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## Self-Promoting Investments

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## **Abstract**

What is the impact on human capital investment when a worker's ability and investments are observed by the labour market only when the worker invests in self-promoting activities? When firms pay spot market wages, high ability workers overinvest in self-promotion. There is no employment contract that attains full efficiency. Constrained efficiency is attained when employment bonds are feasible. The contract that both attains constrained efficiency and minimizes the bond posted offers (i) severance payments, (ii) strategically matching outside offers and (iii) a minimum wage.

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## 1. Introduction

Workers who acquire certain skills on the job make themselves more valuable to their employers, and also more attractive to other firms. Alternative employers who recognize the workers' enhanced expertise may try to bid them away by offering them wage increases. The possibility of workers' investments in certain kinds of human capital leading to such alternative offers generates tension for a firm. On the one hand, an employer would like its employees to acquire skills that enhance the employees' performance on the job, but on the other hand, it would prefer that other employers not observe these skills. When other employers observe certain kinds of skills and not others, workers have a tendency to overinvest in the skills that might be observed relative to those that are not observed. The objective of this paper is to understand the employment relationship that develops under such circumstances.

Since investment decisions of workers are distorted when self-promoting activities are available, we investigate employment contracts that are more efficient than is the spot market contract. The infrequent arrival of outside offers can be exploited by the firm in its design of an optimal contract. In general, full efficiency cannot be achieved. We find that a constrained efficient contract has two essential characteristics: the first is a promise by the firm to match outside offers to any worker who has invested efficiently; the second is a commitment by the firm not to match outside offers of any worker who is revealed to have invested inefficiently. An implication of the firm's commitment is that workers who invest inefficiently generate only low outside offers. The firm's promise and commitment provide incentives for the workers to invest efficiently.

Our work is related to and complements three literatures. In the first of these, research effort has been devoted to understanding the contracts and incentive schemes that emerge when employees may invest in human capital that is valuable both to their current employer and to alternative employers. Carmichael (1983,1988), Malcomson (1984), Waldman (1984,1990), Milgrom and Oster (1987),

Kahn and Huberman (1988), MacLeod and Malcomson (1988), Ricart I Costa (1988), Gibbons and Katz (1991), Bernhardt and Scoones (1993), and Bernhardt (1995) explore how employers manipulate personnel systems when the knowledge they have about their own employees is superior to that of other employers. Our paper complements this literature by looking at the case in which workers may promote themselves directly to the market. In addition, in our initial model, we assume that the knowledge that the incumbent firm has about its employees is identical to that of outside firms. The symmetric information assumption is realistic when, for example, a firm has many branches<sup>1</sup> or when workers are highly specialized<sup>2</sup>. We relax the symmetric information assumption in Section 3.2.

Our model is also related to models of multi-tasking and in uence activities (Milgrom and Roberts (1982) and Milgrom (1982), Holmstrom and Milgrom (1991)). While the basic multi-tasking model ignores the effect of time use on outside options, we apply the multi-tasking model in an environment in which time use affects outside options.

Lastly, our model also fits into the literature in which a forcing contract is used in a principal-agent problem to obtain a particular effort on the part of the agent. (For example, see Gale and Hellwig (1985).)

We now provide the details of the basic model.

## 2. The Model

Workers in a competitive occupation work for two periods and maximize their expected present value of income. The proportion,  $\pi^h$ , of the workers have high ability,  $\theta = h$ , and the proportion,  $\pi^l$ , have low ability,  $\theta = l, l < h$ .

When new workers enter an occupation, their abilities are unknown. At this stage, the worker and firm negotiate the terms of the worker's employment. After

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<sup>1</sup>If a firm is geographically separated from head office, local competitors may have as good an idea of the branch manager's efforts and skills as does head office.

<sup>2</sup>If workers are highly specialized, a manager may not be able to evaluate the contribution of each specialist as well as other specialists in competing firms.

the agreement is made, the employee works at the firm. Workers learn their ability on the job at the beginning of the first period. The employee's knowledge is private. Once workers know their ability, they undertake investment in the first period. This investment affects a worker's productivity in the second period. Investment may also affect a worker's visibility in the labour market.

Output in the first period, denoted by  $Z$ , is independent of a worker's type and investment decision. Let the worker have one unit of time in the first period to divide between two investment activities, labelled  $V$  and  $I$ . (We address the issue of costly investment in a later section.) Each activity enhances the productivity of the worker in the second period. Suppose that a worker of type  $\theta$  invests time  $t \in [0, 1]$  on  $V$  and time  $(1 - t)$  on  $I$ . The value of a worker's investment in the two activities is aggregated into the second period's output,  $y(t, \theta) = Y(V(t, \theta), I(1 - t, \theta))$ . The output,  $y(t, \theta)$ , is assumed to be increasing in  $\theta$ , concave in  $t$ , and single peaked in  $t$ . (If  $H$  is a function of several variables we let  $H_x$  denote its partial derivative with respect to  $x$ .) The derivative  $y_t(t, \theta)$  is assumed to be increasing in  $\theta$ . Note that  $t$  and  $\theta$  are unobserved by employers in the first period.

$V$  encompasses activities (like networking) that simultaneously are valued by the firm and enhance a worker's chance of being seen to be valuable. At the beginning of the second period, a worker's self promoting investments may result in the revelation of a worker's investment and type to the inside firm and an outside firm. When revelation occurs,  $V(t, \theta)$ ,  $I(1 - t, \theta)$  and thus  $t$ ,  $\theta$  and  $y(t, \theta)$ , the second period's output, are revealed to both the inside firm and an outside firm.

Let  $p(t, \theta)$  be the probability that a worker who invests  $t$  on  $V$  is revealed to the inside and an outside firm. The probability,  $p(t, \theta)$ , is assumed to be increasing and concave in its two arguments. The derivative  $p_t(t, \theta)$  is assumed to be increasing in  $\theta$ .

We assume that the value of a worker to a firm depends on the match between the firm and worker as well as on  $y(t, \theta)$ , so that there is an idiosyncratic component to a firm's valuation of a worker. A worker, whose output is  $y(t, \theta)$ , is

valued at  $y(t, \theta) + x$  by the inside firm, and is valued at  $y(t, \theta) + z$  by an outside firm, where both  $x$  and  $z$  are distributed independently and identically on  $[-\epsilon, \epsilon]$  according to the distribution  $F$ . The density of  $F$  is atomless and continuous on  $[-\epsilon, \epsilon]$  and is symmetric about zero. The mean of  $F$  is zero. Different valuations occur due to idiosyncratic differences across firms. We refer to  $x$  and  $z$  as inside and outside idiosyncratic valuations respectively. We refer to  $y(t, \theta) + x$  and  $y(t, \theta) + z$  as inside and outside valuations respectively.

