

# Discounting and Relative Consumption 

## Olof Johansson-Stenman and Thomas Sterner

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#### Abstract

This paper analyzes optimal social discount rates where people derive utility from relative consumption. We identify and compare three separate discount rates: the social rate (taking positional externalities into account), the private rate, and the conventional Ramsey rate. Two main findings resulted for the standard case with a positive growth rate: 1) the social discount rate exceeds the private discount rate if the degree of positionality increases with consumption, and 2) the social discount rate is smaller than the Ramsey rate if preferences are quasi-concave in own and reference consumption, and exhibit risk aversion with respect to reference consumption. Numerical calculations demonstrate that the latter difference may be substantial and economically important for such issues as global warming.


Key Words: discounting, relative consumption, Ramsey rule, degree of positionality, keeping up with the Joneses

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# Discounting and Relative Consumption 

Olof Johansson-Stenman and Thomas Sterner*

## 1. Introduction

The theory and practice of discounting is central to economics (e.g., Arrow and Lind 1970; Frederick et al. 2002; Arrow et al. 1996) and essential for dealing with very long-term phenomena. With the increased attention to climate change, interest in discounting has experienced a revival. (See Gollier (2010), Gollier and Weitzman (2010), and Weitzman (2010) for recent contributions.) For example, virtually the entire economics debate in the wake of the Stern Review (Stern 2006) focused on the discount rate used-not on climate science or the assessment of costs and benefits of mitigation, for which there are still very large uncertainties. (See, e.g., Brekke and Johansson-Stenman 2008; Dasgupta 2007, 2008; Dietz and Stern 2007; Heal 2008; Howarth 2009; Nordhaus 2007; Sterner and Persson 2008; and Weitzman 2007a, 2007b.) The primary reason is, of course, that most of the consequences of climate change will occur far into the future and, thus, the discount rate can have a dramatic effect on their present value.

This paper, as far as we know, is the first to incorporate relative consumption effects into the theory of social discounting. Yet, the idea that humans value consumption in a social context and in relation to others' consumption is far from new. In fact, classical economists-such as Adam Smith, John Stuart Mill, and Alfred Marshall-emphasized such concepts, and modern research on the subject dates back at least to Duesenberry (1949). There is now a substantial body of empirical evidence suggesting that people not only derive utility from their absolute

[^0]consumption but are also concerned with their own consumption relative to that of others. ${ }^{1}$ There is also a growing literature dealing with various kinds of optimal policy responses to such relative consumption effects. ${ }^{2}$ For example, Aronson and Johansson-Stenman (2008; 2010) show, in a static and a dynamic model, respectively, that optimal marginal income taxes are likely to be substantially larger under relative consumption effects than in a conventional case, where people only care about their absolute consumption level.

Arrow and Dasgupta's paper (2009) is closest to ours, as it explicitly deals with the implications of relative consumption effects for intertemporal resource allocation. They show that concern for relative consumption does not necessarily lead people to consume more today than is socially optimal because they are also concerned with relative consumption (and, hence, produce positional externalities) in the future. However, the issue of how discount rates are affected by relative consumption effects has not been analyzed before.

In section 2, we analyze whether and, if so, how the SOCial discount rate (i.e., when positional externalities are taken into account) is smaller or larger than the private discount rate (when people do not take into account that their consumption affects others-although they still take into account that they themselves are affected by others' consumption). We show that an important condition derived by Arrow and Dasgupta (2009), when private and social consumption paths over time coincide, translates to the condition for when the private and social discount rates are equally large. We then explore the conditions when the social discount rate exceeds the private one, and vice versa. We express these conditions in terms of the degree of positionality, a measure reflecting the extent by which relative consumption matters. For a positive growth rate, we show that if the degree of positionality increases with the consumption levels, consistent with some empirical evidence, then the social discount rate exceeds the private one. We also show that this discrepancy can be internalized by time-varying consumption taxes.

[^1]In section 3, we continue by analyzing a related but distinct issue-relevant from a climate policy perspective-namely, whether the conventional optimal social discounting rule, the so-called Ramsey (1928) discounting rule, should be modified; and, if so, how in the presence of relative consumption effects. The conventional instantaneous Ramsey discounting rule says that the optimal discount rate equals the pure rate of time preference plus the product of the individual degree of relative risk aversion multiplied by the growth rate. Hence, this formulation does not take into account that others' consumption will also change in the future. In the standard case, where relative consumption does not matter, this also does not matter.

The formula describing the optimal social discount rate in the presence of relative consumption effects can be written in a form similar to the Ramsey formula. The only difference is that the individual degree of relative risk aversion is replaced by what we denote here as the socia/ degree of relative risk aversion. By this we mean a corresponding measure of risk aversion had the individual made a risky choice on behalf of his or her whole generation. It is shown, moreover, that for a positive growth rate the social discount rate is smaller than the Ramsey discount rate, if preferences are quasi-concave in own and reference consumption (consisting of others' average consumption), and exhibit risk aversion with respect to reference consumption. The latter means that individuals prefer that others have a certain consumption level, compared to the case where others' consumption is uncertain with the same expected value. Assuming a positive consumption growth rate, we show that the social discount rate is larger than the private rate and smaller than the Ramsey rate, if the degree of positionality increases with consumption and preferences reflect risk aversion with respect to reference consumption, and are quasiconcave with respect to own and reference consumption.

Finally, we illustrate quantitatively in section 4 how the overall long-term social discount rates recently suggested by Stern (2006) and Weitzman (2007b) would be modified when taking relative consumption effects into account. We conclude that these modifications may be substantial with commonly used functional forms and reasonable parameter values, and that they are potentially very important for the economics of climate change. Section 5 offers some final remarks and observations.

## 2. Private and Social Discount Rates

In this section, we analyze and compare the private and social discount rates between two arbitrary points in time, when people care about relative consumption. At the end of this section, we also determine the corresponding time-varying internalizing consumption taxes.

### 2.1 Preferences and Objectives

Following Arrow and Dasgupta (2009), society consists of identical individuals with a population size normalized to one, who, at time $t$, consumes $C_{t}$ and who, in addition to absolute consumption, also cares about relative consumption. The latter depends on own consumption, as well as on the reference consumption $z_{t}$ (the average of others' consumption), such that $R_{t}=r\left(c_{t}, z_{t}\right)$. The individual instantaneous utility (or felicity) at time $t$ is:

$$
\begin{equation*}
U_{t}=u\left(c_{t}, R_{t}\right)=u\left(c_{t}, r\left(c_{t}, z_{t}\right)\right)=v\left(c_{t}, z_{t}\right), \tag{1}
\end{equation*}
$$

where $u_{1 t}>0, u_{2 t}>0, v_{1 t}>0, v_{2 t}<0, u_{11 t}<0, v_{11 t}<0, r_{1 t}>0, r_{2 t}<0$; where sub-indices reflect partial derivatives, $u_{1 t} \equiv \partial u\left(c_{t}, R_{t}\right) / \partial c_{t}$, etc.; and where $u, v$, and $r$ are twice continuously differentiable.

