GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 319

Consumer Loss Aversion and the Intensity of Competition

* Heiko Karle
** Martin Peitz

* Université Libre de Bruxelles

** University of Mannheim

May 2010

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.sfbtr15.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Dr. Urs Schweizer. · Department of Economics · University of Bonn · D-53113 Bonn,
Phone: +49(0228)739220 · Fax: +49(0228)739221

Consumer Loss Aversion and the Intensity of Competition*

Heiko Karle[†]
Université Libre de Bruxelles

Martin Peitz[‡]
University of Mannheim

Version: May 2, 2010
(First Version: November 2009)

Abstract

Consider a differentiated product market in which all consumers are fully informed about match value and price at the time they make their purchasing decision. Initially, consumers become informed about the prices of all products in the market but do not know the match values. Some consumers have reference-dependent utilities—i.e., they form a reference-point distribution with respect to match value and price that will make them realize gains or losses if their eventually chosen product performs better or, respectively, worse than their reference point in both dimensions. Loss aversion in the match-value dimension leads to a less competitive outcome, while loss aversion in the price dimension leads to a more competitive equilibrium than a market in which consumers are not subject to reference dependence. Depending on the weights consumers attach to the price and the match-value dimension, a market with loss-averse consumers may be more or less competitive than a market with consumers that do not have reference-dependent utilities. We also show that consumer loss aversion tends to lead to higher prices if the market accommodates a larger number of firms.

*A previous version of this paper was entitled "Consumer Loss Aversion and Competition in Differentiated Product Markets". We are grateful to Rani Spiegler and various seminar audiences for helpful comments and suggestions. Martin Peitz acknowledges financial support from the German Science Foundation (SFB TR 15).

[†]ECARES, Université Libre de Bruxelles (ULB), B-1050 Brussels, Belgium. E-mail: hkarle@ulb.ac.be

[‡]Department of Economics, University of Mannheim, 68131 Mannheim, Germany. E-mail: martin.peitz@googlemail.com. Also affiliated with CEPR, CESifo, ENCORE, and ZEW.

Keywords: Loss Aversion, Reference-Dependent Utility, Behavioral Industrial Organization, Imperfect Competition, Product Differentiation
JEL Classification: D83, L13, L41, M37.

1 Introduction

In this paper, we introduce loss-averse consumers into a differentiated product market and investigate the competitive effects of consumer loss aversion and, more generally, reference-dependent utilities. Our framework applies to inspection goods: Consumers learn about available products and prices but have to inspect products before knowing the match value between product characteristics and consumer taste—consumers often face such a situation because price information can be easily communicated, whereas match value is more difficult for a consumer to assess.

Reference dependence and loss-aversion in consumer choice is a robust empirical phenomenon that has been documented in a variety of laboratory and field settings starting with Kahneman and Tversky (1979). Following Koszegi and Rabin (2006), reference points are expectation-based: A consumer's reference point is her probabilistic belief about the relevant consumption outcome held between the time she first begins to contemplate the consumption plan and the moment she actually makes the purchase. Consumers are loss-averse with respect to prices and match value and have self-fulfilling expectations about equilibrium outcomes to form their reference point, as in Heidhues and Koszegi (2008).[1]

Firms compete in prices for differentiated products. Product differentiation is modeled as in Salop (1979). In addition to the standard business-stealing effect in oligopoly, price affects reference-dependent utilities. In particular, holding the reference-point distribution fixed, a price reduction leads to a gain in the price dimension for consumers who buy this product but to a loss in the price dimension for all consumers who buy the other product. This implies that, due to reference dependence, a consumer's realized net utility depends not only on the price of the product she buys but also on the price of the product she does not buy. Furthermore, price can be seen as an expectation-management tool, as it affects the reference-point distribution in the price and in the match-value dimension. Utility is also affected by the match-value dimension because price changes affect the expected match quality.

We characterize the equilibrium and establish conditions for equilibrium existence and uniqueness. Our model allows for clear-cut comparative statics results.

[1]For evidence that expectation-based counterfactuals can affect the individual's reaction to outcomes, see Blinder, Canetti, Lebow, and Rudd (1998), Medvec, Madey, and Gilovich (1995), and Mellers, Schwartz, and Ritov (1999). The general theory of expectation-based reference points and the notion of personal equilibrium have been developed by Koszegi and Rabin (2006) and Koszegi and Rabin (2007).

Our first main result is that, in markets in which consumers' utility is reference-dependent and, more specifically, features loss aversion, the competitive effect of such a behavioral bias depends on the weight of the price dimension relative to the match-value dimension. In other words, whether the behavioral bias makes the market more or less competitive depends on how gains and losses in the two dimensions enter consumers' utility function. We show that reference dependence with respect to prices leads to lower prices and, thus, is pro-competitive, whereas reference dependence with respect to match value is anti-competitive. This holds even if gains and losses enter with the same weights into the utility function.[2]

We then focus on the utility specification in which the price and match-value dimensions enter with the same weights in the utility function. Consider the *n*-firm oligopoly with localized competition put forward by Salop (1979). We accommodate loss-averse consumers in this model. In this context, we obtain our second main result: Consider a setting in which the number of firms would be neutral to competition if consumers' utility functions did not feature reference dependence. Then, an increase in the number of firms leads to higher prices if consumers are loss-averse.

This paper contributes to the analysis of consumer loss aversion in imperfectly competitive markets and complements our companion paper, Karle and Peitz (2010), as well as Heidhues and Koszegi (2008) and Zhou (2008). More broadly, it contributes to the analysis of behavioral biases in market settings, as in Eliaz and Spiegler (2006), Gabaix and Laibson (2006), and Grubb (2009).

Compared to Heidhues and Koszegi (2008), our model has two distinguishing features. First, firms' marginal costs are identical and common knowledge. This is in line with a large part of the industrial organization literature on oligopoly and constitutes a limiting case of Heidhues and Koszegi (2008). It is approximately satisfied in stationary markets in which firms are well-informed about the technology of their competitors. Assuming the same marginal cost amounts to assuming that all firms use the same technology. Second and more importantly, we postulate that prices are set *before* consumers form their reference point. This property in particular holds in market in which prices are easily observed but in which consumers need time to evaluate the match value—for an elaborate discussion see Section 1 of our companion paper, Karle and Peitz (2010). We also allow for a population mix between consumers with and without reference-dependent utilities, whereas Heidhues and Koszegi (2008) only allow for the two polar cases.

[2] Reference dependence includes this case, while loss aversion requires that gains and losses enter with different weights.

In independent work, Zhou (2008) predicts a pro-competitive effect of consumers being loss averse that contrasts with Heidhues and Koszegi (2008). In Zhou's model and in ours firms can manage consumers' reference point by choosing product prices. A key difference between the two models is that consumers in his model do not use an expectation-based reference point. Instead, he proposes a history-dependent reference point: Consumers consider the product visited last as their reference point.

In this paper, we provide a taxonomy of different market environments and find that the impact of consumer loss aversion on competition depends on the particular specification of the gain-loss utility: If consumers experience a gain-loss utility in the price dimension only, the behavioral bias is pro-competitive; if they experience a gain-loss utility in the match-value dimension only, the behavioral bias is anticompetitive. If both dimensions enter the utility function symmetrically, the result depends on the presence of consumer loss aversion: If gains and losses receive the same weights (i.e., no loss aversion), the bias is competitively neutral; otherwise, with consumer loss aversion, the anti-competitive effect in the taste dimension dominates.

In Karle and Peitz (2010), we analyze a model of asymmetric duopoly and explore the effect of cost asymmetry and the share of ex ante available information in the consumer population on market outcomes. The present paper has a different focus: We analyze symmetric oligopoly and explore how different weights in the price and match-value dimension of the reference-dependent utilities and the number of firms shape competition.

The plan of the paper is as follows. In Section 2, we present the model. In Section 3, we characterize the duopoly equilibrium. We also compare our findings to those of the duopoly model with a different timing of events inspired by Heidhues and Koszegi (2008). In Section 4, we extend our analysis to an n-firm oligopoly. Section 5 concludes. Some of the proofs are relegated to Appendix A. Equilibrium existence in symmetric n-firm oligopoly is established in Appendix B.

2 The Model

Consider a market with n firms and a continuum of loss-averse consumers of mass 1. Firms are located equidistantly on a circle of length $L = n$. The location of firm i is denoted by $y_i = i - 1$ for all $i \in \{1, ..., n\}$. Consumers observe firms' locations ex ante. Each firm i announces its price p_i to all consumers.

Consumers are uniformly distributed on the circle. A consumer's location x, $x \in [0, n)$, represents her taste parameter. Her taste is initially—i.e., before she forms her reference point—not known to herself.

A fraction $(1-\beta)$ of consumers, $0 \leq \beta \leq 1$, has reference-dependent utilities. As will be detailed below, consumers endogenously determine their reference point and then, before making their purchase decision, observe their taste parameter (which is each consumer's private information). At the moment of purchase, all consumers are perfectly informed about product characteristics, prices, and tastes.

All consumers have the same reservation value v for an ideal variety and have unit demand. Their utility from not buying is $-\infty$, so that the market is fully covered.

We note that the circle model allows for the alternative and equivalent interpretation about the type of information consumers initially lack: Consumers do not know the location of the firms; they know only that the two firms are located equidistantly on the circle.

Let the consumer type with standard utilities in $[0, 1]$, who is indifferent between buying good i and good $i + 1$, be denoted by $\hat{x}_i(p_i, p_{i+1})$. The corresponding indifferent loss-averse consumer is denoted by $\hat{\hat{x}}_i^+(p_1, \ldots, p_n)$.[3] Note that the location of the loss-averse consumer who is indifferent between two products depends not only on the prices of the two products she will choose from, but also on the prices of the other products, since they affect the reference-point distribution in the price and taste dimensions. The firms' profits are:

$$\pi_i(p_1, \ldots, p_n) = (p_i - c)\Big(\beta \cdot \frac{\hat{x}_i(p_i, p_{i+1}) - \hat{x}_i(p_{i-1}, p_i)}{n} + (1-\beta) \cdot \frac{\hat{\hat{x}}_i^+(p_1, \ldots, p_n) - \hat{\hat{x}}_i^-(p_1, \ldots, p_n)}{n}\Big).$$

The timing of events is as follows:

Stage 1) Price-setting stage: Firms simultaneously set prices p_i.

Stage 2) Reference-point-formation stage: All consumers observe prices, and consumers with reference-dependent utilities form reference-point distributions over purchase price and match value.

Stage 3) Inspection stage: Consumers observe their taste x.

Stage 4) Purchase stage: Consumers decide which product to buy.

[3] We denote the indifferent loss-averse consumer between buying from firm i and firm $i - 1$ by $\hat{\hat{x}}_i^-(p_1, \ldots, p_n)$.

At stage 1, we solve for subgame perfect Nash equilibrium, where firms foresee that consumers with reference-dependent utilities play a personal equilibrium at stage 2. Consumers with reference-dependent utilities do not know their ideal taste x ex ante and, thus, are ex ante uncertain as to which product they will buy after they have learned their ideal taste x. Ex ante, they face uncertainty about purchase price and match value. This leads to a non-degenerate reference-point distributions in these two dimensions.

Following Koszegi and Rabin (2006) and Heidhues and Koszegi (2008), we assume that consumers experience gains and losses not with respect to net utilities, but with respect to each product "characteristic" separately, where price is then treated as a product characteristic. This is in line with much of the experimental evidence on the endowment effect; for a discussion, see, e.g., Koszegi and Rabin (2006). Following Heidhues and Koszegi (2008), we also assume that consumers evaluate gains and losses *across* products. This appears to be the natural setting for consumers facing a discrete choice problem.

To derive the two-dimensional reference-point distribution of loss-averse consumers, suppose that the price vector $\boldsymbol{p} = (p_1, \ldots, p_n)$ is such that any sub-market between two neighboring firms is served by only these two firms—i.e., the maximum price difference between any two neighboring firms is not too large in absolute terms.[4] The rank order of the price difference, $\Delta p_i^+ = p_{i+1} - p_i$, and distance between firm i and her indifferent loss-averse consumer on the right, $\hat{\hat{x}}_i^+ - y_i = \hat{\hat{x}}_i^+ - (i-1) \in [0, 1]$, are identical.[5] This holds true since the reference comparison induced by reference-dependent utility is, by construction, rank-order maintaining. For example, if $p_i = p_{i+1}$ ($\Delta p_i = 0$), then $\hat{\hat{x}}_i^+ - (i-1) = 1/2$ (by symmetry), while $\hat{\hat{x}}_j^+ - (j-1) > 1/2$ if $p_j < p_{j+1}$ ($\Delta p_j > 0$). The reference-point distribution in the price dimension, $F(p)$, is the probability that the equilibrium purchase price p^* is smaller than or equal to p. Recall that due to consumers' initial taste uncertainty, the equilibrium purchase price is not known when consumers form their reference point, even though firms' prices are already disclosed. Buying from a cheap firm is more likely than buying from an expensive firm, as a cheap firm serves a larger market share in equilibrium. Utilizing the uniform distribution of x, we derive

$$F(p) = \sum_{i \in \{i | p_i \leq p\}} \frac{(\hat{\hat{x}}_i^+ - \hat{\hat{x}}_i^-)}{n}. \tag{1}$$

We next define the distances z_j between an indifferent consumer's location and the loca-

[4]The case in which a single firm serves several sub-markets is considered in Section B.2 in the Appendix.

