Suspense

William Chan

University of Hong Kong

Pascal Courty

London Business School

Li, Hao

University of Toronto

Abstract: In a dynamic model of sports competition, we show that when spectators care only about the level of effort exerted by contestants, rewarding schemes that depend linearly on the final score difference provide more efficient incentives for efforts than schemes based only on who wins and who loses. This is inconsistent with the prevalence of rank order incentive schemes in sports competitions. We provide an explanation by introducing spectators' demand for suspense as greater utility derived from contestants' efforts when the game is closer. As the demand for suspense increases, so does the advantage of rank order schemes.

Corresponding author: Li, Hao, Department of Economics, University of Toronto, 150 St. George Street, Toronto, Ontario, Canada M5S3G7. Phone: (416) 978-4603. Fax: (416) 978-6713. Email: li.hao@utoronto.ca.

Acknowledgements: This paper was presented at the Memorial Conference for Sherwin Rosen at the University of Chicago, and at 2001 Western Economic Association Conference in San Francisco. We thank Dragan Filipovich, Aidan Hollis, Edward Lazear, Sridhar Moorthy, Canice Prendergast, Sherwin Rosen, Allen Sanderson, Wing Suen, and seminar audience at Alberta, Calgary, Chicago GSB, El Collegio De Mexico, Queens, Southampton, Tilburg, and Wissenschaftszentrum Berlin fuer Sozialforschung for helpful comments.

Suspense

1. Introduction

Sport contestants can earn substantial financial prizes. Total 2005 prize money at Wimbledon has topped £10 million for the first time in 2005. The Professional Golf Association has distributed approximately $250 million in prize money across 48 events in the PGA Tour. Even in less popular sports, prize money plays an important role. Nearly $21 million in prize money was awarded in international track and field competitions in 2004. Interestingly, prize money is typically allocated on the basis of who wins and who loses. More informative measures of performance such as score differences rarely matter, even though they are readily available. When there are several contestants in a sporting event, rewards depend on who wins the most games (round robin tournament), or on the sequence of games won (elimination tournament). Total scores and other performance measures matter only in terms of determining who the winner is, not how much the winner gets.[1] Why do win-lose rank order incentives dominate in the design of financial incentives in sports?

The literature on optimal design of rank order tournaments that began with Lazear and Rosen (1981) explains how tournaments provide incentives for efforts. However, the literature has not shed much light on when rank order incentive schemes should be used in lieu of schemes based on other relative performance measures, particularly when the agents are risk neutral. Prendergast (1999, pp 36-37) reviews several reasons for using tournaments, but it remains unclear why prizes in sporting events depend only on rank order of scores and not on scores themselves. Holmstrom (1982) casts doubt on the importance of rank order tournaments in incentive contracts by demonstrating that relative

[1] Although for superstar athletes prize money pales in comparison with income from sponsorship deals (only about $6 million of Tiger Woods' income of close to $90 million in 2004 were prize money), sponsorship money can depend on factors such as charisma and star power other than athletic performance, so prize money is still the main provider of incentives for efforts in a given sport event. Likewise, in recent years contracts for players in some team sports are loaded with incentive clauses that often take on a strong piece rate flavor (e.g., salaries for batters depending on the batting average in baseball), but financial incentives for team efforts remain largely independent of relative performance measures such as score differences.

performance schemes such as rank order tournaments have no intrinsic incentive value if output measures of individual agents are separately observed and are uncorrelated.[2]

One possible reason why rank order schemes dominate in sports is that spectators simply derive great utility from watching rank order contests (see, e.g., O'Keeffe, Viscusi and Zeckhauser, 1984, pp 28-29). Such preference for rank order contests presumably arises from the notion that winner-takes-all tournaments increase the stakes that contestants face through payoff discontinuity, and create the drama that somehow makes the games more appealing to spectators. However, a gripping drama cannot be reduced to mere payoff discontinuity. Even if spectators' interests are piqued by a large payoff discontinuity before a sports game starts, the game can turn lop-sided and spectators may lose their interests. Payoff discontinuity does not capture the intuitive notion that whether a sporting event is involving or not depends on how the game is played out from the beginning to the end.

In this paper, we present an explanation of the dominance of rank order schemes in sports that rests on an analysis of the dynamics of sports competitions and on an understanding of the nature of spectators' demand for drama. The starting point is a dynamic version of Lazear and Rosen's (1981) tournament model, presented in Section 2. In their original static model, tournament participants exert efforts that determine "scores." Within this context, if participants are risk-neutral, rank order tournaments and other schemes based on relative performance measures such as score differences perform equally well: when designed optimally, they all achieve the first best outcome. This conclusion is no longer valid in a sports game with two halves where two contestants choose efforts at the beginning of each half. In a rank order scheme, contestants are rewarded according to whose total score is greater, and therefore they keep up the efforts in the second half only when the game is still close at the end of the first half. In a linear score difference scheme, rewards depend on the final score difference not just in terms of its sign but also linearly in terms of its magnitude, and so contestants face constant incentives to exert effort independent of the stage of the game and of the score difference at the end of the

[2] There are also some works on design of tournaments in a dynamic setting (Aron and Lazear, 1990; Cabral, 2003), but their focus is on risk-taking rather than on effort choice. Dynamic tournament models also appear in the literature of patent races (e.g., Harris and Vickers, 1987), where the issue is whether competitors' R&D efforts increase with the intensity of their rivalry.

first half. In Section 3, we show that under the standard assumption that contestants face convex effort costs, constant allocation of efforts across different states of the game reduces effort costs to the contestants. As a result, linear score difference schemes out-perform rank order schemes. Indeed, under reasonable assumptions, the optimal linear score difference scheme induces the first best efforts.

The result that rank order schemes are dominated by linear score difference schemes suggests that spectators in a sporting event care about other characteristics of the sports game besides contestants' effort levels. In Section 4 we capture a unique feature in the demand for sports, and in the design of incentives in sports, by assuming that spectators enjoy "suspense" in the game: Instead of caring about efforts per se, spectators derive greater utility from contestants' efforts when the game is closer and the outcome is still uncertain. Under a linear score difference scheme, contestants continue to exert efforts in the second half to collect rewards even when the game has become lop-sided and spectators have lost their interests. In contrast, when spectators demand suspense, a rank order scheme provides incentives for continuing efforts exactly when such efforts matter to spectators. We show that as the demand for suspense increases, the optimally designed rank order scheme increases the stake for the contestants. The more spectators demand suspense, the better rank order schemes perform relative to linear score difference schemes. When the demand for suspense is sufficiently high, the optimal rank order scheme dominates all linear score difference schemes. Although the first best efforts cannot be achieved by a rank order scheme when the demand for suspense is sufficiently high, there is a sense that the optimal rank order scheme does the best in terms of satisfying the demand among all schemes based on the final score difference. This is established with further restrictions on the model. We show that the optimal rank order scheme dominates a broad class of incentive schemes that reward contestants on the basis of the final score difference.

In applying the standard contract theory to the world of sports, we submit to the premise that players in a sporting event condition their efforts on expected reward and level of competition throughout the game. There is substantial evidence in sports, both casual and statistical, which supports this premise: in team sports, team effort drops once a margin of victory is established as first-team players are replaced by bench players; Ehrenberg and Bognanno (1990) find significant incentive effects on performance of professional golfers on European circuit by comparing tournaments with different prize money and by comparing players in different positions at the beginning of the final round; Fernie and Metcalf (1996) find similar effects by comparing the performance of British jockeys who are employed on fixed retainers with those who are offered prizes for winning races. Further, in applying contract theory to sports, we make a simplifying assumption that players' effort choice is one-dimensional. In other words, we abstract from strategic issues such as allocation of efforts between offense and defense (Palomino, Rigotti and Rustichini, 2000; Brocas and Carrillo, 2002). There is no doubt that in many sports, particularly team sports, strategic choices are important, and that taking such choices into account can enrich our analysis. However, given our goal of understanding the form of reward schemes in spectator sports, it is a natural first step to concentrate on the simple case of uni-dimensional effort choice. Section 5 contains further discussions of other significant restrictions imposed on the dynamic contract, and relates our dynamic incentive design problem to the broad literature on incentives.

Our concept of demand for suspense contributes to a large existing literature on the determinants of the demand for sport events. In particular, our concept is consistent with the "uncertainty of outcome hypothesis" in the empirical sports literature, which states that spectators are willing to pay more for more uncertain games (Knowles, Sherony and Haupert, 1992). Also related is the finding in the literature that sports leagues try to maintain "competitive balance" by minimizing the disparity between the strong and weak teams (Fort and Quirk, 1995; Sanderson, 2002), and some recent research about its implications to income distribution in sports (Palomino and Rigotti, 2000; Szymanski and Kesenne, 2004). Both the uncertainty of outcome hypothesis and the idea that competitive balance helps provide effective effort incentives can be understood in the standard tournament model of Lazear and Rosen, where contestants supply more effort when the game is closer. What the empirical sports literature has overlooked is the fact that a sport event is an experience good and the spectators' experience depends on the dynamics of contestants' efforts as the game is played out. We model a sport contest as a sequence of effort choices by the contestants, opening up the possibility that the demand for a sport event, measured for example by viewership ratings, changes in a systematic way as the contest unfolds. The main results of the present paper are briefly summarized in Section 6, which concludes with some thoughts on the methodology of the present paper and discussions of possible applications of our framework of dynamic sports competition to other economic issues.

