Proposition 2.2. Adapting (2.8) to take into account the exchanges with bureaucrats, we have the following value functions for private agents:

$$\begin{aligned}(1-\beta)V_H &= (1-\gamma)\{[\alpha + (1-\alpha)X_{HU}]^2 P(u - \beta c) - (1-P)\beta c(1-\alpha)X_{HU}\} \\ &\quad + \gamma \times \max_{(x_{HG}, q^g_{HG}, q^a_{HG})} x_{HG} \left[\begin{array}{l} \alpha Y_{GH}\delta^{GH}_{HG}(u + q^a_{HG} - bQ^a_{GH} - \beta c) \\ +(1-\alpha)Y_{GU}\delta^{GU}_{HG}(u + q^a_{HG} - bQ^a_{GU} - \beta c) \end{array} \right],\end{aligned}$$

$$\begin{aligned}(1-\beta)V_L &= (1-\gamma)(1-\alpha)uPX_{HU} \\ &\quad + \gamma \times \max_{(x_{LG}, q^g_{LG}, q^a_{LG})} x_{LG} \left[\begin{array}{l} \alpha Y_{GL}\delta^{GL}_{LG}(u + q^a_{LG} - bQ^a_{GL}) \\ +(1-\alpha)Y_{GU}\delta^{GU}_{LG}(u + q^a_{LG} - bQ^a_{GU}) \end{array} \right].\end{aligned}$$

Notice that we have already imposed the consistency requirement that if a private agent meets a bureaucrat and cannot recognize the quality of the bureaucrat's good, he must rationally believe that the quality is high (since bureaucrats can only produce high-quality goods).

Every bureaucrat obtains net expected utility u_G from consuming storable goods in each period. This is equal to the total utility from consuming high-quality goods minus the total production cost in the bureaucracy, divided by the size of the bureaucracy. That is,

$$\begin{aligned}u_G &= u\left\{\gamma + (1-\gamma)[\alpha P Y_{GH} X_{HG} \Delta^{HG}_{GH} + (1-\alpha) P Y_{GU} X_{HG} \Delta^{HG}_{GU}]\right\} \\ &\quad - c\left\{1 - (1-\gamma)\left[\begin{array}{l} 1 - \alpha P Y_{GH} X_{HG} \Delta^{HG}_{GH} - \alpha(1-P) Y_{GL} X_{LG} \Delta^{LG}_{GL} \\ -(1-\alpha) Y_{GU}(P X_{HG} \Delta^{HG}_{GU} + (1-P) X_{LG} \Delta^{LG}_{GU}) \end{array} \right]\right\}.\end{aligned}$$

The first bracket in the above expression gives the average rate at which a bureaucrat receives a high-quality good from exchange. Notice that the bureaucrat always exchanges for a high-quality good with another bureaucrat. The second bracket gives the average rate at which a bureaucrat needs to be re-supplied with a high-quality good. The only case in which a bureaucrat does not need to be supplied is when he meets a private agent and does not trade. The probability for this event is given by the terms in the large squared brackets. The value function for a bureaucrat, V_G, is as follows:

$$\begin{aligned}(1-\beta)V_G \quad &= u_G + (1-\gamma)\alpha P \times \max_{(y_{GH}, q^g_{GH}, q^a_{GH})} y_{GH} X_{HG} \delta^{HG}_{GH} (q^a_{GH} - bQ^a_{HG}) \\ &+(1-\gamma)\alpha(1-P) \times \max_{(y_{GL}, q^g_{GL}, q^a_{GL})} y_{GL} X_{LG} \delta^{LG}_{GL} (q^a_{GL} - bQ^a_{LG} - \alpha R) \\ &+(1-\gamma)(1-\alpha) \times \max_{(y_{GU}, q^g_{GU}, q^a_{GU})} y_{GU} \left[\begin{array}{l} P X_{HG} \delta^{HG}_{GU} (q^a_{GU} - bQ^a_{HG}) \\ +(1-P) X_{LG} \delta^{LG}_{GU} (q^a_{GU} - bQ^a_{LG} - \alpha R) \end{array} \right].\end{aligned}$$

A notable feature of this value function is that a bureaucrat's trading decision is driven by bribes. The utility (u_G) from consuming the storable goods does not affect directly a bureaucrat's decision on whether to accept a bribe. The expected punishment is given by the product of the detection probability, α, and the level of punishment, R. Notice that it is impossible to detect bribery when a bureaucrat exchanges for a high-quality good. We impose a tie-breaking restriction that the bureaucrat exchanges with probability 1 if the partner holds a known high-quality good and the surplus from bribes is zero.

Having specified the general forms of the value functions, we now turn to the study of equilibria. Again, we examine only symmetric equilibria where agents with the same type and information choose the same strategies. Not surprisingly, the bureaucracy's trading instructions are consistent with equilibrium if expected punishment to corruption exceeds the benefit of bribery, i.e., if $\alpha R > B$. The following proposition verifies this intuition (see Appendix C for a proof):

Proposition 3.1. *If $\alpha R > B$, no bribes are exchanged in equilibrium. Trades between private agents are as described in Proposition 2.2. A private agent and a bureaucrat trade if and only if the private agent holds a high-quality good, and the bureaucrat recognizes the quality of the private agent's good.*

If the punishment on corruption is not severe enough, however, bribery will occur in certain matches. We assert this result in the following proposition, which is proved in Appendix D.

Proposition 3.2. *If R is sufficiently close to zero, then the following trading strategies form an equilibrium. Trades between private agents are as described in Proposition 2.2. In a match between two bureaucrats, the two swap the storable goods. In a match between a bureaucrat and a private agent, trade occurs except when the private agent's good is of high-quality but is not recognized by the bureaucrat. Moreover, bribery occurs if and only if the private agent holds a low-quality good. Specifically, the trading decisions are as follows:*

$$Q^a_{GL} = Q^a_{GU} = B,\ Q^a_{GH} = 0,\ Q^g_{GL} = Q^g_{GU} = Q^g_{GH} = 0; \tag{3.1}$$

$$Q^a_{LG} = Q^a_{HG} = 0,\ Q^g_{LG} = B,\ Q^g_{HG} = 0; \tag{3.2}$$

$$Y_{GL} = Y_{GU} = Y_{GH} = 1,\ X_{HG} = X_{LG} = 1, \tag{3.3}$$

$$\Delta^{GU}_{HG} = \Delta^{HG}_{GU} = 0,\ \Delta^{HG}_{GH} = \Delta^{GH}_{HG} = 1,\ \Delta^{Gj}_{LG} = \Delta^{LG}_{Gj} = 1\ (j = L, U). \tag{3.4}$$

Eq. (3.1) states that the bureaucrat requests a bribe whenever he is not sure whether the partner's good is of high quality, but a bureaucrat never offers a bribe. Eq. (3.2) states that the private agent never requests a bribe from a bureaucrat, and that he offers a bribe only when he holds a low-quality good. Eq. (3.3) states that the two agents always decide to trade their storable goods. However, a trade does not always occur because the quantities of services proposed by the two agents may be inconsistent. This happens when the private agent holds a high-quality good, and the bureaucrat does not observe it, as indicated by (3.4).

In light of Proposition 3.2, we can re-examine two views expressed in the literature. First, bribes are sometimes deemed "natural" because they are similar to gifts between private agents (Rose-Ackerman, 1999). Under our parameter restrictions, there is a clear distinction between bribes and gifts: private agents offer inefficient services as bribes to bureaucrats which they do not offer to other private agents or receive from bureaucrats. Second, it is sometimes said that bribery improves social welfare because it can speed up exchange. Although it is true in our model that an equilibrium with bribery has more trades than in an equilibrium without bribery, it is the *socially inefficient* exchange that bribery speeds up. This is because only the producers of lemons are willing to bribe, so corruption encourages the production and exchange of lemons and socially inefficient services. As a result, corruption unambiguously reduces welfare.

Propositions 3.1 and 3.2 point to the possibility of multiple equilibria, since they suggest that bribery and corruption are more likely to arise when the punishment on corruption is lower. Because the punishment on corruption in reality often depends on whether there is corruption, it is possible that multiple equilibria can be self-fulfilling when the punishment is endogenous. In the next section we examine this issue and analyze welfare.

4. Endogenous Size of Bureaucracy and Punishment on Corruption

In this section we assume that a bureaucrat who is caught accepting bribes is expelled from the bureaucracy. In this event the bureaucrat loses the current utility, u_G, plus the future benefit from belonging to the bureaucracy. That is, the punishment is now endogenously determined as $R = u_G + \beta(V_G - V_0)$.

The size of the bureaucracy is also endogenous in any equilibrium with corruption because corrupt bureaucrats exit from the bureaucracy once they are caught. In order to obtain a stationary size of the bureaucracy, we introduce exogenous deaths and births. Each agent dies with probability θ at the end of each period, and a measure θ of new agents are born at the beginning of each period. A fraction σ of the newborns are bureaucrats, and the remaining $1 - \sigma$ are private agents. If the size of the bureaucracy is γ in this period, the size in the next period is $\gamma' = (1-\theta)(\gamma-m)+\theta\sigma$, where m is the endogenous mass of corrupt bureaucrats who are expelled as a result of being caught accepting bribes. In a steady state, $\gamma' = \gamma$, and so $\gamma = \sigma-(1-\theta)m/\theta$, which is positive iff $m < \theta\sigma/(1-\theta)$. Given the probability of death, the effective discount factor is $\beta = (1-\theta)\widetilde{\beta}$, where $\widetilde{\beta}$ now stands for the discount factor used previously. All the Bellman equations in the previous section are valid after re-interpreting the discount factor as $\beta = (1-\theta)\widetilde{\beta}$. Again we restrict attention to the case where $0 < X_{HU} < 1$ and $0 < P < 1$. In this case, $V_0 = V_L = V_H - c$.