[5]Note that the index i for Δp_i^+ is modulo n—i.e., $\Delta p_n^+ = p_1 - p_n$.

tions of her two neighboring firms,

$$\forall j \in \{1, ..., 2n\}: \qquad z_j = \begin{cases} \hat{\hat{x}}_i^+ - (i-1), & \text{if } j = 2i-1; \\ 1 - (\hat{\hat{x}}_i^+ - (i-1)), & \text{if } j = 2i. \end{cases} \tag{2}$$

Note that $\max\{z_{2i-1}, z_{2i}\}$ represents the maximum taste difference consumers located between firm i and $i+1$ are willing to accept for given prices. Also note that $\max_{j\in\{1,...,2n\}}\{z_j\}$ reflects consumers' maximum acceptable taste difference in the entire market and corresponds to the largest price difference between two neighboring firms. Distances z_j can be ordered by rank. Let $z[k]$ describe the kth smallest distance in $\{z_j\}_{j=1}^{2n}$ and $\#(z[k])$ the number of distances of size $z[k]$.[6] $\sigma(x)$ describes consumer x's purchase decision (pure-strategy personal equilibrium), which requires that, for given prices, $\boldsymbol{p}$ consumers correctly anticipate the locations of the indifferent consumers $\{\hat{\hat{x}}_i^+\}_{i=1}^n$. The reference-point distribution in the taste dimension, $G(s)$, is the probability that the equilibrium taste difference between the consumer's ideal taste x and the taste of the purchased product $y_{\sigma(x)}$ is smaller than a real number s—i.e., $G(s) = Prob(|x - y_{\sigma(x)}| \leq s)$. We obtain,

$$G(s) = \begin{cases} 2s, & s \in [0, z[1]]; \\ 2s\frac{2n-\#(z[1])}{2n} + a_1, & s \in (z[1], z[2]]; \\ \vdots & \vdots \\ 2s\frac{2n-\sum_{j=1}^{k}\#(z[j])}{2n} + a_k, & s \in (z[k], z[k+1]]; \\ \vdots & \vdots \\ 2s\frac{2n-\sum_{j=1}^{K-1}\sharp(z[j])}{2n} + a_{K-1}, & s \in (z[K-1], z[K]]; \\ a_K = 1, & s \in (z[K], 1]. \end{cases} \tag{3}$$

with $\{a_k\}_{k=1}^K$ being the required constants for the kinked cdf. If all prices are the same, then consumers expect to buy from their closest firm ex post with probability one. The distribution of the expected taste difference, $G(s)$, is not kinked in this case and approaches the uniform distribution: $K = 1$ and $G(s) = 2s$ for $s \in (0, 1/2]$. If there are two or more different prices p_i in the market, then there are at least two different distances z_j. For small realized taste differences, $s \in [0, z[1]]$, consumers expect to buy from their closest firm ex post and, thus, $G(s) = 2s$. However, for a larger taste difference consumers anticipate that they will be attracted with positive probability to the more distant, cheaper firm ex post. For this to happen, given $s \in (z[1], z[2]]$, the realization of x must be sufficiently close to the more expensive firm in the sub-market with the largest price difference. Let, for

[6] Obviously, if there are no ties between price differences and between distances, then $\#(z[k]) = 1$ for all $k \in \{1, ..., K\}$ and $K = 2n$.

instance, $\Delta p_i^+ = p_{i+1} - p_i$ be the (unique) maximum price difference for given $\boldsymbol{p}$. Then, the indifferent consumer, $\hat{\hat{x}}_i^+$, in this sub-market is more closely located to the expensive firm $i+1$ ($y_{i+1} = i$). Moreover, the distance between firm $i+1$ and the indifferent consumer $\hat{\hat{x}}_i^+$ is the smallest distance in the entire market—i.e., $y_{i+1} - \hat{\hat{x}}_i^+ = i - \hat{\hat{x}}_i^+ = 1 - (\hat{\hat{x}}_i^+ - (i-1)) = z[1]$. Thus, if the realization of x lies in the interval $[y_{i+1} - z[2], \hat{\hat{x}}_i^+]$, the consumer will be attracted by the cheaper firm i. Therefore, the consumer will not buy from her closest firm in equilibrium. This means that for $s \in (z[1], z[2]]$, only $2n-1$ sub-markets are relevant for the probability of facing s and $G(s)$, therefore, equals $2s(2n-1)/2n$. This argument carries over to all $s \in (z[k], z[k+1]]$ with $1 \leq k \leq K \leq 2n$. $G(s)$ shows up to $2n$ kinks if there n distinct price differences in the market.

We next turn to the consumers' utility function. Using the reference-point distribution in both dimensions, we can then solve for consumers' personal equilibria. Consider the indirect utility functions of a consumer who has learned, after forming her reference-point distribution given prices, that her ideal taste x lies in the sub-market between firm i and firm $i+1$. Suppose further that this consumer is the indifferent loss-averse consumer on this sub-market—i.e., $x = \hat{\hat{x}}_i^+ \in [i-1, i]$. The consumer faces a distance of $\hat{\hat{x}}_i^+ - (i-1) = z_{2i-1}$ to firm i and $1 - z_{2i-1}$ to firm $i+1$. Her indirect utility if buying from firm i can be expressed as

$$\begin{aligned} u_i(x = \hat{\hat{x}}_i^+, \boldsymbol{p}) =& v - t z_{2i-1} - p_i \\ &+ \alpha_p\Big(-\lambda \sum_{j \in \{j | p_j \leq p_i\}} \frac{(\hat{\hat{x}}_j^+ - \hat{\hat{x}}_j^-)}{n}(p_i - p_j) + \sum_{j \in \{j | p_j > p_i\}} \frac{(\hat{\hat{x}}_j^+ - \hat{\hat{x}}_j^-)}{n}(p_j - p_i)\Big) \\ &+ \alpha_m\Big(-\lambda t \int_0^{z_{2i-1}} (z_{2i-1} - s) dG(s) + t \int_{z_{2i-1}}^1 (s - z_{2i-1}) dG(s)\Big), \end{aligned}$$

where the first line describes the consumer's intrinsic utility from product i. Parameter v represents the common reservation value for one unit of any product, and t scales the disutility from distance between ideal and actual taste on the circle. In the second line, $\alpha_p \geq 0$ measures the degree of reference dependence in the price dimension.[7] The first term in the second line shows the loss in the price dimension from not facing a lower price than p_i, while the second term in this line shows the gain from not facing a higher price than p_i. The weight on losses is $\lambda > 1$, while the weight on gains is normalized to one. This feature, combined with the reference comparison, implements loss aversion in our setup.[8] In the third line, $\alpha_m \geq 0$ measures the degree of the reference dependence in the

[7] α_p is equal to 1 for standard reference-dependent preferences that are considered in Heidhues and Koszegi (2008) and Karle and Peitz (2010).

[8] For $\lambda \to 1$, consumers face no loss aversion but are still reference-dependent.

match-value dimension, which is equal to 1 in the standard case. The two terms in the third line correspond to the loss (gain) from not facing a smaller (larger) distance in the taste dimension than $\hat{\hat{x}}_i^+ - (i-1) = z_{2i-1}$. If buying from firm $i+1$ instead, the indifferent consumer's indirect utility equals

$$\begin{aligned} u_{i+1}(x = \hat{\hat{x}}_i^+, \boldsymbol{p}) \quad =& v - t(1 - z_{2i-1}) - p_{i+1} \\ &+ \alpha_p\Big(- \lambda \sum_{j \in \{j | p_j \le p_{i+1}\}} \frac{(\hat{\hat{x}}_j^+ - \hat{\hat{x}}_j^-)}{n}(p_{i+1} - p_j) + \sum_{j \in \{j | p_j > p_{i+1}\}} \frac{(\hat{\hat{x}}_j^+ - \hat{\hat{x}}_j^-)}{n}(p_j - p_{i+1})\Big) \\ &+ \alpha_m\Big(- \lambda t \int_0^{(1-z_{2i-1})} ((1 - z_{2i-1}) - s) dG(s) + t \int_{(1-z_{2i-1})}^1 (s - (1 - z_{2i-1})) dG(s)\Big). \end{aligned}$$

By setting $u_i - u_{i+1} = 0$ for all i and solving for $\{\hat{\hat{x}}_i^+\}_{i=1}^n$, we determine the locations of indifferent loss-averse consumers (consumers' personal equilibria) for any given $\boldsymbol{p}$ (provided that a solution exists).

Since the focus of this paper is on symmetric firms and symmetric price equilibria, we can restrict our attention to prices that are the same for all firms but one. The variation in the price of one firm is required to determine the symmetric equilibrium price in stage 1 of the game. Let $p_i \neq p'$ be the price set by firm i and $p_j = p'$, $j \neq i$, the price of any other firm in the market. By symmetry, the location of indifferent consumers in any sub-market with zero price difference lies exactly in the middle between the two firms on this sub-market—i.e., $\hat{\hat{x}}_j^+ - (j-1) = 1/2$. The location of indifferent consumers in the two sub-markets around firm i is further apart from firm i than $1/2$, if firm i has set a lower price than any neighboring firm—i.e., $\hat{\hat{x}}_i^+ - (i-1) = (i-1) - \hat{\hat{x}}_i^- > 1/2$ for $p_i < p'$—and vice versa if firm i has set a higher price than any neighboring firm. In the following lemma, we solve for the location of the indifferent consumer $\hat{\hat{x}}_i^+$ as a function of the price difference $\Delta p = p' - p_i \geq 0$, conditional on the number of firms n in the market and the weights α_p and α_m with respect to the two dimensions of loss aversion.

Lemma 1. *Suppose that $\hat{\hat{x}}_i^+ \in [(i-1)+1/2, i]$, $p_i \leq p'$, and $p_j = p'$ for all $j \neq i$. Moreover, $\lambda > 1$ and $\alpha_m > 0$. Then $\hat{\hat{x}}_i^+$, as a function of the price difference $\Delta p = p' - p_i \in [0, \Delta\bar{p}]$, is*

$$\hat{\hat{x}}_i^+(\Delta p) = (i-1) + \left(\frac{2(\alpha_m + 1)}{\alpha_m(\lambda - 1)(n+2)} + \frac{3n+2}{n(n+2)}\right) - \frac{2\alpha_p \cdot \Delta p}{\alpha_m n(n+2)t} - 2S(\Delta p), \qquad (4)$$

where

$$S(\Delta p) = \sqrt{\frac{\alpha_p^2(\lambda-1)^2 \cdot \Delta p^2 - (\lambda - 1)\Lambda t \cdot \Delta p + (1 + \alpha_m \lambda)^2 n^2 t^2}{(\alpha_m(\lambda-1)n(n+2)t)^2}}$$

with $\Lambda = \left(2\alpha_p n + \alpha_m \left(n(n+2) + \alpha_p \left(2(\lambda - 1) + (3\lambda + 1)n + n^2\right)\right)\right)$ *and* $\Delta\bar{p}$ *being the upper bound of* Δp *for which the square root* $S(\Delta p)$ *is defined.*[9]

The proof of Lemma 1 is relegated to Appendix A.1. In the proof, we make use of the fact that there exist only two indifferent consumers whose locations are different from 1/2, the indifferent consumers to the right and the left of firm i. Since their locations are symmetric, it suffices to solve a system of one (quadratic) equation and one unknown—i.e., to solve $u_i - u_{i+1} = 0$ for $\hat{\hat{x}}_i^+$. For $\lambda \to 1$ or $\alpha_m \to 0$, $u_i - u_{i+1} = 0$ collapses to a linear equation and $\hat{\hat{x}}_i^+(\Delta p)$ shows a much simpler form.

From the general form of $\hat{\hat{x}}_i^+(\Delta p)$ in Lemma 1, we can easily derive the demand from loss-averse consumers of firm i, $\hat{\hat{x}}_i(\Delta p)$: Using the uniform distribution of x and symmetry we obtain

$$\hat{\hat{x}}_i(\Delta p) = \frac{\hat{\hat{x}}_i^+(\Delta p) - \hat{\hat{x}}_i^-(\Delta p)}{n} = \frac{2}{n}\left(\hat{\hat{x}}_i^+(\Delta p) - (i-1)\right) = \frac{2}{n} z_{2i-1}. \tag{5}$$

In the next section, we consider duopoly markets varying the weights on the price and taste dimensions of loss aversion. In Section 4, we set both weights equal to one and analyze the n-firm oligopoly.

3 Duopoly

In this section, we characterize equilibrium candidates rearranging first-order conditions. We provide conditions under which an interior equilibrium in a symmetric duopoly exists and under which it is unique. We start by establishing some properties of market demand that will be needed below. Initially, we focus on the case $\alpha_p = \alpha_m = 1$.

3.1 Properties of market demand

We first consider non-biased consumers who do not have reference-dependent utilities. Such a situation will represent our benchmark. For prices p_i and p_{-i}, a non-biased consumer located at x obtains the indirect utility $u_i(x, p_i) = v - t|y_i - x| - p_i$ from buying

[9] For $x \in [i-1, i]$, consumer x's personal equilibrium (determining her product choice) is described by

$$\sigma(x, \Delta p) = \begin{cases} i & \text{if } x \in [y_i, \hat{\hat{x}}_i^+(\Delta p)], \\ i+1 & \text{if } x \in (\hat{\hat{x}}_i^+(\Delta p), y_{i+1}]. \end{cases}$$

product i. The expression $v - t|y_i - x|$ captures the match value of product i for consumer of type x. Denote the indifferent (non-biased) consumer between buying from firm i and $-i$ on the first half of the circle by $\hat{x}_i \in [0, 1]$ and solve for her location given prices. The indifferent non-biased consumer is given by

$$\hat{x}_i(p_i, p_{-i}) = \frac{(t + p_{-i} - p_i)}{2t}. \tag{6}$$

Symmetrically, a second indifferent (non-biased) consumer type is located at $2 - \hat{x}_i(p_i, p_{-i}) \in [1, 2]$. Without loss of generality we focus on demand of consumers between 0 and 1 and multiply by 2.

We next turn to loss-averse consumers. In duopoly with equal weights of one on both dimensions of loss aversion, the location of the indifferent consumer of firm i is equal to

$$\hat{\hat{x}}_i^+(\Delta p) \qquad = (i-1) + \frac{\lambda}{(\lambda - 1)} - \frac{\Delta p}{4t} - \underbrace{\sqrt{\frac{\Delta p^2}{16t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)}\Delta p + \frac{(\lambda + 1)^2}{4(\lambda - 1)^2}}}_{\equiv S(\Delta p)}. \tag{7}$$

This expression is valid for $\Delta p = p_{-i} - p_i$ sufficiently small. The square root, $S(\Delta p)$ in (7), is defined for $\Delta p \in [0, \Delta\bar{p}]$ with

$$\Delta\bar{p} \equiv \frac{2t}{(\lambda - 1)}\left(2(\lambda + 2) - \sqrt{(2(\lambda + 2))^2 - (\lambda + 1)^2}\right), \tag{8}$$

which is strictly positive for all $\lambda > 1$. It can be shown that, for $\lambda \geq 3 + 2\sqrt{5} \approx 7.47$, the indifferent consumer satisfies $\hat{\hat{x}}_i^+(\Delta p) \in [1/2, 1]$ for all $\Delta p \in [0, \Delta\bar{p}]$. If the degree of loss aversion is smaller, $\lambda < 3 + 2\sqrt{5}$, $\hat{\hat{x}}_i^+(\Delta\bar{p})$ rises above one. Therefore, we have to define another upper bound on the price difference, $\Delta\tilde{p}$, with $\Delta\tilde{p} < \Delta\bar{p}$ by the solution to $\hat{\hat{x}}_i^+(\Delta\tilde{p}) = 1$. We can solve explicitly,

$$\Delta\tilde{p} = \frac{(\lambda + 3)t}{2(\lambda + 1)}. \tag{9}$$

The upper bound for the price difference (which depends on the parameters t and λ), for which $\hat{\hat{x}}_i^+$ is defined as in equation (7), is given by:

$$\Delta p^{max} \equiv \begin{cases} \Delta\tilde{p}, & \text{if } 1 < \lambda \leq \tilde{\lambda}; \\ \Delta\bar{p}, & \text{if } \lambda > \tilde{\lambda}. \end{cases} \tag{10}$$

with $\tilde{\lambda} \equiv 3 + 2\sqrt{5} \approx 7.47$.[10]

Since x is uniformly distributed on a circle of length $L = 2$, the demand of firm i from loss-averse consumers, $\hat{\hat{x}}_i$, is equal to $(\hat{\hat{x}}_i^+ - \hat{\hat{x}}_i^-)/2 = 2(\hat{\hat{x}}_i^+ - (i-1))/2 = \hat{\hat{x}}_i^+ - (i-1)$. It can be shown that the derivative of $\hat{\hat{x}}_i(\Delta p)$ with respect to Δp, $\hat{\hat{x}}_i'(\Delta p)$, is strictly positive for all $\Delta p \in [0, \Delta p^{max}]$:

$$\hat{\hat{x}}_i'(\Delta p) = -\frac{1}{4t} - \frac{1}{2 \cdot S(\Delta p)} \cdot \left(\frac{\Delta p}{8t^2} - \frac{(\lambda+2)}{2t(\lambda-1)}\right).$$

Evaluated at $\Delta p = 0$, this becomes

$$\hat{\hat{x}}_i'(0) = -\frac{1}{4t} + \frac{(\lambda+2)}{2t(\lambda+1)}.$$

$\hat{\hat{x}}_i'(0)$ is approaching $1/(2t)$ from below for $\lambda \to 1$ and $1/(4t)$ from above for $\lambda \to \infty$. This implies that, evaluated at $\Delta p = 0$, demand of loss-averse consumers reacts less sensitive to price changes than demand of non-biased consumers—we return to this property in the following subsection. Moreover, $\hat{\hat{x}}_i(\Delta p)$ is strictly convex for all $\Delta p \in [0, \Delta p^{max}]$, as illustrated in Figure 1 below.