2. The Model

There are two players in a sports game that consists of two halves. In each half $k = 1, 2$, the two players $j = A, B$ choose efforts μ_k^j simultaneously. Effort choices are modeled as one-dimensional, non-negative real variables. Let δ_k denote the cumulative score difference at the end of each half, defined as A's score minus B's score. We assume that changes in the score difference in each half are determined by the efforts μ_k^A and μ_k^B of the two players, and a continuously distributed random variable ϵ_k. Specifically, we assume

$$\delta_1 = \mu_1^A - \mu_1^B + \epsilon_1;$$
$$\delta_2 = \delta_1 + \mu_2^A - \mu_2^B + \epsilon_2.$$

where the random variables ϵ_k, $k = 1, 2$, are i.i.d. across the two halves, and have a differentiable, uni-modal density f that is symmetric around 0. Denote as F the corresponding distribution function of ϵ_k. For analytical convenience, we assume that the support of ϵ_k is the real line.

Our formulation of dynamic sports games is admittedly simplistic, even in the context of individual sports. It involves a number of simplifying assumptions, but allows us to focus on the comparison of different incentive schemes. First, we limit the strategic interactions of the two players to dynamic effort choices, and abstract from other strategic issues such as risk-taking. The outcome of the game, that is, the final score difference δ_2, depends only on effort choices of the two players in the two halves, and the random variables ϵ_k. The random variables capture the intrinsic uncertainty in the game, and is assumed to enter the score additively. This means that greater efforts by the players do not make the game outcome more or less uncertain. Another important assumption we made is that the random variable ϵ_k in each half k has a symmetric density function. This symmetry assumption is standard in the literature following Lazear and Rosen (1981), and is especially handy in our model as it simplifies the characterization of the equilibrium effort dynamics. Finally, by adopting a model of continuous scores, we remove the possibility of a tied game and avoid its implications to player incentives and contract design. This simplification does not qualitatively alter our main conclusions.

The two players are risk-neutral, and do not discount. They simultaneously choose effort at the beginning of each half to maximize their expected reward less the sum of effort costs in the two halves. The players observe the first half score difference δ_1 before choosing their efforts in the second half. We assume that the cost of effort, C, is the same in each half and the same for the two players. In addition to the standard assumption that C is increasing and convex, we make the following technical assumption.

Assumption 1: $0 \leq C''' \leq (C'')^2/C'$, *with at least one strict inequality*; $C'(0) = 0$ *and* $C'(\infty) = \infty$.

Assumption 1 imposes two global restrictions on the cost function. These restrictions are satisfied by increasing quadratic cost functions (e.g., $C(\mu) = \frac{1}{2}\mu^2$). The assumption that the two players share the same effort cost function, together with the implicit assumption that their efforts are equally effective in producing scores, yields the interpretation that the two players have equal "abilities." Since Lazear and Rosen (1981), there has been an extensive literature on the optimal design of rank order schemes with heterogeneous players. The present paper instead asks why rank order schemes should be used to provide incentives for efforts in the first place, so we abstract from the issues that arise when players have different abilities.

The incentive designer chooses a reward scheme to maximize spectators' utility minus the expected reward to the players, subject to voluntary participation and equilibrium response by the players. Let $\underline{U}$ be the reservation payoff of each player before entering the game. Spectators derive utility from efforts exerted by players during the game.[3] We define P_k as the rate of spectator utility per unit of effort μ_k^j in half $k = 1, 2$, and we assume that this rate is the same for the two players. Players' efforts are observable to spectators,

[3] We assume that spectators do not derive utility from simply watching their favorite player win the game. That is, in our model spectators are not "fans."

but not contractible. This ensures that the designer's objective function can involve efforts explicitly, but that the designer cannot condition rewards directly on efforts.[4] Moreover, rewards can depend only on the final score difference, not on the score difference δ_1 at the end of the first half. These contractual restrictions are reasonable for incentive design in sports. The implications of relaxing these assumptions are discussed in Section 5.

We will distinguish between the case where P_2 is constant and the case where it depends on the first half score difference $P_2(\delta_1)$. When P_2 is constant, we will say that spectators care only about "excitement." One goal of the model is to capture the idea that spectators care also about "suspense" in addition to excitement. A simple way of modeling demand for suspense is by assuming that spectators care more about efforts when the game is closer at the end of the first half.[5] A game that remains close at the end of the first half is one that has a less predictable outcome, or a greater chance of outcome reversal. We will say that spectators care also about suspense when P_2 as a function of δ_1 is symmetric around and single-peaked at $\delta_1 = 0$. A constant $P_2(\delta_1)$ should be viewed as a polar case corresponding to no preference for suspense. The other polar case occurs when the function $P_2(\delta_1)$ is an indicator function with all the weight at $\delta_1 = 0$ (tied first half), which corresponds to an extreme preference for suspense. The intermediate cases are defined precisely in Section 4. Throughout the paper, we maintain the following assumption:

Assumption 2: $P_1 = \int P_2(\delta_1) f(\delta_1) \mathrm{d}\delta_1$.

In the absence of any presumption regarding how much spectators enjoy the excitement of the game in the first half versus in the second half, Assumption 2 is a natural starting point.

[4] The assumption of observable efforts is standard in the contest literature following Lazear and Rosen (1982). Since the players are risk-neutral in our model, our results remain valid in a modified model where individual scores are observed and used in place of efforts in the objective function of the incentive designer. See Section 5 for a discussion.

[5] Television audience of a sports game change channels or switch off their sets once outcome of the game appears certain. When the Bowl Championship Series eliminated the margin of victory component in its computer ranking formula for the 2002 college football season so that the chance of qualifying for post-season bowl games no longer depends on the margin of victory in regular season games, American Football Coaches Association announced in its online news (www.afca.org) that the change was designed to "end the possibility of teams running up scores in order to improve their positions in the BCS standings." Presumably AFCA understood that a football game loses all its attraction at the point when the outcome becomes certain to spectators, even if the winning side keeps up their effort.

In the case when spectators care only about the excitement of the game, Assumption 2 implies that $P_2 = P_1$. We use separate notation for P_1 and P_2 throughout the paper, to highlight the distinction between the case where P_2 is constant (Section 3) and the case where P_2 depends on δ_1 (Section 4). Assumption 2 is not needed for some of the analysis; its role will become clear later.

Some may argue that the demand for suspense in sports competitions is purely a taste for uncertain outcomes, and that the satisfaction of this demand requires a substantial difference in payoffs between the winner and the loser. It is hard to distinguish this view of suspense from the taste for uncertainty, such as in the case of movies or lotteries, where efforts of participants are not involved or consumers do not derive utility from such efforts. Further, this view allows no role for dynamics of sports games, which intuitively is an ingredient to what makes sports games interesting to spectators. In contrast, our definition of the demand for suspense is tailored to the context of dynamic sports competitions. We take two components in the demand for sports that have been shown to matter, demand for uncertainty and demand for play quality or effort, and combine them by postulating a complementary relation between the two. According to our definition, the marginal utility for spectators derived from additional efforts by the players is enhanced when the game outcome remains uncertain. We recognize that the issue of suspense is multi-faceted, and that there may be other ways to model the consumer demand for suspense. These other dimensions to suspense may influence the incentive design of sport competitions, but do not invalidate our results.

3. Excitement Only

This section deals with the benchmark case where spectators care only about excitement (that is, P_2 is constant). We characterize equilibrium effort dynamics under rank order incentive schemes and under linear score difference schemes. We derive the optimal rank order scheme and the optimal linear score difference scheme, and compare the performance of these two incentive schemes.

A rank order scheme rewards players entirely on the basis of who wins and who loses, regardless of the score difference at the end. Such a scheme is represented by an "incentive

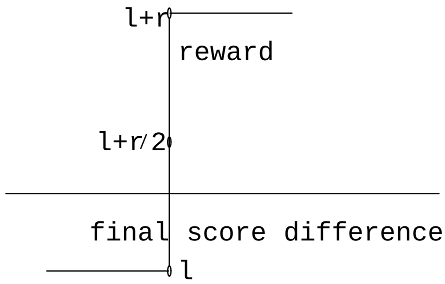

Figure 1. A rank order scheme

prize" r, which is the difference between the winner's and the loser's rewards, and a fixed transfer l, which can be either positive or negative. See Figure 1 for an illustration. To find the optimal rank order scheme, we use backward induction to characterize the equilibrium response to an arbitrary rank order scheme (r, l).

In the second half, given first half score difference δ_1, player A wins if δ_2 is positive, or $\delta_1 + \mu_2^A - \mu_2^B + \epsilon_2 > 0$. The probability that A wins the game is therefore $1 - F(-\delta_1 - \mu_2^A + \mu_2^B)$. Player A chooses μ_2^A to maximize

$$r(1 - F(-\delta_1 - \mu_2^A + \mu_2^B)) - C(\mu_2^A),$$

where μ_2^B is taken as given. The necessary first order condition for A is

$$C'(\mu_2^A) = rf(-\delta_1 - \mu_2^A + \mu_2^B).$$

Given the same first half score difference δ_1, player B wins the game if δ_2 is negative, which occurs with probability $F(-\delta_1 - \mu_2^A + \mu_2^B)$. Therefore, the first order condition for B is:

$$C'(\mu_2^B) = rf(-\delta_1 - \mu_2^A + \mu_2^B).$$

The above two first order conditions imply that $\mu_2^A = \mu_2^B$ regardless of the first half score difference δ_1. That is, in equilibrium the leading player (player A if $\delta_1 > 0$ and B if $\delta_1 < 0$) and the trailing player choose the same level of effort. This result is due to the fact that in the second half the marginal benefit of effort, in terms of increased probability of winning, is the same for the winning player and for the losing player.[6]

[6] We note that the same result obtains even if the noise density function f is asymmetric. Also, it extends to a model with more than two periods. However, with more than two periods, equilibrium effort before the last period is no longer the same for the winning player and the losing player. See the discussion in Section 5.