4.1. An Equilibrium with Corruption

In this equilibrium, agents' decisions are as described by Proposition 3.2. Substituting (3.1)–(3.4) and $R = u_G + \beta(V_G - V_L)$ into the value functions in the last section, we have:

$$(1-\beta)V_H = \alpha(1-\gamma)(1-P)\beta c + \gamma\alpha(u-\beta c),$$

$$(1-\beta)V_L = (1-\gamma)(1-\alpha)uPX_{HU} + \gamma(u-bB),$$

$$(1-\beta)V_G = u_G + (1-\gamma)(1-P)\{B - \alpha[u_G + \beta(V_G - V_L)]\},$$

$$u_G = (u-c)[\gamma + (1-\gamma)\alpha] - (1-\gamma)(1-P)[\alpha u + (1-\alpha)c].$$

The probability X_{HU} is given by (2.9). Solving $V_H - V_L = c$, we have:

$$P = 1 - \frac{\alpha(u-\gamma\beta c) - (1-\beta)c - \gamma(u-bB)}{(1-\gamma)\left[\alpha(u-\beta c) + u\beta c/(u-\beta c)\right]}. \tag{4.1}$$

Notice that the size of the bureaucracy and the level of corruption, as measured by the fraction of bureaucrats accepting bribes, affect private agents' production decisions in equilibrium. This influence comes from the non-negligible role of the bureaucracy in the market.

The equilibrium with corruption imposes three restrictions. First, a bureaucrat must have no incentive to deviate from the equilibrium strategy. That is, a bureaucrat gains nothing by not accepting bribes when the quality of the private agent's good is known to be low or is unknown. The conditions for these two types of matches are the same, and so we consider only the case where a bureaucrat discerns the private agent's good to be of low quality. Suppose that the bureaucrat refuses to accept bribes in this case. His value function, denoted by V_G^d, satisfies

$$(1-\beta)V_G^d = u_G + (1-\gamma)(1-\alpha)(1-P)\{B - \alpha[u_G + \beta(V_G - V_L)]\}.$$

Comparing this value V_G^d with the equilibrium value V_G, it is evident that the deviation is not profitable if and only if $B > \alpha[u_G + \beta(V_G - V_L)]$. Substituting $(V_G, V_L, u_G, P, X_{HU})$, we can express this condition explicitly as follows:

$$\begin{array}{rl} & (1-\beta)B/\alpha \ + \beta\gamma(u-bB) \\ > & (u-c)[\gamma + \alpha(1-\gamma)] + \alpha\beta(1-\gamma)u \\ & -\frac{\alpha u-(1-\beta)c-\gamma(u-bB+\alpha\beta c)}{\alpha(u-\beta c)+\ u\beta c/(u-\beta c)}\left[\alpha u + (1-\alpha)c + \beta u\left(\frac{\beta c}{u-\beta c} + \alpha\right)\right]. \end{array} \tag{4.2}$$

Second, this equilibrium requires that $X_{HU}, P \in (0,1)$. The restrictions $0 < X_{HU} < 1$, which imply $0 < P < 1$, are equivalent to:

$$\frac{(1-\beta)c + \gamma(u - bB - \beta c) + \sqrt{\Delta_1}}{2\left[\gamma(u-\beta c) + (1-\gamma)\beta c\right]} < \alpha < \frac{u[(1-\gamma\beta)c + \gamma(u - bB)]}{\gamma(u-\beta c)^2 + \beta c[(2-\gamma)u - \beta c]}, \tag{4.3}$$

where

$$\begin{aligned} \Delta_1 \quad \equiv \quad & [(1-\beta)c + \gamma(u - bB - \beta c)]^2 \\ & + \tfrac{4\beta c}{u-\beta c}[(1-\beta)c + \gamma(u - bB)][\gamma(u-\beta c) + (1-\gamma)\beta c]. \end{aligned}$$

Third, the mass of bureaucrats must be stationary; i.e., $\gamma = \sigma - (1-\theta)m/\theta$. The mass of corrupt bureaucrats who are caught and, thus, expelled in each period is $m = \gamma(1-\gamma)(1-P)\alpha$, where $(1-\gamma)(1-P)$ is the probability with which a bureaucrat accepts a bribe and α the detection probability. Then, the stationary size of the bureaucracy satisfies:

$$\gamma[\theta + \alpha(1-\theta)(1-\gamma)(1-P)] - \theta\sigma = 0. \tag{4.4}$$

Clearly, $0 < \gamma < \sigma$. Substituting P from (4.1), we can write the above equation as a quadratic equation of γ and show that there is a unique admissible solution for γ if

$$\sigma < \frac{\alpha u - (1-\beta)c}{u - bB + \alpha\beta c}. \tag{4.5}$$

Moreover, the solution for γ approaches 0 when either σ approaches 0 or θ approaches 1.[9] Because the bureaucrats' choice of accepting bribes affects the private sector's production decision, P, it also affects the stationary size of the bureaucracy.

An equilibrium with corruption exists if (4.2), (4.3), and (4.4) hold, together with (2.1) – (2.3). We delay the study of the existence issue until subsection 4.3.

4.2. An Equilibrium without Corruption

In this equilibrium, agents' trade decisions are described by Proposition 3.1. Since no bribery occurs, no agent exits the bureaucracy, and so the stationary size of the bureaucracy is $\gamma = \sigma$.

[9] To show these properties, substitute P from (4.1) to write (4.4) as

$$\begin{aligned} 0 \quad = \quad & (u - bB + \alpha\beta c)\gamma^2 + \tfrac{\theta\sigma}{(1-\theta)\alpha}\left[\alpha(u-\beta c) + \tfrac{u\beta c}{u-\beta c}\right] \\ & - \left\{\alpha u - (1-\beta)c + \tfrac{\theta}{(1-\theta)\alpha}\left[\alpha(u-\beta c) + \tfrac{u\beta c}{u-\beta c}\right]\right\}\gamma. \end{aligned}$$

Under (4.5), the above quadratic function is positive when $\gamma = 0$ and is negative when $\gamma = \sigma$.

The value functions are as follows:

$$(1-\beta)V_H = \alpha(1-\sigma)(1-P)\beta c + \sigma\alpha(u-\beta c),$$

$$(1-\beta)V_L = (1-\sigma)(1-\alpha)uPX_{HU},$$

$$(1-\beta)V_G = u_G = (u-c)[\sigma + (1-\sigma)\alpha P],$$

where X_{HU} is given by (2.9), and P is given by

$$P = 1 - \frac{\alpha(u-\sigma\beta c)-(1-\beta)c}{(1-\sigma)\left[\alpha(u-\beta c)+u\beta c/(u-\beta c)\right]}. \tag{4.6}$$

For an equilibrium without corruption to exist, a bureaucrat must have no incentive to deviate from the equilibrium strategy of not accepting bribes. The required condition is $B < \alpha\left[u_G + \beta(V_G - V_L)\right]$. Although this condition appears to simply reverse the earlier inequality that was required to induce a bureaucrat to accept bribes, the implied restriction on the parameters is quite different. This is because the variables (u_G, V_G, V_L) now have different values. To see this distinction clearly, substitute (u_G, V_G, V_L) in the equilibrium without bribery. We can express the no-deviation condition as follows:

$$\begin{aligned}(1-\beta)B/\alpha \quad < \quad & (u-c)[\sigma+(1-\sigma)\alpha] + \alpha\beta(1-\sigma)u \\ & -\tfrac{\alpha(u-\sigma\beta c)-(1-\beta)c}{\alpha(u-\beta c)+u\beta c/(u-\beta c)}\left[\alpha(u-c)+\beta u\left(\tfrac{\beta c}{u-\beta c}+\alpha\right)\right].\end{aligned} \tag{4.7}$$

This is different from the corresponding condition in the equilibrium with bribery, (4.2).

In addition, the equilibrium requires that $X_{HU}, P \in (0,1)$. These are equivalent to

$$\frac{(1-\beta-\sigma\beta)c+\sqrt{\Delta_2}}{2\left[\sigma(u-\beta c)+(1-\sigma)\beta c\right]} < \alpha < \frac{(1-\sigma\beta)cu}{\sigma(u-\beta c)^2+\beta c[(2-\sigma)u-\beta c]}, \tag{4.8}$$

where

$$\Delta_2 \equiv (1-\beta-\sigma\beta)^2c^2 + \frac{4(1-\beta)\beta c^2}{u-\beta c}[\sigma(u-\beta c)+(1-\sigma)\beta c].$$

An equilibrium without corruption exists if (4.7), (4.8), and (2.1) – (2.3) hold.

4.3. Existence and Co-existence of Equilibria

We first establish the existence and co-existence of the two equilibria analytically for special cases. Then we numerically illustrate the existence in a broader parameter region.

Proposition 4.1. *If $\beta \to 0$, no equilibrium with trade exists. If β is sufficiently close to 1, and σ is sufficiently close to 0, an equilibrium with $X_{HU}, P \in (0,1)$ exists in a non-empty parameter region that satisfies $0 < \alpha < u/(2u-c)$ and (2.1) – (2.3). Moreover, the equilibrium with corruption and the equilibrium without corruption co-exist.*

Proof. If $\beta \to 0$, (2.9) implies $X_{HU} < 0$, and so there is no equilibrium. For the rest of the proposition, set $(\beta, \sigma) \to (1, 0)$ so that $\gamma \to 0$. The conditions for $X_{HU} \in (0,1)$ then become $0 < \alpha < u/(2u-c)$ in both equilibria. For the equilibrium with bribery, the requirement (4.2) becomes $B > -\infty$; for the equilibrium without bribery, the requirement (4.7) becomes $B < +\infty$. Both are trivially satisfied. Thus, an equilibrium with $X_{HU}, P \in (0,1)$ exists if and only if the conditions $0 < \alpha < u/(2u-c)$ and (2.1) – (2.3) hold. These requirements are satisfied by a non-empty set of parameter values. Moreover, the two equilibria coexist. Since the equilibria depend on (β, σ) continuously, they also exist for (β, σ) sufficiently close to $(1, 0)$. QED

The following example illustrates the existence of the two equilibria in a broader region.