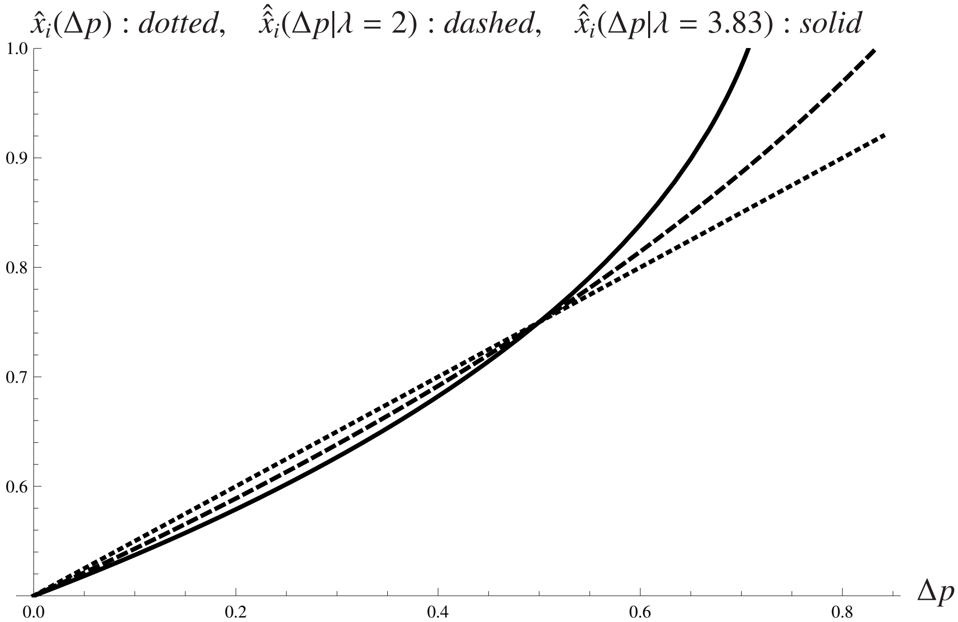
$$\hat{\hat{x}}_i''(\Delta p) = \frac{(3+\lambda)(5+3\lambda)}{64t^2 \cdot (S(\Delta p))^3} > 0.$$

We note that the degree of convexity of $\hat{\hat{x}}_i(\Delta p)$ is strictly increasing in λ.

We also note a continuity property. For $\lambda \to 1$, the indirect utility function of loss-averse consumers differs from the one of non-biased consumers only by a constant.[11] The equation $u_i - u_{-i} = 0$ collapses to a linear equation, and we obtain $\hat{\hat{x}}_i(\Delta p) = \hat{x}_i(\Delta p)$ as a solution in this case. This means that if consumers put equal weights on gains and losses, the effect of comparing expectations with realized values exactly cancels out when a choice between two products is made.

[10] Note that $\Delta\tilde{p} \in \left[t \cdot (\sqrt{5}-1)/2, t\right) \approx \left[0.618t, t\right)$ for $1 < \lambda \leq \tilde{\lambda}$ and $\Delta\tilde{p} \in \left(t \cdot 2(\sqrt{3}-2), t \cdot (\sqrt{5}-1)/2\right) \approx \left(0.536t, 0.618t\right)$ for $\lambda > \tilde{\lambda}$.

[11] This continuity property holds in the present specification where the gain-loss utility in the price and in the match-value dimension enter with equal weights. This does not hold more generally, see the next subsection.



Demand of non-biased and loss-averse consumer as a function of Δp for parameter values of $t = 1$, $\alpha_m = \alpha_p = 1$; $\Delta p^{max}(\lambda = 2) = 0.8333$ and $\Delta p^{max}(\lambda = 3.83) = 0.7070$.

Figure 1: Demand of non-biased and loss-averse consumers

We define the upper bound of firm i's demand of loss-averse consumers as[12]

$$\hat{\hat{x}}_i(\Delta p^{max}) \equiv \begin{cases} \hat{\hat{x}}_i(\Delta\bar{p}) = 1, & \text{if } 1 < \lambda \leq \tilde{\lambda}, \\ \hat{\hat{x}}_i(\Delta\bar{p}) < 1, & \text{if } \lambda > \tilde{\lambda}. \end{cases} \tag{11}$$

Combining (6) and (7), we obtain the market demand of firm i as the weighted sum of the demand by non-biased and loss-averse consumers,

$$q_i(\Delta p; \beta) \qquad = \beta \cdot \hat{x}_i(\Delta p) + (1-\beta) \cdot \begin{cases} \hat{\hat{x}}_i(\Delta p), & \text{if } 0 \leq \Delta p < \Delta p^{max} \\ 1, & \text{if } t \geq \Delta p \geq \Delta p^{max} \end{cases} \tag{12}$$

The demand of firm i is a function in the price difference Δp, which is kinked at Δp^{max}. Furthermore, for $\Delta p^{max} = \Delta\bar{p}$, it is discontinuous at Δp^{max}. It approaches one for $\Delta p = t$.[13] Firm $-i$'s demand is determined analogously by $q_{-i}(\Delta p; \beta) = 1 - q_i(\Delta p; \beta)$. In the following, we are interested in interior equilibria in which both products are purchased

[12] $\hat{\hat{x}}_i(\Delta\bar{p}) = \frac{\lambda}{\lambda-1} - \frac{2(\lambda+2)-\sqrt{4(\lambda+2)^2-(\lambda+1)^2}}{2(\lambda-1)} \in (\sqrt{3}/2, 1)$ for $\lambda > \tilde{\lambda}$—i.e., $\hat{\hat{x}}_i(\Delta\bar{p})$ is less than one for $\lambda > \tilde{\lambda}$. This leads to a jump in demand of loss-averse consumers at $\Delta\bar{p}$ from $\hat{\hat{x}}_i(\Delta\bar{p})$ to one (see the definition of $q_i(\Delta p; \beta)$), since $\hat{x}'_{un}(\Delta p) \to \infty$ for $\Delta p \to \Delta\bar{p}$.

[13] At $\Delta p = t$, firm i serves also all distant non-biased consumers which are harder to attract than distant loss-averse consumers because the former do not face a loss in the price dimension if buying from the more expensive firm $-i$. For $\Delta p > t$ demand of firm i shows a second kink. We ignore this region since we are interested in cases in which both firms face strictly positive demand.

by a strictly positive share of loss-averse consumers—i.e., Δp is lower than Δp^{max}. This holds when firms' prices are not too asymmetric.

We note some properties of the demand of firm i that carry over from $\hat{\hat{x}}_i^+(\Delta p)$:[14] For $0 < \Delta p < \Delta p^{max}$ (i.e., for any price p_i that is lower than its competitor's price), the demand of firm i, $q_i(\Delta p; \beta)$ is strictly increasing and convex in Δp. It is concave for a price above the competitor's price (for $0 > \Delta p$). In the remainder, we often refer to q_i as a short-hand notation for $q_i(\Delta p; \beta)$. The derivative $\partial q_i / \partial \Delta p$ is denoted by q_i'.

3.2 Equilibrium characterization, existence, and uniqueness

We next turn to the equilibrium characterization. At the first stage, firms foresee consumers' purchase decisions and set prices simultaneously to maximize profits. This yields first-order conditions

$$\frac{\partial \pi_i}{\partial p_i} = q_i + (p_i - c)(-q_i') = 0 \quad , i = 1, 2 \tag{FOC_i}$$

We refer to a solution characterized by these first-order conditions as an interior solution. Our framework allows us to explicitly solve for equilibrium markup in symmetric duopoly, in contrast to Heidhues and Koszegi (2008). The following result characterizes the symmetric equilibrium.

Lemma 2. *Any equilibrium is unique and symmetric. Equilibrium prices are given by*

$$p_i^* = c + \frac{t}{1 - \frac{(1-\beta)}{2}\frac{(\lambda-1)}{(\lambda+1)}}, i = 1, 2. \tag{13}$$

Proof. Rearranging the first-order conditions (FOC_i) and using that $q_i(0; \beta) = 1/2$ for all β, we obtain

$$p_i^* - c = \frac{\frac{1}{2}}{q_i'(0; \beta)}, i = 1, 2., \tag{14}$$

where

$$q_i'(0; \beta) = -\frac{1}{4t}(1 - 3\beta) - \frac{(1-\beta)}{2(S(0))}\left(0 - \frac{(\lambda + 2)}{2t(\lambda - 1)}\right)$$

[14]See Appendix A for more details.

$$= -\frac{1}{4t}(1-3\beta) + \frac{(1-\beta)}{2\frac{\lambda+1}{2(\lambda-1)}}\left(\frac{(\lambda+2)}{2t(\lambda-1)}\right)$$

$$= -\frac{1}{4t}(1-3\beta) + \frac{(1-\beta)(\lambda+2)}{2t(\lambda+1)}$$

$$= \frac{1}{4t(\lambda+1)}\Big(2(\lambda+1) - (1-\beta)(\lambda-1)\Big).$$

Substituting into equation (14) yields the unique symmetric equilibrium price in (13). □

For any interior solution, concavity of the profit functions would assure that the solution characterizes an equilibrium.

$$\frac{\partial^2 \pi_A}{\partial p_i^2} = -2q_i' + (p_i - c)q_i'' < 0 \qquad (SOC_A)$$

$$\frac{\partial^2 \pi_B}{\partial p_{-i}^2} = -2q_i' - (p_{-i} - c)q_i'' < 0. \qquad (SOC_B)$$

Given the properties of q_i —particularly that q_i is strictly increasing and convex in Δp for $\beta < 1$— SOC_B holds globally, while SOC_A is not necessarily satisfied. Using that $(p_i - c) = q_i/q_i'$ by FOC_A, SOC_A can be expressed as follows

$$-2(q_i')^2 + q_i q_i'' < 0. \tag{15}$$

It can be easily shown that (17) is satisfied for small Δp while it is violated for $\Delta p \to \Delta\bar{p}$, as q_i'' goes faster to infinity in Δp than $(q_i')^2$.[15] This violation of SOC_A reflects that firm i may have an incentive to non-locally undercut prices to gain the entire demand of loss-averse consumers when Δp is large. The driving force behind this is that loss aversion in the price dimension increasingly dominates loss aversion in the taste dimension if price differences become large. Moreover, excessive losses in the price dimension if buying the expensive product $-i$ make also nearby consumers of $-i$ more willing to opt for product i.

The proof of equilibrium existence in duopoly is non-standard since the profit function is not quasi-concave. If firm i sets a much lower price than firm $-i$ ($\Delta p = p_{-i} - p_i > 0$ and Δp large), firm i's profit becomes increasingly convex due to the increasing convexity of

[15] This implies that π_A is not globally concave. It is easy to check that it is neither globally quasi-concave. Moreover, the non-concavity of π_A is increasing in Δp (resp. $-p_i$) for $\Delta p \le \Delta p^{max}$ (resp. $p_i \ge p_{-i} - \Delta p^{max}$).

its demand with loss-averse consumers.

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2q_i' + (p_i - c)q_i'', \tag{16}$$

where $q_i'' = \partial^2 q_i(\Delta p)/\partial \Delta p^2$ which is positive for $\Delta p > 0$ but negative for $\Delta p \leq 0$ due to symmetry (since $q_i(-|\Delta p|) = 1 - q_i(|\Delta p|)$). Using that $(p_i - c) = q_i/q_i'$ by FOC_i, the second-order condition of firm i can be expressed as

$$-2(q_i')^2 + q_i q_i'' < 0. \tag{17}$$

For $\beta < 1$, equation (17) is satisfied for Δp sufficiently small, while it is violated for $\Delta p \rightarrow \Delta \bar{p}$, as q_i'' goes faster to infinity in Δp than $(q_i')^2$.

The next proposition clarifies the issue of equilibrium existence. It deals with the non-concavity of firm i's profit function by determining critical levels for the degree of loss aversion such that no firm i has an incentive to non-locally undercut prices. We use that the convexity of firm i's profit function is increasing in Δp which yields that stealing the entire demand of loss-averse consumers is the uniquely optimal deviation of firm i. We focus on the most critical case for equilibrium existence, the case in which all consumers are loss-averse.[16]

Proposition 1. *Suppose that all consumers are loss averse ($\beta = 0$) and there are two firms in the market. A symmetric equilibrium with prices p_i^* for all $i \in \{1, 2\}$ exists if and only if*

$$1 < \lambda \leq \lambda^c \text{ with } \lambda^c = 1 + 2\sqrt{2} \approx 3.828. \tag{18}$$

The following proposition extends the existence condition analyzing the relationship between the degree of loss aversion and the share of non-biased consumers on the market. A critical level of β for symmetric equilibria to exist, $\beta^{crit}(\lambda)$, is derived as a function of λ.

