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A Free Lunch in the Commons

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RESOURCES FOR THE FUTURE

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Abstract

We derive conditions under which cost-increasing measures—consistent with either regulatory constraints or fully expropriated taxes—can increase the profits of all agents active within a common-pool resource. This somewhat counterintuitive result is possible regardless of whether price is exogenously fixed or endogenously determined. Consumers are made no worse off and, in the case of an endogenous price, can be made strictly better off. The results simply require that total revenue be decreasing and convex in aggregate effort, which is an entirely reasonable condition, as we demonstrate in the context of a renewable natural resource. We also show that our results are robust to heterogeneity of agents and, under certain conditions, to costless entry and exit. Finally, we generalize the analysis to show its relation to earlier work on the effects of raising costs in a model of Cournot oligopoly.

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1 Introduction

Common-pool resources give rise to an important source of market failure. A large economics literature investigates both positive and normative aspects of the common-pool resource problem. Fundamental insights are most frequently made through a comparison of two extreme cases of property rights: sole ownership and open access. Under sole ownership, externalities are nonexistent and, in the absence of price effects, management of the resource is efficient because rents are maximized. Under open access, individual incentives result in overexploitation (investment or production), and congestion externalities completely dissipate all rents, giving rise to the so-called "Tragedy of the Commons."[1] Comparison of these two polar cases provides the conceptual foundation for understanding a range of policy instruments designed to manage the commons, including, but not limited to, taxes, harvest limits, entry restrictions, technology constraints, and individual transferable quotas.

The starting point of this paper is the intermediate case in which a fixed number of agents is active in the commons. We take a game-theoretic approach that is consistent with either a static game or the analysis of a steady-state.[2] We focus in particular on the ways in which policy instruments that increase marginal costs—including taxes and regulatory constraints—affect strategic incentives and payoffs. We identify what heretofore has been an unrecognized opportunity for a Pareto improvement that does not require redistribution. Under conditions we delineate, we find, for example, that a tax imposed on effort can increase both producer profits and consumer surplus, even when the tax revenue is fully expropriated. We find, in other words, a veritable "free lunch" in the commons.

Our treatment of an exogenous increase in marginal costs is consistent with at least two different types of policy instruments. The first is a tax on each unit of a single input. The second is a regulatory constraint that reduces the efficiency of the input. In the context of a fishery or wildlife extraction, for example, a tax might take the form of a licensing fee, whereas the regulatory constraint might consist of a technology restriction, size limit, or seasonal and area closures. In what follows, we investigate how policies such as these affect equilibrium profits of each agent and consumer surplus, in addition to overall social welfare.

The next section develops the base-case model of a limited-entry commons with linear costs and identical agents. Section 3 derives the main result that a

[1]See Gordon (1954) for the first economics treatment of the problem, and see Hardin (1968) for the description that gave the "Tragedy" its name.

[2]Existing studies that take a similar approach to the study of common-pool resources such a fisheries include Cheung (1970), Dasgupta and Heal (1979), and Heintzelman, Salant, and Schott (2009). Other analyses of contest games, which are isomorphic, include studies by Nitzan (1991), Chung (1996), Baik and Lee (2001), and Baye and Hoppe (2003).

cost-increasing measure will always increase equilibrium profits if total revenue is decreasing and convex in aggregate effort. The intuition is that the cost increase serves as a collusive mechanism while also mitigating a negative congestion externality. Section 4 generalizes the model to account for an endogenous price and shows how the results relate to the study of cost-increasing measures in a model of Cournot oligopoly. Section 5 shows the robustness of our results to heterogeneity of agents and costless entry and exit. Section 6 considers the more general welfare implications of the analysis. Section 7 concludes with policy implications in the context of established literatures on institutional arrangements for managing the commons and on the controversial Porter Hypothesis.

2 The Commons Without Entry

We begin with the case in which there are n identical agents indexed $i = 1, ..., n$. Agent i chooses to exert effort level x_i at cost cx_i. Aggregate effort is $X = \sum_{i=1}^{n} x_i = X_{-i} + x_i$ and produces total product $F(X)$. Average product is defined as $A(X) = F(X)/X$. We assume $A(X)$ is continuous, twice differentiable, $A'(X) < 0$ and bounded away from negative infinity, $A(0) - c > 0$, and $\lim_{X\to\infty} A(X) = 0$. We also assume initially, though relax the assumption later, that price is exogenous and normalized to unity. With this later assumption, the functions $F(X)$ and $A(X)$ also represent total revenue and average revenue, respectively.

We assume that an agent exerting k percent of the total effort receives k percent of the total product (revenue). Conditional on effort level x_i, agent i earns payoff

$$\pi_i = F(X)\frac{x_i}{X} - cx_i.$$

Each agent i thus chooses x_i to solve

$$\max_{x_i} \pi_i = A(x_i + X_{-i})\, x_i - cx_i.$$

The assumption $A(0) - c > 0$ implies that, in any equilibrium, every agent must be active ($x_i > 0$).[3] Accordingly, the following first-order condition must hold for each of the n agents:

$$A(X) + x_i A'(X) - c = 0. \tag{1}$$

This condition implies that, in any equilibrium, all n agents exert the same level of effort, denoted $x = X/n = (A(X) - c) / - A'(X) > 0$.

