GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 308

Unique Equilibrium in Two-Part Tariff Competition between Two-Sided Platforms

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February 2010

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.sfbtr15.de
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Abstract

Two-sided market models in which platforms compete via two-part tariffs, i.e. a subscription and a per-transaction fee, are often plagued by a continuum of equilibria. This paper augments existing models by allowing for heterogeneous trading behavior of agents on both sides. We show that this simple method yields a unique equilibrium even in the limit as the heterogeneity vanishes. In case of competitive bottlenecks we find that in this equilibrium platforms benefit from the possibility to price discriminate if per-transaction costs are relatively large. This is the case because two-part tariffs allow platforms to better distribute these costs among the two sides. Under two-sided single-homing price discrimination hurts platforms if per-transaction fees can be negative.

JEL classification: D43, L13
Keywords: Two-Sided Markets, Per-Transaction Fee, Subscription Fee, Two-Part Tariffs, Unique Equilibrium

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*I thank Helmut Bester, Martin Peitz, Sven Rady, Roland Strausz, Piers Trepper and Glen Weyl as well as seminar participants at the Free University of Berlin and the University of Munich for very helpful discussions and suggestions. Financial support by the German Science Foundation through SFB/TR-15 is gratefully acknowledged.

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1 Introduction

There are many industries that are organized around platforms on which two groups of agents interact and trade with each other. Prominent examples are operating system platforms like Symbian, iPhone OS or Blackberry OS that enable interaction between smart-phone users and application developers, credit card companies like MasterCard or Visa that allow payment by credit card between buyers and merchants, or real estate agents who facilitate trade between house buyers and sellers. Since platforms enable trade between suppliers and consumers, in many two-sided markets, including the examples above, platforms charge two-part tariffs that consist of a subscription fee and a per-transaction fee to at least one of the sides. For example, in the operating system industry developers are charged a fixed fee for getting access to the system's source code and in addition pay royalties for the applications they sell to users.[1] Users of smart-phones just pay a price for the phone but are not charged by the platforms for applications. A similar structure can be observed in the video game market, where game developers also pay a two-part tariff while gamers are just charged for the console. Another example are retail warehouse clubs that bring together suppliers of a variety of products and shoppers. Here shoppers pay a fixed membership fee and a price for each product they buy. Suppliers obtain a price for each good and sometimes pay or receive an upfront payment from the retailer. This widespread use of two-part tariffs in two-sided markets naturally begs the question what the implications of this form of price discrimination on the profits of competing platforms and on the welfare of the two sides are.

However, as first pointed out by Armstrong (2006a), the answer to this question appears to be problematic. He shows that when platforms compete in two-part tariffs, a continuum of equilibria exists, each one with a different profit and surplus for both sides. This causes major problems on the predictive power of such models. The reason for this multiplicity is that, given the prices of the rival, a platform receives the same profit via different combinations of the fixed and the per-transaction fee. In particular, an agent is indifferent between paying a high fixed fee but a small per-transaction fee and a low fixed fee but a high per-transaction fee. Therefore, these combinations attract the same number of agents and a platform obtains the same profit. Since this holds for both platforms, a tremendous multiplicity of equilibria emerges.

The aim of this paper is to provide a simple and tractable framework that resolves this multiplicity problem but is otherwise as close as possible to standard two-sided market models. In addition, the framework yields realistic predictions on prices and makes the platform competition model richer and more realistic. In Armstrong (2006a) and also in most other models of platform competition[2] agents of one side may differ with respect to their indirect network benefit, i.e. the benefit they receive from interacting with an agent from the other group, but they are homogenous with respect to their trading behavior, that is, they all interact with the same number of agents. For simplicity, this number is usually taken as the complete group of agents that joins the same platform. In this model we introduce heterogeneity in the trading behavior of agents in a simple way. There are now two types of agents where one

[1] In the market for smart-phones, application developers for the iPhone sell their apps via the AppStore at which they have to pay a per-transaction charge of 30% of the trading price.

[2] See, for example, Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003) or Hagiu (2006).

type interacts with an agent from the other group only with smaller probability than the other type.[3] Ex ante, platforms cannot distinguish between these two types and therefore charge the same prices to them. This heterogeneity is a realistic feature in many two-sided markets. For example, in the software industry some developers offer very prominent applications that are valued by many buyers while others are less successful or develop more for fun reasons. Similarly, some smart-phone users buy lots of applications while others primarily use the basic options of their phone. A similar heterogeneity is present in many Internet trading markets where some sellers cause more shoppers to buy their products while some shoppers are more likely to react on a seller's offer than others. However, operating systems or Internet trading platforms do not know the type of an agent in advance.

We show that this heterogeneity, although simple and easy to apply, is powerful in reducing the continuum of equilibria that prevails under homogeneity to a unique one. More importantly, even in the limit as the heterogeneity vanishes, the method singles out a unique equilibrium from the continuum. This allows for a meaningful comparison of profit and welfare with a regime in which just pure subscription or per-transaction fees are possible. The intuition behind the uniqueness is that the different types react differently to a change in the tariff combination. For example, if a platform raises the fixed fee but lowers the per-transaction fee in such a way that the type with the lower trading probability is indifferent, the other type is strictly better off because he trades more often and so benefits more from the reduced per-transaction fee. As it turns out, no two combinations of fees now attract the same number of agents which implies that a platform has a unique best response to the rival's prices. Since this holds for both platforms, there is a unique equilibrium. As the heterogeneity vanishes, the equilibrium that is selected is therefore the continuous extension of the equilibrium in the model with two types of agents but where the mass of one type becomes negligible.

This method of equilibrium selection is similar to one of introducing demand uncertainty to pin down a unique equilibrium tariff in a model of supply function competition, an approach that was pioneered by Klemperer and Meyer (1989). They show that if firms compete in supply functions, multiple equilibria exist in a deterministic framework. However, this multiplicity can be reduced via introducing uncertainty about demand and even eliminated if this uncertainty is large enough. Although this approach is very useful, it has appeared to be difficult to apply. This paper shows that in the two-sided markets framework, the method is very powerful because it is sufficient to introduce only a slight amount of heterogeneity in a natural way to select a unique equilibrium.

The equilibrium two-part tariff of our framework has many appealing properties. First, the indirect network externalities determine just the per-transaction fees but not the fixed fees. This is realistic since these externalities measure the benefit of each interaction and only accrue via interaction and not via subscription. Second, the per-transaction fees for side 1 are low if side 2 benefits a lot from an additional member on side 1. This result complements the finding of previous literature that platforms charge low prices to the side with the larger externality. This paper shows that under two-part tariffs this is still true but the lower payment is purely represented in the per-transaction fee. Third, the per-transaction fee of the single-homing side

[3] In case the interaction involves trading of goods, this can also be interpreted as one type trading a smaller amount of goods than the other type.

is often negative in equilibrium—or equal to zero if negative fees are impossible—while this does not hold true for the per-transaction fee of the multi-homing side. This outcome can be observed in many industries like the smart-phone or credit card industry. Phone users or shoppers often use only a single phone or credit card and are not charged for the applications they buy or whenever they use their credit card. By contrast, developers and merchants are multi-homing and have to pay royalties per application or percentage fees per transaction.

Most importantly, the uniqueness of the equilibrium allows for a comparison of platform profits and consumer welfare with the case in which just one of the fees is possible. It is well-known that price discrimination under imperfect competition in one-sided markets hurts firms because the additional pricing instrument opens a new front of competition, see e.g. the surveys by Armstrong (2006b) or Stole (2007). We show that this effect is also present in two-sided markets. However, there is a countervailing effect, which is that via two-part tariffs platforms are able to better distribute the costs per transaction among the two sides. Consider for example the case of pure subscription fees. Here platforms must recoup their per-transaction costs via the subscription fees although the source of these costs are the transactions. If instead per-transaction fees are possible, platforms can levy these costs exactly where they arise. In addition, since these costs are not attributable to just one side, platforms can allocate them optimally among the two sides. This latter effect is not present in a one-sided market where firms receive revenues only from one consumer group, and so the question of cost distribution does not arise. We find that in case of competitive bottlenecks if the per-transaction costs are large, the countervailing effect dominates, and platforms' profits increase under two-part tariffs. Interestingly, this can occur even if both per-transaction fees are negative in equilibrium. Therefore, in two-sided markets it is ultimately a question of the industry in consideration if two-part tariffs increase profits compered to linear pricing. For example, in the credit card industry per-transaction costs are relatively small while in the video game industry these costs are sizeable since game console firms engage in mass production and distribution of the developed games. Thus, in the former industry profits are likely to fall via two-part tariffs while in the latter they are likely to rise. We also look at the case of two-sided single-homing. Here we find that profits are lower under two-part tariffs than under pure subscription fees. The reason is that platforms compete for both sides and so the effect of additional competition through the second fee dominates. However, we also show that if the fees cannot be negative, platforms' profits can never fall through two-part tariffs.

Turning to consumer welfare we find that the utility of an agent of the multi-homing side is often the same under two-part tariffs and under pure subscription or per-transaction fees, while the welfare of the single-homing side falls exactly in case platforms' profits increase. As a consequence, the policy implications from price discrimination between a one- and a two-sided market can differ substantially. While in one-sided markets price discrimination is often beneficial for consumer, the conclusions in a two-sided market are industry specific.

The paper complements and extends previous studies on platform competition by providing a framework to pin down a unique equilibrium when platforms set two-part tariffs. In particular, Armstrong (2006a) works out many principles of pricing in two-sided markets—e.g. prices fall with the indirect network externalities or the single-homing side is treated favorably in case of competitive bottlenecks—by considering several models that fit well with different industries.[4] He focuses on pure subscription charges.[5] In this paper we make use of these models, and in addition consider a different one, and show that under two-part tariffs in each of them a continuum of equilibria exist under homogenous trading behavior of agents but that the equilibrium is unique when trading behavior differs. In contrast to Armstrong (2006a), Rochet and Tirole (2003) mainly focus on pure per-transaction charges. They show, among several other things, how prices on each side depend on the demand elasticities of both sides and how equilibrium prices differ from the consumer optimal Ramsey ones.[6] In their Section 6, Rochet and Tirole (2003) consider two-part tariffs but suppose that platforms compete just in the sum of the two charges. Since, as pointed out by Armstrong (2006a), this is not equivalent to offering two-part tariffs, they obtain a unique equilibrium.

Rochet and Tirole (2006) allow for both subscription and per-transaction fees in a general model but confine their attention to a monopoly platform. They show how the prices obtained in the models above must be modified in case both fees are possible. Weyl (2009a) also analyzes the case of a monopoly platform but allows for general tariffs. He develops the notion of "insulated equilibrium" that helps to overcome the well-known "chicken-and-egg" problem in two-sided markets[7] and derives the profit and welfare maximizing pricing structure.[8]

Caillaud and Jullien (2001 and 2003) and Hagiu (2006) allow for two-part tariffs and platform competition but suppose perfect Bertrand competition. Thus, if agents can only single-home, in equilibrium just one platform is active and platforms make zero profits. In this setting Caillaud and Jullien (2001) demonstrate that competition under two-part tariffs is fiercer than under pure subscription fees because a platform can attract agents from the rival platform more easily.[9] Caillaud and Jullien (2003) show that the possibility of multi-homing may relax competition between platforms, thereby allowing them to reap positive profits. Hagiu (2006) considers the case in which the two sides decide sequentially about their participation and shows under which conditions pricing commitments are beneficial for platforms.

As the present paper, Liu and Serfes (2009) also study the implications of price discrimination in two-sided markets when platforms are differentiated. In contrast to the present paper they analyze the case of perfect price discrimination, where each agent, even within one group, can be charged a different price. They show that perfect price discrimination can be profitable

[4]For a summary of the results appearing in industries with two-sided platforms and its implications on antitrust policy, see Rysman (2009).

[5]As mentioned, in one of his models Armstrong (2006a) analyzes two-part tariffs and finds that a continuum of equilibria exist.

[6]For an extension of Rochet and Tirole's (2003) analysis to the socially optimal Ramsey prices, see Weyl (2009b).

[7]The idea of the insulated equilibrium is that the platform chooses an allocation—a participation rate for both sides—directly instead of choosing a price pair. To avoid failure of the implementation of the desired allocation, the platform changes the price to side 1 if less or more than expected agents of side 2 participate. Thereby, it insulates the participation of side 1 from the participation of side 2.

[8]In Section 6.3 of his article, Weyl (2009a) notes that the insulated equilibrium can also be helpful to tackle the problem of multiple equilibria under platform competition, which is also the goal of the present article. See the conclusion for further discussion of this issue.

[9]Ambrus and Argenziano (2009) also consider perfect Bertrand competition between platforms but focus on pure subscription fees. However, they allow agents within a side to differ with respect to their network benefits. They show that if this heterogeneity is large enough, both platforms are active, earn positive profits and have asymmetric networks, that is, platform A attracts many agents from side 1 and few from side 2 and vice versa for platform B.

for platforms because it reduces the cross-group externalities that intensify competition under linear prices.[10]

The rest of the paper is organized as follows: The next section first sets out a general model of competitive bottlenecks and then analyzes in detail three examples of different industries. Section 3 analyzes a model of two-sided single-homing. Section 4 discusses the relationship of our equilibrium selection method to previous literature and Section 5 concludes.

2 Competitive Bottlenecks

In this section we will first describe a general model of competitive bottlenecks, i.e. agents of group 1 deal only with one platform (single-home), while agents of group 2 wish to deal with each platform (multi-home). There are many examples that fit this description. For example, the video game or smart-phone industry, where developers' applications often run on several systems, while gamers or phone users use just one system, the credit card market, where merchants accept all cards while customers often possess only one card, or Internet trading platforms, where sellers post offers on many platforms while buyers often use just one. After introducing the general model, we will present three specific applications that fit the stylized facts of selected industries relatively well. In each application we will start by analyzing the case in which agents of both sides are homogeneous with respect to their trading behavior and show that there is a continuum of equilibria. We will then introduce heterogeneity in each group, show that this selects a unique equilibrium and analyze the properties of this equilibrium in detail.

2.1 General Framework

There are two platforms denoted by $i = A, B$ that enable interaction between two groups of agents denoted by $k = 1, 2$. Each platform i can set two different sets of prices.[11] The first is a fixed or subscription fee denoted by p_k^i that an agent of group k pays for joining platform i. This fixed fee could be a membership fee in case of credit cards or a fee to make the underlying code available to software developers. The second is a per-transaction charge for each group denoted by γ_k^i. An agent of group k has to pay this charge each time she interacts with a member of the other group via platform i. Examples are fees levied by trading platforms or credit card companies on a transaction between buyers and sellers or royalties charged by game console firms to developers for every game they sell. So overall each platform decides about four different prices.

We now turn to the description of the utilities of the agents in each group. In the following we denote the number of agents of group k who join platform i by n_k^i. Let us start with agents of group 2, the multi-homing side. We will often refer to them as sellers. The utility of a seller

[10] Taking the industry examples given above into account, two-part tariffs are very widespread while perfect price discrimination can be observed only rarely. Thus, one might argue that the policy implications drawn from the two-part tariff analysis are perhaps more important for antitrust considerations.

[11] We suppose that platforms are independent companies that are not owned by the agents of the two sides. For a model that explicitly considers different forms of ownerships or where a platform is integrated with one of the sides, see Nocke, Peitz and Stahl (2007).

who joins platform i is given by[12]

$$U_2^i = b + (\alpha_2 - \gamma_2^i)n_1^i - p_2^i. \tag{1}$$

Here b is the membership benefit of a group-2 agent (which can also be negative if the agent incurs some costs from joining the platform, i.e. application developers may incur time costs to learn the code of the underlying software) whereas α_2 measures the benefit that a group-2 agent enjoys from interacting with a group-1 agent.[13] This can be the margin that a seller receives from selling her product to a buyer. In this utility function sellers are heterogenous, that is, they differ with respect to b or α_2. This differentiation is not observable by the platforms who view each agent as ex ante identical. The parameter b or α_2 is distributed according to a continuous distribution function $F(.)$. Note that an implicit assumption in (1) is that each seller who joins platform i trades with every buyer on platform i, i.e. sellers are homogeneous with respect to their trading behavior. Given that there is a unit mass of sellers, we get that

$$n_2^i = prob\left(b + (\alpha_2 - \gamma_2^i)n_1^i - p_2^i \geq 0\right).$$

Thus, n_2^i does not depend directly on the fees that platform $-i$ charges because sellers multi-home which implies that there is no direct competition for them.