Proposition 2. *Suppose that there are two firms in the market. A symmetric equilibrium with prices p_i^* for all $i \in \{1, 2\}$ exists if and only if*

$$\beta \geq \beta^{crit}(\lambda), \tag{19}$$

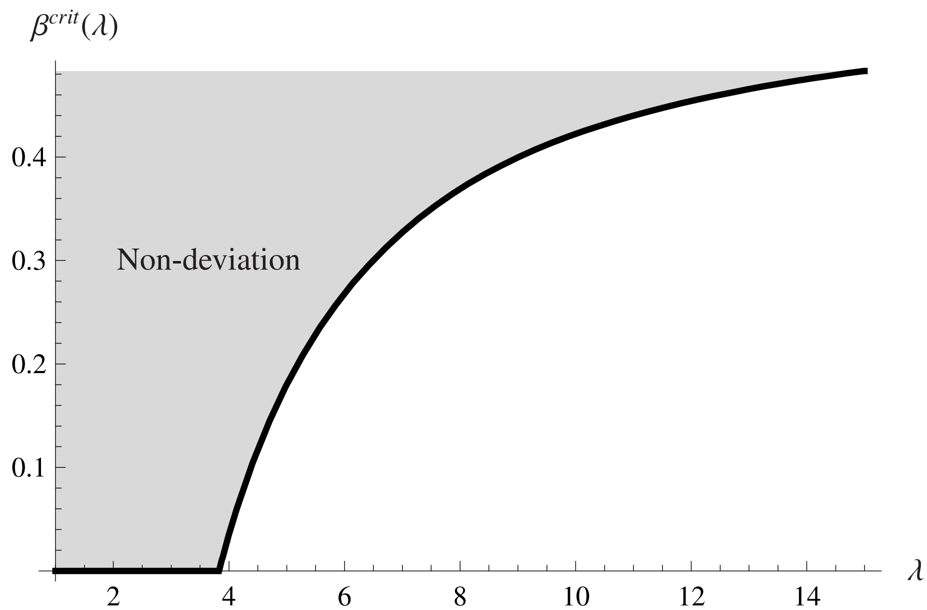
[16] Adding more non-biased consumers always reduces the non-concavity of firm i's profit function since the demand of non-biased consumers is linear. Thus, the upper bound on the degree of loss aversion with only loss-averse consumers is sufficient for existence with a positive share of non-biased consumers. Cf. Proposition 2.

with $\beta^{crit}(\lambda)$ being an increasing function in λ which is expressed by

$$\beta^{crit}(\lambda) \equiv \begin{cases} 0, & \text{if } \lambda \in (1, 1+2\sqrt{2}], \\ \beta_0^{crit}(\lambda), & \text{if } \lambda \in (1+2\sqrt{2}, \tilde{\lambda}], \\ \beta_1^{crit}(\lambda), & \text{if } \lambda > \tilde{\lambda} \approx 7.47, \end{cases} \tag{20}$$

where β_0^{crit} and β_1^{crit} are defined in the proof in Appendix A.2

We note that $\beta_0^{crit}(\lambda) \in (0, 0.349]$ and $\beta_1^{crit}(\lambda) \in (0.349, 0.577)$ in the associated ranges for λ. The proofs of the last two propositions are relegated to Appendix A.2. The critical share of non-biased consumers for symmetric equilibria to exist, $\beta^{crit}(\lambda)$, is depicted in Figure 2. If the share of informed consumers is sufficiently large (above 57.7%) symmetric equilibria exist for all $\lambda > 1$. The existence proof covering different weights α_p and α_m is provided in Appendix B.1.



Critical share of non-biased consumers, $\beta^{crit}(\lambda)$, for which symmetric equilibria exist as a function of the degree of loss aversion $\lambda > 1$, $t = 1$. Non-deviation for $\beta \geq \beta^{crit}(\lambda)$.

Figure 2: Non-deviation in symmetric duopoly

In the remainder of this section, we disentangle pro- and anti-competitive effects of the presence of loss-averse consumers. We also relate our findings on the competitive effects of consumer loss aversion to a setting in which consumers form reference points before the firms have set their prices, as is also the case in Heidhues and Koszegi (2008).

3.3 Comparative statics in the basic duopoly model

We define the equilibrium markup as $m^* \equiv p^* - c$. Using Lemma 2, we obtain comparative static results. In particular, as the share of non-biased consumers increases, the firms' markup decreases. This result follows directly from differentiating (13) with respect to β.

Proposition 3. *For $\lambda > 1$, equilibrium markup is decreasing in the share of non-biased consumers β.*

In other words, non-biased consumers exert a positive externality on loss-averse consumers. With respect to recent work with behavioral biases, our result is of interest in the light of claims that non-biased consumers are cross-subsidized at the cost of biased consumers. This, for instance, holds in Gabaix and Laibson (2006) where only a fraction of consumers are knowledgeable about their future demand of an "add-on service", while other consumers are "naively" unaware of this.

Two additional comparative static results follow immediately from Lemma 2. First, equilibrium markup is increasing in the degree of loss aversion, λ. For $\lambda \to 1$, firms receive the standard Hotelling markup of t. Second, equilibrium markup is increasing in the inverse measure of industry competitiveness, t. For $t \to 0$, firms engage in pure Bertrand competition and markups converge to zero for all levels of loss aversion. This shows that consumer loss aversion does not affect market outcomes in perfectly competitive environments, and our results rely on the interaction of imperfect competition and behavioral bias.

Table 1: Symmetric Equilibrium: Markups

The table shows the variation of $m_i^*(\beta, \lambda) \equiv p_i^*(\beta, \lambda) - c$ for all $i \in \{1, 2\}$ in β and λ.

β \ λ	1	2	3	3.8284	5	7	9	∞
1	1	1	1	1	1	1	1	1
0.8	1	1.03448	1.05263	1.06222	1.07143	1.08108	1.08696	1.11111
0.6	1	1.07143	1.11111	1.1327	1.15385	1.17647	1.19048	1.25
0.4	1	1.11111	1.17647	1.2132	1.25	1.29032	1.31579	-
0.2	1	1.15385	1.25	1.30602	1.36364	-	-	-
0	1	1.2	1.33333	1.41421	-	-	-	-

Table 1 shows the variation of equilibrium markups in the share of informed consumers β and the degree of loss aversion λ. We make the following observation: The highest markup is reached when all consumers are loss-averse and the degree of loss-aversion approaches its critical level for existence in symmetric markets $\lambda = 1 + 2\sqrt{2} \approx 3.82843$—compare Figure 2.

3.4 Pro- and anti-competitive effects of consumer loss aversion: the general duopoly model

In symmetric equilibrium, consumers do not experience gains and losses in the price dimension. In this situation, loss-averse consumers exclusively experience gains and losses in the taste dimension. Due to the potential loss in the taste dimension, loss-averse consumers are more willing to buy next door than non-biased consumers—i.e., they are harder to attract by low prices than their non-biased counterparts and, thus, the demand of loss-averse consumers shows a lower price elasticity of demand. While this suggests consumer loss aversion has an anti-competitive effect, a correct understanding is more nuanced.

To this end, we have to disentangle various effects at play; we consider it useful to analyze the general model in which consumers experience gain-loss utilities in the two dimensions, the price and the match-value dimension, with different weights. We do not see any a priori reasons to exclude the possibility of different weights in the two dimensions.[17]

In this case firm i's demand in duopoly is given by

$$\hat{\hat{x}}_i(\Delta p) = \frac{\alpha_m(2\lambda - 1) + 1}{2\alpha_m(\lambda - 1)} - \frac{\alpha_p}{4\alpha_m t}\Delta p - S(\Delta p), \tag{21}$$

where

$$S(\Delta p) = \sqrt{\frac{\alpha_p^2(\lambda - 1)^2\Delta p^2 - 4(\lambda - 1)(\alpha_p + \alpha_m(\alpha_p(2\lambda + 1) + 2))t\Delta p + 4(\alpha_m\lambda + 1)^2 t^2}{16\alpha_m^2(\lambda - 1)^2 t^2}} \tag{22}$$

for $\lambda > 1$ and $\alpha_m > 0$ and $\Delta p \geq 0$ and not too large. In Figure 6 in Appendix C, we illustrate the demand of loss-averse consumers (with different weights on the two dimensions of reference dependence).

[17] A real world motivation for different weights could be an extra utility for making a better than expected pecuniary deal (independent from any budget considerations). This would be represented in our setup by a relatively higher weight on the price dimension of loss aversion. In a different vein, for household decisions the preferences of the spouse not directly involved in a purchase decision could be more comparison-based concerning prices and enter the buyer's utility function as a positive weight in the price dimension.

The symmetric equilibrium prices in this general case can be derived analogously to Lemma 2. Considering only loss-averse consumers ($\beta = 0$) we obtain

$$p_i^* = c + \frac{t}{1 - \frac{1}{2}\frac{(2\alpha_m\lambda - \alpha_p(\lambda+1))}{(\alpha_m\lambda+1)}}, i = 1, 2, \tag{23}$$

provided an equilibrium exists. In Proposition 6 in Appendix B.1 we show that conditions for equilibrium existence carry over from the case with α-weights being equal to one.

As one polar case we consider markets in which all consumers are uniformed ex ante and experience loss-aversion in the price dimension only. As the other polar case we consider markets in which all consumers are uninformed ex ante and experience loss-aversion in the match-value dimension only.

First, consider the case that consumers experience a gain-loss utility in the price dimension only. Since gains relative to the expected price distribution enter positively and losses negatively the utility function, consumers find lower-priced products relatively more attractive than higher-priced products. Consequently, the price elasticity of demand is larger and the equilibrium is *more* competitive than in the standard Hotelling-Salop model. Formally, the unique symmetric equilibrium (if it exists) is characterized by $p_p^* = c + 2t/(3 + \lambda)$. This proves that a gain-loss utility in the price dimension has a pro-competitive effect. To obtain a better understanding, we take a closer look at loss-averse consumers. Consider a small price decrease by firm i to p_i. Consumers observe the corresponding prices. They expect with some probability $1 - \hat{\hat{x}}_p$ to end up buying the high-price firm. Hence, they have an expected gain of $(1 - \hat{\hat{x}}_p)(p_j - p_i)$ when consuming product i and an expected loss of $\lambda\hat{\hat{x}}_p(p_j - p_i)$ when consuming product j, $j \neq i$. This means that a price decrease yields a stronger utility difference in favor of the low-price product. This increases the price elasticity of demand and, everything else given, makes a price cut more attractive. The presence of loss-averse consumers leads to a downward shift of best-response functions. Consequently (for best-response functions being upward sloping), the equilibrium is more competitive than in the standard Hotelling-Salop model.

In the special case that $\lambda = 1$, we can calculate the indifferent consumer as $\hat{\hat{x}}_p = \frac{1}{2} + \frac{2}{2t}(p_p^* - p_i)$. Hence, $\partial\hat{\hat{x}}_p/\partial p_i = -1/t < -1/(2t)$, where the latter term is the value of the derivative for informed consumers. More generally, for $\lambda > 1$ we can calculate the indifferent consumer holding beliefs about the likelihood of buying product i at x^e fixed. Considering a lower price for product 1, we have that

$$v - tx - p_1 + (1 - x^e)(p_2 - p_1) = v - t(1 - x) - p_2 - \lambda x^e(p_2 - p_1) \tag{24}$$

Hence, for given beliefs x^e, the indifferent consumer x^i is

$$x^i = \frac{1}{2} + \frac{1}{2t}(p_2 - p_1) + \frac{1 + (\lambda - 1)x^e}{2t}. \tag{25}$$

We observe that on top of the effect for $\lambda = 1$, under consumer loss aversion there is an additional positive club effect: The more consumers are expected to buy from firm 1, the better it is for a consumer to buy product 1 instead of 2. Under self-fulfilling expectations $x^e = x^i = \hat{\hat{x}}_p$ we thus see that, due to a positive club effect, the pro-competitive effect in the price dimension becomes larger as the degree of loss aversion λ increases.

Second, consider the case that consumers experience a gain-loss utility in the match-value dimension only. Comparing a market with loss-averse to a market with non-biased consumers reveals that competition is less intense if consumers are loss averse. Straightforward computations show that $p^*_m = c + t(1 + \lambda)$ which leads to a less competitive outcome than in the standard Hotelling-Salop model. A price decrease for firm i implies that consumers are more likely to buy from firm i than firm $-i$. This implies that the marginal consumer more often encounters a worse match from firm i. Since relatively bad matches enter negatively the gain-loss utility, the price elasticity of demand is lower and best-response functions are shifted upward. Effectively, competition is less intense compared to the market populated by non-biased consumers.

The following remark summarizes the insights obtained above.

Remark 1. *If consumers experience a gain-loss utility in the price dimension only, markets with loss-averse consumer are more competitive than markets with non-biased consumers. By contrast, if consumers experience a gain-loss utility in the match-value dimension only, markets with loss-averse consumers are less competitive than markets with non-biased consumers.*

This insight holds more generally; in particular, it does not rely on the assumption that taste parameters are uniformly distributed and that utility depends linearly on match value, defined as the distance between consumer and product. These assumptions are mainly made for computational reasons.

We note that this result does not rely on losses entering the utility function with a different weight than gains; what matters is that the consumers' utility function is reference-dependent. In other words, the result remains to hold true if $\lambda = 1$.

Let us now consider intermediate cases between the two polar cases. In our baseline model, both dimensions entered with equal weights, $\alpha_p = \alpha_m$. For this case we obtain

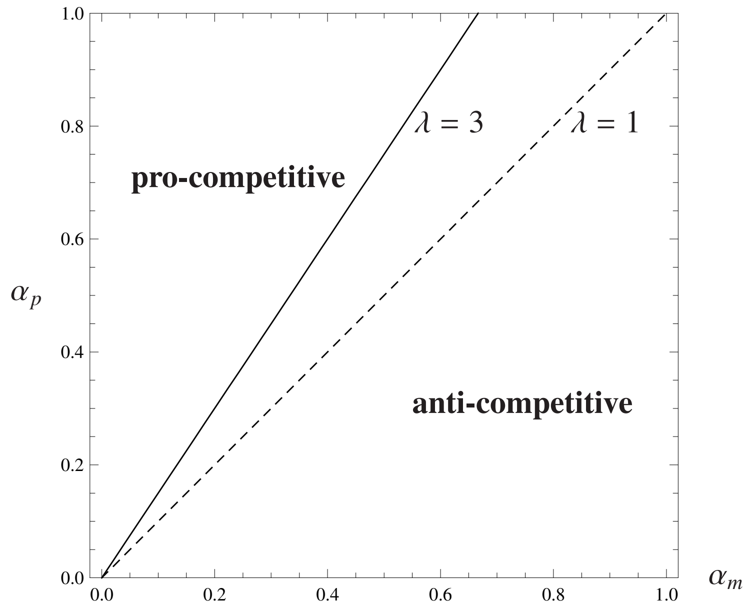


Figure 3: Pro- and anti-competitive effects of loss aversion

that the taste dimension dominates the price dimension (as follows from equation (13)) if consumers are loss-averse.