[3] For if $x_i = 0$ is optimal for some i, then it must hold that $A(X) - c \leq 0$. But under this circumstance, it would be optimal for every other identical agent to be inactive as well, requiring that $A(0) - c \leq 0$, which contradicts one of our initial assumptions.

We now prove that a pure strategy Nash equilibrium exists and make a further assumption to establish its uniqueness. Building on (1), it is useful to define the function

$$J(X) = A(X) + \frac{X}{n} A'(X) - c. \tag{2}$$

It follows from the assumptions on $A(X)$ that $J(X)$ is continuous, differentiable, $J(0) > 0$, and $J(X)$ is strictly negative for sufficiently large X. Hence the intermediate value theorem ensures existence of at least one root X^* that satisfies $J(X^*) = 0$. The only candidates for a Nash equilibrium are these roots, as only they satisfy the first-order condition for each agent. We further assume that $A(\cdot)$ satisfies the following condition:

$$2A'(Z) + zA''(Z) < 0 \tag{3}$$

for any $Z \geq 0$ and any $z \in [0, Z]$. The important implication of (3) is that at $Z = X^*$ and $z = x$, agent i's second-order condition holds globally for any x. It follows that every root satisfying $J(X^*) = 0$ is a Nash equilibrium.

Condition (3) also implies that X^*—and therefore the Nash equilibrium—must be unique. If (3) holds for any root solving $J(X^*) = 0$, then

$$J'(X^*) = [(n+1) A'(X^*) + X^* A''(X^*)] \frac{1}{n} < 0, \tag{4}$$

and this implies the existence of only one X^*. For, if there were more than one X^* satisfying (4), there would have to be another root in between violating (4), and our assumptions imply that no root violates (4). We conclude, therefore, that the Nash equilibrium exists and is unique.[4]

3 Profits and Marginal Costs

We now consider how a change in the common marginal cost affects equilibrium profits. The analysis is consistent with changes in the marginal cost that may arise from either a tax on effort or a regulation that requires the use of a less efficient technology. To capture both possibilities, we expand the marginal cost of the previous section into two terms: $c = \bar{c} + \tau$, where $\bar{c}$ is the marginal cost associated with the most efficient technology, and τ represents either a per unit tax on effort or an additional marginal cost from a technology restriction. In this

[4] A weaker condition than (3) that also guarantees uniqueness is to assume that the payoff function $A(x_i + X_{-i})x_i - cx_i$ is pseudoconcave in x_i. This implies that whenever the first-order condition is satisfied, the payoff function must be locally concave at each root, or equivalently that $2A'(X^*) + x^* A''(X^*) < 0$. This inequality also implies condition (4), which we have already shown guarantees the uniqueness of X^*.

section we focus on the effect of changes in τ on equilibrium profits, and later in the paper we consider the implications of heterogeneity in c.

The first step is to determine how a change in τ affects equilibrium effort for each agent.[5] Totally differentiating (1) at the equilibrium yields

$$\frac{dx^*}{d\tau} = \frac{1}{(n+1)A'(X^*) + X^* A''(X^*)} = \frac{1}{nJ'(X^*)} < 0. \tag{5}$$

This implies, for example, that an increase in the marginal cost of effort decreases equilibrium effort.

To determine how a change τ affects profits, we use the fact that each agent's equilibrium profit can be written as

$$\pi^* = A\left(x^* + X^*_{-i}\right) x^* - (\bar{c} + \tau)\, x^*.$$

After totally differentiating and substituting in the first-order condition, we solve for

$$\frac{d\pi^*}{d\tau} = (n-1)x^* A'(X^*)\frac{dx^*}{d\tau} - x^*. \tag{6}$$

The first term reflects the *strategic effect* and the second term reflects the *direct effect*. The direct effect is negative because a cost increase reduces a given agent's profits in the absence of any response by other agents. The strategic effect is strictly positive, assuming $n > 1$, because a cost increase causes other agents to decrease their aggregate effort, and this effect alone, due to the negative externality of effort, would increase the given agent's profits. An interesting—and somewhat counterintuitive—possibility arises if the strategic effect *outweighs* the direct effect, in which case all agents would benefit from the cost increase. This can never happen if $n = 1$, however, as the strategic effect would be nonexistent under sole-ownership.

More generally, nothing rules out the possibility for the sign of (6) to be positive, and we now consider in more detail the conditions necessary and sufficient for this to occur. Substituting (5) into (6) and rearranging yields

$$\frac{d\pi^*}{d\tau} = x^*(E^* - 2)\frac{A'(X^*)}{nJ'(X^*)}, \tag{7}$$

where

$$E^* = -\frac{X^* A''(X^*)}{A'(X^*)}.$$

[5] Our analytical approach is consistent with comparative–static analysis of the equilibrium of a one-shot game or of the steady state of a dynamic game. Although the approach is well-established in the literature (see, for example, Dasgupta and Heal 1979), it neglects transitions, which are not instantaneous, from one static equilibrium to another or from one steady state to another.