Both the formulations in equation (1) are used extensively in this paper because they allow us to vary which variable is constant, either relative consumption $R_{t}$ or others' consumption $z_{t}$. Throughout the paper, we ignore intragenerational issues and assume the same uniform and normalized population in each period. Moreover, we let $r$ satisfy the criterion that it is unaffected if own consumption and others' consumption are changed equally, in other words, if $r_{1 t}=-r_{2 t} \cdot{ }^{3}$ In addition, we make the common assumption of (weak) keeping-up-with-theJoneses property, such that $V_{12 t} \geq 0$. (See, e.g., Gali 1994; Carrol et al. 1997; and Dupor and Liu 2003.)

This assumption is made largely for convenience and basically implies that people want to consume more when others consume more, and vice versa. It is easy to see that $v_{12 t} \geq 0$ implies that identical individuals will have the same consumption in each period, further implying that $z_{t}=C_{t}$. In contrast, when $V_{12 t}<0$, it may be privately and socially optimal for identical individuals to have different consumption patterns over time. Other properties of these utility functions will be discussed in subsequent sections.

[^2]Each individual maximizes the flow of instantaneous utility over time, subject to a fixed pure rate of time preference-or the utility discount rate $\delta$-implying the following maximization problem:

$$
\begin{equation*}
\max _{c_{\tau}} W^{0} \equiv \int_{0}^{T} u\left(c_{\tau}, r\left(c_{\tau}, z_{\tau}\right)\right) e^{-\delta \tau} d \tau=\int_{0}^{T} v\left(c_{\tau}, z_{\tau}\right) e^{-\delta \tau} d \tau \tag{2}
\end{equation*}
$$

Hence, the individual takes others' consumption $Z_{t}$ as given in each moment in time, implying that the individual takes into account that a changed consumption path also implies a changed path of relative consumption. The social planner, in contrast, maximizes the flow of instantaneous utility for all individuals (and, hence, also takes into account the externalities through relative consumption). As a consequence, since all individuals are identical, the social planner takes $R_{t}$ as given. Thus, the social planner solves the following maximization problem:

$$
\begin{equation*}
\max _{C_{\tau}} W^{s} \equiv \int_{0}^{T} u\left(c_{\tau}, r\left(c_{\tau}, c_{\tau}\right)\right) e^{-\delta \tau} d \tau=\int_{0}^{T} v\left(c_{\tau}, c_{\tau}\right) e^{-\delta \tau} d \tau \tag{3}
\end{equation*}
$$

### 2.2 Private and Social Discount Rates

Consider now a small project undertaken and paid for in terms of reduced consumption at time zero that results in increased consumption at time $t$. If the individual is indifferent between undertaking such a project or not, the private discount rate per time unit between time zero and $t$, given by $\rho^{p}$, is implicitly defined by:

$$
\frac{\partial W^{\rho} / \partial c_{t}}{\partial W^{\rho} / \partial c_{0}}=\exp \left(-\rho^{\rho} t\right)
$$

Note that this is true for any given consumption path, whether optimal or not. ${ }^{4}$ Solving for $\rho^{p}$ gives:

$$
\begin{equation*}
\rho^{p}(t)=-\frac{1}{t} \ln \frac{\partial W^{p} / \partial c_{t}}{\partial W^{p} / \partial c_{0}} \tag{4}
\end{equation*}
$$

[^3]Similarly, if the social planner is indifferent between undertaking such a project or not, the social discount rate per time unit between time zero and $t$, given by $\rho^{s}$, is implicitly defined by $\partial w^{s} / \partial c_{0}=\exp \left(\rho^{s} t\right) \partial w^{s} / \partial c_{t}$, implying that:

$$
\begin{equation*}
\rho^{s}(t)=-\frac{1}{t} \ln \frac{\partial W^{s} / \partial c_{t}}{\partial W^{s} / \partial c_{0}} \tag{5}
\end{equation*}
$$

Next, from equation (2), we have $\partial w^{p} / \partial c_{t}=v_{1 t} \exp (-\delta t)$, which substituted into equation (4) implies:

$$
\begin{equation*}
\rho^{\rho}(t)=\delta-\frac{1}{t} \ln \frac{v_{1 t}}{v_{10}} . \tag{6}
\end{equation*}
$$

Similarly, we have $\partial w^{s} / \partial c_{t}=\left(v_{1 t}+v_{2 t}\right) \exp (-\delta t)$, such that:

$$
\begin{equation*}
\rho^{s}(t)=\delta-\frac{1}{t} \ln \frac{V_{1 t}+V_{2 t}}{V_{10}+V_{20}} \tag{7}
\end{equation*}
$$

By comparing equations (6) and (7), it follows that $\rho^{s}(t)=\rho^{\rho}(t)$, if and only if $\frac{V_{2 t}}{V_{20}}=\frac{V_{1 t}}{V_{10}}$, immediately implying this result:

Proposition 1. $\rho^{s}(t)=\rho^{p}(t) \forall t$, if and only if $v_{2 t}\left(c_{t}\right)=\beta v_{1 t}\left(c_{t}\right) \forall c_{t}$, where $\beta$ is a constant.

Proposition 1 says that the social discount rate equals the private one between all time periods, if and only if the ratio between the marginal disutility of others' consumption and own marginal utility of consumption is a constant. As such, it resembles proposition 1 in Arrow and Dasgupta (2009) -based on a similar but slightly different model—saying that the privately and socially optimal consumption paths coincide, if and only if, for all $t, v_{2 t}=\beta V_{1 t}$.

In order to explore the differences between the social and private discount rates in a way that allows for a straightforward economic interpretation, we introduce a measure (following, e.g., Johansson-Stenman et al. 2002; and Aronsson and Johansson-Stenman 2008) reflecting the extent to which relative consumption matters:

Definition 1. The degree of positionality is defined by:

$$
\begin{equation*}
\gamma_{t} \equiv \frac{u_{2 t} r_{1 t}}{u_{1 t}+u_{2 t} r_{1 t}} . \tag{8}
\end{equation*}
$$

Thus, $\gamma_{t} \in(0,1)$ reflects the fraction of the overall utility increase from the last dollar consumed that is due to increased relative consumption. As $\gamma_{t}$ approaches zero, relative consumption at time $t$ does not matter at all at the margin, taking us back to the standard model. In the other extreme case, where $\gamma_{t}$ approaches one, absolute consumption does not matter at all (i.e., all that matters is relative consumption).