Let μ_2 be the players' common equilibrium second half effort. It satisfies:

$$C'(\mu_2) = rf(\delta_1). \tag{1}$$

Since C is convex with $C'(0) = 0$ and $C'(\infty) = \infty$, the above condition determines a unique μ_2 for any δ_1 and r, denoted as $\mu_2(\delta_1, r)$. As a function of δ_1, the equilibrium effort μ_2 is symmetric around 0 because f is. This means that an individual player's equilibrium effort choice in the second half is the same whether after the first half the player is leading by some score difference δ_1 or trailing by the same score difference. Further, taking derivative of the equilibrium condition (1), we have

$$\frac{\partial \mu_2}{\partial \delta_1}(\delta_1, r) = \frac{rf'(\delta_1)}{C''(\mu_2)}.$$

Since C is convex, under our assumption that f is single-peaked at $\delta_1 = 0$, the sign of $\partial \mu_2/\partial \delta_1$ is determined by f', the slope of the density function: $\partial \mu_2/\partial \delta_1$ is positive if $\delta_1 < 0$ and negative if $\delta_1 > 0$. This means that the second half equilibrium effort increases if the score difference δ_1 gets closer to 0 and decreases if δ_1 drifts away from 0.[7]

The state of the game at the beginning of the second half is summarized by δ_1, so we can write the equilibrium payoff of each player at the beginning of the second half as

$$v(\delta_1) = rF(\delta_1) - C(\mu_2(\delta_1, r)) + l.$$

Taking derivative and using the equilibrium condition (1) for μ_2, we have

$$v'(\delta_1) = rf(\delta_1) - rf(\delta_1)\frac{\partial \mu_2}{\partial \delta_1}(\delta_1, r). \tag{2}$$

In the first half, player A chooses μ_1^A to maximize

$$\int v(\mu_1^A - \mu_1^B + \epsilon_1)f(\epsilon_1)\mathrm{d}\epsilon_1 - C(\mu_1^A),$$

where $\mu_1^A - \mu_1^B + \epsilon_1$ represents the score difference δ_1 at the end of the first half, and μ_1^B is taken as given. In the symmetric equilibrium, both players exert the same effort μ_1 in the first half, which satisfies the following necessary condition:

$$C'(\mu_1) = \int v'(\epsilon_1)f(\epsilon_1)\mathrm{d}\epsilon_1.$$

[7] Ehrenberg and Bognanno (1990) document this dynamics of efforts in golf tournaments.

By the symmetry of $\mu_2(\delta_1, r)$, the integral of the second term in $v'(\delta_1)$ (equation 2) vanishes, and the equilibrium condition for the first half can be simplified as

$$C'(\mu_1) = \int r f^2(\epsilon_1) \mathrm{d}\epsilon_1. \tag{3}$$

Thus, the first half effort is chosen as in a static game with a noise term equal to the sum of the noise in the two halves. Given that the two players exert the same effort μ_1 in the first half, the equilibrium score difference δ_1 is a random variable with the distribution function F. In what follows, we continue to write $\mu_2(\delta_1, r)$ instead of $\mu_2(\epsilon_1, r)$, to stress that μ_2 depends on δ_1, even though in equilibrium δ_1 is equal to ϵ_1.[8]

We can now compare equilibrium efforts μ_1 and μ_2. Our first result shows that in a rank order scheme, the second half can be more exciting than the first half in terms of generating greater efforts from the two players, but on average the second half gets boring. Equilibrium dynamics of efforts are characterized in the following lemma.

Lemma 1. *In a rank order scheme, the second half efforts are higher (lower) than the first half efforts when the first half score difference is small (large), but the expected second half efforts are at most as high as the first half efforts.*

Proof: Compare the equilibrium conditions (1) and (3). Since $\mu_2(\delta_1, r)$ is symmetric around and single-peaked at $\delta_1 = 0$, so is $C'(\mu_2(\delta_1, r))$ (as a function of δ_1). Because (1) and (3) imply

$$C'(\mu_1) = \int C'(\mu_2(\delta_1, r)) f(\delta_1) \mathrm{d}\delta_1,$$

there exists some $d > 0$ such that $C'(\mu_1) > C'(\mu_2(\delta_1, r))$, and hence $\mu_1 > \mu_2(\delta_1, r)$, if and only if $\delta_1^2 < d^2$. Further, Assumption 1 implies that C' is convex, and so by Jensen's inequality we have

$$C'(\mu_1) \geq C'\left(\int \mu_2(\delta_1, r)) f(\delta_1) \mathrm{d}\delta_1 \right).$$

[8] Later we will show that under any linear score difference scheme, the first half score difference is also equal to the random noise. Thus, which scheme is used does not directly affect the score difference. Our main theorem that rank order schemes dominate linear score difference schemes when spectators care about suspense sufficiently does not result because rank order schemes induce closer scores. However, when the game is modeled with more than two periods, the score difference is no longer pure noise under a rank order scheme. See the discussion in Section 5.

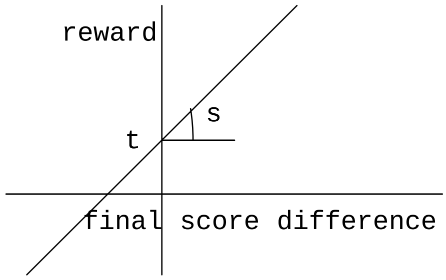

Figure 2. A linear score difference scheme

Thus, $\mu_1 \geq \int \mu_2(\delta_1, r) f(\delta_1) \mathrm{d}\delta_1$. *Q.E.D.*

The incentive designer chooses the rank order scheme (r, l) to maximize "profits" per player:

$$\max_{r,l} \; P_1\mu_1 + \int P_2\mu_2(\delta_1, r) f(\delta_1) \mathrm{d}\delta_1 - \left(l + \frac{1}{2}r\right)$$

subject to the participation constraint

$$l + \frac{1}{2}r - C(\mu_1) - \int C(\mu_2(\delta_1, r)) f(\delta_1) \mathrm{d}\delta_1 \geq \underline{U},$$

where μ_k, $k = 1, 2$, are equilibrium effort functions defined in equations (1) and (3). Since the equilibrium efforts in the two halves depend only on r and not on l, the above optimization problem is solved by first choosing r, which determines μ_1 and μ_2, and then choosing l to bind the participation constraint. The optimal r satisfies the following necessary first order condition:

$$\text{(4)} \qquad \Big(P_1 - C'(\mu_1)\Big)\frac{d\mu_1}{dr} + \int \Big(P_2 - C'(\mu_2(\delta_1, r))\Big)\frac{\partial \mu_2}{\partial r}(\delta_1, r) f(\delta_1) \mathrm{d}\delta_1 = 0.$$

Assumption 1 ensures that the second order condition is satisfied.[9]

Next, we consider the optimal linear score difference scheme. A linear scheme has two parameters: the fixed transfer t, which can be either positive or negative, and a piece rate

[9] To see this, write the objective function as the difference between revenue $i(r)$ and cost $k(r)$, where $i(r) = P_1\mu_1 + \int P_2\mu_2(\delta_1, r) f(\delta_1) \mathrm{d}\delta_1$, and $k(r) = \underline{U} + C(\mu_1) + \int C(\mu_2(\delta_1, r)) f(\delta_1) \mathrm{d}\delta_1$. The first order condition for the optimal r is $i'(r) = k'(r)$, and the second order condition is $i''(r) < k''(r)$. A sufficient condition for the second order condition is that $i''(r) \leq 0$ and $k''(r) \geq 0$ for all r, with at least one strict inequality. Using the equilibrium conditions for $\mu_2(\delta, r)$ and μ_1 (equations 1 and 3), we can show that under the assumption that $C''' \geq 0$, these efforts are weakly concave in r, and so $i''(r) \leq 0$. Similarly, the assumption that $C''' \leq (C'')^2/C'$ implies that the effort cost in each half as a function of r is weakly convex, and so $k''(r) \geq 0$.

s. If the final score difference is δ_2, then A's reward is $t + s\delta_2$ and B's reward is $t - s\delta_2$. See Figure 2 for an illustration.

Under linear score difference schemes, the level of effort is the same for the two players and for the two halves, and independent of the first half score difference. To see this, fix a linear score difference scheme (t, s). Given the score difference δ_1 at the beginning of the second half, player A chooses μ_2^A to maximize

$$\int (t + s(\mu_2^A - \mu_2^B + \epsilon_2 + \delta_1)) f(\epsilon_2) \mathrm{d}\epsilon_2 - C(\mu_2^A),$$

where μ_2^B is taken as given. Given symmetry, the same second half effort μ_2 satisfies the necessary condition:

$$C'(\mu_2) = s. \tag{5}$$

The equilibrium effort level μ_2 is a constant determined entirely by the piece rate s. The equilibrium payoff of each player at the beginning of the second half is therefore

$$v(\delta_1) = t + s\delta_1 - C(\mu_2).$$

In the first half, anticipating this equilibrium payoff, player A chooses μ_1^A to maximize

$$\int v(\mu_1^A - \mu_1^B + \epsilon_1) f(\epsilon_1) \mathrm{d}\epsilon_1 - C(\mu_1^A),$$

where $\mu_1^A - \mu_1^B + \epsilon_1$ represents the random score difference δ_1 at the end of the first half, and μ_1^B is taken as given. In equilibrium both players exert the same effort μ_1 in the first half, which satisfies the following necessary condition:

$$C'(\mu_1) = \int v'(\epsilon_1) f(\epsilon_1) \mathrm{d}\epsilon_1.$$

Since $v' = s$, the first order condition for the first half is the same as (5).

A constant level of effort μ, determined by $C'(\mu) = s$, is exerted by the two players throughout the game. Given this, the designer's profit maximization problem for the optimal linear score difference scheme is

$$\max_{t,s} \ (P_1 + P_2)\mu - t$$

subject to the participation constraint

$$t - 2C(\mu) \geq \underline{U}.$$

The optimal piece rate s is given by $\frac{1}{2}(P_1 + P_2)$, and the fixed transfer t binds the participation constraint.

