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Theories of Leptonic Flavor

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I discuss different theories of leptonic flavor and their capability of describing the features of the lepton sector, namely charged lepton masses, neutrino masses, lepton mixing angles and leptonic (low and high energy) CP phases. In particular, I show examples of theories with an abelian flavor symmetry G_f , with a non-abelian G_f as well as theories with non-abelian G_f and CP.

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1 Introduction

Charged lepton masses have been determined with high precision [1]

$$\begin{aligned} m_e &= (0.5109989461 \pm 0.0000000031) \text{ MeV}, \\ m_\mu &= (105.6583745 \pm 0.0000024) \text{ MeV}, \\ m_\tau &= (1776.86 \pm 0.12) \text{ MeV} \end{aligned} \quad (1)$$

and turned out to be strongly hierarchical. In contrast, the neutrino mass spectrum is not yet completely known, but the solar and the atmospheric mass squared differences Δm_{sol}^2 and $|\Delta m_{\text{atm}}^2|$ have been measured in neutrino oscillation experiments [2]

$$7.03 \times 10^{-5} \text{ eV}^2 \lesssim \Delta m_{\text{sol}}^2 \lesssim 8.09 \times 10^{-5} \text{ eV}^2, \quad 2.41 \times 10^{-3} \text{ eV}^2 \lesssim |\Delta m_{\text{atm}}^2| \lesssim 2.64 \times 10^{-3} \text{ eV}^2 \quad (2)$$

and an upper bound on the sum of the neutrino masses of less than 1 eV is derived from cosmology and beta decay experiments. The 3σ ranges of the three lepton mixing angles, obtained in global fits of the data from neutrino experiments, are [2]

$$0.01934 \lesssim \sin^2 \theta_{13} \lesssim 0.02397, \quad 0.271 \lesssim \sin^2 \theta_{12} \lesssim 0.345, \quad 0.385 \lesssim \sin^2 \theta_{23} \lesssim 0.638. \quad (3)$$

The nature of neutrinos, i.e. whether they are Dirac or Majorana particles, is still unknown. The following discussion assumes them to be Majorana particles. However, most of the results also hold for neutrinos being Dirac particles. For now only hints exist for CP violation in the lepton sector [2].

The strong hierarchy among the charged lepton masses, that is observed among the masses of the up and down type quarks as well, has led to the consideration of abelian flavor symmetries G_f , usually called Froggatt-Nielsen (FN) symmetry $U(1)_{\text{FN}}$ [3]. The different generations of charged leptons carry different charges under $U(1)_{\text{FN}}$. As $\sqrt{\Delta m_{\text{sol}}^2/|\Delta m_{\text{atm}}^2|} \sim 1/6$, neutrino masses exhibit no strong hierarchy and it is thus expected that the neutrino sector is (partly) uncharged under $U(1)_{\text{FN}}$ (see e.g. below the assignment called leptonic anarchy). A disadvantage is that an FN symmetry is only capable to explain the order of magnitude of observables in terms of the (small) symmetry breaking parameter λ .

For a non-abelian G_f many choices of symmetries are available: if G_f should be continuous, potentially suitable choices are $SO(3)$, $SU(2)$ and $SU(3)$,* while for G_f being discrete indeed an infinite number of potentially suitable choices is known, like the series of dihedral groups D_n ($n > 2$), alternating groups A_n ($n = 4, 5$), symmetric groups S_n ($n = 3, 4$), the series $\Delta(3n^2)$ and $\Delta(6n^2)$ for $n > 1$. An advantage of such non-abelian G_f is the possibility to unify the three lepton generations partially, $L_\alpha \sim \mathbf{2} + \mathbf{1}$, or fully, $L_\alpha \sim \mathbf{3}$. Furthermore, if broken to non-trivial residual symmetries [5], like $G_f = S_4$ that is broken to Z_3 in the charged lepton and to $Z_2 \times Z_2$ in the neutrino sector, certain values of the lepton mixing parameters can be predicted, e.g. tri-bi-maximal mixing. Compared to an abelian G_f , model building with a non-abelian G_f is, however, more challenging, e.g. more fields are needed, the construction of the potential in order to achieve the correct symmetry breaking pattern is non-trivial.

Recently, theories with a discrete non-abelian G_f have been extended with a CP symmetry [6], in particular, in order to also predict the Majorana phases α and β and in order

*An example of a model with a continuous non-abelian G_f is found in [4].