Example 4.2. *Let the parameters be given by the following: $\beta = 0.9$, $\sigma = 0.4$, $\theta = 0.1$, $u = 1$, $c = 0.5$, $b = 3$.*

With these parameter values, Figure 1 depicts the existence regions of the two equilibria in the subspace (B, α). The assumptions (2.1) – (2.3) are satisfied when $0.275 < B < 0.333$. For any B in this range, the equilibrium with corruption exists when $\alpha L_b(B) < \alpha < \alpha H_b(B)$, and the equilibrium without corruption exists when $\alpha L_n(B) < \alpha < \alpha H_n(B)$. The two equilibria co-exist when $\alpha L_b(B) < \alpha < \alpha H_n(B)$.

Figure 1 here.

We emphasize that the co-existence is driven by the fact that the punishment on corruption endogenously depends on the level of corruption. When bureaucrats do not accept bribes, they exchange their high-quality goods only when they can see that their partners' goods are of high quality. Since the bureaucracy's goods are never exchanged for lemons, the cost of replacing these goods is low, and so the payoff to belonging to the bureaucracy is high. This high payoff deters corruption because a bureaucrat would risk losing this high payoff if he accepted a bribe and were caught. On the other hand, if bureaucrats accept bribes, a large fraction of the bureaucracy's goods are exchanged for lemons and bribes. In that case, the bureaucracy's consumption of high-quality goods is low, and the cost of replacing goods is high. Both factors reduce the payoff to belonging to the bureaucracy and make corruption more likely.

Although our model is very stylized, casual observations suggest that corruption of government officials is indeed less wide-spread in countries where such officials enjoy high benefits (e.g., salary, status, etc.). Conversely, corruption is widely spread in countries where such officials are treated poorly, and the value of being in the bureaucracy (without bribes) is low.

4.4. Welfare Analysis

A private agent's ex ante welfare is V_0, which is equal to V_L (or $V_H - c$) in the two equilibria in our discussion. A bureaucrat's ex ante welfare is V_G. The society's welfare is $W \equiv \sigma V_G + (1-\sigma)V_0$, which measures the expected utility of an agent before he realizes whether he is a private agent or a bureaucrat. We examine how these welfare measures change with σ and α. The effects of σ are interesting because an increase in σ, the fraction of newborns who are bureaucrats, is likely to increase the stationary size of the bureaucracy. The effects of α are interesting because an increase in α represents an improvement in the ability to recognize quality.[10] To distinguish the two equilibria, we attach the subscript n to variables in the equilibrium without corruption and b to variables in the equilibrium with corruption. We have the following.[11]

[10] Another parameter is the size of bribe, B. We found that welfare decreases with B in all numerical examples.

[11] The proof is straightforward and omitted. To determine the signs of various derivatives, notice that the equilibria exist in the limit case $(\beta, \sigma) \to (1, 0)$ only if $0 < \alpha < u/(2u - c)$.

Proposition 4.3. *When* $(\beta, \sigma) \to (1, 0)$, (V_0, V_G, W) *are all increasing functions of* σ *in both equilibria.*

In other words, all measures of welfare increase with the size of the bureaucracy when this size is sufficiently small. The gain to private agents comes primarily from increased frequency of transactions for private agents who hold high-quality goods. In an economy without bureaucracy, a private agent with a high-quality good sometimes exchanges the good for a low-quality good from which the agent derives little utility. For every private agent that is replaced by a bureaucrat who commits to producing only high-quality goods, the number of agents producing high-quality goods increases by $1-P$. In this case, a private agent with a high-quality good has a higher chance than before to meet another high-quality good holder, and so his expected utility increases. This is the case even in the equilibrium with bribery, because a private agent holding a high-quality good does not offer bribes. Thus, V_H increases in both equilibria as σ increases. Because a bureaucrat is a high-quality good producer by construction, his value function increases for a similar reason. The value function of a low-quality good producer also increases, because $V_L = V_H - c$. Therefore, a private agent's ex ante welfare increases.

To see the welfare effects of σ in a larger parameter region, consider the following example:

Example 4.4. $\beta = 0.9$, $\alpha = 0.4$, $\theta = 0.1$, $u = 1$, $c = 0.5$, $b = 3$, $B = 0.322$ *(*$\equiv BB$*).*

With these parameter values, the two equilibria coexist for $\sigma \in (0, 0.565)$. Figure $2a$ depicts the dependence of (V_0, V_G) on σ in the two equilibria, and Figure $2b$ depicts the dependence of (P, h, G) on σ, where h is the probability with which a private agent with a high-quality good succeeds in obtaining a high-quality good, and G is the solution for the bureaucracy size (γ) in the equilibrium with corruption. An increase in σ increases the size of the bureaucracy in the two equilibria (recall that $\gamma = \sigma$ in the equilibrium without corruption). Both the private agent's and the bureaucrat's welfare levels are increasing functions of σ in the two equilibria. Thus the society's ex ante welfare is increasing in σ. As we discussed above, the welfare gain to a private agent rises because h increases with σ (see Figure 2b). Similarly, as σ increases, a private agent is more likely to exchange a high quality good for a good of unknown quality.

It is important to supplement the above welfare effect of σ with two remarks. First, the positive welfare result of a bureaucracy should not be construed as a statement that a bureaucracy can always improve welfare. Rather, the result serves the purpose of demonstrating that the exogenous advantages for a bureaucracy assumed in the literature are not necessary for the bureaucracy to improve welfare. Because our assumption that a bureaucracy can commit to producing high-quality goods is not always satisfied in actual economies, one should be careful when mapping our welfare result with actual observations.

Second, one may be tempted to conclude that the positive welfare effect in our model is a simple change in the decomposition between bureaucrats and private agents, with some private agents who produce low-quality goods being replaced by bureaucrats who produce high-quality goods. This interpretation misses an interesting feature in Figure 2b. Namely, the increase in the public provision of high-quality goods, caused by the increase in σ, crowds out the private provision of high-quality goods. Not only does the size of the private sector shrink but also private agents produce high-quality goods with a lower probability. This probability falls because an increase in bureaucracy size benefits a holder of a high-quality good more than a holder of a low-quality good. In equilibrium agents must be indifferent between producing high-quality and low-quality goods. Therefore, to restore this mixed-strategy equilibrium, the fraction of low-quality private agents must increase to offset the excess gain to agents with high-quality goods. Despite such crowding-out, the increase in σ increases the overall fraction of high-quality goods in the market. That is, the crowding-out is less than one-for-one.

Figures 2a and 2b here.

We now illustrate the welfare effects of reducing the informational asymmetry, as represented by an increase in α. Consider the parameter values in Example 4.2. In addition, set $B = 0.322$ ($\equiv BB$), in which case the two equilibria co-exist for $\alpha \in (0.257, 0.501)$. Figure 3$a$ depicts the dependence of welfare levels on α in the two equilibria, and Figure $3b$ depicts the dependence of (P, h, G) on α.

Figure $3a$ and $3b$ here.

In the equilibrium without corruption, both private agents' and bureaucrats' welfare increases with α. For a private agent with a high-quality good, an increase in α increases his utility by reducing the number of exchanges with agents holding low-quality goods. Since $V_L = V_H - c$ in the two equilibria, expected utility of agents who hold low-quality goods also increases. For bureaucrats, an increase in α increases their expected utility by increasing the number of exchanges with private agents who hold high-quality goods.

In the equilibrium with corruption, in contrast, an increase in α improves welfare only when α is large. When α is small, an increase in α reduces welfare for both private agents and bureaucrats. This negative welfare effect arises because the equilibrium size of the bureaucracy shrinks with α, as illustrated by the decreasing function G in Figure $3b$. As the detection technology improves, a larger fraction of corrupt bureaucrats is caught and expelled from the bureaucracy. This generates a smaller bureaucracy; thus, the fraction of high-quality goods in the market can be lower. When α is sufficiently large, however, a further increase in α improves welfare by sufficiently reducing the number of low-quality goods exchanged between private agents.

The effects of α on P are similar to the effect of σ. That is, an increase in α benefits a private agent who holds a high-quality good more than a private agent who holds a low-quality good; to restore the indifference between producing the two qualities, P must fall.[12]

5. Conclusion

We have analyzed the exchange patterns in two economies with decentralized exchanges and asymmetric information regarding product quality. In the economy without a bureaucracy, agents do not trade socially inefficient bribes with each other. We introduced bureaucracy as a coalition of agents who have centralized production and consumption and showed that the positive exter-

[12] This result is similar to the one obtained in a monetary model by Wang (2000).

nality created by such a bureaucracy also gives rise to low power incentives for the individual bureaucrats and might lead to corruption. We endogenized the punishment on corruption as the loss of the benefit from being part of the bureaucracy. In the economy with a bureaucracy, there are two self-fulfilling equilibria. In one the bureaucrats accept bribes, and in the other they do not. The equilibrium without corruption generates higher welfare and is able to support a larger bureaucracy than the equilibrium with corruption. Despite the existence of corruption, a bureaucracy may improve the welfare of the society.

Our welfare results should be interpreted with some caution. First, real world bureaucracies have many functions other than alleviating market failures arising from private information, although the latter is an important function emphasized in our paper as well as some other papers on bureaucracy (e.g., Acemoglu and Verdier, 2000). Second, there are features that we abstract from that limit the desirability of a large bureaucracy. Introducing such features will lead to a more realistic model and might overturn some of our welfare conclusions. We consider our welfare results to be of value to the extent that they will remain an important part of such a more complicated story.