Remark 2. *Suppose that losses have a larger weight than gains and that the gain-loss utility enter with the same weights in the price and the match-value dimension. Markets with loss-averse consumers are less competitive than markets with non-biased consumers.*

Depending on the degree of loss aversion λ, there is a critical relationship of gains and losses in the price dimension relative to the match-value dimension such that pro-competitive and anti-competitive effects cancel out each other. This critical relationship is given by

$$\tilde{\alpha}_p(\alpha_m; \lambda) = \frac{2\lambda}{\lambda + 1} \cdot \alpha_m, \tag{26}$$

which turns out to be simply a ratio of weights on the price and match-value dimension α_p and α_m for given λ.[18] This ratio is depicted in Figure 3 for $\lambda = 1$ and $\lambda = 3$. It shows the competitiveness of price equilibria (relative to the benchmark with non-biased consumers) for different weights in the two dimensions of loss aversion. It can be seen that for any positive degree of loss aversion ($\lambda > 1$), markets are anti-competitive if weights are identical on the price and match-value dimension. If the degree of loss aversion is increased, a relatively higher weight on the price dimension is required to balance the

[18]This critical ratio can be derived by setting the symmetric equilibrium price with gain/loss utility and flexible weights equal to the one with intrinsic utility only. In the latter case the symmetric equilibrium price is given by $p_i^* = c + t$, the standard Hotelling result. In the former case it is represented by (23).

anti- and the pro-competitive effect. The figure reveals that even when gains and losses are weighted equally ($\lambda = 1$; so that the utility function features reference dependence but not loss aversion), markets become anti- (resp. pro-) competitive if reference-dependent consumers, for a certain product category, put a relatively higher (resp. lower) weight on the match-value dimension than on the price dimension.

3.5 Comparison to a model in which price information is not available ex ante

In this subsection, we discuss the outcome of the modified model in which consumers do not observe prices before forming their reference point—i.e., firms set prices after consumers form their stochastic reference point. This model is the limit case of Heidhues and Koszegi (2008), when the cost uncertainty has vanished. Since consumers do not observe prices when forming their reference point, deviations from the equilibrium do not affect the consumers' reference-point distribution. To simplify the analysis, we set the share of non-biased consumers equal to zero, $\beta = 0$.

Consider the model in which consumers do not observe price at the time they form their reference-point distribution. If consumers are loss-averse only in the price dimension, there is a continuum of equilibria: any price in the interval $[c + t/(\lambda + 1),\ c + t/2]$ for all $\lambda > 1$. The unique equilibrium price in the setting in which prices are observed ex ante lies within this interval.[19] We note that a market with reference-dependent consumers features a more competitive price under both informational assumptions than a market with non-biased consumers. Also note that, for $\lambda = 1$ the equilibrium under both informational assumption is the same and $p^* = c + t/2$.

If consumers are loss-averse only in the match-value dimension, there is a unique equilibrium $p^* = c + t(\lambda + 1)$.[20] This price is the same that prevails if consumers learn prices before the reference point is formed and thus the timing of the price setting is immaterial to the outcome. The reason is that a local price deviation has only a second-order effect that is induced by consumer loss aversion; the price elasticity of demand remains locally

[19] To derive this result the reference-point distribution in the price dimension has to be adjusted to the simpler form of $F(p^*) = 1$ and zero for $p < p^*$. This leads to a kinked demand curve and a continuum of equilibria.

[20] This price can be derived by adjusting the reference-point distribution in the taste dimension to $G(s) = 2s$, i.e. all consumers expect to buy from their closest firm ex post. This yields a smooth demand function and a single equilibrium price.

unaffected so that we obtain the same solution to the system of first-order conditions of profit maximization.

If consumers are loss-averse in both dimensions, any price in the interval $[c + t/(\lambda + 1), c + (t/2)(\lambda + 1)]$ constitutes an equilibrium. The unique equilibrium price in the setting in which prices are observed ex ante lies within this interval.

More generally, whenever there is a positive weight on the gain-loss utility in the price dimension ($\alpha_p > 0$), there is a continuum of prices that can be supported in symmetric equilibrium. We summarize our observations in the following remark.

Remark 3. *If consumers form reference points before observing price, there is a continuum of equilibria, whenever the weight on the gain-loss utility in the price dimension is strictly positive. The equilibrium price set contains the unique equilibrium price that prevails if consumers observe price ex ante.*

4 n-Firm Oligopoly and Comparative Statics in the Number of Firms

In this section, we analyze an n-firm oligopoly for the case of two-dimensional consumer loss aversion. Suppose that the length of the circle is $L = n$ (while the consumer mass is equal to 1); this implies that the equilibrium markup in the model with non-biased consumers (as in Salop (1979)) are independent of the number of firms. Here, we restrict attention to the case $\alpha_p = \alpha_m = 1$.

4.1 Market demand in oligopoly

Firm i's demand can be expressed by [21]

$$\hat{\hat{x}}_i(\Delta p) = \left(\frac{4}{(\lambda-1)(n+2)} + \frac{3n+2}{n(n+2)}\right) - \frac{2\Delta p}{n(n+2)t} - 2S(\Delta p), \tag{27}$$

where

$$S(\Delta p) = \sqrt{\frac{\Delta p^2(\lambda-1)^2 - (\lambda-1)(\lambda(3n+2) + n(2n+5) - 2)t\Delta p + (1+\lambda)^2 n^2 t^2}{(\lambda-1)^2 \left(2n+n^2\right)^2 t^2}} \tag{28}$$

[21] Cf. the general representation of the indifferent loss-averse consumer in (4).

for $\lambda > 1$ and $\Delta p \geq 0$ and sufficiently small. Figure 7 in Appendix C illustrates the demand of non-biased and loss-averse consumers in a market with more than two firms.

4.2 Equilibrium existence and uniqueness

Establishing equilibrium existence in n-firm oligopoly is rather involved, since in our setup there might arise profitable non-local deviations by stealing consumers in distant sub-markets. Although for large n conditions for equilibrium existence carry over from the duopoly case, stricter conditions are required in markets with a small number of firms. The next proposition reports these conditions, which are derived in detail in Appendix B.2. The equilibrium markup $m^*(n)$ is determined in the next subsection.

Proposition 4. *A unique symmetric equilibrium with n firms and prices* $p^*(n) = m^*(n) + c = ((1+\lambda)nt)/(\lambda - 1 + 2n) + c$ *exists*

1. $\forall \lambda \in (1, \lambda^c]$ *with* $\lambda^c = 1 + 2\sqrt{2} \approx 3.828$ *if* $n = 2$ *or* $n > 6$,
2. $\forall \lambda \in (1, \lambda^{cc}]$ *with* $\lambda^{cc} = 1/4\left(1 + \sqrt{57}\right) \approx 2.137$ *if* $n \in \{3, 4, 5\}$.

4.3 The intensity of competition and the number of firms

In this subsection we will show that the equilibrium mark-up positively depends on the number of firms, whereas in the model with non-biased consumers it does not. The intuition for this non-neutrality result is straightforward. In the duopoly model, consumers expect that they are likely to be affected by a price deviation and thus adjust their reference-point distribution accordingly, while, given a larger number of firms, the reference-point distribution reacts less sensitive to one firm's deviation from the equilibrium strategy.

Before turning to some special cases, we analyze the general case α_m and $\alpha_p > 0$. Analogously to Lemma 2, we can derive firm i's symmetric equilibrium markup for loss-averse consumers ($\beta = 0$). Using firm i's demand function in (27) and that $\hat{x}_i(0) = 1/n$ yields

$$m^*(n|\alpha_m, \alpha_p) = \frac{(1 + \alpha_m \lambda)nt}{\alpha_p(\lambda - 1 + n) + n}. \tag{29}$$

We now turn to some special cases. First, consider the case that consumers are loss-averse only in the price dimension—i.e., $\alpha_m = 0, \alpha_p = 1$. The equilibrium markup is

$$m_p^*(n) = \frac{nt}{(\lambda - 1) + 2n}, \tag{30}$$

which is illustrated in Figure 4.

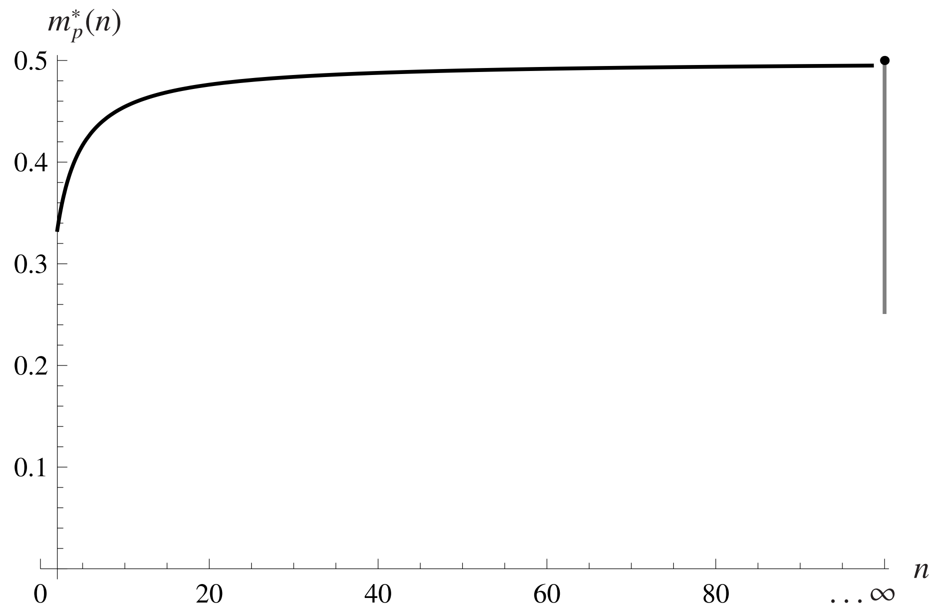


Figure 4: Markups with loss aversion in price dimension only

We find that the equilibrium markup is increasing in the number of firms. This confirms the general insight that, given a larger number of firms, the reference-point distribution reacts less sensitive to individual price deviations. The intuition for this result is that a price change in a market with few firms is more effective in changing the consumers' reference-point distribution. For a small variation of the model in which the circle becomes more crowded if the number of firm increases, by continuity, this implies that consumers may be better off in a market with a small number of firms if they are loss averse, whereas the opposite may hold if they do not have reference-dependent utilities.

For $n \to \infty$, the markup converges to $m_p^*(\infty) = t/2$—this is the upper bound on prices in the duopoly setting in which consumers form their reference-point distribution before observing prices.[22]

Second, consider the case that consumers are loss-averse in both dimensions—i.e., $\alpha_m = 1, \alpha_p = 1$. The equilibrium markup is

$$m^*(n) = \frac{(\lambda + 1)nt}{(\lambda - 1) + 2n}, \tag{31}$$

[22]The set of equilibrium prices would not be affected by the number of firms if consumers did not observe prices ex ante. Formally, treating n as a continuous variable, the equilibrium correspondence is not lower-hemicontinuous in the limit.

which is illustrated in Figure 5. Again, we find that the equilibrium price is increasing in the number of firms because the reference price distribution reacts less sensitive to a price change after an increase of the number of firms. Consequently, the market becomes less competitive. For $n \to \infty$, this markup converges to $m^* = (\lambda + 1)t/2$—this is the upper bound on prices in the duopoly setting in which consumers form their reference-point distribution before observing prices.

Third, consider the case that consumers are loss-averse only in the match-value dimension—i.e., $\alpha_m = 1, \alpha_p = 0$. The equilibrium markup is $m_m^*(n) = (\lambda + 1)t$ and is independent of n. This holds true because the distance between two neighboring firms, which determines the loss aversion in the taste dimension, is kept constant here.

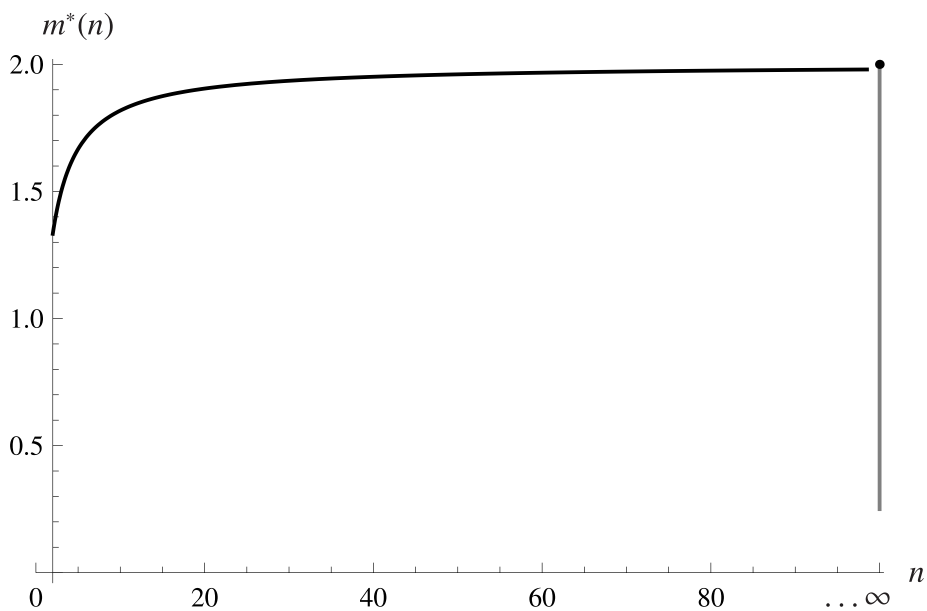


Figure 5: Markups with loss aversion in both dimensions

We summarize our findings as follows:

Proposition 5. *In the Salop model with $L = n$ and non-biased consumers, the number of firms does not affect competition. By contrast, with loss-averse consumers the equilibrium price is increasing in the number of firms. For $n \to \infty$ and $\alpha_p > 0$, the equilibrium price $p^*(\infty) \equiv \lim_{n\to\infty} p^*(n)$ is the upper bound of the equilibrium set that results in the model in which consumers do not observe price before forming their reference-point distribution.*

In this sense, our model provides a rationale for selecting the upper bound in the equilibrium set under the alternative timing.

5 Conclusion

This paper has explored the impact of consumer loss aversion on market outcomes in symmetric imperfectly competitive markets. We did so in a Hotelling-Salop setting, which is a standard work horse in the modern industrial organization literature. Consumer loss aversion only makes a difference compared to a market in which consumers lack this behavioral bias if they are uncertain about product characteristics or associated match value at an initial stage at which they form their reference-point distribution. Since price information is readily available, firms can use price to manage the reference-point distribution of consumers in the price and match-value dimensions.

Our paper provides a nuanced view on the competitive effects of consumer loss aversion in differentiated product markets. Loss aversion, and more generally, a gain-loss utility, in the price dimension leads to more competitive outcomes, while the reverse holds in the match-value dimension. It is the interplay between this pro- and anti-competitive effect that determines whether the market is more or less competitive compared to the standard Hotelling-Salop world. Empirical work may want to uncover the relative strength of those two effects.

In our modeling effort we followed Heidhues and Koszegi (2008) and Karle and Peitz (2010). Consumers learn posted prices before they form their reference points, whereas in Heidhues and Koszegi (2008) consumers form their reference points before knowing posted prices. This means that in our model a price change is observed and, thus, changes the consumers' reference-point distribution. The sensitivity of the reference-point distribution to price changes is particularly pronounced in duopoly. We show that increasing the number of firms in a way that does not affect equilibrium prices in the standard model makes the market less competitive. In the limit, prices converge to the upper bound of the equilibrium set for the model with the timing that consumers form their reference-point distribution before observing prices.