A rank order scheme on average gets boring (generates low efforts from players) in the second half, but can become exciting when the game is close at the end of the first half. In contrast, under a linear score difference scheme, players keep up the same level of effort regardless of whether the game is close or lopsided after the first half. How do the two schemes compare if both are chosen optimally?[10]

PROPOSITION 1. *When spectators care only about excitement, the optimal linear score difference scheme dominates all rank order schemes.*

PROOF: Let (r, l) be the optimal rank order scheme, and let μ_1 and $\mu_2(\delta_1, r)$ be the equilibrium efforts. Define $\mu = \frac{1}{2}(\mu_1 + \int \mu_2(\delta_1, r) f(\delta_1) \mathrm{d}\delta_1)$. Then, since C is convex,

$$C(\mu_1) + \int C(\mu_2(\delta_1, r)) f(\delta_1) \mathrm{d}\delta_1 \geq C(\mu_1) + C\left(\int \mu_2(\delta_1, r) f(\delta_1) \mathrm{d}\delta_1\right) \geq 2C(\mu).$$

From Assumption 2,

$$P_1 \mu_1 + P_2 \int \mu_2(\delta_1, r) f(\delta_1) \mathrm{d}\delta_1 = (P_1 + P_2)\mu.$$

Define $s = C'(\mu)$. Then a linear scheme with the piece rate s induces μ, with a lower effort cost to the players and the same revenue to the designer. Define $t = \underline{U} + 2C(\mu)$, then (t, s) generates greater profits than (r, l). *Q.E.D.*

The conclusion of Proposition 1 can be strengthened by noting that the optimal linear score difference scheme implements the first best efforts. The first best efforts maximize the difference between the revenue and the cost

$$P_1 \mu_1 + \int P_2 \mu_2(\delta_1) f(\delta_1) \mathrm{d}\delta_1 - C(\mu_1) - \int C(\mu_2(\delta_1)) f(\delta_1) \mathrm{d}\delta_1 - \underline{U}.$$

[10] The proof remains valid if Assumption 2 is replaced by the weaker condition $P_1 \leq P_2$. By Lemma 1, if P_1 is sufficiently greater than P_2, then the result of Proposition 1 is reversed. We ignore this possibility as it seems unreasonable in sports tournaments.

The first order necessary conditions for the first best efforts μ_1^* and μ_2^* are therefore

$$C'(\mu_1^*) = P_1; \quad C'(\mu_2^*) = P_2. \tag{6}$$

Under Assumption 2, the optimal linear scheme with $s = \frac{1}{2}(P_1 + P_2)$ achieves the first best.

4. Excitement and Suspense

Given the importance of rank order incentives in the design of financial incentives in sports, the inferiority of rank order schemes to score difference schemes established in the previous section calls for an explanation. Our attempt is motivated by the unique feature in sports that spectators care about the dynamics of the game. We model this by assuming that spectators value player's efforts more when the game is closer. Formally, we assume that P_2 depends on δ_1. In particular, $P_2(\delta_1)$ is symmetric around and single-peaked at $\delta_1 = 0$ (tied first half). This modification of spectators' preference captures the idea that spectators enjoy both excitement and suspense. Spectators do not care about excitement only: a lop-sided game is not appealing even when the losing side keeps up the effort. On the other hand, spectators do not care about suspense only: they do not like it when the leading player slacks off even though it makes the game close. We show in this section that rank order schemes perform better than score difference schemes when spectators have a strong enough preference for suspense.

We capture the concept of increasing demand for suspense by assuming that P_2 is indexed by a one-dimensional parameter a, such that, with a slight abuse of notation, the demand for suspense is greater under $P_2(\delta_1, a)$ than under $P_2(\delta_1, \tilde{a})$ when $a > \tilde{a}$. The functions $P_2(\delta, a)$ satisfy: (i) $\int (P_2(\delta_1, a) - P_2(\delta_1, \tilde{a})) f(\delta_1) \mathrm{d}\delta_1 = 0$ for any a and $\tilde{a}$, and (ii) there exists a function $\alpha(a)$ such that $\partial P_2(\delta_1, a)/\partial a > 0$ if and only if $\delta_1^2 < \alpha^2(a)$. We say that $P_2(\delta_1, a)$ is more "concentrated" (with respect to f) than $P_2(\delta, \tilde{a})$ if $a > \tilde{a}$. Intuitively, $P_2(\delta_1, a)$ is more concentrated in the sense that the two functions have the same expectation under density f, but the value of $P_2(\delta_1, a)$ is larger for close games (middle values of δ_1) and smaller when a player has acquired a strong lead (more extreme values of δ_1 in either direction).

Increasing demand for suspense does not change the design of linear difference scheme. Since the two players exert the same effort in the two halves regardless of the score difference, the optimal piece rate s depends only on the expectation of $P_2(\delta_1)$, which does not change. The fixed transfer t that binds the player's participation constraint is also unchanged. In contrast, intuition suggests that the optimal rank order scheme should change as spectators' demand for suspense increases. As $P_2(\delta_1)$ becomes more concentrated around $\delta_1 = 0$, the designer will want to make $\mu_2(\delta_1, r)$ also more concentrated in order to take advantage of the fact that spectators have a greater demand for suspense. How can this be achieved? From the equilibrium condition for second half effort μ_2, we see that increasing r will raise $\mu_2(\delta_1, r)$ for all δ_1. But since the density function $f(\delta_1)$ is uni-modal, the increase in μ_2 will be more pronounced around $\delta_1 = 0$. Thus, as $P_2(\delta_1)$ becomes more concentrated around $\delta_1 = 0$, the designer will want to increase r. This intuition is confirmed in the following result.

Lemma 2. *As demand for suspense increases, the incentive prize under the optimal rank order scheme increases and the optimal profits also increase.*

Proof: From the equilibrium condition for second half effort μ under rank order scheme (equation 1), we find that

$$\frac{\partial \mu_2}{\partial r}(\delta_1, r) = \frac{f(\delta_1)}{C''(\mu_2(\delta_1, r))}.$$

Since both $f(\delta_1)$ and $\mu_2(\delta_1, r)$ are symmetric around $\delta_1 = 0$, $\partial \mu_2(\delta_1, r)/\partial r$ is also symmetric. Furthermore,

$$\frac{\partial^2 \mu_2}{\partial r \partial \delta_1}(\delta_1, r) = f'(\delta_1) \frac{(C'')^2 - C'''C'}{(C'')^3}.$$

Under Assumption 1, $\partial \mu_2(\delta_1, r)/\partial r$ is also single-peaked around $\delta_1 = 0$.

With P_2 as a function of δ_1 and indexed by a, the first order condition with respect to r in the optimal design problem of rank order schemes (equation 4) becomes

$$(P_1 - C'(\mu_1)) \frac{d\mu_1}{dr} + \int (P_2(\delta_1, a) - C'(\mu_2(\delta_1, r))) \frac{\partial \mu_2}{\partial r}(\delta_1, r) f(\delta_1) \mathrm{d}\delta_1 = 0.$$

Taking derivatives of the above condition with respect to a, we find that, dr/da, the effect of increasing demand for suspense, has the same sign as

$$\int \frac{\partial P_2}{\partial a}(\delta_1, a)\frac{\partial \mu_2}{\partial r}(\delta_1, r)f(\delta_1)\mathrm{d}\delta_1.$$

By condition (i) of increasing demand for suspense, $\int(\partial P_2(\delta_1, a)/\partial a)f(\delta_1)\mathrm{d}\delta_1 = 0$. Then, for any constant α we can write the above integral as

$$\int \frac{\partial P_2}{\partial a}(\delta_1, a)\left(\frac{\partial \mu_2}{\partial r}(\delta_1, r) - \frac{\partial \mu_2}{\partial r}(\alpha, r)\right)f(\delta_1)\mathrm{d}\delta_1.$$

By condition (ii) of the definition of increasing demand for suspense, we can choose $\alpha > 0$ such that $\partial P_2(\delta_1, a)/\partial a$ is positive for all $\delta_1 \in (-\alpha, \alpha)$, and negative for any $\delta_1 < -\alpha$ or $\delta_1 > \alpha$. We have shown that $\partial \mu_2(\delta_1, r)/\partial r$ is symmetric around and single-peaked at $\delta_1 = 0$. Then, the above integral is positive both for $\delta_1 < -\alpha$ and for $\delta_1 > \alpha$, because $\partial P_2(\delta_1, a)/\partial a < 0$ and $\partial \mu_2(\delta_1, r)/\partial r < \partial \mu_2(\alpha, r)/\partial r$. The integral from $-\alpha$ to α is also positive because $\partial P_2(\delta_1, a)/\partial a > 0$ and $\partial \mu_2(\delta_1, r)/\partial r > \partial \mu_2(\alpha, r)/\partial r$. It follows that $dr/da > 0$.