To the contrary, platforms compete for the agents of side 1, the single-homing side, and we will refer to these agents repeatedly as buyers. To capture competition for buyers in a tractable way we model it in a Hotelling fashion, as e.g. in Anderson and Coate (2005) or Armstrong (2006a).[14] There is a mass 1 of buyers that is uniformly distributed on a line of length 1, platform A is located at point 0 while platform B is located at point 1 and the transportation costs are denoted by t. The utility of a buyer who joins platform i gross of transportation costs can then be written as

$$U_1^i = U(n_2^i) - \gamma_1^i n_2^i - p_1^i, \tag{2}$$

where $U(n_2^i)$ is the utility that a buyer obtains from interacting with n_2^i sellers. The slope if this function can either be positive, for example, if group-2 agents are software or game developers, or negative, i.e. if group-2 agents are advertisers and buyers view ads as nuisance. The number of buyers on platform i is therefore given by

$$n_1^i = \frac{1}{2} + \frac{U(n_2^i) - \gamma_1^i n_2^i - p_1^i - U(n_2^j) + \gamma_1^j n_2^j + p_1^j}{2t}, \quad i \neq j,\ i,j = A, B.$$

The costs of platform i depend on the number of agents of each group that platforms i attracts, i.e. they are given by $C(n_1^i, n_2^i)$. These costs can consist of per-transaction costs, e.g.

[12] The per-transaction fee is modelled as an absolute payment here not as a percentage charge. This is done to stay as close as possible to the formulations in Armstrong (2006a) and Rochet and Tirole (2006). However, all results also hold in case of percentage transaction charges, in which case the utility would by given by $U_2^i = b + \alpha_2(1 - \gamma_2^i)n_1^i - p_1^i$. As will become clear later, also in that case the method developed in this paper selects a unique equilibrium that has similar properties as the ones with an absolute per-transaction charge.

[13] In line with most of literature we suppose that there are no direct externalities within the agents of one group. For papers that consider intra-group externalities between sellers, see, among others, Nocke, Peitz and Stahl (2007), Belleflamme and Toulemode (2008) and Hagiu (2009).

[14] A drawback of this formulation is that the population of group-1 agents is kept constant. However, this formulation is widely used in the literature since it is easy to work with. In addition, the insights of our analysis are not be restricted to this formulation.

$cn_1^i n_2^i$ in case these costs are linear—a realistic assumption for video game consoles or credit cards—or $n_1^i c(n_2^i)$ with $c' > 0$ and $c'' \geq 0$. The latter cost function fits well to yellow page directories, where the cost of producing and distributing a directory is $c(n_2^i)$. The costs can also include a fixed component per buyer and seller that we denote by f_1 and f_2. Overall, the profit of platform i in its general form can be written as

$$\Pi^i = p_1^i n_1^i + p_2^i n_2^i + (\gamma_1^i + \gamma_2^i) n_1^i n_2^i - C(n_1^i, n_2^i).$$

2.2 A Linear Example of Operating Systems and Credit Cards

We first consider an example that fits well to an industry in which platforms are operating systems or credit cards. In the first case, the sellers are application developers who are charged for getting access to the underlying source code and pay a royalty for each application they sell, while the buyers are users who pay for getting access to the software but are also potentially charged whenever they buy an application. In the credit card industry, sellers are merchants who pay a per-transaction fee each time a buyer pays with credit card and a fixed fee for being authorized to accept the card. Buyers are customers who pay a yearly fixed fee for the card and can (potentially) also be charged each time they use the card.[15]

To state the main point on how to select a unique equilibrium with competition in two-part tariffs in the clearest way, we set up a framework in which one can explicitly solve for platform prices. In order to do so we consider a model where demand and cost functions are linear. This also has the advantage of being able to consider all kinds of costs and of deriving comparative static results with respect to prices in an easy way. However, as will become clear from the next examples, the selection of a unique equilibrium does not at all depend on the linearity of the model and also works if equilibrium prices are only implicitly given.

We first look at the buyers. In a linear model the utility function $U(n_2^i)$ in (2) can be written as $U(n_2^i) = B + \alpha_1 n_2^i$. For example, each user enjoys some gross benefit from using the software without application, i.e. in the market for cellular phones a user can make calls or send text messages, but benefits if more applications are present for this software. In the credit card example buyers may benefit from withdrawing money via the credit card in an easy way abroad but their utility increases if more merchants accept the card. This yields that the number of buyers is given by[16]

$$n_1^i = \frac{1}{2} + \frac{(\alpha_1 - \gamma_1^i) n_2^i - p_1^i - (\alpha_1 - \gamma_1^j) n_2^j + p_1^j}{2t}, \quad j \neq i, i, j = A, B. \tag{3}$$

Turning to the sellers, suppose that the gross utility of sellers from joining a platform, denoted by b, is uniformly distributed on $[\underline{b}, \bar{b}]$. This can either represent different costs in learning a software's code to write applications or differing costs of shops for installing a device to allow customers to use the card. It can also be the fun that developers enjoy from

[15] To focus on our main point of interest, we abstract from other features of the credit card industry, like the interaction of issuer and acquirer bank. For in-depth studies of these issues see, for example, Rochet and Tirole (2002), Wright (2003) or Bedre-Defolie and Calvano (2009).

[16] We suppose that the utility of a buyer depending on B and α_1 is large enough such that in each price equilibrium all buyers indeed receive a positive utility, which implies that the buyer market is covered.

programming or that it is safer for shops to deal with plastic rather than cash.[17] To ease notation we denote $\bar{b} - \underline{b} \equiv \Delta b$. Thus, a seller joins platform i if

$$b + (\alpha_2 - \gamma_2^i)n_1^i - p_2^i \geq 0.$$

Since b is uniformly distributed, we obtain that

$$n_2^i = \frac{\bar{b} + (\alpha_2 - \gamma_2^i)n_1^i - p_2^i}{\Delta b} \quad \text{and} \quad n_2^j = \frac{\bar{b} + (\alpha_2 - \gamma_2^j)n_1^j - p_2^j}{\Delta b}. \tag{4}$$

Using the fact that $n_1^j = 1 - n_1^i$ in (4) and solving (3) and (4) for n_1^i and n_2^i yields

$$n_1^i = \frac{1}{2} + \frac{\bar{b}(\gamma_1^j - \gamma_1^i) + \Delta b(p_1^j - p_1^i) + \alpha_1(p_2^j - p_2^i + \gamma_2^j - \alpha_2) + \gamma_1^j(\alpha_2 - \gamma_2^j) + p_2^i\gamma_1^i - p_2^j\gamma_1^j}{\alpha_1(\gamma_2^j + \gamma_2^i) + \alpha_2(\gamma_1^j + \gamma_1^i) - \gamma_2^i\gamma_1^i - \gamma_2^j\gamma_1^j - 2\alpha_1\alpha_2} \tag{5}$$

and

$$n_2^i = \frac{1}{\Delta b\left(2t\Delta b + \alpha_1(\gamma_2^j + \gamma_2^i) + \alpha_2(\gamma_1^j + \gamma_1^i) - \gamma_2^i\gamma_1^i - \gamma_2^j\gamma_1^j - 2\alpha_1\alpha_2\right)} \times \tag{6}$$

$$\times\Big(\Delta bt(2\bar{b} + \alpha_2 - \gamma_2^i - 2p_2^i) - (p_1^i - p_1^j)(\gamma_2^i - \alpha_2) + \bar{b}\left(\alpha_1(\gamma_2^j + \gamma_2^i - 2\alpha_2) + 2\alpha_2\gamma_1^j - \gamma_2^i\gamma_1^i - \gamma_2^j\gamma_1^j\right) +$$

$$+(\gamma_1^j - \alpha_1)\left(\alpha_2^2 - \alpha_2(\gamma_2^i + \gamma_2^j + p_2^i + p_2^j) + \gamma_1^i(p_2^j + \gamma_2^j) + \gamma_2^i\gamma_2^j\right)\Big).$$

A platform incurs costs $f_2 \geq 0$ for each seller, e.g. because it has to make the software code available, and $f_1 \geq 0$ for each buyer due to manufacturing of the video game console or smart phone or to issue the credit card. In addition, there are per-transaction costs $c \geq 0$ because the platform has to install devices to monitor the interactions between the two groups to be able to charge per-transaction fees. Therefore, the profit function of platform i is given by

$$\Pi^i = (p_1^i - f_1)n_1^i + (p_2^i - f_2)n_2^i + (\gamma_1^i + \gamma_2^i - c)n_1^i n_2^i, \tag{7}$$

where n_1^i and n_2^i are defined in (5) and (6), respectively. To make the problem interesting, we suppose that costs are small enough so that it is efficient for platforms to be active. In particular, this implies that $\bar{b} > f_2$ and $\alpha_1 + \alpha_2 > c$, that is, the highest fixed benefit of a seller from joining a platform is larger than the fixed per-seller cost and the sum of per-transaction benefits is larger than the per-transaction costs. We do not constrain the fees to be positive to focus on the main point of interest, i.e. the multiplicity of equilibria and how to select one of them.[18]

As a benchmark we start with the analysis in which per-transaction fees are not possible, i.e. because platforms cannot control if agents of the two groups interact. This means that $\gamma_1^i = \gamma_2^i = 0$ in the profit function and in the definitions of n_1^i and n_2^i.

To focus on market sharing equilibria we suppose that the externalities represented by α_1 and α_2 are weak relative to the differentiation parameter t and the heterogeneity of sellers

[17]We consider the case in which sellers differ with respect to their per-transaction valuation in the next subsection.

[18]However, in Appendix A we provide an analysis for the case in which fees are restricted to be positive, given that the unrestricted equilibrium fees are the ones obtained by our selection method.

represented by Δb. If t is small compared to the externalities, all buyers join only the platform with the larger number of sellers, and so sellers in turn find it worthwhile to join this platform exclusively, which implies cornering of the market on both sides. Similarly, if Δb is relatively small, a platform may attract all sellers which gives rise to large externalities and makes the model prone to market cornering. In particular, the necessary and sufficient condition to rule out such a situation is

$$8t\Delta b > (\alpha_1 + \alpha_2 - c)^2 + 4\alpha_1\alpha_2.$$

Maximizing (7) with respect p_1^i and p_2^i and solving the resulting system of equations we obtain a unique symmetric equilibrium, i.e. $p_k^A = p_k^B = p_k$, in which prices are given by

$$p_1 = t + f_1 + \frac{c\left(2(\bar{b} + \alpha_1 - f_2) + \alpha_2\right) - c^2 - \alpha_2(2\bar{b} + 3\alpha_1 + \alpha_2 - 2f_2)}{4\Delta b} \tag{8}$$

and

$$p_2 = \frac{2(f_2 + \bar{b}) + c + \alpha_2 - \alpha_1}{4}. \tag{9}$$

The profit of each platform is

$$\Pi^p = \frac{t}{2} + \frac{4(\bar{b} - f_2)^2 - (\alpha_1 + \alpha_2 - c)^2 - 4\alpha_1\alpha_2}{16\Delta b}.$$

We now turn to the case in which per-transaction fees are possible. As Armstrong (2006a) notes, with competition in two-part tariffs there may exist asymmetric equilibria. Naturally, and in line with Armstrong (2006a), in the following we will focus on symmetric equilibria. We first have to make sure that the objective function of a platform is concave. Since each platform has four choice variables, this can be a tedious matter. However, as Armstrong (2006a) shows, one can easily reduce the number of strategic variables to two. This is the case because, given the prices of its rival, platform i's profit can be written as a function that depends only on the utilities u_1^i and u_2^i that it offers to the two sides. Defining $(\alpha_1 - \gamma_1^i)n_2^i - p_1^i \equiv u_1^i$ and $(\alpha_2 - \gamma_2^i)n_1^i - p_2^i \equiv u_2^i$ and replacing p_1^i, p_2^i, γ_1^i and γ_2^i by u_1^i and u_2^i in the equations determining the number of consumers n_1^i and n_2^i and in the profit function Π^i, we get

$$n_1^i = \frac{1}{2} + \frac{u_1^i - (\alpha_1 - \gamma_1^j)n_2^j + p_1^j}{2t} \quad \text{and} \quad n_2^i = \frac{\bar{b} + u_2^i}{\Delta b}$$

and

$$\Pi^i = n_1^i(\alpha_1 n_2^i - u_2^i - f_1) + n_2^i(\alpha_2 n_1^i - u_1^i - f_2) - c n_1^i n_2^i.$$

To show that Π^i is concave in these utilities, we have to verify (i) that $\partial\Pi^i/\partial u_k^i < 0$, $k = 1, 2$ and (ii) that the matrix of second derivatives of Π^i is positive definite. In a symmetric equilibrium where $\gamma_k^A = \gamma_k^B = \gamma_k$ we get that condition (i) is fulfilled if

$$\frac{\partial\Pi^i}{\partial u_1^i} = -\frac{2\Delta b}{2\Delta bt + (\alpha_1 - \gamma_1)(\gamma_2 - \alpha_2)} < 0$$

and $\partial\Pi^i/\partial u_2^i = -2/\Delta b < 0$. The second inequality is always satisfied while the first one is only satisfied if

$$2\Delta bt > (\alpha_1 - \gamma_1)(\alpha_2 - \gamma_2). \tag{10}$$

Tedious calculations show that condition (ii) holds if

$$8t\Delta b > (\alpha_1 + \alpha_2 - c)^2 + 4(\alpha_1 - \gamma_1)(\alpha_2 - \gamma_2). \tag{11}$$

One can then easily check that (11) implies (10). Thus, a platform's problem is concave if (11) holds which imposes a restriction on the per-transaction fees in equilibrium.

Now maximizing the profit function of each firm with respect to the four strategy variables and solving for the equilibrium we obtain that the fixed fees are given by

$$p_1 = t + f_1 + \frac{(\alpha_1 - \gamma_1)(\alpha_1 - \alpha_2 + 2\gamma_2 - c) - (\alpha_1 + \alpha_2 - c)^2 - 2(\bar{b} - f_2)(\alpha_2 + \gamma_1 - c)}{4\Delta b} \tag{12}$$

and

$$p_2 = \frac{2(f_2 + \bar{b} - \gamma_2) + c + \alpha_2 - \alpha_1}{4}, \tag{13}$$

while the first-order conditions for γ_1 and γ_2 are redundant and so these fees are undetermined. After calculating the profit we get the following result:

Proposition 1 *There is a continuum of symmetric equilibria in the linear framework. In these equilibria the fixed fees are given by* (12) *and* (13) *while the per-transaction fees* γ_1 *and* γ_2 *satisfy* (11) *but are otherwise undefined. The profit of a platform is given by*

$$\Pi = \frac{t}{2} + \frac{4(\bar{b} - f_2)^2 + 4(\alpha_1 - \gamma_1)(\gamma_2 - \alpha_2) - (\alpha_1 + \alpha_2 - c)^2}{16\Delta b}.$$

As Armstrong (2006a) notes, the multiplicity of equilibria stems from the fact that a platform can ensure itself the same profit via different combinations of the subscription and the per-transaction fee. This is the case because the effect on profit of a marginal change in the per-transaction fee is always δ times the effect of a marginal change in the fixed fee, where δ is a constant. Consider for example the fees for the sellers. Since in a symmetric equilibrium $n_1 = 1/2$, a change in γ_2^i has always half the effect on profit compared to a change in p_2^i implying $\delta = 1/2$. On the buyer side, $\delta = n_2$. Thus, for each price quadruple of its rival, a platform has a continuum of best response combinations of the four fees. As a consequence, since a platform is indifferent between a continuum of price quadruples, in equilibrium it picks the one that renders the price quadruple of the rival optimal. Since the rival platform does the same, a continuum of equilibria emerges.