We compare the spot market outcome with a constrained efficient outcome and a contracting outcome.

## 2.1. The Constrained Efficient Allocation of Effort

The efficient solution maximizes the sum of the second period gains to three parties: the inside firm, the worker, and the outside firm. However, it may be impossible for the inside firm and worker to extract all the gains from the outside firm. We are interested in the highest expected second period income that a worker and inside firm (hereafter referred to as the team) can achieve. Under the assumption that the valuations are known to both the team and the outside firm once an offer is made, the final matches are efficient and the surplus from trade<sup>3</sup>,  $\max\{z - x, 0\}$ , is split in some way between the raider and the team. The exact split of the surplus depends on strategic issues that arise between the team and the outside firm. In the case that the team never gains any of the surplus from trade with the outside firm, the best that the team can obtain, is  $\max_t y(t, \theta) + E\{x\} = \max_t y(t, \theta)$  since  $E\{x\} = 0$  where  $E$  denotes the expectation operator with respect to  $F$ . The surplus becomes available with probability  $p(t, \theta)$  when the worker invests  $t$  in activity  $V$ . In the case that the team gains all of the surplus from trade with the outside firm, the best that the worker can obtain is  $\max_t (y(t, \theta) + E\{x\} + p(t, \theta)E \max\{z - x, 0\}) = \max_t (y(t, \theta) + p(t, \theta)E \max\{z - x, 0\})$ . In general, in the case that the team extracts the fraction  $s$  of the maximum surplus available, the best that the team can obtain is the maximum, over  $t \in [0, 1]$ , of

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<sup>3</sup>Trade occurs when the worker leaves the incumbent firm to work for the raider.

$$y(t, \theta) + p(t, \theta)sE \max\{z - x, 0\}$$

Consequently, the constrained efficient investment,  $T^\theta(s)$ , satisfies

$$(y_t(t, \theta)) + (p_t(t, \theta))sE \max\{z - x, 0\} = 0 \quad (2.1)$$

The derivative of equation 2.1 is used in a straightforward fashion to prove the following proposition under the assumptions that  $y$  and  $p$  are concave in  $t$  and that  $p$  increases in  $t$ .

**Proposition 2.1.** *The constrained efficient solution  $T^\theta(s)$  increases in  $s$ .*

The intuition is as follows. The more of the surplus over the output that the team can obtain through an outside offer, the more effort the worker exerts on  $V$ , the self-promoting activity.

Recall that we have assumed that  $y(t, \theta)$  is concave in  $t$ ,  $p(t, \theta)$  increases in  $t$ , and that both  $p_t(t, \theta)$  and  $y_t(t, \theta)$  increase in  $\theta$ . Thus, the solution to 2.1 increases in  $\theta$ , i.e.  $T^h(s) > T^l(s)$ , so that the more able worker allocates more effort to the visible activity than does the less able. One implication is that, at the constrained efficient outcome, the probability that a worker receives an outside offer, is higher if the worker is of type  $h$  rather than  $l$ , i.e.,  $p(T^h(s), h) > p(T^l(s), l)$ . Though the presence of idiosyncratic matching affects the worker's investment decision so that  $T^\theta(s)$  does not maximize output,  $y(t, \theta)$ , we assume that the effect of matching is not so great as to invert the relationship of the output of workers of differing ability. Thus, we assume that heterogeneity in general ability is large relative to that in firm matches so that the constrained efficient output of a worker is higher if the worker is more able (i.e.,  $y(T^h(s), h) > y(T^l(s), l)$ ).

## 2.2. The Spot Market Solution

In order to characterize the spot market solution we need to analyze the bidding process that begins once an outside offer is made. Assume that a worker is earning

a wage of  $w$  if no outside offers have been made. Once a worker shows an outside offer to the incumbent firm, a bidding process begins. We model this bidding process as an ascending bid oral auction with two bidders who have individual valuations. The equilibrium outcome of such a bidding process is that the worker goes to the firm with the higher idiosyncratic valuation and is paid the lower valuation. In this case, a worker presents only initial offers that are above the current wage  $w$ . In addition, outside firms that value the worker above  $w$  make an initial offer that equals the current wage. Thus, with probability  $(1 - p(t, \theta))$ , there is no outside bidder and the worker obtains  $w$ . With probability  $p(t, \theta)$ , there is an outside bidder and the worker obtains  $y(t, \theta) + \min\{x, z\}$  if it exceeds  $w$ .

### 2.2.1. Spot Market Investment in Activity V

Given the current base wage  $w$ , a worker's income is at least as great as  $w$ . With probability  $p(t, \theta)$ , a worker can augment the base wage by the amount  $y(t, \theta) + \min\{x, z\} - w$  whenever this amount is positive. Thus, a worker maximizes expected income by investing, in activity  $V$ , time,  $\tau^\theta(w)$  equal to

$$\arg \max(w + p(t, \theta)E(\max\{y(t, \theta) + \min\{x, z\} - w, 0\}))$$

so that  $\tau^\theta(w)$  satisfies

$$(p_t(t, \theta))E(\max\{y(t, \theta) + \min\{x, z\} - w, 0\}) + p(t, \theta)(E_{R(\theta)}y_t(t, \theta)) = 0 \quad (2.2)$$

where  $E_{R(\theta)}$  denotes the expectation operator conditional on the domain  $R(\theta)$  and  $R(\theta)$  denotes the domain over which  $x, z \in [-\epsilon, \epsilon]$  satisfy

$$y(t, \theta) + \min\{x, z\} - w > 0$$

If we let  $\chi^\theta(t, w) = \arg_{r \in [-\epsilon, \epsilon]} \min \delta(y(t, \theta) + r, w)$ , where  $\delta$  is Euclidean distance, then the domain  $R(\theta)$  can be written as  $R(\theta) = [\chi^\theta(t, w), \epsilon] \times [\chi^\theta(t, w), \epsilon]$ .