By the envelope theorem, the change in the value of the objective function under the optimal rank order scheme has the same sign as

$$\int \frac{\partial P_2}{\partial a}(\delta_1, a)\mu_2(\delta_1, r)f(\delta_1)\mathrm{d}\delta_1.$$

We know that $\mu_2(\delta_1, r)$ is symmetric and single-peaked, just like $\partial \mu_2(\delta_1, r)/\partial r$. By a similar argument as above, the above integral is positive, and therefore the value of the objective function under the optimal rank order scheme increases. *Q.E.D.*

Similar comparative statics about the design and the profits of the optimal rank order scheme can be carried out with respect to the density function f of the noise in the game. A more concentrated f represents an environment of sports competition that is less susceptible to pure luck of players, and therefore more responsive to their efforts in the game.[11] Comparative statics with respect to the role of chance is interesting because

[11] In our model, performance measurement errors decrease the likelihood that the score will stay close in the second half and reduce the utility of the spectators. In the standard principal-agent moral hazard literature, measurement errors increase the risk premium of the agent. In both cases, measurement errors decrease profits by hampering the working of incentive contracts.

characteristics of a sports game can be, and indeed have often been, modified when changes are introduced to the rules of the game, training technology for athletes, or equipment used in the game. Formally, we can define "diminishing role of chance" in the game as follows. Let the density function f of the noise be indexed by a one-dimensional parameter b, such that there exists a function $\beta(b)$ with $\partial f(\delta_1, b)/\partial b > 0$ if and only if $\delta_1^2 < \beta^2(b)$. This condition means that f becomes more concentrated for middle values of δ_1. Intuitively, when f becomes more concentrated, the game is more likely to be closer given any effort levels of the two players, and the incentive designer should respond by increasing the incentive prize, in the same way as when P_2 becomes more concentrated. Indeed, the proof of Lemma 2 can be directly extended to show that with a diminishing role of chance (that is, as b increases), the incentive designer increases the incentive prize and the profits under the optimal scheme also increase. Thus, diminishing role for chance has the same effects on the design and the profits of the optimal scheme as increasing demand for suspense.[12]

We want to show that when spectators care enough about suspense in the game, the optimal rank order scheme eventually dominates the optimal linear score difference scheme. We establish this result indirectly, by noting that there is a rank order scheme that achieves the first best efforts in both halves if and only if $f(\delta_1)/P_2(\delta_1)$ is constant for all δ_1. The conditions for the first best efforts in the two halves are given by equations (6) in Section 3, with P_2 replaced by $P_2(\delta_1)$. These conditions can be replicated by the equilibrium conditions (1) and (3) under a rank order scheme, if and only if $f(\delta_1)/P_2(\delta_1)$ is constant for all δ_1.[13]

---

[12] There is an important difference between the comparative statics with respect to P_2 and with respect to f. In the case of increasing demand for suspense, the design and the profits of the optimal linear score difference schemes are not affected, and therefore the relative advantage of rank order schemes emerges. In the case of diminishing role of chance, one can show that the optimal piece rate s in a linear score difference scheme is given by $\frac{1}{2}(P_1 + \int P_2(\delta_1) f(\delta_1, b) \mathrm{d}\delta_1)$, which increases with b. Similarly, the effect of increasing b on the profits under the optimal linear score difference scheme has the same sign as $\mu(s) \int P_2(\delta_1)(\partial f(\delta_1, b)/\partial b) \mathrm{d}\delta_1$ (where $\mu(s)$ is defined by $C'(\mu(s)) = s$), which can be shown to be positive. Thus, with diminishing role of chance, performance is improved under both the optimal rank order difference scheme and the optimal linear score difference scheme. The net effect on the comparison of the two schemes is generally ambiguous. Note that in the above comparative statics exercise with respect to f, Assumption 2 is no longer satisfied.

[13] The role of Assumption 2 is now apparent. When the assumption is not satisfied, no reward schemes based on final score difference, including rank order schemes and linear score difference schemes, can implement the first best efforts. This follows because the equilibrium conditions for the first best first half efforts and the second half efforts cannot be satisfied at the same time. Intuitively, under our assumption

Now we are ready to show that rank order schemes dominate linear score difference schemes when spectators' demand for suspense is sufficiently high. Consider the problem of designing the optimal rank order scheme, for a given preference function $P_2(\delta_1)$. For simplicity, we assume that the rescaled functions $P_2(\delta_1)/P_1$ and $f(\delta_1)/(\int f^2(x)\mathrm{d}x)$ intersect exactly twice, at d and $-d$. For the following result, we say that the demand for suspense is high relative to the chance in the game, if $P_2(\delta_1)/P_1 > f(\delta_1)/(\int f^2(x)\mathrm{d}x)$ when $\delta_1^2 < d^2$ (i.e., if $P_2(\delta_1)$ is more concentrated than $f(\delta_1)$ after proper rescaling.)

Proposition 2. *If spectators' demand for suspense is high relative to the chance in the game, then the optimal rank order scheme dominates all linear score difference schemes.*

Proof: Consider a class of spectator preference functions $P_2(\delta_1, a)$ indexed by a, given by

$$P_2(\delta_1, a) = \frac{P_1 f(\delta_1)}{\int f^2(x)\mathrm{d}x} + a\left(P_2(\delta_1) - \frac{P_1 f(\delta_1)}{\int f^2(x)\mathrm{d}x}\right).$$

By construction, $\int(\partial P_2(\delta_1, a)/\partial a)f(\delta_1)\mathrm{d}\delta_1 = 0$, so condition (i) of increasing demand for suspense is satisfied. By assumption, $\partial P_2(\delta_1, a)/\partial a$ is positive if and only if $\delta_1^2 < d^2$ for all a, so condition (ii) of increasing demand for suspense is also satisfied. Since $P_2(\delta_1, 0)$ is proportional to $f(\delta_1)$, at $a = 0$ the optimal rank order scheme achieves the first best and therefore dominates linear schemes. Lemma 2 then implies that the optimal rank order scheme continues to dominate linear schemes when $a = 1$. But this is precisely what we need, because $P_2(\delta_1, 1) = P_2(\delta_1)$ by construction. *Q.E.D.*

Proposition 2 can be strengthened. The continuity of the profits under the optimal rank order scheme in a implies that the conclusion of Proposition 2 can hold even if spectators' demand for suspense is "a little" lower than the chance in the game. More precisely, since when $P_2(\delta_1)$ is proportional to $f(\delta_1)$, the optimal rank order scheme implements the first best efforts and therefore dominates linear score difference schemes, if the given $P_2(\delta_1)$ is just a little less concentrated than $f(\delta_1)$, the optimal rank order scheme still

that the reward for the players depends only on the final score difference, the incentives for the first half and for the second half are directly linked (e.g., through equations (1) and (3) under a rank order scheme). When Assumption 2 is not satisfied, such link becomes a binding restriction on what can be achieved under a reward scheme based on the final score difference.

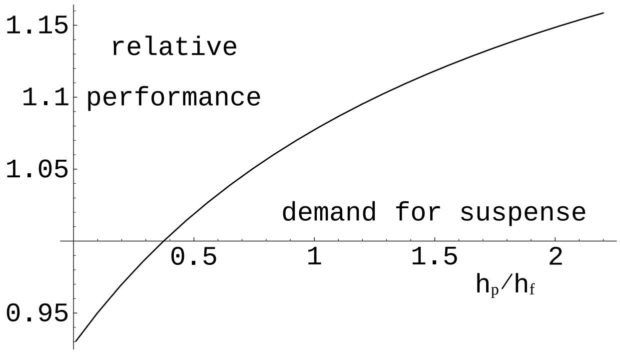

Figure 3. Relative performance: rank order and linear score difference

dominates linear score difference schemes. Figure 3 illustrates the case where the effort cost function is $C(\mu) = \frac{1}{2}\mu^2$, and both the rescaled preference function P_2 and the noise density function f are normal, with precision (inverse of variance) h_p and h_f respectively. The curve represents the ratio of the profit under the optimal rank order scheme to the profit under the optimal linear scheme, as a function of h_p/h_f. Note that the optimal rank order scheme outperforms the optimal linear scheme even when the demand for suspense falls well under what it takes to make the optimal rank order scheme achieve the first best (i.e. when $h_p/h_f = 1$).

Propositions 1 and 2 give us a clear picture of the comparison between linear score difference schemes and rank order schemes. Linear schemes dominate when the demand for suspense is low, while rank order schemes dominate when the demand is high. Indeed, linear schemes and rank order schemes stand at the opposite ends of a continuum: a linear scheme provides constant incentives to increase the final winning margin, while a rank order scheme provides no incentive at all at the margin so long as winning is ensured. The former achieves the first best when there is no demand for suspense ($P_2(\delta_1)$ is independent of δ_1), while the latter achieves the first best when the demand for suspense matches the randomness in the game ($P_2(\delta_1)$ is proportional to $f(\delta_1)$). One can imagine that there are other schemes based on the final score difference that provide incentives intermediate between those under the piece rate and the rank order tournament. Figure 4 illustrates a family of such schemes generated by normal distribution functions of different precision (and zero mean). A natural question to ask is then: can these schemes achieve the first best where linear schemes and rank order schemes fail?

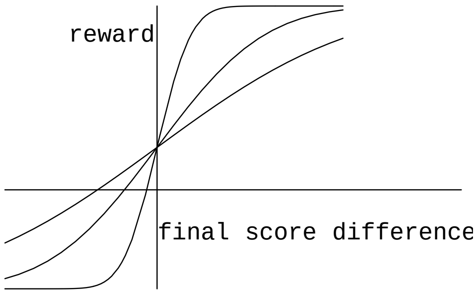

Figure 4. Nonlinear score difference schemes

To understand how linear schemes and rank order schemes compare with other schemes based on the final score difference, we begin with the question of whether the first best efforts can be implemented outside the two polar cases known from Propositions 1 and 2, namely, when there is no demand for suspense ($P_2(\delta_1)$ is independent of δ_1) and the optimal linear scheme achieves the first best, and when the demand for suspense matches the role of chance ($P_2(\delta_1)$ is proportional to $f(\delta_1)$) and the optimal rank order scheme achieves the first best. Consider the class of nonlinear schemes n based on the final score difference δ_2. For analytical convenience, we restrict our attention to functions $n(\delta_2)$ that are differentiable, with derivatives n' that are symmetric around and single-peaked at $\delta_2 = 0$. We refer to n as a "nonlinear" scheme, although nonlinear schemes include linear schemes as a special case with a constant n', and rank order schemes as a limit case with n' arbitrarily close to an indicator function with all weights on the point of $\delta_2 = 0$. Following the previous analysis, we can establish the equilibrium effort condition in the first half:

(7)
$$C'(\mu_1) = \int \int n'(\delta_1 + \epsilon_2) f(\epsilon_2) \mathrm{d}\epsilon_2 f(\delta_1) \mathrm{d}\delta_1,$$

and the condition in the second half:

(8)
$$C'(\mu_2(\delta_1)) = \int n'(\delta_1 + \epsilon_2) f(\epsilon_2) \mathrm{d}\epsilon_2.$$

Compare these conditions to those for the first best efforts (equations 6). A nonlinear score difference scheme $n(\delta_2)$ implements the first best efforts in both halves if and only if for all δ_1,

(9)
$$\int n'(\delta_1 + \epsilon_2) f(\epsilon_2) \mathrm{d}\epsilon_2 = P_2(\delta_1).$$

The above equation may not have a solution for the function n' for given noise density function f and the preference function P_2.