The analysis so far shows that the conjecture of Armstrong (2006a), that there are multiple equilibria in a model of competitive bottlenecks is indeed correct.[19] Since the profit and the welfare of the two sides is different in each of these equilibria, this multiplicity causes major problems on the predictive power of models in which both fixed and per-transaction fees are possible.[20] We will now provide a natural way how to resolve this obstacle while at the same time making the demand structure more realistic.

[19]See Armstrong (2006a), Section 5.

[20]The multiplicity of optimal tariffs also arises in a model with a monopoly platform. However, it is much less of a problem in that case because the profit of the platform and the welfare of both sides is the same independent of the exact tariff that the monopolist selects.

Heterogenous Trading Behavior

Suppose now that there are two different types on each side that are heterogeneous with respect to their trading behavior. In particular, on the buyer side there is a mass q_1 of buyers, with $q_1 > 0$ but small, who interact with each seller only with probability $\beta < 1$. The remaining mass $1 - q_1$ is of the same type as above. A natural interpretation is that there is a small fraction of buyers who assign a positive value to each seller's good only with probability β. Another interpretation is that some buyers do not buy one unit from each seller but instead β units. Here we can allow β to be higher or lower than 1.[21] We refer to this new type of buyers as buyers of type β.

Similarly, on the seller side there is a mass q_2 of sellers who do not sell with probability 1 but just with $\lambda < 1$. The remaining mass $1 - q_2$ of sellers is of the same type as described above. As for the buyers, this can naturally be interpreted as there being a small fraction of sellers who produce a good that only a fraction λ of buyers value while the others abstain from buying.[22] We refer to this new type of sellers as sellers of type λ.

The introduction of different types makes the description of the two-sided market more realistic. For example, some smart-phone users buy more applications than others while some developers are more effective or spend more time on developing applications than others. In general, as Weyl (2009a) notes, heterogeneity between agents almost certainly stems from the value of the interaction to the other side. Thus, it is a natural step to incorporate different trading behavior. Ex ante, when an agent joins a platform, the platform does not know how many applications a particular user or developer will trade, and so it views all agents of the same group as identical. We will suppose that the mass of new types of agents is small because we are especially interested in the limit as q_1 and q_2 go to zero. The reason is that we want to compare platforms' profits and buyers' and sellers' utilities in case of two-part tariffs with the ones when only fixed fees or only per-transaction fees are possible.

As before, each platform i sets four prices, a subscription fee p_k^i and a per-transaction fee γ_k^i, $k = A, B$, to each side. This implies that we abstract from the possibility of price discrimination, i.e. that a platform charges different subscription or per-transaction fees within the same group. The reason for this is twofold. First, since platforms do not know ex ante which agent is a regular seller and which one a seller of type λ, the platform needs some mechanism to elicit this information in order to engage in an optimal price discrimination scheme. Since the fraction q_2 of type λ is very small, it may not be worthwhile for platforms to do so if this mechanism incurs some costs. The same argument holds for the buyers. Second, differing fees within the same group of agents are rarely observed in reality. For example, credit card companies charge merchants the same per-transaction fee independent of the number of transactions. Similarly, in the video game industry game console firms charge a uniform royalty to developers.

As a consequence, the utility (gross of transport costs) of a buyer who is of standard type and joins platform i can now be written as $B + (1 - q_2)(\alpha_1 - \gamma_1^i)n_2^i + q_2(\alpha_1 - \gamma_1^i)\lambda n_{2\lambda}^i - p_1^i$, while the utility of a buyer of type β who joins the same platform is $B + (1 - q_2)(\alpha_1 - \gamma_1^i)\beta n_2^i +$

[21] It is not important for our purposes if β is larger or smaller than 1 but just that it differs from 1.

[22] As on the buyer side, it is also possible to interpret λ as the amount of goods traded by a seller in which case λ can also be larger than one.

$q_2(\alpha_1-\gamma_1^i)\beta\lambda n_{2\lambda}^i-p_1^i$. Similarly, a seller of standard type who joins platform i receives now a benefit of $b+(1-q_1)(\alpha_2-\gamma_2^i)n_1^i+q_1(\alpha_2-\gamma_2^i)\beta n_{1\beta}^i-p_2^i$ while a seller of type λ receives a benefit of $b+(1-q_1)(\alpha_2-\gamma_2^i)\lambda n_1^i+q_1(\alpha_2-\gamma_2^i)\lambda\beta n_{1\beta}^i-p_2^i$

Therefore, the number of buyers of standard type and type β who join platform i can be written as

$$n_1^i=\frac{1}{2}+ \tag{14}$$

$$+\frac{(\alpha_1-\gamma_1^i)(1-q_2)n_2^i+(\alpha_1-\gamma_1^i)q_2\lambda n_{2\lambda}^i-p_1^i-(\alpha_1-\gamma_1^i)(1-q_2)n_2^j-(\alpha_1-\gamma_1^i)q_2\lambda n_{2\lambda}^j+p_1^j}{2t}$$

and

$$n_{1\beta}^i=\frac{1}{2}+ \tag{15}$$

$$+\frac{(\alpha_1-\gamma_1^i)(1-q_2)\beta n_2^i+(\alpha_1-\gamma_1^i)q_2\beta\lambda n_{2\lambda}^i-p_1^i-(\alpha_1-\gamma_1^i)(1-q_2)\beta n_2^j-(\alpha_1-\gamma_1^i)q_2\beta\lambda n_{2\lambda}^j+p_1^j}{2t},$$

while the number of sellers of standard type and type λ who join platform i are given by

$$n_2^i=\frac{\bar{b}+(\alpha_2-\gamma_2^i)(1-q_1)n_1^i+(\alpha_2-\gamma_2^i)q_1\beta n_{1\beta}^i-p_2^i}{\Delta b} \tag{16}$$

and

$$n_{2\lambda}^i=\frac{\bar{b}+(\alpha_2-\gamma_2^i)(1-q_1)\lambda n_2^i+(\alpha_2-\gamma_2^i)q_1\lambda\beta n_{1\beta}^i-p_2^i}{\Delta b}. \tag{17}$$

The profit function of platform i is now given by

$$\Pi^i=(p_1^i-f_1)n_1^i\left((1-q_1)n_1^i+q_1n_{1\beta}^i\right)+(p_2^i-f_2)n_2^i\left((1-q_2)n_2^i+q_2n_{2\lambda}^i\right)+ \tag{18}$$

$$+(\gamma_1^i+\gamma_2^i-c)(1-q_1)n_1^i\left((1-q_2)n_2^i+q_2\lambda n_{2\lambda}^i\right)+(\gamma_1^i+\gamma_2^i-c)q_1\beta n_{1\beta}^i\left((1-q_2)n_2^i+q_2\lambda n_{2\lambda}^i\right).$$

As above, we can determine conditions for the profit function to be concave. However, since we are mainly interested in the case where q_1 and q_2 tend to zero, condition (11) must be fulfilled to guarantee concavity of the profit function in this limiting case. Below we will check under which conditions (11) is fulfilled at the equilibrium fees.

We can now proceed in the same way as above, namely solving (14), (15), (16) and (17) for n_1^i, $n_{1\beta}^i$, n_2^i and $n_{2\lambda}^i$, inserting these values into the profit function (18) and taking derivatives with respect to p_1^i, p_2^i, γ_1^i and γ_2^i. Solving for the symmetric equilibrium, tedious but otherwise routine calculations show that now none of the four first-order conditions is redundant. We obtain equilibrium subscription fees of

$$p_1=t+f_1 \quad \text{and} \quad p_2=\frac{\bar{b}+f_2}{2}$$

and equilibrium per-transaction fees of

$$\gamma_1=c-\alpha_2-\frac{(\alpha_1+\alpha_2-c)^2\left(1-q_1+\beta^2q_1\right)(1-q_2+\lambda q_2)}{2\left(\bar{b}-f_2\right)(1-q_2+\lambda q_2)} \quad \text{and} \quad \gamma_2=\frac{c+\alpha_2-\alpha_1}{2}.$$ [23]

[23] The fact that only γ_1 and not γ_2 depends on β and λ is due to the linear structure of the example. In this linear case via pinning down the per-transaction fee to the single-homing side, the per-transaction fee that ensures the optimal composition of types at the multi-homing side is uniquely determined.

It is now easy to see that even in the limit as $q_1 \to 0$ and $q_2 \to 0$ the equilibrium stays unique and the fees are given by

$$p_1 = t + f_1, \quad p_2 = \frac{\bar{b} + f_2}{2}, \quad \gamma_1 = c - \alpha_2 - \frac{(\alpha_1 + \alpha_2 - c)^2}{2(\bar{b} - f_2)} \quad \text{and} \quad \gamma_2 = \frac{c + \alpha_2 - \alpha_1}{2}. \tag{19}$$

It remains to check if condition (11) is satisfied at this equilibrium. To do we insert the per-transaction fees given by (19) into (11) to get that this is the case if

$$f_2 \left(8\Delta b t - 3(\alpha_1 + \alpha_2 - c)^2\right) + (\alpha_1 + \alpha_2 - c)^3 > 0. \tag{20}$$

After inserting the prices into the profit function we obtain the following result.

Proposition 2 *Suppose that* (20) *holds. In case of different buyer and seller types there is a unique symmetric equilibrium in the linear framework. As $q_1 \to 0$ and $q_2 \to 0$, the equilibrium fixed and per-transaction fees are given by* (19)*. The profit of each platform is given by*

$$\Pi^{p\gamma} = \frac{t}{2} + \frac{\left(\bar{b} - f_2 - \alpha_1 - \alpha_2 + c\right)\left(2(\bar{b} - f_2) + \alpha_1 + \alpha_2 - c\right)^2}{16\Delta b(\bar{b} - f_2)}.$$

The intuition behind the uniqueness of the equilibrium in case of different types in each group is that the two types react differently to a particular combination of the subscription and the per-transaction fee. For example, to keep the utility of a seller of type λ constant, an increase in the per-transaction fee must be coupled with a smaller reduction of the subscription fee than to keep the utility of a seller of regular type constant, because a seller of type λ trades less often. The same holds true when comparing a buyer of type β with a buyer of standard type.[24] Therefore, the effect on profit of a marginal change in γ_k^i is no longer a constant multiple of the effect of a marginal change in p_k^i. Instead, this multiple varies continuously as the fees change because the ratio of the two types that join platform i also varies continuously. As a consequence, a platform has a unique optimal combination of the fees as a reaction to the price quadruple of its rival. Since this holds for both platforms, there is a unique equilibrium. A particular advantage of this formulation is that introducing buyer and seller heterogeneity is a natural and realistic extension. Thus, one does have to rely on more subtle mechanisms of equilibrium selection to predict equilibrium outcomes.[25]

Moreover, the analysis shows that the formulation gives a unique equilibrium even in the limit as the heterogeneity in each group vanishes. Intuitively, if a platform could perfectly discriminate between the two types, the difference in their per-transaction fees would be proportionate to the number of transactions they engage in. Since this is independent of the fraction of each type, it also holds if the mass of one type becomes negligible. This selected equilibrium is the continuation of the equilibrium in the case with a small amount of heterogeneity. It is therefore a natural choice out of the multiple equilibria that occur when working

[24]Put differently, in a fixed-per-transaction-fee plane the indifference curves of the two types of side k cross just once.

[25]In Section 4 we briefly discuss the similarities and differences to other equilibrium selection criteria that involve some kind of perturbation of the game.

directly in the limit, i.e. when agents' trading behavior is homogeneous.[26]

It is also interesting to note that, as will become evident later, this equilibrium is not the Pareto dominant one from the platforms' perspective. In case of homogeneous trading behavior one could argue that platforms may coordinate on the equilibrium that yields the highest profit. Our analysis shows that if this were the case, introducing a tiny amount of uncertainty involves a discrete jump in the equilibrium outcome. Therefore, Pareto dominance may be not the natural selection criterion here.

Before taking a closer look at the obtained equilibrium, we mention that the way how uncertainty must be introduced to obtain a unique equilibrium is not arbitrary. To see this suppose, for example, that agents in each group differ in the benefit they receive from trading, e.g. there are two types of buyers and sellers that differ with respect to α_1 and α_2, respectively. Although there is now heterogeneity in the per-transaction benefit, the effect of a change in the per-transaction fee is still the same for the two types. So a platform is still indifferent between a continuum of tariff combinations. Thus, again a continuum of equilibria emerges in which the fees now depend on the expected indirect externalities.

Let us now analyze the unique equilibrium in more detail. The prices obtained in (19) have an intuitive interpretation and fit well with those observed in real markets. First, look at the prices for the buyers, p_1 and γ_1. We obtain that the fixed per-buyer costs f_1 and the differentiation parameter t enters just the fixed charge because these elements are not relevant for transactions. To the contrary, the per-transaction charge is mainly determined by the externalities and the per-transaction costs. It is evident that if c is small relative to the externalities, the per-transaction charge to the buyers is negative (provided that α_1 and α_2 are positive which is a realistic assumption in the credit card or the operating system industry). This feature can be observed for example in the credit card industry where buyers just pay a yearly fixed fee but often receive gifts if they use the credit card by a substantial amount. Turning to the prices for the sellers, the fixed elements involved when a seller joins the platform, i.e. the fixed costs and benefits, also just affect the fixed fee p_2 but not the per-transaction fee γ_2. This per-transaction fee just depends on parameters governing the interaction between both sides and is positive as long as the per-transaction costs plus the externality of buyers on sellers is larger than the reverse externality. For example, in the credit card industry we observe that the per-transaction fee of the sellers is positive which fits with the obtained results. Also in the software industry, like in the markets for video games, platforms charge developers a positive fee for access to the source code and a royalty per transaction while gamers pay only a fixed price when buying the console.

We can now evaluate if the possibility of price discrimination is beneficial for platforms and/or consumers. Comparing the profit in case of pure subscription fees with the one in case of two-part tariffs we obtain

$$\Pi^p - \Pi^{p\gamma} = \frac{2(\bar{b} - f_2)\left((\alpha_1 + \alpha_2 - c)^2 - 2\alpha_1\alpha_2\right) + (\alpha_1 + \alpha_2 - c)^3}{16\Delta b(\bar{b} - f_2)}. \tag{21}$$

[26]The feature that the equilibrium stays unique as the uncertainty vanishes is also present in Klemperer and Meyer (1989). In contrast to the present paper, in supply function competition the distribution of the demand function must have full support in the first place to obtain a unique equilibrium. However, this equilibrium remains unchanged as the distribution becomes more and more sharply peaked.

Since $\bar{b} - f_2 > 0$, it is easy to check that the right-hand side of (21) is positive if c is close to zero but negative if $c \to \alpha_1 + \alpha_2$. Thus, price discrimination is profitable for platforms in case per-transaction costs are large. It is possible to show numerically that for any parameter constellation there is a unique solution for $c \in (0, \alpha_1 + \alpha_2)$ above which the profit under two-part tariffs is larger than under pure subscription charges and below which the opposite holds true. So compared to the literature on one-sided markets, that reaches the conclusion that price discrimination lowers profits if firms compete, the result is mixed in case of two-sided markets. The intuition is the following: As in a one-sided market, the possibility of charging a per-transaction fee in addition to the subscription fee opens a new front of competition between platforms. This lowers their profits. However, since a platform charges per-transaction fees to both sides, it can optimally distribute the per-transaction costs c among the two sides. This is important since these costs arise only if the two sides interact and can therefore not be attributed to just one side. Such an effect is not present in a one-sided market, where there is only one consumer group. Therefore, if c is sizeable, platforms benefit from two-part tariffs. Interestingly, platforms can benefit from two-part tariffs even if both per-transaction fees are negative. One can check from (19) and (21) that such a constellation can occur if α_1 is large relative to α_2 and $c + \alpha_2$ is only slightly smaller than α_1. Thus, even if the two additional fees that platforms charge are negative, profits can nevertheless rise, since the ability to better allocate c among the two sides is dominating.