**Proposition 2.2.** *The worker in the spot market over-invests in  $V$  relative to the constrained efficient solution when the team's share of the gains to trade is zero. The size of the over-investment diminishes as the share of the maximum gains to trade increases.*

**Proof.** Since the left-hand side of equation 2.2 is positive when evaluated at  $t = T^\theta(0)$  (due to the facts that  $y_t(T^\theta(0), \theta) = 0$ , and  $p_t(t, \theta) > 0$  for all  $t \in [0, 1]$ ), it is immediate that the worker over-invests in activity  $V$  in the spot market relative to the constrained efficient level  $T^\theta(0)$  when the team extracts the fraction 0 of the maximum gains to trade with the outside firm. An appeal to the previous proposition completes the proof. ■

### 2.2.2. Spot Market Base Wage

We first find the wage in the second period and then work backwards to find the market-clearing wage in the first period. All workers who either receive an offer that is less than the base wage, or who do not have the chance to reveal themselves credibly, receive the base wage. The base wage must equal the expected output of a randomly chosen worker who receives such a wage. If the base wage is greater than the expected output, the firm is losing money on those who receive the base wage. If it is less, another firm can offer more than the base wage to a worker who has not been revealed and be assured, on average, of attracting someone of higher value than their offer. The base wage is received by a worker of type  $\theta$  who invests  $\tau^\theta(w)$  in  $V$  and who either did not receive an offer (this happens with probability  $(1 - p(\tau^\theta(w), \theta))$ ) or who received an offer that was below the base wage (this happens with probability  $p(\tau^\theta(w), \theta)F(\chi^\theta(\tau^\theta(w), w))$ ).

Let

$$S(w) = \frac{\sum_{\theta \in \{h,l\}} \pi^\theta y(\tau^\theta(w), \theta) [1 - p(\tau^\theta(w), \theta) + p(\tau^\theta(w), \theta) F(\chi^\theta(\tau^\theta(w), w))]}{\sum_{\theta \in \{h,l\}} \pi^\theta [1 - p(\tau^\theta(w), \theta) + p(\tau^\theta(w), \theta) F(\chi^\theta(\tau^\theta(w), w))]}$$

so that  $S(w)$  is the conditional expected output of a randomly chosen worker who receives the base wage  $w$ . The equilibrium spot market base wage,  $w_0$ , satisfies  $w_0 = S(w_0)$ . By continuity, there is always a solution to  $w - S(w) = 0$ . Note that

$$y(\tau^l(w_0), l) < w_0 < y(\tau^h(w_0), h) \quad (2.3)$$

Now, we determine the first period wage. The firm makes zero expected profits on workers who receive  $w_0$ . However, the firm makes positive profit on those workers who receive an outside offer from an outside firm whose valuation is higher than  $w_0$  and whose idiosyncratic valuation  $z$  is less than the inside idiosyncratic valuation  $x$ . In order to clear the market<sup>4</sup> for employment in the first period, competitive firms offer their workers a first period wage<sup>5</sup> equal to

$$Z + \sum_{\theta \in \{h, l\}} \pi^\theta p(\tau^\theta(w_0), \theta) \int_{\chi^\theta(\tau^\theta(w), w)}^\varepsilon \int_z^\varepsilon (x - z) dF(x) dF(z)$$

### 2.3. Constrained Efficient Contract

We now consider long-term, constrained-efficient contracts in which workers can post a bond in the first period. If profits in the second period are expected to be negative, workers can choose to post a bond,  $B$ , in the first period, that is earn  $Z - B$ , in order to render the constrained efficient contract attractive to the firm. If there is no bound to the bond that workers can pay in the first period, there are many contracts that are constrained efficient. For example, suppose that the firm guarantees the highest potential wage that any worker may receive, to all workers in the second period. All workers are then indifferent among their choices of  $t \in [0, 1]$ . It is then optimal for workers of type  $\theta$  to choose the constrained efficient amount of investment. However, such a contract may not achieve the constrained efficient outcome when effort is costly.

In addition, under such a contract, the firm loses money on every worker in the second period. The bond that workers must pay in the first period to sustain this contract may be substantial. If workers have limited access to capital markets, this contract may not be feasible. We now find the minimum bond associated with

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<sup>4</sup>We note that the raider also makes positive profits on those workers whose idiosyncratic valuations are  $z > x$ . We assume that search costs dissipate the profits.

<sup>5</sup>We set the interest rate to zero for convenience.

a contract that achieves constrained efficiency. Conceptually, it is easier to look at the problem under the assumption that the firm buys the services of the worker and then sells the services of the worker to a raider whenever there are gains to trade. The class of contracts that we consider stipulates payment to the worker as a function of the information that the firm has about the worker. Either the worker has an offer, in which case the firm knows  $z, x, \theta$ , and the investment level; or, the worker has no offer. Thus, a contract needs to stipulate what the worker receives in each case.

Let  $\mathfrak{R}$  be the set of real numbers. We consider the set of contracts  $\mathcal{C} = \{(w, C^h(z, x), C^l(z, x)) : w \in [0, \infty), C^\theta(z, x) \in \mathfrak{R}, \theta \in \{h, l\}\}$ . The interpretation of a contract in  $\mathcal{C}$  is as follows. The employer offers to pay (1)  $w$  to all who either do not reveal themselves or reveal themselves to be of type  $\theta$  with investment  $t \neq T^\theta(s)$ , (2)  $w + C^\theta(z, x)$  to all who reveal themselves to be of type  $\theta$  with output  $y(T^\theta(s), \theta)$  and idiosyncratic valuations  $(z, x)$ . Note that, by the terms of any contract in  $\mathcal{C}$ , a worker can earn at least  $w$  by never revealing an outside offer, so that consistency requires  $w + C^\theta(z, x) \geq w$  for  $\theta \in \{h, l\}$ . Thus, the relevant set of contracts is  $\mathcal{C}^+ = \{(w, C^h(z, x), C^l(z, x)) : w, C^h(z, x), C^l(z, x) \in [0, \infty)\}$ . We can interpret  $w$  to be a base wage;  $C^\theta(z, x)$ , a counteroffer (over and above the base wage) for  $\theta \in \{h, l\}$ ; and  $y^\theta(T^\theta(s), \theta)$ , a target for  $\theta \in \{h, l\}$  that dictates who receives the counteroffer among those who receive an outside offer.

Each contract in  $\mathcal{C}^+$  is associated with a bond payment in the initial period. We look for the contract that minimizes the bond payment among incentive compatible, individually rational, constrained efficient contracts in  $\mathcal{C}^+$ .

In order that a contract in  $\mathcal{C}^+$  elicit the constrained efficient outcome, three sets of individual rationality constraints must be satisfied. Firstly, workers must want to invest in the constrained efficient level of each activity. Secondly, workers who invest constrained efficiently must want to accept the payments from the firm rather than negotiate separately with a raider. Thirdly, workers must be sorted efficiently among firms.