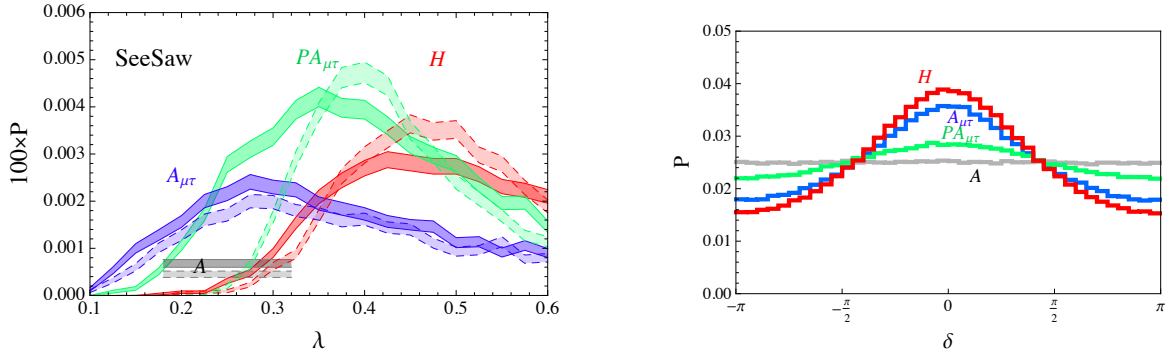


Figure 1: The left plot shows the success of the different charge assignments in describing correctly the lepton masses and mixing angles with respect to λ , assuming the seesaw mechanism responsible for neutrino masses. The right plot shows the probability distribution of the values of δ for a fixed value of λ , $\lambda = 0.2$ for A , $A_{\mu\tau}$ and $\lambda = 0.4$ for H and $PA_{\mu\tau}$. For further details see [8].

to obtain non-trivial values for the Dirac phase δ . If such a theory comprises right-handed (RH) neutrinos, it is possible to generate the baryon asymmetry Y_B of the Universe via (unflavored) leptogenesis. In this case the sign of Y_B can be directly correlated with the results for the low energy CP phases α , β and δ [7].

2 Theories with abelian G_f

In [8] the following charge assignments of the three generations of the left-handed (LH) lepton doublets L_α , RH charged leptons α_L^c and RH neutrinos ν_i^c have been analyzed

$$\begin{aligned}
 \text{leptonic anarchy (A)} & : L_\alpha \sim (0, 0, 0), \quad \alpha_L^c \sim (3, 2, 0), \quad \nu_i^c \sim (0, 0, 0), \\
 \mu\tau\text{-anarchy (A}_{\mu\tau}) & : L_\alpha \sim (1, 0, 0), \quad \alpha_L^c \sim (3, 2, 0), \quad \nu_i^c \sim (2, 1, 0), \\
 \text{pseudo } \mu\tau\text{-anarchy (PA}_{\mu\tau}) & : L_\alpha \sim (2, 0, 0), \quad \alpha_L^c \sim (5, 3, 0), \quad \nu_i^c \sim (1, -1, 0), \\
 \text{hierarchy (H)} & : L_\alpha \sim (2, 1, 0), \quad \alpha_L^c \sim (5, 3, 0), \quad \nu_i^c \sim (2, 1, 0).
 \end{aligned}$$

The structure of the charged lepton mass matrix m_l and the light neutrino mass matrix m_ν , arising from leptonic anarchy, is

$$m_l \sim \begin{pmatrix} \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad \text{and} \quad m_\nu \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (4)$$

In figure 1 we display the success of the different charge assignments in describing correctly the lepton masses and mixing angles with respect to λ as well as the probability distribution of the values of δ for a fixed value of λ , assuming the seesaw mechanism responsible for neutrino masses.

The realization of a model with an FN symmetry is simple. In particular, the breaking of $U(1)_{\text{FN}}$ is easily engineered. Furthermore, an FN symmetry is also often used for the description of the quark sector, quark masses as well as mixing angles. Hence, such a symmetry can be suitable for both leptons and quarks. In addition, it has been shown that

it can also be compatible with the particle assignment in a grand unified theory. Results, similar to those obtained with an FN symmetry, can also be achieved in extra-dimensional models in which particles are localized differently in the additional dimension(s). For models with an FN symmetry see [9].