Turning to the effects on the two sides, it is easy to see that the overall payment of sellers is the same in case of price discrimination and in case of pure subscription fees. This holds because $n_1 = 1/2$ and so $p_2 + \gamma_2/2$, with p_2 and γ_2 defined in (19), equals $(2(f_2 + \bar{b}) + c + \alpha_2 - \alpha_1)/4$, the payment of a seller in case of pure subscription fees. Thus, there is no effect on the multi-homing side which implies that the number of agents joining each platform is the same under both regimes. This implies that a group-1 agent enjoys the same externality benefit under both regimes. Therefore, her utility with price discrimination increases exactly in the case when the platform loses through price discrimination. As a consequence, we find that the benefit of price discrimination for the platforms and the single-homing side are opposed to each other while the multi-homing side is not affected.

We can provide a similar analysis for the case of pure per-transaction fees, i.e. where subscription fees are not possible. In this case the equilibrium per-transaction fees are given by

$$\gamma_1 = \frac{c + \alpha_1 - \alpha_2}{2} + (f_1 + t)\left(\frac{\Delta b}{\bar{b}}\right) - \frac{(\alpha_1 + \alpha_2 - c)(\alpha_1 + \alpha_2 - c - 2f_2)}{\bar{b}} \tag{22}$$

and

$$\gamma_2 = \bar{b} + f_2 + \frac{c + \alpha_2 - \alpha_1}{2} \tag{23}$$

and the profit of a platform is

$$\Pi^\gamma = \frac{t - f_1}{4} + \frac{\bar{b}^2}{4\Delta b} + \frac{\bar{b}(\alpha_1 + \alpha_2 - c - 4f_2)}{8\Delta b} - \frac{(\alpha_1 + \alpha_2 - c + 2f_2)(\alpha_1 + \alpha_2 - c - 2f_2)}{16\Delta b} +$$
$$+ \frac{(t + f_1)(\alpha_1 + \alpha_2 - c - 2f_2)}{8\bar{b}} - \frac{(\alpha_1 + \alpha_2 - c)(\alpha_1 + \alpha_2 - c - 2f_2)}{32\bar{b}\Delta b}.$$

From (23), the payment of the sellers is the same as under two-part tariffs. Thus, price discrimination does not change the utility of a seller compared to any regime with pure fees.

As above, this implies that the profits of the platforms and the utilities of group-1 agents change diametral with each other. Now comparing the profit under pure per-transaction fees with the one under two-part tariffs we get that the sign of this difference is given by

$$\operatorname{sign}\{\Pi^{\gamma} - \Pi^{p\gamma}\} = \tag{24}$$

$$= \operatorname{sign}\Big\{4\bar{b}^3(\alpha_1+\alpha_2-c) - \bar{b}^2\left[8\Delta b(t+f_1) - (\alpha_1+\alpha_2-c)(\alpha_1+\alpha_2-c-f_2)\right] +$$
$$+\bar{b}(\alpha_1+\alpha_2-c)\left[(\alpha_1+\alpha_2+2f_2-c)(\alpha_1+\alpha_2-2f_2-c)+4\Delta b(t+f_1)\right] -$$
$$-f_2(\alpha_1+\alpha_2-c-2f_2)\left[4\Delta b(f_1+t)+f_2(\alpha_1+\alpha_2-c)(\alpha_1+\alpha_2-c-2f_2)\right]\Big\}.$$

If $c \to \alpha_1+\alpha_2$, the right-hand side of this equation becomes $-8\Delta b(\bar{b}-f_2)(\bar{b}+f_2)(t+f_1) < 0$ while if $c = 0$ and e.g. Δb is relatively small, the right-hand side is positive. Thus, we again have that if c is large, two-part tariffs are beneficial for platforms while if c is small, this is not necessarily the case.[27] One can provide a similar analysis for f_1 and f_2 which yields that the profit under price discrimination is larger if the fixed costs per agent are relatively large. The intuition is, as above, that two-part tariffs allow platforms to better distribute these costs among the two sides. This effect dominates the increased competition effect if these costs are relatively large. The discussion is summarized in the next proposition:

Proposition 3 *Platforms' profits are larger under two-part tariff than under pure subscription or per-transaction fees if c is relatively large. The utility of an agent of the multi-homing side is unchanged with two-part tariffs while the utility of an agent of the single-homing side increases if platforms' profits fall.*

In this subsection we considered a model with a uniform distribution of benefits and a linear cost function to obtain explicit solutions. We now go one to analyze a model that fits well with media and Internet trading platforms and allows for more general cost and externality functions. We show that our method singles out a unique equilibrium in this case as well.

2.3 Media Platforms and Internet Trading Platforms

Consider the situation where there is competition between two symmetric media platforms, like yellow page directories, or Internet trading platforms, like Amazon.com and eBay. In this case the sellers are producers or retailers who wish to make contact with consumers by placing ads in the media outlets or offers on the trading platform.[28] For simplicity, we suppose that $b = 0$ in (1); so the utility of a seller is given by $(\alpha_2 - \gamma_2^i)n_1^i - p_2^i$.[29] Sellers are differentiated

[27] Calculating the critical c numerically reveals that there is a unique solution for c given that it exists. The solution may not exist in this case since even at $c = 0$ the profit under two-part tariffs may be larger which implies that for some parameter constellations, two-part tariffs dominate pure per-transaction fees for any c.

[28] The fixed fee for producers in this case is a lump-sum charge for devoting space to the advertisement or offer while the per-transaction charge can be interpreted as a per-reader charge, if platforms are yellow page directories, or as the fee that a producer pays each time a buyer purchases her good, in case of Internet trading platforms. Similarly, the buyers pay a price for the yellow page directory that consists of a fixed part and can (potentially) rise in the number of advertisements that the outlet contains. In case of Internet trading platforms buyers pay a membership fee and (potentially) a per-transaction fee for each product they purchase.

[29] We consider the same model of media platforms as Armstrong (2006a) but allow for two-part tariffs.

with respect to α_2, the profit that a seller receives per buyer.[30] In particular, α_2 is, for each seller, independently drawn from a distribution function $F(\alpha_2)$. Therefore, we get that

$$n_2^i = 1 - F\left(\frac{p_2^i}{n_1^i} + \gamma_2^i\right). \tag{25}$$

The single-homing group are the buyers. Their utility is defined in (2) and so we have

$$n_1^i = \frac{1}{2} + \frac{U(n_2^i) - \gamma_1^i n_2^i - p_1^i - U(n_2^j) + \gamma_1^j n_2^j + p_1^j}{2t}. \tag{26}$$

Since $n_2^i = n_2^i(n_1^i)$ and $n_2^j = n_2^j(1 - n_1^i)$, (26) can in principle have multiple solutions. As in Armstrong (2006a), we sidestep this issue and suppose that the underlying parameters are such that there is a unique solution to n_1^i for the relevant prices. In the last subsection, the linearity of the externality function and the uniform distribution of the sellers' fixed benefits ruled out this possibility. Although it is hard to give precise conditions to guarantee a unique solution, it is clear what is needed: t should be large compared to $U'(n_2^i)$ and $F'(p_2^i/n_1^i + \gamma_2^i)$, $i = A, B$. The slope of the right-hand side of (26) with respect to n_2^i is then relatively flat.

A yellow page directory incurs costs for producing and distributing a copy of the directory of $c(n_2^i)$ given that it contains n_2^i ads. Thus, the overall costs of a yellow page directory are $C(n_1^i, n_2^i) = n_1^i c(n_2^i)$. For an Internet trading platform the main bulk of its (variable) costs arise from governing and monitoring the transaction between buyers and sellers. Thus, its cost function is also proportional to n_1^i. If there are no other cost, we can therefore write the profit of a platform as[31]

$$\Pi^i = p_1^i n_1^i + p_2^i n_2^i + (\gamma_1^i + \gamma_2^i) n_1^i n_2^i - n_1^i c(n_2^i), \tag{27}$$

where n_1^i and n_2^i are defined in (25) and (26).[32]

As before, by replacing the prices to the two sides with their utilities, we can determine conditions for the profit function to be concave. It turns out that these conditions are that t is large relative to U' and F' and that the per-transaction fees are not too large in absolute value. Thus, the requirements are similar to the ones stated in (11) in the last subsection. The interpretation is also the same, namely that the differentiation between platforms is large relative to the network externalities. The precise conditions in the present case are unwieldy and not very enlightening, so we do not explicitly state them here. However, in Appendix B we provide the calculations for determining these conditions and explicitly derive them for the equilibrium fees in case of different types on each side.

Differentiating (27) with respect to the four strategy variables we get first-order conditions of

$$\frac{\partial \Pi^i}{\partial p_k^i} = n_k^i + p_1^i \frac{dn_1^i}{dp_k^i} + p_2^i \frac{dn_2^i}{dp_k^i} + (\gamma_1^i + \gamma_2^i)\left(n_2^i \frac{dn_1^i}{dp_k^i} + n_1^i \frac{dn_2^i}{dp_k^i}\right) - c(n_2^i)\frac{dn_1^i}{dp_k^i} - n_1^i c'(n_2^i)\frac{dn_2^i}{dp_k^i} = 0,$$

[30] For empirical studies about the strength of the indirect network effects in media markets, see Rysman (2004) for the yellow page market or Kaiser and Wright (2006) for the magazine market.

[31] We abstract from fixed per-agent costs in this case since the effects of these costs were already analyzed in the last subsection, and such costs are likely to be small in the examples considered here.

[32] Apart from the cost function the description also fits well the television or radio broadcasting industry in which producers make contact to consumers via commercials. See, for example, Anderson and Coate (2005), Gabszewicz, Laussel and Sonnac (2003) or Peitz and Valletti (2008) for in-depth studies of the television industry.

and

$$\frac{\partial \Pi^i}{\partial \gamma_k^i} = p_1^i \frac{dn_1^i}{d\gamma_k^i} + p_2^i \frac{dn_2^i}{d\gamma_k^i} + n_1^i n_2^i + (\gamma_1^i + \gamma_2^i)\left(n_2^i \frac{dn_1^i}{d\gamma_k^i} + n_1^i \frac{dn_2^i}{d\gamma_k^i}\right) - c(n_2^i)\frac{dn_1^i}{d\gamma_k^i} - n_1^i c'(n_2^i)\frac{dn_2^i}{d\gamma_k^i} = 0,$$

with $k = 1, 2$. To determine the solutions we first have to calculate the derivatives of n_1^i and n_2^i with respect p_k^i and γ_k^i, $k = 1, 2$, respectively. In a symmetric equilibrium, prices are the same on both platforms, i.e. $p_k^i = p_k^j = p_k$ and $\gamma_k^i = \gamma_k^j = \gamma_k$, which implies market sharing at the buyer market, $n_1^i = n_1^j = 1/2$, and an equal number of sellers on each platform, $n_1^i = n_2^j = n_2$. Totally differentiating (25) yields that, in equilibrium, $dn_2^i = -2F'dp_2^i - F'd\gamma_2^i + 4p_2F'dn_1^i$ and $dn_2^j = -4p_2F'dn_1^i$,[33] where $F' = F'(2p_2 + \gamma_2)$, while totally differentiating (26) yields

$$dn_1^i = \frac{(U'(n_2^i) - \gamma_1)dn_2^i - (U'(n_2^j) - \gamma_1)dn_2^j - dp_1^i - n_2^i d\gamma_1^i}{2t}.$$

We can now use these equations to calculate the derivatives of the number of buyers and sellers with respect to p_1^i and γ_1^i to get

$$\frac{dn_1^i}{dp_1^i} = -\frac{1}{2\rho}, \quad \frac{dn_1^i}{d\gamma_1^i} = -\frac{n_2}{2\rho}, \quad \frac{dn_2^i}{dp_1^i} = -\frac{2p_2F'}{\rho} \quad \text{and} \quad \frac{dn_2^i}{d\gamma_1^i} = -\frac{2n_2p_2F'}{\rho},$$

where $\rho = (t - 4F'p_2(U'(n_2) - \gamma_1))$. In the same way we can determine the derivatives with respect to p_2^i and γ_2^i. Here we obtain

$$\frac{dn_1^i}{dp_2^i} = -\frac{\psi}{\rho}, \quad \frac{dn_1^i}{d\gamma_2^i} = -\frac{\psi}{2\rho}, \quad \frac{dn_2^i}{dp_2^i} = -\frac{2F'(t - 2p_2\psi)}{\rho} \quad \text{and} \quad \frac{dn_2^i}{d\gamma_2^i} = -\frac{F'(t - 2p_2\psi)}{\rho},$$

where $\psi = F'(U'(n_2) - \gamma_1)$.

Inserting the derivatives for dn_k^i/dp_1^i and $dn_k^i/d\gamma_1^i$ into the first-order conditions for p_1^i and γ_1^i and rearranging, we get

$$t + c(n_2) - p_1 - \gamma_1 n_2 - \gamma_2 n_1 - 2p_2F'(2p_2 + \gamma_2)(2p_2 + 2U'(n_2) - \gamma_1 + \gamma_2 - c'(n_2)) = 0$$

in both equations. As a consequence, there exists a continuum of combinations of p_1 and γ_1 that fulfill both first-order conditions. The relation of p_1 to γ_1 is given by

$$p_1 = t + c(n_2) - \gamma_1 n_2 - \gamma_2 n_1 - 2p_2F'(2p_2 + 2U'(n_2) - \gamma_1 + \gamma_2 - c'(n_2)). \tag{28}$$

Inserting the respective values for dn_k^i/dp_2^i and $dn_k^i/d\gamma_2^i$ into the first-order conditions for p_2^i and γ_2^i and using (28) yields that the first-order condition for γ_2 is satisfied for any γ_2 while the solution for p_2 is given by

$$p_2 = \frac{n_2}{2F'} - \frac{U'(n_2) + \gamma_2 - c'(n_2)}{2}. \tag{29}$$

Thus, there also exists a continuum of p_2-γ_2-combinations that fulfill both first-order conditions. Solving (28) and (29), we obtain that p_1 and p_2 are implicitly given by

$$p_1 = t + c(n_2) - n_2c'(n_2) + F'(U'(n_2) + \gamma_2 - c'(n_2)) - \frac{n_2}{F'} \quad \text{and} \quad p_2 = \frac{n_2}{2F'} - \frac{U'(n_2) + \gamma_2 - c'(n_2)}{2}. \tag{30}$$

[33] Note that because $n_1^j = 1 - n_1^i$, we have that $dn_1^j = -dn_1^i$.

After inserting (30) into the profit function (27) we obtain the following result:

Proposition 4 *There is a continuum of symmetric equilibria in the model of media or Internet trading platforms. In these equilibria the fixed fees are implicitly defined by* (30) *while γ_1 and γ_2 fulfill the second-order conditions but are otherwise undefined. The profit of a platform is given by*

$$\Pi = \frac{t - n_2(U'(n_2) - \gamma_1) + F'(U'(n_2) + \gamma_2 - c'(n_2))}{2} - \frac{n_2(1-n_2)}{2F'}.$$

Heterogenous Trading Behavior

As in the last section, let us now consider the case in which there are two types on each side with different trading behavior. That is, on the buyer side there is a mass q_1 of buyers who purchase the good of a producer only with probability β while on the producer side there is a mass q_2 of producers who sell their goods only with probability λ. The remaining masses $1-q_1$ and $1-q_2$ on each side are as above, i.e. they buy and sell with probability 1. Again, we suppose that q_1 and q_2 are small. To simplify notation in the following we denote $q_1\beta n^i_{1\beta} + (1-q_1)n^i_1 \equiv \bar{n}^i_1$ and $q_2\lambda n^i_{2\lambda} + (1-q_2)n^i_2 \equiv \bar{n}^i_2$. So $\bar{n}^i_1$ represents how many goods a producer of standard type sells when placing an offer on platform i while $\bar{n}^i_2$ represents the number of goods that a buyer of standard type who joins platform i purchases. Therefore, we can write the the number of buyers of different types as

$$n^i_1 = \frac{1}{2} + \frac{U(\bar{n}^i_2) - \gamma^i_1\bar{n}^i_2 - p^i_1 - U(\bar{n}^j_2) - \gamma^j_1\bar{n}^j_2 + p^j_1}{2t} \tag{31}$$

and

$$n^i_{1\beta} = \frac{1}{2} + \frac{U(\beta\bar{n}^i_2) - \beta\gamma^i_1\bar{n}^i_2 - p^i_1 - U(\beta\bar{n}^j_2) - \beta\gamma^j_1\bar{n}^j_2 + p^j_1}{2t}, \tag{32}$$

and number of producers of different types as

$$n^i_2 = 1 - F\left(\frac{p^i_2}{\bar{n}^i_1} + \gamma^i_2\right) \quad \text{and} \quad n^i_{2\lambda} = 1 - F\left(\frac{p^i_2}{\lambda\bar{n}^i_1} + \gamma^i_2\right). \tag{33}$$

The profit function of platform i is then given by

$$\Pi^i = p^i_1(q_1 n^i_{1\beta} + (1-q_1)n^i_1) + p^i_2(q_2 n^i_{2\lambda} + (1-q_2)n^i_2) + (\gamma^i_1 + \gamma^i_2)(q_1\beta n^i_{1\beta} + (1-q_1)n^i_1)(q_2\lambda n^i_{2\lambda} + (1-q_2)n^i_2)$$

$$-(q_1 n^i_{1\beta} + (1-q_1)n^i_1)c(q_2 n^i_{2\lambda} + (1-q_2)n^i_2),$$

where n^i_1, $n^i_{1\beta}$, n^i_2 and $n^i_{2\lambda}$ are defined in (31), (32) and (33), respectively.