The first individual rationality constraint requires that workers choose the con-

strained efficient level of investment in  $V$ . A worker who makes the constrained efficient investment earns at least  $w$  under a contract in  $\mathcal{C}^+$  since the counteroffer  $C^\theta(z, x) \geq 0, \theta \in \{h, l\}$ . A worker who invests inefficiently makes at most  $w$  since a raider need not offer more than  $w$  to a worker who receives no counteroffers. Thus, a worker prefers to make the constrained efficient investment when facing a contract in  $\mathcal{C}^+$ .

The second individual rationality constraint requires that workers have no incentive to negotiate separately with the outside firm when the opportunity arises. For  $\theta \in \{h, l\}$ , let  $p^\theta$  denote  $p(T^\theta(s), \theta)$ , the probability that an offer is received given that a worker has chosen the constrained efficient investment, and let  $y^\theta$  denote  $y(T^\theta(s), \theta)$ , the output given the constrained efficient investment. Under a contract in  $\mathcal{C}^+$ , the worker is paid  $w + C^\theta(z, x)$  whenever the worker is revealed to have produced  $y^\theta$ . In the event that  $z > x$ , the outside firm obtains the worker and pays  $y^\theta + x + s(z - x)$  to the inside firm. If instead,  $z < x$ , the worker remains with the inside firm and the outside firm is willing to pay  $y^\theta + z$  to obtain the worker. Now recall that the worker can always earn  $w$  by not revealing the existence of an outside offer. Consequently, the required individually rational constraint is that  $w + C^\theta(z, x) \geq \max\{w, y^\theta + \min\{z, x\} + s \max\{z - x, 0\}\}$ .

The last individually rational constraint requires that workers who make the constrained efficient investment are sorted efficiently. Recall that when a worker is revealed with idiosyncratic valuations  $z > x$ , the worker is valued more highly by the raider than by the inside firm; when  $z < x$ , the worker is less highly valued by the raider. When a worker of type  $\theta$  is revealed to have invested  $T^\theta(s)$  and  $z > x$ , a contract in  $\mathcal{C}^+$  that achieves efficient sorting must stipulate that the worker leaves; when  $z < x$ , the contract must stipulate that the worker stays. Any worker who does not obtain an offer remains at the firm. Therefore, a contract in  $\mathcal{C}^+$  achieves efficient sorting conditional on the worker having made the constrained efficient investment provided that it stipulates that the worker leave when it is efficient to do so.

If the base wage is to be effective, the base wage must be greater than or

equal to the expected output of those who receive it. Otherwise, an outside firm can offer more than the base wage to a random worker who receives it and expect to earn a profit. Let  $E^C(w)$  represent the conditional expected output of those who receive the base wage,  $w$ , and stay at the firm when the constrained efficient outcome is achieved using a contract  $(w, C^h, C^l) \in \mathcal{C}^+$ . Thus,  $E^C(w)$  is the expected output of workers of type  $\theta \in \{h, l\}$  who either do not receive a financial offer or who are revealed with idiosyncratic valuations  $z < x$ , such that  $C^\theta(z, x) = 0$ . Let  $G^C(w)$  be the probability, conditional on revelation, that a worker is revealed with idiosyncratic valuations  $z < x$  such that  $C^\theta(z, x) = 0$ . Thus,

$$E^C(w) = \frac{\sum_{\theta \in \{h, l\}} y^\theta \pi^\theta [1 - p^\theta + p^\theta G^C(w)]}{\sum_{\theta \in \{h, l\}} \pi^\theta [1 - p^\theta + p^\theta G^C(w)]}$$

It remains for us to discuss the bond associated with a constrained efficient contract in  $\mathcal{C}^+$ . For any contract in  $\mathcal{C}^+$ , the bond posted by a worker in the first period equals the expected loss of the firm in the second period. The bond equals the difference between the firm's expected expenditure and its expected revenue. The firm's expected expenditure is the payment to the worker. The firm's expected revenue is the expected revenue from the worker's services. This revenue consists of internal production plus the sale of the worker's services to a raider whenever it is profitable to do so.

Consider the following problem (hereafter referred to as problem 1).

$$\begin{aligned} \min_{(w, C^h, C^l) \in \mathcal{C}^+} B(w, C^h(z, x), C^l(z, x)) = \\ \sum_{\theta \in \{h, l\}} \pi^\theta [w + p^\theta EC^\theta(z, x)] \\ - \sum_{\theta \in \{h, l\}} \pi^\theta [y^\theta + p^\theta \int_{-\varepsilon}^{\varepsilon} \int_{-\varepsilon}^z s(z - x) dF(x) dF(z)] \end{aligned}$$

subject to the base wage constraint

$$w \geq E^C(w)$$

and the individual rationality constraints

$$w + C^\theta(z, x) \geq \max\{w, y^\theta + \min\{z, x\} + s \max\{z - x, 0\}\}, \theta \in \{h, l\}$$

For  $\theta \in \{h, l\}$ , let  $C^{\theta*}(z, x, w) = \max\{w, y^\theta + \min\{z, x\} + s \max\{z - x, 0\}\} - w$ . Note that  $C^{\theta*}$  decreases in  $w$  until  $w = y^\theta + \min\{z, x\} + s \max\{z - x, 0\}$ .

**Proposition 2.3.** *The solution to problem 1 is*

$$w^* = \min\{w \geq 0 : w = \mathbf{E}^{C^*}(w)\}, C^{\theta*}(z, x, w^*), \theta \in \{h, l\}$$

where

$$\mathbf{E}^{C^*}(w) = \frac{\sum_{\theta \in \{h, l\}} y^\theta \pi^\theta [1 - p^\theta + p^\theta (\frac{1}{2}) F_{\chi^\theta}(T^\theta(s), w)]}{\sum_{\theta \in \{h, l\}} \pi^\theta [1 - p^\theta + p^\theta (\frac{1}{2}) F_{\chi^\theta}(T^\theta(s), w)]}$$

**Proof.** We cannot use the Kuhn-Tucker conditions to find a solution for problem 1 as we have a continuum of constraints. However, since the objective function is linear and increasing in its choice variables we can provide a direct proof. Note that the bond increases in  $w$  and  $C^\theta(z, x)$ . No feasible triple  $(w, C^h(z, x), C^l(z, x))$  such that

$$C^\theta(z, x) > \max\{w, y^\theta + \min\{z, x\} + s \max\{z - x, 0\}\} - w$$

for any  $\theta \in \{h, l\}$  can be optimal since the feasible triple  $(w, C^{h*}(z, x, w), C^{l*}(z, x, w))$  is associated with a lower bond than is  $(w, C^h(z, x), C^l(z, x))$ . This is immediate since the bond increases in  $C^\theta(z, x)$ , for  $\theta \in \{h, l\}$ .