3 Theories with non-abelian G_f

If a discrete non-abelian G_f is broken to (non-trivial) residual symmetries G_e in the charged lepton and to $G_\nu = Z_2 \times Z_2$ in the neutrino sector, lepton mixing angles and the Dirac phase (up to π) can be fixed [5]. G_e is chosen in such a way that the three lepton generations can be distinguished, while G_ν is always fixed to the maximal residual symmetry for three Majorana neutrinos that does not lead to any constraints on their masses. The requirement that the charged lepton mass matrix m_l should be invariant under G_e determines the contribution U_e of charged leptons to the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix U_{PMNS} , while the request that m_ν is invariant under G_ν fixes U_ν . So, also the form of the PMNS mixing matrix $U_{PMNS} = U_e^\dagger U_\nu$ is given by G_e and G_ν , up to possible permutations of rows and columns of U_{PMNS} , since lepton masses are not predicted from G_f , G_e and G_ν . Consequently, the lepton mixing angles and the Dirac phase are determined up to these permutations of rows and columns of U_{PMNS} . Furthermore, the columns of U_e and U_ν can be re-phased, so that Majorana phases are in general not fixed.

One of the very first implementations of this approach that leads to non-zero θ_{13} and non-maximal θ_{23} has been discussed in [10]. The choice of G_f is $G_f = \Delta(96)$. The residual symmetries are $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$ and lead to a PMNS mixing matrix whose elements have the absolute values

$$||U_{PMNS}|| = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}(\sqrt{3}+1) & 1 & \frac{1}{2}(\sqrt{3}-1) \\ 1 & 1 & 1 \\ \frac{1}{2}(\sqrt{3}-1) & 1 & \frac{1}{2}(\sqrt{3}+1) \end{pmatrix}. \quad (5)$$

The results for the lepton mixing angles are

$$\sin^2 \theta_{12} = \sin^2 \theta_{23} = \frac{8 - 2\sqrt{3}}{13} \approx 0.349 \quad \text{and} \quad \sin^2 \theta_{13} = \frac{2 - \sqrt{3}}{6} \approx 0.045. \quad (6)$$

The Dirac phase is predicted to be trivial, $\sin \delta = 0$. A grand unified theory with $G_f = \Delta(96)$ has been constructed in [11]. In several studies [12], in particular in [13], series of G_f , possible choices of G_e and G_ν and the resulting mixing patterns have been analyzed. It has been observed that $\sin \delta = 0$ follows, if the lepton mixing angles are in accordance with the experimental data.

This approach can be combined with an FN symmetry so that a simultaneous understanding of lepton mixing parameters as well as charged lepton masses becomes possible. For G_f being discrete, the symmetry breaking scale can be as low as the electroweak scale or even larger than the scale of grand unification which offers great freedom in building models. Explicit model realizations with a non-abelian discrete G_f are discussed in e.g. [14].

4 Theories with non-abelian G_f and CP

In a scenario with a discrete non-abelian G_f and a CP symmetry, in which both symmetries are broken to (non-trivial) residual groups, it becomes possible to not only determine the lepton mixing angles and δ , but also the Majorana phases α and β . The CP symmetry that is imposed in the fundamental theory acts in general non-trivially on flavor space [15], i.e. for a set of scalars ϕ_i that transform in the same way under the gauge symmetries (and form a multiplet of G_f) a CP transformation X acts as

$$\phi_i \rightarrow X_{ij} \phi_j^* \quad \text{with} \quad XX^\dagger = XX^* = 1. \quad (7)$$

In order to consistently combine G_f and CP certain conditions have to be fulfilled [6, 16]. In the following examples all such conditions are fulfilled. The approach for fixing lepton mixing angles and predicting leptonic CP phases, presented in [6], assumes G_f and CP and as residual symmetries G_e and G_ν . While G_e has to fulfill the same constraints as in the approach without CP, G_ν is chosen as the direct product of a Z_2 symmetry, contained in G_f , and the CP symmetry. Thanks to the latter choice it becomes possible to also predict the Majorana phases. Furthermore, one real free parameter, which affects in general all lepton mixing parameters, is introduced in the PMNS mixing matrix, since G_ν is no longer the maximal residual symmetry $Z_2 \times Z_2$. A consequence of this free real parameter is the possibility to obtain results for lepton mixing angles in agreement with experimental data and, at the same time, to achieve non-trivial values of the Dirac phase δ . The actual form of the PMNS mixing matrix in this approach is obtained from the contribution U_e to lepton mixing from charged leptons, determined by G_e , and the contribution U_ν from neutrinos, which is subject to $G_\nu = Z_2 \times CP$. It can be shown that U_ν can be written as $U_\nu = \Omega_\nu R(\theta) K_\nu$ and thus U_{PMNS} reads