We believe that the search model with lemons is a rudimentary but appropriate framework that captures the link between market failure and corruption. This framework endogenously generates trading patterns that distinguish bribery from gifts; the former is a corrupt exchange between a private agent and a bureaucrat, while the latter is an exchange between two private agents. In addition, the model illustrates that corruption is not merely a transfer from private agents to bureaucrats, but rather an exchange that incurs a social deadweight loss. There is a delicate trade-off between the cost of corruption and the potential social benefit from a bureaucracy that can commit to producing high-quality goods.

As we mentioned earlier, bureaucracy in existing models is typically exogenous in the sense that, by assumption, it controls certain elements of the exchange. The bureaucracy in our model is not fully endogenous either since we do not model explicitly how agents choose to form it, and how it commits to producing high-quality goods. These exogenous features notwithstanding, our model makes a step forward by assuming that the bureaucracy does not control directly any element of the exchange process. We hope that our analysis will provide a step forward to shifting the focus of future research towards a fully endogenous bureaucracy. For such an analysis, we believe that the feature of a bureaucracy that we emphasized in this paper, namely, centralized production and consumption, will continue to be important.

Many extensions are possible. We mention two here. The first involves allowing agents to acquire technologies that improve the ability to recognize the quality of goods. We have already shown that a higher ability is not necessarily welfare-improving, and so there might be a socially desirable level of such ability. Wang (2000) has analyzed information acquisition in a search model without corruption. It would be interesting to see whether the presence of corruption changes his results. A second extension is to examine corruption at different levels of a hierarchical bureaucracy. More generally, one can examine how different ways of organizing a bureaucracy affect corruption, a topic emphasized by Shleifer and Vishny (1993).

References

[1] Acemoglu, D. and T. Verdier, 2000, "The choice between market failures and corruption," *American Economic Review* 90, 194-211.

[2] Akerlof, G., 1971, "The market for lemons: qualitative uncertainty and the market mechanism," *Quarterly Journal of Economics* 84, 488-500.

[3] Becker, G., 1968, "Crime and punishment: an economic approach," *Journal of Political Economy* 76, 169-217.

[4] Cadot, O., 1987, "Corruption as a gamble," *Journal of Public Economics* 33, 223-244.

[5] Engineer, M. and S. Shi, 1998, "Asymmetry, imperfectly transferable utility, and the role of fiat money in improving terms of trade," *Journal of Monetary Economics* 41, 153-183.

[6] Kiyotaki, N. and R. Wright, 1989, "On money as a medium of exchange," *Journal of Political Economy* 97, 927-954.

[7] Li, Y. and R. Wright, 1998, "Government transaction policy, media of exchange, and prices," *Journal of Economic Theory* 81, 290-313.

[8] Lui, F.T., 1985, "An equilibrium queuing model of bribery," *Journal of Political Economy* 93, 760-781.

[9] Lui, F.T., 1986, "A dynamic model of corruption deterrence," *Journal of Public Economics* 31, 215-236.

[10] Murphy, K.M., Shleifer, A. and R. Vishny, 1993, "Why is rent-seeking so costly to growth?" *American Economic Association Papers and Proceedings* 83, 409-414.

[11] Rose-Ackerman, S., 1978, *Corruption: A Study in Political Economy*, Academic Press, New York.

[12] Rose-Ackerman, S., 1999, *Corruption and Government: Causes, Consequences, and Reform*, Cambridge University Press, Cambridge, UK.

[13] Shleifer, A. and Vishny, R., 1993, "Corruption", *Quarterly Journal of Economics* 108: 599-617.

[14] Wang, W., 2000, *Essays on Search Monetary Economy*, Ph.D. dissertation, Queen's University (Kingston, Canada).

[15] Williamson, S. and R. Wright, 1994, "Barter and monetary exchange under private information," *American Economic Review* 84, 104-123.

Appendix

A. Proof of Proposition 2.1

First, we establish the following Lemma.

Lemma A.1. *A symmetric equilibrium with* $P > 0$ *has the following properties: (i)* $X_{LH} = \Delta_{LH}^{HL} = \Delta_{LH}^{HU} = 1$*; (ii)* $Q_{iH}^a = B$ *implies* $X_{Hj} = 0$ *for all* $i, j \in \{H, L, U\}$*; (iii)* $X_{HL} = 0$ *and* $Q_{LH}^a = Q_{HU}^g$.

Proof. Similar to (2.6), we express the value function V_L as follows:

$$\begin{aligned}(1-\beta)V_L = {} & \alpha P \times \max_{(x_{LH}, q_{LH}^g, q_{LH}^a)} x_{LH} \left\{ \begin{array}{c} \alpha X_{HL}\delta_{LH}^{HL}[u + q_{LH}^a - bQ_{HL}^a + \beta(V_0 - V_L)] \\ +(1-\alpha)X_{HU}\delta_{LH}^{HU}[u + q_{LH}^a - bQ_{HU}^a + \beta(V_0 - V_L)] \end{array} \right\} \\ & +\alpha(1-P) \times \max_{(x_{LL}, q_{LL}^g, q_{LL}^a)} x_{LL} \left\{ \begin{array}{c} \alpha X_{LL}\delta_{LL}^{LL}[q_{LL}^a - bQ_{LL}^a + \beta(V_0 - V_L)] \\ +(1-\alpha)X_{LU}\delta_{LL}^{LU}[q_{LL}^a - bQ_{LU}^a + \beta(V_0 - V_L)] \end{array} \right\} \\ & +(1-\alpha) \times \max_{(x_{LU}, q_{LU}^g, q_{LU}^a)} x_{LU} \left\{ \begin{array}{c} P \left[\begin{array}{c} \alpha X_{HL}\delta_{LU}^{HL}[u + q_{LU}^a - bQ_{HL}^a + \beta(V_0 - V_L)] \\ +(1-\alpha)X_{HU}\delta_{LU}^{HU}[u + q_{LU}^a - bQ_{HU}^a + \beta(V_0 - V_L)] \end{array} \right] \\ +(1-P) \left[\begin{array}{c} \alpha X_{LL}\delta_{LU}^{LL}[q_{LU}^a - bQ_{LL}^a + \beta(V_0 - V_L)] \\ +(1-\alpha)X_{LU}\delta_{LU}^{LU}[q_{LU}^a - bQ_{LU}^a + \beta(V_0 - V_L)] \end{array} \right] \end{array} \right\}. \end{aligned} \tag{A.1}$$

Part (i) of the Lemma states that an L agent who sees the partner's quality as H always wants to trade. The L agent's surplus is $u + q_{LH}^a - bQ_{HL}^a + \beta(V_0 - V_L)$ if the partner knows his good's quality and $u + q_{LH}^a - bQ_{HU}^a + \beta(V_0 - V_L)$ if he does not. Even under the worst terms of trade, $q_{LH}^a = 0$ and $Q_{Hj}^a = B$, the surpluses in the two types of matches are no less than $u - bB$, which is positive under assumption (2.1). Thus, the agent always wants to trade. Part (ii) of the Lemma states that an H holder will not trade with a partner who asks for a bribe. If the H holder trades and offers a bribe, his surplus is $u + q_{Hj}^a - bB + \beta(V_0 - V_H)$, where $j \in \{H, L, U\}$ is the agent's perception of the quality of the partner's good. Since $q_{Hj}^a \le B$ and $V_0 - V_H \le -c$, the surplus is no more than $u - (b-1)B - \beta c$, which is negative under assumption (2.2). For part (iii), consider an agent with an H good who sees that the partner holds an L good. If he trades, his surplus is $q_{HL}^a - bQ_{Lj}^a + \beta(V_0 - V_H)$, where $j \in \{H, U\}$. Since $V_0 - V_H \le -c$ if $P > 0$, the surplus is no more than $B - \beta c < u/b - \beta c < 0$. Thus, $x_{HL} = X_{HL} = 0$, as in part (iii). With $X_{HL} = 0$, the surplus associated with the decision x_{LH} is

$$(1-\alpha)X_{HU}\delta_{LH}^{HU}(u + q_{LH}^a - bQ_{HU}^a + \beta(V_0 - V_L)).$$

To maximize this surplus, the choices (q_{LH}^a, q_{LH}^g) must ensure $\delta_{LH}^{HU} = 1$. For $\delta_{LH}^{HU} = 1$, it is necessary that $q_{LH}^a \le Q_{HU}^g$ and $q_{LH}^g \ge Q_{HU}^a$. Under the latter restrictions, the surplus is maximized by setting $q^a_{LH} = Q^g_{HU}$. This completes the proof of Lemma A.1. QED

Next, we move to the proof of Proposition 2.1. Since $P = 1$ in a type a equilibrium, we can use (2.7) to write the functional equation for V_H as follows:

$$(1-\beta)V_H = \max_{(x_{HH}, q^g_{HH}, q^a_{HH})} x_{HH} X_{HH} \delta^{HH}_{HH} (u + q^a_{HH} - bQ^a_{HH} - \beta c).$$

Agents do not request bribes in this equilibrium; i.e., $Q^a_{HH} = 0$. If, to the contrary, $Q^a_{HH} = B$, then $x_{HH} = 0$ by Lemma A.1, and so $V_H = 0$, which violates the requirement $V_H - V_L > c$. Similarly, $X_{HH} > 0$. Given $Q^a_{HH} = 0$ and $Q^g_{HH} \geq 0$, an individual agent earns the maximum surplus $X_{HH}(u + Q^g_{HH} - \beta c)$ if he chooses $q^a_{HH} = Q^g_{HH}$ and $q^g_{HH} \geq 0$. This surplus is positive, and so agents trade with probability 1; i.e., $x_{HH} = 1$. Also, in equilibrium, $Q^g_{HH} = q^a_{HH} = Q^a_{HH} = 0$.