We now argue that no feasible triple  $(w, C^{h*}(z, x, w), C^{l*}(z, x, w))$  such that  $w > \mathbf{E}^{C^*}(w)$  can be optimal since the feasible triple

$$(\mathbf{E}^{C^*}(w), C^{h*}(z, x, \mathbf{E}^{C^*}(w)), C^{l*}(z, x, \mathbf{E}^{C^*}(w)))$$

is associated with a lower bond. To see this, first note that  $C_w^{\theta*}(z, x, w) = -1$  for  $w \leq y^\theta + \min\{z, x\} + s \max\{z - x, 0\}$  for  $\theta \in \{h, l\}$ . This implies

that  $B(w, C^{\theta*}(z, x, w))$  is increasing in  $w$ . Next, note that when  $C^\theta(z, x) = C^{\theta*}(z, x, w)$ ,  $G^{C^*}(w)$  is the probability that  $y^\theta + z \leq w$  and  $z < x$  conditional on revelation. We can calculate the conditional probability as follows. Conditional on revelation, the probability that the idiosyncratic valuation of the raider is less than that of the inside firm, equals  $\frac{1}{2}$ . To complete the calculation, we need to know the conditional probability that the outside valuation is less than the base wage. Recall that  $\chi^\theta(t, w) = \arg_{r \in [-\epsilon, \epsilon]} \min \delta(y(t, \theta) + r, w)$ , where  $\delta$  is Euclidean distance. It follows that  $G^{C^*}(w) = \frac{1}{2} F_{\chi^\theta}(T^\theta(s), w)$  so that  $G^{C^*}$  increases in  $w$ . This implies that

$$E^{C^*}(w) = \frac{\sum_{\theta \in \{h, l\}} y^\theta \pi^\theta [1 - p^\theta + p^\theta (\frac{1}{2}) F_{\chi^\theta}(T^\theta(s), w)]}{\sum_{\theta \in \{h, l\}} \pi^\theta [1 - p^\theta + p^\theta (\frac{1}{2}) F_{\chi^\theta}(T^\theta(s), w)]}$$

so that  $E^{C^*}$  increases in  $w$ , since  $y^h > y^l, p^h > p^l$ , and<sup>6</sup>  $F$  is increasing in  $w$ .

Kuhn-Tucker conditions show that the solution to the problem of minimizing  $B(w, C^{h*}(z, x, w), C^{l*}(z, x, w))$  subject to only the first constraint in problem 1 must satisfy

$$w = E^{C^*}(w). \quad (2.4)$$

Existence of a base wage that solves equation 2.4 is obtained as follows. The right-hand side of equation 2.4 is larger than the left-hand side when  $w = 0$ ; the right-hand side is smaller when  $w > y^h + y^l$ . There may be multiple solutions to equation 2.4. The fact that  $B(w, C^{h*}(z, x, w), C^{l*}(z, x, w))$  increases in  $w$  implies that the base wage in the solution to problem 1 is the smallest solution to equation 2.4. That is,  $w = w^*$ . Thus, the feasible triple  $(w^*, C^{h*}(z, x, w^*), C^{l*}(z, x, w^*))$  is the solution to problem 1. ■

The interpretation of the bond minimizing contract is as follows.

- (i) All workers are guaranteed a based wage of  $w^*$ .
- (ii) If  $z > x$  the outside firm pays the worker  $y^\theta + x + s(z - x)$  and the worker leaves to work for the outside firm. If  $y^\theta + x + s(z - x) \leq w^*$ , the inside firm pays a

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<sup>6</sup>Recall that the implications that  $p^h > p^l$  and  $y^h > y^l$  follow from the assumptions that  $T^h(s) > T^l(s)$ , that  $p(t, \theta)$  increases in  $t$  and  $\theta$  and that the partial derivative of  $y(t, \theta)$  and  $p(t, \theta)$  with respect to  $t$  increases in  $\theta$ .

severance of  $w^* - (y^\theta + x + s(z - x))$  when the worker leaves and so the inside firm makes negative profits in the second period in this case. If  $y^\theta + x + s(z - x) > w^*$ , the inside firm pays zero and breaks even.

(iii) If  $x > z$ , and  $y^\theta + z > w^*$ , the worker stays at the inside firm and earns  $y^\theta + z$ . The firm earns positive profits on these workers in the second period.

(iv) If  $x > z$ , and  $y^\theta + z \leq w$  or if the worker receives no offers, the worker stays at the inside firm and earns  $w^*$ . The firm breaks even in expected value terms in this case.

The bond paid by the worker in the first period may be negative. The inside firm collects a positive bond to pay severance in case (ii) and it collects a negative bond in case (iii).

Under the bond minimizing contract, the average earnings of workers who leave are higher than that of those who stay when earnings include both severance payments and the wages paid by the outside firm. The argument is as follows. As  $z$  and  $x$  are drawn independently according to  $F$ , the probability that  $z > x$  is equal to the probability that  $x > z$ . Thus, the probability that a worker leaves equals the probability that a worker stays given that the worker has received an outside offer. The probability that  $y^\theta + x + s(z - x) > w$  given  $z > x$  is greater than the probability that  $y^\theta + z > w$  given  $x > z$ . That is, conditional on a worker receiving outside offer, the probability that a worker who leaves earns more than  $w$  is greater than the probability that a worker who stays earns more than  $w$ . In addition, the expected value of  $y^\theta + x + s(z - x)$  (given both  $z > x$  and  $y^\theta + z + s(z - x) > w$ ) is larger than the expected value of  $y^\theta + x$  (given both  $x > z$  and  $y^\theta + x > w$ ). That is, conditional on receiving an offer, the average earnings of workers who leave and earn more than  $w$ , are greater than those of workers who stay and earn more than  $w$ . Since a worker always earns at least  $w$  if we include severance payments as part of the earnings, we conclude that the unconditional average earnings of those who leave are higher than of those who stay.

If we compare wages and exclude severance payments then we find that the



average wages of workers who leave may be greater or lesser than of those who stay<sup>7</sup>.

One incentive in the minimum bond contract is provided by the threat of the incumbent firm to pay  $w^*$  to any worker who is revealed to have invested inefficiently. In our model a worker's productivity is revealed to a single outside firm and the inside firm. The implication of the incumbent firm's threat in this context is that a sole outside firm who knows the worker's type and investment level need not offer more than  $w^*$  to such a worker. Thus the incumbent firm exploits the scarcity of knowledgeable outside firms in the optimal contract. The assumption that knowledgeable outside firms are scarce is reasonable when information is costly and workers have to invest in self-promoting activities to attract outside offers. Another incentive in the contract is provided by the promise of the firm to match outside offers. Note that the ability to commit is necessary for the contract to work. In the context of a repeated game, commitment can be enforced by reputation arguments. In the absence of commitment, as in the spot market case, the incumbent firm bids for the worker's services whenever it is profitable to do so.