From equation (1), we have that $v_{2 t}=u_{2 t} r_{2 t}$ and $v_{1 t}=u_{1 t}+u_{2 t} r_{1 t}$, together implying that:

$$
\begin{equation*}
-\frac{V_{2 t}}{V_{1 t}}=-\frac{r_{2 t}}{r_{1 t}} \gamma_{t}=\gamma_{t} . \tag{9}
\end{equation*}
$$

Substituting equation (9) into equation (7) gives, after some straightforward manipulations:

$$
\begin{equation*}
\rho^{s}(t)=\delta-\frac{1}{t} \ln \left(\frac{V_{1 t}}{V_{10}}-\frac{V_{1 t}}{V_{10}+V_{20}}\left(\gamma_{t}-\gamma_{0}\right)\right)=\delta-\frac{1}{t} \ln \left(\frac{V_{1 t}}{V_{10}} \frac{1-\gamma_{t}}{1-\gamma_{0}}\right), \tag{10}
\end{equation*}
$$

implying that the social discount rate increases in the positionality difference $\left(\gamma_{t}-\gamma_{0}\right)$.
Combining equations (6) and (10), and denoting the instantaneous growth rate at time $t$ as $g_{t} \equiv \dot{c}_{t} / c_{t}$, where a dot denotes time derivative, the following result follows immediately:

Proposition 2. For $g_{t}>0 \forall t$, it follows that $\rho^{s}(t)>(<) \rho^{p}(t) \forall t$, if and only if $\partial \gamma_{t} / \partial c_{t}>(<) 0 \forall t$.

Proof: If $\partial \gamma_{t} / \partial c_{t}>0 \forall t$ and $g_{t}>0 \forall t$ then $\gamma_{t}>\gamma_{0}$; and by comparing equation (6) and equation (10), $\rho^{s}(t)>\rho^{p}(t) \forall t$. Similarly, if $\rho^{s}(t)>\rho^{p}(t) \forall t$ then $\gamma_{t}>\gamma_{0} \forall t$. Since equations (6) and (10) hold, regardless of the initial time 0 , this implies that $\partial \gamma_{t} / \partial c_{t}>0 \forall t$. The proof for the case where $\partial \gamma_{t} / \partial c_{t}<0 \forall t$ is analogous. The case of $\partial \gamma_{t} / \partial c_{t}=0 \forall t$ follows directly from proposition 1 , where $\gamma_{t}=\beta \forall t$.

In other words, if relative consumption becomes more important (compared to absolute consumption) when consumption increases then the social discount rate is larger than the private one. The intuition is straightforward: when relative consumption becomes more important for higher consumption levels, a larger share of overall consumption increases is "waste" in terms of positional externalities; and since consumption increases over time, this waste share will increase over time too. This implies that future consumption becomes less valuable, compared to the present one, from a social point of view, and compared to the case where this waste is not taken into account. Specifically, the social discount rate is larger than the private one.

The empirical evidence, although not conclusive, seems indeed to suggest that $\partial \gamma_{t} / \partial C_{t}>0$, such that relative consumption becomes more important, compared to absolute consumption, when the individuals get richer. (See, e.g., Clark et al. 2008; and Corazzini, Esposito, and Majorano, 2011, in press; and references therein.) This implies that, based on the present model, we may conclude that the social discount rate tends to exceed the private one under relative consumption effects.

### 2.3 Optimal Time-Dependent Consumption Taxes

Although it is not the main purpose of the paper, it makes sense to derive the set of consumption taxes that would internalize the time-dependent positional externalities. Let us assume a set of consumption taxes, such that $q_{t}$ is the consumption tax at time $t$; where we normalize, such that $q_{0}=0$; and where the tax revenues are distributed back in a lump-sum manner. We can then simply derive $q_{t}$ as follows: if the individual is indifferent about undertaking a small project that will result in increased consumption at time $t$-which is paid for in terms of reduced consumption at time zero, and where consumption at time zero is untaxed and at time $t$ is taxed by $q_{t}$-then the private discount rate per time unit is implicitly defined by:

$$
\frac{\partial w^{\rho} / \partial c_{t}}{\partial w^{\rho} / \partial c_{0}}=\left(1+q_{t}\right) \exp \left(-\rho^{\rho} t\right)
$$

implying that:

$$
\rho^{p}=\delta-\frac{1}{t} \ln \left(\frac{V_{1 t}}{V_{10}} \frac{1}{1+q_{t}}\right)
$$

If we set this private discount rate equal to the social one, given by equation (10), then this implies:

$$
\begin{equation*}
q_{t}=\frac{\gamma_{t}-\gamma_{0}}{1-\gamma_{t}} . \tag{11}
\end{equation*}
$$

Thus, the tax is directly related to the difference in consumption positionality at time $t$, compared to the untaxed baseline at time zero. The intuition is straightforward, since $\gamma_{t}$ is also a measure of social waste associated with consumption at time $t$. (Compare the corresponding static optimal tax results in Dupor and Liu 2003; and Aronsson and Johansson-Stenman 2008.)

## 3. Comparisons with the Conventional Ramsey Discounting Rule

In the previous section, we analyzed the conditions for when the social discount rate is smaller or larger than the private one and concluded that the social discount rate exceeds the private one when the degree of positionality increases with the consumption level. We based this on a simple continuous time model, where we discounted over an arbitrary discrete time period, from zero to $t$.

Yet, much of the discounting literature, including recent work on climate change, is based on the Ramsey discount rate, according to which the instantaneous discount rate consists of the pure rate of time preference plus the individual coefficient of relative risk aversion multiplied by the growth rate. It clearly has great policy relevance to compare the optimal social discount rate under relative consumption effects with the Ramsey discounting rule, which is the main purpose of this section.

Since the Ramsey discounting rule is normally derived over an infinitesimally short period of time, ${ }^{5}$ we solely consider instantaneous discount rates (hereafter, discount rates). We also compare the private and the social discount rates, as well as the private and the Ramsey discount rates, and, hence, provide conditions for ordering the sizes of the three discount rates. (For previous studies that have analyzed modifications of the Ramsey discounting rule, see, e.g., Hoel and Sterner 2007; Howarth 2009; and Gollier 2010.)