$$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu \quad (8)$$

with Ω_ν being determined by the CP transformation X and the residual Z_2 flavor symmetry, $R(\theta)$ being a rotation in one plane through the free parameter θ , $0 \leq \theta < \pi$, and K_ν a diagonal matrix with entries ± 1 and $\pm i$. The latter is related to the request to achieve positive neutrino masses. Like the approach given in the preceding section, also here lepton masses are unconstrained. Hence, all statements made hold up to possible permutations of rows and columns of the PMNS mixing matrix.

One example that shows the predictive power of this approach has been discussed in [6]. For $G_f = S_4$, $G_e = Z_3$ and $G_\nu = Z_2 \times CP$ the PMNS mixing matrix is of the form

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \cos \theta & \sqrt{2} & 2 \sin \theta \\ -\cos \theta + i\sqrt{3} \sin \theta & \sqrt{2} & -\sin \theta - i\sqrt{3} \cos \theta \\ -\cos \theta - i\sqrt{3} \sin \theta & \sqrt{2} & -\sin \theta + i\sqrt{3} \cos \theta \end{pmatrix} K_\nu \quad (9)$$

which leads to lepton mixing angles

$$\sin^2 \theta_{13} = \frac{2}{3} \sin^2 \theta, \quad \sin^2 \theta_{12} = \frac{1}{2 + \cos 2\theta}, \quad \sin^2 \theta_{23} = \frac{1}{2} \quad (10)$$

and CP phases

$$|\sin \delta| = 1, \quad \sin \alpha = 0 \quad \text{and} \quad \sin \beta = 0. \quad (11)$$

s	$\sin^2 \theta_{13}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin \delta$	$\sin \alpha = \sin \beta$
$s = 1$	0.0220	0.318	0.579	0.936	$-1/\sqrt{2}$
	0.0220	0.318	0.421	-0.936	$-1/\sqrt{2}$
$s = 2$	0.0216	0.319	0.645	-0.739	1
$s = 4$	0.0220	0.318	0.5	∓ 1	0

Table 1: Results for lepton mixing parameters from $G_f = \Delta(6n^2)$ with $n = 8$, $m = 4$ and different CP transformations $X(s)$. The matrix K_ν is chosen as trivial. The absolute value of $\sin \delta$ is large and the two Majorana phases α and β take different values for different s .

The Dirac phase is thus maximal, whereas both Majorana phases are trivial. Furthermore, the atmospheric mixing angle is fixed to be maximal. The reactor and the solar mixing angle depend on the free parameter θ and for $\theta \approx 0.18$ or $\theta \approx 2.96$ both, θ_{13} and θ_{12} , are in agreement with experimental data. In [17] a supersymmetric model for the lepton sector with the gauge group of the Standard Model has been constructed. In this model LH leptons are unified in a(n irreducible, faithful) triplet, whereas RH charged leptons are singlets of S_4 . Both symmetries, S_4 and CP, are broken spontaneously at a high energy scale. The above-estimated size of θ , needed for achieving values of θ_{13} and θ_{12} consistent with experimental data, can be naturally explained in this model. Furthermore, neutrinos are predicted to follow normal mass ordering (NO) and the values of the neutrino masses m_i are

$$m_1 \approx 0.016 \text{ eV} , \quad m_2 \approx 0.018 \text{ eV} , \quad m_3 \approx 0.052 \text{ eV} . \quad (12)$$

In addition, the Majorana phases are fixed to the values $\alpha = \pi$ and $\beta = \pi$ so that m_{ee} , the quantity measurable in neutrinoless double beta decay, is $m_{ee} \approx 0.003 \text{ eV}$. The charged lepton mass hierarchy is also naturally described, since charged lepton masses arise from operators of different dimension.