We may think of the functions f and rescaled n' and P_2 as density functions of random variables X, Y and Z respectively. Then condition (9) means that for the nonlinear scheme n to achieve the first best, n' is such that the random variable Z is the convolution of X and Y (i.e., the sum of independent random variables X and Y). This suggests that for certain specifications of the density function f and the preference function $P_2(\delta_1)$, explicit forms of the nonlinear scheme that induces the first best efforts can be found. Suppose that both f and P_2 (after proper rescaling) are normal, with mean 0 and precision h_f and h_p respectively. Then, the nonlinear scheme n achieves the first best if n' is proportional to the normal density function with mean 0 and precision $(h_p^{-1} - h_f^{-1})^{-1}$, as long as $h_p < h_f$. For any fixed h_f, as h_p increases to h_f, the optimal nonlinear reward function n converges to a rank order scheme. In the limiting case of $h_p = h_f$, the rank order scheme achieves the first best efforts. This result serves as a special case of Proposition 2. If $h_p > h_f$, there is no reward function n that achieves the first best. The demand for suspense is too high relative to the noise distribution, and even rank order schemes fail to limit players' incentives in the second half when the game becomes lop-sided at the end of the first half. In this case the designer wants to reduce the incentives for continuing second half efforts to a minimum when the first half score difference is sufficiently large. Since rank order schemes give no reward to the winner for the margin of victory, one naturally conjectures that the optimal rank order scheme achieves the second best among all schemes based on the final score difference, but additional assumptions are necessary to validate the intuition.

We focus on the "normal-quadratic" case, where the noise density function is normal with mean 0 and precision h_f, the preference function is proportional to a normal density function with mean 0 and precision h_p, and the effort cost function is quadratic. The following proposition assumes that $h_p > h_f$. By a "normal" scheme we mean any nonlinear scheme n such that n' is proportional to a zero-mean density function up to a constant.

PROPOSITION 3. *If spectators' demand for suspense is high relative to the chance in the game, then in the normal-quadratic case the optimal rank order scheme dominates any normal incentive scheme.*

PROOF: Without loss of generality, assume that $C(\mu) = \frac{1}{2}\mu^2$. First consider the class of differentiable schemes with symmetric derivatives, which includes schemes that provide normal incentives. Suppose that a scheme n is optimal in the class. Then, n maximizes the profit

$$P_1\mu_1 + \int P_2(\delta_1)\mu_2(\delta_1)f(\delta_1)\mathrm{d}\delta_1 - \frac{1}{2}\mu_1^2 - \int \frac{1}{2}\mu_2^2(\delta_1)f(\delta_1)\mathrm{d}\delta_1 - \underline{U},$$

subject to the equilibrium conditions for efforts (equations 7 and 8). Consider an alternative scheme $\tilde{n}(\delta_2) = n(\delta_2) + K\delta_2$. Since n is optimal, the derivative of the profit with respect to K vanishes at $K = 0$:

$$P_1 + \int P_2(\delta_1)f(\delta_1)\mathrm{d}\delta_1 - \mu_1 - \int \mu_2(\delta_1)f(\delta_1)\mathrm{d}\delta_1 = 0. \tag{10}$$

The equilibrium effort conditions (equations 7 and 8) imply

$$\int \mu_2(\delta_1)f(\delta_1)\mathrm{d}\delta_1 = \mu_1.$$

Therefore, under Assumption 2 equation (10) implies $\mu_1 = P_1$ and

$$\int \mu_2(\delta_1)f(\delta_1)\mathrm{d}\delta_1 = P_1. \tag{11}$$

As a result, the scheme n maximizes

$$\int P_2(\delta_1)\mu_2(\delta_1)f(\delta_1)\mathrm{d}\delta_1 - \int \frac{1}{2}\mu_2^2(\delta_1)f(\delta_1)\mathrm{d}\delta_1 \tag{12}$$

subject to equations (11) and (8).

Now suppose that the optimal scheme n provides normal incentives for efforts. Let $h_{n'}$ be the precision of the normal density function corresponding to n'. Since f is normal, equation (8) implies that up to some constant the equilibrium second half effort function $\mu_2(\delta_1)$ under n is also proportional to a zero-mean normal density function. Let h_μ be the precision of the corresponding normal density function. Then equation (8) implies that $h_\mu^{-1} = h_{n'}^{-1} + h_f^{-1}$. Thus, $h_\mu < h_f$ and h_μ increases with $h_{n'}$. Since by $h_f < h_p$ by assumption, we have $h_\mu < h_p$. Consider the effect on the second half profit (equation 12) of making $\mu(\delta_1)$ more concentrated. This effect is given by the following derivative:

$$\int \frac{\partial \mu_2(\delta_1)}{\partial h_\mu}(P_2(\delta_1) - \mu_2(\delta_1))f(\delta_1)\mathrm{d}\delta_1. \tag{13}$$

Since $h_\mu < h_p$, equation (11) implies that $P_2(\delta_1) - \mu_2(\delta_1)$ is symmetric and single-peaked. Moreover, $\partial\mu_2(\delta_1)/\partial h_\mu$ is also symmetric and single-peaked. An argument similar to the proof of Lemma 2 then implies that the derivative (equation 13) is positive. Since h_μ increases with $h_{n'}$, we have established that the incentive designer can increase profit by increasing $h_{n'}$. This contradicts the assumption that n is optimal among all schemes that provide normal incentives. The proposition follows immediately from the fact that h_μ achieves the greatest possible value of h_f under a rank order scheme. *Q.E.D.*

Now we briefly comment on the restrictions imposed in Proposition 3. The assumption of quadratic effort cost function allows us to obtain explicit functional forms for the equilibrium efforts. More importantly, together with Assumption 2, the quadratic cost assumption enables us to use a variational method to separate profit maximization in the two halves. Without this assumption, the marginal cost of effort is nonlinear. Instead of equation (10), we have

$$\frac{P_1}{C''(\mu_1)} + \int \frac{P_2(\delta_1)}{C''(\mu_2(\delta_1))} f(\delta_1)\mathrm{d}\delta_1 - \frac{C'(\mu_1)}{C''(\mu_1)} - \int \frac{C'(\mu_2(\delta_1))}{C''(\mu_2(\delta_1))} f(\delta_1)\mathrm{d}\delta_1 = 0.$$

Equilibrium efforts in the two halves cannot be separated, and optimal nonlinear schemes would in general trade off the first half effort against the second half effort. The normality restriction for the noise density function, the preference function and the nonlinear schemes reduces the comparison among f, P_2 and different nonlinear schemes n to a one-dimensional parameter, and allows us to examine the effect on the second half profit (equation 12) through $\mu_2(\delta_1)$, instead of through n' and equation (8). As an alternative to the normality restriction, one could imagine that there is a one-dimensional parameter, say i, which indexes f, P_2 and n', with a greater value of i implying greater concentration. But in general $\mu_2(\delta_1)$ as determined by equation (8) as a convolution of n' and f is not indexed by the same parameter i.[14] This makes it impossible to evaluate the effect of making $\mu_2(\delta_1)$ more concentrated on the second half profit (equation 13).

[14] The convolution of two symmetric and single-peaked functions is always symmetric and single-peaked, but even if the original two functions are ordered according to some one-dimensional parameter, the result of convolution in general cannot be compared in terms of concentration. To see this, suppose that g_1 and g_2 are symmetric and single-peaked (around 0), and let g be the convolution of the two, given by $g(x) = \int g_1(x+y)g_2(y)\mathrm{d}y$. Symmetry of g can be shown by a change of variable. Single-peakedness

5. Discussions

In our model of sports games individual scores play no role in the design of incentive schemes; only score differences matter. Since in most sports individual scores are observable and verifiable, one may wonder whether our results are robust if individual scores are used in incentive schemes. To address this issue, we need to modify our model. Suppose that θ_1^j is the first half score of player $j = A, B$, determined according to:

$$\theta_1^j = \mu_1^j + \eta_1^j,$$

where η_1^j is a random variable. The second half score θ_2^j is determined similarly:

$$\theta_2^j = \theta_1^j + \mu_2^j + \eta_2^j.$$

Let $\delta_i = \theta_i^A - \theta_i^B$ be the score difference for each half i. We assume that η_i^j's are i.i.d. (across players and across time), with the distribution function H. To facilitate comparison with earlier results, we maintain the other main assumption on the incentive contracts: rewards to each player can depend only on the final scores θ_2^A and θ_2^B. Assume also the designer's objective function is the same as in Section 4.[15]

The question we want to answer is whether there is any loss of generality to restrict incentive schemes to those conditioned on score differences.. However, the opposite question seems more immediate: why should player A's incentive depend on B's performance at all? The answer is clearly that it should not if P_2 is constant. That is, when there is no demand for suspense, the two players should be independently rewarded according to their

follows from the result that $\int g_1'(x+y)g_2(y)\mathrm{d}y < 0$ if and only if $x > 0$ (write the integral as the sum of $\int_{y \geq x} g_1'(y)(g_2(y-x) - g_2(y+x))\mathrm{d}y$ and $\int_{0 \leq y < x} g_1'(y)(g_2(y-x) - g_2(y+x))\mathrm{d}y$; both parts are negative because single-peakedness of g_1 implies $g_1'(-y) = -g_1'(y) > 0$ for any $y > 0$.) Further, if g_1 becomes more concentrated according to a parameter i, an argument similar to that in the proof of Lemma 2 shows that $\int (\partial g_1(y)/\partial\gamma)g_2(y)\mathrm{d}y$ increases with i, and therefore $g(0)$ becomes greater. However, in general the function g can change arbitrarily with i at other points, so that the concentration of g cannot be measured according to i.