### 3.1 Three Different Discount Rates and Three Measures of Relative Risk Aversion

We will undertake our analysis based on equation (1), the same underlying instantaneous utility function as before; as well as equations (2) and (3), again, the same time-consistent objective functions as before; and we also keep the assumption of identical individuals with a population size normalized to one. Note first that, when $t$ approaches zero, we have:

$$
\begin{aligned}
& \partial w^{p} / \partial c_{t}=\partial w^{p} / \partial c_{0}+t \partial\left(\partial w^{p} / \partial c_{t}\right) / \partial t, \text { and } \\
& \partial w^{s} / \partial c_{t}=\partial w^{s} / \partial c_{0}+t \partial\left(\partial w^{s} / \partial c_{t}\right) / \partial t
\end{aligned}
$$

Substituting these into equations (4) and (5), respectively, and applying l'Hôpital's rule implies the following private and social instantaneous discount rates:

[^4]\[

$$
\begin{align*}
& \rho_{t}^{p}=-\frac{\partial\left(\partial W^{p} / \partial c_{t}\right) / \partial t}{\partial w^{p} / \partial c_{t}}, \text { and }  \tag{12}\\
& \rho_{t}^{s}=-\frac{\partial\left(\partial W^{s} / \partial c_{t}\right) / \partial t}{\partial W^{s} / \partial c_{t}} . \tag{13}
\end{align*}
$$
\]

In order to provide an economic interpretation to the comparison between these different discount rates, we introduce some useful risk aversion definitions:

Definition 2. The individual coefficient of relative risk aversion is given by:

$$
\begin{equation*}
\sigma_{t} \equiv-C_{t} V_{11 t} / V_{1 t} . \tag{14}
\end{equation*}
$$

The social coefficient of relative risk aversion is given by:

$$
\begin{equation*}
\psi_{t} \equiv-c_{t} u_{11 t} / u_{1 t} . \tag{15}
\end{equation*}
$$

The coefficient of reference consumption relative risk aversion is given by:

$$
\begin{equation*}
\theta_{t} \equiv Z_{t} V_{22 t} / V_{2 t} . \tag{16}
\end{equation*}
$$

Thus, whereas $\sigma_{t}$ reflects the conventional measure of relative risk aversion (or the elasticity of marginal utility for the individual of increased consumption where others' consumption, $z_{t}$, is held fixed), $\psi_{t}$ is a measure on the curvature of the instantaneous utility function, where relative consumption, $R_{t}$ is held fixed. It can, therefore, be thought of as a measure that reflects risk aversion had the individual made a risky choice on behalf of the whole population. Thus, it is the social coefficient of relative risk aversion. Finally, $\theta_{t}$ is a measure of the concavity and, hence, risk aversion, with respect to reference consumption. Its definition is strictly analogous to the conventional measure of individual relative risk aversion, except that it refers to others' consumption.

When $\theta_{t}>0$, an individual prefers that others have consumption $\tilde{z}_{t}$ with certainty, over a situation where others' consumption is uncertain (with the expected value $\hat{z}_{t}$ ), whereas the individual is indifferent when $\theta_{t}=0 .{ }^{6}$

We can now define the conventional instantaneous Ramsey discount rate as follows:

[^5]Definition 3. The discount rate according to the Ramsey discounting rule is given by:

$$
\begin{equation*}
\rho_{t}^{R}=\delta+\sigma_{t} g_{t} \tag{17}
\end{equation*}
$$

Hence, the Ramsey discount rate simply consists of the pure rate of time preference plus the product of the individual coefficient of risk aversion times the growth rate.

### 3.2 The Relationship between the Private and the Ramsey Discount Rate, and Keeping Up with the Joneses

From equation (2), it follows that:

$$
\begin{aligned}
& \partial w^{p} / \partial c_{t}=v_{1 t} e^{-\delta t}, \text { and } \\
& \partial\left(\partial w^{p} / \partial c_{t}\right) / \partial t=\left(v_{11 t} \dot{c}_{t}+v_{12 t} \dot{c}_{t}-\delta v_{1 t}\right) \exp (-\delta t),
\end{aligned}
$$

which substituted into equation (12) implies the optimal private discount rate as follows: ${ }^{7}$

$$
\begin{equation*}
\rho_{t}^{p}=\delta-\frac{V_{11 t}}{V_{1 t}} c_{t} g_{t}-\frac{V_{12 t}}{V_{1 t}} c_{t} g_{t} \tag{18}
\end{equation*}
$$

By comparing equations (17) and (18) and using definition 3, we immediately obtain the following result:

Proposition 3. The private discount rate can be written as:

$$
\begin{equation*}
\rho_{t}^{p}=\delta+\sigma_{t} g_{t}-\frac{V_{12 t}}{V_{1 t}} c_{t} g_{t}=\rho_{t}^{R}-\frac{V_{12 t}}{V_{1 t}} c_{t} g_{t} \tag{19}
\end{equation*}
$$

such that, for $g_{t}>0, \rho_{t}^{p}<\rho_{t}^{R}$.
Thus, if the growth rate is positive and the preferences are characterized by the keeping-up-with-the-Joneses property then the private discount rate falls short of the Ramsey discount rate. The intuition is that if my preferences are characterized by the keeping-up-with-the-Joneses property, then I will perceive future consumption, relative to the present one, as more valuable if others consume more in the future. But, this is the same as saying that my private discount rate will decrease as others' consumption grows.

[^6]
### 3.3 The Relationship between the Private and the Social Discount Rate

Consider next the social discount rate. From equation (3), it follows that:

$$
\begin{aligned}
& \partial w^{s} / \partial c_{t}=\left(v_{1 t}+v_{2 t}\right) \exp (-\delta t), \text { and } \\
& \frac{\partial\left(\partial W^{s} / \partial c_{t}\right)}{\partial t}=\left(v_{11 t} \dot{c}_{t}+v_{12 t} c_{t}+v_{21 t} \dot{c}_{t}+v_{22 t} c_{t}-\delta\left(v_{1 t}+v_{2 t}\right)\right) \exp (-\delta t) ;
\end{aligned}
$$

which substituted into equation (13) implies the following optimal social discount rate:

$$
\begin{equation*}
\rho_{t}^{s}=\delta-\frac{v_{11 t}+2 v_{12 t}+v_{22 t}}{V_{1 t}+V_{2 t}} c_{t} g_{t}=\delta+\sigma_{t} g_{t}-\frac{v_{12 t}}{V_{1 t}} c_{t} g_{t}+\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t} g_{t} \tag{20}
\end{equation*}
$$

where, in the second step, we have used $\frac{d \gamma_{t}}{d c_{t}}=-\left(\frac{V_{22 t}}{V_{1 t}}+\frac{V_{21 t}}{V_{1 t}}-\frac{V_{2 t}}{V_{1 t}{ }^{2}} v_{11 t}-\frac{V_{2 t}}{V_{1 t}{ }^{2}} v_{12 t}\right)$. By comparing equations (19) and (20), we obtain:

$$
\begin{equation*}
\rho_{t}^{s}=\rho_{t}^{p}+\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t} g_{t} \tag{21}
\end{equation*}
$$

implying that we can again confirm proposition 2.
In order to simplify this expression further, we can define the degree of non-positionality as:

$$
\begin{equation*}
\varsigma_{t} \equiv 1-\gamma_{t}=\frac{u_{1 t}}{u_{1 t}+u_{2 t} r_{1 t}} \tag{22}
\end{equation*}
$$

and the corresponding consumption elasticity of non-positionality as:

$$
\begin{equation*}
\Gamma \equiv \frac{d \varsigma_{t}}{d c_{t}} \frac{c_{t}}{\varsigma_{t}} \tag{23}
\end{equation*}
$$

Hence, the degree of non-positionality is defined by the fraction of the overall utility increase from the last dollar consumed that is due to increased absolute consumption, whereas the consumption elasticity of non-positionality can be interpreted as (approximately) the percentage change in non-positionality that arises from a percentage consumption increase.