In [18] (see also [19]) the series $\Delta(3n^2)$ and $\Delta(6n^2)$ combined with CP have been analyzed in detail. The residual symmetries G_e and G_ν are fixed to $G_e = Z_3$ and $G_\nu = Z_2 \times CP$. It has been shown that for these types of residual symmetries only four cases of mixing patterns can arise that lead to lepton mixing angles potentially compatible with experimental data. One particularly interesting case, called Case 3 b.1) in [18], has the following features: the first column of the PMNS mixing matrix is fixed via the choice of the residual flavor symmetry $Z_2(m)$ (m integer); the solar mixing angle constrains m to fulfill $m \approx n/2$; the free parameter θ is fixed by the reactor mixing angle and for $m = n/2$ a lower limit on the CP violation via the Dirac phase is found

$$|\sin \delta| \gtrsim 0.71 \quad (13)$$

and both Majorana phases α and β depend on the CP transformation $X(s)$ only

$$|\sin \alpha| = |\sin \beta| = |\sin 6\phi_s| \quad \text{with} \quad \phi_s = \frac{\pi s}{n} \quad \text{and} \quad s = 0, \dots, n-1. \quad (14)$$

In table 1 results for the lepton mixing parameters are shown for $\Delta(6n^2)$ with $n = 8$ and $m = 4$ and different values s .

The fact that both, lepton mixing angles and Majorana phases, are strongly constrained leads also to strong restrictions on m_{ee} , even if the neutrino mass spectrum is not constrained

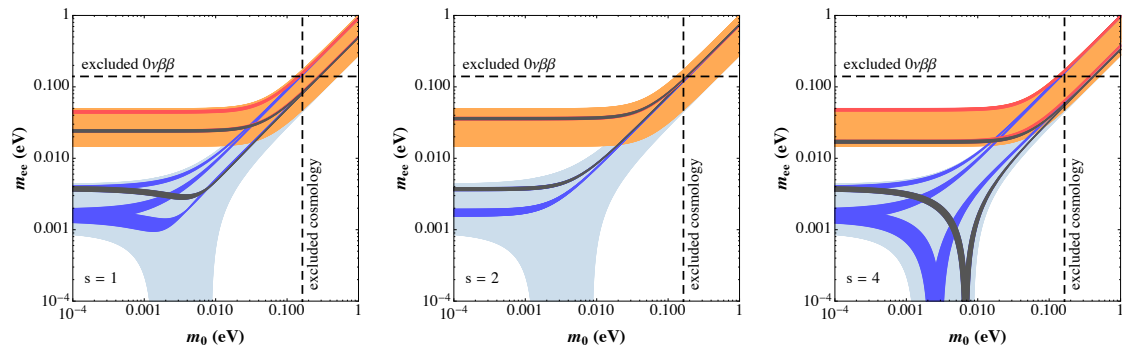


Figure 2: Results for m_{ee} with respect to the lightest neutrino mass m_0 for the choices of G_f , CP, G_e and G_ν used in table 1. Blue areas indicate m_{ee} for NO, while orange areas refer to m_{ee} for inverted mass ordering. In dark colors the impact of the restrictions on the lepton mixing parameters on m_{ee} is displayed, assuming for neutrino masses only the experimental constraints. For comparison in light colors the ranges of m_{ee} are shown, as obtained from the experimentally preferred 3σ intervals of the lepton mixing parameters and neutrino masses. The darkest color highlights K_ν trivial.

beyond the request to reproduce experimental bounds on the sum of the neutrino masses and to match the measured mass squared differences Δm_{sol}^2 and Δm_{atm}^2 . This is exemplified in figure 2 for the choices of G_f , CP, G_e and G_ν used in table 1.

Given the predictive power of the approach with G_f and CP regarding leptonic CP phases it has been applied in [7] to a scenario with three RH neutrinos N_i . N_i transform in the same triplet as LH leptons L_α . They give masses to light neutrinos via the type-I seesaw mechanism. For $10^{12} \text{ GeV} \lesssim M_i \lesssim 10^{14} \text{ GeV}$ the baryon asymmetry Y_B of the Universe can be generated via unflavored leptogenesis [20], $Y_B \sim 10^{-3} \epsilon \eta$. A value of Y_B in accordance with experimental data [21], $Y_B = (8.65 \pm 0.09) \times 10^{-11}$, can be achieved for CP asymmetries $10^{-4} \gtrsim \epsilon \gtrsim 10^{-7}$ for efficiency factors $10^{-3} \lesssim \eta \lesssim 1$. In order to implement the breaking scheme of G_f and CP, as described before, the charged lepton sector is taken to be invariant under G_e , while the mass matrix M_R of RH neutrinos preserves G_ν and the Dirac Yukawa coupling Y_D is invariant under G_f and CP. As a consequence, light neutrino masses m_i are inversely proportional to RH neutrino masses M_i and the contribution U_ν from neutrinos to the PMNS mixing matrix is $U_\nu = U_R = \Omega_\nu R(\theta) K_\nu$. Since charged leptons do not contribute to lepton mixing in the chosen basis, $U_{PMNS} = U_\nu$. Computing the CP asymmetries ϵ_i , arising from the decay of N_i , they are found to vanish. This has already been observed in scenarios with G_f only [22]. Thus, non-zero ϵ_i can be achieved, if corrections are included. A particularly interesting case is that corrections to Y_D are considered that are proportional to a (small) symmetry breaking parameter κ and are invariant under G_e , the residual symmetry in the charged lepton sector. Taking these corrections into account,

