Lambertian Source Modelling of Free Space Optical Ground-to-Train Communications

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Abstract—There is a increasing demand for high speed data services while travelling on trains and other means of public transportation. Hence, an alternative technology to radio frequency (RF) communications called free space optics (FSO) could be used to provide truly high-speed broadband services in trains. This paper presents a Lambertian source modelling of FSO ground-to-train communication link along with its link performance analysis. Here, we investigate the use of a single optical base station (BS) that can provide the coverage range up to 75 m at 10 Mbps using on off keying (OOK) non-return to zero (NRZ) data signal with a bit error rate (BER) of $10^{-8}$.

I. INTRODUCTION

FSO communications (also known as optical wireless communications (OWC)) is a promising technology that complements the widely adopted and accepted RF communication links. FSO technology offers a huge unregulated bandwidth supporting data rates beyond Gigabits per second, a license free spectrum, immunity to the electromagnetic interference and low bit error rates. Since optical signals don’t penetrate through walls and optical footprint can be defined precisely, the transmission can be made highly secure [1-3]. These features are some significant benefits of FSO links over other wireless links.

The ever-increasing demand for the higher capacity in wireless communication links has led to the usage of FSO communications in various static and dynamic environments such as home, offices or on the move. The demand for the quality of service for wireless communications while travelling would be one important and major application of FSO in the near future. This paper deals a mathematical modelling and analysis of a FSO link for train communications and establishes the minimum requirements for transmitter (e.g. divergence angle, transmission power) and receiver (field of view (FOV), data rate, tilting angle) in order to offer a quality data service to the passengers on-board. Travelling in trains could be made so much fun by providing passengers with high-speed wireless internet services (i.e. creating the office or home working environment). Existing infrastructure can’t suffice the present need for good quality data services due to slow connection problems. At present, the service providers employ RF links with the WiMax where the theoretical data rate is 75 Mbps [4] and around 10 Mbps for trains moving at high speed [5]. The practical achievable data rate ranges from few hundred kb/s to a few Mb/s depending on the signal strength and the locations of the trains. To address these problems one solution would be to adopt the FSO technology, which is capable of providing broadband services on the train. The FSO link or base stations (BS) located along the tracks would be connected to the high capacity fibre-optic backbone networks via fibre trackside cables. In this work we propose to use light emitting diodes (LED) as the optical source mounted at BSs to flood the incoming trains. LEDs are capable of delivering very high data rates in excess of 500 Mbps [6] and 1 Gbps [7], which would be ideal for ground-to-train communication links. Also, reliability, simple drive circuitry and low cost are some distinct advantages of LEDs over lasers [8]. Additionally, as LED is treated as an extended source, they are considered as essentially diffuse and hence safe regarding eye and skin safety as compared to lasers [9]. LED beam profile could be modelled as Lambertian emission hence a Lambertian model for FSO ground-to-train communications is proposed along with a mathematical model describing the configuration of the proposed system. The paper is organised as follows: Section II discusses the proposed system model along with the Lambertian modelling. Section III presents results along with the discussions and finally Section IV concludes the paper.

II. SYSTEM MODEL

A typical FSO ground-to-train communication links is shown in Fig. 1(a). The system consists of BSs positioned alongside the train track mounted on to the high voltage electrical cables carrying poles located approximately over a metre from the track; see Fig. 1(b). The spacing between BSs is 75 m (covering up to 3 train carriages of 22 m length each), which would be used in our analysis since the typical separation of gantries is 70-75 m [10]. LEDs are used as the transmitter and the photodetectors are used as the receiver mounted on the side or top of the train carriages. The relative receiver height is approximately 4 m above the ground.