To complete the proof we must show that it is optimal for agents to always produce H goods. Suppose that an agent deviates to producing an L good. With $P = 1$, the deviator always meets a partner who holds an H good. By Lemma A.1, he successfully trades only if the partner does not recognize the quality of his good, and if $q^a_{LH} \leq Q^g_{HU} = 0$ and $q^g_{LH} \geq Q^a_{HU} = 0$. Thus $q^a_{LH} = 0$. Substituting these conditions and (2.7) into the functional equation for V_L yields $V_L = (1-\alpha)(u - \beta c)/[(1-\beta\alpha)(1-\beta)]$. We have $V_L < V_H - c$ if and only if $\alpha > c/u$, in which case the deviation is not profitable. QED

B. Proof of Proposition 2.2

We establish a series of Lemmas first.

Lemma B.1. $X_{LL} = 0$.

Proof. Suppose, to the contrary, that $X_{LL} > 0$. We proceed through the following steps.

Step 1. $X_{LL} > 0$ in a symmetric equilibrium only if $X_{LU} > 0$, $Q^a_{LL} = Q^g_{LU} = B$, and $Q^a_{LU} = Q^g_{LL} = 0$.

In a symmetric equilibrium, $X_{LL} > 0$ iff $x_{LL} > 0$. From the Bellman equation, for $x_{LL} > 0$, the following expected surplus associated with the decision x_{LL} must be positive:

$$\alpha X_{LL} \delta^{LL}_{LL} (q^a_{LL} - bQ^a_{LL}) + (1-\alpha) X_{LU} \delta^{LU}_{LL} (q^a_{LL} - bQ^a_{LU}).$$

Since $q^a_{LL} - bQ^a_{LL} = (1-b)Q^a_{LL} \leq 0$ in any symmetric equilibrium, the surplus is strictly positive only if the second term is positive, which requires that $X_{LU} > 0$, $q^a_{LL} = B \leq Q^g_{LU}$ and $Q^a_{LU} = 0$. Thus, in a symmetric equilibrium $Q^a_{LL} = Q^g_{LU} = B$ and $Q^a_{LU} = 0$. Then, $q^a_{LL} - bQ^a_{LL} < 0$. If

$Q_{LL}^g = B$, the agent can increase his expected surplus by deviating to $q_{LL}^g = 0 < Q_{LL}^a$, which yields $\delta_{LL}^{LL} = 0$ and neutralizes the potential loss. This cannot be an equilibrium, and so $Q_{LL}^g = 0$.

Step 2. If $X_{LL} > 0$, $Q_{LL}^a = Q_{LU}^g = B$ and $Q_{LL}^g = Q_{LU}^a = 0$, as in Step 1, then $X_{LU} > 0$ only if $X_{HU} > 0$, $Q_{HU}^g = 0$ and $Q_{HU}^a = B$.

Let us examine the decision on x_{LU}. Recalling that $X_{HL} = 0$, the expected surplus associated with the decision x_{LU} is

$$\begin{aligned} &P(1-\alpha)X_{HU}\delta_{LU}^{HU}(u + q_{LU}^a - bQ_{HU}^a) \\ &+(1-P)[\alpha X_{LL}\delta_{LU}^{LL}(q_{LU}^a - bQ_{LL}^a) + (1-\alpha)X_{LU}\delta_{LU}^{LU}(q_{LU}^a - bQ_{LU}^a)]. \end{aligned} \tag{B.1}$$

If the agent chooses $(q_{LU}^g, q_{LU}^a) = (Q_{LU}^g, Q_{LU}^a) = (B, 0)$, as in the supposed equilibrium, then $\delta_{LU}^{HU} = \delta_{LU}^{LL} = \delta_{LU}^{LU} = 1$, and the above surplus becomes

$$P(1-\alpha)X_{HU}(u - bQ_{HU}^a) - (1-P)\alpha X_{LL}bB.$$

For $x_{LU} > 0$, the surplus must be non-negative, which requires that $X_{HU} > 0$. If $Q_{HU}^g = B$, the surplus in (B.1) can be increased by choosing $q_{LU}^a = B$ rather than $q_{LU}^a = Q_{LU}^a = 0$: the choice $q_{LU}^a = B$ ($> Q_{LL}^a$) neutralizes the loss $q_{LU}^a - bQ_{LL}^a$ by inducing $\delta_{LU}^{LL} = 0$, while maintaining $\delta_{LU}^{HU} = 1$. Similarly, if $Q_{HU}^a = 0$, by choosing $q_{LU}^g = 0$ ($< Q_{LL}^a$) rather than $q_{LU}^g = Q_{LU}^g = B$, the agent can neutralize the loss $q_{LU}^a - bQ_{LL}^a$ without affecting other possible trades. Thus, for $x_{LU} > 0$, we must have $Q_{HU}^g = 0$ and $Q_{HU}^a = B$.

Step 3. If $X_{LL} > 0$, $X_{LU} > 0$, $Q_{LL}^a = Q_{LU}^g = Q_{HU}^a = B$ and $Q_{LL}^g = Q_{LU}^a = Q_{HU}^g = 0$, as in Step 2, then $X_{HU} > 0$ only if $X_{HH} > 0$, $Q_{HH}^a = 0$ and $Q_{HH}^g = B$.

With $Q_{HU}^g = 0$, Lemma A.1 implies that $Q_{LH}^a = 0$. With $Q_{LH}^a = Q_{HU}^a = B$ and $Q_{LU}^a = 0$, the expected surplus associated with the choice x_{HU} is

$$\begin{aligned} &P\alpha X_{HH}\delta_{HU}^{HH}(u + q_{HU}^a - bQ_{HH}^a - \beta c) + P(1-\alpha)X_{HU}\delta_{HU}^{HU}(u + q_{HU}^a - bB - \beta c) \\ &+(1-P)\left[\alpha X_{LH}\delta_{HU}^{LH}(q_{HU}^a - \beta c) + (1-\alpha)X_{LU}\delta_{HU}^{LU}(q_{HU}^a - \beta c)\right]. \end{aligned}$$

Under (2.1) and (2.2), the term multiplied by $(1-P)$ is negative, and so is the term $u + q_{HU}^a - bB - \beta c$. For $x_{HU} > 0$, the first term in the above surplus must be positive under the supposed equilibrium strategy $(q_{HU}^g, q_{HU}^a) = (0, B)$. For this we need $X_{HH} > 0$ and $\delta_{HU}^{HH} = 1$. For $\delta_{HU}^{HH} = 1$, in turn, we need $Q_{HH}^a \le q_{HU}^g = 0$ and $Q_{HH}^g \ge q_{HU}^a = B$. Thus, $Q_{HH}^a = 0$ and $Q_{HH}^g = B$.

Step 4. If $X_{HH} > 0$, $X_{HU} > 0$, $Q_{HU}^a = B$, and $(Q_{HH}^g, Q_{HH}^a) = (B, 0)$, as in Step 3, there is a gain for an agent to deviate to $q_{HH}^g = 0$.

With $Q_{HH}^a = 0$ and $Q_{HU}^a = B$, the expected surplus associated with the choice of x_{HH} is

$$\alpha X_{HH}\delta_{HH}^{HH}(u + q_{HH}^a - \beta c) + (1-\alpha)X_{HU}\delta_{HH}^{HU}(u + q_{HH}^a - bB - \beta c).$$

Note that $X_{HU}(u + q^a_{HH} - bB - \beta c) < 0$, for any $q^a_{HH} \geq 0$. Since $Q^a_{HU} = B$, this potential loss can be neutralized by deviating to $q^g_{HH} = 0$, which induces $\delta^{HU}_{HH} = 0$. This deviation does not change the first term of the above surplus. Thus, the deviation is profitable.

Therefore, there cannot be any symmetric equilibrium with $X_{LL} > 0$. QED

Lemma B.2. *(i) $X_{HU} = 0$ implies $X_{LU} = 0$ and $V_L = 0$; (ii) $X_{HU} > 0$ implies $X_{HH} > 0$.*

Proof. If $X_{HU} = 0$, the expected surplus associated with the choice of x_{LU} is

$$(1 - P)(1 - \alpha)X_{LU}\delta^{LU}_{LU}(q^a_{LU} - bQ^a_{LU}).$$

Since $q^a_{LU} - bQ^a_{LU}$ is negative for an exchange with $Q^a_{LU} > 0$, the best the two agents in such a match can achieve is to swap the L-quality goods and exchange no services. This yields zero surplus. By our tie-breaking rule, there is no exchange. Thus, $x_{LU} = X_{LU} = 0$. With $X_{LU} = X_{HU} = 0$, it is easy to verify that $V_L = 0$. To prove that $X_{HU} > 0$ implies $X_{HH} > 0$, suppose $X_{HU} > 0$. The expected surplus associated with the choice x_{HU} is

$$\begin{aligned}
&P\alpha X_{HH}\delta^{HH}_{HU}(u + q^a_{HU} - bQ^a_{HH} - \beta c) + P(1 - \alpha)X_{HU}\delta^{HU}_{HU}(u + q^a_{HU} - bQ^a_{HU} - \beta c) \\
&+(1 - P)\left[\alpha X_{LH}\delta^{LH}_{HU}(q^a_{HU} - bQ^a_{LH} - \beta c) + (1 - \alpha)X_{LU}\delta^{LU}_{HU}(q^a_{HU} - bQ^a_{LU} - \beta c)\right] \\
&\leq P\alpha X_{HH}\delta^{HH}_{HU}(u + q^a_{HU} - bQ^a_{HH} - \beta c) + P(1 - \alpha)X_{HU}\delta^{HU}_{HU}(u + q^a_{HU} - bQ^a_{HU} - \beta c).
\end{aligned}$$