### 3. Variations of the Model

In this section we consider two variations of the model. In one variation, investment is costly. In another variation, we suppose that the inside firm has an informational advantage over the outside firm vis à vis the productivity of the worker. The basic insights remain. The role of a specialized activity such as  $V$  is to allow more able individuals to separate themselves from the less abled. The

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<sup>7</sup> This is due to the fact that while the wage of a worker who stays is greater than or equal to  $w$ , the wage of one who leaves may be less than  $w$ . The wage is less than  $w$  when  $y^\theta + x + s(z - x) < w$  and  $z > x$ . In this case, conditional on receiving an offer, the average wages of workers who leave may be more or less than those of workers who stay. Thus, the unconditional average wages of workers who leave may be more or less than those of workers who stay. Workers who leave earn more than those who stay if, for example, the proportion of highly skilled workers is relatively high and the variance of  $F$  is low.

spot market provides private incentives for more able individuals to over-invest in the specialized activity relative to the efficient investment. In order to achieve efficient investment in such an activity, the firm promises a counteroffer in the event that a worker is shown to have invested efficiently.

### 3.1. Costly Investment

Our basic model focuses on the allocation of one unit of time to two activities. It ignores the cost of time spent in either activity. In particular, it ignores the problem of shirking on the part of the workers. Let us now consider a version of our model in which we assume that individuals face a disutility of effort exerted. Suppose that  $\varepsilon = 0$  and that  $s = 0$  in the model so that the efficient investment is the one that maximizes  $y(t, \theta)$ . We now offer a way of incorporating a cost of investment in the model.

Workers initially choose between effort and leisure and then choose how to allocate the effort between activities  $V$  and  $I$ . Suppose that an individual of type  $\theta$  who exerts effort  $\lambda$  faces the cost  $\kappa(\lambda, \theta)$ . Further assume that an individual of type  $\theta$  who exerts effort  $\lambda$  and invests time  $t$  in activity  $V$  produces  $\lambda y(t, \theta)$ . We have normalized total investment so that an employee who works 0 of the time is doing the minimum necessary to stay on the job. The worker's maximization problem is analogous to that which elicits the efficient outcome in the basic model. The efficient outcome is one in which the worker maximizes  $\lambda y(t, \theta) - \kappa(\lambda, \theta)$ , over  $\lambda \in [0, 1], t \in [0, 1]$ ,

The solution is that the efficient investment decision ( $T^*(\theta)$ ) is as in the basic model while the efficient labour decision ( $\lambda^*(\theta)$ ) sets  $y(t, \theta)$  equal to the marginal cost of investment.

Now consider the spot market solution. Assume that an individual of type  $\theta$  who exerts effort  $\lambda$  and who invests time  $t$  in activity  $V$  is revealed with probability  $P(\lambda, t, \theta)$  where  $P$  increases in each of its variables. The worker's spot market maximization problem is analogous to that in the basic model. When the base wage is  $w$ , an individual of type  $\theta$  maximizes  $w + P(\lambda, t, \theta)(\lambda y(t, \theta) - w) - \kappa(\lambda, \theta)$ ,

over  $\lambda \in [0, 1], t \in [0, 1]$ , at  $\hat{\tau}(\theta, w), \hat{\lambda}(\theta, w)$ .

It is immediate that  $\hat{\tau}(\theta, w) > T^*(\theta)$  whenever  $\hat{\lambda}(\theta, w)y(\hat{\tau}(\theta, w), \theta) > w$ . Thus, any individual who chooses to work over-invests in activity  $V$  relative to the efficient level.

Now let us consider the question of shirking in the spot market. Shirking was not an available option in the basic model. As in the basic model, the spot market base wage equals the expected output of those who receive it. It is immediate that it cannot be the case that both types exert zero effort and thereby produce zero output. (Recall that the units are normalized so that zero output is just enough to stay employed.) If so, then the base wage equals zero. The best response of the high ability worker is to exert positive effort when facing a zero base wage. However, in this case, the base wage of zero no longer equals the expected output of those who receive it. Thus, it must be the case that the high ability worker exerts positive effort. In equilibrium, the less able worker may exert zero effort, positive effort, or each with some probability. Each of these cases is associated with an expected output of those who receive the base wage. An equilibrium exists if the solution to the equation (that equates the base wage with the expected output) elicits the effort associated with the equation. As the parameters vary, the equation (equating the base wage with the expected output) whose solution elicits the associated effort varies. We find that some low types always exerts zero effort in equilibrium. The remaining low types exert zero or positive effort, depending on the costs and benefits of doing so<sup>8</sup>.

Lastly, we discuss the minimum bond that allows the firm to achieve the efficient outcome subject to the wage being greater than the expected output of those who receive it and subject to individual rationality constraints. Here the results are analogous to those in Section 5. The minimum bond is achieved when the base wage equals the expected output of those who receive it and the counteroffers make the workers indifferent between pursuing the efficient outcome and negotiating directly with the outside firms. A notable difference is that, since

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<sup>8</sup>See the Appendix for the details.

low ability workers exert costly effort, even those of low valuation need to be paid a counteroffer when they are revealed to have exerted the efficient labour and investment choices. The reason is that the base wage is a weighted sum of the two outputs. In this case the output of the less abled worker who exerts the efficient amount of effort is less than the base wage. When the counteroffer is zero, the less abled receive only the base wage and this is independent of their investment levels. But then the less abled have an incentive to exert zero effort unless there is some reward for exerting costly effort.

### **3.2. Matching with Asymmetric Information about Worker Ability**

We now consider altering the informational assumptions regarding the ability and investment of the worker. In the basic model, the inside firm has no informational advantage over the outside firm regarding the worker's type and investment. With probability  $p(t, \theta)$ , the worker's type and investment is made known to the inside and outside firms simultaneously. We now make the alternative polar assumption that the inside firm knows a worker's type and investment as soon as the worker knows. Thus, it is the outside firm who is alerted to a worker's type and investment level with probability  $p(t, \theta)$ .

When bonds are available and long-term contracts can be written, the worker and firm can agree that the firm pays each worker according to output. In this case, each worker chooses investments so as to maximize output.