In the same way as above we can now solve for the equilibrium prices. A detailed description can be found in Appendix C. Doing so yields, after letting $q_1 \to 0$ and $q_2 \to 0$, that the prices of side 1 are implicitly given by

$$p_1 = t + c(n_2) \quad \text{and} \quad \gamma_1 = \frac{2F'p_2(2U' + \gamma_2 + 2p_2 - c'(n_2)) + \gamma_2 n_2}{2F'p_2 - n_2}$$

while those of side 2 are given by

$$p_2 = 0 \quad \text{and} \quad \gamma_2 = \frac{tn_2 - F'((U'(n_2) - \gamma_1)(p_1 - c(n_2)) - tc'(n_2))}{F'(t + n_2(U'(n_2) - \gamma_1))} - \gamma_1.$$

Simplifying these four equations yields

$$p_1 = t + c(n_2), \quad p_2 = 0, \quad \gamma_1 = U'(n_2) - c'(n_2) - \frac{n_2}{F'}, \quad \text{and} \quad \gamma_2 = -(U'(n_2) - c'(n_2)) + \frac{n_2}{F'}. \tag{34}$$

It remains to check that the second-order conditions are satisfied at these prices. In Appendix B we show that this is the case if

$$(F')^2 \left[2 - F'(U'' - c'')\right] + n_2 F'' > 0 \tag{35}$$

and

$$F't\left\{(F')^2 \left[2 - F'(U'' - c'')\right] + n_2 F''\right\} + 2n_2 F'' \left[2 - F'(U'' - c'')\right] \left[n_2 - F'(U' - c')\right]^2 > 0, \tag{36}$$

where we have skipped the arguments of the derivatives of $U(n_2)$ and $c(n_2)$. Inserting the equilibrium prices into the profit function gives the following result:

Proposition 5 *Suppose* (35) *and* (36) *hold. In the model of media or Internet trading platforms with two different types on each side there is a unique symmetric equilibrium. As* $q_1 \to 0$ *and* $q_2 \to 0$, *platform fees in equilibrium are implicitly given by* (34)*. The profit of a platform is* $\Pi = t/2$.

The intuition behind the uniqueness of the equilibrium is the same as above. The two types on each side react differently to a change in the combination of subscription and per-transaction fee because the marginal rates of substitution between the two fees are different for the two types. As a consequence, a platform cannot ensure itself the same profit with multiple combinations of the fees, but has a unique optimal combination on each price combination of its rival.

The proposition also shows that, although we do not get explicit solution for some prices, the profit can be determined explicitly. As above, it is of particular interest to determine if and how this profit and the rents of the two sides change compared to the case in which platforms can charge just one of the two fees. We start with the case where per-transaction fees are not possible and so platforms can charge only fixed fees. As Armstrong (2006a) shows, in this case the optimal fees are given by

$$p_1 = t + c(n_2) - n_2 c'(n_2) + F'U'(n_2)(U'(n_2) - c'(n_2)) - \frac{(n_2)^2}{F'} \tag{37}$$

and

$$p_2 = -\frac{U'(n_2) - c'(n_2)}{2} + \frac{n_2}{2F'}, \tag{38}$$

yielding a profit of

$$\Pi = \frac{t + U'(n_2)(F'(U'(n_2) - c'(n_2)) - n_2)}{2}$$

for each platform.[34]

Since $n_1 = 1/2$, it is easy to see that the payment of side 2 is the same as in the case in which both fees are available. Thus, we obtain the same result as in the linear case above. Turning to the buyers, their overall payment goes down in case of two-part tariffs if $U'(n_2)\,[F'(U'(n_2) - c'(n_2)) - n_2] > 0$. It is evident that if $U'(n_2) < 0$, the latter inequality surely holds because $F' > 0$. So if buyers are overloaded by advertisements and view them as nuisance in equilibrium, they benefit from two-part tariffs because their per-transaction charge is negative. However, if $U'(n_2) > 0$, the result is ambiguous and depends, among other things, on the marginal costs. If $c'(n_2)$ is relatively large, platforms demand a higher payment from the buyers because they can recoup their marginal costs in a better way by using per-transaction fees. This intuition is similar to the one in the previous subsection. Since the payment of sellers stays unchanged, platforms benefit from the possibility to charge two-part tariffs if the overall payment of the buyers goes up, i.e. if $U'(n_2)\,[F'(U'(n_2) - c'(n_2)) - n_2] < 0$.

In the case where only per-transaction fees are possible we obtain equilibrium fees of

$$\gamma_1 = \frac{t + c(n_2)}{n_2} + U'(n_2) - c'(n_2) - \frac{n_2}{F'} \quad \text{and} \quad \gamma_2 = -(U'(n_2) - c'(n_2)) + \frac{n_2}{F'}.$$

It is easy to see that the payment of each group is the same as in the case of two-part tariffs. Therefore, the profit is also unchanged. The reason is that there are no fixed costs or benefits if an agent joins a platform. Therefore, subscription fees are not helpful for platforms to extract utility or recoup costs but do also not destroy profits.[35] They just shift the revenue source between the two fees. The following proposition summarizes these results:

Proposition 6 *In the model of media or Internet trading platforms, profits are larger in case of two-part tariffs than in case of pure subscription fees if and only if*

$$U'(n_2)\left[F'(U'(n_2) - c'(n_2)) - n_2\right] < 0. \tag{39}$$

The utility of sellers is unchanged while the one of buyers is higher if the inequality in (39) *is reversed. Comparing the case of two-part tariffs with the one of pure per-transaction fees, platforms' profits and utilities of both groups are the same.*

We abstain from analyzing the case in which fees are restricted to be positive here because the results are very similar to the ones obtained in the last subsection.

2.4 Retail Warehouse Clubs

Another application of competitive bottlenecks are retail warehouse clubs. These clubs sell products of suppliers to their customers, and customers need to become members of the club to be able to buy in a company's retail warehouses. Prominent examples are Cosco and Sam's Club in the U.S. or Makro in Europe. In the following we will refer to the retail warehouse clubs

[34] The notation of these prices differs slightly from the one in Armstrong (2006a). His solution is written in the form $p_1 = t + c(n_2) - r(n_2) - (n_2'(1/2)U'(n_2))/2$, where $r(n_2) = (1 - F(p_2/n_1))p_2/n_1$. The solution in (37) can be obtained by inserting $n_1 = 1/2$, $n_2 = 1 - F(p_2/n_1)$, $n_2' = F'p_2/(n_1)^2$ and the equilibrium expression for p_2 given in (38) into the formula for p_1. In a similar way, we can derive (38) from his notation of p_2.

[35] This can also be seen from (24) of the last subsection. If $\bar{b} = 0$ and $f_2 = 0$, we obtain that $\Pi^{\gamma} = \Pi^{p\gamma}$.

simply as retailers.[36] Consumers register with just one retailer over the relevant time period while suppliers sell their products via both retailers. We suppose that there is a continuum of monopoly suppliers and each supplier faces a unit cost of α_2 for her good, where α_2 is independently and identically drawn from a distribution function $F(\alpha_2)$. The unit costs of a product are unknown to retailers; hence, suppliers are ex ante identical for retailers. Consumers value variety and wish to buy one unit of each product as long as the price charged by the retailer is below the reservation value denoted by α_1. We assume that retailers have all the bargaining power vis-a-vis consumers and suppliers. A retailer incurs a cost of c for selling a unit of each good.

Retailers can again set four different prices. Retailer i sets a retail price of γ_1^i per unit to consumers and a fixed fee p_1^i for membership.[37] On the supplier side, retailer i pays a price of γ_2^i per unit to suppliers.[38] In addition, retailers sometimes pay a lump-sum fee p_2^i to suppliers, either to ensure the service of the suppliers, in which case the fee would be positive, or to extract more profits from the suppliers, in which case the fee would be negative.

We can now determine the equilibria of this game. Again we find that there exists a continuum of symmetric equilibria.[39] In each of these equilibria the fixed fees are implicitly defined by

$$p_1 = (\alpha_1 - \gamma_1)(\alpha_1 - \gamma_2 - c)F' + t - \frac{F^2}{F'} \quad \text{and} \quad p_2 = \frac{\alpha_1 - c - \gamma_2}{2} - \frac{F}{2F'},$$

where we dropped the argument of F and F', while the per-unit prices satisfy the second-order conditions but are otherwise undefined. The property that the per-unit prices are left undefined is especially undesirable in case of retailers since in this industry the fixed fees are often less important because consumers and suppliers mainly care about per-unit prices.

Now let us use the same method as in the last two subsections and introduce a second type on each side. As before, suppose that there is mass q_2 of suppliers who sell their goods to consumers only with a probability of $\lambda < 1$, and that there is a mass q_1 of consumers who buy each product just with a probability of $\beta < 1$. We can then write the number of suppliers of each type who sell via retailer i as

$$n_2^i = F\left(p_2^i/\hat{n}_1^i + \gamma_2^i\right) \quad \text{and} \quad n_{2\lambda}^i = F\left(p_2^i/(\lambda\hat{n}_1^i) + \gamma_2^i\right), \tag{40}$$

where $\hat{n}_1^i \equiv (1 - q_1)n_1^i + q_1\beta n_{1\beta}^i$.

The number of consumers of each type who shop at retailer i can be written as

$$n_1^i = \frac{1}{2} + \frac{\hat{n}_2^i(\alpha_1 - \gamma_1^i) - p_1^i - \hat{n}_2^j(\alpha_1 - \gamma_1^j) + p_1^j}{2t} \tag{41}$$

and

$$n_{1\beta}^i = \frac{1}{2} + \frac{\hat{n}_2^i\beta(\alpha_1 - \gamma_1^i) - p_1^i - \hat{n}_2^j\beta(\alpha_1 - \gamma_1^j) + p_1^j}{2t}, \tag{42}$$

[36] The model is the same as the one of supermarkets in Armstrong (2006a). However, since we explicitly allow for membership fees here, retail warehouse clubs fit the case of two-part tariffs better than supermarkets where customers usually do not pay an entry or membership fee.

[37] For example, Sam's Club or Makro charge consumers a yearly membership fee.

[38] Note that in this case the payment is from the retailer to the suppliers.

[39] A detailed derivation can be found in Appendix D.

with $\hat{n}_2^i \equiv (1-q_2)F\left(p_2^i/\hat{n}_1^i + \gamma_2^i\right) + q_2\lambda F\left(p_2^i/(\lambda\hat{n}_1^i) + \gamma_2^i\right)$. The profit function of retailer i is given by

$$\Pi^i = p_1^i\left((1-q_1)n_1^i + q_1 n_{1\beta}^i\right) - p_2^i\left((1-q_2)n_2^i + q_2 n_{2\lambda}^i\right) +$$
$$+(\gamma_1^i - \gamma_2^i - c)\left((1-q_1)n_1^i + q_1\beta n_{1\beta}^i\right)\left((1-q_2)n_2^i + q_2\lambda n_{2\lambda}^i\right),$$

where n_2^i, $n_{2\lambda}^i$, n_1^i and $n_{1\beta}^i$ are defined in (40), (41) and (42), respectively.

In exactly the same way as in the last subsection we can build the first-order conditions and then derive the solution. We find that also in this case there exists a unique symmetric solution in which, as $q_1 \to 0$ and $q_2 \to 0$, the prices are implicitly given by

$$p_1 = t, \quad p_2 = 0, \quad \gamma_1 = \alpha_1 - \frac{F(\gamma_2)}{F'(\gamma_2)} \quad \text{and} \quad \gamma_2 = (\alpha_1 - c) - \frac{F(\gamma_2)}{F'(\gamma_2)}. \tag{43}$$

In the same way as in the last subsection we can determine the conditions for the profit function to be concave. Here we get that these conditions can be written as (again dropping the argument of F and its derivatives)

$$2(F')^2 - FF'' > 0 \tag{44}$$

and

$$t(2(F')^2 - FF'') - 2(\alpha - c)\left[F - F'(\alpha_1 - c)\right]^2\left[(\alpha_1 - c)(F')^3 + FF''\right] > 0, \tag{45}$$

where the first inequality ensures that the second derivative with respect to u_k^i, $k = 1, 2$, is negative while the second inequality ensures that the determinant of the matrix of second derivatives is positive. After determining the profit we get the following result:

Proposition 7 *Suppose* (44) *and* (45) *hold. In the model of retail warehouse clubs there exists a unique equilibrium in case of different supplier and consumer types. As* $q_1 \to 0$ *and* $q_2 \to 0$*, the prices in this unique equilibrium are given by* (43)*. The profit of each platform is* $\Pi = t/2$*.*

The equilibrium prices in (43) exhibit several realistic features. First, as demonstrated in several cases, retailers often contract with suppliers by way of linear prices. For example, Smith and Thanassoulis (2009) and Inderst and Valletti (2009) report that in the U.K. grocery industry contracts between retail chains and suppliers of liquid milk, carbonated soft drinks and bakery products are linear. This is in line with the result that the fixed fee to suppliers equals zero. Second, as in the previous cases, the differentiation parameter t just affects the fixed fee for consumers but not the per-unit price since t does not influence the value of a transaction. Finally, if consumers derive large benefits from suppliers, i.e. if α_1 is large, platforms set both a high final good and a high input price. Thus, if retailers obtain high revenues from customers, they pass on these revenues to some extent to their suppliers, which fits with the observation that retailers vary the margins given to their suppliers if consumers' willingness-to-pay changes.

We can now compare the equilibrium under price discrimination with the one in which only per-unit prices are possible.[40] Calculating the equilibrium for pure per-unit prices yields

$$\gamma_1 = \alpha_1 + 2t - \frac{F}{F'} \quad \text{and} \quad \gamma_2 = 2t + \alpha_1 - c - \frac{t}{F} - \frac{F}{F'}, \tag{46}$$

[40]The case with pure subscription charges is unrealistic in case platforms are retailers and is therefore not considered here.

and a profit for each platform of $\Pi = t/2$.[41] Thus, concerning platforms' profits we obtain the same result as in the last subsection, i.e. they do not change with the possibility to price discriminate because there are no fixed costs or benefits when an agent joins a platform.