The inequality follows from the fact that the term multiplied by $1 - P$ is non-positive under (2.1) and (2.2). For $x_{HU} > 0$, it is necessary that the above surplus be non-negative for some (q^a_{HU}, q^g_{HU}). Setting (q^a_{HH}, q^g_{HH}) to be equal to those (q^a_{HU}, q^g_{HU}), the surplus associated with the decision x_{HH} is non-negative, and so $x_{HH} > 0$. QED

Lemma B.3. *There is no bribe exchanged in an equilibrium with $0 < P < 1$ and $V_L > 0$. In particular, (i) $Q^a_{HU} = Q^g_{HU} = Q^a_{HH} = Q^a_{LH} = 0$, (ii) $X_{HH} = \Delta^{HH}_{HH} = \Delta^{HU}_{HH} = \Delta^{HH}_{HU} = \Delta^{HU}_{HU} = 1$, and (iii) $Q^a_{LU} = 0$, $X_{LU} = 1$, $\Delta^{HU}_{LU} = \Delta^{LU}_{LU} = 1$.*

Proof. For part (i), suppose first, to the contrary, that $Q^a_{HH} = B$. As shown in the proof of Lemma B.2, the expected surplus associated with the decision x_{HU} is less than or equal to

$$P\alpha X_{HH}\delta^{HH}_{HU}(u + q^a_{HU} - bQ^a_{HH} - \beta c) + P(1 - \alpha)X_{HU}\delta^{HU}_{HU}(u + q^a_{HU} - bQ^a_{HU} - \beta c).$$

Since $X_{HH} > 0$ when $X_{HU} > 0$, with $Q^a_{HH} = B$, the first term is non-positive and is strictly negative if $\delta^{HH}_{HU} = 1$. For $X_{HU} > 0$, the second term must be non-negative, and this can be achieved in a symmetric equilibrium only if $q^a_{HU} = Q^a_{HU} = 0$. It is also necessary that

$q^g_{HU} = Q^g_{HU} = 0$. This is because $u + q^a_{HU} - bQ^a_{HH} - \beta c < 0$ when $q^a_{HU} = 0$ and $Q^a_{HH} = B$; if $Q^g_{HU} = B$; a deviation to $q^g_{HU} = 0$ induces $\delta^{HH}_{HU} = 0$ and neutralizes this potential loss without affecting the possible expected surplus from trading with an HU agent. However, if $Q^g_{HU} = Q^a_{HU} = 0$ and $Q^a_{HH} = B$, we have $\delta^{HU}_{HH} = 0$, in which case the expected surplus associated with the decision x_{HH} is

$$\alpha X_{HH}\delta^{HH}_{HH}(u + q^a_{HH} - bB - \beta c) < 0.$$

This induces $x_{HH} = 0$, contradicting the result $X_{HH} > 0$. Thus, $Q^a_{HH} = 0$. Set $Q^a_{HH} = 0$ and suppose, contrary to the Lemma, that $Q^a_{HU} = B$. The expected surplus associated with the decision x_{HH} is

$$\alpha X_{HH}\delta^{HH}_{HH}(u + q^a_{HH} - \beta c) + (1-\alpha)X_{HU}\delta^{HU}_{HH}(u + q^a_{HH} - bB - \beta c).$$

This surplus is maximized by setting $q^a_{HH} = Q^g_{HH}$. Since $q^a_{HH} = Q^a_{HH} = 0$ in any symmetric equilibrium, $Q^g_{HH} = 0$. With $Q^a_{HU} = B$ and $X_{HU} > 0$, the expected surplus associated with the decision x_{HU} is strictly less than

$$P\alpha X_{HH}\delta^{HH}_{HU}(u + q^a_{HU} - \beta c).$$

For $x_{HU} > 0$, the terms (q^a_{HU}, q^g_{HU}) must ensure $\delta^{HH}_{HU} = 1$, which requires $q^a_{HU} \leq Q^g_{HH}$ $(= 0)$. This is not possible in a symmetric equilibrium with $Q^a_{HU} = B$. Therefore, $Q^a_{HU} = 0$. With $Q^a_{HH} = Q^a_{HU} = 0$, an H agent who does not know the quality of the partner's good will secure the trade with the partner with an H good no matter whether he chooses $q^g_{HU} = 0$ or B, provided that $q^a_{HU} = 0$. If $Q^g_{HU} = B$, however, Lemma A.1 implies that $Q^a_{LH} = Q^g_{HU} = B$, and the choice $q^g_{HU} = B$ would invite a trade with an L partner. That is, the term $(1-P)\alpha X_{LH}\delta^{LH}_{HU}(q^a_{HU} - bQ^a_{LH} - \beta c)$ in the surplus associated with the choice x_{HU} is negative if $q^g_{HU} = B$ and can be reduced to zero if $q^g_{HU} = 0$. The deviation does not reduce the surpluses in other types of trade that the HU agent might have. This shows that $Q^g_{HU} = 0$ in all symmetric equilibria, which implies $Q^a_{LH} = 0$ by Lemma A.1. Given that part (i) of the Lemma holds, the corresponding quantities of trade imply that $\delta^{HH}_{HH} = \delta^{HU}_{HH} = 1$. The expected surplus associated with the decision x_{HH} is strictly positive, and so $X_{HH} = 1$. Similarly, $\delta^{HH}_{HU} = \delta^{HU}_{HU} = 1$.

We now prove part (iii). Since $X_{HL} = X_{LL} = 0$, and $Q^a_{HU} = 0$, the expected surplus associated with the decision x_{LU} is

$$P(1-\alpha)X_{HU}\delta^{HU}_{LU}(u + q^a_{LU}) + (1-P)(1-\alpha)X_{LU}\delta^{LU}_{LU}(q^a_{LU} - bQ^a_{LU}).$$

Consider the choices $q^a_{LU} = 0$ and $q^g_{LU} = 0$. Since $(Q^a_{HU}, Q^g_{HU}) = (0, 0)$, these choices satisfy $q^a_{LU} \leq Q^g_{HU}$ and $q^g_{LU} \geq Q^g_{HU}$. Thus, $\delta^{HU}_{LU} = 1$, and the first term of the surplus is strictly positive.

If $Q^a_{LU} = B$, the choice $q^g_{LU} = 0$ induces $\delta^{LU}_{LU} = 0$ and neutralizes the second term of the surplus; if $Q^a_{LU} = 0$, then the choice $q^g_{LU} = 0$ generates $q^a_{LU} - bQ^a_{LU} = 0$. In both cases, the expected surplus is strictly positive, and so $x_{LU} = 1$. This argument also shows that $q^a_{LU} = Q^a_{LU} = B$ cannot be an equilibrium: if $Q^a_{LU} = B$, following the strategy $q^a_{LU} = Q^a_{LU} = B$ would generate at most zero surplus, which can be improved by setting $q^a_{LU} = 0$. With $Q^a_{LU} = 0$ and $Q^a_{HU} = 0$, it is immediately clear that $\delta^{HU}_{LU} = \delta^{LU}_{LU} = 1$. QED

Now we complete the proof of Proposition 2.2. The above Lemmas have established the trading patterns described in the Proposition. In particular, there is no bribe exchange in any equilibrium. With these trading patterns, we can simplify the value functions as (2.8). Then the requirement $0 < P < 1$, which is equivalent to $V_H - c = V_L$, becomes

$$(1-\beta)c = P[\alpha + (1-\alpha)X_{HU}]^2(u - \beta c) - (1-\alpha)X_{HU}[Pu + (1-P)\beta c]. \tag{B.2}$$

Also, the equilibrium requirement $X_{HU} > 0$ becomes

$$P[\alpha + (1-\alpha)X_{HU}](u - \beta c) \geq (1-P)\beta c. \tag{B.3}$$

If this condition holds with strict inequality, then $X_{HU} = 1$; if this condition holds with equality, then $X_{HU} \in (0,1)$. For the type b equilibrium, set $X_{HU} = 1$ in (B.2) to solve for P. Then, verify the equilibrium requirements that (B.3) holds with strict inequality, and that $0 < P < 1$. These requirements are equivalent to $c/u < \alpha < \alpha_2$. For the type c equilibrium, set (B.3) as equality, and solve this equation jointly with (B.2) for (P, X_{HU}). Imposing the equilibrium requirements $0 < P < 1$ and $0 < X_{HU} < 1$ yields $\alpha_3 < \alpha < \alpha_2$. (In fact, the requirement $0 < X_{HU} < 1$ implies $0 < P < 1$ in this case.) QED

C. Proof of Proposition 3.1

Suppose that $\alpha R > B$. We follow the following steps.

Step 1. $X_{LG} = 1$ and $Y_{GL} = 0$. If $Y_{GU} = 0$, then no bribes are exchanged in equilibrium.

Since $bB < u$, the surplus to an L agent from the exchange with a bureaucrat is positive, no matter whether the bureaucrat recognizes the L quality. Thus, $X_{LG} = 1$. Since $\alpha R > B$, the surplus to the bureaucrat in such a match is negative if he recognizes quality. Thus $y_{GL} = 0$. Suppose $Y_{GU} = 0$. If either $Y_{GH} = 0$ or $X_{HG} = 0$, then there is no exchange between private agents and the bureaucracy, in which case the Lemma is trivially true. Thus, let us suppose $Y_{GH} > 0$ and $X_{HG} > 0$. For $y_{GH} > 0$, we need $Q^a_{HG} = 0$; otherwise $q^a_{GH} - bQ^a_{HG} < 0$, and so $y_{GH} = 0$. Similarly, for $x_{HG} > 0$, we need $Q^a_{GH} = 0$. Since Q^a_{HG} and Q^a_{GH} are both zero, no bribes are exchanged.

Step 2. $Y_{GU} > 0$ implies $Q^a_{HG} = 0$ and $Y_{GH} = 1$.