The geometry layout of the proposed system model is depicted in Fig. 2. The BS is positioned at a distance $d_i$,
from the train track at the ground and $d_2$ is the position of the BS from point C (see Fig. 2) and the effective coverage length along the track is $L$. If $\delta$ and $\beta$ are the angle of the coverage beam at the longest point B and the shortest point C, respectively and $\theta$ is the beam divergence of the transmitter at the BS, then from the geometry, we get the following equations:

$$\theta = \beta - \delta,$$

$$\tan \beta = \frac{d_1}{d_2}.$$  

Using the triangle ABD:

$$\tan \delta = \frac{d_1}{d_2 + L}.$$  

Using (2), (3) and (4), the estimation of the transmitter beam divergence can be given as:

$$\theta = \tan^{-1} \left( \frac{d_1 L}{d_1^2 + d_2 L + d_2^2} \right).$$  

From Fig. 2, the length along the axis AO denoted by $z$ can be written as $z = AH + HO$ and length AH can be written as $AG + GH$. Similarly, length HO can be written as $HO = (L - CH) \cos \gamma$. Hence $z$ can be given as:

$$z = AG + GH + (L - CH) \cos \gamma.$$  

Performing basic geometry calculations, we get the following equations:

$$AG = x \cos \theta_{1/2},$$

$$GH = \frac{x \sin \theta_{1/2}}{\tan \gamma},$$

$$CH = \frac{x \sin \theta_{1/2}}{\sin \gamma}.$$  

where $\theta_{1/2}$ is the half angle divergence ($\theta = 2\theta_{1/2}$). Combining (5) - (8), the transmission axis $z$ can be written in terms of the coverage length $L$ as:

$$z = L \cos \gamma + x \cos \theta_{1/2}.$$  

The BS at the transmitter consists of a signal generator, a direct current (DC) driver and an LED. The LED is assumed to have a Lambertian radiant intensity as given by:

$$R_o(\phi) = \left( \frac{m + 1}{2\pi} \right) \cos^m \phi.$$  

where $\phi$ is the irradiance angle and $m$ is the order of Lambertian emission which is related to the LED semi-angle at half power $\phi_{1/2}$ as given by:

$$m = -\frac{\ln 2}{\ln (\cos \phi_{1/2})}.$$
The optical receiver front end mainly consists of an optical concentrator, an optical filter, a detector and a transimpedance amplifier (TIA). The LOS channel DC gain can be written as:

\[ H(0)_\text{LOS} = \frac{A_{\text{eff}}(\psi)}{\pi^2} R_0(\phi) \cos(\psi), \quad 0 \leq \psi \leq \psi_C \]
\[ \psi > \psi_C \] \quad \text{(12)}

where \( z \) is the distance between the transmitter BS and the train receiver which is dynamic and changes with the coverage length \( L \), \( \psi \) is the angle of incidence with respect to the receiver axis, \( \psi_C \) is the FOV of the receiver, \( A_{\text{eff}}(\psi) \) is the effective collection area of the receiver on the train tilted at an angle \( \gamma \) as given by:

\[ A_{\text{eff}}(\psi) = \left\{ \begin{array}{ll} A_{\text{det}} T_s(\psi) g(\psi) \cos(\psi), & 0 \leq \psi \leq \psi_C \\ 0, & \psi > \psi_C \end{array} \right\} \]
\quad \text{(13)}

where \( A_{\text{det}} \) is the physical area of the detector, \( T_s(\psi) \) is the filter transmission factor which is given as 0.8 [11] and \( g(\psi) \) is the gain of an optical concentrator employed at the receiver given by:

\[ g(\psi) = \left\{ \begin{array}{ll} \frac{n^2}{\sin^2\psi_C}, & 0 \leq \psi \leq \psi_C \\ 0, & \text{elsewhere} \end{array} \right\} \]
\quad \text{(14)}

where \( n \) is the refractive index of the concentrator. The receiver optics system is shown in Fig. 3. The power collected at the receiver is proportional to the effective receiver collection area. The FOV of an optical concentrator can be evaluated as given by the following relation [12]:

\[ A_{\text{coll}} \sin^2\psi_C = n^2 A_{\text{det}} \]
\quad \text{(15)}

where \( A_{\text{coll}} \) is the collection area of the optical concentrator. The received optical power at the train receiver is given as:

\[ P_r = H(0)_\text{LOS} P_{\text{tx}} \]
\quad \text{(16)}

where \( P_{\text{tx}} \) is the optical transmitted power. Combining equations (9) - (16), the received power can be written as:

\[ P_r = A_{\text{det}} P_{\text{tx}} (m + 1) T_s(\psi) \cos^m(\phi) n^2 \cos(\psi) \]
\quad 2\pi \sin^2\psi_C (L \cos \gamma + x \cos \theta_{1/2})^2 \]
\quad \text{(17)}

where \( L \) is the length of the track considered, \( x \) is the length AC (see Fig. 2(b)), \( \theta_{1/2} \) is the semi angle divergence and \( \gamma \) is the tilting angle of the transmitter and/or receiver given by \( \theta_{1/2} + \delta \).

The noise induced at the receiver is due to the shot noise from the received signal, noise due to the dark current flowing in the detector, thermal noise due to the amplifier and the noise variance due to the background radiation. The current due to the shot noise variance is given by:

\[ \sigma_{\text{shot}}^2 = 2 q R P_r B \]
\quad \text{(18)}

where \( q \) is the electronic charge, \( R \) is the Responsivity of the photodiode (A/W), \( P_r \) is the power received along the track length and \( B \) is the electronic bandwidth. The noise resulting from the dark current of the photodiode can be given as [8]:

\[ \sigma_{\text{dc}}^2 = 2 q i_{\text{dc}} B \]
\quad \text{(19)}

where \( i_{\text{dc}} \) is the dark current in the photodiode. Noise variance resulting from the background radiation can be given as [13]:

\[ \sigma_{\text{bg}}^2 = 2 q B R (P_{\text{sky}} + P_{\text{sun}}) \]
\quad \text{(20)}

where \( P_{\text{sky}} \) and \( P_{\text{sun}} \) represent the radiation from the diffused source (like sky) and the sun, respectively which

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameters & Symbol & Value \\
\hline
Optical transmit power & \( P_{\text{tx}} \) & 15 mW \\
Operating wavelength & \( \lambda \) & 850 nm \\
Transmitter divergence & \( \theta \) & 1.20° \\
Active area of the detector & \( A_{\text{det}} \) & 7 mm\textsuperscript{2} \\
Responsivity & \( R \) & 0.59 A/W \\
Receiver sensitivity & \( S_r \) & -36 dBm @ 10 Mbps \\
Concentrator focal length & \( f \) & 50 mm \\
Concentrator radius & \( R_{\text{coll}} \) & 25 mm \\
Concentrator semi FOV & \( \Psi \) & 5.15° \\
Coverage length & \( L \) & 75 m \\
Vertical separation & \( d_z \) & 1 m \\
Horizontal separation & \( d_x \) & 15 m \\
Refractive index of lens & \( n \) & 1.5 \\
Load Resistance & \( R_L \) & 50 Ω \\
Tx/Rx tilting angle & \( \gamma \) & 2.25° \\
Background noise & \( P_{\text{bg}} \) & 0.01 mW \\
Filter transmission factor & \( L_s(\psi) \) & 0.8 \\
\hline
\end{tabular}
\caption{Simulation Parameters}
\end{table}
can be denoted by $P_{bg}$. Typical background noise power used is 0.01 mW \cite{14, 15}. The noise in the photodetector due to the load resistor $R_L$ gives rise to the thermal noise as given by \cite{8}:

$$\sigma_{th}^2 = \frac{4KB}{R_L}, \quad (21)$$

where $K$ is the Boltzmann’s constant, $T$ is the equivalent temperature and $R_L$ is the load resistance. Hence, the total noise variance is the combination of all the noise variances using (18), (19), (20) and (21) and is given as:

$$\sigma_{tot}^2 = \sigma_{shot}^2 + \sigma_{dc}^2 + \sigma_{bg}^2 + \sigma_{th}^2. \quad (22)$$

Usually, the noise variance due to the dark current is small compared to the shot noise and the thermal noise and can be ignored. In general, a communicating system is evaluated in terms of the electrical signal-to-noise ratio (SNR), which is defined as \cite{2}:

$$SNR = \frac{(R_P)^2}{\sigma_{tot}^2}. \quad (23)$$

The BER for OOK-NRZ is then evaluated as \cite{2}:

$$BER = Q(\sqrt{SNR}), \quad (24)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{y^2}{2}} dy$.