Although profits do not change, this does not hold true for the utilities of the two sides. In general these utilities can rise or fall and it seems hard to obtain general conditions when the utility of one or the other side rises. This is the case because the number of group-2 agents may differ in the two regimes which changes the utility if a group-1 agent over and above the pure payment change of that agent—an issue that was not at work in the first two examples. However, it is possible to make progress on this comparison by simplifying the distribution function of suppliers' costs to a uniform distribution. Suppose that F is uniformly distributed between 0 and $\bar{\alpha}_2$. In this case, in equilibrium $F = \gamma_2/\alpha_2$ and $F' = 1/\bar{\alpha}_2$. Thus, prices under two-part tariffs can be written as

$$p_1 = t, \quad p_2 = 0, \quad \gamma_1 = \frac{\alpha_1 + c}{2} \quad \text{and} \quad \gamma_2 = \frac{\alpha_1 - c}{2}.$$

Calculating the equilibrium under pure per-unit prices we obtain

$$\gamma_1 = \frac{6t + c + 3\alpha_1 + \sqrt{(2t + \alpha_1 - c)^2 - 8t\bar{\alpha}_2}}{4} \quad \text{and} \quad \gamma_2 = \frac{2t - c + \alpha_1 - \sqrt{(2t + \alpha_1 - c)^2 - 8t\bar{\alpha}_2}}{4}.$$ [42]

Since we know that $n_1 = 1/2$ in both cases, we can compare the utility of suppliers by comparing the payments in both cases. Suppliers receive a larger payment under two-part tariffs if

$$\frac{\alpha_1 - c}{2} - \frac{2t - c + \alpha_1 - \sqrt{(2t + \alpha_1 - c)^2 - 8t\bar{\alpha}_2}}{4} > 0$$

which can be simplified to $\alpha_1 - c > \bar{\alpha}_2$. Thus, if the gains from trade are large relative to the distribution of costs, suppliers benefit from two-part tariffs. The intuition for this result is the following: Platforms set prices to their buyers such that the per-transaction fee is smaller under two-part tariffs than under pure per-transaction fees, because the differentiation parameter t is now incorporated only in the fixed fee. Therefore, platforms obtain a smaller profit per transaction. If now $\bar{\alpha}_2$ is large, there are relatively few suppliers on each platform and so there are only few transactions. As a consequence, platforms lose on the buyer side relative to linear prices. Since transactions are less valuable for the platform, it is less important to attract suppliers and so platforms pay a lower per-transaction price to suppliers.

The utility of a buyer (gross of transport costs) is given by

$$\frac{\gamma_2(\alpha_1 - \gamma_1)}{\bar{\alpha}_2} - p_1 = \frac{(\alpha_1 - c)^2}{4\bar{\alpha}_2} - t$$

in case of two part tariffs while in case of pure per-unit prices it is

$$\frac{(6t + c - \alpha_1 + \sqrt{(2t + \alpha_1 - c)^2 - 8t\bar{\alpha}_2})(c - 2t - \alpha_1 + \sqrt{(2t + \alpha_1 - c)^2 - 8t\bar{\alpha}_2})}{4}.$$

[41] Again the notation of the equilibrium prices differs slightly from the one in Armstrong (2006a). For example, he obtains $\gamma_1 = c + \gamma_2 + t/F$. Inserting γ_2 from the second equation in (46) and rearranging then yields γ_1 written in the form of the first equation in (46).

[42] There is also a second solution that solves (46) but it is easy to check that the second-order conditions are not satisfied at this solution.

Interestingly, comparing these utilities yields that buyers benefit from two-part tariffs if $\alpha_1 - c > \bar{\alpha}_2$, which is the same condition as the one for suppliers. So, if platforms grant a higher payment to their suppliers via two-part tariffs in order to attract more of them, buyers benefit as well. Although their total payment is larger (since platforms' profits are the same), their utility increase from the larger number of suppliers is more important. Thus, we find that the change in surplus of the two sides goes in the same direction with two-part tariffs while platforms' profits are unchanged. The following proposition summarizes this result:

Proposition 8 *In case suppliers' costs are uniformly distributed on* $[0, \bar{\alpha}_2]$, *the utility of both sides is larger under two-part tariffs than under pure per-transaction fees if an only if* $\alpha_1 - c > \bar{\alpha}_2$.

3 Two-Sided Single-Homing

In this section we analyze the case in which each agent can only join one platform. We model this in the same way as Armstrong (2006a) by assuming that there are two Hotelling lines, where platform A is located at point 0 on each line while platform B is located at point 1 on each line. We denote the transport costs by t_1 for group 1 and by t_2 for group 2.

As before we start with the case in which there is just a single type of agent on each side. This implies that each agent of group k, $k = 1, 2$, interacts with probability 1 with an agent of group $-k$ who has joined the same platform. Therefore, the market share of platform i at each group is given by

$$n_1^i = \frac{1}{2} + \frac{(\alpha_1 - \gamma_1^i)n_2^i - p_1^i - (\alpha_1 - \gamma_1^j)n_2^j + p_1^j}{2t_1}$$

and

$$n_2^i = \frac{1}{2} + \frac{(\alpha_2 - \gamma_2^i)n_1^i - p_2^i - (\alpha_2 - \gamma_2^j)n_1^j + p_2^j}{2t_2}.$$

Using the fact that $n_1^j = 1 - n_1^i$ and $n_2^j = 1 - n_2^i$, we can solve the two equations for n_1^i and n_2^i to get

$$n_1^i = \frac{1}{2} + \frac{(2\alpha_1 - \gamma_1^i - \gamma_1^j)(2p_2^j - 2p_2^i + \gamma_2^j - \gamma_2^i) + 2t_2(2p_1^j - 2p_1^i + \gamma_1^j - \gamma_1^i)}{4t_1t_2 - (2\alpha_1 - \gamma_1^i - \gamma_1^j)(2\alpha_2 - \gamma_2^i - \gamma_2^j)} \tag{47}$$

and

$$n_2^i = \frac{1}{2} + \frac{(2\alpha_2 - \gamma_2^i - \gamma_2^j)(2p_1^j - 2p_1^i + \gamma_1^j - \gamma_1^i) + 2t_1(2p_2^j - 2p_2^i + \gamma_2^j - \gamma_2^i)}{4t_1t_2 - (2\alpha_1 - \gamma_1^i - \gamma_1^j)(2\alpha_2 - \gamma_2^i - \gamma_2^j)}. \tag{48}$$

Turning to the profit function of a platform, suppose that each platform incurs a per-agent cost of f_k for serving group k and a per-transaction cost of c for each transaction that the platform governs. The profit function of platform i can then be written as

$$\Pi^i = (p_1^i - f_1)n_1^i + (p_2^i - f_2)n_2^i + (\gamma_1^i + \gamma_2^i - c)n_1^i n_2^i,$$

where n_1^i and n_2^i are defined in (47) and (48).[43]

[43]The model is the same as the one in Armstrong (2006a) with the exception that we allow for c to be greater than zero while $c = 0$ in Armstrong (2006a).

As in the case of competitive bottlenecks we start with the case in which platforms cannot charge per-transaction fees, i.e. $\gamma_k^i = 0$, $k = 1, 2$ and $i = A, B$. To guarantee a market sharing equilibrium we assume that

$$16t_1t_2 > (4\alpha_1 + 4\alpha_2 - 3c)^2.$$

This condition is the counterpart of (11) in the case of competitive bottlenecks and is derived in the same way as described there. Here, the differentiation parameters must be large enough compared to the network externality parameters to avoid equilibria where one platform corners the market on both sides. Calculating the equilibrium fixed fees yields

$$p_1 = f_1 + t_1 - \alpha_2 + \frac{c}{2} \quad \text{and} \quad p_2 = f_2 + t_2 - \alpha_1 + \frac{c}{2}$$

which gives each platform a profit of

$$\Pi^p = \frac{2\left(t_1 + t_2 - \alpha_1 - \alpha_2\right) + c}{4}.$$

We now turn to the case in which both fees are possible. Here, we can use the same method as in the last section, i.e. replacing the payment of each group by its utility (gross of transport costs), to determine under which conditions the profit function is concave. Similarly to Section 2.2, we obtain that the condition for the Hessian matrix of second derivatives to be positive definite implies that both second derivatives with respect to the utilities are negative. The condition for the former to hold is

$$16t_1t_2 > \left(2\alpha_1 + 2\alpha_2 - \gamma_1 - \gamma_2 - c\right)^2. \tag{49}$$

Maximizing platforms' profits and calculating the symmetric equilibrium yields fixed fees of

$$p_1 = f_1 + t_1 - \alpha_2 + \frac{c}{2} + \frac{\gamma_2 - \gamma_1}{2} \quad \text{and} \quad p_2 = f_2 + t_2 - \alpha_1 + \frac{c}{2} + \frac{\gamma_1 - \gamma_2}{2}$$

while the per-transaction fees γ_1 and γ_2 fulfill (49) but are otherwise undefined. Thus, we again obtain a continuum of equilibria. The profit of each platform depends on the selected equilibrium and is given by

$$\Pi = \frac{2\left(t_1 + t_2 - \alpha_1 - \alpha_2\right) + c + \gamma_1 + \gamma_2}{4}.$$

Now suppose as above that there are two types on each side, i.e. that there is a fraction q_1 of sellers that trade only with a probability λ and there is a fraction q_2 of buyers that trade only with a probability β. Therefore, the number of types that join platform i on each side are given by

$$n_1^i = \frac{1}{2} + \frac{(\alpha_1 - \gamma_1^i)\left((1-q_2)n_2^i + q_2\beta n_{2\beta}^i\right) - p_1^i - (\alpha_1 - \gamma_1^j)\left((1-q_2)n_2^j + q_2\beta n_{2\beta}^j\right) + p_1^j}{2t_1} \tag{50}$$

and

$$n_{1\lambda}^i = \frac{1}{2} + \frac{\lambda(\alpha_1 - \gamma_1^i)\left((1-q_2)n_2^i + q_2\beta n_{2\beta}^i\right) - p_1^i - \lambda(\alpha_1 - \gamma_1^j)\left((1-q_2)n_2^j + q_2\beta n_{2\beta}^j\right) + p_1^j}{2t_1} \tag{51}$$

for the sellers, and by

$$n_2^i = \frac{1}{2} + \frac{(\alpha_2 - \gamma_2^i)\left((1-q_1)n_1^i + q_1\beta n_{1\lambda}^i\right) - p_2^i - (\alpha_2 - \gamma_2^j)\left((1-q_1)n_1^j + q_1\beta n_{1\lambda}^j\right) + p_2^j}{2t_2} \tag{52}$$

and

$$n_{2\beta}^i = \frac{1}{2} + \frac{\beta(\alpha_2 - \gamma_2^i)\left((1-q_1)n_1^i + q_1\beta n_{1\lambda}^i\right) - p_2^i - \beta(\alpha_2 - \gamma_2^j)\left((1-q_1)n_1^j + q_1\beta n_{1\lambda}^j\right) + p_2^j}{2t_2} \tag{53}$$

for the buyers. The profit function of platform i is given by

$$\Pi^i = (p_1^i - f_1)n_1^i\left((1-q_1)n_1^i + q_1 n_{1\lambda}^i\right) + (p_2^i - f_2)n_2^i\left((1-q_2)n_2^i + q_2 n_{2\beta}^i\right) +$$

$$+(\gamma_1^i + \gamma_2^i - c)(1-q_1)n_1^i\left((1-q_2)n_2^i + q_2\beta n_{2\beta}^i\right) + (\gamma_1^i + \gamma_2^i - c)q_1\lambda n_{1\lambda}^i\left((1-q_2)n_2^i + q_2\beta n_{2\beta}^i\right),$$

where now n_1^i, $n_{1\lambda}^i$, n_2^i and $n_{2\beta}^i$ are defined by (50), (51), (52) and (53), respectively.

In the same way as above, we can calculate the prices in the symmetric equilibrium as $q_1 \to 0$ and $q_2 \to 0$ to get

$$p_1 = f_1 + t_1 - \alpha_1 + \frac{c}{2}, \quad p_2 = f_2 + t_2 - \alpha_2 + \frac{c}{2}, \quad \gamma_1 = c - 2\alpha_2 \quad \text{and} \quad \gamma_2 = c - 2\alpha_1. \tag{54}$$

To guarantee that the second-order conditions are satisfied at the these prices we insert them into (49) which yields

$$16t_1t_2 > (4\alpha_1 + 4\alpha_2 - 3c)^2. \tag{55}$$

After determining the profit we get the following result:

Proposition 9 *Suppose that* (55) *holds. In case of different buyer and seller types there is a unique symmetric equilibrium under two-sided single-homing, in which, as* $q_1 \to 0$ *and* $q_2 \to 0$*, prices are given by* (54)*. The profit in this unique symmetric equilibrium is given by*

$$\Pi^{p\gamma} = \frac{t_1 + t_2}{2} - (\alpha_1 + \alpha_2) + \frac{3c}{4}.$$

It is evident from (54) that in the unique equilibrium each side's per-transaction fee is falling in the per-transaction externality that it exerts on the other side while the fixed fee is independent of this externality. This feature is also present under competitive bottlenecks—see e.g. the equilibrium fees of Section 2.1 given by (19)—and carries over to two-sided single-homing. Interestingly, fixed fees on each side fall in the own externality parameter. This is a consequence of the increased competition effect under two-part tariffs, namely that each platform sets low (or negative) per-transaction fees which has a price reducing effect not only on the rival's per-transaction fees but also on the fixed fees.

We can now compare the profit under price discrimination with the one under pure per-transaction fees. We obtain that

$$\Pi^p - \Pi^{p\gamma} = \alpha_1 + \alpha_2 - c > 0,$$

where the inequality stems from the assumption that $\alpha_1 + \alpha_2 > c$, since otherwise the per-transaction costs would outweigh the benefits. Therefore, in contrast to the case of competitive bottlenecks, under two-sided single-homing we obtain that two-part tariffs unambiguously reduce platforms profits and increase consumer welfare. The reason is that if both sides single-home, platforms have to compete for agents on both sides which renders the effect that two-part tariffs open an additional front of competition more detrimental than under competitive bottlenecks. As a consequence, under two-sided single-homing the implications of price discrimination in two-sided markets are similar to the ones in a one-sided market where there is necessarily competition for consumers.

Since in our obtained equilibrium $\gamma_{-k} = c - 2\alpha_k$, we have that at least one of the per-transaction fees is negative because $\alpha_1 + \alpha_2 - c > 0$. It is therefore of particular interest to analyze the case of non-negative per-transaction fees.[44] In the same way as in Appendix A we can calculate the optimal price structure if $c < 2\alpha_k$ which would imply that $\gamma_{-k} < 0$ in the unconstrained problem. Under the non-negativity restriction we obtain that

$$p_k = t_k + f_k, \quad p_{-k} = t_{-k} + f_{-k} + c - \alpha_1 - \alpha_2, \quad \gamma_k = c - 2\alpha_{-k} \quad \text{and} \quad \gamma_{-k} = 0. \tag{56}$$

Thus, the per-transaction fee to side k is unchanged even if γ_{-k} is restricted to be non-negative. This necessarily implies that if both per-transaction fees were negative in the unconstrained case, the constrained case involves both of them to be equal to zero. Therefore, profits are unchanged compared to the case of pure subscription fees. So let us suppose that $\gamma_k > 0$. Inserting the prices in (56) into the profit gives

$$\Pi = \frac{t_1 + t_2 + c - \alpha_k}{2} - \alpha_{-k}. \tag{57}$$

Subtracting (57) from Π^p yields $(2\alpha_{-k} - c)/4$ which is negative since $\gamma_k = c - 2\alpha_{-k} > 0$. This implies that the profit in case of one-sided price discrimination is larger than under pure fixed fees. Thus, the result that price discrimination hurts platforms under two-sided single-homing is due to the fact that one or both per-transaction fees are negative in equilibrium. If such negative fees are impossible, profits can never fall but would in fact rise if one externality parameter is not too large relative to c. In this case the possibility to distribute the per-transaction costs among the two sides in a better way dominates the increased competition effect. This discussion is summarized in the following proposition:

Proposition 10 *If per-transaction fees are unrestricted, in the unique symmetric equilibrium, as $q_1 \to 0$ and $q_2 \to 0$, platform profits are lower under two-part tariffs than under pure fixed fees. By contrast, if per-transaction fees are restricted to be weakly positive, platform profits under two-part tariffs in this equilibrium are weakly larger than under pure fixed fees.*

The analysis also shows that if $c = 0$—the case that Armstrong (2006a) considers—and per-transaction fees cannot be negative, it is not problematic to concentrate solely on fixed fees because platforms optimally set per-transaction fees equal to zero.