If $E^* < 2$, we have what is perhaps the more intuitive case in which the sign of (7) is negative, meaning that profits are decreasing in the marginal cost of effort. Here again we can see that under sole ownership of the commons (i.e., $n = 1$), the sign of (7) must be negative, as it is straightforward to verify that the inequality in (4) requires $E^* < 2$.

The more interesting possibility—when profits are increasing in the marginal cost of effort, even with price held constant—requires $E^* > 2$. Though we can see that strict convexity of the average product at X^* is necessary for the result, it is not sufficient. A condition that is both necessary and sufficient, however, is strict convexity of the total product (total revenue) at X^*. To see this, recall that $F(X^*) = X^* A(X^*)$ and therefore

$$\begin{aligned} F''(X^*) &= 2A'(X^*) + X^* A''(X^*) \\ &= A'(X^*)[2 - E^*], \end{aligned}$$

which implies $F''(X^*) > 0$ if and only if $E^* > 2$.[6] But is the possibility that $d\pi^*/d\tau > 0$ implausible, or does it arise in models widely accepted as realistic? To begin answering this question, we turn to the canonical model of a renewable natural resource.

4 Application to Renewable Resources

Consider a renewable resource (fish, wildlife, etc.) harvested by n independent agents. The standard approach is to specify a biological growth function $G(S)$, where S is the biological stock, and a harvest function $H(X, S)$, where X is effort. In the steady state, the rate of biological growth is exactly offset by the rate of harvest, implying that

$$\dot{S} = G(S) - H(X, S) = 0.$$

This condition implicitly defines the steady-state stock as a function of effort, $S(X)$. Substituting this back into the harvest function gives sustained yield as a function of effort, or what is simply the aggregate production function:

$$F(X) = H(X, S(X)).$$

With common linear costs of effort for all n agents, this renewable resource model is entirely consistent with the analysis of the previous sections.

To demonstrate the possibility for $d\pi^*/d\tau > 0$, we consider the most commonly used functional forms in the renewable resource model (see Conrad and

[6] The inequality in (4) also requires that $E^* < n + 1$, which once again confirms that profits cannot be increasing in marginal costs if $n = 1$, as this would contradict $E^* > 2$.

Clark 1987). The biological growth function is logistic, $G(S) = rS\left(1 - \frac{S}{k}\right)$, and the harvest function is Cobb-Douglas, $H(X,S) = \gamma X^{\alpha} S^{\beta}$. We make the simplifying assumption that $\gamma = \beta = 1$. Steady state requires

$$rS\left(1 - \frac{S}{k}\right) = X^{\alpha} S.$$

Solving for S yields

$$S(X) = k\left(1 - \frac{X^{\alpha}}{r}\right).$$

Substituting this back into the harvest function gives sustained yield as a function of effort (i.e., the aggregate production function):

$$F(X) = k\left(X^{\alpha} - \frac{X^{2\alpha}}{r}\right).$$

We have shown that the primary result of interest depends critically on the properties of $F(X)$. In particular, at the equilibrium level of aggregate effort, local convexity of the production function is sufficient for cost-increasing measures to increase profits. The simplest way to demonstrate the possibility for $F''(X^*) > 0$ is to provide an example.[7] Suppose $k = 10$, $\alpha = .2$, and $r = 2$. Note that our assumption of $\alpha < 1$ implies diminishing returns to effort. With these numerical values, $F(X) = 10(X^{.2} - .5X^{.4})$. It follows that $F'(X) \gtreqless 0$ if $X \lesseqgtr 1$, and $F''(X) \lesseqgtr 0$ if $X \lesseqgtr \left(\frac{.4}{.3}\right)^5 \equiv \rho$. That is, the steady-state yield rises to a maximum at $X = 1$ and then declines. The function is strictly concave until $X = \rho \approx 4.214$, inflects there, and is strictly convex for $X > \rho$. Average product is also monotonically decreasing, that is, $A'(X) < 0$.

We illustrate in Figure 1 the average and the marginal product curves for the numerical example. In addition, we assume that $c = .5$. As Figure 1 reflects, it is socially efficient to set aggregate effort at approximately .48, the level at which marginal product equals marginal cost. Under open access, however, aggregate effort expands to approximately 7.5, the level at which all rents are dissipated. Under restricted access, it is well-known that aggregate effort monotonically increases as the number of agents is exogenously increased, reaching the rent-dissipating level as $n \to \infty$.[8] Hence, for any number of agents sufficiently large that aggregate effort exceeds ρ, it holds that $F''(X^*) > 0$. In such cases,

[7] Though we consider a numerical example, it is straightforward to verify that the first and second derivatives have signs according to sign$\{F'(X)\}$ = sign$\{r - 2X^{a}\}$ and sign$\{F''(X)\}$ = sign$\{r(\alpha - 1) - 2X^{\alpha}(2\alpha - 1)\}$.

[8] This can also be shown formally. Using (2) combined with the fact that X^* satisfies $J(X^*) = 0$, we can solve for $dX^*/dn = x^* A'(X^*)/nJ'(X^*) > 0$, which shows that equilibrium, aggregate effort is monotonically increasing in the number of agents.