Suppose $Y_{GU} > 0$. The expected surplus associated with the decision y_{GU} is

$$\begin{aligned} & PX_{HG}\delta^{HG}_{GU}(q^a_{GU} - bQ^a_{HG}) + (1-P)X_{LG}\delta^{LG}_{GU}(q^a_{GU} - bQ^a_{LG} - \alpha R) \\ \leq \ & PX_{HG}\delta^{HG}_{GU}(q^a_{GU} - bQ^a_{HG}). \end{aligned}$$

The inequality follows from the fact that $q^a_{GU} - bQ^a_{LG} - \alpha R \leq B - \alpha R < 0$. Thus, for $y_{GU} > 0$, it is necessary that $X_{HG} > 0$ and $q^a_{GU} - bQ^a_{HG} \geq 0$ for such (q^a_{GU}, q^g_{GU}) that $q^a_{GU} \leq Q^g_{HG}$ and $q^g_{GU} \geq Q^a_{HG}$. This is impossible if $Q^a_{HG} = B$. Let $Q^a_{HG} = 0$. The same quantities (q^a_{GU}, q^g_{GU}) are feasible for the choices (q^a_{GH}, q^g_{GH}), which yield a non-negative expected surplus for the choice y_{GH}. Thus, $y_{GH} = 1$.

Step 3. If $Y_{GU} > 0$, then $Q^a_{LG} = Q^g_{GU} = 0$, and $Q^g_{LG} = Q^a_{GU} = B$.

Suppose $Y_{GU} > 0$. Then the best choices for an L-agent in a match with a bureaucrat are $q^a_{LG} = Q^g_{GU}$ and $q^g_{LG} \geq Q^a_{GU}$. If $q^a_{GU} = 0$, this demand can be met by the private agent, yielding $\delta^{LG}_{GU} = 1$. In this case, the surplus associated with the choice y_{GU} is strictly negative, contradicting the supposition $Y_{GU} > 0$. Thus, $q^a_{GU} = Q^a_{GU} = B$ in equilibrium, which implies $Q^g_{LG} = B$. If $Q^g_{GU} = B$, then $Q^a_{LG} = q^a_{LG} = B$ from the decision on q^a_{LG}. Again, this generates $\delta^{LG}_{GU} = 1$ and a loss to the bureaucrat in an exchange with an L agent. By deviating to $q^g_{GU} = 0$ $(< Q^a_{LG} = B)$, the bureaucrat can induce $\delta^{LG}_{GU} = 0$ and neutralize this loss. The deviation does not reduce the chance for the bureaucrat to exchange with an H agent since $Q^a_{HG} = 0$ as shown in Step 2. Therefore, in equilibrium it must be true that $Q^g_{GU} = 0$, which induces $Q^a_{LG} = 0$.

Step 4. If $Y_{GU} > 0$, then $Q^a_{GH} = Q^g_{HG} = 0$, which yields a contradiction.

With $Y_{GU} > 0$, we have $Q^a_{GU} = B$, as shown in Step 3, and so $u + q^a_{HG} - bQ^a_{GU} - \beta c < 0$. The expected surplus associated with the choice of x_{HG} is at most $\alpha Y_{GH}\delta^{GH}_{HG}(u + q^a_{HG} - bQ^a_{GH} - \beta c)$. Since $Y_{GH} = 1$ when $Y_{GU} > 0$, the above surplus is strictly negative if $Q^a_{GH} = B$. This would imply $x_{HG} = 0$, which has been shown above to be inconsistent with trade between the private sector and the bureaucracy. Thus, $Q^a_{GH} = 0$. Next, notice that when $Q^a_{GH} = 0$, the best choice of q^g_{HG} for a private agent holding an H good in a match with a bureaucrat is $q^g_{HG} = 0$. This is because the private agent in such a match gets a positive surplus from trade when the government recognizes H, and a loss when the government does not recognize H. The choice $q^g_{HG} = 0 < Q^a_{GH}$ keeps the possible gain and neutralizes the possible loss. In contrast, the choice $q^g_{HG} = B$ invites trade in both cases and reduces the expected surplus. Now that $Q^g_{HG} = 0 < q^a_{GU}$, the surplus associated with the decision y_{GU} is strictly negative, contradicting the supposition $Y_{GU} > 0$. This completes the proof of the Lemma. QED

D. Proof of Proposition 3.2

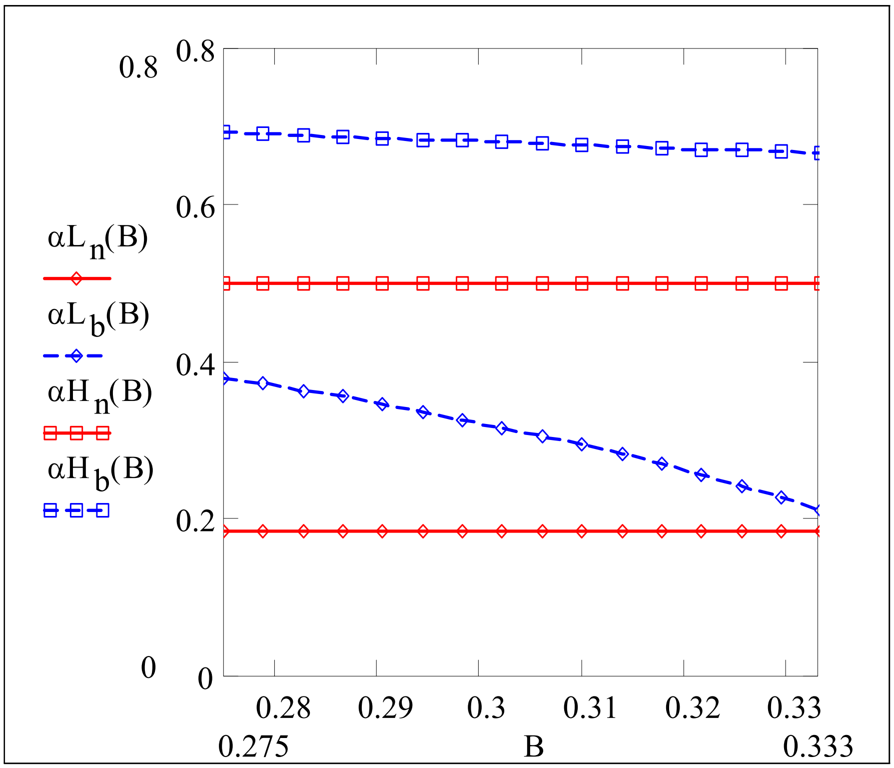
Consider a bureaucrat's decision on y_{GL}. Given $Q^g_{LG} = B$ and $Q^a_{LG} = 0$, the surplus associated with y_{GL} is $X_{LG}\delta^{LG}_{GL}q^a_{GL}$, where $\delta^{LG}_{GL} = 1$ iff $q^a_{GL} \le Q^g_{LG} = B$ and $q^g_{GL} \ge Q^a_{LG} = 0$. By choosing the supposed equilibrium quantity $q^a_{GL} = Q^a_{GL} = B$, the bureaucrat obtains a surplus $X_{LG}B$; if he deviates to $q^a_{GL} = 0$, the surplus is 0. Since $X_{LG} = 1$, there is no incentive for the bureaucrat to deviate from the equilibrium choices $q^a_{GL} = B$ and $y_{GL} = 1$. Also, following the equilibrium choice $q^g_{GL} = Q^g_{GL} = 0$ is not dominated by other choices (although $q^g_{GL} = B$ yields the same surplus). Similarly, consider a bureaucrat's decision on y_{GU}. Since $Q^a_{HG} = Q^a_{LG} = 0$, $\max\{Q^a_{HG}, Q^a_{LG}\} = 0 \le q^g_{GU}$. Also, $q^a_{GU} \le B = Q^g_{LG}$. Thus $\delta^{LG}_{GU} = 1$ for any choice $q^a_{GU}, q^g_{GU} \in \{0, B\}$. Noting that $X_{LG} = 1$, as supposed in the equilibrium, the surplus associated with the choice y_{GU} is $PX_{HG}\delta^{HG}_{GU}q^a_{GU} + (1-P)q^a_{GU}$. This is 0 if the bureaucrat chooses to deviate to $q^a_{GU} = 0$. If the bureaucrat adheres to the supposed equilibrium strategy $q^a_{GU} = Q^a_{GU} = B$, the surplus is at least $(1-P)B > 0$. Thus, $q^a_{GU} = B$ and so $y_{GU} = 1$, as in the supposed equilibrium. Like the quantity q^g_{GL}, the decision $q^g_{GU} = 0$ is consistent with the bureaucrat's incentive, but $q^g_{GU} = B$ yields the same surplus. Next, consider a private agent's decision on x_{LG}. With $Y_{GL} = Y_{GU} = 1$ and the quantities $(Q^a_{GL}, Q^a_{GU}) = (B, B)$, the surplus associated with x_{LG} is

$$\alpha\delta^{GL}_{LG}(u + q^a_{LG} - bB) + (1-\alpha)\delta^{GU}_{LG}(u + q^a_{LG} - bB).$$

Since $Q^g_{GL} = Q^g_{GU} = 0$ and $Q^a_{GL} = Q^a_{GU} = B$, the above surplus is zero if either $q^a_{LG} = B$ or $q^g_{LG} = 0$. In contrast, if $q^a_{LG} = 0$ and $q^g_{LG} = B$ as described in equilibrium, then $\delta^{GL}_{LG} = \delta^{GU}_{LG} = 1$, and the surplus is $u - bB > 0$. Thus, $q^a_{LG} = 0$, $q^g_{LG} = B$, and $x_{LG} = 1$. Finally, consider a meeting between a bureaucrat and a private agent with a (known) H good. Given that $Q^g_{HG} = Q^a_{HG} = 0$, clearly the choices $q^a_{GH} = q^g_{GH} = 0$ maximize the bureaucrat's surplus associated with y_{GH} and yield $y_{GH} = 1$. Given $Q^g_{GH} = Q^a_{GH} = Q^g_{GU} = 0$, $Q^a_{GU} = B$ and $Y_{GH} = Y_{GU} = 1$, the choices $q^a_{HG} = q^g_{HG} = 0$ yield $\delta^{GH}_{HG} = 1$, $\delta^{GU}_{HG} = 0$, and a surplus $\alpha(u - \beta c) > 0$ for the private agent associated with the choice x_{HG}. A deviation to $q^a_{HG} = B$ reduces the surplus to 0. A deviation to $q^g_{HG} = B$ induces $\delta^{GU}_{HG} = 1$ and reduces the surplus to

$$\alpha(u - \beta c) + (1-\alpha)(u - (b-1)B - \beta c) < \alpha(u - \beta c),$$

where the inequality follows from (2.2). Thus, the optimal choices for the private agent in such a meeting are $q^a_{HG} = q^g_{HG} = 0$ and $x_{HG} = 1$. QED



The equilibrium with bribery exists if $\alpha L_b(B) < \alpha < \alpha H_b(B)$,
The equilibrium without bribery exists if $\alpha L_n(B) < \alpha < \alpha H_n(B)$.