It is the spot market outcome that is interesting in this case. When long-term contracts are not available, the firm cannot commit to pay each worker according to output. The firm pays the wage that maximizes current profits. It pays a base wage to all workers who are unable to reveal themselves. The base wage is greater than or equal to the output of the low ability worker. Otherwise a raider can profitably hire away such workers at a wage equal to this low output. However, the base wage may be below the expected output of those who receive it. The reason is that, as in Greenwald (1986), if a raider randomly attempts to hire a worker of unknown quality, the inside firm can respond by matching the

wage offer whenever the worker is of high ability. Thus, a raider only wins those workers of low ability. In this case, a raider offers a low wage to those of unknown quality even when the wage offered by the inside firm is below the average output of workers who receive the base wage. So, what is the base wage that the inside firm pays in this case? Let  $\varepsilon = 0$  so that  $s = 0$  in the basic model. In this case, there are no idiosyncratic valuations. Since the base wage is at least as great as the output of the low ability worker, a worker who is identified as low ability by an outside firm never receives an offer that is higher than the base wage. In this case, the worker is indifferent among investment levels when facing a given base wage. In this case, the base wage must be at least as great as the efficient output of the low ability worker. Let  $y^l$  represent the efficient level of output of the low ability worker. Let  $y(t)$  represent the output of the high ability worker. Let  $p(t)$  represent the probability that a worker of high ability obtains an outside offer when the worker invests the amount  $t$ . For any given base wage  $w \geq y^l$  a worker of high ability chooses  $\tau(w)$  to maximize  $p(t)y(t) + (1 - p(t))w$ . Since a high ability worker receives  $y(\tau(w))$  with probability  $p(\tau(w))$ , the incumbent firm makes positive profit on high ability workers only with probability  $(1 - p(\tau(w)))$ . The firm's profit function is  $\Pi(w) = \pi(y(\tau(w)) - w)(1 - p(\tau(w))) + (1 - \pi)(y^l - w)$ . (If  $H$  is a function of one variable,  $x$ , we let  $H'(x)$  denote the derivative of  $H$  with respect to  $x$ ). The derivative  $\Pi'(w) = \pi(y'(\tau(w))\tau'(w) - (1 - p(\tau(w)))) - (1 - \pi)$ . We can find examples in which the firm should set the base wage equal to  $y^l$ , the minimum feasible base wage. We can also find examples in which the firm should set the base wage above the expected output of those who receive it<sup>9</sup>. This contrasts with the results of Greenwald (1986). Greenwald assumes that the market's wage offer to a worker the firm does not lay off equals the worker's expected ability conditional on the worker choosing to accept the market's offer rather than the firm's. In the terminology of auction theory, Greenwald allows for the winner's curse. If only the lesser abled can be hired away then the wage is driven down to the output of the less abled. In our setting, the base wage has

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<sup>9</sup>See the Appendix for the details.

two roles to play. It affects not only the probability that a signal is sent but also affects the output that a worker produces. The higher the wage, the higher the output produced and the lower the probability that an offer is received. Thus, although the winner's curse is present, it does not drive the base wage down to the output of the low ability worker.

## 4. Conclusion

Our model explains why it is rational for the firm to respond strategically to outside offers in the presence of self-promoting activities. It is also rational for the employee to seek outside offers. In the presence of costly investment the absence of outside offers leads to inefficiencies in both matches and human capital investment. We conclude by offering an application of the results to academia.

University administrators usually know less about a professor's productivity than do a professor's peers.<sup>10</sup> Even when a particular department may be informed of a professor's productivity, it may not be prudent for the university administration to rely on a department's internal evaluation. Frequently, professors at other universities have a more accurate perception of an individual's performance relative to that of a professor's departmental colleagues. Thus, the polar assumption of no insider informational advantage in the basic model is not unreasonable in academia.

Academic research corresponds to the self-promoting investment in the model. Research increases both the productivity and visibility of the professor. Teaching and administrative activities corresponds to the non-visible activity of the model.

Due to the lack of information about how to evaluate the productivity of their professors, universities pay most of their professors relatively uniform salaries. Consistent with the model, professors, in some disciplines, spend a lot of effort trying to generate outside offers. When a professor receives an outside offer, the university begins to renegotiate the professor's salary. Whether the raider

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<sup>10</sup>The problems which academic employers face are discussed more broadly in Siow (1996).

succeeds in attracting the professor or not, the professor ends up with a higher salary.

The process of raising salaries on the evidence of an outside offer can explain the finding of a negative seniority wage premium in academia<sup>11</sup> (Ransom (1993), Hallock(1994), Moore et. al. (1994)). These authors find that in cross section data, controlling for years of experience but without controlling for productivity, professors with less seniority in a university earn higher wages. This finding contrasts with the positive seniority wage premium found in the general working population (e.g. Altonji and Shakoto (1987), Topel (1991)).

In general, the base wage in the minimum bond contract is different from the wage in the spot market. This base wage is an element of academic tenure in that tenure enables a university to guarantee a minimum wage for professors<sup>12</sup>. The most common criticism of academic tenure is that it encourages tenured professors to shirk. (See e.g. Alchian (1959).) Our model suggests that this argument is incomplete. As discussed in our extension, shirking can be deterred, even when the university offers a minimum wage to its employees, so long as universities respond optimally to outside offers received by their tenured professors. Our model does highlight the importance that a university's response to outside offers has in the deterrence of shirking.

## Appendix

In this appendix, we first provide some of the details of the equilibrium spot market wage in the variation of the basic model in which investment is costly. We then provide an example of the variation of the basic model in which informational asymmetries exist and the incumbent firms cannot exploit the winners' curse fully.

### A. Costly Investment

As in the basic model, the equilibrium spot market wage equals the expected output of those who receive the base wage. When investment is costly, the expected

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<sup>11</sup>In our model, a negative seniority wage premium obtains if, for example, the proportion of highly skilled workers is relatively high and the variance of  $F$  is low.