We can then use equation (23) and rewrite equations (20) and (21) as follows:

$$
\begin{equation*}
\rho_{t}^{s}=\delta+\sigma_{t} g_{t}-\frac{V_{12 t}}{V_{1 t}} c_{t} g_{t}-\Gamma_{t} g_{t}=\rho_{t}^{p}-\Gamma_{t} g_{t} \tag{24}
\end{equation*}
$$

Thus, the social discount rate is equal to the private one minus the consumption elasticity of non-positionality times the consumption growth rate. Note that $\Gamma_{t} g_{t}$ reflects the growth rate
of the fraction of consumption that, from a social point of view, is not waste. If this growth rate is positive, it implies a reason for society to consume more in the future, implying a lower social discount rate. If it is negative, and more in line with the empirical evidence mentioned, then the opposite applies.

### 3.4 The Relationship between the Social and the Ramsey Discount Rate

Let us now turn to the central part of this section, the comparison between the social and the Ramsey discount rate. So far in equation (19), we have expressed the relationship between the private and the Ramsey discount rate, in terms of a keeping-up-with-the-Joneses measure; and in equation (24), expressed the relationship between the private and the social discount rate, in terms of how the degree of positionality depends on the consumption level. By combining equations (19) and (24), we can clearly present the relationship between the social and the Ramsey discount rate, in terms of a keeping-up-with-the-Joneses measure and how the degree of positionality depends on the consumption level, as follows:

$$
\begin{equation*}
\rho_{t}^{s}=\rho_{t}^{R}-\frac{v_{12 t}}{V_{1 t}} c_{t} g_{t}+\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t} g_{t}=\rho_{t}^{R}-\frac{v_{12 t}}{V_{1 t}} c_{t} g_{t}-\Gamma_{t} g_{t} \tag{25}
\end{equation*}
$$

Thus, according to this formulation, the social discount rate exceeds the Ramsey discount rate, if the effects-through increased positionality (or decreased non-positionality) with consumption and (hence), over time, given a positive growth rate-exceed the keeping-up-with-the-Joneses effect. Both assumptions are fairly intuitive and, if we accept them, we cannot say anything regarding the relative size of $\rho_{t}^{s}$ and $\rho_{t}^{R}$ without adding information regarding the relative strengths of these mechanisms. Yet, we will indirectly show conditions for when the keeping-up-with-the-Joneses effect dominates the increasing positionality effect.

In doing this, let us use the alternative formulation of equation (3), where $W^{s}$ is expressed in terms of the $u$ function. It then follows that:
$\partial \omega / \partial c_{t}=u_{1 t} \exp (-\delta t)$, and

$$
\partial\left(\partial w / \partial c_{t}\right) / \partial t=\left(u_{11 t} c_{t}-\delta u_{1 t}\right) \exp (-\delta t)
$$

which substituted into equation (12) implies the following alternative formulation of the optimal social discount rate:

$$
\begin{equation*}
\rho_{t}^{s}=\delta+\psi_{t} g_{t} \tag{26}
\end{equation*}
$$

By comparing equations (17) and (26), we immediately have:

Proposition 4. For $g_{t}>0, \rho_{t}^{s}>(<) \rho_{t}^{R}$ if and only if $\psi_{t}>(<) \sigma_{t}$.
In words, given a positive growth rate, the welfare maximizing social discount rate in the presence of relative consumption effects exceeds the conventional Ramsey discount rate, if the social coefficient of relative risk aversion exceeds the individual coefficient of relative risk aversion, and vice versa.

The next question is obvious. Under which conditions does $\psi_{t}$ exceed $\sigma_{t}$, and vice versa? In order to interpret the conditions for this in economic terms, we introduce a measure on the complementarity between own consumption and (reduction of) others' consumption:

Definition 4. The elasticity of substitution between $C_{t}$ and $z_{t}$ is given by:

$$
\begin{equation*}
\Phi_{t} \equiv \frac{-\frac{V_{11 t}}{V_{1 t}{ }^{2}}-\frac{V_{22 t}}{V_{2 t}{ }^{2}}+2 \frac{V_{12 t}}{V_{1 t} V_{2 t}}}{\frac{1}{V_{1 t} c_{t}}+\frac{1}{V_{2 t} z_{t}}}=\left(-\frac{V_{11 t}}{V_{1 t}{ }^{2}}-\frac{V_{22 t}}{V_{2 t}{ }^{2}}+2 \frac{V_{12 t}}{V_{1 t} V_{2 t}}\right) \frac{V_{1 t} V_{2 t}}{V_{1 t}+V_{2 t}} c_{t} . \tag{27}
\end{equation*}
$$

This definition is standard (although it is not often used between one good and one bad in the utility function). It reflects the degree of quasi-concavity of $v$ and, hence, the degree of convexity of the indifference curves in $c_{t}, z_{t}$ space. When $\Phi_{t}>0$ everywhere, an individual who could hypothetically buy a reduction in others' consumption at a fixed per unit price would then face a unique optimum; whereas $\Phi_{t}=0$ everywhere would give linear indifference curves, such that the increase in consumption necessary to compensate an individual for other people's increase in consumption would be constant. We can now specify the following crucial relation between $\psi_{t}$ and $\sigma_{t}$ :

Lemma 1. The social coefficient of relative risk aversion is given by:

$$
\begin{equation*}
\psi_{t}=\sigma_{t}-\theta_{t}-\Phi_{t} \tag{28}
\end{equation*}
$$

Proof: Let us first express the social coefficient of relative risk aversion $\psi_{t}$ in terms of the $v$ function. By comparing equations (18) and (23), we have:

$$
\begin{equation*}
\psi_{t}=-\frac{V_{11 t}+2 V_{12 t}+V_{22 t}}{V_{1 t}+V_{2 t}} c_{t} . \tag{29}
\end{equation*}
$$

We can now combine equations (14) and (27) and, after some straightforward algebraic manipulations, obtain:

$$
\begin{equation*}
\psi_{t}-\sigma_{t}=-c_{t} \frac{V_{22 t}}{V_{2 t}}-c_{t} \frac{V_{1 t} V_{2 t}}{V_{1 t}+V_{2 t}}\left(-\frac{V_{11 t}}{V_{1 t}^{2}}-\frac{V_{22 t}}{V_{2 t}{ }^{2}}+2 \frac{V_{12 t}}{V_{1 t} V_{2 t}}\right) . \tag{30}
\end{equation*}
$$

Substituting, finally, equations (16) and (27) into equation (30) gives equation (28).
Directly from lemma 1 and equation (10), we can specify the social discount rate in terms of individual risk aversion and, hence, also the conditions for when the social discount rate exceeds the one based on the conventional Ramsey discounting rule, and vice versa.