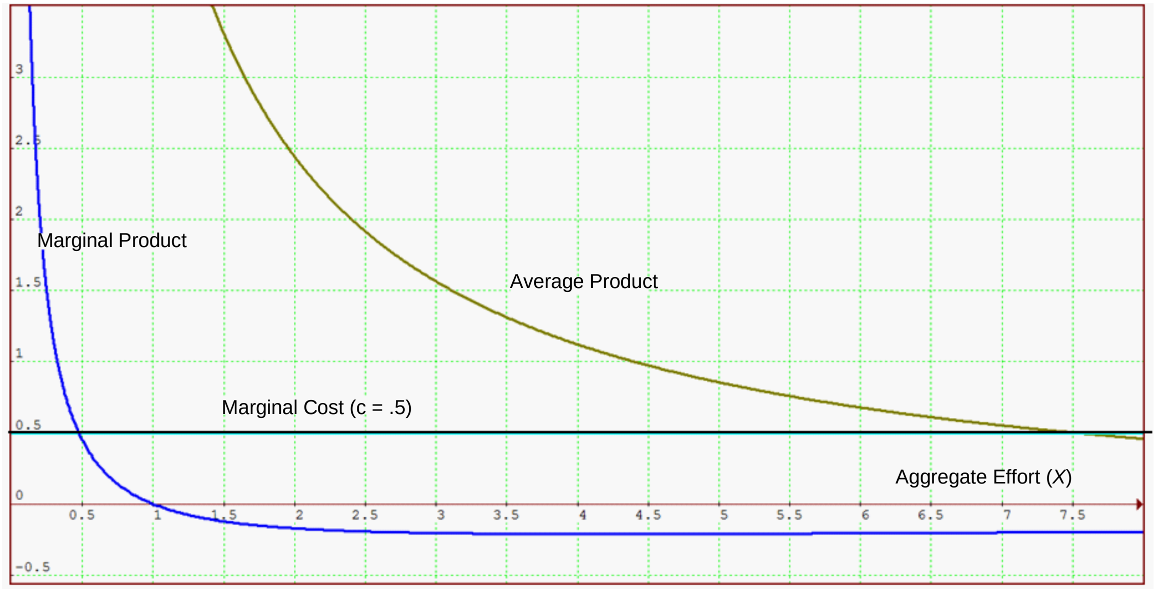

Figure 1: Example in which a cost-increasing measure increases equilibrium profits for all agents

cost-increasing measures, as shown above, will always increase each agent's equilibrium profits. The possibility for $d\pi^*/d\tau > 0$, therefore, is not a degenerate case and is worthy of further inquiry.

5 A More General Model

This section generalizes the model to allow for an endogenous price of output from the commons. We modify the results of the previous sections to reflect the generalization and show how they continue to hold. We also demonstrate how the model and results relate to the study of cost-increasing measures in a model of Cournot oligopoly.

5.1 Generalization

The additional feature of the model is a monotonically decreasing inverse demand function of aggregate output Q, written as $P(Q)$ where $Q = F(X)$. The payoff to each agent is now

$$\pi_i = P(F(X))\,F(X)\,\frac{x_i}{X} - cx_i, \tag{8}$$

and each agent solves

$$\max_{x_i} \pi_i = P\left(F\left(x_i + X_{-i}\right)\right) A\left(x_i + X_{-i}\right) x_i - cx_i. \tag{9}$$

A straightforward way to see the parallel with our previous analysis is to define $\tilde{A}(X) = P(F(X)) A(X)$. It follows that all of the previous results hold as long as we replace $A(X)$ everywhere with $\tilde{A}(X)$. To avoid confusion, we denote aggregate effort in the Nash equilibrium with price endogenous as $\tilde{X}$, the counterpart to X^* in the exogenous price case discussed previously. In the generalized model, aggregate effort $(\tilde{X})$ solves

$$\tilde{A}(\tilde{X}) + \frac{\tilde{X}}{n}\tilde{A}'(\tilde{X}) - c = 0.$$

As for the main result—that a cost-increasing measure can increase profits—the sufficient conditions can be written in parallel with the previous analysis. An increase in τ, as defined previously, will increase each agent's profits if and only if $\tilde{A}'(X) < 0$ and $\tilde{E} > 2$, where $\tilde{E} = -\tilde{X}\tilde{A}''(\tilde{X})/\tilde{A}'(\tilde{X})$.

We can once again interpret the condition $\tilde{E} > 2$ with an equivalent expression for strict convexity of total revenue. By definition, we have

$$P(F(X)) F(X) = P(F(X)) XA(X) = X\tilde{A}(X),$$

so the derivatives of the expression on the left equal the derivatives of the expression on the right, evaluated at $\tilde{X}$. The second derivative of the expression on the right is $2\tilde{A}'(\tilde{X}) + \tilde{X}\tilde{A}''(\tilde{X})$, which is positive if and only if $\tilde{E} > 2$. It follows that

$$\tilde{E} > 2 \Leftrightarrow \frac{d^2[P(F(\tilde{X}))F(\tilde{X})]}{dX^2} > 0. \tag{10}$$

Our earlier result can be seen as the special case where $P(\cdot) = 1$, implying that $\tilde{A}(X) = A(X)$, $\tilde{E} = E$, $\tilde{X} = X^*$, and the necessary and sufficient condition for a cost-increasing measure to increase profits is $d^2F(X^*)/dX^2 > 0$.