$$\epsilon_i \propto \kappa^2. \quad (15)$$

Hence $\kappa \sim 10^{-(2\div 3)}$ explains correctly the size of the CP asymmetries. Most importantly, the sign of ϵ_i (and consequently also Y_B) can be fixed, because all CP phases are determined in this approach. In figure 3 the results for Y_B as function of the lightest neutrino mass m_0 are shown. The light-blue, red and green areas arise from the variation of order one parameters appearing in the correction to Y_D . The choice of G_f , CP, G_e and G_ν is the

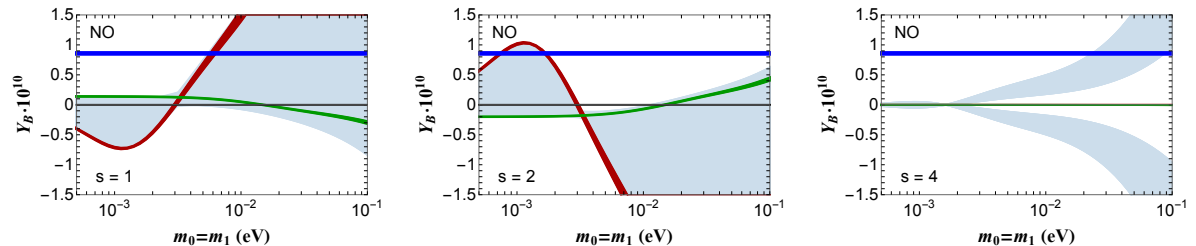


Figure 3: Results for the baryon asymmetry Y_B of the Universe with respect to the lightest neutrino mass m_0 for the choice of G_f , CP, G_e and G_ν as in table 1 and figure 2. Light neutrino masses have NO. Light-blue, red and green areas refer to different choices of the parameters in the correction to Y_D . The dark-blue area indicates the experimentally measured value of Y_B . For the choices $s = 1$ and $s = 2$ (predominantly) positive or negative Y_B is achieved for certain ranges of m_0 .

same as in table 1 and figure 2. As can be clearly seen, for certain choices of CP, $s = 1$ and $s = 2$, and certain ranges of m_0 , Y_B is (predominantly) positive or negative, whereas for the choice $s = 4$ no such preference is visible. The explanation for this observation is that for $s = 1$ the Majorana phase α fulfills $\sin \alpha < 0$, whereas for $s = 2$ we find $\sin \alpha > 0$. For $s = 4$ the CP phases α and β are trivial and only $\sin \delta$ is non-vanishing. Studies of flavored leptogenesis in scenarios with G_f and CP can be found in [23].

5 Conclusions

I have discussed for different flavor symmetries G_f (abelian and non-abelian, continuous and discrete, combined with CP or not) their predictive power regarding lepton masses and lepton mixing parameters, in particular leptonic CP phases. While an FN symmetry is suitable for (charged lepton) mass hierarchies and for explaining the gross structure of the mixing pattern, non-abelian G_f , especially if chosen to be discrete and broken non-trivially, can explain all three lepton mixing angles and the Dirac phase δ . However, their predictive power regarding CP phases is limited, since only one CP phase can be determined. A combination of non-abelian discrete G_f and CP is most powerful in constraining all lepton mixing parameters and can also restrict high energy CP phases that are relevant for the baryon asymmetry Y_B of the Universe in leptogenesis scenarios. I have also briefly shown that in concrete models the predictive power can be further increased, e.g. the neutrino mass ordering is predicted and the Majorana phases are entirely fixed.

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