Proposition 5. The social discount rate can be written as:

$$
\begin{equation*}
\rho_{t}^{s}=\delta+\left(\sigma_{t}-\theta_{t}-\Phi_{t}\right) g_{t}=\rho_{t}^{R}-\left(\theta_{t}+\Phi_{t}\right) g_{t}, \tag{31}
\end{equation*}
$$

such that, for $g_{t}>0, \rho_{t}^{s}<(>) \rho_{t}^{R}$, if $\theta_{t}+\Phi_{t}>(<) 0$.
Hence, a sufficient, but not necessary, condition for $\rho_{t}^{s}<\rho_{t}^{R}$ is that $v$ is strictly quasiconcave $\left(\Phi_{t}>0\right)$ and individuals are weakly reference-consumption risk averse ( $\left.V_{22 t} \leq 0, \theta_{t} \geq 0\right)$ or, alternatively, that $V$ is weakly quasi-concave $\left(\Phi_{t} \geq 0\right)$ and individuals are strictly referenceconsumption risk averse $\left(V_{22 t}<0, \theta_{t}>0\right)$.

The intuition behind proposition 5 is not straightforward enough to see easily. Roughly speaking, reference-consumption risk aversion and quasi-concavity both contribute to a decrease in the social marginal utility of consumption when consumption increases. Yet, at the same time, they contribute to a decrease in the percentage change in social marginal utility of consumption for a percentage increase in consumption.

In order to gain some further insights, let us consider some special cases where we deal with reference-consumption risk aversion and quasi-concavity separately. Starting with the case of reference-consumption risk aversion (i.e., that $v_{22 t}<0$ ) and perfect substitution (i.e., linear indifference curves, such that $\Phi_{t}=0$ ), we can write instantaneous utility as:

$$
f\left(c_{t}-a z_{t}\right)=f\left((1-a) c_{t}+a\left(c_{t}-z_{t}\right)\right),
$$

where $0<a<1$ reflects the degree of positionality, and where $f^{\prime}>0$ and $f^{\prime \prime}<0$. Then, $v_{1 t}=f^{\prime}$, $v_{11 t}=f^{\prime \prime}, u_{1 t}=(1-a) f^{\prime}$, and $u_{11 t}=(1-a)^{2} f^{\prime \prime}$; implying that $\sigma_{t}=-c_{t} f^{\prime \prime} / f^{\prime}$ and $\psi_{t}=-(1-a) c_{t} f^{\prime \prime} / f^{\prime}$, such that $\psi_{t}-\sigma_{t}=a c_{t} f^{\prime \prime} / f^{\prime}<0$ and, hence, $\rho_{t}^{s}<\rho_{t}^{R}$.

The reason is simply that when $a$ increases, the curvature of $u$ decreases. When $a$ approaches 1 , such that only relative consumption matters, both $u_{1 t}$ and $u_{11 t}$ approach zero, but $u_{11 t}$ approaches zero more rapidly. A 1 percent change of a subset of consumption (consumption minus the effect of reference consumption) will cause a smaller than 1 percent change of
consumption, per se. Hence, the corresponding changes in marginal instantaneous social utility will be smaller too.

Consider next the case with quasi-concavity and reference consumption risk neutrality, such that $\Phi_{t}>0$ and $v_{22 t}=\theta_{t}=0$. A simple functional form that fulfills this is:

$$
c_{t}^{1-\alpha} /(1-\alpha)-\beta z_{t}=c_{t}^{1-\alpha} /(1-\alpha)+(1-\beta) c_{t}+\beta\left(c_{t}-z_{t}\right),
$$

where $\alpha, \beta>0$ and $\beta<1+\frac{1}{1+\alpha} c_{t}^{-\alpha}$ to ensure quasi-concavity. $\alpha$ is the individual coefficient of relative risk aversion, such that $\sigma_{t}=\alpha$, and the degree of positionality increases with $\beta$, but is not constant.

It then follows that $u_{1 t}=c_{t}^{-\alpha}+(1-\beta)$ and $u_{11 t}=-\alpha C_{t}^{-\alpha-1}$, implying that:

$$
\begin{aligned}
& \psi_{t}=\frac{\alpha}{1+(1-\beta) c_{t}^{\alpha}}, \text { and } \\
& \psi_{t}-\sigma_{t}=-\alpha \frac{(1-\beta) c_{t}^{\alpha}}{1+(1-\beta) c_{t}^{\alpha}}=-\frac{\alpha}{1+(1-\beta)^{-1} c_{t}^{-\alpha}}<0
\end{aligned}
$$

Hence, again, $\rho_{t}^{s}<\rho_{t}^{R}$. Here there is no direct effect on the curvature from $Z$, since we have reference-consumption risk neutrality, but $Z$ will still affect the curvature of the instantaneous utility function differently, depending on whether relative consumption or others' consumption is held fixed.

However, it should be noted that the condition in equation (31) is not independent of equation (25). Indeed, we can directly show that:

$$
\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t}=-\Gamma_{t}=\frac{v_{12 t}}{v_{1 t}} c_{t}-\left(\theta_{t}+\Phi_{t}\right)
$$

clearly implying that, if $\theta_{t}+\Phi_{t}>0$ then $\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t} g_{t}<\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t}$. Thus, if $v$ is quasi-concave and individuals are reference-consumption risk averse then the effects through increased positionality over time in equation (31) cannot exceed the keeping-up-with-the-Joneses effect.

### 3.5 Summary of Main Findings

By combining equation (20) and proposition 5, we can summarize our main findings in this section regarding the ordering of our three different discount rates, as follows: ${ }^{8}$

Corollary 1. For $g_{t}>0$, if $d \gamma_{t} / d c_{t}>0$ and $\theta_{t}+\Phi_{t}>0$ then $\rho_{t}^{p}<\rho_{t}^{s}<\rho_{t}^{R}$.
Spelled out, this means that, for a positive consumption growth rate, the social discount rate is larger than the private rate, but smaller than the Ramsey rate, if the degree of positionality increases with consumption and preferences reflect risk aversion with respect to reference consumption, and are quasi-concave with respect to own and reference consumption.