5.2 Renewable Resources Again

We now verify, with a continuation of the renewable resource example, that a cost-increasing measure can increase profits, even if price is endogenous. To see why this is important, recall that in the previous example an increase in the common marginal cost caused agents to reduce their effort, which, in turn, increased output. It followed that, with price held constant, profits could increase if the increase in output (equivalent to total revenue in that case) was sufficiently large. This condition is more difficult to satisfy, however, if price is decreasing in output. Nevertheless, the following argument demonstrates how the result can still hold.

Suppose that price in the renewable resource example is a decreasing linear function of aggregate output with intercept 1 and $|\text{slope}| = \theta$, so that $P(Q) = 1 - \theta Q$, where $Q = F(X)$. We can write the equilibrium level of aggregate effort as a continuous function of the slope parameter. We denote this function $X(\theta)$.[9] When $\theta = 0$, we have the earlier case in which $P = 1$ and $X(0) = X^*$. The generalized condition in (10) requires that total revenue be strictly convex at the equilibrium level of effort. Total revenue is $TR(X(\theta)) = [1 - \theta F(X(\theta))] F(X(\theta))$ and, solving for the second derivative with respect to effort, we have

$$TR''(X(\theta)) = F''(X(\theta)) - 2\theta \left[F(X(\theta)) F''(X(\theta)) + F^{2\prime}(X(\theta)) \right]. \qquad (11)$$

To see that this expression can be strictly positive with $\theta > 0$, assume that $F''(\cdot)$ is continuous, and notice that at $\theta = 0$ the condition collapses to $F''(X^*) > 0$, where the inequality was shown previously in our numerical example for sufficiently large n. It then follows by continuity that (11) must be strictly positive over some nonempty interval of $\theta > 0$.

We have thus verified that, even if price is sensitive to output, profits may increase in response to a cost-increasing measure. Moreover, in this case, the response is an *increase* in steady-state output that causes a *decrease* in price. As a result, consumer surplus increases as well. Later in the paper we further discuss the welfare implications of this result and others.

5.3 Relation to Cournot Oligopoly

In a well-known working paper, Jésus Seade (1985) delineated the circumstances under which a tax on the output of Cournot oligopolists with identical linear costs will increase each firm's profits. At a superficial level, Seade's results seem unrelated to ours. In his model, output is taxed; in our model, effort is (implicitly or explicitly) taxed. In his model, output declines in response to the tax; in our model, output increases in response to the tax. In his model, price increases, and consumers are made worse off; in our model, price either does not change or decreases, and consumers are either unaffected or made better off.

Despite these apparent differences, we can show that Seade's results are a special case of ours. To see this, let $F(X) = X$, meaning that X units of effort produce X units of output. In this case, and in parallel with (8) and (9), each agent's payoff reduces to

$$\pi_i = P(X) x_i - c x_i,$$

[9] We omit the "tilde" when writing aggregate effort as a function of θ.

and each agent solves

$$\max_{x_i} \pi_i = P\left(x_i + X_{-i}\right) x_i - c x_i.$$

This, however, is the standard setup of the Cournot model of oligopoly with identical, linear costs. In this special case, it follows that $A(X) = 1$ and $\tilde{A}(X) = P(X)$, so here again, all of the previous results continue to hold, but the conditions apply directly to the inverse demand function, as shown in Seade (1985). In particular, $\tilde{E} = -\tilde{X}P''(\tilde{X})/P'(\tilde{X})$ and a cost-increasing measure will increase the oligopolists' profits if and only if demand is downward sloping and $\tilde{E} > 2$, which is equivalent to having total revenue, $P(\tilde{X})\tilde{X}$, locally convex.[10]

Intuition for the oligopoly result is straightforward. The cost increase induces every firm to reduce output, and this increases the market price. If price increases enough, firms will earn greater profits despite the lower output and increased marginal cost.[11] Though the conditions are analytically equivalent, the mechanism at work in the commons interpretation differs. The cost increase still induces all agents to reduce effort, but this, in contrast, increases output because the biological resource is able to reach a higher steady-state level of growth. As a result, if the increased growth is sufficiently large, then profits will increase even if price falls or remains constant.

6 Extensions

We have thus far assumed that all agents have identical linear cost functions and that agents cannot freely enter the market. We now consider extensions to the model that introduce heterogeneity and allow for an endogenous number of agents. We focus on whether our main result—the possibility that cost-increasing measures will increase profits—continues to hold.

6.1 Introducing Heterogeneity

Let us return to the base-case model of the commons in which price is exogenous and normalized to unity.[12] Building on the previous expansion of marginal costs,

[10] Seade (1985) shows a simple example in which the result will always hold: isoelastic demand $P(X) = \phi X^{-\frac{1}{\varepsilon}}$ with $\varepsilon \in (1/n, 1)$.