<sup>12</sup>Models of tenure include those of Chen and Ferris (1995), Carmichael (1988), Freeman (1977), and McPherson and Winston (1983). See Siow (1996).

output of those who receive the base wage depends on the investments made by the workers in the spot market equilibrium. The equation for the expected output depends on whether the less able workers produce zero or positive effort in equilibrium. The parameters determine the equilibrium spot market wage as follows. Recall that the pair  $(\hat{\tau}(\theta, w), \hat{\lambda}(h, w))$  is the optimal response of a worker of type  $\theta$  to a spot market wage of  $w$ . Let  $\hat{P}(w, \theta) = P(\hat{\lambda}(\theta, w), \hat{\tau}(\theta, w), \theta)$  be the probability that a worker of type  $\theta$  (who exerts effort  $\hat{\lambda}(\theta, w)$  and devotes time  $\hat{\tau}(\theta, w)$  to  $V$ ) is revealed. The right-hand side of

$$w = \frac{\pi^h \hat{\lambda}(h, w) y(\hat{\tau}(h, w), h) (1 - \hat{P}(w, h))}{\pi^h \hat{\lambda}(h, w) (1 - \hat{P}(w, h)) + \pi^l} \quad (\text{A.1})$$

is the expected output when the more able worker produces positive output and the less able produces zero output. Let  $\tilde{w}$  be the solution to equation A.1. In the case that

$$P(\lambda, t, l)(\lambda y(t, l) - \tilde{w}) < \kappa(\lambda, l)$$

for  $\lambda \in (0, 1]$ ,  $t \in [0, 1]$ , the less able worker finds it too costly to exert effort when facing  $\tilde{w}$ . In this case, the spot market base wage is  $\tilde{w}$  and those of type  $l$  exert zero effort and so exert less than the efficient amount of effort. Those of type  $h$  may over- or under-exert relative to the efficient amount.

In the case that

$$\hat{P}(\tilde{w}, l)(\hat{\lambda}(l, \tilde{w}) y(\hat{\tau}(l, \tilde{w}), l) - \tilde{w}) > \kappa(\hat{\lambda}(l, \tilde{w}), l)$$

$\tilde{w}$  cannot be the spot market wage. The reason is due to the following contradiction. In this case,  $\tilde{w}$  equals an expected output that is based on the assumption that the less able worker exerts zero effort but the less able worker has an incentive to provide effort when facing  $\tilde{w}$ . We thus look for a solution in which the fraction  $q$  of the less able workers exerts effort and the fraction  $1 - q$  exerts zero effort. Since some of the less able workers exert positive effort and some exert zero effort, it must be the case the less able worker is indifferent between these two options. In this case, we look for a solution, in  $w$  and  $q$ , to the simultaneous equations

$$w = \frac{\pi^h \hat{\lambda}(h, w) y(\hat{\tau}(h, w), h) (1 - \hat{P}(w, h)) + \pi^l \hat{\lambda}(l, w) y(\hat{\tau}(l, w), l) (1 - \hat{P}(w, l)) q}{\pi^h (1 - \hat{P}(w, h)) + \pi^l [1 - q + (1 - \hat{P}(w, l)) q]} \quad (\text{A.2})$$

and

$$\hat{P}(w, l)(\hat{\lambda}(l, w) y(\hat{\tau}(l, w), l) - w) = \kappa(\hat{\lambda}(l, w), l) \quad (\text{A.3})$$

When  $q = 0$ ,  $w = \tilde{w}$  solves equation A.2, (i.e. the solution to A.1 solves A.2 when  $q = 0$ ), and the value of the left-hand side of A.3 is greater than that of the right. When  $q = 1$ , say that the solution to A.2 is  $\check{w}$ . It is clear that the left-hand



side of A.3 is smaller than the right-hand side when the wage is  $\check{w}$ . (The reason is that when  $q = 1$ ,  $\check{w}$  is a linear combination of  $y(\hat{\tau}(h, \check{w}), h)$  and  $y(\hat{\tau}(l, \check{w}), l)$ . Thus,  $\check{w} > y(\hat{\tau}(l, \check{w}), l)$ .) Continuity then guarantees the existence of simultaneous solutions to equations A.2 and A.3. Let  $\bar{w}, \bar{q}$  be the solution to the simultaneous equations A.2 and A.3 in this case. When such a solution,  $\bar{w}, \bar{q}$ , exists then those of type  $l$  are indifferent between working  $\hat{\lambda}(l, \bar{w})$  of the time and not working at all. In this case, we assume that the proportion  $\bar{q}$  of those of type  $l$  work  $\hat{\lambda}(l, \bar{w})$  of the time while the proportion  $1 - \bar{q}$  exert minimal effort.

We note that it is always the case that some low types exert zero effort in equilibrium. If all low types and high types exerted positive effort then the wage would be so high that it would be in the interest of the low types to exert no effort.

## B. Informational Asymmetries and the Winner's Curse

We now provide an example of the variation of the basic model in which informational asymmetries exist and the incumbent firms cannot exploit the winner's curse fully.

Let  $p(t) = .5 + .5t, y(t) = 4 - t, y^l < 1$ . In this case,  $y'(t) = -1$  and

$$\tau(w) = \left\{ \begin{array}{ll} 1 & \text{if } 0 < w < 1 \\ (3 - w)/2 & \text{if } 1 < w < 3 \\ 0 & \text{if } 3 < w \end{array} \right\}, y'(\tau(w))(\tau'(w)) = \left\{ \begin{array}{ll} 0 & \text{if } 0 < w < 1 \\ 1/2 & \text{if } 1 < w < 3 \\ 0 & \text{if } 3 < w \end{array} \right\}$$

$$p(\tau(w)) = \left\{ \begin{array}{ll} 1 & \text{if } 0 < w < 1 \\ (5 - w)/4 & \text{if } 1 < w < 3 \\ 1/2 & \text{if } 3 < w \end{array} \right\}, 1 - p(\tau(w)) = \left\{ \begin{array}{ll} 0 & \text{if } 0 < w < 1 \\ (w - 1)/4 & \text{if } 1 < w < 3 \\ 1/2 & \text{if } 3 < w \end{array} \right\}$$

Thus, using the derivative of the profit function derived in Section 3.2 of the text, we see that

$$\Pi'(w) = \left\{ \begin{array}{ll} \pi - 1 & \text{if } 0 < w < 1 \\ (\pi(7 - w)/4) - 1 & \text{if } 1 < w < 3 \\ \pi/2 - 1 & \text{if } 3 < w \end{array} \right\}, W(\pi) = \left\{ \begin{array}{ll} y^l & \text{if } \pi < 2/3 \\ 7 - 4/\pi & \text{if } \pi > 2/3 \end{array} \right\}$$

where  $\Pi$  denotes the firm's profit and  $W(\pi)$  is the profit-maximizing base wage set by the inside firm. Note that if  $\pi < 2/3, \Pi'(w) < 0$  so that  $\Pi$  is maximized at  $w = y^l$  if  $\pi < 2/3$ . However, if  $\pi > 2/3$ , then  $\Pi$  first decreases then increases then decreases. Thus, the maximizer of  $\Pi$ , in this case, is at  $w = 7 - 4/\pi$  since the value of  $\Pi$  is higher here than at  $w = 0$ .

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