## 4. Practical Significance and Orders of Magnitude

So far we have concluded that, under plausible assumptions, the social discount rate tends to exceed the private discount rate (sections 2 and 3), and we have also shown that the social discount rate tends to be smaller than the one corresponding to the conventional discounting rule (section 3), under relative consumption comparisons. The latter finding is more important from a policy perspective, for example, as it relates to greenhouse effects. Yet, while it is clearly important to identify whether the optimal social discount rate exceeds or falls short of the one corresponding to the conventional rule, it is also important to analyze whether the discrepancies between the two are likely to be sufficiently large to be economically important.

To do this, we utilize a simple, albeit quite flexible, functional form characterized by constant elasticity of substitution and constant relative risk aversion, similar to one used by Dupor and Liu (2003), as follows:

$$
\begin{equation*}
U_{t}=\frac{1}{1-\alpha}\left((1-a) c_{t}^{1-\omega}+a\left(c_{t}^{1-\omega}-z_{t}^{1-\omega}\right)\right)^{\frac{1-\alpha}{1-\omega}}=\frac{1}{1-\alpha}\left(c_{t}^{1-\omega}-a z_{t}^{1-\omega}\right)^{\frac{1-\alpha}{1-\omega}} \tag{32}
\end{equation*}
$$

This functional form has some convenient properties. The degree of positionality (see definition 1 ) is constant and given by $\gamma_{t}=a$. The elasticity of substitution between own consumption and reference consumption (see definition 4) is constant and equal to $\omega$. The

[^7]coefficient of reference-consumption relative risk aversion (see definition 2) is constant and given by $\theta_{t}=\frac{a \alpha-\omega}{1-a}$, implying reference-consumption risk aversion for $\omega<a \alpha$.

The individual and the social coefficients of relative risk aversion (see definition 2) are also constant and given by $\sigma_{t}=\frac{\alpha-\omega a}{1-a}$ and $\psi_{t}=\alpha$, respectively, implying that:

$$
\begin{equation*}
\psi_{t}-\sigma_{t}=(\omega-\alpha) \frac{a}{1-a}, \tag{33}
\end{equation*}
$$

which is clearly weakly negative (such that $\rho_{t}^{s} \leq \rho_{t}^{R}$ ) as long as $\omega \leq \alpha$, which is required for the weak keeping-up-with-the-Joneses property assumed.

The constant elasticity of substitution functional form in equation (32) includes as special cases the two most commonly used comparison-consumption functional forms. When $\omega=0$, we obtain the simple difference comparison form, so that own consumption and (the negative of) others' consumption are perfect substitutes, as follows:

$$
\begin{equation*}
U_{t}=\frac{1}{1-\alpha}\left((1-a) c_{t}+a\left(c_{t}-z_{t}\right)\right)^{(1-\alpha)}=\frac{1}{1-\alpha}\left(c_{t}-a z_{t}\right)^{(1-\alpha)} . \tag{34}
\end{equation*}
$$

For this functional form, we find that:

$$
\psi_{t}-\sigma_{t}=-\alpha \frac{a}{1-a}<0 .
$$

For example, if $a=0.5$ and $\alpha=1$ then $\psi_{t}-\sigma_{t}=-1$, implying that the private coefficient of relative risk aversion $\sigma_{t}=2$, whereas the corresponding social coefficient $\psi_{t}=1$. Hence, it is clear that the effects of relative consumption may be substantial.

Similarly, we obtain the ratio-comparison form by letting $\omega$ approach unity and applying l'Hôpital's rule, implying that equation (32) converges to:

$$
\begin{equation*}
U_{t}=\frac{1}{1-\alpha}\left(c_{t}\left(\frac{c_{t}}{z_{t}}\right)^{a /(1-\alpha)}\right)^{(1-\alpha)}=\frac{1}{1-\alpha} c_{t}^{(1-\alpha) /(1-\alpha)} z_{t}^{-(1-\alpha) a /(1-\alpha)}, \tag{35}
\end{equation*}
$$

such that $\psi_{t}-\sigma_{t}=(1-\alpha) \frac{a}{1-a}$, which is clearly negative if $\alpha>1$, and positive if $\alpha<1$. Note that reference-consumption risk aversion implies that $\alpha>1 / a$, and the keeping-up-with-theJoneses assumption implies that $\alpha \geq 1$. Here, too, it is clear that the effects of relative consumption may be substantial as we illustrate further below.

Yet, our main interest concerns the orders of magnitude of the effects on the discount rates. Substituting equation (33) into equation (25) yields:

$$
\begin{equation*}
\rho_{t}^{s}=\delta+\left(\sigma+(\omega-\alpha) \frac{a}{1-a}\right) g . \tag{36}
\end{equation*}
$$

Below, we calculate the optimal social discount rate as a function of the degree of positionality for two commonly discussed sets of assumptions in the economic climate change literature, associated with Stern (2006) and Weitzman (2007b), respectively. Weitzman (2007b) used a model with constant parameters, $\delta=\sigma=g=2$, leading to a Ramsey discount rate of 6 percent.

In figure 1, we plot the optimal social discount rate, taking relative consumption effects into account, as a function of $a$ for different values of the elasticity of substitution $\omega$. As can be seen, the optimal social discount rate is always below the Ramsey discount rate, except for when $\omega=2$, where the optimal discount rate equals the Ramsey discount rate. Here only the weak keeping-up-with-the-Joneses property is fulfilled, such that an individual's consumption is independent of the consumption of others. Moreover, we also have that the individual is reference-consumption risk-loving, since $\omega>a \alpha$. It is easy to show that this condition is fulfilled, if and only if $\omega>\sigma a /(1+a)$ and, hence, that the individual is reference-consumption risk averse when $\omega<\sigma a /(1+a)$. Thus, a necessary (but not sufficient) condition for referenceconsumption risk aversion is that $\omega<\sigma$, for $0<a<1$.

In figure 1, we illustrate the parts of the relationships where the individual is referenceconsumption risk-loving by dotted lines, and the remaining parts (reference-consumption risk aversion) by solid lines. As can be seen, for non-negligible levels of $a$, the optimal discount rate tends to be substantially smaller than the Ramsey rate, if we assume reference-consumption risk aversion. For some $(a, \omega)$ combinations, the optimal discount rate even approaches the values used in the Stern Review (see below).

Figure 1. The Effect of the Degree of Postionality a on the Optimal Social Discount Rates Associated with the Assumptions in Stern (2006) and Weitzman (2007b).