[11] Although Seade's result is intriguing, one might question the rationale for imposing a tax (or other cost-increasing measure) in an oligopolistic industry. Without any other source of market failure, the market power results in too little production, and imposing a tax will only exacerbate the inefficiency. But the tax could improve both efficiency and profits if the industry is associated with a negative externality. Hence, as we discuss more later, Seade's result may have a particularly useful implication in the context of environmental policy.

[12] We do this for simplicity, as our results with endogenous price can also be extended.

$c = \bar{c} + \tau$, we assume there are two agents: one with low costs $c_l = \bar{c} + \tau - \kappa$ for $\kappa > 0$, and one with high costs $c_h = \bar{c} + \tau + \kappa$. Notice that this formulation includes homogeneity as a special case with $\kappa = 0$. Denote the equilibrium level of effort for the two agents as x_l^* and x_h^*. Because the sum of the marginal costs is $2c$ in both the homogeneous and heterogeneous cases, aggregate effort remains the same X^* either way and is therefore independent of κ.[13] For the same reason, any change $d\tau$ will induce a different level of aggregate effort that is also independent of κ.

Following the steps in Section 3, we use the first-order conditions to solve for the change in the effort of agent $i = l, h$ given a change in τ:

$$\frac{dx_i^*}{d\tau} = \frac{A' + \left[x_{-i}^* - x_i^*\right] A''}{\left[2A' + x_i^* A''\right] \left[2A' + x_{-i}^* A''\right] - \left[A' + x_{-i}^* A''\right] \left[A' + x_i^* A''\right]}. \tag{12}$$

Though it involves a bit a tedium, one can verify that with homogeneity, which requires $\kappa = 0$ and implies $x_i^* = x_{-i}^*$, this expression reduces for both agents to

$$\frac{dx_i^*}{d\tau} = \frac{1}{3A'(X^*) + X^* A''(X^*)} < 0,$$

which is simply equation (5) for the case of $n = 2$, and we have already shown that it must be strictly negative. Then, since $dx_i^*/d\tau$ is a continuous function of κ, it must also be true that (12) is strictly negative for both $i = l, h$ over some nonempty interval of $\kappa > 0$. It follows in this case that, even with heterogeneity of agents, a cost-increasing measure decreases the equilibrium level of effort for both agents.

A similar line of reasoning can demonstrate the possibility that profits will increase as well. The change in profit for agent $i = l, h$ given a change in τ, the counterpart to equation (6), is

$$\frac{d\pi_i^*}{d\tau} = x_i^* A'(X^*) \frac{dx_{-i}^*}{d\tau} - x_i^*, \tag{13}$$

which consists, once again, of the strategic effect and the direct effect, respectively. We know, in the case of homogeneous agents, that (13) simplifies to (6) for the case of $n = 2$, and we have already shown that (6) can be strictly positive. By continuity of $d\pi_i^*/d\tau$ in κ, therefore, there exists a nonempty interval of $\kappa > 0$ over which (13) is strictly positive as well. We thus conclude that—even with heterogeneity of agents—a cost-increasing measure can increase equilibrium profits for all agents.

[13] This result is a special case of the theorem proved in Bergstrom and Varian (1985).

6.2 Costless Entry and Exit

We have assumed throughout that the number of agents in the commons, n, is exogenously fixed. We now consider whether our main result depends on this assumption, or whether, when costless entry and exit are permitted, a cost-increasing measure can still increase profits.

Consider a general setup in which each of a fixed number of players $N > n$ simultaneously decides whether to be an extractor or to engage in an alternative activity. Assume that each agent who chooses extraction, after the number making this choice is publicly disclosed, receives the equilibrium payoff we have specified for this activity. Assume that each agent who chooses the alternative activity receives a payoff that is weakly decreasing in the number of agents making this choice. In a Nash equilibrium, the N players would allocate themselves between the two activities so that no player could unilaterally switch activities and earn a strictly greater payoff. In the absence of an integer problem, therefore, profits per player would equalize across the two activities.

Now consider the case in which a cost-increasing measure increases profits for all of the initial players in the commons. With the measure in place, the profit per player in the commons would be higher than the profit per player in the alternative activity unless more of the N players migrate from the alternative activity to the commons. This follows because, as we have shown, profits per player in the commons are monotonically decreasing in the the number of players. It must be the case, therefore, that the cost-increasing measure induces a new Nash equilibrium with more players in the commons and fewer players pursuing the alternative activity.

The effect on profits then depends on the underlying structure of the alternative activity. If the migration of players out of the alternative activity raises the payoff per person of those who remain, then payoffs for both activities will equalize at a higher level, making all N players better off. If instead the alternative activity has a constant payoff per person independent of the number pursuing the activity, then the entry of players into the commons would drive profits back to the initial level, with no change in the payoff to any of the N players. In this case, the paradoxical result about profits does not hold, but a new paradoxical result about entry emerges: a cost-increasing measure in the commons promotes entry.

7 Welfare Implications

We have focused throughout the paper on how cost-increasing measures can affect equilibrium profits. We now discuss some of the more general welfare implications of our finding that higher marginal costs can increase profits. To begin, recall that two different interpretations are consistent with our treatment of increased marginal costs. The first is a tax on each unit of effort, and the second is a regulatory constraint that reduces the efficiency of effort. In terms of welfare implications, one obvious difference between the two interpretations is that a tax generates tax revenue, whereas a regulatory constraint does not.