[15] We note that in this alternative setup a player's score is independent of the efforts of his opponent. There is hardly any sport where this is completely accurate. Even for non-confrontational sports such as sprint and swimming, it makes a difference whether an athlete is performing by himself or competing against other athletes.

own scores. Of course, this is a just a special case of Holmstrom's (1982) celebrated result that tournaments have no intrinsic value in providing incentives in a team production problem, if individual output can be measured and measurement errors are independent. In our dynamic model, the players are risk-neutral, so under Assumption 2 the argument used in proving Proposition 1 shows that the first best efforts can be achieved with a pair of individualized, linear incentive schemes. Clearly, the two individualized schemes are identical, implying that a linear scheme based on the score difference achieves the first best as well, which is precisely the import of Proposition 1. Thus, there is no reason to tie the incentives of the two players together when there is no demand for suspense, but symmetry and Assumption 2 imply that there is no loss of generality in using a scheme based on score differences.

The situation is different when P_2 is a function of δ_1. Now, the objective function of the incentive designer introduces a link between the incentives of the two players. But we will argue that there is no loss of generality in assuming that the incentives for one player depend on the final score of his opponent only through the final score difference. To begin, let $m^j(\theta_2^A, \theta_2^B)$ be the reward scheme for player $j = A, B$. It is natural to assume symmetry, with $m^A(\theta, \tilde{\theta}) = m^B(\tilde{\theta}, \theta)$. As usual, we look at the equilibrium conditions in the second half, given the first half scores θ_1^A and θ_1^B (and hence $\delta_1 = \theta_1^A - \theta_1^B$). Player A chooses μ_2^A to maximize

$$\int\int m^A(\theta_1^A + \mu_2^A + \eta_2^A, \theta_1^B + \mu_2^B + \eta_2^B)\mathrm{d}H(\eta_2^A)\mathrm{d}H(\eta_2^B) - C(\mu_2^A),$$

taking as given μ_2^B. This yields a necessary condition for player A's best response function in the second half:

$$\int\int \frac{\partial m^A}{\partial \theta_2^A}\mathrm{d}H(\eta_2^A)\mathrm{d}H(\eta_2^B) = C'(\mu_2^A).$$

An analogous condition for player B holds. The symmetry assumption implies that the incentives facing player A when the first half scores are $(\theta, \tilde{\theta})$ are the same as the incentives facing player B when the scores are $(\tilde{\theta}, \theta)$. The two conditions determine the second half equilibrium efforts μ_2^A and μ_2^B, which in general depend on both θ_1^A and θ_1^B.

Relative to the nonlinear schemes $n(\delta_2)$ analyzed in Section 4, a pair of individualized schemes m^A and m^B can achieve second half equilibrium efforts that are more general in two ways. First, for a given first half score difference δ_1, the total score $\theta_1^A + \theta_1^B$ may differ, and (m^A, m^B) can create different incentives for the two players. Second, for a given first half score difference δ_1, the individualized scheme (m^A, m^B) can create different incentives for the leading player and the trailing player.[16] However, with the preference function P_2 depending only on the first half score difference, and the effort cost function C being identical and convex, there is no reason for the designer to exploit these new possibilities allowed by individualized schemes (m^A, m^B). An argument similar to the proof of Proposition 1 shows that any individualized scheme (m^A, m^B) that creates different incentives for the same first half score difference is dominated by another scheme that homogenizes the incentives among different levels of total scores and between the two players.

Another restriction we have imposed on the reward schemes is that they do not depend on the first half score difference. This restriction is not binding when there is no demand for suspense, as the first best efforts can be implemented with a linear scheme that depends only on the final score difference (Proposition 1). This result is related to Holmstrom and Milgrom's (1987) theory of linear incentive contracts in a model of a single risk-averse agent with no wealth effect in the utility function. They show that in a dynamic environment in which the agent can adjust his efforts according to a commonly observed history of output, the principal cannot do better than making the payment conditional only on some aggregated output measure. In particular, the two-wage payment schemes proposed by Mirrlees (1974) to approximate the first best do not work well because the agent can game such schemes by conditioning his efforts on the output path. Our Proposition 1 establishes a related result with a different logic in a multiple-agent setting. Rank order schemes correspond to the two-wage payment schemes of Mirrlees, because the equilibrium effort function in the second half depends on the first half score difference, while linear score difference schemes correspond to linear contracts of Holmstrom and Milgrom, because the incentives facing the players are constant throughout the game. Linear schemes dominate

[16] This is impossible under any nonlinear scheme n with symmetric n'. Note also that the symmetry assumption imposed on (m^A, m^B) does not mean $\partial m^A(\theta_2^A, \theta_2^B)/\partial\theta_2^A = \partial m^B(\theta_2^A, \theta_2^B)/\partial\theta_2^B$. Symmetry means role reversal when the individualized scores are reversed not that the leading player faces the same incentives as the losing player.

rank order schemes because constant incentives are cost-efficient in our setting, not because the players can exploit the dependence of incentives on the history of the game.

When the demand for suspense is so high that no incentive scheme based on the final score difference achieves the first best, one naturally suspects that the restriction to final score difference schemes becomes binding. Indeed, we now show that the first best can always be achieved with a "conditional" linear scheme in which the intensity of incentives depends on the first half score difference. If a conditional linear scheme with a piece rate $s(\delta_1)$ and a fixed transfer t achieves the first best, then the second half equilibrium condition for effort (equation 5) and the condition for first best second half effort (the second equation in 6) must coincide. Thus,

$$s(\delta_1) = P_2(\delta_1).$$

The equilibrium payoff function of each player at the end of the first half is therefore:

$$v(\delta_1) = t + P_2(\delta_1)\delta_1 - C(\mu_2^*(\delta_1)),$$

where $\mu_2^*(\delta_1)$ is the first best second half effort. This implies

$$v'(\delta_1) = P_2(\delta_1) + P_2'(\delta_1)\delta_1 - C'(\mu_2^*(\delta_1))\frac{d\mu_2^*(\delta_1)}{d\delta_1}.$$

The equilibrium condition for the first half effort is

$$C'(\mu_1) = \int v'(\epsilon_1) f(\epsilon_1) \mathrm{d}\epsilon_1.$$

Compare the above with the condition for the first best first half effort (the first equation in 6). For the conditional linear scheme with $s(\delta_1)$ to achieve the first best, the expectation of the second and third term in $v'(\delta_1)$ must vanish. The third term does have a zero integral with respect to f, because $\mu_2^*(\delta_1)$ is symmetric with respect to 0. However, the expectation of the second term in $v'(\delta_1)$ is negative, because $P_2'(\delta_1)$ is positive (negative) when δ_1 is negative (positive). Thus, if the piece rate $s(\delta_1)$ of the conditional linear scheme is chosen to make the second half effort the first best, the first half effort will be smaller than the corresponding first best. The reason for this result is clear. For the second half effort to be the first best, the piece rate of the linear scheme must be aligned with the preference function P_2. Through the effect on the first half score difference, the players anticipate the impact of their first half effort on their reward. When choosing the first half effort, players not only factor in the impact on their reward given the intensity of the incentives, which is what the designer wants, but also the impact on their reward due to changes in the intensity when the first half score difference changes, which is unwanted. The effect of the unwanted anticipation by the players is negative on their first half effort, because a greater first half score difference reduces the intensity of the incentives.

But this problem can be easily fixed, if the designer can also condition the fixed transfer t on the first half score difference δ_1. In this case, there is an additional term in $v'(\delta_1)$, which is $t'(\delta_1)$, and the function $t(\delta_1)$ can always be chosen to exactly cancel the unwanted second term in $v'(\delta_1)$. Other schemes can be conditioned on the first half score difference to achieve the first best. For example, a similar analysis shows that a conditional rank order scheme $(l(\delta_1), r(\delta_1))$ achieves the first best regardless of how concentrated P_2 is relative to f. Again, it is essential that both the fixed transfer l and the incentive prize r depend on δ_1.

If conditional schemes help align players' incentives to spectators' interests, why are such schemes seldom seen in practice? Conditional schemes are more difficult to administer than schemes based on the final score differences. The gain from using conditional schemes may not be significant enough to outweigh the inconvenience in their use. This is especially true if the demand for suspense is not too high relative to the chance in the game, if there are many periods in one game, or if there are frequent changes in scores, so that aligning players' incentives to spectators' interests requires frequent fine-tuning of the prizes throughout the game. A separate reason why complicated conditional schemes are not used in practice may be that, when the demand for suspense is high relative to the chance in the game, the incentive designer can use other tools to help the rank order scheme manage players' incentives according to the state of the game. One such tool is to stratify the aggregation of points in the calculation of the final score difference. For example, in a tennis match, points are aggregated into games, games into sets, and the first player to win a given number of sets (three or five) wins the match and the winner's purse. Similar scoring schemes are used in volleyball and in playoff series in basketball. A related tool is to end each period in a game or the entire game when the scores become lop-sided, while prolonging the game when the score differences are small. Thus, a set in tennis can be decided in six love-games, or it can continue indefinitely, at least theoretically, with tie-breaking in each game. These tools are simple to administer, and can be effective in economizing on incentives and aligning them to spectators' interests.