Appendix

A Relegated Proofs

A.1 Relegated proofs of Section 2

Proof of Lemma 1. In this proof we first use that the more symmetric price structure simplifies the reference-point distributions in (1) and (3). We then update the indirect utility functions for the indifferent consumer to the right of firm i and solve for her location (personal equilibrium).

Since the price differences in the sub-market between firm $i-1$ and firm i and in the one between firm i and firm $i+1$ are the same, the taste differences which the two indifferent consumers of firm i face are the same, i.e. $\hat{\hat{x}}_i^+ - (i-1) = (i-1) - \hat{\hat{x}}_i^-$. We therefore can simplify $\hat{\hat{x}}_i^+ - \hat{\hat{x}}_i^-$ in $F(p)$ to $2(\hat{\hat{x}}_i^+ - (i-1))$ or equivalently to $2z_{2j-1}$. Furthermore using that $p_j = p'$ for all $j \neq i$, we receive

$$F(p) = \begin{cases} \frac{2z_{2j-1}}{n} & \text{if } p \in [p_i, p') \\ 1 & \text{if } p \geq p'. \end{cases}$$

$p_i < p'$ implies that $\hat{\hat{x}}_i^+ - (i-1) = z_{2j-1} > 1/2$. Thus, the smallest critical taste distance in the market exists between $\hat{\hat{x}}_i^+$ and firm $i+1$ (and resp. between $\hat{\hat{x}}_i^-$ and firm $i-1$). This distance is equal to $1 - z_{2j-1}$. The next larger critical taste distance is the one in sub-markets with symmetric prices. It is equal to $1/2$. Finally, only the consumers that will be attracted by firm i ex post face up to the maximum critical taste distance which is z_{2j-1}. Hence, $G(s)$ can be rewritten as

$$G(s) = \begin{cases} 2s & \text{if } s \in [0, 1 - z_{2j-1}] \\ 2s\frac{n-1}{n} + a_1 & \text{if } s \in (1 - z_{2j-1}, \frac{1}{2}] \\ 2s\frac{1}{n} + a_2 & \text{if } s \in (\frac{1}{2}, z_{2j-1}], \end{cases}$$

where $a_1 = \frac{1-2z_{2j-1}}{n}$ and $a_2 = (1 - \frac{2z_{2j-1}}{n})$. Using the properties of the reference-point distributions, we rewrite the indirect utility functions of consumers buying from firm i or $i+1$,

$$u_i(x = \hat{\hat{x}}_i, \boldsymbol{p}) \quad = v - tz_{2i-1} - p_i + \alpha_p(1 - \frac{2z_{2i-1}}{n})(p' - p_i) + \frac{\alpha_m \lambda t}{4n}\left(8z_{2i-1}^2 - 4(2+n)z_{2i-1} + 2 + n\right)$$

$$u_{i+1}(x = \hat{\hat{x}}_i, \boldsymbol{p}) = v - t(1 - z_{2i-1}) - p' - \alpha_p \lambda \frac{2z_{2i-1}}{n}(p' - p_i) \\ + \frac{\alpha_m t}{4n}\left(n\left((2z_{2i-1} - 1)^2 - 4\lambda(z_{2i-1} - 1)^2\right) + 2(2z_{2i-1} - 1)^2\right).$$

If buying from firm i, the indifferent consumer only faces a gain in the price dimension (there is no λ in the price dimension term) because p_i is the lowest price in the market. For the same reason she faces a pure loss in the price dimension when buying from firm $i + 1$. On the other hand, the indifferent consumer experiences a pure loss in the taste dimension if she purchases the cheap product i.

Next, we determine the location of the indifferent loss-averse consumer by setting $u_i = u_{i+1}$. Rearranging leads to the following quadratic equation in z_{2j-1},

$$4\alpha_m(\lambda - 1)(n + 2)t \cdot z_{2i-1}^2 + \Big(8\alpha_p(\lambda - 1)\Delta p - 4(2n + \alpha_m(2(\lambda - 1) + (3\lambda - 1)n))t\Big) \cdot z_{2i-1} \\ + \Big(4(1 + \alpha_p)n\Delta p + 2\alpha_m(\lambda - 1)t + (4 + \alpha_m(5\lambda - 1))nt\Big) = 0$$

Solving this quadratic equation w.r.t. z_{2j-1} and adding $(i - 1)$ leads to $\hat{\hat{x}}_i^+(\Delta p)$, the expression given in the lemma. The second solution to the quadratic equation can be ruled out because does not lie in the interval $[1/2, 1]$. □

A.2 Relegated material of Section 3

Properties of firm i's demand in duopoly:

$$\begin{aligned} q_i' &= \frac{\partial q_i(\Delta p; \beta)}{\partial \Delta p} = -\frac{\partial q_i(\Delta p; \beta)}{\partial p_i} = -\frac{\partial q_{-i}(\Delta p; \beta)}{\partial \Delta p} = -\frac{\partial q_{-i}(\Delta p; \beta)}{\partial p_{-i}} \\ &= \beta \cdot \hat{x}_{in}'(\Delta p) + (1 - \beta) \cdot \hat{x}_{un}'(\Delta p) \\ &= -\frac{1}{4t}(1 - 3\beta) - \frac{(1 - \beta)}{2(S(\Delta p))}\underbrace{\left(\frac{\Delta p}{8t^2} - \frac{(\lambda + 2)}{2t(\lambda - 1)}\right)}_{\ominus} > 0 \end{aligned}$$

$q_i' > 0 \quad \forall \Delta p$ feasible and $\forall \beta$. At the boundaries we have

$$\begin{aligned} q_i'(0; \beta) &= -\frac{1}{4t}(1 - 3\beta) + (1 - \beta)\frac{(\lambda + 2)}{2t(\lambda - 1)} > 0 \\ \lim_{\Delta p \uparrow \Delta \bar{p}} q_i'(\Delta; \beta) &= \infty \quad \text{for } \beta < 1 \text{ since } S(\bar{\Delta p}) = 0. \end{aligned}$$

For $0 \leq \Delta p < \Delta p^{max}$ the demand of i is convex in Δp.

$$q_i''(\Delta p;\beta) = (1-\beta)\cdot \hat{\hat{x}}_i''(\Delta p) = (1-\beta)\cdot \frac{(3+\lambda)(5+3\lambda)}{64t^2\cdot (S(\Delta p))^3} \geq 0$$

$q_i'' > 0 \quad \forall \Delta p$ feasible and $\forall \beta < 1$ since $S(\Delta p) \geq 0$. At the boundaries we have

$$\begin{aligned} q_i''(0;\beta) &= (1-\beta)\cdot \frac{(3+\lambda)(5+3\lambda)}{32t^2\cdot \frac{(\lambda+1)^3}{(\lambda-1)^3}} > 0 \\ \lim_{\Delta p \uparrow \Delta \bar{p}} q_i''(\Delta p;\beta) &= \infty \quad \text{for } \beta < 1. \end{aligned}$$

It can be also show that $q_i''(\Delta p;\beta,\lambda)$ is increasing in Δp and λ).

Equilibrium existence in duopoly with constant weights equal to one:

Proof of Proposition 1. In this proof we rule out non-local deviations from symmetric price equilibrium in the duopoly case, i.e. when firms only compete in their neighboring sub-markets.[23] Let firm i be the deviating firm. It is shown above that firm i's profit is concave if the price difference $\Delta p = p_{-i} - p_i$ is sufficiently low, i.e. Δp is negative or not too positive. Therefore non-local price increases are never profitable. Since the convexity of firm i's profit increases in Δp, firm i's most profitable price deviation is a price reduction stealing the entire demand of loss-averse consumers. The intuition behind this result is that for sufficiently large price differences loss-averse consumers excessively avoid to buy the more expensive product. Furthermore, this avoidance is the larger the higher the degree of loss aversion. This holds true because the convexity of firm i's demand increases in the degree of loss aversion, i.e. $\partial q_i''/\partial\lambda > 0$.

We next derive the critical upper bound of the degree of loss aversion for which stealing the entire demand of loss-averse consumers is no longer profitable. For stealing the entire market firm i sets a deviation price p_i^d s.t. $\Delta p^d = \Delta p^{max}$, i.e. $p_i^d = p_{-i}^* - \Delta p^{max} = p^* - \Delta p^{max}$. For $\beta = 0$ the firm i's deviation profit, π_i^d, can be expressed as follows,

$$\begin{aligned} \pi_i^d &= (p_i^d - c)\cdot 1 \\ &= (p^* - c) - \Delta p^{max} \end{aligned} \tag{32}$$

Firm i's profit in symmetric equilibrium is equal to

$$\pi_i^* = (p_i^* - c)\cdot q_i(0)$$

[23] Compare the equilibrium existence proof for n-firm oligopoly in Proposition 4.

$$= \frac{(p^* - c)}{2} \tag{33}$$

Thus, a deviation from symmetric equilibrium is not profitable if and only if

$$\begin{aligned}
\pi_i^*(\lambda) &\geq \pi_i^d(\lambda) \\
\Delta p^{max}(\lambda) &\geq \frac{p^*(\lambda) - c}{2} && \text{by (32) and (33)} \\
\frac{(\lambda+3)t}{2(\lambda+1)} &\geq \frac{t}{2 - \frac{(\lambda-1)}{(\lambda+1)}} && \text{by (10) and (13)} \\
(\lambda+3)^2 &\geq 2(\lambda+1)^2 \\
\lambda &\lesseqqgtr 1 \pm 2\sqrt{2}.
\end{aligned}$$

Since $\lambda > 1$, we receive the unique solution $\lambda \leq \lambda^c \equiv 1 + 2\sqrt{2}$. □

Proof of Proposition 2. For $\beta > 0$, firm i's demand from setting the uniquely optimal deviation price p_i^d extents to $q_i(\Delta p^{max}) = (1-\beta) + \beta(1/2 + \Delta p^{max}/(2t))$.[24] Therefore, her deviation profit becomes,

$$\pi_i^d(\lambda,\beta) = \Big((p^*(\lambda,\beta) - c) - \Delta p^{max}(\lambda)\Big)\Big((1-\beta) + \beta(\frac{1}{2} + \frac{\Delta p^{max}(\lambda)}{2t})\Big), \tag{34}$$

where $p^*(\lambda,\beta)$ is given by (13) and $\Delta p^{max}(\lambda)$ by (10). Moreover, firm i's profit in symmetric equilibrium is equal to $\pi_i^*(\lambda,\beta) = (p^*(\lambda,\beta) - c)/2$. This yields the following non-deviation condition in $\lambda > 1$ and $\beta \in [0, 1)$,

$$\pi_i^*(\lambda,\beta) \geq \pi_i^d(\lambda,\beta)$$

$$\Delta p^{max}(\lambda)\Big(1 + \frac{\beta}{2}(\frac{\Delta p^{max}(\lambda)}{t} - 1)\Big) \geq \frac{p^*(\lambda,\beta) - c}{2}\Big(1 + \beta(\frac{\Delta p^{max}(\lambda)}{t} - 1)\Big)$$

Solving for β^{crit} as a function of λ such that $\pi_i^*(\lambda,\beta) \geq \pi_i^d(\lambda,\beta)$ for $\beta \geq \beta^{crit}$, leads to the following result:

1. for $\lambda \in (1, 1 + 2\sqrt{2}]$

$$\pi_i^*(\lambda,\beta) > \pi_i^d(\lambda,\beta) \text{ for all } \beta \geq 0,$$

[24] Since q_i is a convex combination of $\hat{\hat{x}}_i$ and $\hat{x}_i$, $p_i^d = p^* - \Delta p^{max}$ remains the uniquely optimal deviation for $\beta < 1$. Cf. the proof of Proposition 1.

2. for $\lambda \in (1 + 2\sqrt{2}, \tilde{\lambda}]$, (i.e. $\Delta p^{max} = \Delta\tilde{p}$)

$$\beta_0^{crit}(\lambda) \equiv 1 - \frac{-\lambda(5\lambda + 14) + \sqrt{(3\lambda + 5)(\lambda(11\lambda(\lambda + 5) + 113) + 77)} - 13}{2(\lambda - 1)(\lambda + 3)}, \tag{35}$$

3. for $\lambda > \tilde{\lambda}$, (i.e. $\Delta p^{max} = \Delta\bar{p}$)

$$\beta_1^{crit}(\lambda) \equiv 1 - \frac{37\lambda^3 - 21\Gamma\lambda^2 + 177\lambda^2 - 54\Gamma\lambda + 247\lambda - 21\Gamma - \Omega + 83}{2\,(12\lambda^3 - 7\Gamma\lambda^2 + 46\lambda^2 - 10\Gamma\lambda + 8\lambda + 17\Gamma - 66)} \tag{36}$$

with $\Omega \equiv (4\lambda^6 - 2\Gamma\lambda^5 + 1596\lambda^5 - 918\Gamma\lambda^4 + 19848\lambda^4 - 9316\Gamma\lambda^3 + 91384\lambda^3 - 31228\Gamma\lambda^2 + 197268\lambda^2 - 42618\Gamma\lambda + 201868\lambda - 20366\Gamma + 78880)^{1/2}$ and $\Gamma \equiv \sqrt{3\lambda^2 + 14\lambda + 15}$.

For $\lambda \to \infty$ it holds that $\beta_1^{crit}(\lambda) \to 1 - \frac{-37+21\sqrt{3}+\sqrt{4-2\sqrt{3}}}{-24+14\sqrt{3}} \approx 0.577$.

□

B Equilibrium Existence

B.1 Equilibrium Existence in Duopoly with Varying Weights

Proposition 6 (Existence in duopoly with varying weights)**.** *Suppose that there are two firms in the market and all consumers are loss averse ($\beta = 0$) with varying weights on the two dimensions of loss aversion ($1 \geq \alpha_p, \alpha_m > 0$). A symmetric equilibrium with prices*

$$p_i^* = c + \frac{t}{1 - \frac{1}{2}\frac{(2\alpha_m\lambda - \alpha_p(\lambda+1))}{(\alpha_m\lambda+1)}}$$

for all $i \in \{1, 2\}$ exists if and only if $1 < \lambda \leq \lambda^c(\alpha_p, \alpha_m)$ with

$$\lambda^c(\alpha_p, \alpha_m) = 1 + \frac{\sqrt{2}(1 + \alpha_m)(1 + \alpha_p)}{\sqrt{\alpha_m(1 + \alpha_m)\alpha_p(1 + \alpha_p)}}. \tag{37}$$

Moreover, $\lambda^c(\alpha_p, \alpha_m) \geq \lambda^c(1, 1) = \lambda^c = 1 + 2\sqrt{2}$.