III. RESULTS AND DISCUSSION

In order to analyse the system performance, simulation is carried out using Matlab where the BER performance is analysed at any point along the train track. The parameters used for the simulation is listed in Table I. In order to observe the variation of effective FOV at the receiver when an optical lens is used, a plot of receiver half-angle FOV against the photodetector area is shown in Fig. 4 for a range of lens radius. For the proposed system, although the desired receiver FOV would be high but a large FOV will collect more background noise compared with small FOV receivers. Size of the lens at the receiver will change the effective FOV of the receiver. Using a large radius lens will offer a wider collection area and a higher concentrator gain, thereby requiring lesser power at the transmitter. Hence there is an optimisation between the transmit power, the receiver FOV and the link length \cite{16}. The minimum FOV required for the system is equivalent to the half angle divergence angle $\theta_{1/2}$. The detector area considered is 7 mm$^2$. Using a lens radius below 20 mm results in higher semi FOV and requirement of higher transmit power. When the radius used is over 20 mm, the light collection area is higher at the same time, FOV is smaller around 6° resulting for lower transmit power. Hence a bigger lens radius of 25 mm is chosen, which seems realistic for a real train system thus resulting in half-angle FOV of 5.15° leading to lower transmit power requirement at the BS.

The power profile of the received signal is plotted based on (17) as shown in Fig. 5. The received power profile peaks at a track length of 2 m, which is -18.25 dBm and the worst case value corresponds to -33.8 dBm at a track length of 75 m. The dynamic range of the received signal is 15.55 dBm. The SNR variation along the track is plotted in Fig. 6 for different data rates. In order to achieve a BER of $10^{-6}$, the required SNR value for OOK-NRZ is 13.6 dB. It can be inferred from Fig. 6 that required value of SNR is achieved up to a length of 75 m for a data rate of 10 Mbps. The coverage length reduces to 42 m at 100 Mbps and nearly 22 m at 1 Gbps. For all the
data rates, the dynamic range of SNR is approximately 31 dB. This drop in SNR at a higher bit rate is due to the increase in the total noise variance as the SNR is inversely proportional to the total noise variance (see eqn. (23)). The noise variance is directly proportional to the minimum bandwidth of the system and the minimum bandwidth for OOK-NRZ is half of data rate.

The BER analysis for the ground-to-train system is shown in Fig. 7, showing BER plot along the train track length of 75 m covered by a single BS. The plot shown is for three different data rates of 10 Mbps, 100 Mbps and 1 Gbps, respectively in order to compare and analyse the system performance. The effective coverage length drops down to around 42 m for a required BER of $10^{-6}$ when the data rate increases from 10 Mbps to 100 Mbps and further drops down at a bit rate of 1 Gbps at the same level of transmit power. This lowering in the coverage length is due to the increase in the noise bandwidth as the bit rate increases, which is also validated by the SNR variation as shown in Fig. 6. Hence, for a fixed transmit power, to achieve the required BER, the BS can provide coverage up to 3 train coaches at 10 Mbps while the coverage is limited to 2 train coaches at 100 Mbps and further down to 1 coach at 1 Gbps. In order to increase the coverage length at higher bit rates, the transmit power at the BS could be increased so that a longer coverage length is achieved using a single BS.

IV. CONCLUSIONS

FSO ground-to-train communications was proposed, which utilises Lambertian source that could provide coverage to the receiver mounted on top of the train. A mathematical model based on the Lambertian transmitter model was proposed. We have shown the analysis of the received signal and the error free performance ($BER = 10^{-6}$) up to a track length of 75 m with a single BS at a data rate of 10 Mbps. Also, the effect of increasing data bit rate was observed as the trade-off factor in the reduction of effective coverage length for a fixed transmit power at a specified BER.

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