Figure 1. Co-existence of equilibria

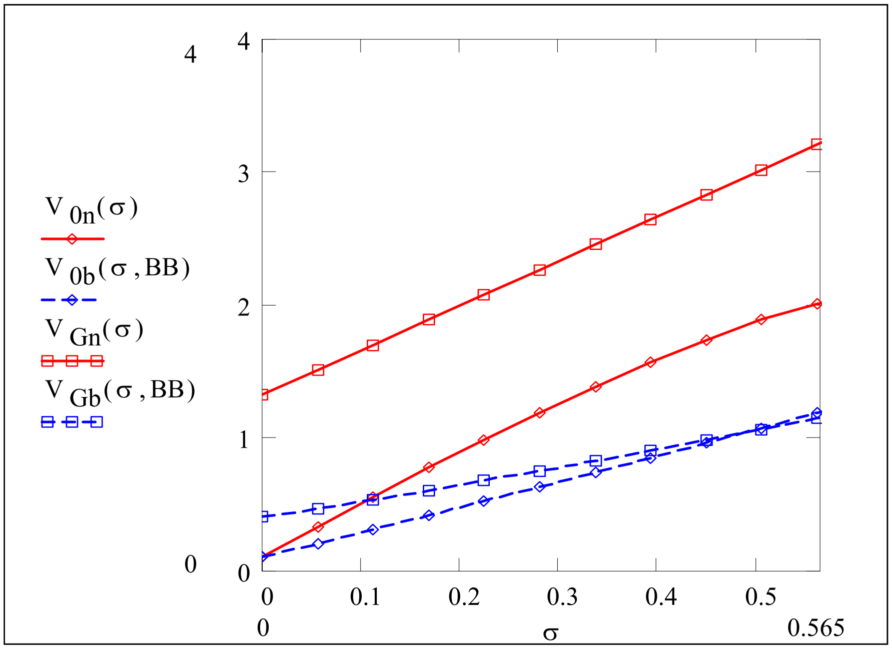


Figure 2a Dependence of welfare on σ

σ: fraction of newborns who are bureaucrats; BB: a fixed size of bribe; V_0: private agent's value function; V_G: bureaucrat's value function; b (n): subscript for the equilibrium with bribery (without bribery); P: prob. for a private agent to produce high-quality goods; h: prob. for a private agent holding a high-quality good to get a high-quality good in exchange; G: stationary size of the bureaucracy.

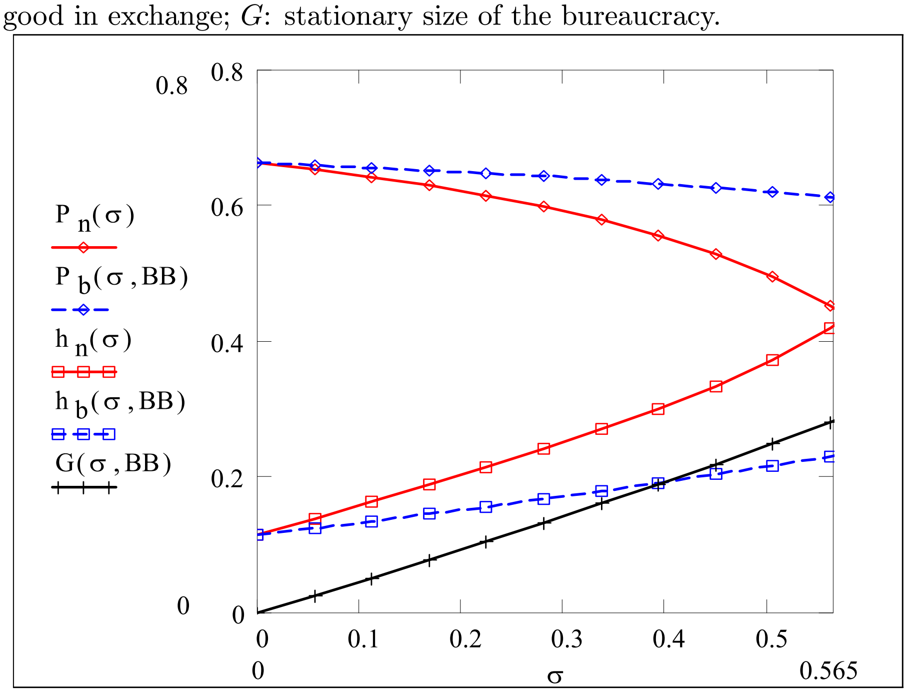


Figure 2b Dependence of P and h on σ

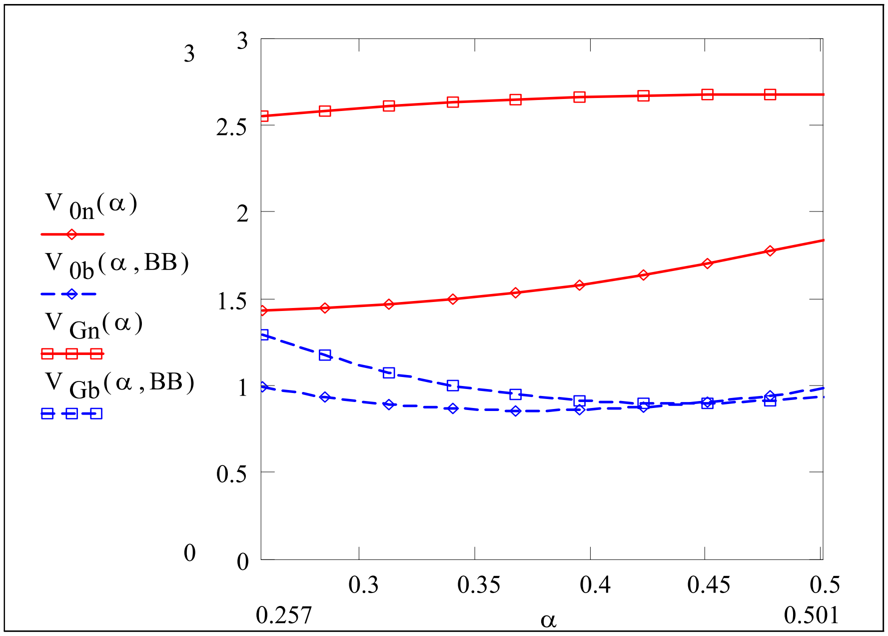


Figure 3a Dependence of welfare on α

α: prob. of recognizing a good's quality; BB: a fixed size of bribe; V_0: private agent's value function; V_G: bureaucrat's value function; b (n): subscript for the equilibrium with bribery (without bribery); P: prob. for a private agent to produce high-quality goods; h: prob. for a private agent holding a high-quality good to get a high-quality good in exchange; G: stationary size of the bureaucracy.

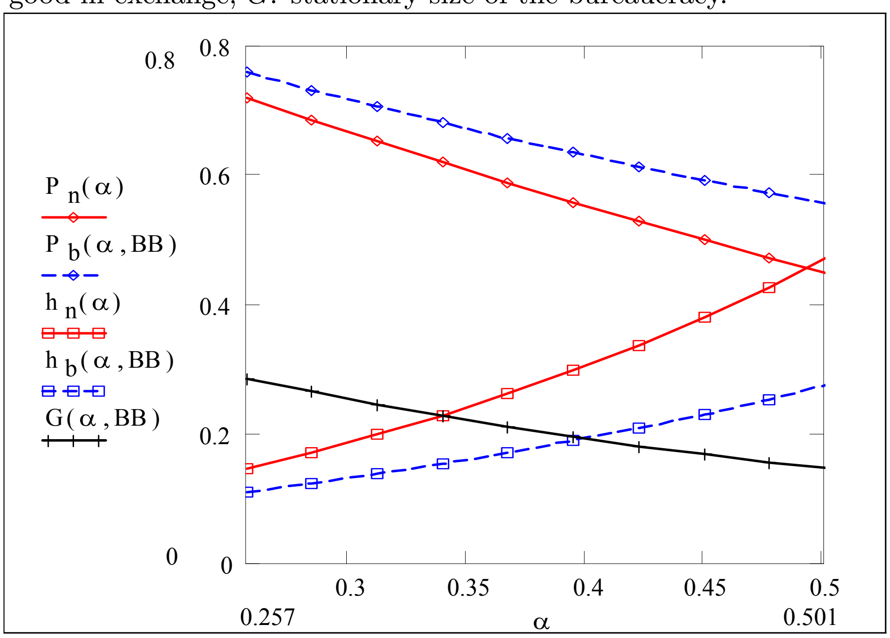


Figure 3b Dependence of P and h on α