Proof of Proposition 6. Analogously to the proof of Proposition 1, a deviation from symmetric equilibrium is not profitable if and only if

$$\pi_i^*(\lambda, \alpha_p, \alpha_m) \geq \pi_i^d(\lambda, \alpha_p, \alpha_m)$$

$$\Delta p^{max}(\lambda,\alpha_p,\alpha_m) \geq \frac{p_i^*(\lambda,\alpha_p,\alpha_m)-c}{2} \qquad \text{by (32) and (33)}$$
$$\frac{(2+\alpha_m+\alpha_m\lambda)t}{2+2\alpha_p\lambda} \geq \frac{t}{2-\frac{(2\alpha_m\lambda-\alpha_p(\lambda+1))}{(\alpha_m\lambda+1)}}$$
$$\lambda \lesseqqgtr 1 \pm \frac{\sqrt{2}(1+\alpha_m)(1+\alpha_p)}{\sqrt{\alpha_m(1+\alpha_m)\alpha_p(1+\alpha_p)}}.$$

The unique solution equals $\lambda \leq \lambda^c(\alpha_p,\alpha_m) \equiv 1+(\sqrt{2}(1+\alpha_m)(1+\alpha_p))/\sqrt{\alpha_m(1+\alpha_m)\alpha_p(1+\alpha_p)}$.

□

B.2 Equilibrium Existence in n-Firm Oligopoly

In n-firm oligopoly there might arise profitable non-local deviations by stealing consumers in distant sub-markets. We next establish existence of symmetric equilibria in this setup. Although conditions for existence carry over from the duopoly case for n sufficiently large, there might arise additional existence problems in markets with a small number of firms when consumers are loss averse up to the level that constitutes the upper bound of the duopoly case. As mentioned before, in contast to Heidhues and Koszegi (2008), in our setup consumers observe prices ex ante and adjust their reference-point distributions to price deviations. Therefore, a large observed price deviation increases the probability of buying from the deviating firm.

In this subsection, we restrict the analysis to the most demanding case: All consumers are loss-averse ($\beta = 0$). Divide the circle of length $L = n$ into $2n$ sub-markets of length $1/2$. Thus, there are n sub-markets on each half of a circle and 2 between each pair of neighboring firms. In a symmetric equilibrium , a firm located at y_i serves all consumers on the left and the right neighboring sub-market—i.e., all consumers x within $[y_i-1/2; y_i+1/2]$.[25] Due to symmetry, it suffices to consider deviations on one half of the circle only. Suppose firm i located at $y_i = 0$ deviates from the symmetric equilibrium by lowering its price. If it attracts consumers up to the mth sub-market (on the first half of the circle), firm i's (right) indifferent consumer is located at $\hat{\hat{x}} \in [\frac{(m-1)}{2}, \frac{m}{2}]$ with $2 \leq m \leq n$. Its total demand equals $2\hat{\hat{x}}/n$ due to the uniform distribution of x. Loss-averse consumers who expect $\hat{\hat{x}}$ to be located in the mth sub-market for given prices, form the following reference-point distribution with respect to the match-value dimension,

[25] Since the set of consumers is restricted to mass one and x is uniformly distributed on $[0; n]$, the demand of firm i on $[y_i - 1/2; y_i + 1/2]$ is equal to $1/n$.

- for even m:

$$G_m(s|n) = \begin{cases} \frac{2}{n}(n-(m-2))s, & s \in [0, 1-(\hat{\hat{x}} - \frac{m-2}{2})]; \\ \frac{2}{n}(n-(m-1))s + a(\hat{\hat{x}}, m, n), & s \in (1-(\hat{\hat{x}} - \frac{m-2}{2}), \frac{1}{2}]; \\ \frac{2}{n}s + b(\hat{\hat{x}}, m, n), & s \in (\frac{1}{2}, \hat{\hat{x}}]. \end{cases}$$

with $a(\hat{\hat{x}}, m, n) = (m-1)/n - 2\hat{\hat{x}}/n$ and $b(\hat{\hat{x}}, m, n) = 1 - 2\hat{\hat{x}}/n$ being the required constants for the kinked cdf.

- for odd m:

$$G_m(s|n) = \begin{cases} \frac{2}{n}(n-(m-1))s, & s \in [0, \hat{\hat{x}} - \frac{m-1}{2}]; \\ \frac{2}{n}(n-(m-2))s + \tilde{a}(\hat{\hat{x}}, m, n), & s \in (\hat{\hat{x}} - \frac{m-1}{2}, \frac{1}{2}]; \\ \frac{2}{n}s + \tilde{b}(\hat{\hat{x}}, m, n), & s \in (\frac{1}{2}, \hat{\hat{x}}]. \end{cases}$$

It can be easily seen that both distributions coincide for $\hat{\hat{x}}$ reaching the boundaries between two neighboring sub-markets: e.g., for $\hat{\hat{x}} = 1$ $G_2(s|n) = G_3(s|n)$ and for $\hat{\hat{x}} = 3/2$ $G_3(s|n) = G_4(s|n)$ and so on. For $n = m = 2$, we are back in the duopoly case.

To see how the reference-point distributions can be derived, consider the case of $m = 3$ and $n \geq 3$: $\hat{\hat{x}} \in [1; 3/2]$ means that the deviating firm i steals all consumers up to the location of its right neighbor (firm $i+1$ located at $y_{i+1} = 1$) and some even in the neighbor's backyard market. Therefore, an equilibrium taste difference s within $[0; \hat{\hat{x}} - 1] \subseteq [0; 1/2]$ can be expected by consumers on each of the n sub-markets on the first half of the circle, except for the two sub-markets neighboring firm $i+1$ ($m = 2, 3$). This holds true since consumers who turn out to be located in these two sub-markets, will be attracted by the deviating firm i which is located further apart, while consumers on all other sub-markets will buy from the firm closest by. The resulting probability of facing a taste difference in this interval equals $(2/n)(n-2)s$. An equilibrium taste difference $s \in (\hat{\hat{x}} - 1; 1/2]$ can be expected on $n-1$ sub-markets (on the first half of the circle) since also consumers on sub-market $m = 3$ with $x \in (\hat{\hat{x}}; 3/2]$ will be buying from their closest firm, which is firm $i+1$ located at $y_{i+1} = 1$. Thus, $G_3(s|n)$ is equal to $2/n(n-1)s$ plus a constant in this interval. Facing an equilibrium taste difference $s \in (1/2; \hat{\hat{x}} - 1] = (1/2; 1] \cup (1; \hat{\hat{x}} - 1]$, there is each time one particular sub-market consumers expect to be located in: $m = 2$ for $s \in (1/2; 1]$ and $m = 3$ for $s \in (1; \hat{\hat{x}} - 1]$. Hence, the probability of $s \in (1/2; \hat{\hat{x}} - 1]$ is equal to $2/n \cdot s$ plus a constant.

From the functional form of $G_m(s|n)$ it follows directly that, for given n, a distribution with a higher m first-order stochastically dominates the ones with lower m. This is because consumers expect to be attracted by the deviating firm with a higher probability when it steals a large market share. Therefore, buying from the closest firm becomes less likely: Consumers put less weight on taste differences less than $1/2$ and positive weight on taste differences greater than $1/2$.[26] An increase in the number of firms has exactly the opposite effect to an increase in the number of stolen sub-markets by the deviating firm: For a given m, the reference-point distribution puts more mass on small taste differences if the number of firms n increases. Here, the chance of being affected by a price cut of a single firm simply washes out if the total number of firms increases without bound.

The probability of buying from the deviating firm i (=probability of facing purchase price p_i) is $\hat{\hat{x}}$ in the duopoly and generalizes to $2\hat{\hat{x}}/n$ in the n-firm case. The intuition for this mirrors the one just given above: If the number of firms rises, any firm is less likely to be affected by a price cut of a single firm. Using the generalized reference-point distribution in both dimensions, we can derive a generalized demand function for symmetric markets with n firms. Consider, for instance, the indirect utility functions of a consumer x who has learned to be located in sub-market m (with m even) which is the sub-market consumers ex ante expected the indifferent loss-averse consumer to be located in,[27] given prices $(p_i < p_{-i} = p^*)$. Moreover, suppose this consumer is the indifferent loss-averse consumer on this side of the circle, $x = \hat{\hat{x}} \in [(m-1)/2; m/2]$. Then, her indirect utility if buying from the deviating firm i can be expressed as follows,[28]

$$
\begin{aligned}
u_i(x = \hat{\hat{x}}, p_i, p^*) \quad =& v - t\hat{\hat{x}} - p_i + \left(1 - \frac{2\hat{\hat{x}}}{n}\right)(p^* - p_i) \\
& - \lambda t \Bigg(\int_0^{1-(\hat{\hat{x}} - \frac{(m-2)}{2})} (\hat{\hat{x}} - s)\frac{2}{n}(n - (m-2))ds \\
& + \int_{1-(\hat{\hat{x}} - \frac{(m-2)}{2})}^{1/2} (\hat{\hat{x}} - s)\frac{2}{n}(n - (m-1))ds + \int_{1/2}^{\hat{\hat{x}}} (\hat{\hat{x}} - s)\frac{2}{n} ds \Bigg) \\
=& v - t\hat{\hat{x}} - p_i + \left(1 - \frac{2\hat{\hat{x}}}{n}\right) - \frac{\lambda t}{4n}\left(-8\hat{\hat{x}}^2 + 4(m+n)\hat{\hat{x}} - ((m-1)m + n)\right).
\end{aligned}
$$

It can be seen that the indifferent loss-averse consumer faces only a gain in the price dimension (last term in the first line) when purchasing the product of the deviating firm.

[26]For this updating behavior the observability of prices is crucial. In contrast to this, consumers in Heidhues and Koszegi (2008) cannot adjust their reference point to price deviations because prices become observable only after forming their reference point.

[27]We use this latter condition here, since, as we show later, the mapping from $\Delta p = p^* - p_i \in \mathbb{R}_0^+$ into $m \in [2, 3, ..., n-1, n]$ is not a function but a correspondence—i.e., for given price difference Δp, there may exist several personal equilibria $\hat{\hat{x}}$ within different sub-markets.

[28]Compare the indirect utility function for $m = 2$ in the proof of Lemma 1 and consult Section 2 for a detailed description of the utility function with reference dependence.

In the taste dimension she faces the maximum loss (second and third line). If buying from firm $i + m/2$ instead, her indirect utility equals

$$\begin{aligned} u_{i+m/2}(x = \hat{\hat{x}}, p_i, p^*) \quad &= v - t(1 - (\hat{\hat{x}} - \frac{(m-2)}{2})) - p^* - \lambda\left(\frac{2\hat{\hat{x}}}{n}\right)(p^* - p_i) \\ &- \lambda t \int_0^{1-(\hat{\hat{x}}-\frac{(m-2)}{2})} (1 - (\hat{\hat{x}} - \frac{(m-2)}{2}) - s)\frac{2}{n}(n - (m-2))ds \\ &+ t\Big(\int_{1-(\hat{\hat{x}}-\frac{(m-2)}{2})}^{1/2} (s - (1 - (\hat{\hat{x}} - \frac{(m-2)}{2})))\frac{2}{n}(n - (m-1))ds \\ &+ \int_{1/2}^{\hat{\hat{x}}} (s - (1 - (\hat{\hat{x}} - \frac{(m-2)}{2})))\frac{2}{n}ds\Big) \\ &= v - t(1 - (\hat{\hat{x}} - \frac{(m-2)}{2})) - p^* - \lambda\left(\frac{2\hat{\hat{x}}}{n}\right)(p^* - p_i) \\ &+ \frac{t}{4n}\Big(4(2 - (\lambda-1)\bar{n})\hat{\hat{x}}^2 + 4(((\lambda-1)\bar{n} - 1)m + n)\hat{\hat{x}} \\ &+ ((1 - (\lambda-1)\bar{n})m - 2n - 1)m + n\Big), \end{aligned}$$

with $\bar{n} = ((n-m)+2)$. Here, the indifferent loss-averse consumer only faces a loss in the price dimension but losses and gains in the taste dimension.[29] By setting $u_i = u_{i+m/2}$, we can solve the consumers' personal equilibrium and determine $\hat{\hat{x}}$ for given n and given that ex ante consumers expect $\hat{\hat{x}} \in [(m-1)/2; m/2]$ for given prices.[30] Firm i's demand from loss-averse consumers in even sub-market m, $q_i(\Delta p|m, n, \beta = 0)$, is then characterized by $2\hat{\hat{x}}/n$. Firm i's demand for odd sub-markets m can be derived analogously.

To analyze whether deviations to sub-markets m, $m \geq 3$, are profitable, we first consider consumers located on the boundaries of the sub-markets, $\hat{\hat{x}} = 1, 3/2, ..., (n-1)/2, n/2$. For $\hat{\hat{x}}$ being an integer, firm i attracts consumers up to the location of a competing firm, while for $\hat{\hat{x}} = j + 1/2$, $j \in \mathbb{N}$, it also attracts the entire backyard market of competitor j. As is known from the standard Salop oligopoly, the price differences for $\hat{\hat{x}} = j$ and $\hat{\hat{x}} = j + 1/2$ coincide. This means that firm i's demand has a discontinuous jump of size

[29] Cf. the proof of Lemma 1 where $m = 2$.

[30] $0 = u_i - u_{i+m/2}$ is equivalent to

$$\begin{aligned} 0 = &\Big((n-m)+4\Big)(\lambda-1)t \cdot \hat{\hat{x}}^2 - \Big((((\lambda-1)m + \lambda + 3)n - (\lambda-1)(m-3)m)\,t - 2(\lambda-1)\Delta p\Big) \cdot \hat{\hat{x}} \\ &+ \frac{1}{4}\Big(8n\Delta p + nt\Big((\lambda-1)m^2 + \lambda + 4m - 1\Big) - (\lambda-1)m((m-3)m+1)t\Big). \end{aligned}$$

We do not present the functional form of $\hat{\hat{x}}_i(\Delta p|m, n)$ here for two reasons. First of all, it is lengthy and tedious to derive, as $u_i - u_{i+m/2} = 0$ describes a quadratic equation in $\hat{\hat{x}}$. Secondly, since we are mainly interested in deviations to the boundaries of a sub-market m, we can fix $\hat{\hat{x}}$ at $(m - 1/2)$ or $m/2$ and solve for the corresponding price difference Δp. This is much simpler as $u_i - u_{i+m/2} = 0$ describes a linear equation in Δp.