In this paper we have assumed that the game is divided into two halves and players choose efforts simultaneously at the beginning of each half. Ideally one would like to study a model where efforts are continuously adjusted as the game proceeds. In such a model, equilibrium efforts under rank order schemes can be asymmetric between the two players as well as history-dependent, even if we maintain the assumption of symmetry between the players in terms of the effort cost function. To see this, consider a variation of our two-period model where there is a handicap score δ_0 for one player at the beginning of the first half. Second half efforts are characterized as before and continue to be symmetric. If the symmetry in the first half efforts still holds, we must have

$$\int v'(\epsilon_1 + \delta_0) f(\epsilon_1) \mathrm{d}\epsilon_1 = \int v'(\epsilon_1 - \delta_0) f(\epsilon_1) \mathrm{d}\epsilon_1.$$

The above condition will not hold because v' is not symmetric.

More precisely, using the equilibrium expression of v' (equation 2), and noting that the first part of v' is symmetric, we find that a necessary condition for equilibrium first half efforts μ_1^A and μ_1^B is given by

$$C'(\mu_1^B) - C'(\mu_1^A) = 2r \int \frac{\partial \mu_2}{\partial \delta_1}(\epsilon_1 + \delta_0 + \mu_1^A - \mu_1^B) f(\epsilon_1 + \delta_0 + \mu_1^A - \mu_1^B) f(\epsilon_1) \mathrm{d}\epsilon_1.$$

When $\delta_0 = 0$, the above condition is satisfied if $\mu_1^A = \mu_1^B$. If $\delta_0 \neq 0$, symmetric solutions cannot obtain. Indeed, one can show that if $\delta_0 > 0$ so that A is leading B, then either μ_1^A is strictly greater than μ_1^B, or μ_1^A is less than μ_1^B by at least the score difference δ_0. The first scenario is the more reasonable one: if A is leading B, then starting from the position that A and B are exerting the same efforts, the marginal benefit of asserting additional efforts is greater for A than for B, because keeping the score difference to the final period saves efforts for both A and B while reducing the score difference forces greater efforts in the final period from both players. This scenario is consistent with Dixit's (1987) result that in a static tournament the favored player has incentives to over-commit efforts in order to preempt his opponent. But stronger conditions on the density function f and the cost function C are needed to exclude the other possibility of $\mu^A < \mu^B - \delta_0$, and to characterize the effort dynamics. Nevertheless, this does not invalidate the conclusion that spectators' demand for suspense is a necessary ingredient in explaining why the rank order tournament is the dominant form of rewarding scheme in sports competitions.

6. Conclusion

This paper answers a fundamental question in the economics of sports: why are rank order schemes the dominant form of incentive mechanism used in sports? Our answer is that spectators of a sporting event care about efforts of contestants when there is suspense about the outcome, not about the efforts per se. This conclusion is reached by considering a dynamic version of tournaments first studied by Lazear and Rosen (1981). When spectators care only about efforts in the game, we find that linear score difference schemes dominate rank order schemes, in the sense that the former induces greater effort with lower expected reward. But when we incorporate the preference for suspense, which type of scheme is better depends on how much spectators demand suspense. The more spectators enjoy suspense, the better rank order schemes perform relative to linear score difference schemes. When spectators' demand for suspense is sufficiently high, the optimal rank order scheme not only dominates linear score difference schemes, but can also outperform a broad class of schemes based on the final score difference.

The theory of contract has made much progress since the 1970s. Optimal design of incentive schemes has become a standard exercise, and applications of the theory have found success in many fields, including labor economics, industrial organization, and sports (Prendergast, 1999). However, empirical validation of the theory has lagged behind.[17] In

[17] Prendergast (1999) identifies this problem as he concludes his survey: "The typical theoretical paper addresses how a certain institution may be optimal. Comparative statics, when offered, are usually of the form that institutions or contracts are likely to vary with certain parameters. However, almost no theoretical work has distinguished among plausible theories there." Chiappori and Salanie (2003) echo the same sentiment at the start of their survey of recent empirical works based on the contract theory: "Many papers consist of theoretical analyses only. Others state so-called stylized facts often based fragile anecdotal evidence and go on to study a model from which these stylized facts can be derived."

the present paper we take the theory of contract to task and contrast the theory with observed practices in incentive design. Our purpose is not to construct another theoretical model based on some assumptions about the contractual environment and use the model to derive optimal incentive design. Instead, we argue that by contrasting what contract theory predicts with the incentive schemes that are used in practice, we can learn much about the contractual environment. Our fundamental assumption is that incentive schemes that are used in practice should outperform those that are not.

We apply this methodology of "reverse engineering" to sports, where incentives are an important motivating factor for participating agents, and observations about actual incentive designs are indisputable. We show that the observation that rank order incentive schemes are preferred to linear incentive schemes implies that spectators do not care only about contestants' effort. We identify a class of consumer preferences which incorporates the demand for suspense, and under which rank order schemes can dominate linear schemes. This class of consumer preferences is consistent with what can be directly estimated from revealed preferences in the empirical sports literature. By identifying consumer tastes from observed incentive design, this work also contributes to the recent literature that explores the structure of consumer preferences (Becker, 1996).

An important assumption in our model of dynamic sports competitions is that contestants have the same abilities. This is a simplification made to facilitate characterization of the equilibrium under rank order schemes. Incorporating heterogeneity among contestants removes symmetry in equilibrium and complicates the analysis, but it can also bring potentially important insights. Incentive schemes that recognize differential abilities, such as handicapping, have been discussed by Lazear and Rosen (1981) in the context of the optimal rank order scheme, and extended more recently by Moldovanu and Sela (2001) to situations where abilities are private information. It would be interesting to explore these implications in a dynamic context, particularly since our model offers a general framework for comparing rank order schemes and schemes based on other relative performance measures. Quite apart from its function as an incentive device, rank order tournament may also serve a sorting function when contestants are heterogeneous and information is imperfect. Indeed, it may be argued that in actual competitions, identifying the athletes with superior abilities is as important as rewarding those who exert great efforts. This is also true in promotion tournaments in internal labor markets, where efficiency often dictates the identification and assignment of superior talents to more senior positions in the hierarchy, as well as in patent races, where fixed financial benefits are awarded based on rank order comparisons of the proposed designs, even though the social value of a patented design depends on how "innovative" it is relative to other designs. In these cases, the mechanism designer is faced with the challenge of structuring the contest to efficiently differentiate the contestants. When effort is costly and the value of continuing effort depends on the state of the competition as well as on the abilities of the contestants, a model of dynamic competition that incorporates both sorting of abilities and incentives for efforts may offer insights that are otherwise lost in a static analysis of the problem.

References

Aron, D.J., Lazear, E.P., 1990. The introduction of new products. American Economic Review 80, 421-426.

Becker, G.S., 1996. Accounting for Tastes. Harvard University Press, Cambridge and London.

Brocas, I., Carrillo, J., 2002. Do the "three-point victory' and "golden goal" rules make soccer more exciting? A theoretical analysis of a simple game. Forthcoming Journal of Sports Economics.

Cabral, L.M., 2003. R&D competition when firms choose variance. Journal of Economics and Management Strategy 12, 139-150.

Chiappori, P., Salanie, B., 2003. Testing contract theory: A survey of some recent work, In: M. Dewatripont, L.P. Hansen and S.J. Turnovsky (Eds.), Advances in Economics and Econometrics, Vol. 1. Cambridge University Press, London.

Dixit, A., 1987. Strategic behavior in contests. American Economic Review 77, 891-898.

Ehrenberg, R.G., Bognanno, M.L., 1990. Do tournaments have incentive effects? Journal of Political Economy 98, 1307-1324.

Fernie, S., Metcalf, D., 1990. It's not what you pay it's the way that you pay it and that's what gets results: Jockeys pay and performance. Labour 13, 704-789.

Fort, R., Quirk, J., 1995. Cross-subsidization, incentives, and outcomes in professional team sports leagues. Journal of Economic Literature 33, 1265-1299.

Harris, C., Vickers, J., 1987. Racing with uncertainty. Review of Economic Studies 54, 1-21.

Holmstrom, B., 1982. Moral hazard in teams. Bell Journal of Economics 13, 324-340.

Holmstrom, B., Milgrom, P., 1987. Aggregation and linearity in the provision of intertemporal incentives. Econometrica 55, 303-328.

Knowles, G., Sherony, K., Haupert, M., 1992. The demand for major league baseball: A test of the uncertainty of outcome hypothesis. American Economist 36, 72-80.

Lazear, E.P., Rosen, S., 1981. Rank order Tournaments as Optimum Labor Contracts. Journal of Political Economy 89, 841-864.

Mirrlees, J., 1974. Notes on welfare economics, information and uncertainty, In: M. Balch, D. McFadden, and S. Wu, (Eds.), Essays in Economic Behavior under Uncertainty, pp. 243-258.

Moldovanu, B., Sela, A., 2001. The optimal allocation of prizes in contests. American Economic Review 91, 542-558.

O'Keeffe, M., Viscusi, W.K., Zeckhauser, R.J., 1984. Economic contests: Comparative reward schemes. Journal of Labor Economics 2, 27-56.

Palomino, F., Rigotti, L., 2000. The sport league's dilemma: Competitive balance and incentives to win. Woking paper, Tilburg University.

Palomino, F., Rigotti, L., Rustichini, A., 2000. Skill, strategy, and passion: An empirical analysis of soccer. Working paper, Tilburg University.

Prendergast, C., 1999. The provision of incentives in firms. Journal of Economic Literature 37, 7-63.

Sanderson, A., 2002. The many dimensions of competitive balance. Journal of Sports Economics 3, 204-228.

Szymanski, S., Kesenne S., 2004. Competitive balance and gate revenue sharing in team sports. Journal of Industrial Economics 52, 165-177.