Our result for the common-pool resource model identifies a new opportunity to produce a Pareto improvement, even without redistribution. In the case where price is exogenously fixed, a regulatory constraint can increase profits at no cost to consumers. This is equivalent to a scenario in which a tax is imposed and the revenue completely discarded. But with a tax, of course, the revenue can be used in socially beneficial ways and is therefore typically treated as neutral from a social welfare perspective. Perhaps the most intriguing possibility occurs when price is endogenous. In this case, we have shown that price will fall, meaning that consumer surplus will increase as well. It follows that cost-increasing measures in the commons have the potential to make both producers and consumers better off, in addition to raising distortion-free tax revenue.

It is important to emphasize that our analysis thus far has focused on local results in the neighborhood of a Nash equilibrium. We now consider the question of whether the same result—an increase in profits—can hold if a cost-increasing measure were imposed to implement the first-best level of effort. This would be consistent with an optimal Pigouvian tax where none of the tax revenue was returned to the agents in the commons.[14] We begin with consideration of a tax and then contrast the findings to that of a regulatory constraint. We also focus, for simplicity, on the base-case model in which price is exogenous and normalized to unity.

The first observation is that our main result cannot hold locally in the neighborhood of the socially optimal (i.e., sole-owner) level of effort. To see this formally, define $\hat{X} = \arg\max \{F(X) - cX\}$. Satisfying the second-order condition thus requires $F''(\hat{X}) \leq 0$, which violates the necessary and sufficient condition for profits to increase locally with a cost-increasing measure. Intuitively, this follows because, for the sole owner, convexity of total revenue at $\hat{X}$ would require mar-

[14] Dasgupta and Heal (1979) consider the same question in the context of a static model. In particular they write, "The question arises whether firms are better or worse off at the free access equilibrium than they are at the tax equilibrium if the entire tax revenue is expropriated from them by the government. Rather surprisingly, perhaps, it is easy to show that they are unambiguously better off at the free access equilibrium" (p. 70). By "free access" they mean the Nash equilibrium with any finite number of agents. Their conclusion, however, depends on the assumption that aggregate production is globally concave, meaning (in our notation) that $F''(\cdot) < 0$. Conditional on that assumption, we would reach the same conclusion. But the payoff of each extractor may be strictly concave even when aggregate production is locally convex. In that case, as we show, the opposite conclusion follows.

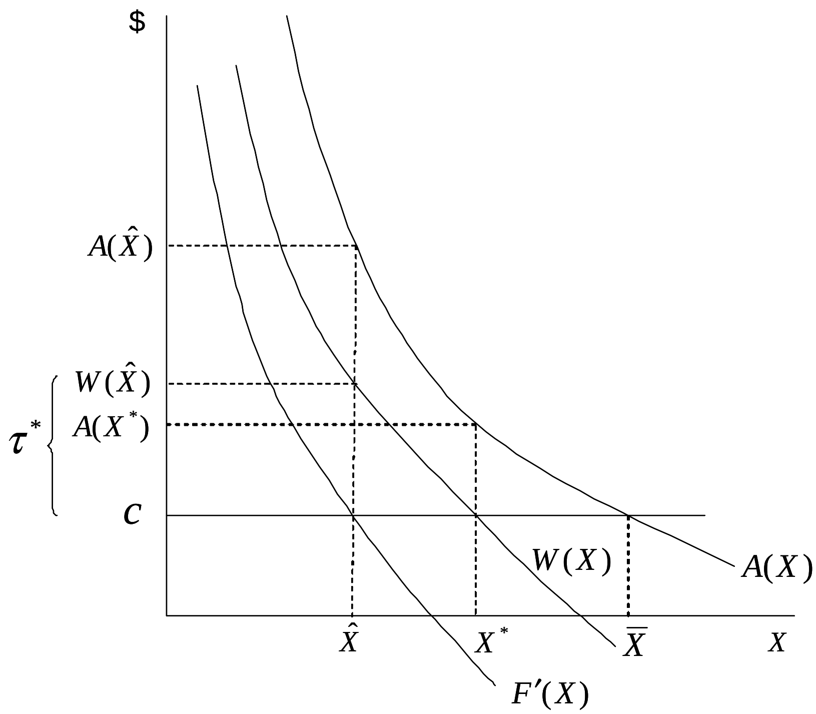

Figure 2: The effect of an optimal tax on profits

ginal revenue to be increasing, which is not compatible with $\hat{X}$ being the optimal level of effort.

But the question still remains about whether, with $n \geq 2$, profits can ever be higher with a tax at the socially optimal level—but with no tax revenue returned—than without a tax. Figure 2 is useful to demonstrate the mechanism at work. The sole owner's level of effort occurs at the intersection of marginal product $(F'(X))$ and marginal cost, yielding profits $\hat{X}(A(\hat{X}) - c)$. The open-access level of effort, denoted $\bar{X}$, occurs at the intersection of average product $(A(X))$ and marginal cost, yielding zero profits. The new feature on the graph is the curve

$$W(X) = \frac{F'(X)}{n} + \frac{n-1}{n} A(X) \tag{14}$$

which is simply a rearranged expression for $J(X) + c$. Essentially, $W(X)$ is a weighted average between the marginal product and the average product, and its intersection with marginal cost determines the Nash equilibrium level of aggregate effort for any n, with corresponding aggregate profits equal to $X^*(A(X^*) - c)$.