$1/2 \cdot 2/n = 1/n$ at this price difference. It can be shown, however, that despite this feature non-local deviation are never profitable in the standard Salop model. To check this in a world with loss-averse consumers, we next derive the deviation price differences for $\hat{\hat{x}} = 1, 3/2, ..., (n-1)/2, n/2$. For a deviation covering an even number of sub-markets m (resp. an odd number of sub-markets $m' = m-1$), replace $\hat{\hat{x}}$ in $u_i - u_{i+m/2} = 0$ by $m/2$ (resp. $(m-1)/2$) and solve for Δp.

$$\Delta p^{even}(m,n) = \frac{\Big(2(\lambda+1)m - (\lambda-1)\Big)n + (\lambda-1)(m-1)m}{4(\lambda-1)m + 8n} t, \quad m \text{ even and } n \geq m \geq 2,$$
$$\Delta p^{odd}(m',n) = \frac{\Big(2(\lambda+1)n - (\lambda-1)(m'-1)\Big)(m'-1)}{4(\lambda-1)m' + 8n} t, \quad m' \text{ odd and } n \geq m' \geq 3.$$

It can be shown that both deviation price differences are increasing in m and n. The first implication of this is very intuitive: For a given number of firms n, attracting consumers on more sub-markets m requires a larger price difference—i.e., a larger price cut by the deviating firm. Secondly and more interestingly, if the number of firms n increases, a larger price cut is necessary to steal a given number of sub-markets m. The intuition for this is that, for a larger number of firms, consumers expect to be less often affected by a certain price cut of a single firm and, therefore, expect their equilibrium taste difference to be low. This increases the loss in the taste dimension for those consumers who ex post happen to buy from the more distant deviating firm, and this makes it more difficult for the deviating firm to steal a large share of the market. Consider for example two markets with $n = 3$ and 5, ($\lambda = 3, t = 1$): $\Delta p^{even}(2,3) = 19/20 < \Delta p^{even}(2,5) = 33/28 < \Delta p^{even}(4,5) = 7/4$. Similarly, $\Delta p^{odd}(3,3) = 5/6 < \Delta p^{odd}(3,5) = 9/8 < \Delta p^{odd}(5,5) = 8/5$. It can also be seen here that the price difference necessary to steal the entire backyard sub-market of a competitor is lower than the one necessary to steal consumers up to the location of this competitor—i.e., $\Delta p^{odd}(m+1,n) < \Delta p^{even}(m,n)$. This demonstrates a violation of the *law of demand* which is caused by the fact that consumer's indirect utility functions if buying the cheap or the most-liked product are decreasing in consumer's location x on odd sub-markets. Hence, to describe a personal equilibrium, $\hat{\hat{x}}$ must be decreasing in Δp on odd sub-markets. This makes deviations under which the deviating firm steals an odd number of sub-markets particularly profitable, as will be shown in the next paragraph. In the example, the demand of the deviating firm is given by $m/2 \cdot 2/n = m/n$ and the corresponding markup in symmetric equilibrium equals $m^*(3) = 3/2$ and $m^*(5) = 5/3$. This illustrates that the deviation price difference might become larger than the equilibrium markup if the number of firms n and the number of deviations m become sufficiently large: In the example we find $m^*(5) = 5/3 < \Delta p^{even}(4,5) = 7/4$. Therefore,

Table 2: Deviation profits with n firms

The table shows the variation of $\pi^{odd}(m,n)/t$ and $\pi^{even}(m,n)/t$ in n and m for $\lambda = \lambda^c = 1 + 2\sqrt{2}$ (and $\beta = 0$).

n \ m	2	3	4	5	6	7	8
2	0.7071						
3	0.4676	0.8360					
4	0.3400	0.5877	0.3694				
5	0.2624	0.4373	0.1444	0.3506			
6	0.2111	0.3388	0.0096	0.1418	-0.1676		
7	0.1751	0.2705	-0.0750	0.0060	-0.3607	-0.2384	
8	0.1486	0.2211	-0.1296	-0.0851	-0.4861	-0.4151	-0.7769

those kind of deviations generate losses for the deviating and are, therefore, never optimal.

We next evaluate whether there exist profitable deviations from the symmetric equilibrium with $n > 2$ firms and $\lambda \leq \lambda^c = 1 + 2\sqrt{2} \approx 3.828$ (compare Prop. 1). The equilibrium profit, $\pi^*(n)$, can be expressed by

$$\pi^*(n) = m^*(n) \cdot \frac{1}{n} = \frac{(1+\lambda)t}{(\lambda - 1 + 2n)},$$

with equilibrium markup, $m^*(n)$, derived in Section 4.[31] The deviation profits for even and odd deviations are equal to

$$\pi^{even}(m,n) = \Big(m^*(n) - \Delta p^{even}(m,n)\Big) \cdot \frac{m}{n}$$
$$\pi^{odd}(m',n) = \Big(m^*(n) - \Delta p^{odd}(m',n)\Big) \cdot \frac{m'}{n}.$$

Deviation profits change monotonously in n and m: $\pi^{odd}(m,n)$ and $\pi^{even}(m,n)$ are monotonously decreasing in n and m. This is shown in Table 2, where we restrict attention to $\lambda = \lambda^c$, the highest level of loss aversion at which a symmetric equilibrium exists for $n = 2$.[32] The table demonstrates that deviating becomes less profitable if the number of firms n in the market increases[33] and that within the class of odd (resp. even) deviations stealing a small number of sub-markets m is preferable to stealing a larger number of sub-markets. Moreover, it is depicted that for a given number of firms n stealing an odd number of

[31] Cf. equation (31).

[32] For smaller levels of loss aversion $\lambda > 1$ deviating is less profitable, but the monotonicity in n and m is preserved.

[33] This also implies that non-local deviations in the home market ($m = 2$), as considered in the duopoly case, are less profitable if n raises.

Table 3: Extra profit from deviating

The table shows the variation of $(\pi^{odd}(m,n) - \pi^*(n))/t$ and $(\pi^{even}(m,n) - \pi^*(n))/t$ in n and m for $\lambda = \lambda^c = 1 + 2\sqrt{2}$ (and $\beta = 0$).

n | m	2	3	4	5	6	7	8
2	0						
3	-0.0793	0.2891					
4	-0.1059	0.1418	-0.0765				
5	-0.1139	0.0610	-0.2320	-0.0258			
6	-0.1145	0.0131	-0.3160	-0.1839	-0.4932		
7	-0.1118	-0.0165	-0.3619	-0.2809	-0.6476	-0.5253	
8	-0.1078	-0.0354	-0.3860	-0.3415	-0.7426	-0.6715	-1.033

sub-markets $m' = m + 1$ is more profitable than stealing an even number of sub-markets m. Thus, the deviation profit is highest in a three-firm oligopoly when the deviating firm steals the entire market ($m = 3$).[34]

To identify the deviations that are the most critical for existence, the difference between deviation and equilibrium profit are presented in Table 3.[35] It can be seen that there exist profitable deviations from symmetric equilibrium for $\lambda = \lambda^c$. However, only deviations stealing $m = 3$ sub-markets are profitable if the number of firms is not too large—i.e., $n \in \{3, 4, 5, 6\}$. More generally, this can be shown by solving for the critical number of firms $n^{odd}(m, \lambda)$ in $\pi^{odd}(m, n) - \pi^*(n) = 0$.[36]

$$n^{odd}(m,\lambda) = (\lambda - 1)\frac{(\lambda + m)m + \sqrt{(m\lambda^2 + 2(3(m-2)m+4)\lambda + (m-2)(m+6)m + 8)\,m}}{4(\lambda+1)(m-2)}$$

Deviating is profitable for given λ, m, and n if $n < n^{odd}(m, \lambda)$ and $m \leq n$. Moreover, $n^{odd}(m, \lambda)$ is strictly decreasing in m for $n^{odd}(m, \lambda) > m$ and strictly increasing in λ. Therefore, $m = 3$ is the most critical deviation and profitable for $n < n^{odd}(3, \lambda^c) \approx 6.3890$.[37] To rule out deviations from symmetric equilibrium for all $n \geq 2$, the maximum degree of loss aversion λ has to be reduced below $\lambda^c = 1 + 2\sqrt{2} \approx 3.828$.

Before stating the conditions for symmetric equilibrium to exist, we return to the issue of multiple personal equilibria for given prices. Since $\Delta p^{odd}(3, n) < \Delta p^{even}(2, n)$, consumers

[34] $m = 1$ can be excluded since $\Delta p^{odd}(1, n)$ coincides with $\Delta p^*(n) = 0$, the symmetric equilibrium .

[35] By construction $\pi^{even}(2, 2) = \pi^*(2)$ at $\lambda = \lambda^c$ (cf. Prop. 1).

[36] $n^{odd}(m, \lambda)$ being the only positive solution.

[37] A critical n can be derived for even deviations analogously. We skip this step here since even deviations are dominated by odd ones in any case.

facing a price difference $\Delta p = \Delta p^{odd}(3, n)$ between the deviating firm and non-deviating firms could expect $\hat{\hat{x}}$ to be located either on the second or the third sub-market (on the first half of the circle). Expecting $m = 3$ rather than $m = 2$ given $\Delta p = \Delta p^{odd}(3, n)$ is preferable for the deviating firm because it receives a strictly larger market share but is not necessarily preferable for consumers. For instance, consumers who do not buy the lower-priced product will ex post experience a higher loss in the price dimension since the probability of low purchase price increases in $\hat{\hat{x}}$.[38] Therefore the deviations considered above use the most conservative personal equilibrium and deliver the strictest conditions for an equilibrium to exist.

Lemma 3. *A unique symmetric equilibrium with n firms and prices* $p^*(n) = m^*(n) + c = ((1 + \lambda)nt)/(\lambda - 1 + 2n) + c$ *exists if* $n \geq n^{odd}(3, \lambda)$ *with* $\lambda > 1$.

The derivation of $n^{odd}(m, \lambda)$ and the relevance of $n^{odd}(3, \lambda)$ is provided in the text. We finally provide a proof of Proposition 4.

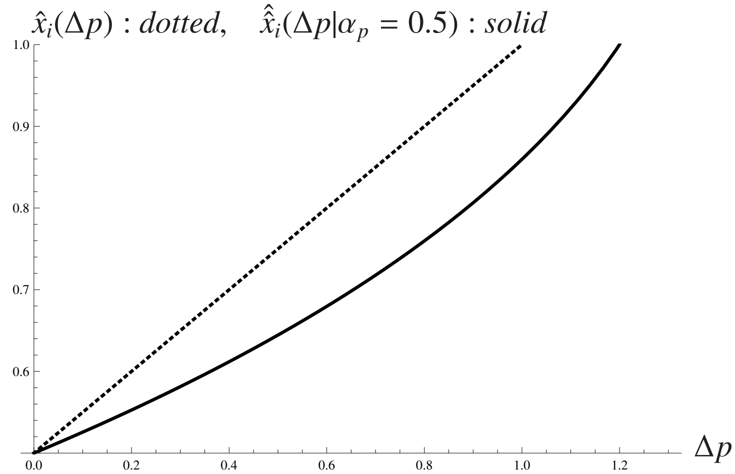
Proof of Proposition 4. $n^{odd}(3, \lambda^c) \approx 6.3890$. Thus, $n = 2$ or $n > 6$ suffice for existence at $\lambda = \lambda^c$.[39] Equilibrium existence holds for $1 < \lambda < \lambda^c$ since $n^{odd}(3, \lambda)$ is increasing in λ. Existence for $n \in \{3, 4, 5, 6\}$ follows from the same property: $n^{odd}(3, \lambda) = 3$ for $\lambda = \lambda^{cc} = 1/4\left(1 + \sqrt{57}\right) \approx 2.137$. □

Hence, existence in the duopoly case carries over to the n-firm oligopoly case in the limit. For symmetric markets with a small number of firms, however, equilibrium might fail to exist for intermediate values of λ ($\lambda < \lambda^c$).

C Figures

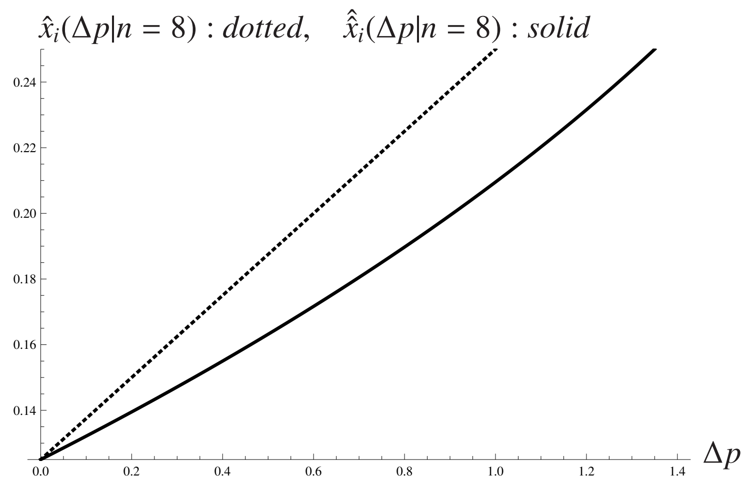
[38] Cf. the concept of (consumer's) preferred personal equilibrium of Koszegi and Rabin (2006) and Koszegi and Rabin (2007).

[39] The former case is proofed in Prop. 1.



Duopoly demand of non-biased and loss-averse consumers as a function of Δp for parameter values of $t = 1$, $\lambda = 3$, $\alpha_m = 1$, and $\alpha_p = 0.5$.

Figure 6: Demand of non-biased and loss-averse consumers ($\alpha_p = 0.5$)



Oligopoly demand of non-biased and loss-averse consumer as a function of Δp for parameter values of $t = 1$, $\lambda = 3$, $\alpha_m = \alpha_p = 1$, and $n = 8$.

Figure 7: Demand of non-biased and loss-averse consumers ($n = 8$)

References

Blinder, A. S., E. R. D. Canetti, D. E. Lebow, and J. B. Rudd (1998): "Asking About Prices: A New Approach to Understanding Price Stickiness," *Russell Sage Foundation*.

Eliaz, K., and R. Spiegler (2006): "Contracting with Diversely Naive Agents," *Review of Economic Studies*, 73(3), 689–714.

Gabaix, X., and D. Laibson (2006): "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets," *Quarterly Journal of Economics*, 121, 505–540.

Grubb, M. D. (2009): "Selling to Overconfident Consumers," *American Economic Review*, 99, 1770–1807.

HEIDHUES, P., AND B. KOSZEGI (2008): "Competition and Price Variation when Consumers are Loss Averse," *American Economic Review*, 98(4), 1245–1268.

KAHNEMAN, D., AND A. TVERSKY (1979): "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 47, 263–291.

KARLE, H., AND M. PEITZ (2010): "Pricing and Information Disclosure in Markets with Loss-Averse Consumers," *CEPR Discussion Paper*, 7785.

KOSZEGI, B., AND M. RABIN (2006): "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, 121, 1133–1165.

——— (2007): "Reference-Dependent Risk Attitudes," *American Economic Review*, 97(4), 1047–1073.

MEDVEC, V. H., S. F. MADEY, AND T. GILOVICH (1995): "When Less Is More: Counterfactual Thinking and Satisfaction Among Olympic Medalists," *Journal of Personality and Social Psychology*, 69, 603–610.

MELLERS, B., A. SCHWARTZ, AND I. RITOV (1999): "Emotion-Based Choice," *Journal of Experimental Psychology*, 128, 332–345.

SALOP, S. C. (1979): "Monopolistic Competition with Outside Goods," *The Bell Journal of Economics*, 10(1), 141–156.

ZHOU, J. (2008): "Reference Dependence and Market Competition," *MPRA Paper No. 9370*.