[44] We abstract here from the case that fixed fees are negative. For a detailed discussion of the non-negativity restriction on fixed fees, see Armstrong and Wright (2007). In our case imposing this restriction would only complicate the analysis without giving new insights over and above Armstrong and Wright (2007). In addition, the range of parameters in which fixed fees are negative but per-transaction fees are positive is very small.

4 Discussion on Equilibrium Selection

The paper showed that there is a unique equilibrium in two-part tariff competition between platforms when we allow for different trading behavior of agents in each group. Since this result even holds as the heterogeneity in trading behavior vanishes, our framework selects a unique equilibrium from the continuum of equilibria that occurs under trading homogeneity. Therefore, the question arises how our method is related to other equilibrium selection techniques that involve the introduction of uncertainty. In this section we now briefly discuss the similarity and differences to other methods.

First, the idea of using uncertainty to select a unique equilibrium tariff was pioneered by Klemperer and Meyer (1989). They show in the context of supply function competition that introducing demand uncertainty reduces the set of equilibria. This is the case because demand uncertainty gives firms a strict preference over their possible supply functions for some range. If this demand uncertainty is large enough, i.e. if the support of the demand function is unbounded, Klemperer and Meyer (1989) show uniqueness for a class of demand and cost functions. Although their approach is very useful in oligopoly models, it has proved challenging to implement in applications.[45]

This paper shows that Klemperer and Meyer's (1989) idea of introducing demand uncertainty is particularly powerful in the context of two-sided market. A very small amount of uncertainty introduced via heterogeneity in trading behavior in each group is sufficient to pin down a unique equilibrium from a continuum of equilibria. In addition, our paper also shows how uncertainty must be introduced for the method to work. As mentioned, the method would have no bite if agents on each side differ in their trading benefit but not in their trading behavior.[46] This implies that, to make the method work in two-sided markets, only a slight amount of uncertainty is needed but it is important how this uncertainty is incorporated.

Second, our method is also related to the well-known trembling-hand perfection refinement of the Nash equilibrium concept developed by Selten (1975). Trembling-hand perfection, by requiring a player to play each of his strategies with some (small) probability, selects Nash equilibria that are robust to mistakes by players. Therefore, under trembling-hand perfection the actions of the players are perturbed. By contrast, our formulation introduces uncertainty in the original game directly but does not perturb actions. An advantage of our method is that is has a natural interpretation and allows for a more realistic description of the market. Equally important, our concept just involves the introduction of a second type on each side and thus is relatively easy to work with.[47] The concept of trembling-hand perfection involves putting a positive probability on each of the players actions which can be a cumbersome technique to select an equilibrium, in particular if the action space is continuous.

Finally, our technique is reminiscent to the one used in general equilibrium theory to

[45]There are a few papers that use the supply function approach in models of different industries. See, for instance, Green and Newberry (1992), Green (1996) and Green (1999) for the electricity market or Hendricks and McAfee (2009) for vertical mergers in the gasoline industry.

[46]If per-transaction fees are levied as a percentage charge and agents differ in their trading benefit, e.g. in α_1 and α_2 in the context of Subsection 2.2, introducing heterogeneity in the trading benefit would also work, since in this case a change in the per-transaction fee has different implications on the two types.

[47]Naturally, our method can be applied to any market situation with imperfect competition in which multiple equilibria exist, not only to two-sided markets.

guarantee that there is a finite number of equilibria. To ensure that a system of equations that determines equilibrium prices is a regular one—which implies a finite number of equilibria—one can perturb this system slightly in an arbitrary manner (see e.g. Mas-Colell, 1985). The difference to the method used in this paper is that in the case of two-sided markets we cannot only reduce the number of equilibria to a finite one but obtain a unique equilibrium starting from a continuity. This is of importance for predictions on market outcomes and welfare. In addition, our results are not only of interest in the limit as the heterogeneity between types vanishes but also when explicitly considering types with different trading behavior. Although we did not focus on this aspect in the paper, it can be worthwhile to consider, for example, price discrimination between these types.

5 Conclusion

This paper first provided a framework how to single out a unique equilibrium in platform competition with two-part tariffs based on the idea of introducing heterogeneity in trading behavior. We showed that the method makes the two-sided market more realistic, is easy to use and works both under competitive bottlenecks and two-sided single-homing. We then analyzed the predictions on prices and profits given by this equilibrium in further detail. We showed that parameters governing the gains from trade between the two sides mainly determine the per-transaction fees while parameters that govern the fixed benefits and costs from joining a platform mainly determine the subscription fees. Two-part tariffs allow platforms to better allocate per-transaction costs among the two sides but also open a new channel for competition. Thus, under competitive bottlenecks platforms gain from two-part tariffs if these costs are relatively high in which case the conclusion of one-sided markets that price discrimination reduces profits under competition is not validated. Under two-sided single-homing per-transaction fees tend to be negative and platforms are hurt by the possibility to price discriminate.

We concentrated our analysis on the case of competition in two-part tariffs since this pricing scheme is prevalent in many industries. However, sometimes platforms use more complicated schemes. For example, in the initial stages of a market's development, platforms may engage in penetration pricing and raise their prices once they have succeeded in gaining a critical number of agents. This implies that fixed and per-transaction fees may vary with the number of agents that a platform attracts. Therefore, an interesting topic for future research is to analyze if heterogeneity in the trading behavior can also solve the problem of multiple equilibria for general pricing schemes. Due to the complexity of the model, this is likely to be a difficult problem. However, in exciting way to tackle it could be using Weyl's (2009) concept of insulated equilibrium which allows to work on a quantity rather than a price basis.

In our analysis we obtain different conclusions on the effects of two-part tariffs under competitive bottlenecks and under two-sided single-homing. However, we kept the difference between these two models exogenous. An interesting direction for further research could be to consider under which conditions one or the other case arises endogenously due to different values of differentiation perceptions or network externalities in each group. This can provide further insights under which conditions price discrimination is helpful for platforms stemming from the primitives of the market. (Armstrong and Wright (2007) provide a first step into endogenizing the participation behavior of agents but focus on only one of the two fees.)

6 Appendix

6.1 Appendix A

In this appendix we consider the case in which equilibrium fees are restricted to be non-negative. This can be the case because negative fees lead to moral hazard problems that arise when agents are paid for trading with the other side.

From (19) it is evident, that both subscription fees are positive in equilibrium. Thus, we can concentrate on the case in which γ_1 or γ_2 are negative. Let us first look at the case where $c - \alpha_2 - (\alpha_1 + \alpha_2 - c)^2/(2\bar{b} - 2f_2) < 0$ but $c + \alpha_2 - \alpha_1 > 0$, so that in the unrestricted case $\gamma_1 < 0$ but $\gamma_2 > 0$. Solving the model in the same way as above with two types on each side and then letting q_1 and q_2 go to zero we obtain restricted equilibrium fees of[48]

$$p_1 = t + f_1 - \frac{(\alpha_1 + \alpha_2 - c)^2 - 2(\alpha_2 - c)(\bar{b} - f_2)}{4\Delta b},\ \gamma_1 = 0,\ p_2 = \frac{\bar{b} + f_2}{2},\ \gamma_2 = \frac{c + \alpha_2 - \alpha_1}{2}, \quad (58)$$

which gives an equilibrium profit to each platform of

$$\frac{t}{2} + \frac{c(2(\alpha_1 + 2\alpha_2) - c) + 4\bar{b}(\bar{b} - 2f_2)f_2^2 - (\alpha_1 + \alpha_2)(3\alpha_1 + \alpha_2)}{16\Delta b}.$$

A comparison with the case with pure subscription fees yields that the profit with two-part tariffs is strictly larger if $c + \alpha_2 - \alpha_1 > 0$ which is indeed the case since $\gamma_2 > 0$. The economic rationale behind this result is that the increased competition effect is less dramatic because there is only one additional front of competition and not two. Since $\gamma_2 > 0$ requires c to be large enough, the cost distribution effect is larger, and so platforms benefit from the possibility to charge two-part tariffs.

Now suppose that $c + \alpha_2 - \alpha_1 < 0$ but $c - \alpha_2 - (\alpha_1 + \alpha_2 - c)^2/(2\bar{b} - 2f_2) > 0$, so that in the unrestricted case $\gamma_1 > 0$ but $\gamma_2 < 0$. In this case our solution method yields restricted equilibrium fees of

$$p_1 = t + f_1,\ \gamma_1 = \frac{c(\alpha_1 + 2\alpha_2 - c) - \alpha_2(3\alpha_1 + \alpha_2) - 2(\alpha_2 - c)(\bar{b} - f_2)}{2(\bar{b} - f_2) + \alpha_1 - \alpha_2 - c}, \quad (59)$$

$$p_2 = \frac{2(\bar{b} + f_2) + \alpha_2 - \alpha_1 + c}{4},\ \gamma_2 = 0,$$

and a profit to each platform of

$$\frac{t}{2} + \frac{(2\bar{b} - 2f_1 - 3\alpha_1 - \alpha_2 + c)(2\bar{b} - 2f_1 + \alpha_1 + \alpha_2 - c)^2}{16\Delta b(2\bar{b} - 2f_1 - \alpha_1 + \alpha_2 - c)}.$$

Comparing this profit with the one under pure subscription fess we obtain that the profit with two-part tariffs is larger if

$$c(\alpha_1 + 2\alpha_2 - c) - \alpha_2(3\alpha_1 + \alpha_2) - 2(\alpha_2 - c)(\bar{b} - f_2) > 0.$$

[48] Since $\gamma_1 = 0$ we obtain the same equilibrium when working with a second type only on side 2, the side where price discrimination is indeed relevant.

From (59), it is obvious that this holds true if $\gamma_1 > 0$. So we obtain a similar conclusion as in the last case, namely if platforms set only three fees and these fees are strictly positive, their profits are larger than with pure subscription fees. Thus, the analysis shows that if platforms can price discriminate with respect ot only one group and set strictly positive fees to this group, they benefit since the cost-distribution effect dominates the increased-competition effect.

Overall the solution under restricted fees can be written as follows:

Suppose $c + \alpha_2 - \alpha_1 \geq 0$. Then, if $c - \alpha_2 - (\alpha_1 + \alpha_2 - c)^2/(2\bar{b} - 2f_2) \geq 0$, the solution is given by (19) while if $c - \alpha_2 - (\alpha_1 + \alpha_2 - c)^2/(2\bar{b} - 2f_2) < 0$, the solution is given by (58).

Suppose $c + \alpha_2 - \alpha_1 < 0$. Then, if $c(\alpha_1 + 2\alpha_2 - c) - \alpha_2(3\alpha_1 + \alpha_2) - 2(\alpha_2 - c)(\bar{b} - f_2) \geq 0$ the solution is given by (59) while if $c(\alpha_1 + 2\alpha_2 - c) - \alpha_2(3\alpha_1 + \alpha_2) - 2(\alpha_2 - c)(\bar{b} - f_2) < 0$, the solution is given by (8) and (9) and $\gamma_1 = \gamma_2 = 0$.

6.2 Appendix B

To reduce the number of platform i's strategic variables we replace $U(n_2^i) - p_1^i - \gamma_1^i n_2^i$ by u_1^i and $-p_2^i - \gamma_2^i n_1^i$ by u_2^i. Therefore, the number of buyers and sellers joining platform i can be written as $n_1^i = 1/2 + (u_1^i - U(n_2^j) + p_1^j + \gamma_1^j n_2^j)/(2t)$ and $n_2^i = 1 - F(-u_2^i/n_1^i)$. The profit function of platform i is given by

$$\Pi^i = n_1^i(U(n_2^i) - u_1^i) - n_2^i u_2^i - n_1^i c(n_2^i).$$

This profit function is concave if $\partial^2\Pi^i/\partial(u_k^i)^2 < 0$ and the determinant of the Hessian is positive. Calculating second derivatives we get

$$\frac{\partial^2\Pi^i}{\partial(u_1^i)^2} = -2\frac{\partial n_1^i}{\partial u_1^i}\left(1 - U'(n_2^i)\frac{\partial n_2^i}{\partial u_1^i}\right) + \frac{\partial^2 n_1^i}{\partial(u_1^i)^2}(U(n_2^i) - u_1^i) + n_1^i\left(U''(n_2^i)\left(\frac{\partial n_2^i}{\partial u_1^i}\right)^2 + U'(n_2^i)\frac{\partial^2 n_2^i}{\partial(u_1^i)^2}\right) -$$

$$-u_2^i\frac{\partial^2 n_2^i}{\partial(u_1^i)^2} - 2c'(n_2^i)\frac{\partial n_1^i}{\partial u_1^i}\frac{\partial n_2^i}{\partial u_1^i} - \frac{\partial^2 n_1^i}{\partial(u_1^i)^2}c(n_2^i) - n_1^i\left(c''(n_2^i)\left(\frac{\partial n_2^i}{\partial u_1^i}\right)^2 + c'(n_2^i)\frac{\partial^2 n_2^i}{\partial(u_1^i)^2}\right), \quad (60)$$

$$\frac{\partial^2\Pi^i}{\partial(u_2^i)^2} = -2\frac{\partial n_2^i}{\partial u_2^i}\left(1 - U'(n_2^i)\frac{\partial n_1^i}{\partial u_2^i}\right) + \frac{\partial^2 n_1^i}{\partial(u_2^i)^2}(U(n_2^i) - u_1^i) + n_1^i\left(U''(n_2^i)\left(\frac{\partial n_2^i}{\partial u_2^i}\right)^2 + U'(n_2^i)\frac{\partial^2 n_2^i}{\partial(u_2^i)^2}\right) -$$

$$-u_2^i\frac{\partial^2 n_2^i}{\partial(u_2^i)^2} - 2c'(n_2^i)\frac{\partial n_1^i}{\partial u_2^i}\frac{\partial n_2^i}{\partial u_2^i} - \frac{\partial^2 n_1^i}{\partial(u_2^i)^2}c(n_2^i) - n_1^i\left(c''(n_2^i)\left(\frac{\partial n_2^i}{\partial u_2^i}\right)^2 + c'(n_2^i)\frac{\partial^2 n_2^i}{\partial(u_2^i)^2}\right) \quad (61)$$

and

$$\frac{\partial^2\Pi^i}{\partial u_1^i\partial u_2^i} = -2\frac{\partial n_1^i}{\partial u_2^i}\left(1 - U'(n_2^i)\frac{\partial n_2^i}{\partial u_1^i}\right) + \frac{\partial^2 n_1^i}{\partial u_1^i\partial u_2^i}(U(n_2^i) - u_1^i) + n_1^i\frac{\partial n_2^i}{\partial u_2^i}\left(U''(n_2^i)\frac{\partial n_2^i}{\partial u_1^i} + U'(n_2^i)\frac{\partial n_1^i}{\partial u_1^i}\right) -$$

$$-\frac{\partial n_2^i}{\partial u_1^i} - u_2^i\frac{\partial^2 n_2^i}{\partial u_1^i\partial u_2^i} - \frac{\partial^2 n_1^i}{\partial u_1^i\partial u_2^i}c(n_2^i) - c'(n_2^i)\left(\frac{\partial n_1^i}{\partial u_2^i}\frac{\partial n_2^i}{\partial u_1^i} + \frac{\partial n_1^i}{\partial u_1^i}\frac{\partial n_2^i}{\partial u_2^i}\right) - n_1^i\left(c''(n_2^i)\frac{\partial n_2^i}{\partial u_2^i}\frac{\partial n_2^i}{\partial u_1^i} + c'(n_2^i)\frac{\partial^2 n_2^i}{\partial u_1^i\partial u_2^i}\right). \quad (62)$$

From the equations determining the number of agents we can determine the first and second derivatives that are needed in (60), (61) and (62). Doing so yields that in a symmetric equilibrium, i.e. $p_k^i = p_k^j = p_k$ and $\gamma_k^i = \gamma_k^j = \gamma_k$ with $k = 1, 2$, we have

$$\frac{\partial n_1^i}{\partial u_1^i} = \frac{1}{2t - 4F'p_2(U' - \gamma_1)}, \quad \frac{\partial n_1^i}{\partial u_2^i} = 0, \quad \frac{\partial n_2^i}{\partial u_2^i} = 2F', \quad \frac{\partial n_2^i}{\partial u_1^i} = -\frac{4F'u_2}{2t - 4F'p_2(U' - \gamma_1)}$$

for the first derivatives,

$$\frac{\partial^2 n_1^i}{\partial (u_1^i)^2} = \frac{16p_2\left(U''(F')^2p_2 + (U' - \gamma_1)F' + (U' - \gamma_1)F''p_2\right)}{(2t - 4F'(U' - \gamma_1))^3}, \quad \frac{\partial^2 n_1^i}{\partial u_1^i \partial u_2^i} = 0, \quad \frac{\partial^2 n_1^i}{\partial (u_2^i)^2} = 0$$

for the second derivatives of n_1^i, and

$$\frac{\partial^2 n_2^i}{\partial (u_2^i)^2} = -4F'', \quad \frac{\partial^2 n_2^i}{\partial u_1^i \partial u_2^i} = -\frac{2F' + 4F''u_2}{t - 2F'p_2(U' - \gamma_1)},$$

$$\frac{\partial^2 n_2^i}{\partial (u_1^i)^2} = -\frac{2F'u_2\left(4\left((U' - \gamma_1)F'p_2\right)\left(1 + p_2\frac{F''}{F'}\right) - U''p_2u_2(F')^2\right) - (2t - 4(U' - \gamma_1)F'p_2)\left(1 - u_2\frac{F''}{F'}\right)}{(t - 2F'p_2(U' - \gamma_1))^3}$$

for the second derivatives of n_2^i.