The other useful implication of (14) is that $W(\hat{X}) - c = \tau^*$ is the tax that implements the first-best level of effort. If imposed, we can see from Figure 2 that profits remain even after the tax revenue is paid, as indicated by the area $\hat{X}(A(\hat{X})-c-\tau^*)$. The question of interest, therefore, is whether these profits can be greater than the initial profits at the Nash equilibrium. The general answer is that this is certainly possible. A simple example can be constructed with a slight modification of the one considered in Section 4.[15] More generally, Figure 2 indicates that the result is more likely to arise if average product is both steeper and more convex, which is also reflected in our necessary and sufficient condition for the local result that $E = -XA''(X)/A'(X) > 2$.

Clearly the effects on profits are unchanged regardless of whether the cost-increasing measure arises from a tax or a regulatory constraint. But the two policy instruments do differ from a social welfare perspective. The fact that a tax raises tax revenue implies that a social planner would always seek to impose a tax that implements the first-best level of effort, provided the planner could capture and utilize the tax revenue. With a regulatory constraint, however, the social planner would never target the first-best level of effort. This follows because the social planner would only seek to impose further cost-increasing measures as long as profits increase on the margin, and that will never occur all the way down to the first-best level of effort. Accordingly, with a regulatory constraint, if the social planner would ever impose a cost-increasing measure, the level of aggregate effort would still be less than first-best. Specifically, the socially optimal level of effort would be at the inflection point, the smallest aggregate effort level for which the total revenue function is strictly convex. The difference between the two policy instruments of a tax and a regulatory constraint is that agents' costs due to the tax are not social costs, but with the regulatory constraint they are.

8 Conclusion

This paper focuses on the potential for a veritable "free lunch" in the commons. We identify necessary and sufficient conditions under which cost-increasing measures—consistent with either taxes or regulatory constraints—can increase the profits of all agents operating in the commons. This somewhat counter-intuitive result is possible regardless of whether price is exogenously fixed or endogenously determined. Moreover, consumers are made no worse off and, in the case of an endogenous price, can be made strictly better off. In general,

[15] If we assume that marginal cost is zero (rather than 0.5), then it is straightforward to verify that after-tax profits can be greater than profits at the initial Nash equilibrium. Profit per agent with the optimal tax (the revenue from which is not returned) is simply $5/n^2$. It follows that, in three cases with n equal to 2, 5, and 10, profit with the tax is 1.25, 0.2, and 0.05, respectively. In contrast, using numerical simulation, profit without the tax in each of the three cases is approximately 1.22, 0.17, and 0.04, respectively.

the results simply require that total revenue be decreasing and convex, which is not an unreasonable condition, as we demonstrate in the context of a renewable natural resource. Finally, the results are robust to heterogeneity of agents and certain conditions of costless entry and exit. The basic intuition for our main result is that cost-increasing measures, although costly, can have more than offsetting benefits as a collusive mechanism that also mitigates a negative congestion externality.

One way to see the importance of this result is to consider the standard policy instrument of imposing a tax to manage the commons. The results of this paper show how both producers and consumers can be made better off, compared to a situation with no tax, even if the tax revenue is fully expropriated. In such cases, our analysis suggests that agents operating in the commons should not oppose implementation of the tax if the alternative is no tax. Similarly, they should welcome other forms of regulation that increase the costs of all agents, as each stands to earn greater profits when compared to situations without any regulation. The same reasoning applies in the context of more decentralized forms of governance in the commons. An extensive literature investigates the emergence various self-governing institutional arrangements for managing the commons. The literature is most commonly associated with the extensive works of Elinor Ostrom, and two standard references are Ostrom (1990) and Ostrom, Roy, and Walker (1994). In the context of this literature, it is straightforward to see how self-governing institutions designed to limit effort are advantageous. This paper provides a rigorous foundation for the ways in which self-governance based on increasing costs can be advantageous as well.

The results of this paper also relate to another literature on the effects of environmental policy on competitiveness and profitability. The so-called "Porter Hypothesis" claims that tighter environmental regulations can enhance competitiveness and profitability over the long run. The argument is that tighter regulations induce innovation that ultimately lowers costs and increases profits (Porter and van der Linde 1995), but the claim is not widely accepted and rather controversial (see, for example, Palmer, Oats, and Portney 1995). Our results contribute to this literature because we find instances where tighter regulations do in fact increase profitability; but rather surprisingly, it is because of cost increases rather than induced innovation. While we have focused primarily on common-pool resources, our more general version of the model demonstrates how closely related results apply in oligopolistic markets of imperfect competition. When more stringent environmental policy is being considered in these markets, perhaps due to negative externalities, the analysis of this paper provides a starting-point for understanding when profitability need not suffer.

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