Stern (2006) assumed instead that $\delta=0.01, \sigma=1$, and $g=1.3$, implying an overall Ramsey discount rate of 1.4 percent annually. ${ }^{9}$ Again, the optimal social discount rate is always equal to or below the Ramsey discount rate, given the weak keeping-up-with-the-Joneses property, where an individual's consumption is independent of the consumption of others for $\omega=1$. In addition, in figure 1 , if we are willing to assume reference-consumption risk aversion (solid lines), we find again that the effects of relative consumption concerns are substantial for non-negligible values of $a$.

Unfortunately, the empirical literature does not provide any precise estimates of the relevant parameters. The most central one, $\gamma$ (or $a$ ), is obviously difficult to measure and it is, therefore, not surprising that the available estimates vary considerably. However, most estimates

[^8]are substantially above zero. According to the survey-experimental evidence of Solnick and Hemenway (1998; 2005), Johansson-Stenman et al. (2002), Alpízar et al. (2005), and Carlsson et al. (2007), the average degree appears to be in the order of magnitude of 0.5 . Wendner and Goulder (2008) argue, based on existing empirical evidence, for a value between 0.2 and 0.4 ; whereas evidence from happiness studies, such as Luttmer (2005), suggests a much larger value close to unity. To our knowledge, there are no quantitative estimates of either $\theta$ or $\Phi$, in other words, of either reference-consumption risk aversion or the degree of quasi-concavity between own consumption and reference consumption, beyond what is implied by keeping-up-with-theJoneses behavior.

There are in contrast many studies trying to estimate $\sigma$, which is relatively less important for the results here. ${ }^{10}$ Estimates of $\delta$ are highly controversial, in particular for ethical reasons, when dealing with intergenerational issues (Stern 2006), but as shown above $\delta$ does not affect the difference between the optimal discount rate and the one corresponding to the Ramsey rule. Future growth rates are of course also difficult to predict.

Overall, the discrepancy between the optimal and the Ramsey discount rate due to relative consumption effects is clearly difficult to quantify, but may well be substantial and could even exceed $1-2$ percentage points.

## 5. Conclusion and Discussion

There are several reasons why one may argue that social discount rates should in practice be lower than individual ones: individuals are more risk averse than society in the presence of uncertainty, and societal time horizons are longer than individual ones (cf. Arrow and Lind 1970). In this paper, we show that relative consumption effects do not provide another reason. On the contrary, the social discount rate tends, under positional concern, to exceed the private one, provided that the degree of positionality increases as we get richer and consumption increases (for which there is some empirical evidence).

[^9]Yet, from a climate policy perspective, it is presumably more important whether the optimal social discount rate should be modified, compared to the one corresponding to the conventional Ramsey discounting rule. We show that, for a positive growth rate, the social discount rate is smaller than the Ramsey discount rate if preferences are quasi-concave in own and reference consumption (consisting of others' average consumption), and exhibit risk aversion with respect to reference consumption. We also demonstrate numerically that the discrepancies may be substantial, although the underlying parameter estimates are highly uncertain. Yet, since the impacts of the discount interest rates on the economics of long-term phenomena, such as global warming, are so large, even for modest adjustments of the discount rate, it is fair to conclude that taking relative consumption effects into account may have a profound effect of the economics of phenomena like global warming.

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[^1]:    ${ }^{1}$ This includes happiness research (e.g., Easterlin 2001; Blanchflower and Oswald 2004; Ferrer-i-Carbonell 2005; Luttmer 2005), questionnaire-based experiments (e.g., Johansson-Stenman et al. 2002; Solnick and Hemenway 2005; Carlsson et al. 2007), and more recently brain science (Fliessbach et al. 2007). There are also recent evolutionary models consistent with relative consumption concerns (Samuelson 2004; Rayo and Becker 2007). Stevenson and Wolfers (2008) constitute a recent exception in the happiness literature, claiming that the role of relative income is overstated.
    ${ }^{2}$ See, e.g., Boskin and Sheshinski (1978), Layard (1980), Oswald (1983), Ng (1987), Blomquist (1993), Corneo and Jeanne (1997; 2001), Brekke and Howarth (2002), Abel (2005), and Wendner and Goulder (2008). Clark et al. (2008) provide a good overview of both the empirical evidence and economic implications of relative consumption concerns.

[^2]:    ${ }^{3}$ This assumption is fairly innocuous as long as individuals are identical within each generation. For example, it encompasses the most commonly used comparison forms, i.e., the difference comparison form where $R_{t}=c_{t}-z_{t}$; the ratio comparison case where $R_{t}=c_{t} / z_{t}$; and the more flexible functional form suggested by Dupor and Liu (2003), which includes both the difference and the ratio forms as special cases.

[^3]:    ${ }^{4}$ Note also that since we are only concerned with given consumption paths, as is common in the discounting literature, our results are independent of any production assumptions, including issues of technical change.

[^4]:    ${ }^{5}$ That is, over $t$ to $t+d t$ (for any $t$ ). Alternatively, the formula can be derived over a discrete period of time, if one is willing to assume constant relative risk aversion preferences, as well as a constant growth rate and pure rate of time preference. Here, we do not want to limit ourselves to particular functional forms.

[^5]:    ${ }^{6}$ Note that reference consumption risk aversion $\left(v_{22 t}<0\right)$ is neither necessary nor sufficient for the more frequently discussed concept of comparison concavity ( $U_{22 t}<0$ ). See Clark and Oswald (1998) for some relationships between comparison-concavity and keeping-up-with-the-Joneses behavior.

[^6]:    ${ }^{7}$ It is easy to show that the same rule applies for discounting over a discrete time period (in continuous time), provided that $\psi_{t}$ and $g_{t}$ are constant in the interval considered.

[^7]:    ${ }^{8}$ Note that if the conditions in corollary 1 are fulfilled, then it also follows that $v_{12 t}>0$, i.e., the keeping-up-with-the-Joneses property is fulfilled.

[^8]:    ${ }^{9}$ It may seem that Stern selected these values partly to compensate for a number of simplifications and omissions. Stern (2006) mentioned combinations of ethical and distributional issues, and deep uncertainty, as well as the effects of different growth rates in different sectors. Relative consumption issues were not mentioned, however.

[^9]:    ${ }^{10}$ For example, Friend and Blume (1975) concluded that it generally exceeds unity and is probably greater than 2, Blundell et al. (1994) and Attanasio and Browning (1995) found, in most of their estimates, order of magnitude of 1 or slightly above, whereas Vissing-Jørgensen (2002) found that $\sigma$ differs between stockholders (approximately $2.5-3$ ) and bond holders (approximately 1-1.2). Halek and Eisenbauer (2001) estimate values for a large sample of individuals and found a very skewed distribution with a median of 0.9 but a mean over 3 .