Inserting these expressions into (60), (61) and (62) and using the equilibrium prices given by (34), we obtain, after simplifying,

$$\frac{\partial^2 \Pi^i}{\partial (u_1^i)^2} = -\frac{2t(F')^3 + \left\{(F')^2\left[2 - F'(U'' - c'')\right] + n_2F''\right\}\left[n_2 - F'\left(U' - c'\right)\right]^2}{2t^2(F')^3},$$

$$\frac{\partial^2 \Pi^i}{\partial (u_2^i)^2} = -\frac{(F')^2\left[2 - F'(U'' - c'')\right] + n_2F''}{F'}$$

and

$$\frac{\partial^2 \Pi^i}{\partial u_1^i \partial u_2^i} = \frac{\left[n_2 - F'\left(U' - c'\right)\right]^2\left\{(F')^2\left[2 - F'(U'' - c'')\right] + n_2F''\right\}}{t(F')^2}.$$

It is then easy to see that (35) implies that $\partial^2\Pi_i/\partial(u_1^i)^2 < 0$ and $\partial^2\Pi_i/\partial(u_2^i)^2 < 0$. Calculating the determinant of the Hessian matrix, $(\partial^2\Pi_i/\partial(u_1^i)^2)(\partial^2\Pi_i/\partial(u_2^i)^2) - (\partial^2\Pi_i/\partial u_1^i\partial u_2^i)^2$, and simplifying yields that it is larger than zero if (36) holds.

6.3 Appendix C

Differentiating Π^i with respect to p_1^i and γ_1^i and using the fact that in a symmetric equilibrium $p_1^i = p_1^j = p_1$ and $\gamma_1^i = \gamma_1^j = \gamma_1$ which implies that $n_1^i = n_{1\beta}^i = 1/2$, $n_2^i = n_2^j = n_2$ and $n_{2\lambda}^i = n_{2\lambda}^j = n_{2\lambda}$ we obtain first-order conditions of

$$\frac{\partial \Pi^i}{\partial p_1^i} = \frac{1}{2} + p_1\left(q_1\frac{dn_{1\beta}^i}{dp_1} + (1 - q_1)\frac{dn_1^i}{dp_1}\right) + p_2\left(q_2\frac{dn_{2\lambda}^i}{dp_1} + (1 - q_2)\frac{dn_2^i}{dp_1}\right) +$$

$$+(\gamma_1+\gamma_2)\left(q_1\beta\frac{dn^i_{1\beta}}{dp_1}+(1-q_1)\frac{dn^i_1}{dp_1}\right)(q_2\lambda n_{2\lambda}+(1-q_2)n_2)+$$

$$+(\gamma_1+\gamma_2)\left(\frac{1-q_1+q_1\beta}{2}\right)\left(q_2\frac{dn^i_{2\lambda}}{dp_1}+(1-q_2)\frac{dn^i_2}{dp_1}\right)-\left(q_1\frac{dn^i_{1\beta}}{dp_1}+(1-q_1)\frac{dn^i_1}{dp_1}\right)c(\hat{n}_2)-$$

$$-\frac{c'(\hat{n}_2)}{2}(q_2n_{2\lambda}+(1-q_2)n_2)\left(q_2\frac{dn^i_{2\lambda}}{dp_1}+(1-q_2)\frac{dn^i_2}{dp_1}\right)=0$$

and

$$\frac{\partial\Pi^i}{\partial\gamma^i_1}=p_1\left(q_1\frac{dn^i_{1\beta}}{d\gamma_1}+(1-q_1)\frac{dn^i_1}{d\gamma_1}\right)+p_2\left(q_2\frac{dn^i_{2\lambda}}{d\gamma_1}+(1-q_2)\frac{dn^i_2}{d\gamma_1}\right)+$$

$$+\frac{1-q_1+q_1\beta}{2}(q_2\lambda n_{2\lambda}+(1-q_2)n_2)+(\gamma_1+\gamma_2)\left(q_1\beta\frac{dn^i_{1\beta}}{d\gamma_1}+(1-q_1)\frac{dn^i_1}{d\gamma_1}\right)(q_2\lambda n_{2\lambda}+(1-q_2)n_2)+$$

$$+(\gamma_1+\gamma_2)\left(\frac{1-q_1+q_1\beta}{2}\right)\left(q_2\frac{dn^i_{2\lambda}}{d\gamma_1}+(1-q_2)\frac{dn^i_2}{d\gamma_1}\right)-\left(q_1\frac{dn^i_{1\beta}}{d\gamma_1}+(1-q_1)\frac{dn^i_1}{d\gamma_1}\right)c(\hat{n}_2)-$$

$$-\frac{c'(\hat{n}_2)}{2}(q_2n_{2\lambda}+(1-q_2)n_2)\left(q_2\frac{dn^i_{2\lambda}}{d\gamma_1}+(1-q_2)\frac{dn^i_2}{d\gamma_1}\right)=0,$$

with $\hat{n}_2=q_2n^i_{2\lambda}+(1-q_2)n^i_2$.

As above, in these first-order conditions we need to determine dn^i_m/dp^i_j and $dn^i_m/d\gamma^i_j$, where now $m=1,1\beta,2,2\lambda$ and $j=1,2$. This can be done via totally differentiating (31), (32) and (33). Totally differentiating n^i_1 and $n^i_{1\beta}$ given by (31) and (32), respectively, yields

$$dn^i_1=$$

$$=\frac{(U'(\bar{n}^i_2)-\gamma_1)(q_2\lambda dn^i_{2\lambda}+(1-q_2)dn^i_2)-(U'(\bar{n}^i_2)-\gamma_1)(q_2\lambda dn^j_{2\lambda}+(1-q_2)dn^j_2)-dp^i_1-\bar{n}^i_2d\gamma^i_1}{2t}$$

and

$$dn^i_{1\beta}=$$

$$=\frac{(U'(\beta\bar{n}^i_2)-\gamma_1)\beta(q_2\lambda dn^i_{2\lambda}+(1-q_2)dn^i_2)-(U'(\beta\bar{n}^i_2)-\gamma_1)\beta(q_2\lambda dn^j_{2\lambda}+(1-q_2)dn^j_2)-dp^i_1-\beta\bar{n}^i_2d\gamma^i_1}{2t}.$$

Totally differentiating n^i_2 and $n^i_{2\lambda}$ given by (33) yields

$$dn^i_2=-\frac{2F'}{1-q_1+\beta q_1}dp^i_2-F'd\gamma^i_2+\frac{4p_2F'}{(1-q_1+\beta q_1)^2}(q_1\beta dn^i_{1\beta})+(1-q_1)dn^i_1)$$

and

$$dn^i_{2\lambda}=-\frac{2F'_\lambda}{\lambda(1-q_1+\beta q_1)}dp^i_2-F'd\gamma^i_2+\frac{4p_2F'_\lambda}{\lambda(1-q_1+\beta q_1)^2}(q_1\beta dn^i_{1\beta})+(1-q_1)dn^i_1).$$

Finally, differentiating n^j_2 and $n^j_{2\lambda}$ with respect to n^i_1 and $n^i_{1\beta}$ gives

$$dn^j_2=-\frac{4p_2F'}{(1-q_1+\beta q_1)^2}(q_1\beta dn^i_{1\beta})+(1-q_1)dn^i_1)$$

and

$$dn_{2\lambda}^j = -\frac{4p_2 F'_\lambda}{\lambda(1-q_1+\beta q_1)^2}(q_1\beta dn_{1\beta}^i) + (1-q_1)dn_1^i),$$

where

$$F' = F'\left(\frac{2p_2}{1-q_1+\beta q_1}+\gamma_2\right) \quad \text{and} \quad F'_\lambda = F'\left(\frac{2p_2}{\lambda(1-q_1+\beta q_1)}+\gamma_2\right).$$

Tedious but routine calculations then allow us to determine dn_m^i/dp_j^i and $dn_m^i/d\gamma_j^i$, $m = 1, 1\beta, 2, 2\lambda$ and $j = 1, 2$.

Inserting the respective values into the first-order conditions and solving the expressions for p_1 and γ_1 yields

$$p_1 = t + c(n_2)$$

and

$$\gamma_1 = \frac{\gamma_2(1-q_1+\beta q_1)^2\lambda(n_2(1-q_2)+n_{2\lambda}q_2\lambda)}{(1-q_1+\beta q_1)\left(p_2F' - (1-q_1+\beta q_1)\lambda(n_2(1-q_2)+n_{2\lambda}q_2\lambda)\right)}+$$

$$+\frac{2F'p_2\left(2\lambda U'(\bar{n}_2)(1-q_1) - 2\lambda\beta q_1 U'(\beta\bar{n}_2) + (\gamma_2 - c'(n_2))\lambda(1-q_1+\beta q_1) - 2p_2(q_2+\lambda(1-q_2))\right)}{(1-q_1+\beta q_1)\left(p_2F' - (1-q_1+\beta q_1)\lambda(n_2(1-q_2)+n_{2\lambda}q_2\lambda)\right)}.$$

In the limit as $q_1 \to 0$ and $q_2 \to 0$ and therefore $F'_\lambda \to F'$ and $\bar{n}_2 = n_{2\lambda} = n_2$, we obtain

$$p_1 = t + c(n_2) \quad \text{and} \quad \gamma_1 = \frac{2p_2F'(2U'(n_2) + 2p_2 + \gamma_2 - c'(n_2)) - \gamma_2 n_2}{2p_2F' - n_2}.$$

Proceeding in the same way for the prices to side 2, we get

$$p_1 = 0 \quad \text{and} \quad \gamma_1 = \frac{t(n_2 - F'(\gamma_1 - c'(n_2))) - F'(U'(n_2) - \gamma_1)(p_1 - c'(n_2) + \gamma_1 n_2)}{F'(t + (U'(n_2) - \gamma_1)n_2)}.$$

Solving the last four expressions for p_1, p_2, γ_1 and γ_2, we obtain (34).

6.4 Appendix D

Given the description of the model the profit of a supplier with cost draw α_2 who sells via retailer i is given by

$$n_1^i(\gamma_2^i - \alpha_2) + p_2^i. \tag{63}$$

Since α_2 is drawn from a distribution $F(\alpha_2)$ and a supplier joins if (63) is positive, the number of suppliers can be written as

$$n_2^i = F\left(\frac{p_2^i}{n_1^i} + \gamma_2^i\right). \tag{64}$$

The utility of a consumer who shops at retailer i is given by $U_1^i = n_2^i(\alpha_1 - \gamma_1^i) - p_1^i$ which, by using (64), yields that the number of consumers of retailer i is given by

$$n_1^i = \frac{1}{2} + \frac{F\left(p_2^i/n_1^i + \gamma_2^i\right)(\alpha_1 - \gamma_1^i) - p_1^i - F\left(p_2^j/n_1^j + \gamma_2^j\right)(\alpha_1 - \gamma_1^j) + p_1^j}{2t}. \tag{65}$$

The profit of retailer i is

$$\Pi^i = p_1^i n_1^i - p_2^i n_2^i + n_1^i n_2^i(\gamma_1^i - \gamma_2^i - c), \tag{66}$$

where n_1^i and n_2^i are defined in (64) and (65).

Differentiating (66) with respect to the four prices gives first-order conditions of

$$\frac{\partial \Pi^i}{\partial p_k^i} = n_k^i I_k + p_1^i \frac{dn_1^i}{dp_k^i} - p_2^i \frac{dn_2^i}{dp_k^i} + (\gamma_1^i - \gamma_2^i - c)\left(n_1^i \frac{dn_2^i}{dp_k^i} + n_2^i \frac{dn_1^i}{dp_k^i}\right) = 0,$$

$$\frac{\partial \Pi^i}{\partial \gamma_k^i} = p_1^i \frac{dn_1^i}{d\gamma_k^i} - p_2^i \frac{dn_2^i}{d\gamma_k^i} + n_1^i n_2^i I_k + (\gamma_1^i - \gamma_2^i - c)\left(n_1^i \frac{dn_2^i}{d\gamma_k^i} + n_2^i \frac{dn_1^i}{d\gamma_k^i}\right) = 0,$$

with $k = 1, 2$, $I_1 = 1$ and $I_2 = -1$. In the same way as in the example in the last subsection we can determine from (64) and (65) how the number of consumers and suppliers vary with each of the four prices. Here we get that in a symmetric equilibrium

$$\frac{dn_1^i}{dp_1^i} = -\frac{1}{\phi}, \quad \frac{dn_1^i}{d\gamma_1^i} = -\frac{F}{\phi}, \quad \frac{dn_1^i}{dp_2^i} = \frac{2F'(\alpha_1 - \gamma_1)}{\phi}, \quad \frac{dn_1^i}{d\gamma_2^i} = \frac{F'(\alpha_1 - \gamma_1)}{\phi}, \quad \frac{dn_2^i}{dp_1^i} = -\frac{4p_2 F'}{\phi},$$

$$\frac{dn_2^i}{d\gamma_1^i} = -\frac{4p_2 F F'}{\phi}, \quad \frac{dn_2^i}{dp_2^i} = \frac{2F'(4t + 2p_2 F'(\alpha_1 - \gamma_1))}{\phi} \quad \text{and} \quad \frac{dn_2^i}{d\gamma_2^i} = \frac{F'(4t + 2p_2 F'(\alpha_1 - \gamma_1))}{\phi},$$

with $\phi \equiv 2t + 8p_2 F'(\alpha_1 - \gamma_1)$, where we abbreviated $F(2p_2 + \gamma_2)$ by F and $F'(2p_2 + \gamma_2)$ by F'.

Solving for symmetric equilibria now yields that there is again a continuum of symmetric equilibria. In each of these equilibria the fixed fees are given by

$$p_1 = (\alpha_1 - \gamma_1)(\alpha_1 - \gamma_2 - c)F' + t - \frac{F^2}{F'} \quad \text{and} \quad p_2 = \frac{\alpha_1 - c - \gamma_2}{2} - \frac{F}{2F'},$$

while the per-unit prices satisfy the second-order conditions but are otherwise undefined. As in Subsection 2.3, the second-order conditions are fulfilled if t is large relative to F' and if the absolute values of the per-transaction fees are not too large.

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